Performance of GEOID09 for Height Conversion in Ohio

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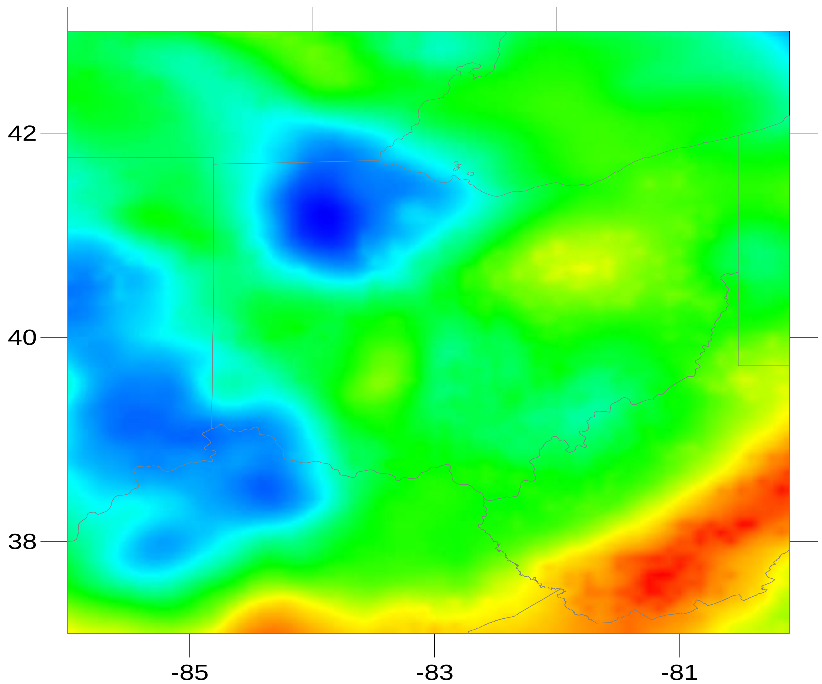

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16. Abstract

This study evaluates Height Modernization issues related to NGS hybrid geoid performance (specifically GEIOD09 and GEOID03) for height conversions between NAVD88 and NAD83 for the state of Ohio and quality of gravity and height data needed to produce a cm-accurate gravimetric geoid. The hybrid geoid is of particular significance in this study, because an accurate hybrid geoid would support both accurate height conversions as well as improve orthometric (MSL) height determination, facilitating the almost-total replacement (except in GPS-antagonistic environments) of traditional leveling exercises by GPS leveling, a much more cost- and labor-efficient heighting technique than its traditional counterpart.

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1. Introduction

Height Modernization is a term which describes the upgrading of height determination techniques by using GPS and other modern positioning technology in addition to (and where possible, instead of) traditional line-of-sight leveling techniques. This report has been formulated to support some of the decision-making processes in which the Ohio Department of Transportation (ODOT) will have to engage in regard to Height Modernization issues relevant to the state of Ohio. Such matters include height modernization support of:

- Faster and more accurate surveying and engineering projects particularly as pertain to roadway and bridge development and maintenance
- Faster and more efficient densification and quality assessment of:
  - vertical control networks using, for example, real time kinematic (RTK) GPS
  - aerial photogrammetric control using, for example, airborne LiDAR – this will allow for more dynamic update of state topographic maps
- Disaster preparedness and mitigation for Ohio landowners particularly in areas where there:
  - Is land subsidence resulting from mining or other types of geo-exploration or where
  - Are other types of land alteration, property loss or damage due to slow-moving, long-term geophysical phenomena such as Glacial Isostatic Adjustment (GIA)
- Precision farming and pollution control in that farmers would be able to formulate and use more effective irrigation and chemical (e.g. fertilizers and pesticides) runoff strategies
- Safer approach and landing strategies for aircraft particularly where pilot visibility is low.

Notably, some states have documented the *potential* positive economic impact of height modernization efforts. For example, Michigan DOT foresees savings of more than $30 million over 5 years. In one of its case studies a photogrammetric control establishment campaign was performed over a distance of 5 miles. This campaign, which would typically have taken 5 weeks using traditional surveying and rapid-static GPS techniques, was condensed to a mere 4 days using roving GPS units in RTK mode. Wisconsin expects to see savings of $1.5 million per year by using airborne-LiDAR and height modernization products (such as the geoid) to facilitate more cost effective establishment of photogrammetric control.

The National Height Modernization Program is currently being spearheaded by the National Oceanic and Atmospheric Administration / National Geodetic Survey (NOAA/NGS). An ultimate objective (among others) of this program is the development and maintenance of a high accuracy, accessible national height reference system which capitalizes on the availability and accuracy of GPS and other existent geodetic infrastructure. One of the key components of this national height reference infrastructure is the hybrid geoid for the continental U.S. (CONUS) regions, the most recent of which is GEOID09 - a key subject of analysis in this study.

Key to understanding why the hybrid geoid is a major component of the Height Modernization project is the knowledge of its benefits. Ultimately, there is a height conversion demand in that accurate conversion between GPS-observed, NAD83-referenced ellipsoidal heights and NAVD88-reference orthometric heights (typically obtained by traditional leveling operations) is needed. An accurate hybrid geoid would support this conversion and, as importantly, would improve orthometric (MSL) height determination. Ideally, this would facilitate the almost-total replacement (except in GPS-antagonistic environments) of traditional leveling exercises by GPS leveling. GPS leveling will, without a doubt, be a much more cost- and labor-efficient heighting technique than its traditional counterpart.

It is noteworthy that it was originally stated that the *Deliverables* of this final report should include:

1. Analysis of the discrepancies observed between GEOID03 undulations and LiDAR- and GPS-derived geometric undulations.
2. Analysis of the quality of existent gravity data in the region and gravity coverage
3. Optional data to support LiDAR feature extraction for traffic corridors in Ohio
4. All MATLAB code developed for fitting of the LiDAR geometric undulations to the GEOID03 and USGG2003 surfaces.

As pertains to these *Deliverables*, when this project was undertaken in November 2006, GEOID03 was the most current NGS hybrid model. However, GEOID09 was formally released by NGS in September 2009. Therefore, the reader will find that all results documented in the ensuing pages of this report will emphasize evaluation of the performance of GEOID09 (as indicated in the report title) instead of GEOID03.

Furthermore a significant objective was to observe additional GPS benchmarks (GPSBMs) not used in the current geoid model as an independent check on the NGS geoid performance in the state. It was envisaged that the ODOT Aerial Engineering Services' LiDAR system could be used to observe multiple swaths of areas. Unfortunately, the department's airplane was grounded during the period of ODOT's data collection. Therefore, no LiDAR profiles were observed (invalidating Deliverables identified by bullet point 3 and 4). However, as was originally determined, GPS ellipsoidal heights were observed at 50 existent benchmarks. While these 50 benchmarks are not widely distributed across the entire state of Ohio, they do provide some insight into the performance of GEOID09 in the regions of their observation.

Given the data collection difficulties experienced in this project the SPIN Lab opted to utilize publicly-available data sets to undertake the computation of a local gravimetric geoid over the state of Ohio. This will also be further described in the pages that follow.

2. Research Objectives

Recall the definition of a geoid undulation (or geoid height), N. As can be seen from Figure 2.1, it is the difference between the ellipsoidal height (in this context, relative to NAD83) denoted by h, and orthometric height (relative to NAVD88) denoted by H:

$N_{GEOM} = h_{NAD83} - H_{NAVD88}$ [2.1]

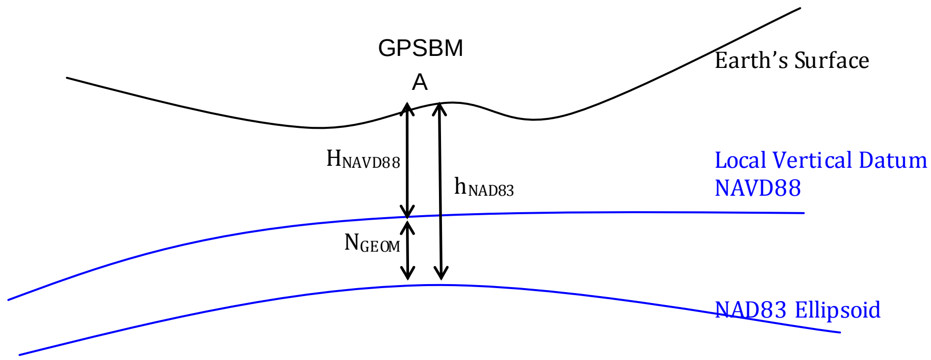


Figure 2.1: Definition of the geoid, ellipsoidal and orthometric heights.

The implication of equation [2.1] is that if positions are known for which co-located GPS observations and orthometric heights (known as GPS benchmarks, GPSBMs) exist, then one should be able to compute an independent non-gravimetric geoid undulation, often referred to as a geometric geoid undulation, N_{GEOM}.

NGS computes 2 types of geoid models:

- A gravimetric geoid and
- A hybrid geoid which is essentially the gravimetric geoid model optimally-warped (using least squares collocation) to fit the aforementioned geometric geoid heights which are inferred by the GPSBMs distributed throughout a region. Therefore, it is the hybrid geoid (not the gravimetric) which is most suitable for the NAD83 / NAVD88 height conversions for CONUS.

In an ideal world, the gravimetric geoid would show close agreement with the geometric geoid heights. However, due to gravity, leveling, GPS and other errors associated with the geoid development process, systematic effects infect the hybrid geoid model.

In this study, therefore, a significant objective is to use geometric geoid undulations in Ohio in order to evaluate the extent to which the hybrid geoid models (GEOID09 and GEOID03) agree with the geoid inferred by the GPSBMs. In so doing, the goal is to enumerate the accuracy with which these models can truly be used for statewide height conversion needs.

Furthermore, the SPIN Lab used publicly available datasets to compute an Ohio-specific gravimetric geoid. This model was used as the basis of a study, the objective of which was to determine the quality of gravity and height data needed to produce a cm-accurate geoid in Ohio.

This ODOT research initiative will inevitably provide a useful insight into the National Height Modernization objectives, a major part of which involves the computation of a cm-accurate national geoid (NGS, 2007).

3. General description of the research

This Height Modernization research study consists of 3 sections, namely:

1. Performance of NGS hybrid geoids in Ohio
2. Gravimetric geoid development for Ohio using publicly-available data
3. Random error influence on gravimetric geoid solution

More comprehensive descriptions of each segment of the study are outlined below.

3.1 Performance of NGS geoids in Ohio

Development of NGS hybrid models involves fitting of the gravimetric geoid result to GPS benchmarks (i.e. sites where GPS observations and orthometric levels are co-located) using least squares collocation (Roman et al., 2004). Given that GEOID03 (ibid.) and GEOID09 (Roman et al., 2009) are the result of "warping" the gravimetric geoid to fit the GPS benchmarks (GPSBMs), the NGS geoid accuracy quotes for Ohio (seen in Table 3.1.1) do not necessarily reflect the performance of the hybrid models at GPSBMs **not** included in their development. Therefore, one way in which to truly evaluate the performance of GEOID03 and GEOID09 is to evaluate their height conversion precision at stations **not** included in the GEOID03 and GEOID09 model development.

Table 3.1.1: Level of NGS hybrid model fit at GPSBMs used in GEOID03 and GEOID09 (Roman et al., 2004 and 2009)

	Geoid Discrepancy: Δ_N = GEOIDxx – (h_{NAD83} – H_{NAVD88})			
Hybrid Model	**Year xx**	**Number of GPSBMs**	**Average [m]**	**STD [m]**
GEOID03	03	254	0.001	± 0.032
GEOID09	09	297	0.000	± 0.022

To this end, NAD83 latitude, longitude and ellipsoidal heights (φ_{VRS}, λ_{VRS}, h_{VRS}) of 50 published NGS benchmarks were re-observed by ODOT staff using the VRS (Virtual Reference Station) GPS technique making fifteen 1Hz observations at each station. These GPSBMs are identified in Figure 3.1.1 relative to the GPSBMs which were used by NGS in the GEOID03 and GEOID09 models. As can be seen, most of the 50 ODOT GPSBMs are different from those used by the hybrid geoid models. They were specifically chosen by ODOT to ensure that an independent set of GPSBMs could be used to validate the **true** performance of the hybrid models at those positions.

In order to evaluate the performance of GEOID03 at the VRS-observed GPSBMs, "GPS-leveled" orthometric heights (H_{VRS}) were computed by using the GEOID09 heights ($N_{GEOID09}$) where:

$H_{VRS} = h_{VRS} - N_{GEOID09}$

These VRS orthometric heights (H_{VRS}) were compared to the NGS published heights (H_{NGS}) and their discrepancies computed according to:

$\Delta_H = H_{VRS} - H_{NGS}$

The geometric geoid undulation (N_{VRS}) was also computed according to:

$N_{VRS} = h_{VRS} - H_{NGS}$

Finally, the discrepancies of N_{VRS} relative the geoid heights of GEOID03 and GEOID09 were computed. These results are documented in Section 4.1.

The reader is directed to Appendix A for plots and summaries of GEOID03 and GEOID09 and their GPSBM fit.

Table 3.1.2: GPS Benchmark Use in hybrid geoid models

Dataset	NGS Model	Number of GPSBMs used
1	Both GEOID09 and GEOID03	155
2	GEOID09 only	38
3	GEOID03 only	96
4	Rejected from both GEOID09 and GEOID03	5

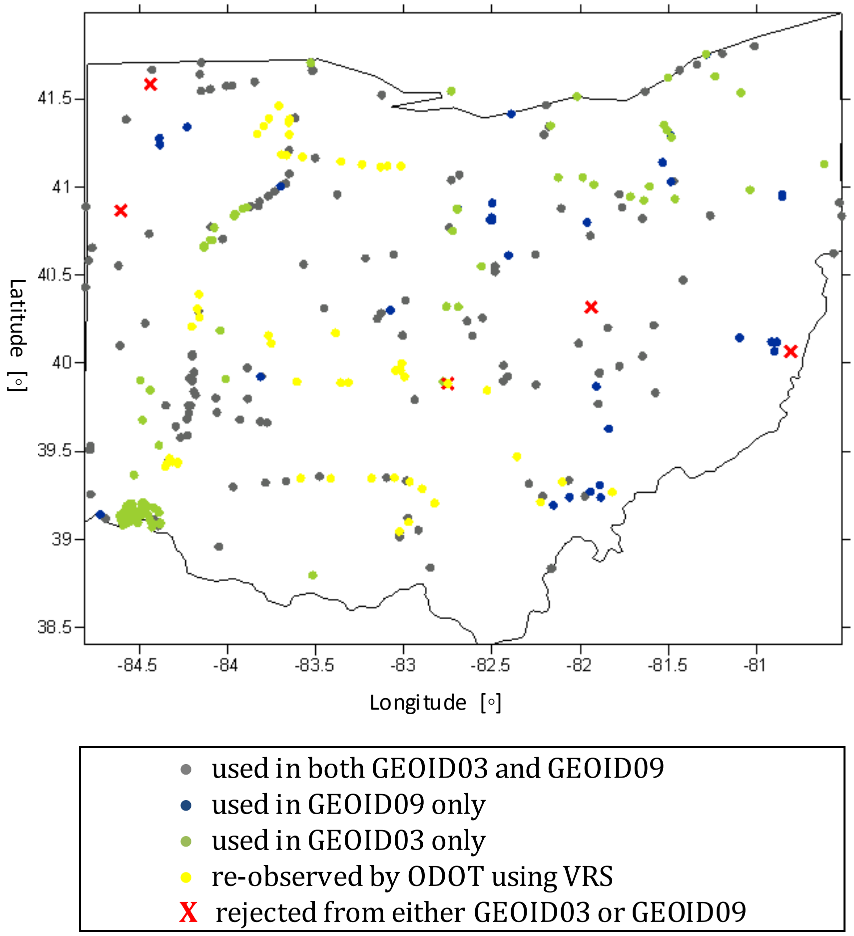


Figure 3.1.1: Distribution of GPSBMs used in GEOID09 and GEOID03 and those re-observed by ODOT using 15 second VRS GPS

3.2 Gravimetric geoid development for Ohio using publicly-available data

The first stage of NGS hybrid geoid development is the formulation of a gravimetric geoid solution. Given the public availability of both gravity and the GTOPO30 digital elevation model, the SPIN Lab opted to develop a gravimetric geoid over the Ohio region. The attempt at the relative gravity quality control evaluation and the process of geoid development are summarized below. Results pertaining to this section can be found in Section 4.2.

3.2.1 Comparison of Relative Gravity in the vicinity of Absolute Gravity Data

Relative Gravity Data Set

Approximately 43150 spot gravity data throughout Ohio and its environs (cf. Fig. 3.2.1a) were downloaded from the Pan American Center for Earth and Environmental Studies (PACES) GeoNet gravity database[1]. When this study was initiated in November 2006 the GEONET database was said to be the most up-to-date gravity data set available for the USA (Roman, 2007, e-mail communication).

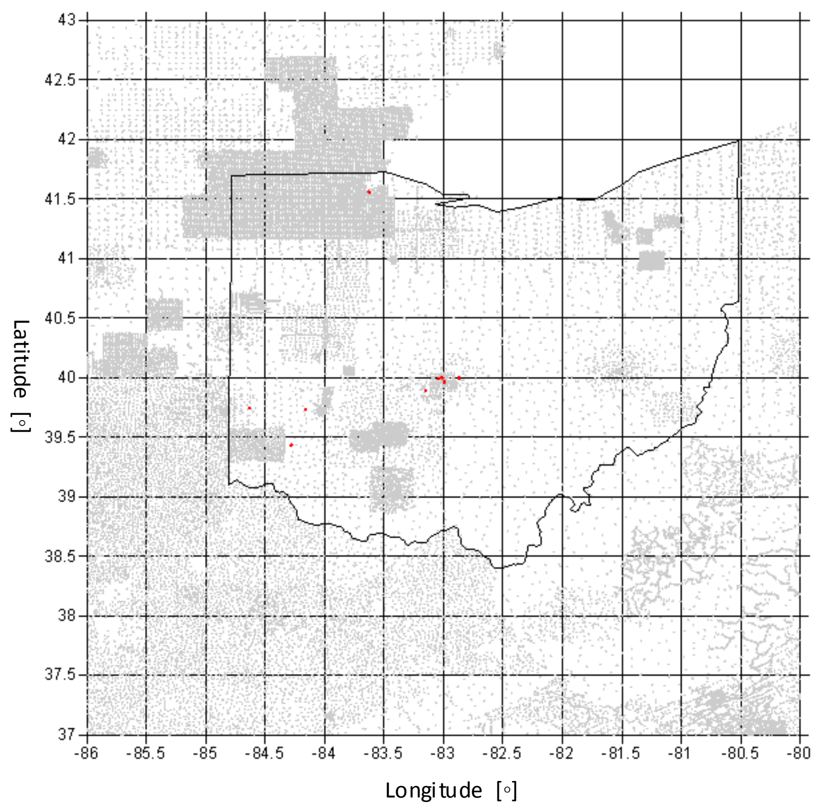


Figure 3.2.1a: Distribution of GEONET relative gravity

As can be seen from Fig. 3.2.1a there is no marine or altimetry-derived gravity data over the Lake Erie region. Furthermore, it is clear that the data distribution is not spatially-homogeneous. In fact, some areas evidence gravity data density which is as low as 1 point per 4km^2 to as many as 10 points per km^2 (such as in the north-west and south-west corners of the state of Ohio). Given the gravity data density, the gravimetric geoid was computed with 5′ x 5′ grid resolution. This means

[1] http://paces.geo.utep.edu/gdrp/

that there will be a few cells (cf. Figs. B.1 and B.2), even in the non-lake region, that contain zero gravity data points. In such a case, the gravity implied by the EGM2008 model (described in Section 3.2.2) will automatically be used in these empty cells. The reader is directed to Appendix B for information about the gravity density per grid cell.

According to NGS (2007) the typically-requested level of accuracy for relative gravity ranges from about 0.1mGal to 1mGal. However, there is no indication in the GEONET database as to the type of relative gravity meters that were used or the accuracy of the data – the data accuracy remains unknown.

Absolute Gravity Data stations

As can be seen from Figure 3.2.1b absolute gravity measurements at about 6 locations in Ohio were obtained. Dan Winester (a gravity expert at NGS) made measurements at Bolton and Perrysburg in 1986 and 1987 respectively. During 2005, he observed and compiled absolute gravity measurements at 4 other sites (Columbus, Lebanon, Dayton and Eaton) in Ohio with error estimates on the order of 1 – 5 μGal.

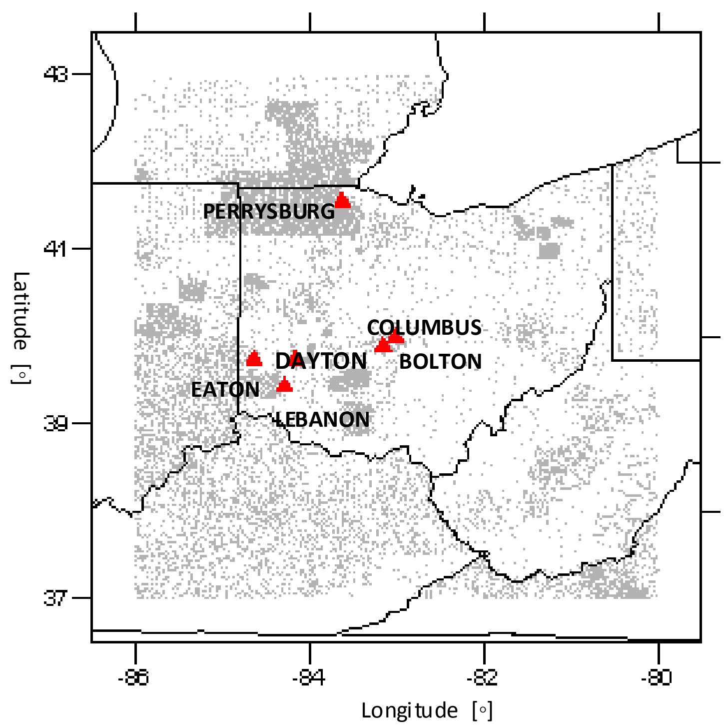


Figure 3.2.1b: Distribution of absolute gravity stations relative to the GEONET data.

Description of Relative Gravity Comparison Study

Given that no information about the quality of the relative gravity data was presented in the GEONET database, an attempt was made to evaluate the quality of the relative gravity **in the vicinity** of absolute gravity stations. This comparative study was undertaken with the understanding that the absolute gravity measurements are several orders of magnitude more precise than those of the relative gravity – a reasonable assumption.

In this regard, a 30´ x 30´ region of relative gravity data around each absolute gravity station was extracted from the GEONET gravity data set and their height and gravity values were interpolated to the single absolute gravity position denoted by (ϕ_{abs} ,λ_{abs} ,H_{abs}) where H_{abs} is the MSL elevation of the absolute gravity station. To gravimetrically-compensate for the difference between the interpolated height and H_{abs} the typically-used **free air** radial gravity gradient[2] estimated at about -0.3086 mGal / m (hereinafter referred to as $\frac{\partial g}{\partial H}$) was applied to the gravity data.

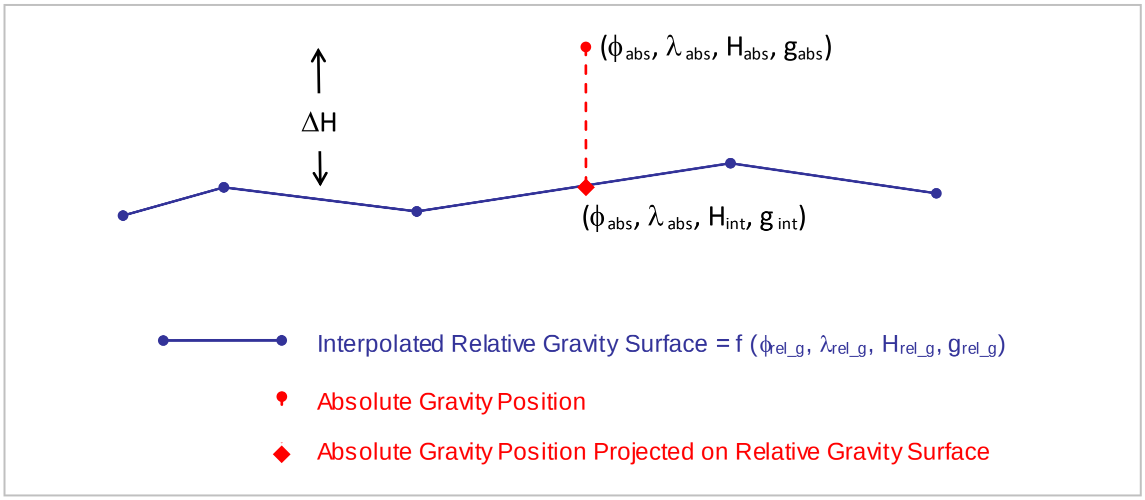


Figure 3.2.2: Relationship between Interpolated Relative Gravity and Absolute Gravity

Therefore, a gravimetric discrepancy was computed using the following steps:

1. Evaluation of the interpolated height, $H_{int} = H_{int}(\phi_{abs}, \lambda_{abs})$ based on interpolation of all the elevations within the 30´ x 30´ relative gravity dataset to the absolute gravity station position (ϕ_{abs} ,λ_{abs}). The height difference ΔH will be used with the gravity gradient to compute a comparison gravimetric value. Note that the height difference is defined by: $\Delta H = H_{int} - H_{abs}$
2. Evaluation of the interpolated gravity value $g_{int} = g_{int}(\phi_{abs}, \lambda_{abs}, H_{int})$ based on interpolation of all the gravity values within the 30´ x 30´ region to (ϕ_{abs} ,λ_{abs}).
3. Gravimetric compensation based on ΔH using the free air gradient:
$$g_{comp} = g_{\text{int}} + \frac{\partial g}{\partial H} \times \Delta H$$
4. Comparison of the latter with the gravity value of the absolute station to render a discrepancy according to:
$$g_{\Delta} = g_{abs} - g_{comp}$$

The results of the comparative relative / absolute gravity study can be found in Section 4.2.1 of this report. A list of all the absolute gravity stations (including excenters) which were used in this study can be found in Table C.1 (see page 48).

[2] This gradient estimate is valid in air not in media (such as water or soil) whose densities are different.

3.2.2 "Remove-Restore" Gravimetric Geoid Development

Fig. 3.2.3 summarises the process involved in computing a gravimetric geoid using input data of:

- a global gravity potential model
- point gravity data (described above in Section 3.2.1)
- a digital elevation model (DEM), specifically the GTOPO30 model

Earth Gravitational Model 2008 (EGM2008)
Gravity attraction is a global (not a local) phenomenon. Theoretically, geoid determination over even the smallest region of the Earth, requires that gravity for the entire Earth be known. Therefore, an underlying assumption of the geoid computation process is that a global and continuous gravity data set is needed. This, of course, is an invalid assumption. There are vast regions of the earth where gravity observations have never been made or are not available for public use. As was seen in Fig 3.2.1a, in Ohio alone there are several regions where there is absolutely no gravity data.

In lieu of this, a global gravity data set is "simulated" by using what is called a global geopotential model such as Earth Gravitational Model 2008 (EGM2008). EGM2008 is a model of the Earth's gravitational potential field which was computed by Pavlis et al. (2008) of National Geospatial Intelligence Agency (NGiA). It was developed from various global data sets including (terrestrial gravity data, altimetry-derived gravity data, a global DEM, just to name a few).

EGM2008 is a spherical harmonic model computed to degree 2159 (with additional spherical harmonic coefficients up to degree 2190 and order 2159) (ibid.). The Earth's gravitational potential, V, at some point (r, θ, λ) on or above the surface of the Earth can be approximated by:

$$V(r,\theta,\lambda) \approx \sum_{n=0}^{n\max}\left(\frac{1}{r^{n+1}}\right)\sum_{m=0}^{\eta}\left(\bar{s}_{nm}\bar{P}_{nm}(\cos\theta)\sin m\lambda + \bar{c}_{nm}\bar{P}_{nm}(\cos\theta)\cos m\lambda\right) \qquad [3.2.1]$$

$$\text{for } n\max = 2190 \quad \text{and} \quad m\max = 2159 \quad \begin{cases} n \le 2159, \eta = n \\ 2159 < n \le 2190, \eta = 2159 \end{cases}$$

where:

- r, θ, λ are the spherical coordinates; specifically r is the radial distance, θ is the co-latitude[3] and λ is the longitude
- $\bar{P}_{nm}$ is the fully-normalized Legendre function of degree and order n and m, respectively.
- $\bar{s}_{nm}$ and $\bar{c}_{nm}$ are the constants which are used to express the gravitational potential of the Earth, also called fully-normalized spherical harmonic coefficients of degree and order n and m, respectively.

The EGM2008 WGS84-referenced geoid undulations (or more accurately, height anomalies) and gravity anomalies can be derived from the potential model described by equation [3.2.1]. By using EGM2008, the gravity in "empty" 5′ x 5′ gravity cells (alluded to in Sec 3.2.1) will automatically be assigned the EGM2008-derived gravity values.

[3] The co-latitude is the angle complement of the latitude ϕ.

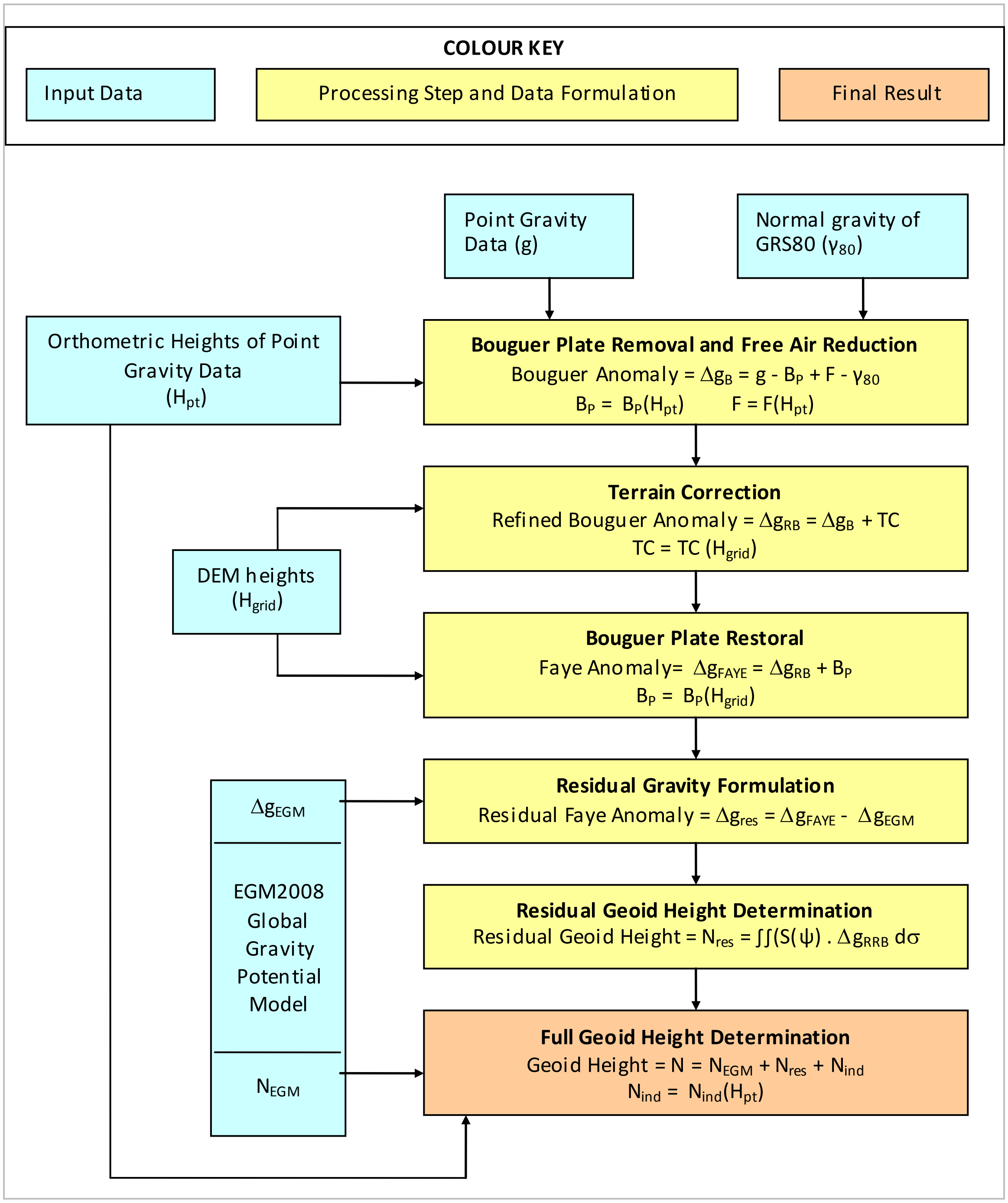


Figure 3.2.3: Summary of the Geoid Computation Process

It is worth noting that the maximum degree and order of EGM2008 imply that the model has a spatial resolution better than about 0.083° (≈ 9km). This essentially means that, the smallest features of gravitational potential represented by EGM2008 are about 9km. This is about 5 times better than the resolution of the previous geopotential model EGM96 (Lemoine et al., 1998). Figure 3.2.4 shows EGM2008 height anomalies over Ohio. This will be used as the medium-to-long wavelength foundation of the gravimetric geoid computed by the SPIN Lab. It was also used by NGS in the computation of GEOID09 (Roman et al, 2009).

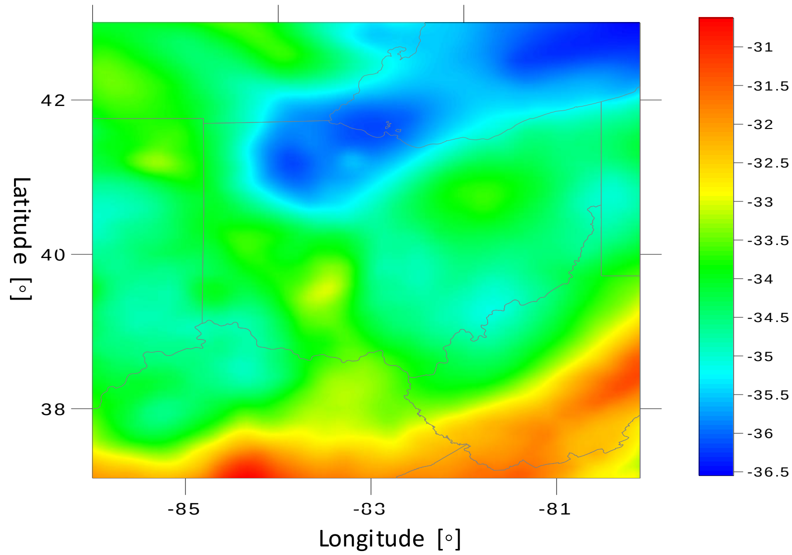


Figure 3.2.4: EGM2008 Height Anomalies over Ohio and environs
Colorbar in units of meters

Gravity Reductions

Because EGM2008 is a global model, it is not sufficiently high resolution to capture the high frequency gravimetric signature of the Earth. To account for the components of the gravimetric features which EGM2008 is unable to capture, observed gravity data must be used in conjunction with height data and subjected to gravity reductions (described below).

As pertains to these gravity reductions, some underlying assumptions which support geoid computation theory are:

1. All masses of the Earth lie within the boundary of the geoid.
2. The density outside the geoid surface is zero.
3. Gravity **on** the geoid surface is physically-accessible and therefore observable.

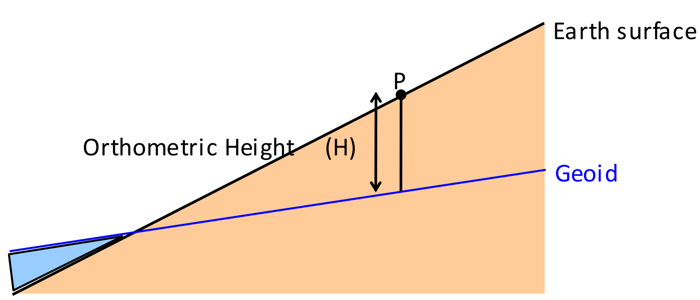


Figure 3.2.5: Possible Geoid / Earth surface configuration

As can be seen from Figure 3.2.5 none of these assumptions is universally-valid. As regions of the geoid are indeed known to exist below the surface of the earth, (1) the density outside of the geoid surface can not possibly be zero and (2) gravity observation **on** the surface of the geoid is not possible. This means that the gravity observed at the surface of the earth must be subjected to a series of gravity reduction processes (two of which are Bouguer plate removal and application of the free air correction to the gravity observed at the surface of the Earth) which help to simulate the scenario which supports the aforementioned assumptions.

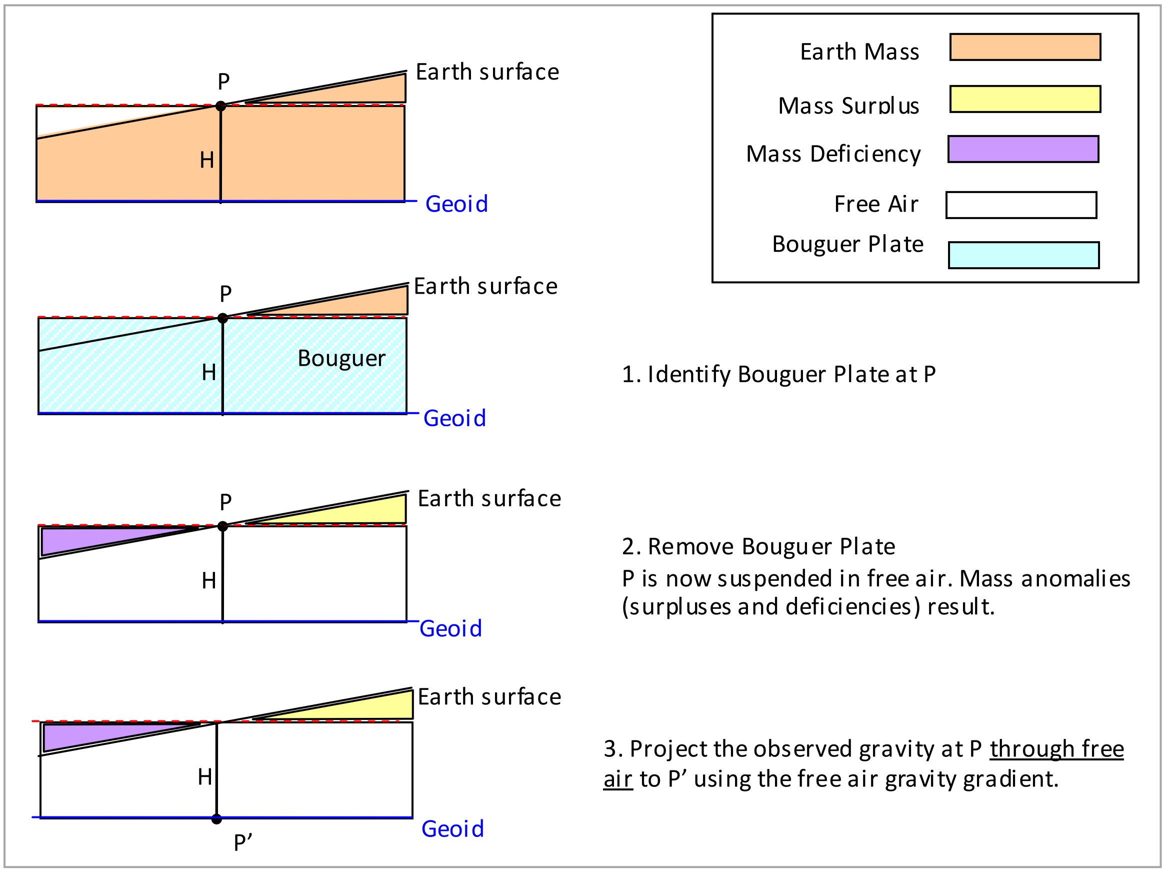


Figure 3.2.6 Summary of the Bouguer Plate Removal and Free Air Reduction

Bouguer Plate Removal and Free Air Reduction

The Bouguer plate at the point P is defined as an imaginary cylinder of constant density, ρ, of infinite radius and of height equal to the orthometric height of the point P (referred to as H). It represents a "slab" of the Earth between the geoid and the surface of the earth. Its gravitational attraction, B_P, is given by:

$$B_P = 2\pi G\rho H$$

where G is the gravitational constant.

This Bouguer plate "slab" is removed from between the point P and the geoid surface leaving the point P suspended in "free air". The gravity at P must be reduced to the point P' on the surface of the geoid based on a free air normal gravity gradient, $\frac{\partial \gamma}{\partial H}$:

$$\frac{\partial \gamma}{\partial H} = -0.3086 \quad \left[mGal\, m^{-1}\right]$$

This downward continuation of the point P through free air is called the free air reduction whose free air correction (F) is defined as:

$$F \approx -\frac{\partial \gamma}{\partial H} H \approx 0.3086 H$$

where γ is the normal (symmetric) gravity generated by a best-fitting Earth ellipsoid of homogeneous mass distribution.

The result of Bouguer plate removal and free air correction (summarized in Fig. 3.2.6) is the Bouguer anomaly, Δg_B which is defined as:

$$\Delta g_B = g - B_P + F - \gamma$$

where g is the observed gravity (which was described in Section 3.2.1 above).

$$TC = G\rho \int_{-\infty}^{\infty}\int \int_{h_P}^{h} \frac{z - h_P}{r^3} dxdydz$$

where G is the gravitational constant, ρ is the mass density, (x_P, y_P, h_P) refers to position of the computation point P, (x, y, z) refers to the integration point and:

$$r^2 = (x_P - x)^2 + (y_P - y)^2 + (h_P - z)^2 .$$

Given that Ohioan topography is predominantly flat, the approximation:

y

can be used, giving rise to the planar approximation (a convolution integral) of the terrain correction:

$$TC \approx \frac{G\rho\, dx\, dy}{2} \sum_{x=x_{\min}}^{x_{\max}} \sum_{y=y_{\min}}^{y_{\max}} \frac{(h - h_P)^2}{r_0^3} .$$

Therefore, in keeping with common practice, the local terrain corrections were computed using a 2D FFT algorithm (Forsberg 1985 and 1997). The terrain correction result and its impact on the gravimetric geoid solution can be found in Section 4.2.2.

Having computed the terrain corrections (depicted in Figure 3.2.7), the terrain-corrected Bouguer anomaly (Δg_{RB}), also known as the **Refined Bouguer anomaly**, can be computed using:

$$\Delta g_{RB} = g - B_P + F + TC - \gamma$$

The refined Bouguer anomaly is not considered suitable for geoid computations because it is large and strongly-correlated with height. Therefore, one more gravity reduction procedure is performed to produce the so-called **Faye Anomaly**. This is formed by restoring the Bouguer Plate as a condensed infinitesimally-thin mass on the surface of the geoid. The Faye anomaly is defined as:

$$\Delta g_{FAYE} = \Delta g_{RB} + B_P'$$

Notice that here B_P' is used for the Bouguer Plate **restoral** to distinguish it from B_P which is used for the Bouguer Plate **removal**. Also worth noting is the fact that the Faye anomaly now represents a gravimetric measurement which makes the 3 aforementioned assumptions plausible. Notably, other terrain reduction procedures exist (cf. Heiskanen and Moritz, 1967) which are comparable with the aforementioned. However, for the purposes of this study, the methodology used by NGS for GEOID03 determination was followed as closely as possible (cf. Smith and Roman, 2001, http://www.ngs.noaa.gov/GEOID/GEOID03/tech.html).

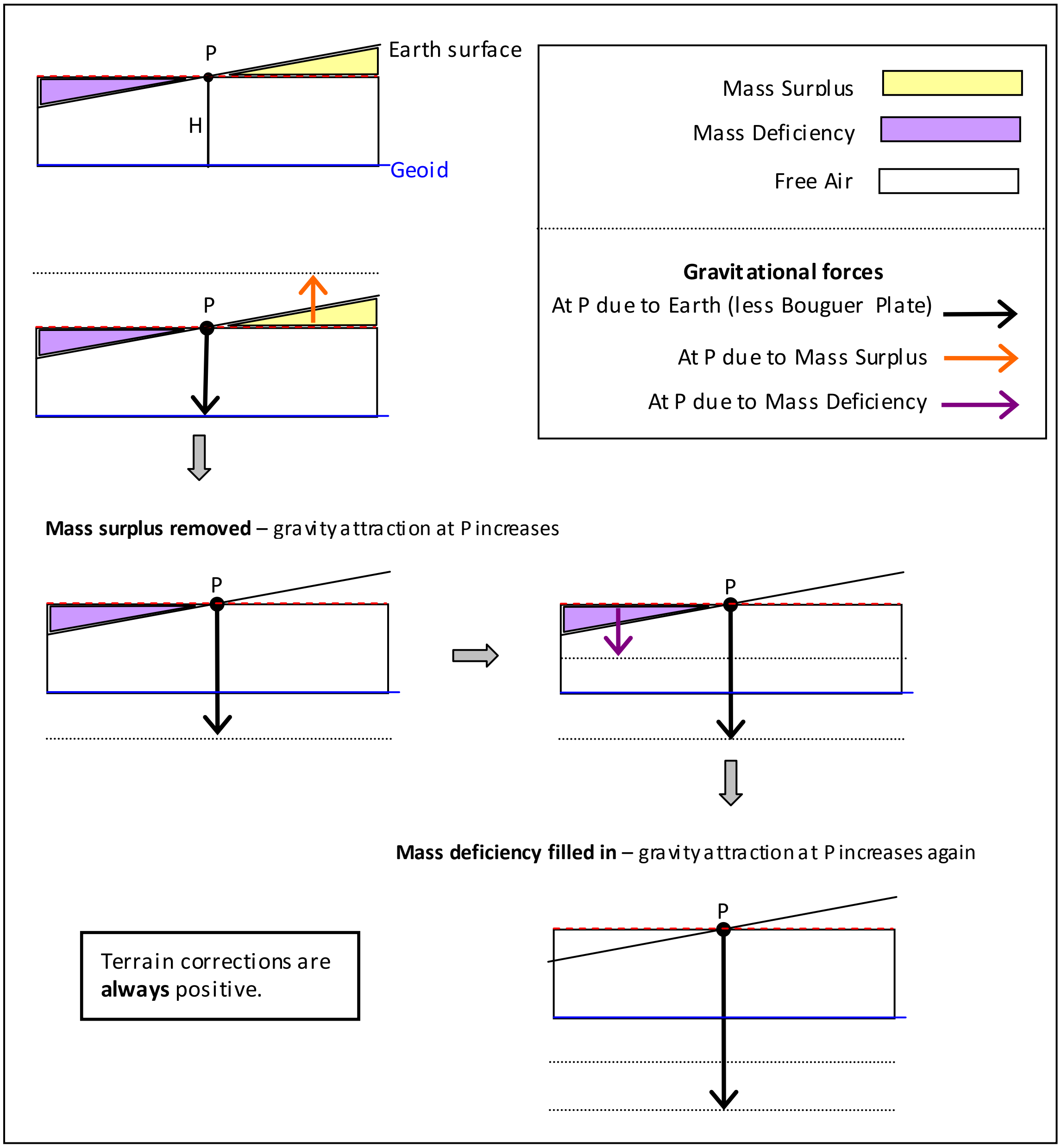


Figure: 3.2.7 Effect of the terrain correction on the gravimetric attraction at P due to the mass of the Earth.

Classical Terrain Correction

To account for the gravimetric effect caused by the mass anomalies or "residual" topography which would have resulted from the Bouguer plate removal, the classical terrain correction (with planar approximation) was applied. This "residual" topography (depicted in Figure 3.2.7) consists of mass surpluses which must be removed and / or mass deficiencies which must be filled in. In both cases, the terrain correction is positive.

The classical terrain correction (TC) is defined as:

GTOPO30

The terrain correction differs from the Bouguer and Free Air corrections in that it is dependent on heights over a region, as opposed to a single point. Therefore the orthometric heights for the terrain correction computations were obtained using the 30″ x 30″ GTOPO30 digital elevation model (DEM). DEM heights in Ohio and its environs vary from about 0 to 2000m with an average elevation of 294.109m ± 186.894m. When compared to the orthometric heights associated with the PACES gravity data (Section 3.2.1) GTOPO30's nominal RMSE of ±18m was validated, the average difference between the two being 2.973m ± 19.940m (1σ). Notably, no bathymetry in the Great Lakes region was used. By being used for the terrain reductions, GTOPO30 (like the gravity data set) provides short-to-medium wavelength gravimetric features to the geoid solution. GTOPO30 over the region of Ohio is shown in Fig 3.2.8.

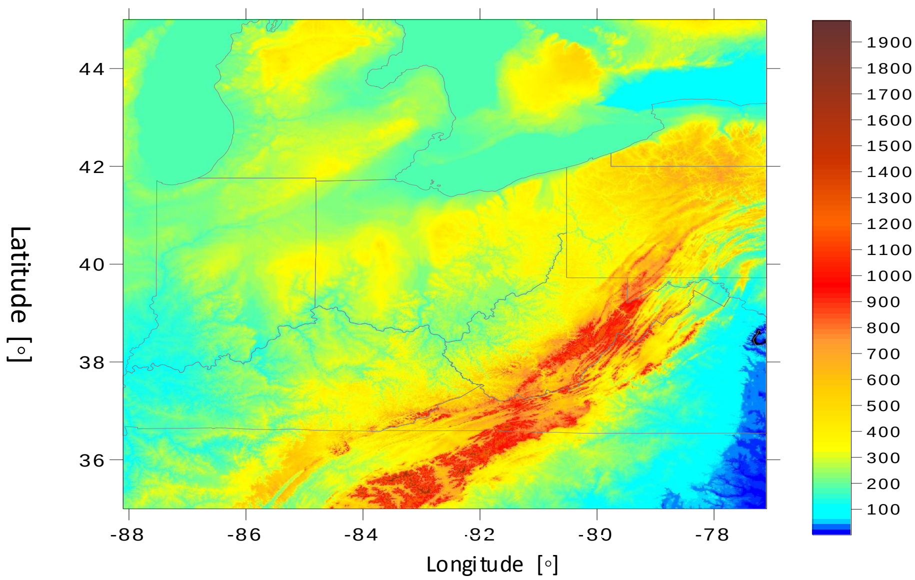


Figure3.2.8: GTOPO30 over Ohio and surrounding states
Colorbar in units of meters

The Stokes' Integral and Gravimetric Geoid Determination

A pivotal part of the "remove-restore" geoid computation process involves removal of the medium-to-long wavelength contribution of the global gravity data set simulated by the aforementioned global geopotential model (EGM2008). The residual Faye anomaly (Δg_{res}) is computed according to:

$\Delta g_{res} = \Delta g_{FAYE} - \Delta g_{EGM}$

where Δg_{EGM} is the gravity anomaly derived from EGM2008.

It is this residual gravity anomaly which is used in the Stokes' Integral to derive a "partial" geoid undulation N_{res}:

$$N_{res} = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g_{res} S(\psi) d\sigma \approx \frac{R}{4\pi\gamma} \sum\sum_{\sigma} \left(\Delta g_{res} S(\psi) d\sigma \right)$$

where R is the earth's radius and γ is the normal gravity of the GRS80 ellipsoid. S(ψ) is the Stokes function which essentially "weights" the gravity anomaly (Δg_{res}) inversely to the spherical distance, ψ, between the computation and integration points. The Stokes' Integral was evaluated using the well-known 1D FFT technique of Haagmans et al. (1993).

The Indirect Effect

Thus far, the terrain reductions (implicitly contained in the Faye anomaly) have been performed by effecting theoretical re-distribution of the Earth's masses. However, the Earth's mass distribution as it existed prior to these gravimetric reductions must be restored. This will inevitably have an effect on the gravity field, and by extension, the geoid undulation. Therefore, another correction needs to be made to ensure that the computed geoid undulation reflects the actual mass and mass distribution of the Earth. This correction is called the indirect effect and it is defined as:

$$N_{IE} = -\frac{\pi G \rho H^2}{\gamma}$$

The Gravimetric Geoid Undulation

Ultimately, the geoid undulation can be considered as the summation of:

- a medium-to-long wavelength component due to the global potential model EGM2008 restored in the form of N_{EGM}
- the residual undulation (N_{res}) due to the terrain and local gravity data which contribute the short and medium wavelength gravimetric characteristics of the field and
- the indirect effect (N_{IE}) which results from the restoral of the Earth's masses to their state prior to application of all the gravity and terrain reductions.

Therefore the geoid undulation can finally be defined and computed as:

$N = N_{EGM} + N_{res} + N_{IE}$

The final result of the gravimetric geoid computation can be found in Section 4.2.3 and serves as the reference geoid solution (referred to as M_0) in the random error study described in Section 3.3.

Table 3.2.1 summarises the underlying assumptions of the gravimetric geoid process and the means by which these assumptions are compensated for.

All algorithms developed to perform the gravimetric geoid computations were developed using MATLAB.

Table 3.2.1: Assumptions which Underlie Gravimetric Geoid Computations

ASSUMPTIONS	FACTS	CONCESSIONS
A global continuous gravity data set is available	This dataset is non-existent. Regional data sets of discrete gravity data are available in many areas	Simulation of a global gravity field using a global potential model (e.g. EGM96 or the more recent EGM2008)
Gravity values on the geoid are known or estimable The geoid is an accessible surface outside of which no masses exist	Gravity is observed on or above the surface of the earth The "on-land" geoid can be found by leveling It is difficult to access in marine environments Masses exist outside the geoid surface	Terrain reductions must be performed. These include: Bouguer Plate removal Terrain corrections Free Air corrections Redistribution of the earth's masses using a suitable theory (e.g. Isostatic compensation, Helmert's 2nd Method of Condensation) Application of the indirect effect for mass restoral
Gravity potential and mass of both the geoid and reference ellipsoid are the same	The NAD83 ellipsoid mass and normal potential vary from that of the mean earth ellipsoid	Computation of the N_0 term

3.3 Random error influence on gravimetric geoid solution

In order to determine the level of influence of the gravity and height information on the gravimetric geoid computation, a study was conducted in which random errors were imposed on the GTOPO30 gridded elevations and on the spot gravity (and their associated heights). Zero mean (1σ) Gaussian-distributed uncorrelated random errors were applied to each of these data sets.

The errors imposed on the PACES spot gravity and corresponding height data (referred to as σ_g and σ_{Hspot} respectively) were intuitively-assigned because the data downloaded from the PACES website did not include information concerning the type of gravimeter used, date of observation or the station heighting technique used. σ_g ranged from 0.5mGal – 5.0mGal while σ_{Hspot} ranged from 5 – 20m. On the other hand, the GTOPO30 model has a nominal RMSE of ±18m. Therefore the errors (referred to as σ_{Hgrid}) of no more than ±20m were assigned to the GTOPO30 heights. The contribution of the EGM2008 geopotential model was assumed error-free.

For a chosen combination of σ_g, σ_{Hspot} and σ_{Hgrid} , 100 error-prone gravity and height data sets were created from which 100 error-prone gravimetric models (referred to as M_k for k = 1, 2, … , 100) were computed using the same processing stream as was used to produce the SPIN gravimetric geoid (cf. Section 4.2). The latter, referred to as M_0 for this study, was used as the reference solution relative to which the behavior of the other models was evaluated.

A mean error-prone geoid ($\overline{M}$) was computed from the 100 models evaluated for each combination of σ_g, σ_{Hspot} and σ_{Hgrid} where for each pixel (i,j):

$$\overline{M}(i,j) = \frac{1}{100}\sum_{k=1}^{100} M_k(i,j)$$

The differenced geoid grid ($\Delta\overline{M}$) of the mean error-imposed geoid relative to the reference solution was evaluated:

$$\Delta\overline{M}(i,j) = \overline{M}(i,j) - M_0(i,j)$$

Similarly, the rmse geoid grid (rmseM) of the 100 error-imposed geoids (M_k) relative to the reference solution (M_0) was evaluated:

$$rmseM(i,j) = \sqrt{\frac{\sum_{k=1}^{100}\left[M_k(i,j) - M_0(i,j)\right]^2}{100}}$$

Results pertaining to this aspect of the study are documented in Section 4.3.

4. Results: Findings of the research effort

4.1 Evaluation of hybrid geoid performance in Ohio at VRS-derived GPSBMs

H_{VRS}, Δ_H and N_{VRS} (as defined in Section 3.1) are all documented in Table 4.1.2. Δ_H gives an indication of the quality of orthometric height that would be obtained if the GEOID09 model were used with NAD83 heights to benchmark new stations in the vicinity of the ODOT GPSBMs. This suggests then that benchmarking precision (1σ level) would be about ±5cm, which clearly is not good enough for precise cm-accurate GPS heighting procedures.

Table 4.1.3 identifies how the NGS hybrid geoid values compare with N_{VRS}, the results of which are summarized in Table 4.1.1. As can be seen from Table 4.1.1 both GEOID09 and GEOID03 appear to agree with the geometric geoid undulations at about the ±5cm to ±6cm level. This means that if VRS techniques were to be used for GPS benchmarking, the precision of the derived orthometric heights will likely be **at least** about 5cm, being further-degraded by the precision of the ellipsoidal height from which it would also be derived.

Table 4.1.1: Summary of Geoid Height Discrepancies (Δ_H)

	$N_{GEOID09} - N_{VRS}$	$N_{GEOID03} - N_{VRS}$
Mean Δ_H [m]	-0.016	0.010
1σ STD Δ_H [m]	0.0520	0.058
Min Δ_H [m]	-0.213	-0.252
Max Δ_H [m]	0.076	0.119

That the absolute discrepancies for GEOID09 exceed as much as 10cm suggests that further checks need to be performed to identify the source of the discrepancy (which could be any combination of the hybrid geoid, itself or the GPS and NGS-published heights). While the stations' ellipsoidal heights were observed by ODOT, the orthometric heights were not re-levelled. It is possible that some of the NGS published heights are now invalid (due to monument displacement or other natural phenomena) but GEOID09 may also be weak in some of the areas where these discrepancies have been observed.

Figures 4.1.1a and 4.1.1b depict the spatial distribution of these discrepancies and may imply that some segments of some level lines appear to be weaker than other segments. The blue icons indicate all stations whose discrepancies fall within the ideal ±3cm range. Clearly, there are pockets of high performance in every area where ODOT has done its field work. However, it may still be useful to extend this study to cover a larger part of Ohio. It is also noteworthy that in the area of LEBANON AA (an absolute gravity station whose value differed from the interpolated relative gravity by about 3mGal) GEOID09 demonstrated strong height conversion performance suggesting that the relative gravity data which was used by NGS in this region is sound.

What is clear from Figures 4.1.1a and 4.1.1.b is that overall the performance of GEOID09 is better at the 50 ODOT GPSBMs than is GEOID03 but additional investigation are needed to determine how best to improve GEOID09 height conversion capability in Ohio.

Table 4.1.2 : Stations used as an independent check on the NGS hybrid models

Station Name	VRS NAD83 Coordinates				Published by NGS	Orthometric height discrepancy	Geometric Geoid Height
	φ_{VRS} [°]	λ_{VRS} [°]	h_{VRS} [m]	H_{VRS} [m]	H_{NGS} [m]	$\Delta_H = H_{VRS} - H_{GEOID09}$ [m]	N_{VRS} [m]
Q347	39.41377	-84.3512	211.221	244.912	244.892	0.020	-33.671
R347	39.42698	-84.3432	189.898	223.574	223.556	0.018	-33.658
WAR 63 AE002	39.44032	-84.3115	174.084	207.741	207.731	0.011	-33.647
WAR 63 AE005	39.43779	-84.2797	221.806	255.451	255.469	-0.018	-33.663
DIS GAR	39.43041	-84.2834	222.629	256.288	256.273	0.015	-33.644
V347	39.45848	-84.3265	166.989	200.628	200.623	0.005	-33.634
W171	41.17098	-83.5736	191.001	226.343	226.391	-0.048	-35.390
R171	41.18008	-83.6653	185.613	221.060	221.087	-0.027	-35.474
K 312	41.18286	-83.6934	183.909	219.384	219.417	-0.033	-35.507
A217	39.04506	-83.0253	144.663	178.042	178.016	0.026	-33.353
S310	39.09864	-82.9724	141.744	175.278	175.193	0.084	-33.449
A218	39.20554	-82.825	147.147	181.009	180.869	0.140	-33.722
U113	39.28626	-82.8971	162.906	196.761	196.648	0.112	-33.743
Y310	39.3265	-82.9679	154.026	187.814	187.754	0.060	-33.728
J338	39.35035	-83.0531	169.579	203.183	203.171	0.012	-33.592
R338	39.34652	-83.1824	284.446	317.645	317.432	0.213	-32.986
K339	39.34614	-83.4137	244.820	277.496	277.439	0.058	-32.618
W339	39.34717	-83.5841	277.619	310.180	310.150	0.030	-32.531
M310	39.92276	-82.9956	193.769	227.623	227.653	-0.030	-33.883
RINGLE	39.99971	-83.0114	193.429	227.228	227.250	-0.023	-33.822
DRA2008	39.96008	-83.019	185.940	219.724	219.716	0.009	-33.776
CNTRLGAR	39.95901	-83.0454	183.719	217.426	217.451	-0.025	-33.732
ALBANY	39.21157	-82.226	200.273	234.404	234.416	-0.012	-34.143
N232	39.26858	-81.8189	147.325	181.751	181.670	0.081	-34.345
D76X	39.32562	-82.1036	167.845	202.066	202.019	0.047	-34.173

V68	39.46971	-82.3597	190.995	225.183	225.117	0.066	-34.122
Q190	39.84652	-82.5296	242.188	276.375	276.396	-0.022	-34.209
F191	39.88378	-82.7508	221.925	256.079	256.097	-0.017	-34.171
S33	39.88973	-83.3138	267.374	300.244	300.320	-0.077	-32.946
R33	39.88879	-83.3574	261.393	294.239	294.282	-0.043	-32.889
H34	39.89482	-83.6047	326.105	359.065	359.080	-0.015	-32.975
T34 RESET	40.15657	-83.7668	284.929	318.385	318.456	-0.070	-33.526
V37	40.17118	-83.3873	271.792	305.201	305.191	0.011	-33.399
Z311	41.11227	-83.1312	202.520	237.551	237.571	-0.020	-35.051
W311	41.11702	-83.0163	232.227	267.263	267.246	0.017	-35.019
M173	41.11826	-83.0943	213.989	249.020	249.040	-0.020	-35.052
R344	41.12726	-83.2375	198.497	233.525	233.486	0.038	-34.990
E312	41.14469	-83.3576	198.643	233.831	233.822	0.010	-35.179
C351	41.3857	-83.646	170.438	205.766	205.740	0.025	-35.302
C352	41.36571	-83.6527	177.321	212.664	212.644	0.020	-35.323
J351	41.29741	-83.6502	172.055	207.443	207.426	0.017	-35.371
P312	41.3002	-83.8302	174.801	210.335	210.315	0.019	-35.514
T116	41.34479	-83.7948	171.803	207.275	207.207	0.069	-35.404
S170	41.38681	-83.7642	170.332	205.744	205.667	0.077	-35.336
Z170	41.45944	-83.7088	167.117	202.437	202.400	0.037	-35.282
J165	40.20744	-84.2009	262.259	295.268	295.266	0.002	-33.008
J349	40.26137	-84.1581	282.695	315.761	315.786	-0.025	-33.091
K349	40.30941	-84.1704	288.748	321.873	321.866	0.006	-33.118
G350	40.39125	-84.1606	291.246	324.574	324.573	0.001	-33.327
WARD	40.11196	-83.7519	287.391	320.832	320.854	-0.023	-33.463

Table 4.1.3: Comparison of NGS hybrid geoid models and the VRS-derived geometric geoid undulation

Station Name	Geometric Geoid Height	GEOID09	GEOID03	Hybrid / Geometric geoid discrepancy	
	N_{VRS} [m]	$N_{GEOID09}$ [m]	$N_{GEOID03}$ [m]	$N_{GEOID09} - N_{VRS}$ [m]	$N_{GEOID03} - N_{VRS}$ [m]
Q347	-33.671	-33.691	-33.658	-0.020	0.013
R347	-33.658	-33.676	-33.64	-0.018	0.018
WAR 63 AE002	-33.647	-33.657	-33.612	-0.010	0.035
WAR 63 AE005	-33.663	-33.646	-33.598	0.017	0.065
DIS GAR	-33.644	-33.659	-33.613	-0.015	0.031
V347	-33.634	-33.639	-33.588	-0.005	0.046
W171	-35.390	-35.343	-35.326	0.047	0.064
R171	-35.474	-35.447	-35.422	0.027	0.052
K 312	-35.507	-35.475	-35.448	0.032	0.059
A217	-33.353	-33.378	-33.372	-0.025	-0.019
S310	-33.449	-33.533	-33.521	-0.084	-0.072
A218	-33.722	-33.862	-33.828	-0.140	-0.106
U113	-33.743	-33.855	-33.84	-0.112	-0.097
Y310	-33.728	-33.788	-33.793	-0.060	-0.065
J338	-33.592	-33.604	-33.634	-0.012	-0.042
R338	-32.986	-33.199	-33.238	-0.213	-0.252
K339	-32.618	-32.676	-32.687	-0.058	-0.069
W339	-32.531	-32.561	-32.543	-0.030	-0.012
M310	-33.883	-33.854	-33.859	0.029	0.024
RINGLE	-33.822	-33.798	-33.808	0.024	0.014
DRA2008	-33.776	-33.785	-33.793	-0.009	-0.017
CNTRLGAR	-33.732	-33.708	-33.715	0.024	0.017
ALBANY	-34.143	-34.131	-34.098	0.012	0.045
N232	-34.345	-34.426	-34.327	-0.081	0.018
D76X	-34.173	-34.221	-34.15	-0.048	0.023

V68	-34.122	-34.188	-34.091	-0.066	0.031
Q190	-34.209	-34.187	-34.176	0.022	0.033
F191	-34.171	-34.154	-34.144	0.017	0.027
S33	-32.946	-32.87	-32.868	0.076	0.078
R33	-32.889	-32.846	-32.834	0.043	0.055
H34	-32.975	-32.96	-32.925	0.015	0.050
T34 RESET	-33.526	-33.456	-33.407	0.070	0.119
V37	-33.399	-33.409	-33.368	-0.010	0.031
Z311	-35.051	-35.031	-35.028	0.020	0.023
W311	-35.019	-35.036	-35.036	-0.017	-0.017
M173	-35.052	-35.032	-35.031	0.020	0.021
R344	-34.990	-35.028	-35.021	-0.038	-0.031
E312	-35.179	-35.189	-35.171	-0.010	0.008
C351	-35.302	-35.328	-35.267	-0.026	0.035
C352	-35.323	-35.343	-35.282	-0.020	0.041
J351	-35.371	-35.388	-35.336	-0.017	0.035
P312	-35.514	-35.533	-35.48	-0.019	0.034
T116	-35.404	-35.473	-35.414	-0.069	-0.010
S170	-35.336	-35.413	-35.353	-0.077	-0.017
Z170	-35.282	-35.32	-35.261	-0.038	0.021
J165	-33.008	-33.009	-32.973	-0.001	0.035
J349	-33.091	-33.066	-33.042	0.025	0.049
K349	-33.118	-33.125	-33.107	-0.007	0.011
G350	-33.327	-33.328	-33.317	-0.001	0.010
WARD	-33.463	-33.441	-33.384	0.022	0.079

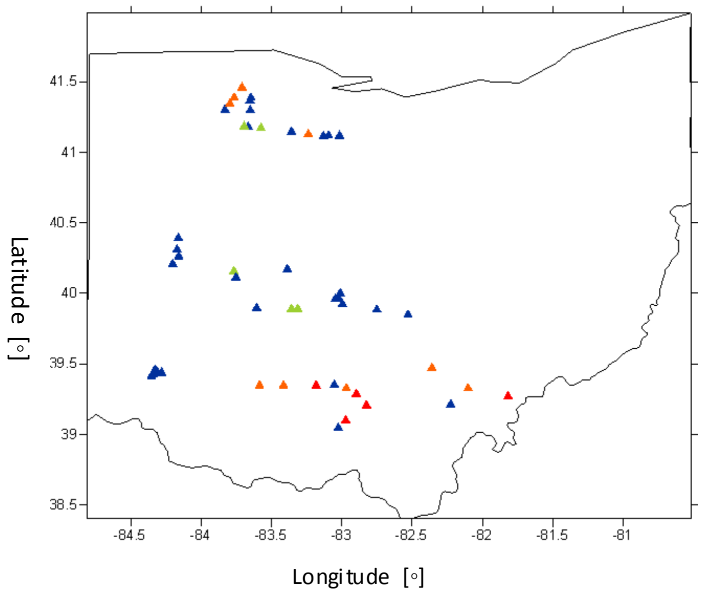
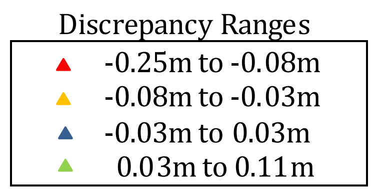


Figure 4.1.1a: GEOID09 discrepancy at ODOT VRS stations



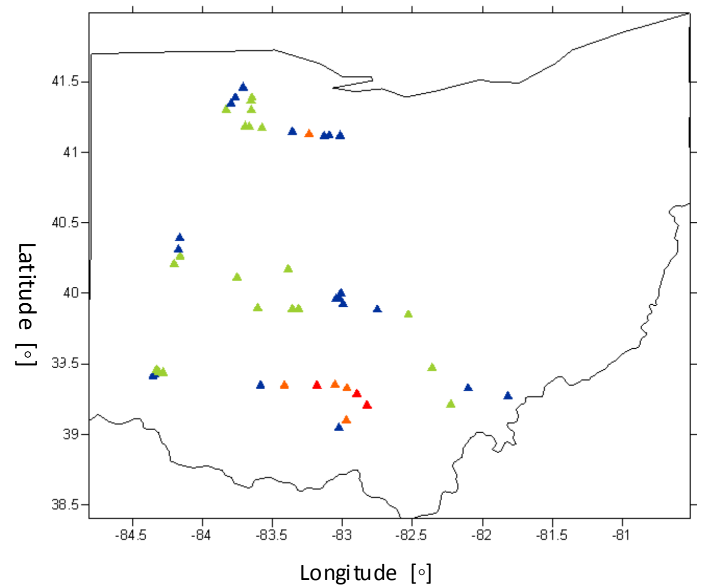


Figure 4.1.1b: GEOID03 discrepancy at ODOT VRS stations

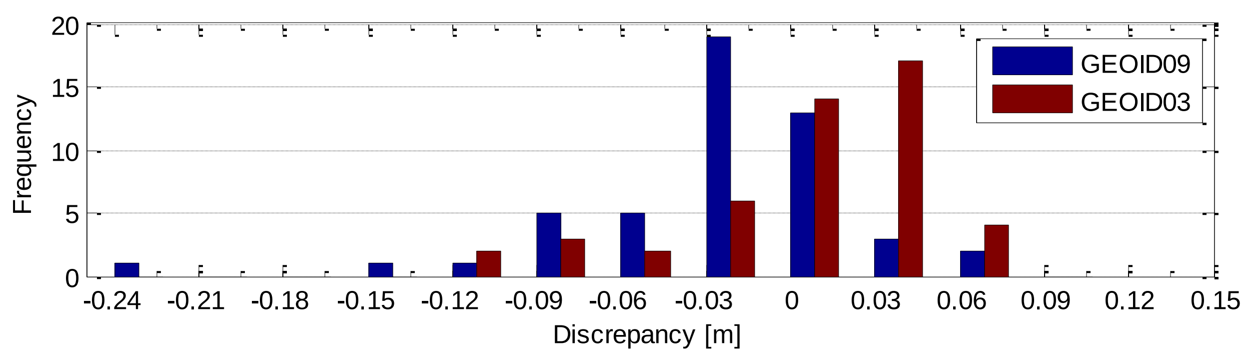


Figure 4.1.1c: Histogram of Discrepancies of the Geoid Heights inferred by ODOT VRS Techniques relative to GEOID09 and GEOID03

4.2 Gravimetric geoid over Ohio using publicly-available data

4.2.1 Absolute Relative Gravity Comparison

As mentioned previously, this study assumes that the absolute gravity measurements are several orders of magnitude more precise than those of the relative gravity. Hence the discrepancies between the relative and absolute gravity which are documented in Table 4.2.1. are likely to be indicative of the quality of the relative gravity in the vicinity of the absolute gravity station, rather than vice versa.

As can be seen the discrepancies at all, except Lebanon AA, are less than an absolute value of 0.6mGal, part of which must be attributable to the interpolation error. This suggests that the relative and absolute gravity agree to within the possible relative gravity accuracies generally desired by agencies such as NGS.

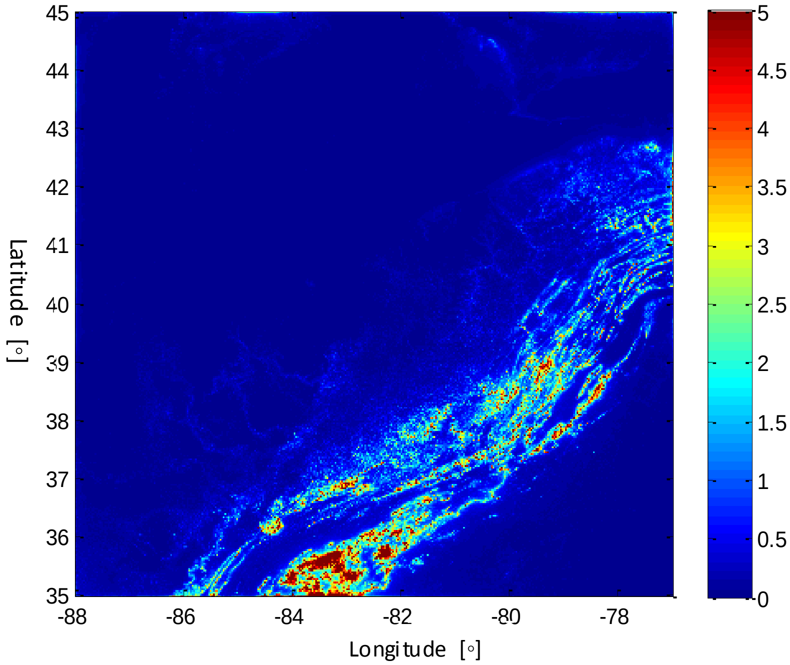
However, a significant discrepancy occurs at Lebanon AA. The authors have not been able to resolve it. It is also possible that the variable data density near Lebanon negatively impacted the interpolation process.

Table 4.2.1: Discrepancy between the interpolated relative gravity point and the its adjacent absolute gravity station

		Absolute Gravity Stn			Interpolated Gravity Stn		Discrepancy [mGal]
Gravity Station	Survey Date	H_{abs} [m]	g_{abs} [mGal]	N_{grav}	H_{int} [m]	g_{int} [mGal]	
COLUMBUS AA (131 CM)	07/02/2005	227.41	980079.8555	562	226.207	980080.5096	0.2828
DAYTON AA (131 CM)	06/24/2005	271.61	980065.1505	495	265.862	980066.3412	-0.5833
EATON AA (131 CM)	06/28/2005	317.53	980032.6446	466	320.077	980032.3405	0.4820
LEBANON AA (131 CM)	06/29/2005	257.51	980020.1823	317	222.599	980027.8398	**-3.1160**
BOLTON CAL BL 0	09/30/1987	272.57	980084.5400	560	272.099	980084.7327	0.0473
PERRYSBURG TT 16 WO	12/01/1986	192.397	980228.3590	1190	193.163	980228.1240	0.0014

4.2.2 GTOPO30 Terrain Corrections and the Gravimetric Geoid

Figure 4.2.1 summarizes the result of the 2D FFT terrain correction computation. As can be seen the terrain corrections are mostly sub-mGal – a testament to Ohio's extreme flatness. Understandably, in approaching the Appalachian mountain chain, the terrain corrections increase to as much as 30 mGal.

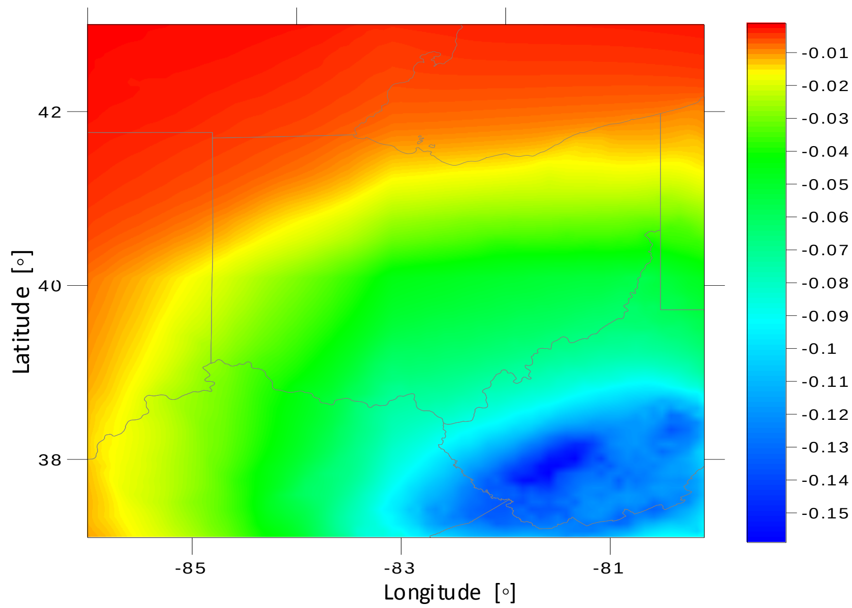


Mean [mGal]	STD [mGal]	Min [mGal]	Max [mGal]
0.290538	±0.724375	0.001024	30.608375

Figure 4.2.1: Terrain Corrections in Ohio and its environs
Colorbar in units of mGal

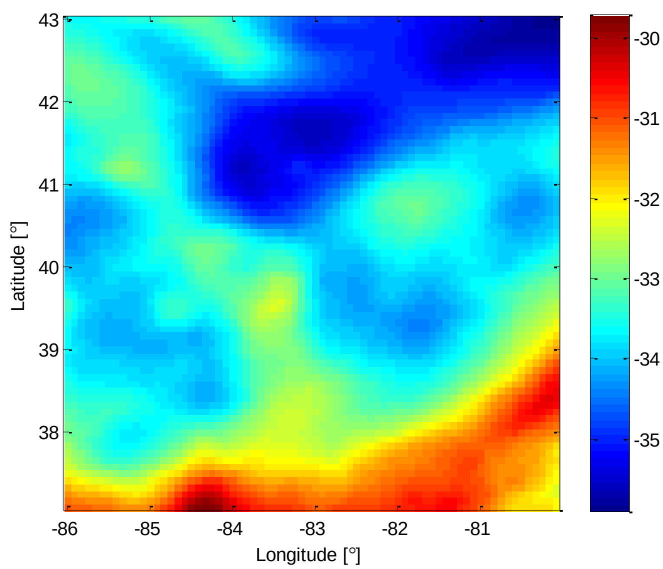
The difference between the terrain-corrected and non-terrain corrected local geoid solution is shown in Fig. 4.2.2. Note that in spite of Ohio's flatness, the neighboring Appalachian mountain range contributes a terrain effect which, in Ohio, ranges from as little as 1mm to about 7 cm (which is not negligible).

Figure 4.2.3 depicts the gravimetric geoid solution computed in this study. When compared to its EGM2008 foundation, it definitely evidences higher frequency height anomaly features which are attributable to the contributions of the GEONET relative gravity data set and the GTOPO30DEM. This is the model which was referred to as M_0 and used as the reference solution in the ensuing random error study (Section 4.3). Notice as well the similarity between the relative topography of the local gravimetric geoid and GEOID09 (shown in Figure 4.2.4) over the region. The difference between these two surfaces is shown in Figure 4.2.5. Therefore, conversions between the gravimetric geoid and GEOID09 at non-grid points can be effected by using a suitable interpolation technique and the surface in Figure 4.2.5.



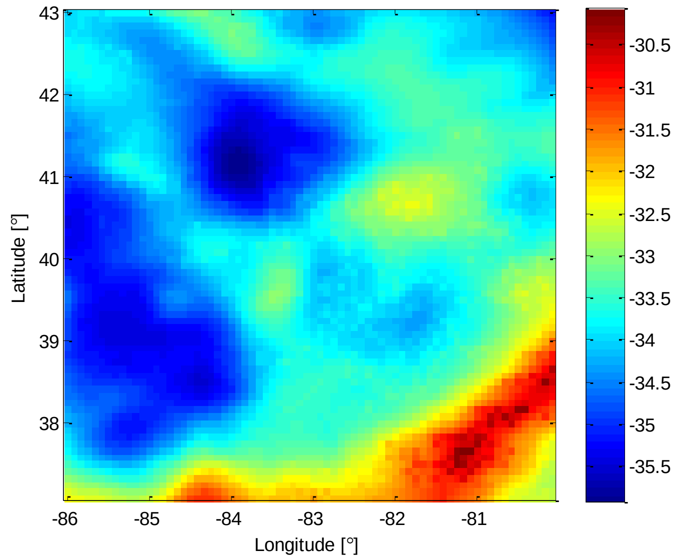
Mean [m]	STD [m]	Min [m]	Max [m]
-0.039	± 0.038	-0.001	-0.159

Figure 4.2.2: Influence of the Terrain Corrections on the local gravimetric geoid in Ohio and its environs Colorbar in units of meters



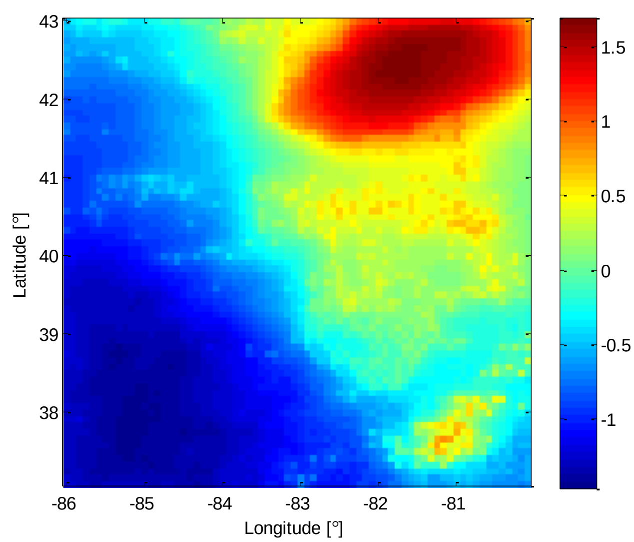
Mean [m]	STD [m]	Min [m]	Max [m]
-33.750	± 1.037	-35.929	-30.077

Figure 4.2.3: Gravimetric geoid in Ohio computed using publicly available data Colorbar in units of meters



Mean [m]	STD [m]	Min [m]	Max [m]
-33.506	± 1.182	-35.923	-29.715

Figure 4.2.4: GEOID09 heights in Ohio
Colorbar in units of meters



Mean [m]	STD [m]	Min [m]	Max [m]
-0.244	± 0.813	-1.466	1.699

Figure 4.2.5: Difference between the local gravimetric geoid and GEOID09
Colorbar in units of meters

4.3 Evaluation of the impact of imposed random errors on the gravimetric solution

Several combinations of σ_g, σ_{Hspot} and σ_{Hgrid} were applied to the gravity and GTOPO30 heights. Table 4.3.1 summarises the mean and STD of all the pixels in the differenced mean geoid grid ($\Delta\overline{M}$) for various error combinations. Similarly it identifies the mean and STD of all the pixels in the rmseM geoid grid. Error combinations were chosen heuristically.

Table 4.3.1: Summary of error influences on the gravimetric geoid based on various error combinations

	Standard Deviation of Applied Errors			$\Delta\overline{M}$		rmseM	
Model [4]	σ_{Hgrid} [m]	σ_{Hspot} [m]	σ_g [mGal]	Mean [m]	STD [m]	Mean [m]	STD [m]
	5	-	-	0.001	0.001	0.004	0.000
1	**10**	**-**	**-**	**0.004**	**0.002**	**0.010**	**0.001**
	15	-	-	0.009	0.002	0.016	0.002
	20	-	-	0.014	0.004	0.023	0.004
	-	1	-	0.000	0.000	0.001	0.000
	-	5	-	0.000	0.000	0.003	0.001
	-	10	-	0.000	0.001	0.007	0.002
2	**-**	**15**	**-**	**0.000**	**0.001**	**0.010**	**0.003**
	-	20	-	-0.001	0.001	0.013	0.004
	-	-	0.5	0.000	0.000	0.002	0.001
	-	-	1	0.000	0.000	0.003	0.001
3	**-**	**-**	**3**	**0.000**	**0.001**	**0.010**	**0.003**
	-	-	5	0.000	0.002	0.016	0.005
	15	10	-	0.009	0.003	0.018	0.002
	15	5	5	0.009	0.003	0.023	0.005
	15	3	5	0.009	0.003	0.024	0.005
	15	3	3	0.010	0.003	0.019	0.003
	10	3	2	0.004	0.002	0.012	0.002
	10	3	3	0.003	0.002	0.014	0.003
	10	**3**	**1**	**0.004**	**0.001**	**0.010**	**0.001**
4	**10**	**2**	**1**	**0.004**	**0.002**	**0.010**	**0.001**

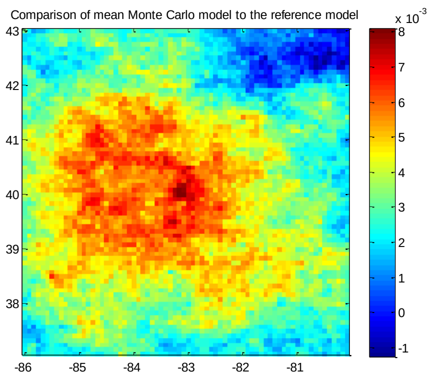
Ideally, one would hope that the errors do not introduce biases into the solution (as evidenced by $\Delta\overline{M}$) neither would one want the *spread* of the 100 error-prone geoid solutions (as evidenced by rmseM) to exceed more than 1 cm (in support of the cm-accurate gravimetric geoid goals expressed by NGS). As can be seen from this table:

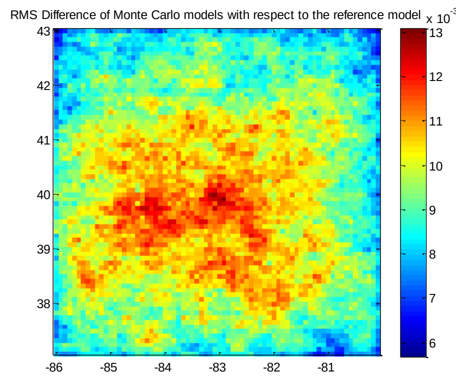
- Biases infect the solution when errors are applied to GTOPO30 elevations, not when errors are applied to the gravity. This bias exceeds, on average, 1cm when **only** GTOPO30 errors in excess of 15m are applied.
- The spread of the geoid solutions exceeds 1cm when

[4] These model numbers refer to the Figures ??? - ???

   - Only GTOPO30 height errors are applied and exceed 10m
   - Only gravity height errors are applied and exceed 15m
   - Only gravity errors are applied and exceed 3 mGal
- These can be considered as the magnitude of the errors which would determine whether one is able to successfully compute a cm-precise geoid.

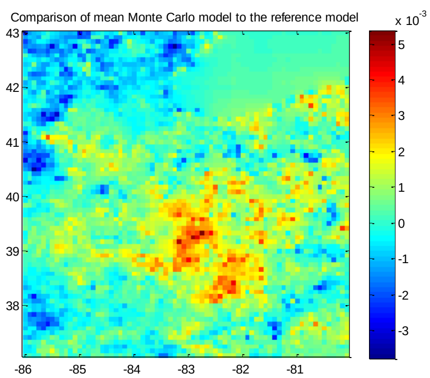
Figures 4.3.1 – 4.3.4 demonstrate the spatial impact of the applied errors. As can be seen from Fig. 4.3.1 the largest propagated GTOPO30 error evidences itself in the regions of lowest elevation (i.e. in the plains). This is most likely due to the fact that the ratio of GTOPO30 error relative to actual GTOPO30 height will be smaller in the areas of low elevation as opposed to areas of greater height (such as in the region of the West Virginian mountain chain). It is noteworthy that in Figures 4.3.2 and 4.3.3 which involve application of gravity and spot height errors, the areas of heightened rmse coincide with areas of sparse gravity data. Understandably, the areas over the lake region show the lowest and near-zero propagated error because of the absence of gravity data there.

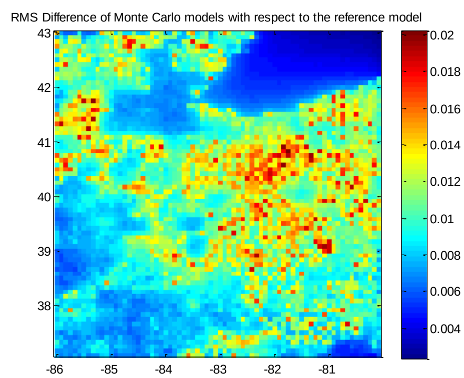




σ_{Hgrid} = ±10m, σ_{Hspot} = 0m, σ_g = 0 mGal

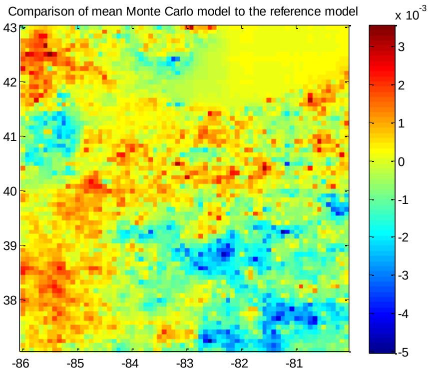
Fig 4.3.1a: Model 1 ΔMCG [m] **Fig 4.3.1b: Model 1 rmseMCG grids [m]**

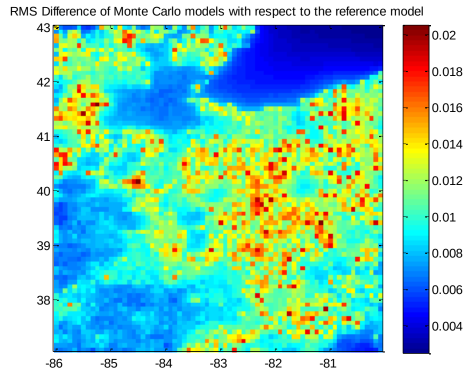




σ_{Hgrid} = 0m, σ_{Hspot} = ±15m, σ_g = 0 mGal

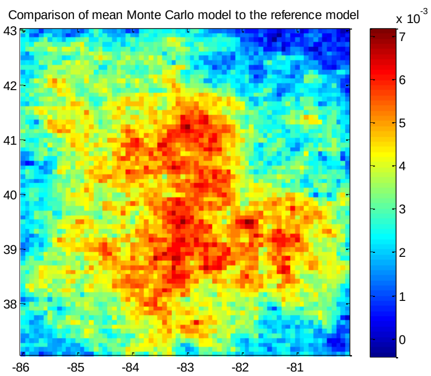
Fig 4.3.2a: Model 2 ΔMCG [m] **Fig 4.3.2b: Model 2 rmseMCG grids [m]**

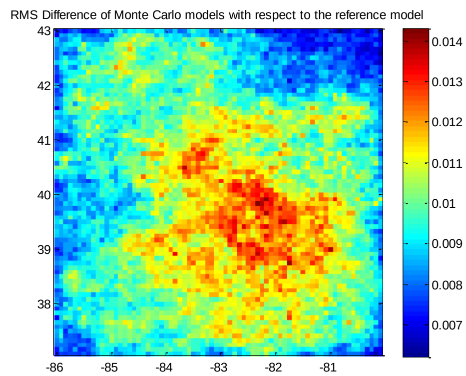




$\sigma_{Hgrid} = 0m, \sigma_{Hspot} = 0m, \sigma_g = \pm 3mGal$

Fig 4.3.3a: Model 3 ΔMCG [m] **Fig 4.3.3b: Model 3 rmseMCG grids [m]**





$\sigma_{Hgrid} = \pm 10m, \sigma_{Hspot} = \pm 2m, \sigma_g = \pm 1mGal$

Fig 4.3.4a: Model 4 ΔMCG [m] **Fig 4.3.4b: Model 4 rmseMCG grids [m]**

5. Conclusions and Recommendations

The results of this study confirm the following:

- GEOID09 performs better than GEOID03 at the 50 stations at which ODOT used VRS. Until a more permanent height transformation solution can be found, GEOID09 should be used for height conversions in Ohio.
- If VRS techniques are used along with the current GEOID09 model to perform GPS / geoid orthometric height determination (i.e. GPS leveling), then one can expect that the orthometric height will have an inherent error of about ±5cm (1σ). This means that, if cm-level height determinations are to be achieved, a more precise geoid over the Ohio region would be needed.
- Before undertaking terrestrial gravity observation over Ohio, an attempt should be made to evaluate the amount and spatial-distribution of gravity which NGS used in its geoid models. One should also keep in mind that NGS has already committed to undertaking a nationwide airborne gravity survey. Therefore any decision by ODOT to observe additional gravity in the state of Ohio should be considered with NGS collaboration in mind – it may prove to be economically more feasible.
- Furthermore, in light of the fact that, unless there are significant changes to the geoid model over a region (as was the case for Louisiana and other non-contiguous US territories such as Alaska), NGS only computes upgraded geoid models every few years. Any ODOT-led gravity undertaking and submission of the data to NGS will not necessarily yield an automatic geoid upgrade in the region.
- The SPIN Lab continues to strongly recommend that ODOT collaborate with National Geodetic Survey in its Height Modernization efforts so that efforts of both agencies can be conducted in a mutually-beneficial manner.

Given the results of the study involving the imposition of random errors on the input (cf. Section 4.3) ODOT would be well-advised to:

- Densify gravity data in the areas consisting of sparse gravity data – further studies into the optimal data density needed would have to be conducted.
- Utilise a higher accuracy DEM, since not only do the GTOPO30 height errors in excess of 15m introduce cm-bias into the solution, but the errors in excess of 10m introduce results which are worse than 1cm.
- Utilize gravity observation techniques which are better than ±3 mGal and heighted with accuracy better than a 2m to 3m, which should be possible if GPS (rather than height extraction from maps) is used.
- Fill in the gravity data gap over the lake region, it will inevitably result in a more accurate solution.

It should be emphasized that all conclusions and recommendations made herein are specific to Ohio and the data which has been used in this study.

6. Implementation Plan

GEOID09 performance at the 50 ODOT VRS stations suggests that the geoid in Ohio needs to be improved to facilitate precise GPS leveling. To avoid implementation of a separate state-based geoid model for the state of Ohio, ODOT is strongly encouraged to communicate hybrid geoid performance to NGS and to collaborate with them to improve the model. Height Modernization is a significant component of the NGS Ten Year Plan, hence NGS may be willing to provide grants to state agencies desirous of undertaking the upgrading of their existent vertical geodetic infrastructure.

Without a doubt, gravity densification will improve the local geoid performance (moreso than the DEM data). However, one can not be sure what gravity data holdings were used by NGS over Ohio and its environs to build it hybrid model. Therefore, ODOT would be well-advised to communicate with NGS to find out whether there were areas of Ohio which were data deficient and which could benefit from gravity data densification. However, ODOT is cautioned that NGS computes updated geoid models every few years and unless significant change is expected by the provision of new and better data sets, a recomputation will likely not be immediate. Again, this is a matter for further discussion with NGS.

There will undoubtedly be great value in re-leveling some of the 50 ODOT GPSBMs at which the GEOID09 discrepancies were in excess of a few centimeters in a bid to determine the source of the discrepancy. Ideally, GPS benchmarking should be conducted throughout the state preferably with a similar resolution to that of GEOID09 (which is approximately 3.7km x 3.7km grid spacing) – a costly exercise, but one which will be needed to get a better indication of the height conversion consistency of GEOID09 throughout the state.

It may be worthwhile to discuss with NGS, the possibility of ODOT creating a *customized* height conversion surface for the state (a *pseudo* GEOID09 surface, as it were) which allows for the transformation of the *true* geoid heights in the region to GEOID09. In this way, heights obtained in Ohio could somehow be referenced to the nationally held GEOID09 standard. However, the SPIN Lab suggests this stop gap solution with some reservation as we do not wish to encourage a *hodge podge* of conversion surfaces throughout the conterminous USA. Naturally, the introduction of yet another transformation tool would require timely communication to and updating of the rest of the surveying and engineering communities in Ohio.

Without a doubt, though, as was intimated in the introductory comments of this report, there are inevitable economic and labor-saving benefits to be had by Ohio *investing* in Height Modernization efforts for the state. The size of this investment and potential benefits would best be determined through consultation with the technical staff of ODOT's Surveying and Aerial Engineering Divisions who are well-versed in the costs incurred in precision re-leveling, GPS observation and aerial surveying.

7. Bibliography

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Appendix A: GEOID03 and GEOID09 models in Ohio

The figures below show the hybrid geoid topography over Ohio. Geoid discrepancies of GEOIDxx (where xx is either 03 or 09) are defined as:
Discrepancy = $N_{GEOIDxx} - (h_{NAD83} - H_{NAVD88})$

Table A.1 shows the level of fit between the corresponding gravimetric and hybrid models to the GPSBMs which were used for the hybrid model development. Notice that while USGG2003 shows a 73cm bias, USGG2009 shows only a 2cm bias indicating the model improvement made in the upgrade from the 2003 to the 2009 model.

Table A.1: NGS Geoid Model Fit to GPSBMs used in their Hybrid Models

Geoid Model Name	Geoid Type	Average Fit [m]	1σ STD Fit [m]
USGG2003	Gravimetric	0.734	± 0.058
GEOID03	Hybrid	0.001	± 0.032
USGG2009	Gravimetric	0.022	± 0.047
GEOID09	Hybrid	0.000	± 0.022

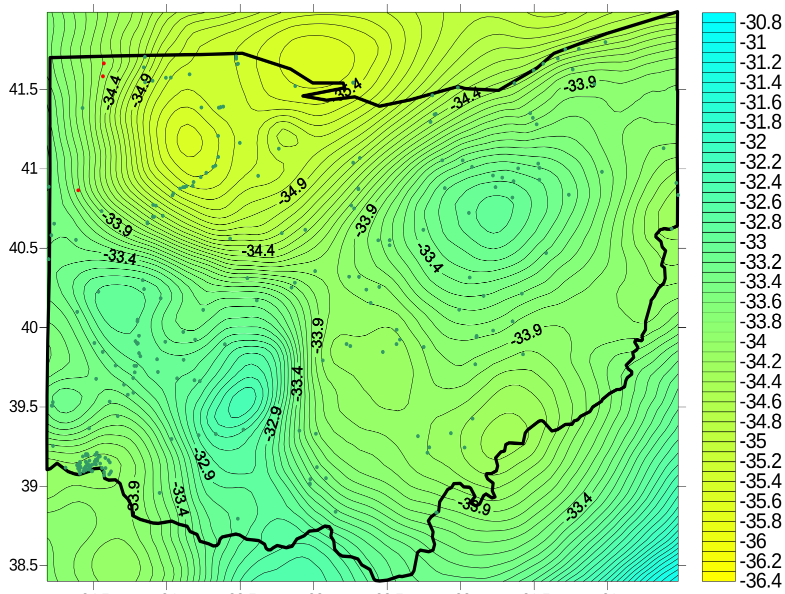

Figure A.1a: GEOID03 over Ohio

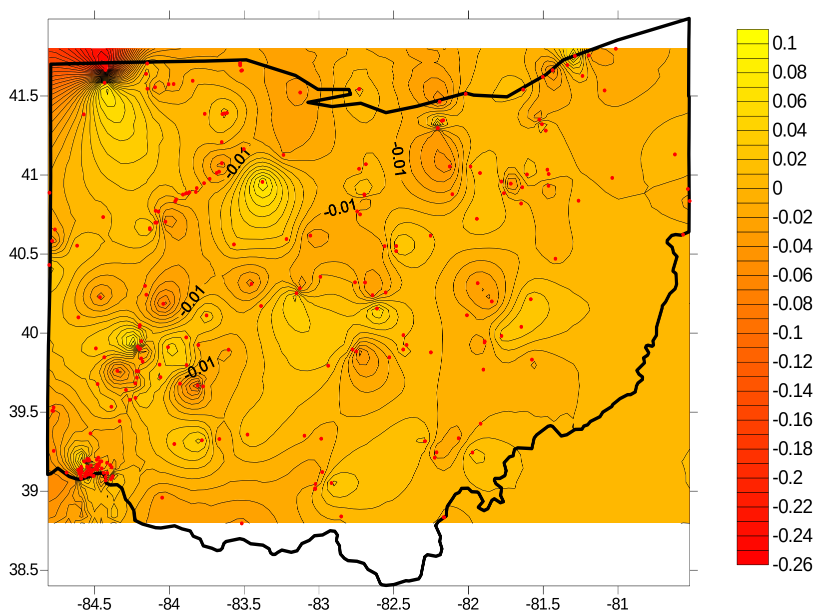

Figure A.1b: GEOID03 discrepancy over Ohio

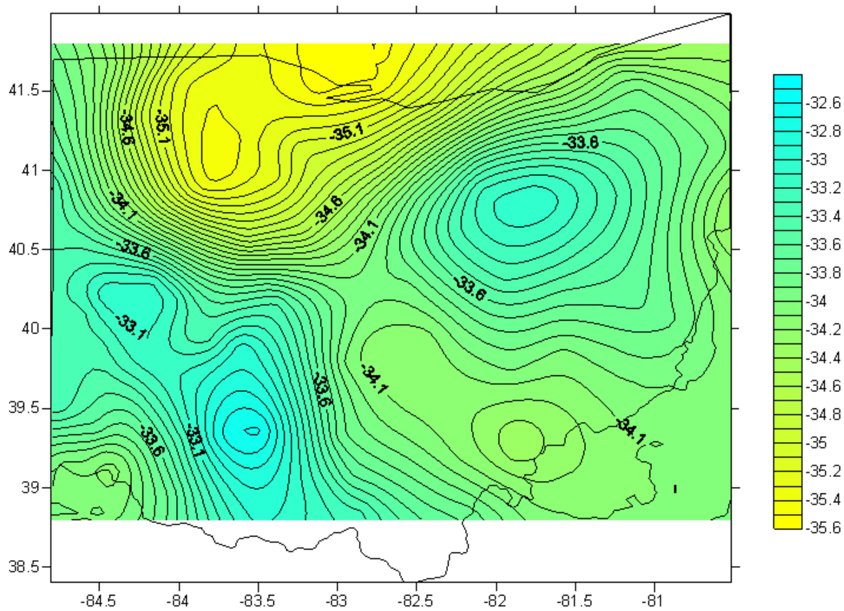

Figure A.2a: GEOID09 over Ohio

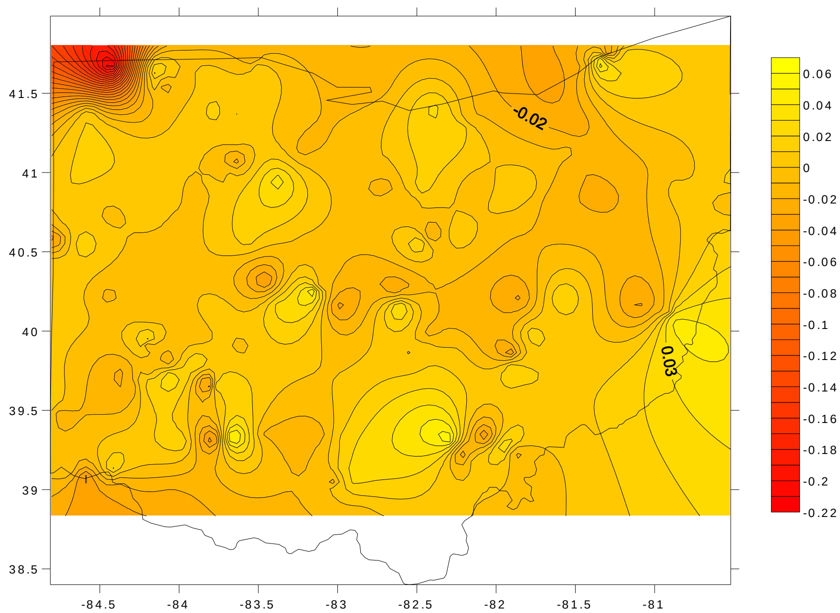

Figure A.2b: GEOID09 discrepancy over Ohio

Appendix B: Relative Gravity Data Density

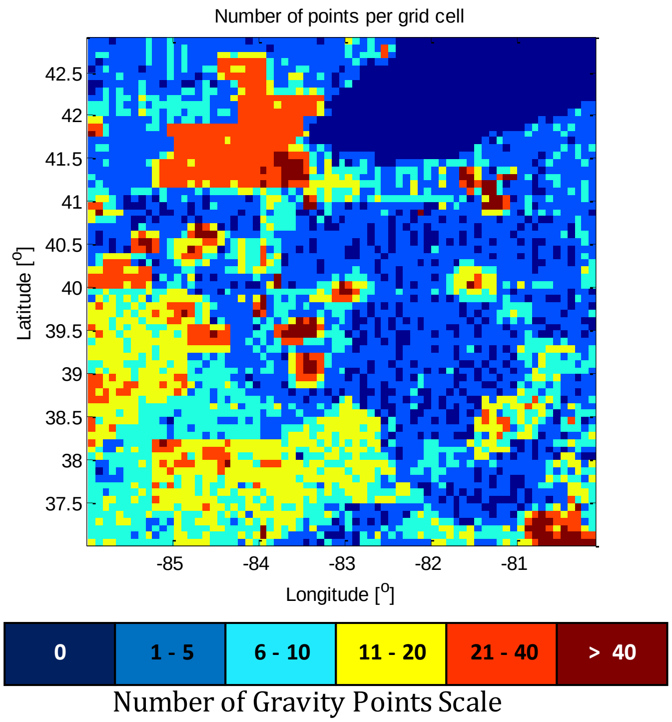

Figure B.1: Number of gravity points in (Ohio and its environs) per grid cell

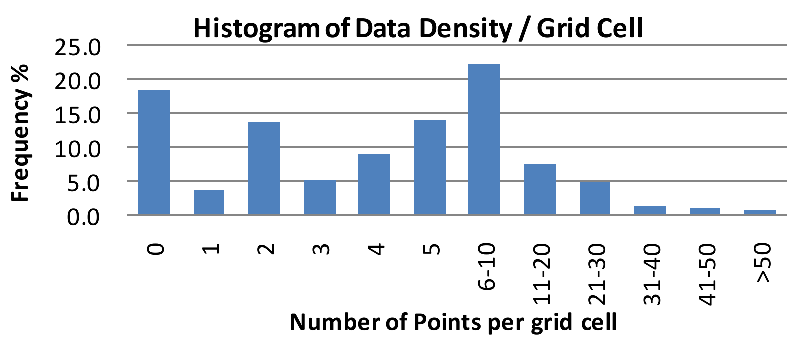

Figure B.2: Histogram of 43159 GEONET gravity points in Ohio and its environs

Appendix C: About Absolute Gravity Data Density

The FG5 measurement actually occurs at about 1.31m above the ground (Winester, D., 2007, e-mail communication) and gravity values are then transferred to the ground and a height of 91 cm above the ground, based on in-situ gravity gradient values, $\left(\frac{\partial g}{\partial H}\right)_{local}$. With regard to the gravity station designations (ibid.):

- The first letter indicates whether it is an absolute gravimeter site (given by 'A') or an excenter (given by 'C').
- The second letter indicates the sequence of gravity observation in a city.
- An excenter could be established on a pre-existing station (from a previous survey) and would retain its original name (e.g. LEBANON DIS GAR).

For example, for the designation:

- Columbus AA – this refers to the first absolute gravity station in Columbus and is a point marked on the floor.
- Columbus AB – this refers to the second absolute gravity station in Columbus.
- Columbus CA – refers to its excenter.

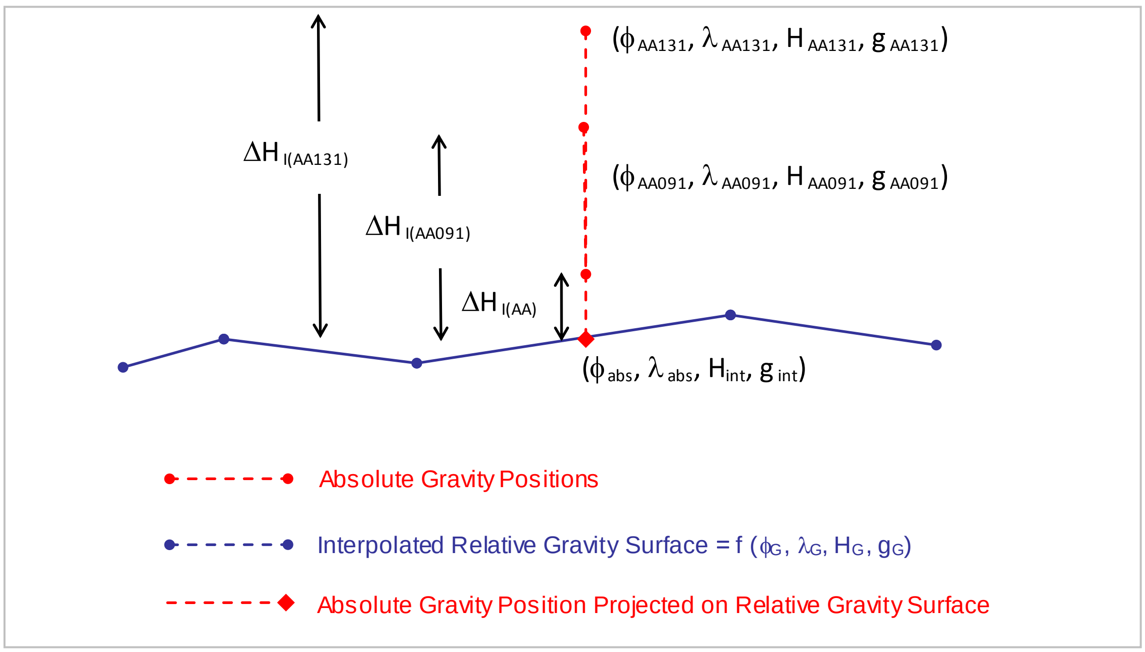

Figure C.1: Relationship between Interpolated Relative Gravity and Absolute Gravity

Items in this table have been defined in Section 3.2.1. Note that Daniel Winester also computed a site-specific **local** free air gravity gradient, $\left(\frac{\partial g}{\partial H}\right)_{local}$ which differs slightly from the **average** free air gravity gradient of -0.3086 mGal / m.

Table C.1: **Absolute Gravity Stations in Ohio**
N_{grav} = number of gravity points in 30' x 30' interpolated region

Gravity Station	H_{abs} [m]	Survey Date	g_{abs} [mGal]	$\left(\frac{\partial g}{\partial H}\right)_{local}$ [mGal/m]	N_{grav}	H_{int} [m]	g_{int} [mGal]	$(g_\Delta)_{avg}$ [mGal]	$(g_\Delta)_{local}$ [mGal]
BOLTON CAL BL 0	272.57	09/30/1987	980084.5400		560	272.099	980084.7327	0.0473	
COLUMBUS	231.3	10/01/1987	980072.2525		553	231.260	980072.2984	0.0334	
COLUMBUS AA	226.10	07/02/2005	980080.2505		562	226.207	980080.5096	0.2921	0.2912
COLUMBUS AA (091 CM)	227.01	07/02/2005	980079.9782	-0.2992	562	226.207	980080.5096	0.2835	0.2902
COLUMBUS AA (131 CM)	227.41	07/02/2005	980079.8555	-0.3015	562	226.207	980080.5096	0.2828	0.2927
COLUMBUS B-1	231.9	07/01/2005	980079.6409		562	231.320	980079.7913	-0.0286	
COLUMBUS C	245.6	09/30/1987	980081.3748		562	245.498	980081.3763	-0.0300	
COLUMBUS J	244.8	12/01/1986	980064.2020		517	245.423	980063.7015	-0.3082	
COLUMBUS LATITUDE	231.167	07/02/2005	980080.0571		561	231.208	980080.2176	0.1731	
COLUMBUS V 189	248.210	12/01/1986	980064.2700		517	245.445	980063.6844	-1.4390	
DAYTON AA	270.3	06/24/2005	980065.4677		495	265.862	980066.3412	-0.4962	-0.2038
DAYTON AA (091 CM)	271.21	06/24/2005	980065.2463	-0.2433	495	265.862	980066.3412	-0.5556	-0.2033
DAYTON AA (131 CM)	271.61	06/24/2005	980065.1505	-0.2421	495	265.862	980066.3412	-0.5833	-0.2046
DAYTON CA	272.595	06/24/2005	980065.3333		495	265.720	980066.3797	-1.0753	
EATON AA	316.22	06/28/2005	980033.0112		466	320.077	980032.3405	0.5197	0.4081
EATON AA (091 CM)	317.13	06/28/2005	980032.7568	-0.2795	466	320.077	980032.3405	0.4932	0.4080
EATON AA (131 CM)	317.53	06/28/2005	980032.6446	-0.2798	466	320.077	980032.3405	0.4820	0.4083
EATON CA	318.104	06/28/2005	980032.6347		468	320.083	980032.3373	0.3133	
LEBANON AA	256.2	06/29/2005	980020.5843		317	222.599	980027.8398	-3.1138	-2.9862
LEBANON AA (091 CM)	257.11	06/29/2005	980020.3088	-0.3028	317	222.599	980027.8398	-3.1191	-2.9880
LEBANON AA (131 CM)	257.51	06/29/2005	980020.1823	-0.3068	317	222.599	980027.8398	-3.1160	-2.9835
LEBANON DIS GAR (5 CM)	256.265	06/27/2005	980020.3637		310	222.562	980027.6850	-3.0796	
PERRYSBURG TT 16 WO	192.397	12/01/1986	980228.3590		1190	193.163	980228.1240	0.0014	
							MEAN	**-0.6097**	**-0.6226**
							STD	**1.2687**	**1.4452**