

Modeling Damage in Concrete Pavements and Bridges

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16. Abstract This project focused on micromechanical modeling of damage in concrete under general, multi-axial loading. A continuum-level, three-dimensional constitutive model based on micromechanics was developed. The model accounts for damage in concrete by statistically averaging the response (opening and shear) of an ensemble of microcracks under a three-dimensional stress state. The model is implemented in ABAQUS analysis code and can be utilized by ALDOT engineers to make an informed assessment of the damage in concrete pavements and bridges.			
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Executive Summary

Concrete is widely used as a construction material in our infrastructure (highway pavements, airport runways, and highway bridges). Damage in concrete due to microcracking contributes significantly to the deterioration of the infrastructures, especially as they age. Replacing a severely damaged concrete structure or a key component of a structure can avoid the loss of property or life often associated with failure of the damaged structure. However, replacing damaged concrete pavements and bridges can be a costly alternative if damage in the pavements and bridges will not grow under the design load. Given limited resources, the highway maintenance engineer is faced with the difficult decision as to whether a damaged concrete bridge or pavement is still safe under the design load or whether it needs to be replaced. Finite element analysis of concrete structures can help the engineer to make a better assessment of the criticality of the damaged structure, provided the material model used in the analysis can accurately represent the evolution of damage in concrete under three-dimensional, cyclic mechanical and thermal loading.

The objective of the research project was to develop a micromechanics-based constitutive model capable of predicting damage and failure in concrete pavements and bridges under three-dimensional loading conditions.

The objective of the research project was achieved. A micromechanics-based constitutive model was developed for damage and failure in concrete pavements and bridges under general (three-dimensional) loading conditions. The model is also implemented in ABAQUS analysis code and can be utilized by ALDOT engineers to make an informed assessment of the damage in concrete pavements and bridges.

The material model will be maintained and supported by The University of Alabama in Huntsville.

Section 1

Introduction

Due to its high compressive strength and affordability, concrete is widely used as a construction material in infrastructure (highway pavements, airport runways, and highway bridges). However, concrete is brittle under normal conditions and is subject to cracking and damage under low tensile stress from traffic loads or thermal loading.

Replacing a severely damaged concrete structure or a key component of a structure can avoid the loss of property and/or life often associated with possible failure of the damaged structure. On the other hand, replacing a damaged concrete pavements and bridges can be a very costly alternative, if the damage in the pavements and bridges will not grow under the design load. Given a limited resource in hand, the highway maintenance engineer today is faced with the difficult decision as to whether a damaged concrete bridge or pavement is still safe under the design load or it needs to be replaced. Finite element analysis of concrete structures can help the engineer to make a better assessment of the criticality of the damaged structure, provided the material model used in the analysis can accurately represent the evolution of damage in concrete under three-dimensional, cyclic mechanical and thermal loading.

Predictive modeling of concrete is challenging (e.g. Yazdani and Schreyer 1988, 1990, 2003; Hansen and Schreyer 1994, 1995; Dube, *et al.* 1996). This is mainly because concrete is a heterogeneous engineering material that often contains a variety of microdefects (microvoids, microcracks) prior to the application of mechanical/thermal loading. As a result, the mechanical response of the material under three-dimensional state of stress is extremely complicated. For example, the microcracks in concrete can open, shear, become unstable, grow in size, and coalesce into macroscopic cracks, giving rise to the complicated response of the material observed macroscopically (for example, Dienes, *et al.* 2006; Zuo, *et al.* 2008). Mechanical damage in concrete is the result of the nucleation, growth, and coalescence of microcracks.

The models available in engineering analysis codes are phenomenological in nature (many of them are based on plasticity theories developed for and applicable to metals and alloys) and fail to properly account for the actual microstructure and defects. Consequently, these models cannot accurately predict the damage in concrete structures under general loading conditions. New models that take into full consideration the behavior of microdefects in concrete are needed if the numerical results from finite element analysis of concrete structures are to be truly predictive.

The objective of the research project is to develop a micromechanics-based constitutive model capable of predicting damage and failure in concrete pavements and bridges under general (three-dimensional) loading conditions. It is also the objective of the project to numerically implement the constitutive model into ABAQUS finite element code for engineering analysis.

Section 2 Model formulation

2.1 Previous Work

Many engineering materials (e.g. ceramics, concrete, rocks, and explosives) contain brittle constituents that are subject to microcracking under loading. Predictive modeling of the mechanical response of brittle materials under a general (three-dimensional), dynamic loading are of practical interest to the designer of structures and systems containing brittle materials.

Much research has been done in recent years on the fundamental understanding and development of advanced constitutive models for brittle and quasi-brittle materials (e.g. Ortiz 1985; Simo and Ju 1987, 1989; Yazdani and Schreyer 1988, 1990, 2003; Hansen and Schreyer 1994, 1995; Dube, *et al.* 1996; Zhang, *et al.* 2003) and on experimentally measuring the material responses under dynamic loading conditions (e.g. Grady and Kipp 1985, 1989; Kipp and Grady 1989a, 1989b; Feng, *et al.* 1996, 1998; Vogler, *et al.* 2010).

A number of micromechanics-based theories and models have been developed recently to predict the behavior of brittle materials under dynamic loading (e.g. Dienes 1978, 1983, 1985; Costin 1983; Taylor, *et al.* 1986; Addessio and Johnson 1990; Rajendran 1994; Rajendran and Grove 1996; Dienes, *et al.* 2006). These models typically assume a distribution of microcracks in the material prior to loading and calculate damage accumulation using the growth of microcracks under stress. In particular, Dienes has developed the theory of Statistical CRACK Mechanics (SCRAM) for modeling the damage and failure of brittle materials (Dienes, *et al.* 2006; Zuo, *et al.* 2008).

The SCRAM theory assumes an ensemble of randomly distributed microcracks in the material in the unloaded state and evaluates the evolution of the probability distribution function (pdf) of the cracks in various orientations as a function of the loading. The damage in the material is obtained by statistically averaging the responses of the microcracks of various sizes and orientations. A unique feature of the SCRAM model is the modeling of anisotropic damage of the material by tracking the mean crack sizes along a set of pre-determined orientations. Based on the SCRAM, Addessio and Johnson (1990) proposed a continuum damage model (ISOSCM) in which the distribution of the crack size is assumed to remain isotropic. An important aspect of the ISOSCM model is its numerical efficiency, as the evolution of damage is based on the average crack size over all orientations.

Following ISOSCM, other models have been proposed to account for additional physical mechanisms that can affect the response of brittle materials (Bennett, *et al.* 1998; Lee, *et al.* 2004). For example, Bennett, *et al.* (1998) proposed a damage model (Visco-SCRAM) for plastically bonded explosives (PBX), which includes the viscous effect of the plastic binder in the explosives (Hackett and Bennett 2000).

2.2 Current Model

Notation

The following direct tensor notations (e.g. Gurtin 1981) are used in the report:

$$\begin{aligned}
\mathbf{i} &\equiv \delta_{ij} \mathbf{e}_i \otimes \mathbf{e}_j & \mathbf{I} &\equiv \frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_k \otimes \mathbf{e}_l \\
\mathbf{u} \otimes \mathbf{v} &\equiv u_i v_j \mathbf{e}_i \otimes \mathbf{e}_j & \mathbf{A} \otimes \mathbf{B} &\equiv A_{ij} B_{kl} \mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_k \otimes \mathbf{e}_l \\
\mathbf{u} \cdot \mathbf{v} &\equiv u_k v_k & \mathbf{A} \mathbf{u} &\equiv A_{ik} u_k \mathbf{e}_i & \mathbf{A} \mathbf{B} &\equiv A_{ik} B_{kj} \mathbf{e}_i \otimes \mathbf{e}_j \\
\mathbf{T} \boldsymbol{\varepsilon} &\equiv T_{ijkl} \varepsilon_{kl} \mathbf{e}_i \otimes \mathbf{e}_j & \mathbf{C} \mathbf{D} &\equiv C_{ijkl} D_{klmn} \mathbf{e}_i \otimes \mathbf{e}_j \otimes \mathbf{e}_m \otimes \mathbf{e}_n \\
tr \mathbf{A} = \mathbf{i} : \mathbf{A} &= A_{ii} & \mathbf{A} : \mathbf{B} &= A_{ik} B_{ki}
\end{aligned}$$

where \mathbf{i} is the second-order identity tensor; \mathbf{I} is the fourth-order (symmetric) identity tensor; δ_{ij} is the Kronecker delta; $\{\mathbf{e}_i\}$ ($i = 1, 2, 3$) has an arbitrary orthonormal basis; " \otimes " is the tensor product; \mathbf{u} and \mathbf{v} are vectors; \mathbf{A} and \mathbf{B} are symmetric second-order tensors; \mathbf{T} , \mathbf{C} , and \mathbf{D} are fourth-order tensors; " $:$ " is the scalar product of second-order tensors; and tr is the trace of a second-order tensor.

We consider the mechanical response of a quasi-brittle material under a general, three-dimensional stress state. The material is assumed to contain a large number of penny-shaped cracks with different sizes and orientations. As in previous work (Addessio and Johnson 1990; Zuo, *et al.* 2006), the size distribution of the cracks is assumed to remain isotropic (i.e. independent of the crack orientation) and exponential during loading. Under such assumptions, the probability distribution function of the crack numbers can be written as (Addessio and Johnson 1990)

$$n(c, t) = \frac{N_o}{\bar{c}(t)} \exp(-c / \bar{c}(t)) \quad (1)$$

where $\bar{c}(t)$ is the mean crack radius and N_o is the initial crack number density per solid angle. In the current model, N_o is kept as a material constant and the damage in the material is reflected through the evolution of $\bar{c}(t)$.

In the DCA model (Zuo, *et al.* 2006), the total strain rate $\dot{\boldsymbol{\epsilon}}$ is decomposed into the contributions from the matrix (uncracked solid) and from the response (open, shear, and growth) of microcracks. Here, to include plastic deformation, the decomposition is modified as

$$\dot{\boldsymbol{\epsilon}} = \dot{\boldsymbol{\epsilon}}_m + \dot{\boldsymbol{\epsilon}}_c^d + \dot{\boldsymbol{\epsilon}}_c^{gr} + \dot{\boldsymbol{\epsilon}}^p \quad (2)$$

where the strain rates related to the matrix, the opening and shear of cracks (with the current sizes), and the growth of cracks are respectfully given by

$$\begin{aligned} \dot{\boldsymbol{\epsilon}}_m &= \mathbf{C}_m \dot{\boldsymbol{\sigma}} \\ \dot{\boldsymbol{\epsilon}}_c^d &= \mathbf{D}(\bar{c}) \dot{\boldsymbol{\sigma}} \\ \dot{\boldsymbol{\epsilon}}_c^{gr} &= \frac{\partial \mathbf{D}(\bar{c})}{\partial \bar{c}} \dot{\bar{c}} \dot{\boldsymbol{\sigma}} \end{aligned} \quad (3)$$

where \mathbf{C}_m is the compliance tensor of the matrix; $\mathbf{D}(\bar{c})$ is the damage tensor; and $\dot{\boldsymbol{\epsilon}}^p$ is the plastic strain rate, which is to be defined later. The plastic deformation considered in the current work refers to deformation that cannot be recovered upon removal of the stress on the material. For a crystalline solid, the physical (micromechanical) origin of such deformation is the moment of dislocations (slip) on the slip planes in the material. Deformation mechanisms such as mechanical twinning and phase transformation are not considered.

If the matrix is modeled by linear isotropic elasticity, then the compliance tensor \mathbf{C}_m of the matrix can be written as

$$\mathbf{C}_m = \frac{1}{3K} \mathbf{P}^{sp} + \frac{1}{2G} \mathbf{P}^d \quad (4)$$

where K and G are, respectively, the bulk and shear moduli of the matrix, which are constants for the model. The spherical and deviatoric projection operators are (e.g. Hansen and Schreyer 1994)

$$\mathbf{P}^{sp} \equiv \frac{1}{3} (\mathbf{i} \otimes \mathbf{i}) \quad \mathbf{P}^d \equiv \mathbf{I} - \mathbf{P}^{sp}$$

with \mathbf{i} and \mathbf{I} denoting, respectively, the 2nd- and (symmetric) 4th- order identity tensors defined in *Notation*.

The damage tensor developed in the DCA model is used in the current work,

$$\mathbf{D}(\bar{c}) = \beta^e N_o \bar{c}^3 \left(\frac{3}{2-\nu} \mathbf{P}^d + \mathbf{P}^+ \left(\mathbf{P}^d + \frac{5}{2} \mathbf{P}^{sp} \right) \mathbf{P}^+ \right) \quad (5)$$

where ν is the Poisson's ratio of the matrix and $\beta^e \equiv 64\pi(1-\nu)/(15G)$ is a material constant that depends on the elastic properties of the matrix. The quantity $N_o \bar{c}^3$ can be thought of as a dimensionless scalar representation of the material damage.

In Eq. (5), \mathbf{P}^+ is the positive projection operator (a 4th-order tensor) defined by the stress state: $\mathbf{P}^+ = \mathbf{I}$ if the stress state is tensile (all three principal stresses are positive) and $\mathbf{P}^+ = \mathbf{0}$ if it is compressive (all three principal stresses are negative). When the principle stresses are of mixed signs, \mathbf{P}^+ eliminates the contributions of the compressive principal stresses to the crack opening strain, making the formulation consistent with crack mechanics (Yazdani and Schreyer 1988, 1990, 2003; Wen and Yazdani 2008). The details of the definition for \mathbf{P}^+ are described by Zuo, *et al.* (2006).

In Eq. (3c), $\dot{\bar{c}}$ is the crack growth rate given by (Zuo, *et al.* 2006)

$$\frac{\dot{\bar{c}}}{\dot{\bar{c}}_{\max}} = 1 - \frac{1}{1 + \langle F(\boldsymbol{\sigma}, \bar{c}) \rangle} \quad (6)$$

where $F(\boldsymbol{\sigma}, \bar{c})$ is the damage function based on the stability of the cracks along the critical orientation. The damage surface $F(\boldsymbol{\sigma}, \bar{c}) = 0$ divides the stress space into two regions: the elastic region corresponding to $F(\boldsymbol{\sigma}, \bar{c}) < 0$ in which $\dot{\bar{c}} = 0$ and the damage-accumulation region for $F(\boldsymbol{\sigma}, \bar{c}) > 0$ in which $\dot{\bar{c}} > 0$.

In Eq. (6) the angled bracket is the Macaulay bracket, which takes the value of the argument when positive and zero otherwise. The expression for $F(\boldsymbol{\sigma}, \bar{c})$ is given in Zuo, *et al.* (2006). The maximum growth rate $\dot{\bar{c}}_{\max}$ is the terminal speed for crack growth (e.g. Freund 1990) and is either the shear wave speed of the matrix for closed cracks or the Rayleigh wave speed C_R for open cracks (which is only slightly less than the shear wave speed). The choice depends on whether the crack with the critical orientation is open or closed.

For simplicity, the von Mises theory with associated flow rule is used to model the plastic response of the material. In the von Mises theory, the yield surface is given by (e.g. Lubliner 1990, Simo and Hughes 1998)

$$f(\boldsymbol{\sigma}) = \bar{\sigma} - \sigma_y = 0 \quad (7)$$

where $\bar{\sigma} \equiv \sqrt{3(\boldsymbol{\sigma}^d : \boldsymbol{\sigma}^d)}/2$ (with $\boldsymbol{\sigma}^d$ being the stress deviator) is the equivalent (von Mises) stress and σ_y is the yield stress of the material, which in general is a function of the plastic strain, strain rate, and temperature. Also, for simplicity, the yield stress σ_y is assumed to remain constant (i.e. perfect plasticity). Following an associated flow rule, the plastic strain rate is given by (e.g. Lubliner 1990, Simo and Hughes 1998)

$$\dot{\boldsymbol{\epsilon}}^p = \dot{\lambda} \frac{\partial f(\boldsymbol{\sigma})}{\partial \boldsymbol{\sigma}} = \dot{\lambda} \mathbf{N} = \dot{\lambda} \frac{3}{2} \frac{\boldsymbol{\sigma}^d}{\bar{\sigma}} \quad (8)$$

where \mathbf{N} denotes the normal to the yield surface. The substitution of Eq. (3) and Eq. (8) into Eq. (2) yields

$$(\mathbf{C}_m + \mathbf{D}(\bar{c})) \dot{\boldsymbol{\sigma}} + \dot{\bar{c}} \frac{\partial \mathbf{D}(\bar{c})}{\partial \bar{c}} \boldsymbol{\sigma} + \dot{\lambda} \frac{3}{2} \frac{\boldsymbol{\sigma}^d}{\bar{\sigma}} = \dot{\boldsymbol{\epsilon}} \quad (9)$$

For a prescribed total strain rate $\dot{\boldsymbol{\epsilon}}$, with the damage tensor defined in Eq. (5), Eq. (9) is a tensorial equation for the stress rate $\dot{\boldsymbol{\sigma}}$, the crack growth rate $\dot{\bar{c}}$, and the plastic parameter $\dot{\lambda}$. When supplemented by the consistency equation, $\dot{f}(\boldsymbol{\sigma}) = 0$, where $f(\boldsymbol{\sigma}) = 0$ is the yield surface given by Eq. (7), and by Eq. (6) for crack growth, Eq. (9) can be solved for $\dot{\boldsymbol{\sigma}}$, $\dot{\bar{c}}$, and $\dot{\lambda}$. In practice, however, solving such coupled nonlinear equations poses a serious technical challenge. As an approximation, we have developed a simpler algorithm.

Section 3 Numerical Algorithm

Consider a typical time step $\Delta t \equiv t^{n+1} - t^n$, where t^n and t^{n+1} are, respectively, the times at the beginning and at the end of the step. The computational algorithm for the step Δt is summarized in the following. Suppose that the material state (i.e. stress, mean crack size, and plastic strain) is known at the beginning of the step. The total strain rate $\dot{\boldsymbol{\varepsilon}}_{n+1}$ for the step is prescribed, and the objective here is to update the stress, mean crack size, and plastic strain.

The integration of Eq. (2) over the time step gives the incremental form

$$\Delta \boldsymbol{\varepsilon} = \left(\Delta \boldsymbol{\varepsilon}_m + \Delta \boldsymbol{\varepsilon}_c^d + \Delta \boldsymbol{\varepsilon}_c^{gr} \right) + \Delta \boldsymbol{\varepsilon}^p \quad (10)$$

where $\Delta \boldsymbol{\varepsilon} = \dot{\boldsymbol{\varepsilon}}_{n+1} \Delta t$ and so on. For convenience, we define

$$\Delta \boldsymbol{\varepsilon}^{DCA} \equiv \Delta \boldsymbol{\varepsilon}_m + \Delta \boldsymbol{\varepsilon}_c^d + \Delta \boldsymbol{\varepsilon}_c^{gr} = \Delta \boldsymbol{\varepsilon} - \Delta \boldsymbol{\varepsilon}^p \quad (11)$$

Then it follows from Eqs. (2) and (3) that

$$\left(\mathbf{C}_m + \mathbf{D}(\bar{c}) \right) \dot{\boldsymbol{\sigma}} = \dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p - \dot{\boldsymbol{\varepsilon}}_c^{gr} \quad (12)$$

Integrating Eq. (12) over the time step gives

$$\boldsymbol{\sigma}^{n+1} = \boldsymbol{\sigma}^n + \left(\mathbf{C}_m + \mathbf{D}(\bar{c}^n) \right)^{-1} \left(\Delta \boldsymbol{\varepsilon} - \Delta \boldsymbol{\varepsilon}^p - \Delta \boldsymbol{\varepsilon}_c^{gr} \right) \quad (13)$$

As an approximation, we assume that over a small time step the damage (crack growth) and plasticity calculations can be done separately. That is, in each step, the crack growth $\Delta \bar{c}$ and the strain increment due to crack growth $\Delta \boldsymbol{\varepsilon}_c^{gr} = \Delta \bar{c} \left(\partial \mathbf{D}(\bar{c}) / \partial \bar{c} \right) \boldsymbol{\sigma}$ are first calculated assuming the step does not involve plasticity. The stress at the end of this sub-step calculation can then be used to calculate the plastic strain for the step, which in turn is used to update (correct) the stress predicted by the first sub-step calculation.

Let us define the stress found by assuming that the step does not involve plasticity as the trial stress for the second sub-step, which involves plasticity only:

$$\boldsymbol{\sigma}^{tr} \equiv \boldsymbol{\sigma}^n + (\mathbf{C}_m + \mathbf{D}(\bar{c}^n))^{-1} (\Delta \boldsymbol{\varepsilon} - \Delta \boldsymbol{\varepsilon}_c^{gr}) \quad (14)$$

Then it follows from Eqs. (13) and (14) that

$$\boldsymbol{\sigma}^{n+1} = \boldsymbol{\sigma}^{tr} - \mathbf{E}^n \Delta \boldsymbol{\varepsilon}^p \quad (15)$$

where, for convenience, $\mathbf{E}^n \equiv (\mathbf{C}_m + \mathbf{D}(\bar{c}^n))^{-1}$ was introduced to represent the (4th-order) elasticity tensor of the material at the beginning of the step. An implicit integration (Simo and Hughes, 1998) of Eq. (8) over the step gives

$$\Delta \boldsymbol{\varepsilon}^p = \Delta \lambda \mathbf{N}^{n+1} = \Delta \lambda \frac{3}{2} \left(\frac{\boldsymbol{\sigma}^d}{\bar{\boldsymbol{\sigma}}} \right)^{n+1} \quad (16)$$

Substitution of Eq. (16) into Eq. (15) gives

$$\boldsymbol{\sigma}^{n+1} + \mathbf{E}^n \Delta \lambda \frac{3}{2} \left(\frac{\boldsymbol{\sigma}^d}{\bar{\boldsymbol{\sigma}}} \right)^{n+1} = \boldsymbol{\sigma}^{tr} \quad (17)$$

Since the distribution of the cracks is assumed to remain isotropic during loading, it is reasonable to assume that the elasticity tensor of the material also remains isotropic. Let

$$\mathbf{E}^n = 3K^n \mathbf{P}^{sp} + 2\mu^n \mathbf{P}^d \quad (18)$$

where K^n and μ^n are the current (damaged) bulk and shear moduli of the material at the beginning of the step. It follows from Eqs. (17) and (18) that

$$\left(1 + \frac{3\mu^n \Delta \lambda}{\bar{\boldsymbol{\sigma}}^{n+1}} \right) (\boldsymbol{\sigma}^{n+1})^d = (\boldsymbol{\sigma}^{tr})^d \quad (19a)$$

$$p^{n+1} = p^{tr} \quad (19b)$$

where $(\boldsymbol{\sigma}^{n+1})^d$ and p^{n+1} are respectively the stress deviator and the pressure at the end of the step and $(\boldsymbol{\sigma}^{tr})^d$ and p^{tr} are the corresponding values for the trial stress:

$$p^{tr} = -\frac{1}{3}tr(\boldsymbol{\sigma}^{tr}) \quad (20a)$$

$$(\boldsymbol{\sigma}^{tr})^d = \boldsymbol{\sigma}^{tr} + p^{tr}\mathbf{i} \quad (20b)$$

It follows from Eq. (19a) that

$$(\boldsymbol{\sigma}^{n+1})^d = \frac{(\boldsymbol{\sigma}^{tr})^d}{\left(1 + \frac{3\mu^n \Delta\lambda}{\bar{\sigma}^{n+1}}\right)} \quad (21a)$$

$$\left(\frac{\boldsymbol{\sigma}^d}{\bar{\sigma}}\right)^{n+1} = \frac{(\boldsymbol{\sigma}^{n+1})^d}{\bar{\sigma}^{n+1}} = \frac{\boldsymbol{\sigma}^{tr}}{\bar{\sigma}^{tr}} \quad (21b)$$

where $\bar{\sigma}^{tr} \equiv \left(3/2(\boldsymbol{\sigma}^{tr} : \boldsymbol{\sigma}^{tr})\right)^{1/2}$ is the trial von Mises stress. Eq. (21a) implies that the direction of the stress deviator at the end of the step is the same as that at the trial state, which is available once the trial state is found, and that the normal to the yield surface can be calculated solely based on the trial state. It follows from Eq. (21a) that

$$\bar{\sigma}^{n+1} = \left(\frac{3}{2}(\boldsymbol{\sigma}^{n+1})^d : (\boldsymbol{\sigma}^{n+1})^d\right)^{1/2} = \frac{\bar{\sigma}^{tr}}{\left(1 + \frac{3\mu^n \Delta\lambda}{\bar{\sigma}^{n+1}}\right)} \quad (22)$$

Or,

$$\bar{\sigma}^{n+1} = \bar{\sigma}^{tr} - 3\mu^n \Delta\lambda \quad (23)$$

That is, the plastic parameter $\Delta\lambda$ is proportional to the distance from the trial state to the final state.

In an implicit algorithm, the final stress state (at the end of the step) is required to be on the yield surface

$$f(\boldsymbol{\sigma}^{n+1}) = \bar{\sigma}^{n+1} - \sigma_y = 0 \quad (24)$$

The substitution of Eq. (23) to Eq. (24) solves for the plastic parameter $\Delta\lambda$:

$$\Delta\lambda = \frac{\bar{\sigma}^{tr} - \sigma_y}{3\mu^n} = \frac{f^{tr}}{3\mu^n} \quad (25)$$

The above formulations can be summarized in the following steps:

1. Assume the step does not involve plastic deformation. The resulting state (stress, mean crack size, and plastic strain) is defined as the trial state. Since $(\Delta\epsilon^p)^{tr} = 0$,

$$(\Delta\epsilon^{DCA})^{tr} = \Delta\epsilon - (\Delta\epsilon^p)^{tr} = \Delta\epsilon \quad (26)$$

2. Call the DCA routines to calculate the stress and mean crack size at the end of the step using $(\Delta\epsilon^{DCA})^{tr}$. The numerical algorithm for this part of the calculation is implicit and has described in detail (Zuo, *et al.* 2006). The stress so calculated is defined as the trial stress for the step, σ^{tr} .

3. Check if the trial state lies outside the yield surface $f(\sigma) = \bar{\sigma} - \sigma_y = 0$. If $f^{tr} = \bar{\sigma}^{tr} - \sigma_y \leq 0$, then the current step does not involve plastic deformation and the calculation for the step is complete. However, if $f^{tr} > 0$, then the step involves plasticity and corrections to the stress must be made. Go to step 4.

4. Solve for the plastic parameter $\Delta\lambda$ using Eq. (25). The plastic strain increment for the time increment is then

$$\Delta\epsilon^p = \Delta\lambda \frac{3}{2} \left(\frac{\sigma^d}{\bar{\sigma}} \right)^{n+1} = \Delta\lambda \frac{3}{2} \frac{(\sigma^{tr})^d}{\bar{\sigma}^{tr}} \quad (27)$$

It follows from Eq. (21b) that

$$(\sigma^{n+1})^d = \frac{(\sigma^{tr})^d}{\bar{\sigma}^{tr}} \bar{\sigma}^{n+1} = \frac{\sigma_y}{\bar{\sigma}^{tr}} (\sigma^{tr})^d \quad (28)$$

That is, in the deviatoric plane, the final stress can be found by returning the trial stress back onto the yield surface along a radial direction (the direction of the trial stress). With the pressure and the deviatoric parts known, the final stress is

$$\sigma^{n+1} = (\sigma^{n+1})^d - p^{n+1} \mathbf{i} = \frac{\sigma_y}{\bar{\sigma}^{tr}} (\sigma^{tr})^d - p^{tr} \mathbf{i} \quad (29)$$

The stress has been corrected and the plastic strain calculated; hence the calculation for the step is complete.

A set of computer subroutines were written to implement the numerical algorithm. A modified form of the algorithm that does not include the plasticity deformation has also been implemented into ABAQUS, a three-dimensional analysis program widely used to analyze the response of structures (pavements and bridges) under thermal-mechanical loads. The numerical results using the model are presented next.

Section 4 Model Results

To illustrate the key features of the model and to validate the model using experimental data (a future effort), the predicted responses of the new model under several loading paths have been studied. The model material is the plain concrete studied by Schreyer and coworkers (Schreyer and Bean 1987; Hansen and Schreyer 1994, 1995), and some of the model constants are based on those used by Schreyer and Bean (1987). For convenience, the model constants are listed in Table 4-1.

Table 4-1. Model constants, based in part on Schreyer and Bean (1987)

$\rho(g / cm^3)$, mass density	2.3
$G(dyn / cm^2)$, shear modulus	8.5×10^{10} (8.5 GPa)
ν , Poisson's ratio	0.2
$\bar{c}_0(cm)$, initial crack size	20×10^{-4}
$N_0 (cm^{-3})$, crack number density	1.0×10^5
$\gamma(erg / cm^2)$, surface energy	1.5×10^2 (0.15 J / m ²)

Consider the response of a brittle concrete under uniaxial (strain) tensile loading. The evolutions of the stress and the average crack radius (damage) with the applied uniaxial strain are shown, respectively, in Figures 4-1 and 4-2. The material is initially stress free and is loaded monotonically to the strain of 0.01.

It can be seen in Figure 4-1 that during the loading part of the path (**A- A*-B-C**) the material behaves first elastically (**A-A***), with a slightly damaged modulus corresponding to an initial crack size of $\bar{c}_0 = 20 \mu m$. Crack growth is initiated at point **A*** ($\epsilon_{11} \approx 0.76 \times 10^{-3}$, $\sigma_{11} \approx 16.2 MPa$), when the stress state reaches the initial damaged surface with \bar{c}_0 . Immediately

following point **A***, the strain rate due to crack growth, $\dot{\epsilon}^{gr}$, is still too low to cause a significant effect in the response. As a result, the stress keeps increasing until it reaches its maximum value at point **B** [$\sigma_{11} \approx 19.5 \text{ MPa}$ and $\epsilon_{11} \approx 1. \times 10^{-3}$]. Following point **B**, the strain rate due to crack growth overcomes the total prescribed strain rate; consequently, the stress starts to decrease while the strain continues to increase ($\dot{\epsilon}_{11} > 0$) and the material strain-softens (from **B** to **C**).

The corresponding evolution of damage in the material (in terms of the mean crack size) is shown in Figure 4-2. It is seen that the mean crack size remains at the initial value ($\bar{c} / \bar{c}_0 = 1$) between A-A*, and that crack growth initiate at **A***. It is also seen that during the crack growth (path **A*-B-C**), though the applied strain rate is constant, the crack growth rate is not constant. Immediately following the initiation point **A***, crack growth is slow, but it quickly gains the speed between **A*B**. In fact, the fast crack growth shown in Figure 4-2 is the physical mechanism for the softening response of the concrete, as seen in Figure 4-1. At the final strain of $\epsilon_{11} = 0.01$ (Point **C**), the mean crack size has grown to $\bar{c} \approx 8.7\bar{c}_0$. Since the damage in the material is proportional to \bar{c}^3 , the model predicts a significant amount of damage accumulation in the material under 1% of tensile strain.

The response of the material under shear loading ($\dot{\epsilon}_{23} > 0$), which is shown in Figures 4-3 and 4-4, has similar features as those for uniaxial (strain) tensile loading.

The results indicate that the micromechanical model is capable of predicting the softening response of concrete due to growth of microcracks, which is the dominant feature of concrete (Yazdani and Schreyer 1988, 1990, 2003; Hansen and Schreyer 1994, 1995).

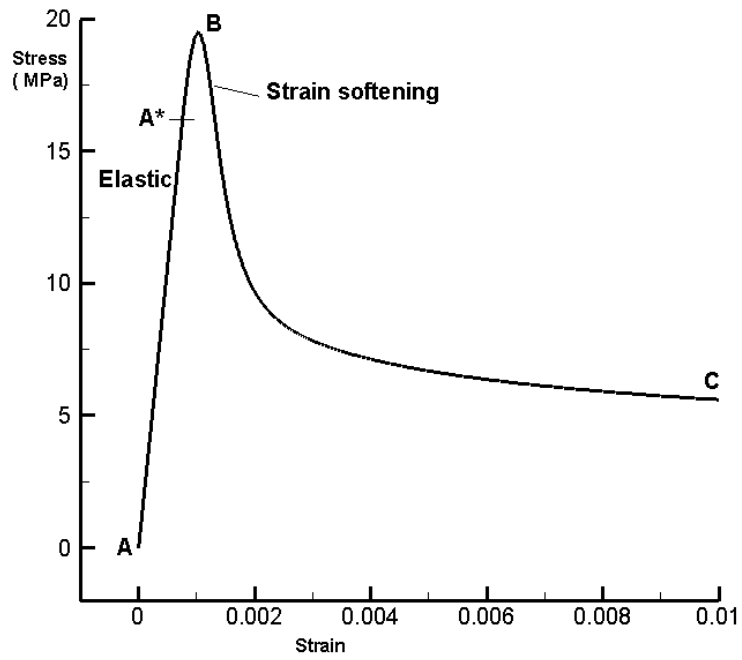


Figure 4-1. The predicted stress-strain response under uniaxial (strain) loading

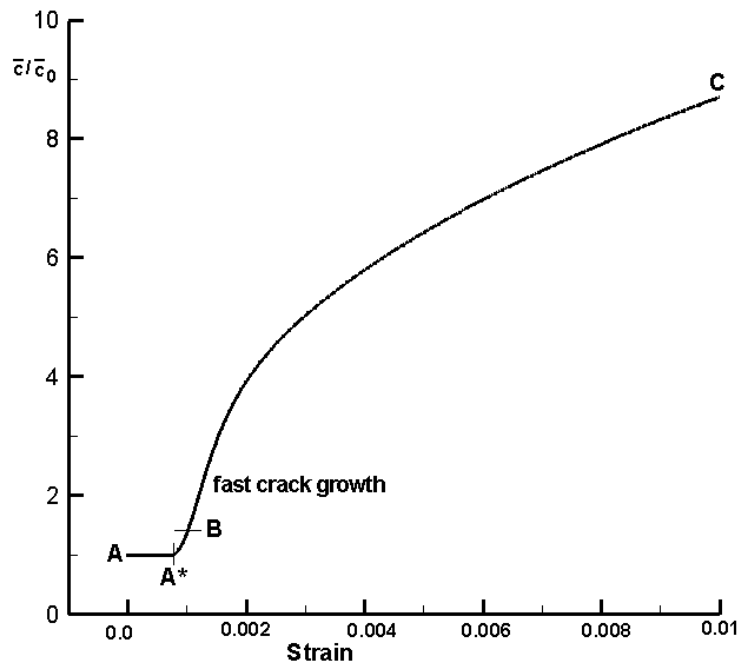


Figure 4-2. Evolution of damage under uniaxial (strain) loading

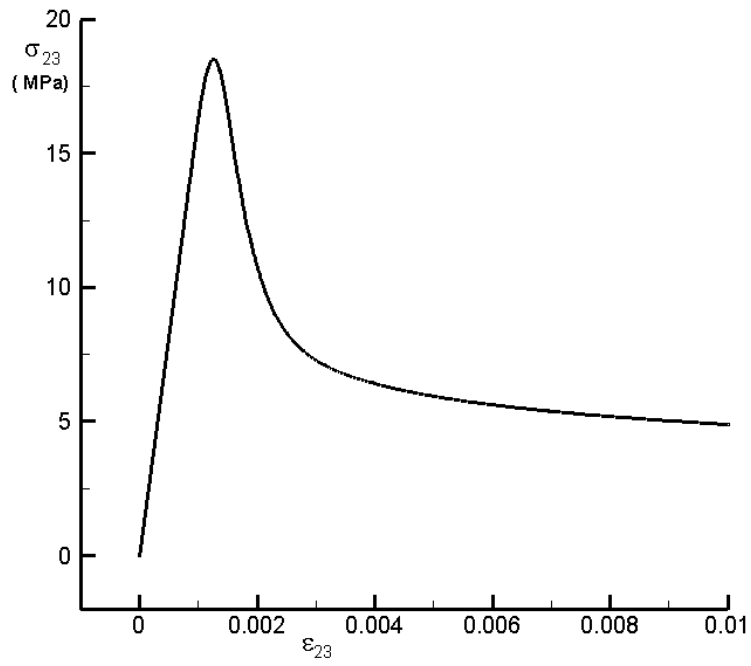


Figure 4-3. The predicted stress-strain response under shear loading

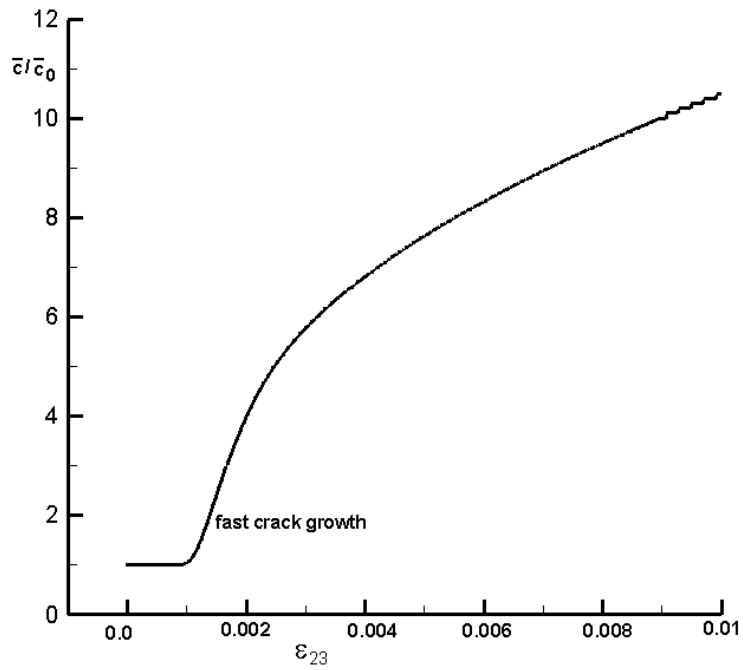


Figure 4-4. Evolution of damage as a function of the strain under shear loading

Section 5 Summary

The objective of the research project was achieved and a micromechanics-based constitutive model was developed for damage and failure in concrete pavements and bridges under general (three-dimensional) loading conditions. The model is also implemented in ABAQUS analysis code and can be utilized by ALDOT engineers to make an informed assessment of the damage in concrete pavements and bridges.

Section 6

Future Work

The focus of the project was on the development of a three-dimensional damage model based on micromechanics and its numerical implementation; as such, no quantitative comparisons of the model predictions with experimental data were attempted during this research. We recommend as a future research topic to conduct a systematic comparison of the model predictions and existing data.

Section 7

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Appendix

List of Journal Publications

UTCA project #09301 has resulted in the following publications:

- Zuo, Q.H., D. Disilvestro, and J.D. Richter. "A crack-mechanics based model for damage and plasticity of brittle materials under dynamic loading." *International Journal of Solids and Structures* Vol. 47, no. 20, pp. 2790-2798. 2010.
- Zuo, Q.H. and H.L. Schreyer. "Effect of deviatoric nonassociativity on the failure prediction for elastic-plastic materials." *International Journal of Solids and Structures* Vol. 47, no. 11-12, pp. 1563-1571. 2010.
- Zuo, Q.H. and D.J. Alldredge. 2010. "Stability and well-posedness of a rate-dependent strain-softening plasticity model." *International Journal of Theoretical and Applied Multiscale Mechanics* Vol. 1, no. 3, pp. 195-218. 2010.
- Zuo, Q.H. "Modified formulation of a rate-dependent damage model for ductile materials." *Journal of Applied Physics* Vol. 107, no. 5, pp. 1-5. 2010.