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FINAL REPORT

BEND LOSSES IN RECTANGULAR CULVERTS

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16 Abstract

This study investigated bend losses for open channel flow in rectangular channels or culverts. Laboratory experiments were performed for sub-critical flow in rectangular channels with abrupt bends. Bend angles of approximately 30, 45, 60, 75 and 90 degrees were tested. The experimental results were expressed as empirical equations, charts and tables relating the bend loss coefficient to the bend angle. The velocity and depth are at a location sufficiently downstream from the bend to allow the flow to return to gradually varied.

A procedure was developed to estimate head loss for gradual bends in rectangular channels flowing as free surface flow. The bend loss coefficients for gradual bends in conduits flowing full were used to develop an adjustment factor. This factor was used to estimate a gradual bend loss coefficient for free surface flow in rectangular channels. The equation used to estimate the gradual bend loss coefficient was where is the abrupt bend loss coefficient, as defined in this study.

Examples problems were presented illustrating the computation of bend loss for both an abrupt bend and a gradual bend in a rectangular open channel.

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Final Report

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A Report on Research Sponsored By

THE KANSAS DEPARTMENT OF TRANSPORTATION
TOPEKA, KANSAS

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PREFACE

The Kansas Department of Transportation's (KDOT) Kansas Transportation Research and New-Developments (K-TRAN) Research Program funded this research project. It is an ongoing, cooperative and comprehensive research program addressing transportation needs of the state of Kansas utilizing academic and research resources from KDOT, Kansas State University and the University of Kansas. Transportation professionals in KDOT and the universities jointly develop the projects included in the research program.

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ABSTRACT

This study investigated bend losses for open channel flow in rectangular channels or culverts. Laboratory experiments were performed for subcritical flow in rectangular channels with abrupt bends. Bend angles of approximately 30, 45, 60, 75 and 90 degrees were tested. The local head loss due to an abrupt bend can be expressed by the equation $h_{bend} = K_b \frac{V_{ds}^2}{2g}$ where K_b = the bend loss coefficient, V_{ds} = velocity downstream from the bend and g = gravitational acceleration. The experimental results were expressed as empirical equations, charts and tables relating the bend loss coefficient to the bend angle, θ and the velocity and depth in the channel downstream from the bend expressed as the dimensionless Froude Number, $Fr = \frac{V_{ds}}{\sqrt{gy_{ds}}}$, where y_{ds} = the flow depth downstream from the bend . The velocity and depth are at a location sufficiently downstream from the bend to allow the flow to return to gradually varied.

A procedure was developed to estimate head loss for gradual bends in rectangular channels flowing as free surface flow. The bend loss coefficients for gradual bends in conduits flowing full were used to develop an adjustment factor, β , This factor was used to estimate a gradual bend loss coefficient, K_{gb} , for free surface flow in rectangular channels. The equation used to estimate the gradual bend loss coefficient was $K_{gb} = \beta K_b$ where K_b is the abrupt bend loss coefficient, as defined in this study.

Example problems were presented illustrating the computation of bend loss for both an abrupt bend and a gradual bend in a rectangular open channel.

ACKNOWLEDGEMENTS

This project was supported by the Kansas Department of Transportation. Special thanks go to Jim Richardson of KDOT, the project monitor, and Brad Rognlie. We would also like to express our appreciation to Jim Weaver and Jay Barnard for their help and advice on this project.

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CHAPTER 1 - BEND LOSSES IN RECTANGULAR CULVERTS

1.1 Introduction

This study considers bend losses for open channel flow in rectangular channels. Figure 1.1 is a definition sketch showing abrupt and gradual bends where θ is the bend angle.

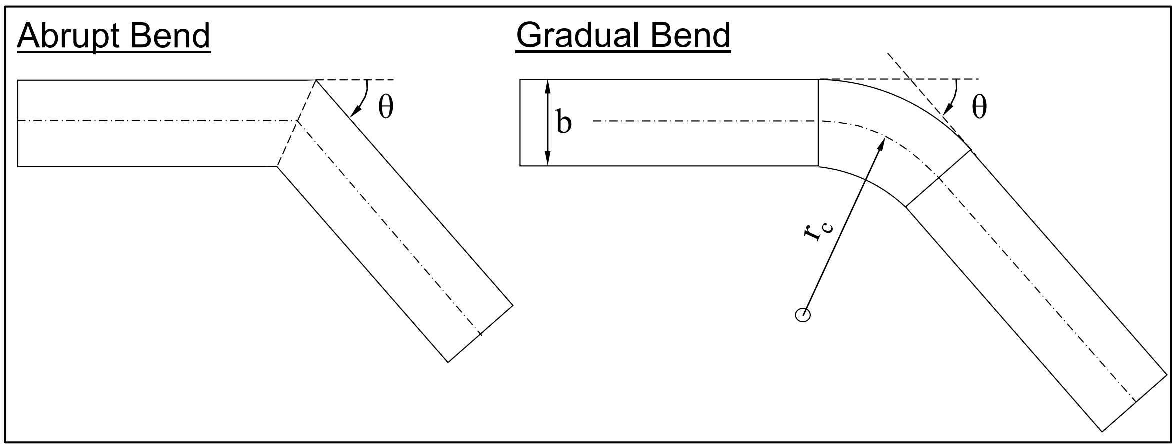

Figure 1.1: Abrupt and Gradual Bends in Rectangular Channels

Laboratory experiments were performed for subcritical flow in rectangular channels with abrupt bends. Bend angles of approximately 30, 45, 60, 75 and 90 degrees were tested. A laboratory flume was constructed specifically for this project. The flume was modified (rebuilt) for each of the five bend angles considered. Experimental results were expressed as empirical equations, charts and tables that can be used to determine bend loss as functions of bend angle and the velocity and depth in the channel downstream from the bend. Specifically, an abrupt bend loss coefficient, K_b, was derived that is a function of the bend angle and the Froude number at the downstream end of the abrupt bend.

A procedure was developed for determining the head loss for gradual bends in rectangular channels. Loss coefficients for gradual bends in conduits flowing full (4, 5) were used to develop an adjustment factor, β. The gradual bend loss coefficient, K_{gb}, is obtained by multiplying K_b by β.

CHAPTER 2 - LABORATORY MODELS

Figure 2.1 shows the plan view of the laboratory flume for the 45 degree bend experiments. The flume was constructed mostly of marine plywood. The flume cross section is 6 inches wide and 12 inches deep. Water was supplied to the flume from the constant head tank of the KU hydraulics laboratory. The flume is equipped with upstream and downstream head tanks. The headwater (upstream) tank contains a level, fixed overflow weir. Water flows from the headwater tank to the rectangular channel through a 10.5-inch long transition section. The tailwater (downstream) tank has an adjustable-height 16.9-inch long overflow weir that is used to control the tailwater elevation in the channel. After overtopping the adjustable overflow weir in the tailwater tank, the water returns to the laboratory sump system.

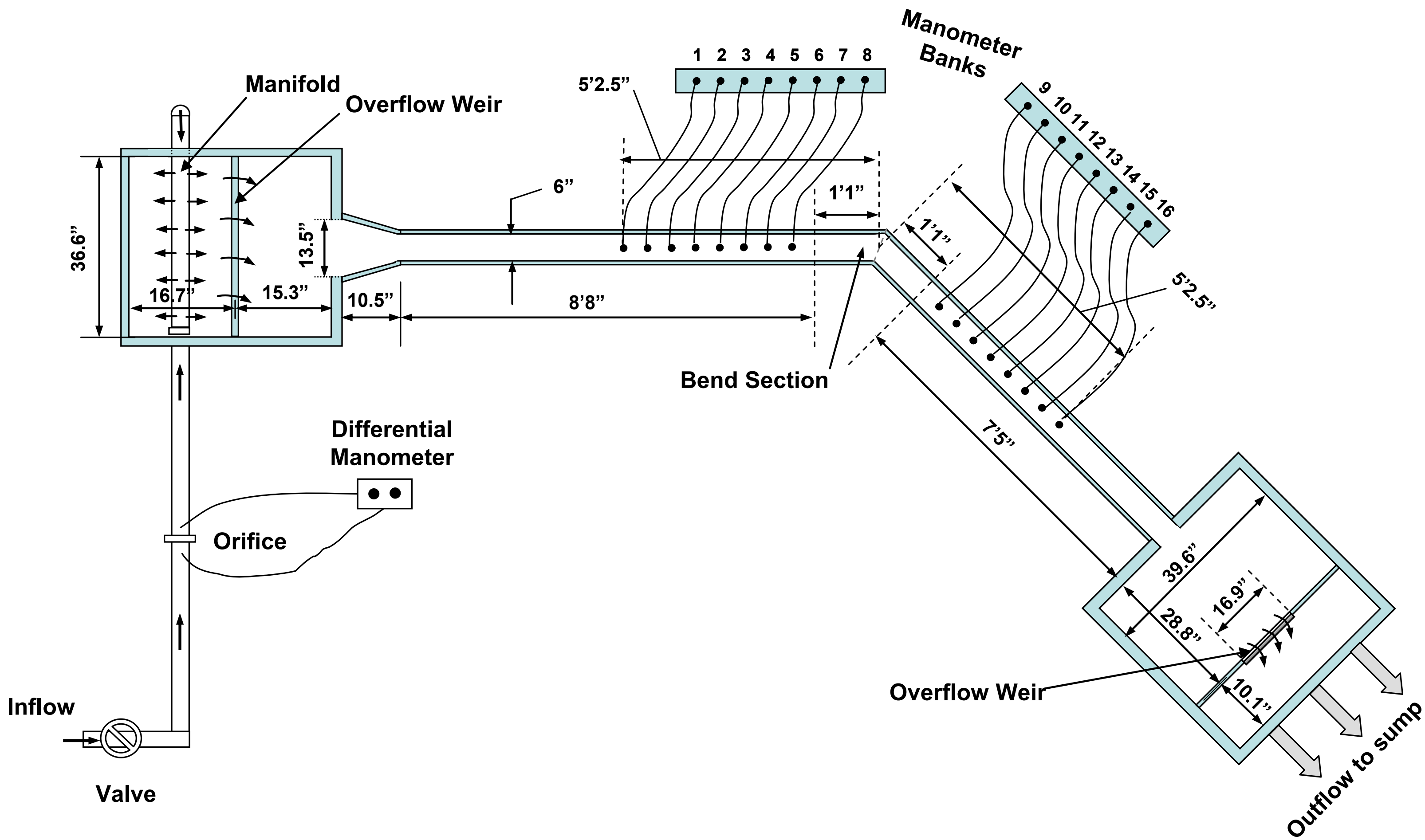

Figure 2.1: Laboratory Set-up.

A different bend section was constructed for each bend angle – 30, 45, 60, 75 and 90 degrees. (The actual constructed angles were 30, 45.2, 58.5, 76 and 90.5 degrees.) The centerline length of each bend section was 2'2"- long with 1'1" upstream and downstream from the bend, respectively. Changing a bend section was not a simple process since the channel and tailwater tank downstream from the bend section had to be moved to different positions for each bend angle. The head tank and the channel upstream from the bend section remained stationary.

A 4-inch PVC line carried water to the flume. The discharge was controlled with a gate valve at the upstream end of the supply line. An orifice plate was constructed and installed in the supply line to facilitate flow calibration and, during experiments, to establish the desired discharge. Piezometers installed just upstream and downstream from the orifice plate were connected to a 2-tube manometer in order to read the head loss across the orifice. The orifice was calibrated volumetrically using a tank built specifically for that task. A rating curve of discharge versus head loss across the orifice plate is shown in Figure 2.2.

The water surface elevations upstream and downstream from the bend were measured using a series of piezometers installed in the flume bed. (See Figure 2.1.) The piezometer readings were read from a manometer bank then the readings were used to determine the local depths of flow. The piezometers were located from 1.71 to 5.21 feet upstream from the bend and from 1.71 to 5.21 feet downstream from the bend and were spaced at 6-inch intervals. (A total of 16 piezometers were installed but the next to last piezometer, No. 15, did not work.) Knowing the flume discharge and the flow depth and bed elevation of each piezometer, the total head $(H = z + p/\gamma + V^2/2g)$ could be calculated for each piezometer location.

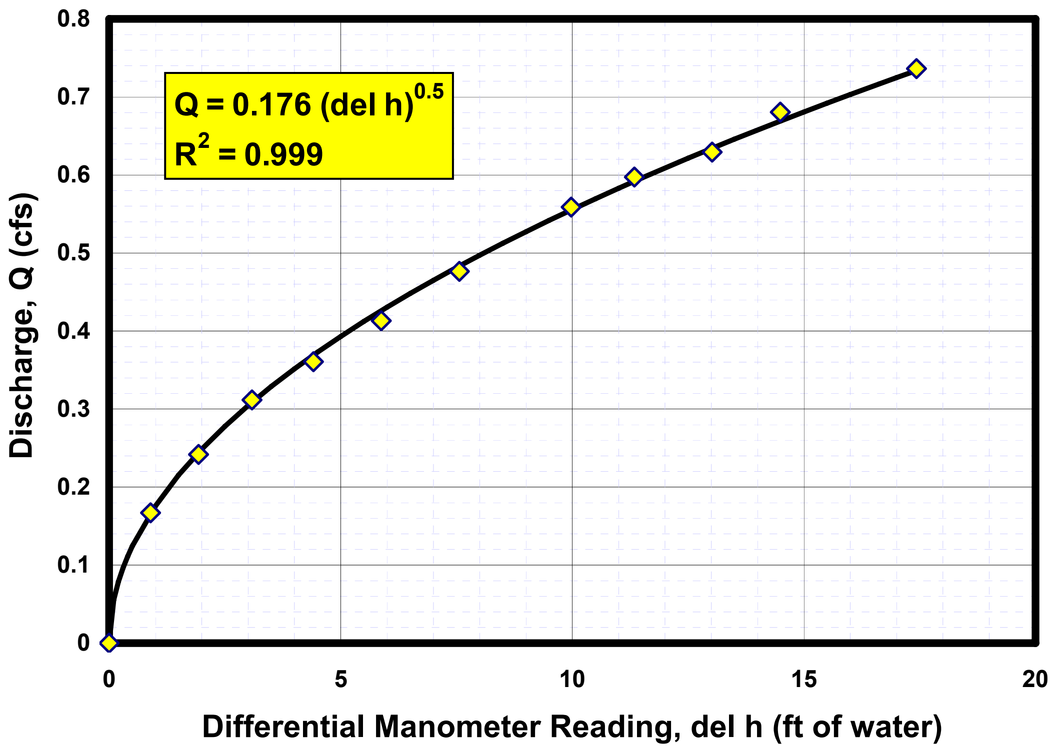

Figure 2.2: Flume Calibration Curve

The Bernoulli equation is only valid at cross sections where the pressure distribution is hydrostatic. This condition is met for free surface flow when the water surface is level across the section and the streamlines are straight and parallel (or nearly so). Hydrostatic pressure distribution occurs in open channels for regions of uniform or gradually varied flow. Rapidly varied flow regions do not meet the conditions for hydrostatic pressure distribution and the Bernoulli equation should not apply in those regions.

Initially readings were taken at all 15 working piezometers to determine the behavior of the flow through the channel. Rapidly varied flow was observed in a region that extended downstream from the bend to the first few (or more) piezometers. Strong transverse and vertical variations in the velocity distribution were evident as the flow came around the bend. The length of the rapidly varied flow region appeared to lengthen as the velocity increased. The flow "smoothed-out" and became gradually varied by the time it reached the most downstream piezometer (Piezometer 16) for all experiments. Consequently, the total head loss was determined for the reach between Piezometers 2 and 16.

The total head loss between Piezometers 2 and 16 consisted of friction loss and bend loss. Thus, the bend loss was the total head loss minus the estimated friction loss between Piezometers 2 and 16. The equations below show the method used to compute the total head loss between Piezometers 2 and 16.

$$H_2 = H_{16} + h_{loss} \qquad \text{Equation 2.1}$$

$$y_2 + z_2 + \frac{V_2^2}{2g} = y_{16} + z_{16} + \frac{V_{16}^2}{2g} + h_{loss} \qquad \text{Equation 2.2}$$

Where y = depth (feet), z = bed elevation (feet) and V = velocity (feet/sec) = Q/(0.5y) where Q is in cubic feet per second. Due to the construction materials, the flume was not exactly horizontal nor was it the same for each series of experiments. Thus, the bed elevations for Piezometers 2 and 16 were determined for each set-up by establishing a pooled-water condition in the flume then measuring the depths at each location. The distance between Piezometers 2 and 16 was 9.92 feet.

The loss term includes the bend loss and the friction loss. The friction loss was estimated using the Manning's equation in the form shown below.

$$h_f = \frac{n^2 \overline{V}^2 L}{(1.49)^2 \overline{R}^{4/3}} \qquad \text{Equation 2.3}$$

Where

$$\overline{V} = \frac{Q}{\overline{A}} = \frac{Q}{(0.5\overline{y})} \qquad \text{Equation 2.4}$$

$$\overline{R} = \frac{\overline{A}}{\overline{P}_w} = \frac{(0.5\overline{y})}{(0.5 + 2\overline{y})} \qquad \text{Equation 2.5}$$

$$\overline{y} = \frac{y_2 + y_{16}}{2} \qquad \text{Equation 2.6}$$

A Manning n-value of 0.010 was used. The sides of the flume were constructed of sanded marine grade plywood and painted with epoxy paint. This provided a very smooth sidewall condition. This n-value corresponds to table values for planed wood. Also, the n-value of 0.010 provided reasonable results in allocating the total head loss to friction and to bend loss. L was equal to 9.92 feet - the distance between Piezometers 2 and 16.

Figure 2.3 shows the bend loss flume from the tailwater tank. Figure 2.4 shows pictures of the water surface elevations for three 90.5 degree experiments. These pictures were taken with the camera directed in the upstream direction toward the bend.

Figure 2.3: Experimental Flume.

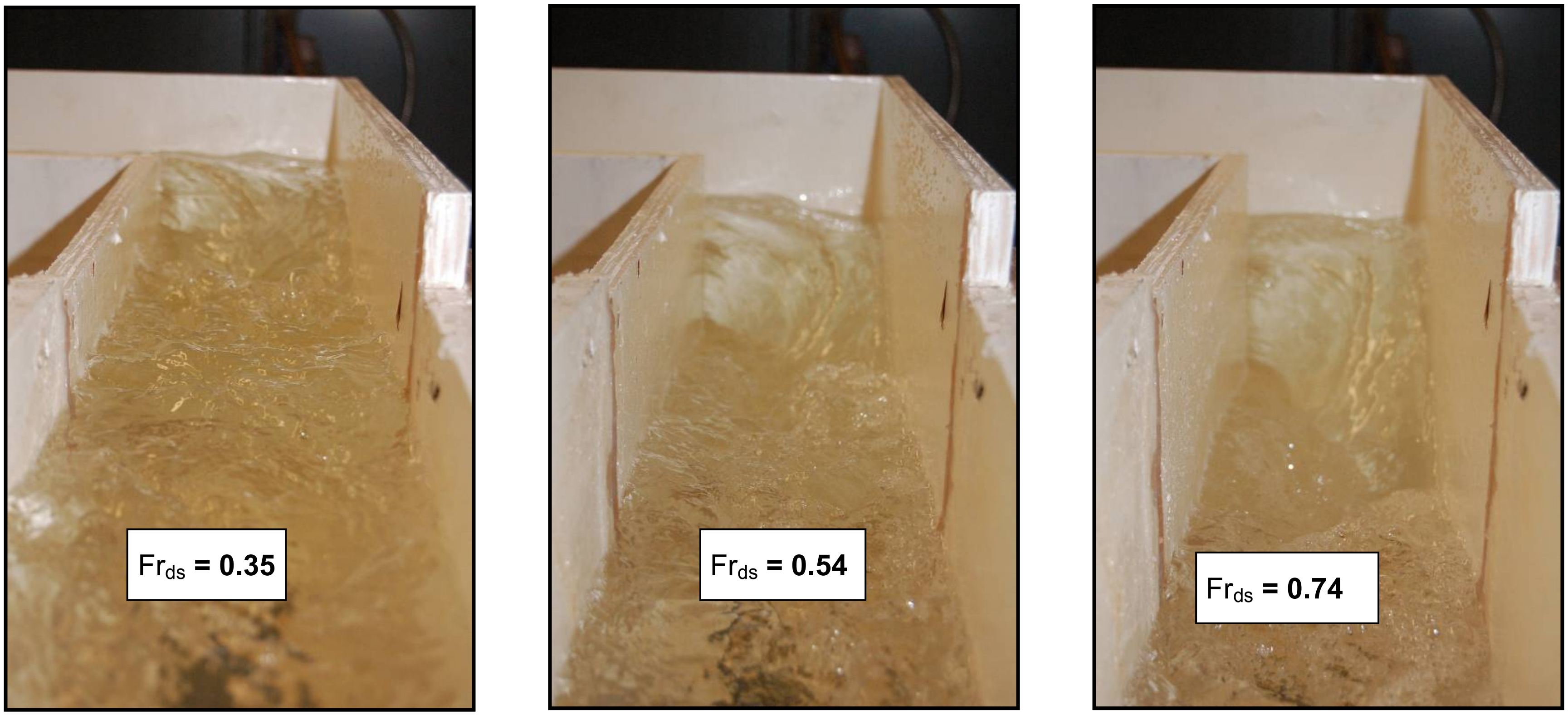

Figure 2.4: Water Surface Through Bend for 90° Experiments

CHAPTER 3 - EXPERIMENTAL RESULTS

A total of 190 experiments were performed. The results are summarized in Tables 3.1-3.5 in Appendix A. The bend loss, h_b, is the total loss, h, minus the estimated friction loss, h_f, between Piezometers 2 and 16. Figure 3.1 is a plot of bend loss versus the downstream velocity head (Piezometer 16).

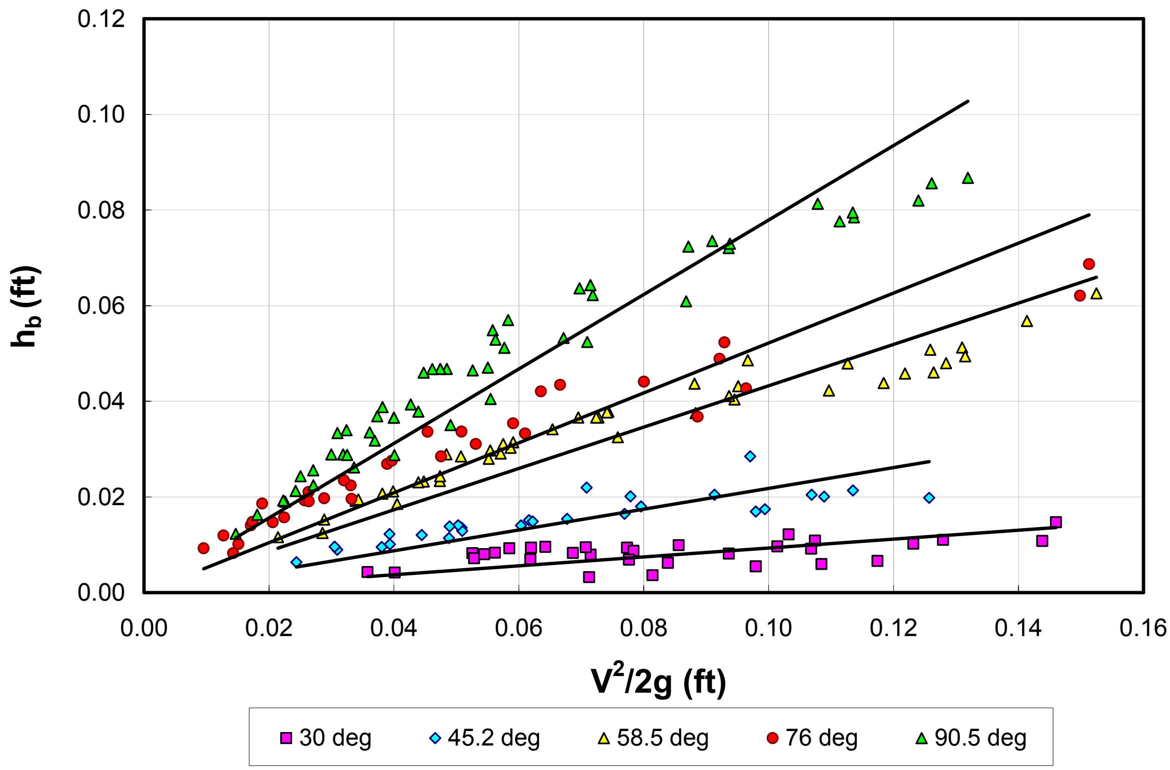

Figure 3.1: Plot of Bend Loss versus Velocity Head at Piezometer 16 with Linear, Zero-Intercept Trend Lines

The standard bend loss equation has the form shown below in Equation 3.1 where Kb is the bend loss coefficient. Typically, Kb is assumed to be constant with respect to velocity. This assumption would dictate that the data would vary linearly with the velocity head. It is apparent from Figure 3.1 that only the data for the 30 degree angle experiments follows a linear trend line.

$$h_b = K_b \frac{V^2}{2g}$$ Equation 3.1

Figure 3.2 is a plot of dimensionless bend loss versus the downstream Froude number squared. A second possible independent variable would be the downstream depth, y_{ds}, divided by the flume width, W. For this study the parameter y_{ds}/W ranged from 0.6 to 1.87 and over that range the parameter appeared to have little effect. Many types of trend lines were examined until the second-order polynomial shown in Equation 3.2 was selected. The trend lines were forced through the (0,0) point which is physically reasonable.

$$h^* = \frac{h_b}{y_{ds}} = a\left(Fr_{ds}^2\right)^2 + b\left(Fr_{ds}^2\right) = aFr_{ds}^4 + bFr_{ds}^2$$ Equation 3.2

Where $Fr^2_{ds} = V^2_{ds}/gy_{ds}$ and y_{ds} = the depth at Piezometer 16. The coefficients a and b are functions of the bend angle, θ. Each of the trend line equations is shown on Figure 3.1 with y representing h* and x representing Fr^2_{ds}.

The coefficients a and b for the trend lines of Figure 3.2 shown in Table 1.1 are plotted in Figure 3.2 versus bend angle.

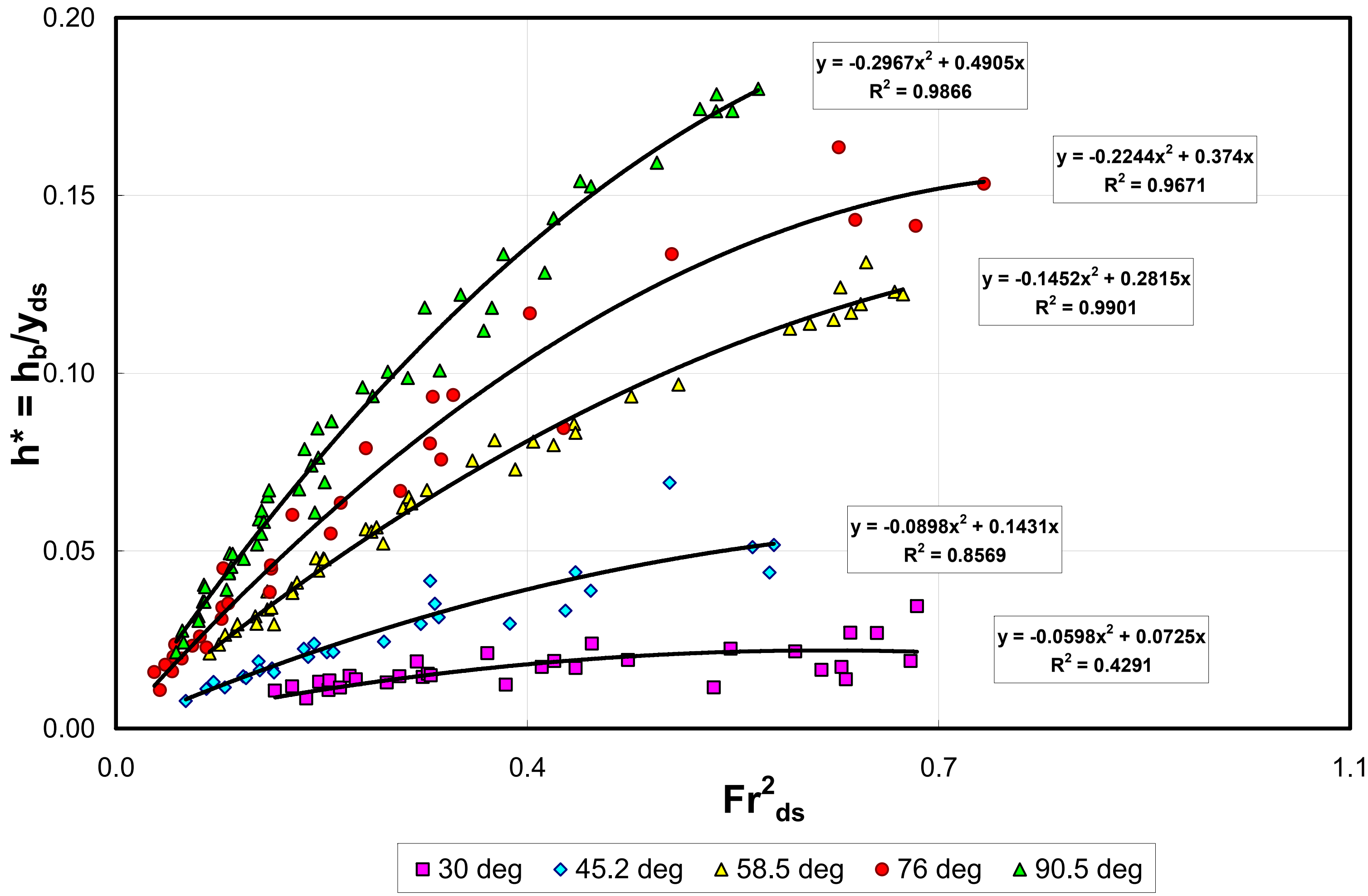

Figure 3.2: Dimensionless Bend Loss versus Downstream Froude Number Squaerd
(Note: The trend lines were created in Excel 2003. Excel 2007 produces different coefficients.)

Table 3.1: Coefficients for Trend Lines of Figure 1.7

Bend Angle, θ	From Figure 1.7	
	a *(1)*	b *(2)*
90.5	-0.2967	0.4905
76	-0.2244	0.3740
58.5	-0.1452	0.2815
45.2	-0.0898	0.1431
30	-0.0598	0.0725

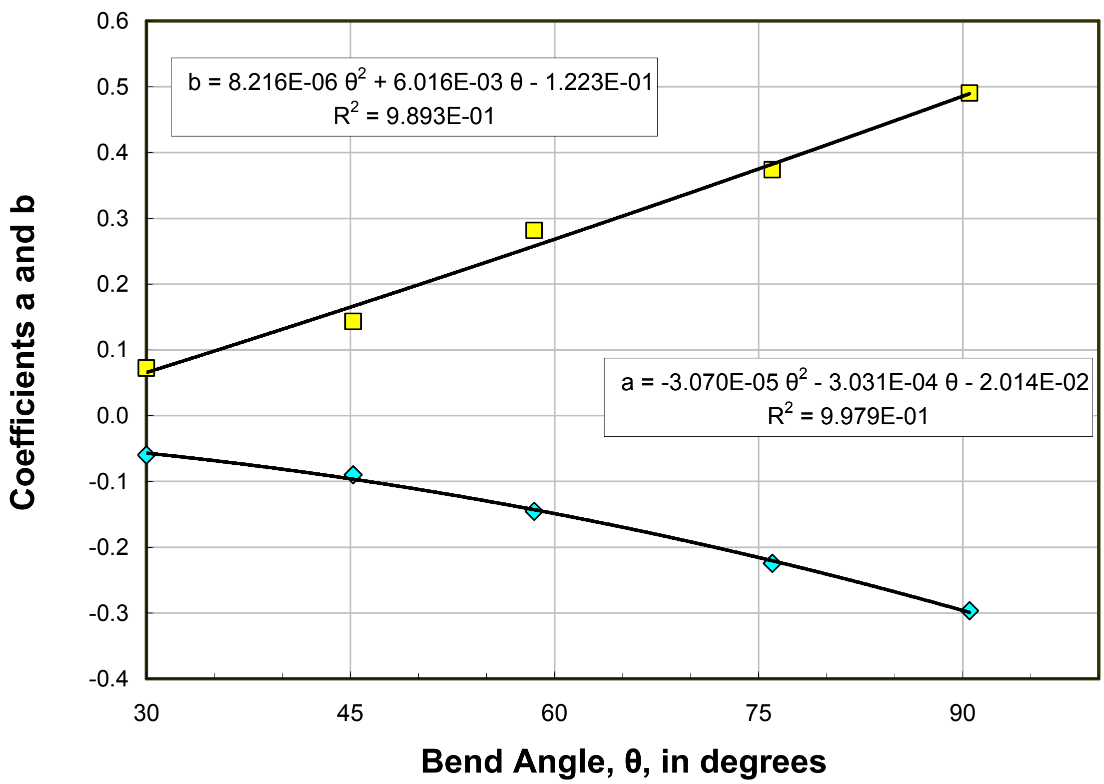

Figure 3.3: Coefficients for Figure 1.6 Trend Lines as Functions of Bend Angle.

The coefficients for the equations shown in Figure 3.3 are presented in Table 3.2. These values will be used to generate a predictive equation for angles other than the ones tested in the flume.

Table 3.2: Coefficients for Trend Lines of Figure 3.3

Parameters for Coefficient a			Parameters for Coefficient b		
m	**n**	**o**	**p**	**q**	**r**
-3.0702 E-05	-3.0310E-04	-2.0139E-02	8.2164E-06	6.0159E-03	-1.2229E-01

The predictive equation is given below.

$$\frac{h_b}{y_{ds}} = aFr_{ds}^4 + bFr_{ds}^2 = \left(m\theta^2 + n\theta + o\right)Fr_{ds}^4 + \left(p\theta^2 + q\theta + r\right)Fr_{ds}^2 \qquad \text{Equation 3.3}$$

Where θ = bend angle in degrees; $Fr_{ds} = V^2_{ds}/y_{ds}$; h_b= bend loss and y_{ds} = depth downstream from the bend. Figure 3.4 shows the experimental data plotted as h_b/y_{ds} versus Fr^2_{ds} on curves generated using Equation 3.3 for the angles tested. The fit for Equation 3.3 is not as good as the fits for the individual trend lines shown in Figure 3.2. Nonetheless it shows that Equation 3.3 can be used to generate data for intermediate bend angles. Figure 3.5 is a plot of hb/yds versus Frds rather than Fr2ds. The use of the Froude number rather than the Froude number squared for the abscissa is probably better since HEC-RAS output tables have the Froude number as an optional table parameter.

Figure 3.6 shows Equation 3.3 plotted for bend angles of 30, 40, 50, 60, 70, 80 and 90 degrees. This curve can be used to determine the head loss for a culvert given the bend angle and the depth and Froude number for the culvert in the region downstream from the bend where flow is deemed hydrostatic.

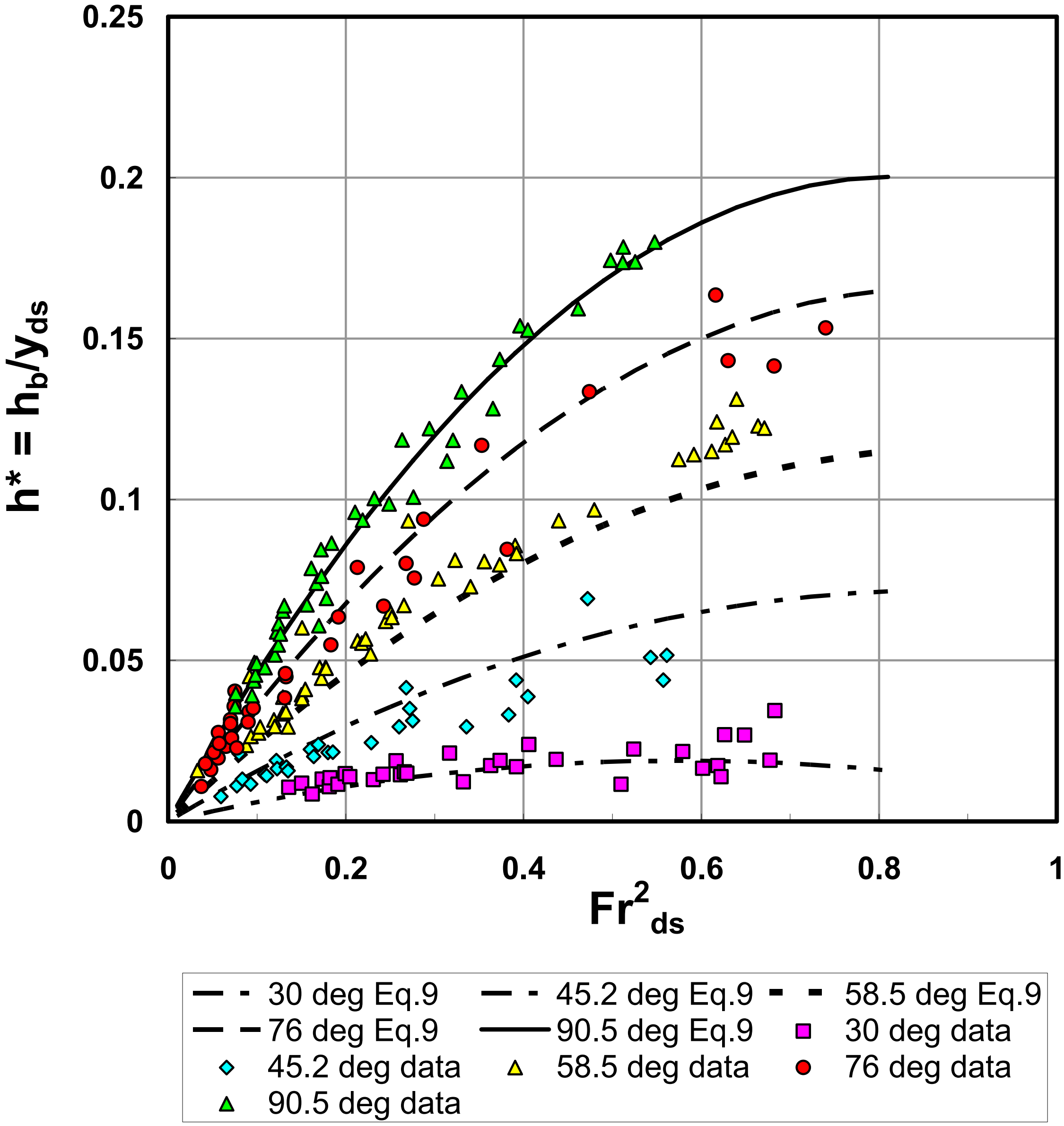

Figure 3.4: Equation 1.9 and Experimental Data h_b/y_{ds} versus Fr^2_{ds}

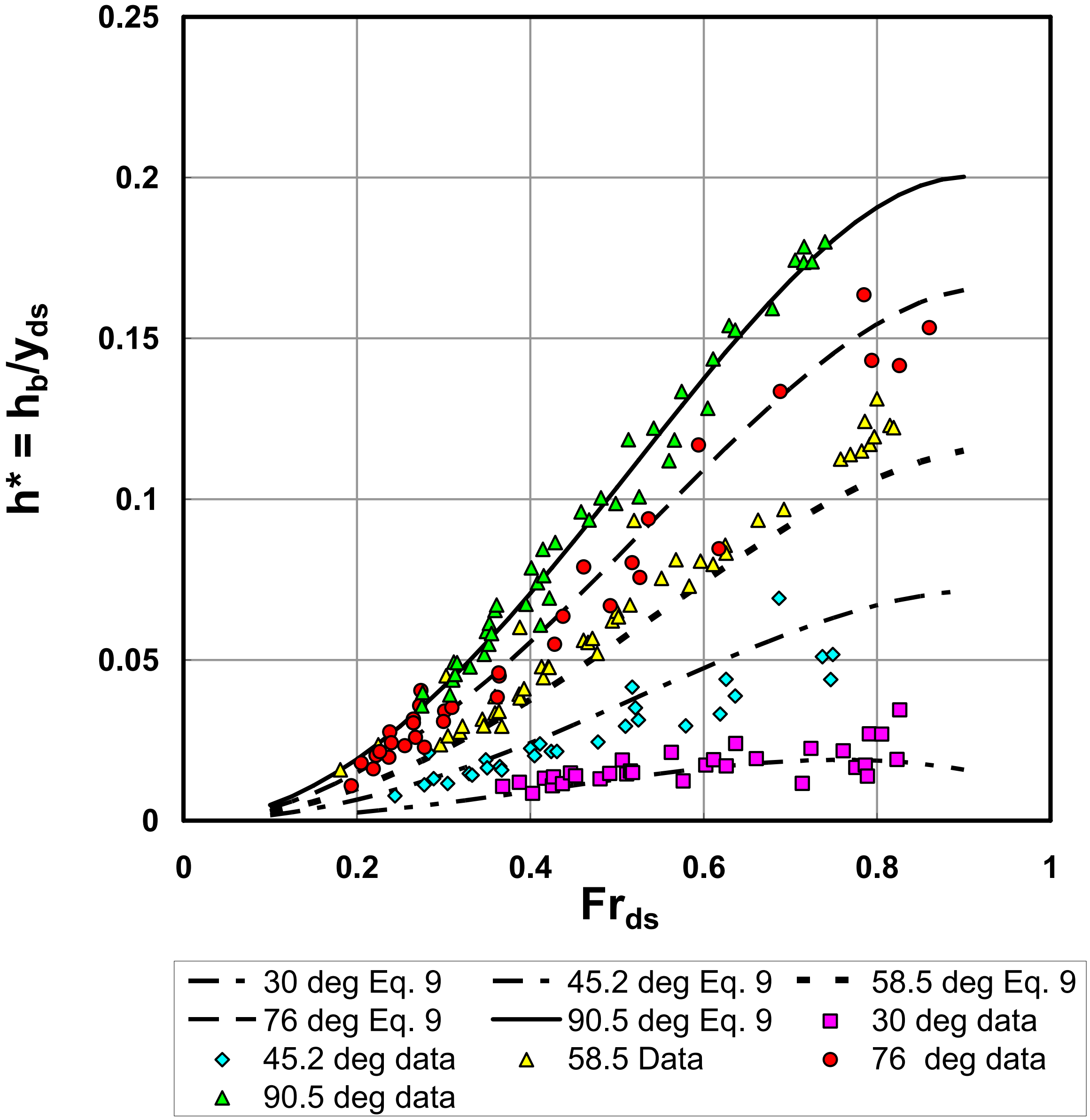

Figure 3.5: Equation 3.3 and Experimental Data h_b/y_{ds} versus Fr_{ds}

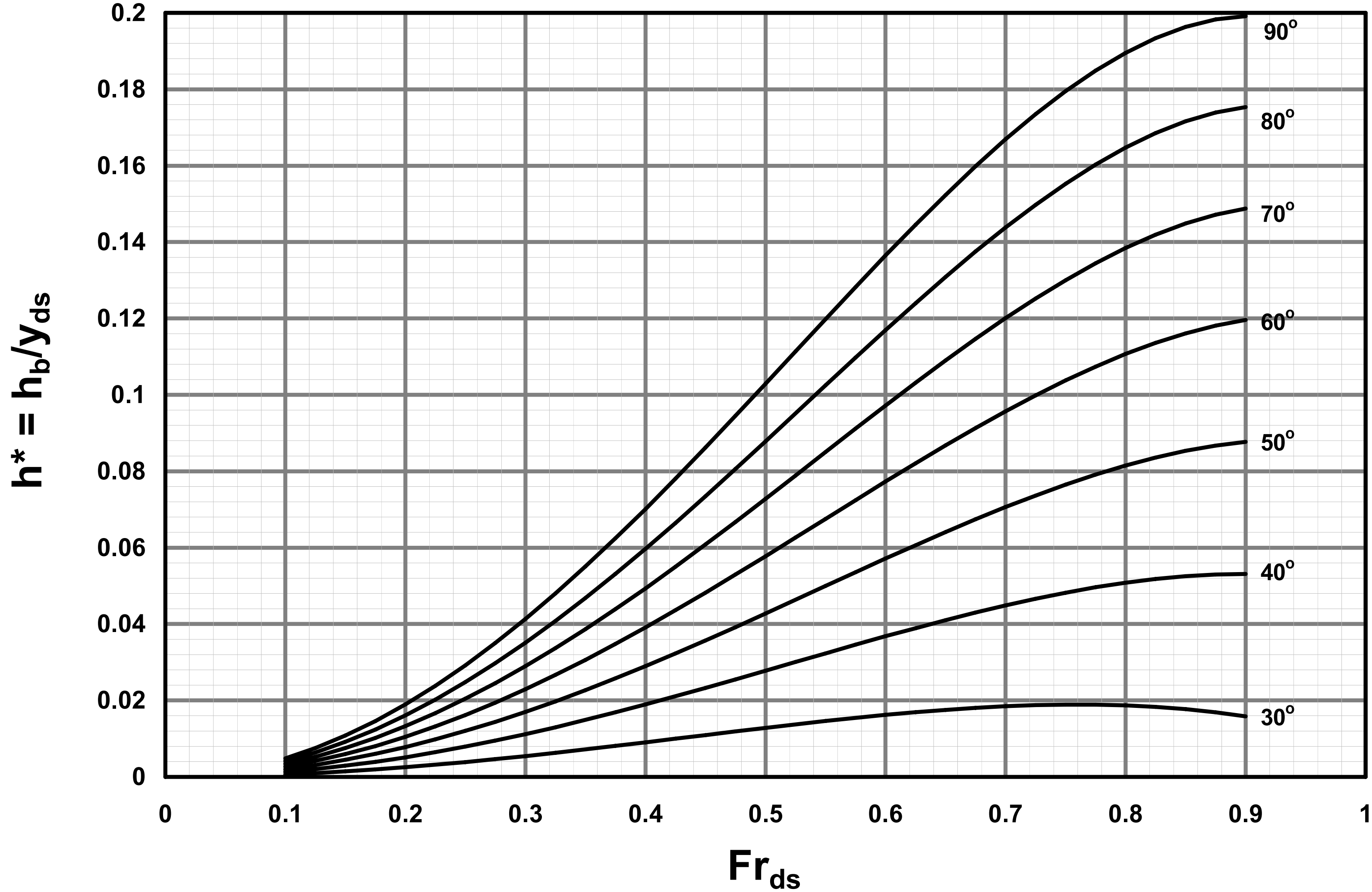

Figure 3.6: Dimensionless Head Loss versus Froude Number (Eq. 1.9)

3.1 Bend Loss Coefficient

Equation 2.4 can be expressed as a function of the Froude number as follows.

$$h_b = K_b \frac{V_{ds}^2}{2g} \rightarrow \frac{h_b}{y_{ds}} = \frac{K_b}{y_{ds}} \frac{V_{ds}^2}{2g}$$

$$\frac{h_b}{y_{ds}} = \frac{K_b}{2} \frac{V_d^{\ 2}}{gy_{ds}} = \frac{K_b}{2} F_r^{\ 2}$$

Or rearranging

$$K_b = \frac{2}{Fr_{ds}^2}\left(\frac{h_b}{y_{ds}}\right) \qquad \text{Equation 3.4}$$

Substituting Equation 3.3 from the experimental data analysis into Equation 3.4 yields.

$$\frac{h_b}{y_{ds}} = aFr_{ds}^4 + bFr_{ds}^2 \quad \textit{(Equation 3.3)}$$

$$K_b = \frac{2}{Fr_{ds}^2}\left(aFr_{ds}^4 + bFr_{ds}^2\right)$$

$$K_b = (2a)Fr_{ds}^2 + (2b) \qquad \text{Equation 3.5}$$

Rearranging Equation gives the following expression that is used to determine the bend loss coefficient for the experimental data.

$$K_b = h_b / \left(V_{ds}^2 / 2g\right) \qquad \text{Equation 3.6}$$

The experimental values of K_b are shown in Column 16 in the Data Tables included in the Appendix A. The K_b experimental values are plotted versus Fr^2_{ds} in Figure 3.7. The solid lines represent Equation 3.5 using the a- and b-values from the previous analysis of the data shown in Figure 3.2. Linear trend line fits for each of the bend angles are shown as dashed lines and represented by the equation below.

$$K_b = \alpha Fr_{ds}^2 + \beta \qquad \text{Equation 3.7}$$

Where α and β are the trend line coefficients.

Table 3.3 below compares α and β with the values obtained by multiplying the coefficients a and b by 2. Equation 3.5 indicates that the values in Columns 3 and 4 should, perhaps, equal the respective values in Columns 5 and 6. They are not quite the same due to differences in the curve fitting schemes used to derive the values. Parameters a and b were based on a second-degree polynomial trend lines while α and β were based on linear trend lines. Both processes used Fr^2_{ds} as the independent variable. Comparing the solid and dashed lines in Figure 3.7 shows that the differences are not large in the regions of interest. We will use the values in Columns 1 and 2 of Table 3.3 to develop the design charts summarizing the experimental data. Figure 3.8 shows a plot of values predicted using Eq. 3.5 versus experimental values of K_b. The R^2 for the plot is 0.951.

Table 3.3: Coefficients for Trend Lines of Figures 6 and 11

Bend Angle, ?	From Fig. 3.2		Doubled Values		Trend Lines, Fig. 3.6	
	a *(1)*	**b** *(2)*	**2a** *(3)*	**2b** *(4)*	**?** *(5)*	**?** *(6)*
90.5	-0.2967	0.4905	-0.5934	0.9810	-0.6041	0.9857
76	-0.2244	0.3740	-0.4488	0.7480	-0.5245	0.7814
58.5	-0.1452	0.2815	-0.2904	0.5630	-0.3063	0.5708
45.2	-0.0898	0.1431	-0.1796	0.2862	-0.1879	0.2891
30	-0.0598	0.0725	-0.1196	0.1450	-0.1488	0.1594

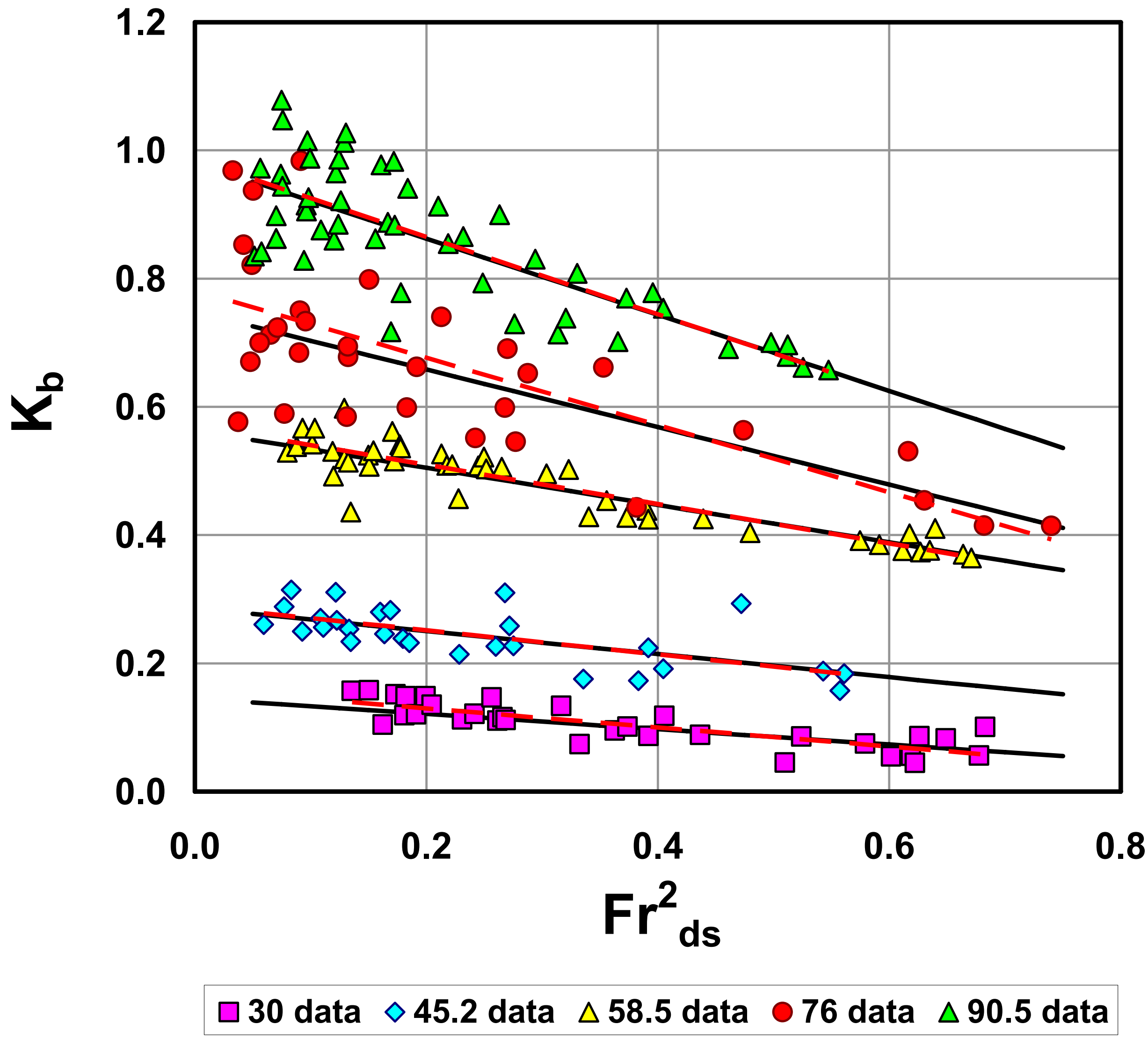

Figure 3.7: Experimental Data, Eq. 11 and Linear Trend Lines

The predictive equation for K_b is given below by combining Equation 3.5 and the parameters from Table 3.2 (repeated below for reference).

$$K_b = 2aFr_{ds}^2 + 2b = 2\left(m\theta^2 + n\theta + o\right)Fr_{ds}^2 + 2\left(p\theta^2 + q\theta + r\right) \qquad \text{Equation 3.8}$$

Where θ = bend angle in degrees; $Fr_{ds} = V^2_{ds}/y_{ds}$; h_b= bend loss and y_{ds} = depth downstream from the bend. Figure 3.8 is a plot of measured versus values predicted using Eq. 3.8. The R^2 value is 0.950.

Table 3.2: (repeated) Coefficients for Plots of Figure 3.3

Parameters for Coefficient a			Parameters for Coefficient b		
m	**n**	**o**	**p**	**q**	**r**
-3.0702 E-05	-3.0310E-04	-2.0139E-02	8.2164E-06	6.0159E-03	-1.2229E-01

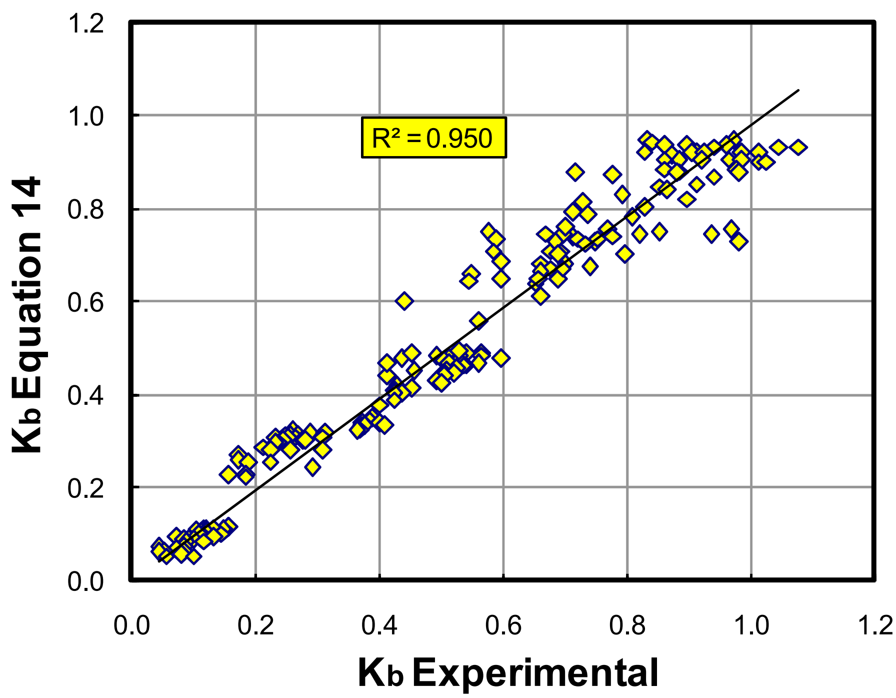

Figure 3.8: Comparison of K_b from Equation 14 and Experimental Data

3.2 Design Curves and Charts for Abrupt Bend Loss Coefficients

Figures 3.9 and 3.10 show the bend loss coefficient K_b versus Fr^2_{ds} and Fr_{ds}, respectively, based on Equation 3.8. Users may prefer Figure 3.10 since it involves only the downstream Froude number. Alternatively, user's can read K_b directly from Table 3.4.

<u>Example Problem</u>

Assume a 12x10 RCB carries 750 cfs and has a depth downstream from the bend of 7.5 feet. Determine the bend loss if a 40 degree abrupt bend exists in the culvert. (*The depth and velocity can be determined using HEC-RAS by modeling the culvert as a channel with a lid. This works very well if you make sure the lid is thick enough to contain the energy grade line elevation. In other words, the top of the lid must be higher than the energy grade line throughout the culvert. If you do not make the lid high enough, some of the water will "flow" above the culvert.*)

$$V_{ds} = \frac{Q}{A} = \frac{700}{(12*7))} = 8.33\, fps$$

$$Fr = \frac{V_{ds}}{\sqrt{gy}} = \frac{8.33}{\sqrt{(32.2)(7)}} = 0.555$$

From Figure 3.10 or Table 3.4.

$$K_b = 0.213$$

$$h_L = K_b \frac{V_{ds}^2}{2g} = (0.213)\frac{8.33^2}{2(32.2)} = 0.230\, ft$$

Alternately, from Figure 3.6

$$Fr_{ds} = 0.555 \rightarrow h^* = h_b / y_{ds} = 0.0329$$

$$h_b = y_{ds}\left(h_b / y_{ds}\right) = 7(0.0329) = 0.230\, ft$$

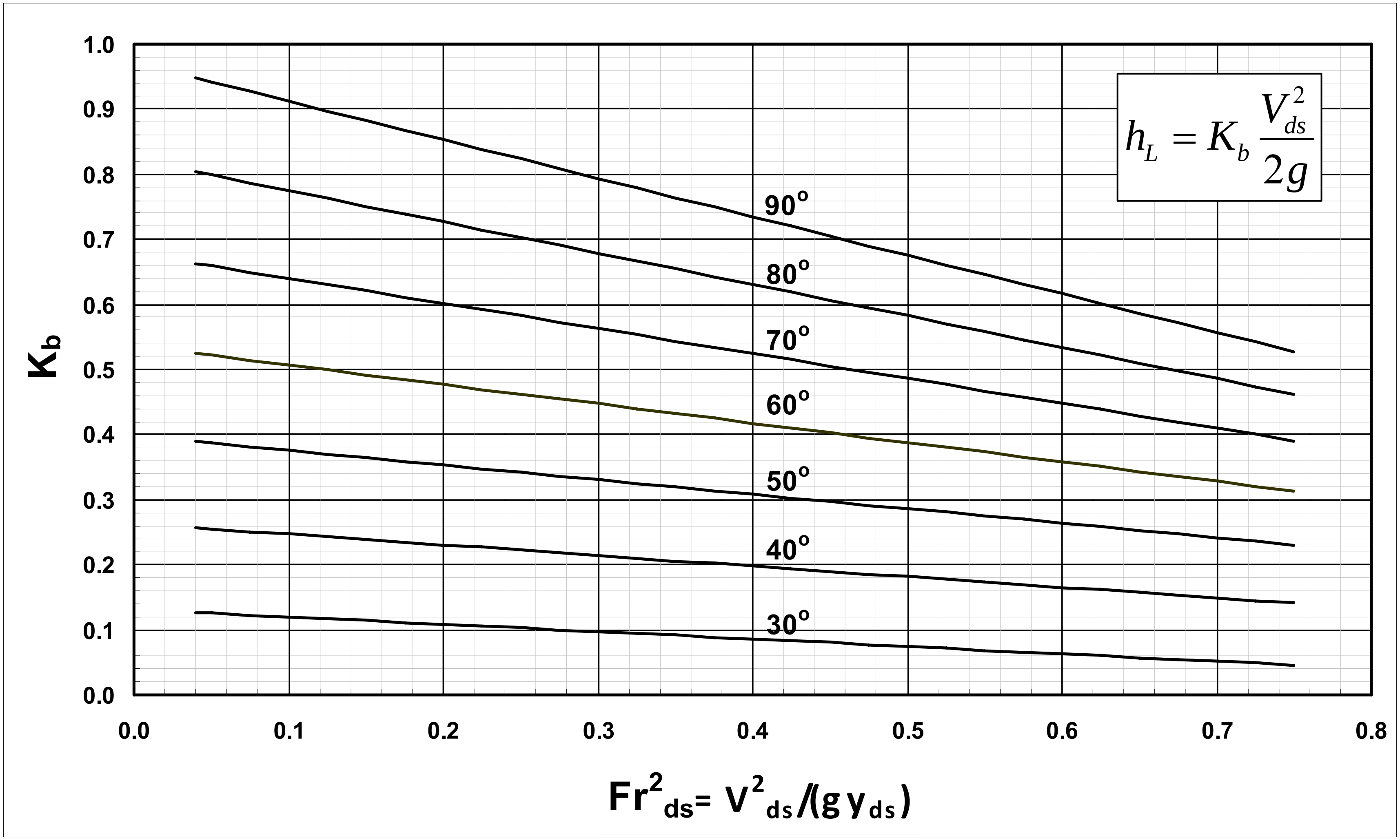

Figure 3.9: K_b versus Downstream Froude Number Squared.

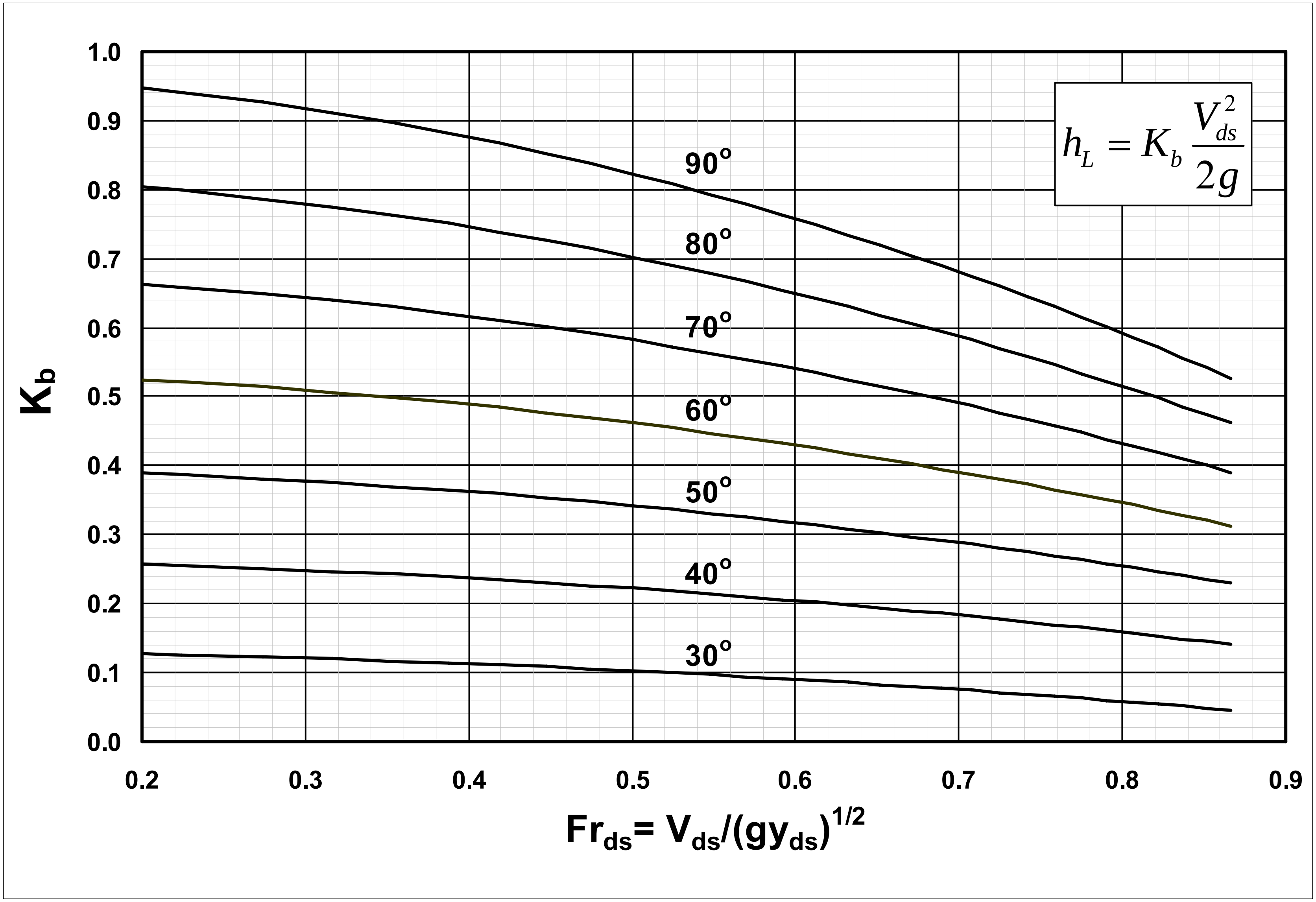

Figure 3.10: K_b versus Downstream Froude Number.

Table 3.4: Loss Coefficients, K_b, for Abrupt Bends in Rectangular Channels.

Fr	Abrupt Bend Angle, θ, in degrees												
	30	35	40	45	50	55	60	65	70	75	80	85	90
0.20	0.127	0.191	0.256	0.322	0.389	0.457	0.525	0.593	0.663	0.733	0.804	0.875	0.948
0.21	0.126	0.191	0.256	0.322	0.388	0.455	0.523	0.592	0.661	0.731	0.802	0.873	0.945
0.22	0.126	0.190	0.255	0.321	0.387	0.454	0.522	0.591	0.660	0.729	0.800	0.871	0.943
0.23	0.125	0.189	0.254	0.320	0.386	0.453	0.521	0.589	0.658	0.727	0.798	0.869	0.940
0.24	0.125	0.189	0.254	0.319	0.385	0.452	0.519	0.587	0.656	0.725	0.795	0.866	0.937
0.25	0.124	0.188	0.253	0.318	0.384	0.451	0.518	0.586	0.654	0.723	0.793	0.863	0.934
0.26	0.123	0.187	0.252	0.317	0.383	0.449	0.516	0.584	0.652	0.721	0.791	0.861	0.931
0.27	0.123	0.187	0.251	0.316	0.382	0.448	0.515	0.582	0.650	0.719	0.788	0.858	0.928
0.28	0.122	0.186	0.250	0.315	0.381	0.447	0.513	0.580	0.648	0.716	0.785	0.855	0.925
0.29	0.122	0.185	0.249	0.314	0.379	0.445	0.511	0.578	0.646	0.714	0.783	0.852	0.922
0.30	0.121	0.184	0.248	0.313	0.378	0.444	0.510	0.576	0.644	0.711	0.780	0.849	0.918
0.31	0.120	0.184	0.247	0.312	0.377	0.442	0.508	0.574	0.641	0.709	0.777	0.845	0.914
0.32	0.120	0.183	0.246	0.310	0.375	0.440	0.506	0.572	0.639	0.706	0.774	0.842	0.911
0.33	0.119	0.182	0.245	0.309	0.374	0.439	0.504	0.570	0.636	0.703	0.771	0.839	0.907
0.34	0.118	0.181	0.244	0.308	0.372	0.437	0.502	0.568	0.634	0.700	0.767	0.835	0.903
0.35	0.117	0.180	0.243	0.307	0.371	0.435	0.500	0.565	0.631	0.697	0.764	0.831	0.899
0.36	0.116	0.179	0.242	0.305	0.369	0.433	0.498	0.563	0.628	0.694	0.761	0.827	0.895
0.37	0.116	0.178	0.241	0.304	0.367	0.431	0.496	0.560	0.626	0.691	0.757	0.824	0.890
0.38	0.115	0.177	0.239	0.302	0.366	0.429	0.493	0.558	0.623	0.688	0.754	0.820	0.886
0.39	0.114	0.176	0.238	0.301	0.364	0.427	0.491	0.555	0.620	0.685	0.750	0.815	0.881
0.40	0.113	0.175	0.237	0.299	0.362	0.425	0.489	0.553	0.617	0.681	0.746	0.811	0.877
0.41	0.112	0.174	0.236	0.298	0.360	0.423	0.486	0.550	0.614	0.678	0.742	0.807	0.872
0.42	0.111	0.173	0.234	0.296	0.359	0.421	0.484	0.547	0.611	0.674	0.738	0.802	0.867
0.43	0.110	0.171	0.233	0.295	0.357	0.419	0.481	0.544	0.607	0.671	0.734	0.798	0.862
0.44	0.109	0.170	0.231	0.293	0.355	0.417	0.479	0.541	0.604	0.667	0.730	0.793	0.857
0.45	0.108	0.169	0.230	0.291	0.353	0.414	0.476	0.538	0.600	0.663	0.726	0.788	0.851
0.46	0.107	0.168	0.229	0.290	0.351	0.412	0.473	0.535	0.597	0.659	0.721	0.784	0.846
0.47	0.106	0.166	0.227	0.288	0.349	0.410	0.471	0.532	0.593	0.655	0.717	0.779	0.841
0.48	0.105	0.165	0.225	0.286	0.346	0.407	0.468	0.529	0.590	0.651	0.712	0.773	0.835
0.49	0.104	0.164	0.224	0.284	0.344	0.405	0.465	0.525	0.586	0.647	0.707	0.768	0.829
0.50	0.103	0.162	0.222	0.282	0.342	0.402	0.462	0.522	0.582	0.642	0.703	0.763	0.823
0.51	0.102	0.161	0.221	0.280	0.340	0.399	0.459	0.519	0.578	0.638	0.698	0.758	0.817
0.52	0.100	0.160	0.219	0.278	0.337	0.397	0.456	0.515	0.574	0.634	0.693	0.752	0.811
0.53	0.099	0.158	0.217	0.276	0.335	0.394	0.453	0.512	0.570	0.629	0.688	0.746	0.805
0.54	0.098	0.157	0.216	0.274	0.333	0.391	0.450	0.508	0.566	0.625	0.683	0.741	0.799
0.55	0.097	0.155	0.214	0.272	0.330	0.388	0.446	0.504	0.562	0.620	0.677	0.735	0.792
0.56	0.095	0.154	0.212	0.270	0.328	0.386	0.443	0.501	0.558	0.615	0.672	0.729	0.786
0.57	0.094	0.152	0.210	0.268	0.325	0.383	0.440	0.497	0.554	0.610	0.667	0.723	0.779
0.58	0.093	0.151	0.208	0.266	0.323	0.380	0.436	0.493	0.549	0.605	0.661	0.717	0.772
0.59	0.092	0.149	0.206	0.263	0.320	0.377	0.433	0.489	0.545	0.600	0.655	0.710	0.765

Table 3.4: Loss Coefficients, K_b, for Abrupt Bends in Rectangular Channels. (cont.)

Fr	Abrupt Bend Angle, θ, in degrees												
	30	35	40	45	50	55	60	65	70	75	80	85	90
0.59	0.092	0.149	0.206	0.263	0.320	0.377	0.433	0.489	0.545	0.600	0.655	0.710	0.765
0.60	0.090	0.147	0.204	0.261	0.317	0.374	0.429	0.485	0.540	0.595	0.650	0.704	0.758
0.61	0.089	0.146	0.202	0.259	0.315	0.370	0.426	0.481	0.535	0.590	0.644	0.698	0.751
0.62	0.087	0.144	0.200	0.256	0.312	0.367	0.422	0.477	0.531	0.585	0.638	0.691	0.744
0.63	0.086	0.142	0.198	0.254	0.309	0.364	0.418	0.472	0.526	0.579	0.632	0.684	0.736
0.64	0.085	0.141	0.196	0.252	0.306	0.361	0.415	0.468	0.521	0.574	0.626	0.678	0.729
0.65	0.083	0.139	0.194	0.249	0.303	0.357	0.411	0.464	0.516	0.568	0.620	0.671	0.721
0.66	0.082	0.137	0.192	0.247	0.300	0.354	0.407	0.459	0.511	0.562	0.613	0.664	0.713
0.67	0.080	0.135	0.190	0.244	0.297	0.350	0.403	0.455	0.506	0.557	0.607	0.656	0.706
0.68	0.079	0.133	0.188	0.241	0.294	0.347	0.399	0.450	0.501	0.551	0.600	0.649	0.698
0.69	0.077	0.132	0.185	0.239	0.291	0.343	0.395	0.445	0.496	0.545	0.594	0.642	0.689
0.70	0.075	0.130	0.183	0.236	0.288	0.340	0.391	0.441	0.490	0.539	0.587	0.634	0.681
0.71	0.074	0.128	0.181	0.233	0.285	0.336	0.386	0.436	0.485	0.533	0.580	0.627	0.673
0.72	0.072	0.126	0.179	0.231	0.282	0.332	0.382	0.431	0.479	0.527	0.573	0.619	0.664
0.73	0.071	0.124	0.176	0.228	0.279	0.329	0.378	0.426	0.474	0.520	0.566	0.612	0.656
0.74	0.069	0.122	0.174	0.225	0.275	0.325	0.373	0.421	0.468	0.514	0.559	0.604	0.647
0.75	0.067	0.120	0.171	0.222	0.272	0.321	0.369	0.416	0.462	0.508	0.552	0.596	0.638
0.76	0.065	0.118	0.169	0.219	0.269	0.317	0.365	0.411	0.457	0.501	0.545	0.588	0.629
0.77	0.064	0.116	0.166	0.216	0.265	0.313	0.360	0.406	0.451	0.495	0.537	0.579	0.620
0.78	0.062	0.113	0.164	0.213	0.262	0.309	0.355	0.401	0.445	0.488	0.530	0.571	0.611
0.79	0.060	0.111	0.161	0.210	0.258	0.305	0.351	0.395	0.439	0.481	0.522	0.563	0.602
0.80	0.058	0.109	0.159	0.207	0.255	0.301	0.346	0.390	0.433	0.474	0.515	0.554	0.592
0.81	0.057	0.107	0.156	0.204	0.251	0.297	0.341	0.384	0.426	0.467	0.507	0.546	0.583
0.82	0.055	0.105	0.154	0.201	0.247	0.292	0.336	0.379	0.420	0.460	0.499	0.537	0.573
0.83	0.053	0.102	0.151	0.198	0.244	0.288	0.331	0.373	0.414	0.453	0.491	0.528	0.563
0.84	0.051	0.100	0.148	0.195	0.240	0.284	0.326	0.368	0.408	0.446	0.483	0.519	0.554
0.85	0.049	0.098	0.145	0.191	0.236	0.279	0.321	0.362	0.401	0.439	0.475	0.510	0.544
0.86	0.047	0.096	0.143	0.188	0.232	0.275	0.316	0.356	0.394	0.431	0.467	0.501	0.533
0.87	0.046	0.094	0.141	0.186	0.230	0.272	0.313	0.353	0.390	0.427	0.462	0.495	0.527

CHAPTER 4 - GRADUAL CULVERT BENDS

The head loss for gradual culvert bend can be determined by the equation.

$$h_b = K_{gb} \frac{V_{ds}^2}{2g}$$ Equation 4.1

Where K_{gb} is the gradual bend loss coefficient. Values of K_{gb} for conduits flowing full are given in Table 4.1 as a function of the radius of curvature of the bend, r, divided by the equivalent culvert diameter, D_{eq} (4, 5). For circular culverts, D_{eq} is the pipe diameter. For noncircular culverts the following equation is used to define D_{eq}.

$$D_{eq} = \sqrt{4A/\pi}$$ Equation 4.2

Where A is the cross sectional area. The values in Table 4.1 are plotted in Figure 4.1.

Table 4.1: Loss Coefficients for Gradual Bends in Conduits Flowing Full (4, 5)

r_c/D_{eq}	K_{gb}		
	90°	**45°**	**22.5°**
1	*2*	*3*	*4*
1	0.5	0.37	0.25
2	0.3	0.22	0.15
4	0.25	0.19	0.12
6	0.15	0.11	0.08
8	0.15	0.11	0.08

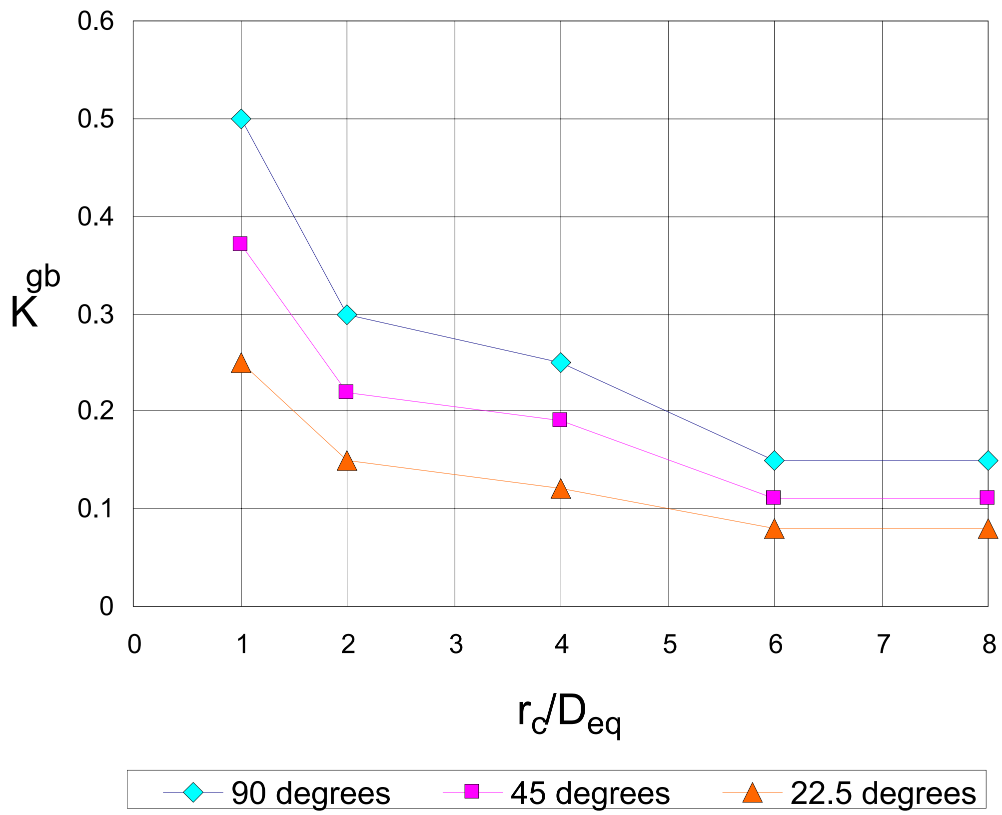

Figure 4.1: Loss Coefficients for Gradual Bends in Conduits Flowing Full (4, 5)

The K_{gb}-values in the top row of Table 4.1 are 0.5, 0.37 and 0.25, respectively, for the 90, 45 and 22.5 degree bends. These values will be called $K_{gb,1}$-values and are the bend loss coefficients for r/D_{eq} = 1. The values in Table 4.2 are the values in Table 4.1 divided by the appropriate $K_{gb,1}$-value . The β-values in Table 4.2 are plotted in Figure 4.2. Both Table 4.2 and Figure 4.2 show that the variation of β with r/D_{eq} is nearly the same for each of the three bend angles. The dashed line in the Figure 4.2 is a plot of the average β-values shown in Column 5 of Table 4.2. These average values will be used herein to estimate bend losses for gradual bends in rectangular open channels using the results of this study.

Table 4.2: $K_{gb}/K_{gb,1}$ and β Factor for Gradual Bends

r_c/D_{eq}	K_{gb}			β Average
	90°	**45°**	**22.5°**	
1	*2*	*3*	*4*	*5*
1	1	1	1	1
2	0.600	0.595	0.600	0.598
4	0.500	0.514	0.480	0.498
6	0.300	0.297	0.320	0.306
8	0.300	0.297	0.320	0.306

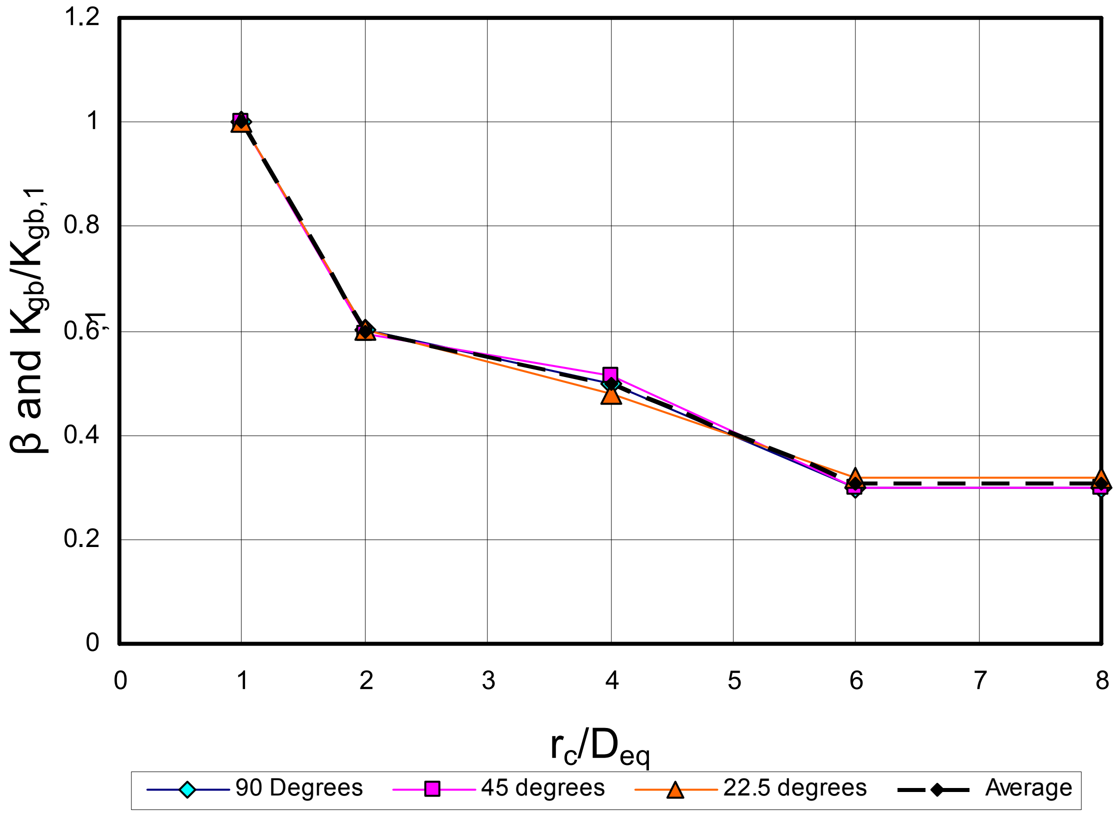

Figure 4.2: Plot of $K_{gb}/K_{gb,1}$ and β Factor for Gradual Culvert Bends

Table 4.3 shows β -values for r/D_{eq} ranging from 1 to 8. The table was developed from the values in Column 5 of Table 4.2 by linear interpolation. It will be assumed that the values in Table 4.2 can be applied to the abrupt bend loss coefficient defined by Eq. 3.8. The abrupt bend loss coefficient, K_b, is assumed to be equivalent to the parameter $K_{gb,1}$ in Table 4.2. Equation 4.3 shows Equation 4.1 expressed in terms of β and K_b where K_b is the bend loss coefficient for abrupt bends developed in Chapter 3 and given by Eq. 3.8.

$$h_b = K_{gb}\frac{V_{ds}^2}{2g} = K_b\left[K_{gb}/K_{gb,1}\right]\frac{V_{ds}^2}{2g} = \beta K_b \frac{V_{ds}^2}{2g} \qquad \text{Equation 4.3}$$

Table 4.3: β Factor versus r/D_{eq} for Gradual Bends

r_c/D_{eq}	$\beta=K_{gb}/K_{gb,1}$									
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	1.000	0.960	0.920	0.879	0.839	0.799	0.759	0.719	0.679	0.638
2	0.598	0.593	0.588	0.583	0.578	0.573	0.568	0.563	0.558	0.553
3	0.548	0.543	0.538	0.533	0.528	0.523	0.518	0.513	0.508	0.503
4	0.498	0.488	0.479	0.469	0.459	0.450	0.440	0.431	0.421	0.411
5	0.402	0.392	0.383	0.373	0.363	0.354	0.344	0.335	0.325	0.315
6	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306
7	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306
8	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306

The example problem below illustrates the procedure recommended for determining the bend loss for gradual bends used in rectangular culverts using the relationships developed in this chapter.

Example Problem

A 10-ft wide rectangular culvert has a gradual 50 degree bend with a radius curvature of 45 feet. The depth and velocity at the downstream end of the bend is known to be 8 feet and 6 feet per second, respectively. Estimate the bend loss. (Note that the shaded cells in Table 4.3 below are used in this example.)

$$Fr = \frac{V}{\sqrt{gy}} = \frac{6}{\sqrt{(32.2)8}} = 0.374 \xrightarrow{\text{Table 3.4 for } 50^o} K_b = 0.367$$

$$\text{Assume } K_b = K_{\text{gb},1} = 0.367$$

$$D_{eq} = \sqrt{4A/\pi} = \sqrt{4(8*10)/\pi} = \sqrt{4(80)/\pi} = 10.09\, ft$$

$$r / D_{eq} = 45/10.09 = 4.46 \xrightarrow{\text{Interpolated from Table 4.3}} \beta = 0.454$$

$$h_L = \beta \mathrm{K_b} \frac{V^2}{2g} = (0.454)(0.367)\frac{V^2}{2g} = (0.167)\left(\frac{6^2}{2(32.2)}\right) = 0.0934\, ft$$

r_c/D_{eq}	$\beta = K_{gb}/K_{gb,1}$									
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	1.000	0.960	0.920	0.879	0.839	0.799	0.759	0.719	0.679	0.638
2	0.598	0.593	0.588	0.583	0.578	0.573	0.568	0.563	0.558	0.553
3	0.548	0.543	0.538	0.533	0.528	0.523	0.518	0.513	0.508	0.503
4	0.498	0.488	0.479	0.469	0.459	0.450	0.440	0.431	0.421	0.411
5	0.402	0.392	0.383	0.373	0.363	0.354	0.344	0.335	0.325	0.315
6	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306
7	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306
8	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306

CHAPTER 5 - SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

A series of experiments were performed to develop head loss due to abrupt bends in rectangular culverts. Surprisingly, there was little information in the literature about abrupt bends in open channels. The experiments were performed in the Hydraulics Laboratory of the University of Kansas. A special flume was constructed for this study. Details of the flume are given in the body of the report. The channel section was approximately 20 feet long and 0.5-ft wide. Abrupt channel bend sections were installed near the middle of the flume. Five sections were built with the intention of testing 30, 45, 60, 75 and 90 degree bends. The exact angles were not quite met since the flume was constructed of marine plywood. The actual angles built and tested were 30, 45.2, 58.5, 76 and 90.5 degrees. All tests were performed for subcritical flow conditions.

A total of 190 experimental runs were performed with the Froude number varying from 0.18 to 0.86 and y_{ds}/W varying from 0.60 to 1.87. Water was supplied to the flume from the constant tank system of the hydraulics laboratory. Flow was controlled using a valve and a calibrated orifice in the supply line to the flume. The total head loss was measured from a section 4.72 feet upstream from the bend to a section that is 5.20 feet downstream from the bend. Piezometers were used at those locations to measure the hydrostatic head and, in turn, to determine the depths, velocities and total energy. The head loss due to the bend was then determined by subtracting the estimated head loss due to friction from the total measured head loss.

The experimental data were plotted and analyzed by statistical methods. Several iterations were performed to obtain reasonable fits. Plots were presented of dimensionless head loss versus the downstream Froude number. However, the more useful plots involved expressing the bend loss coefficient, K_b, as functions of the bend angle and the downstream Froude number.

Head loss for abrupt bends can be determined as follows.

(a) Determine the depth, y_{ds}, and velocity, V_{ds}, at a section far enough downstream from the culvert to assume hydrostatic pressure distribution exists. In this study the section was located about 10 culvert widths downstream from the bend. Typically, the section will not need to be that far downstream.

(b) Calculate the downstream Froude number.

$$Fr = \frac{V}{\sqrt{gy}}$$

(c) Read the bend loss coefficient, K_b, from either a design chart (Figure 3.10) or from the Table 3.4.

(d) Compute the loss due to the bend using the equation.

$$h_b = K_b \frac{V_{ds}^2}{2g}$$

The equations below are sometimes recommended for determining bend losses in culverts.

(http://epg.modot.mo.gov/index.php?title=750.2_Culverts and

http://www.fcd.maricopa.gov/Pub/Manuals/downloads/Hydraulics%20Design%20Manual.

pdf , respectively)

$$h_b = K_b^* \frac{V^2}{2g} = \left(0.5\sqrt{\theta/90}\right)\frac{V^2}{2g}$$ *(Equation 4.3)*

$$h_b = K_b^* \frac{V^2}{2g} = \left(0.0033\,\theta\right)\frac{V^2}{2g}$$ (Equation 5.1)

Where θ is the bend angle in degrees and K_b^* is the bend loss coefficient. These equations are compared with Eq. 3.8 for a range of downstream Froude numbers in Figure 5.1 The bend loss coefficients for r/D_{eq} = 1 are also shown as points (diamonds) for bend angles of 45° and 90°. It is apparent that the results of this study provide bend loss coefficients that differ significantly from other commonly used equations. The flume experiments demonstrated that head loss at abrupt bends in rectangular channels is influenced by both flow conditions and geometry.

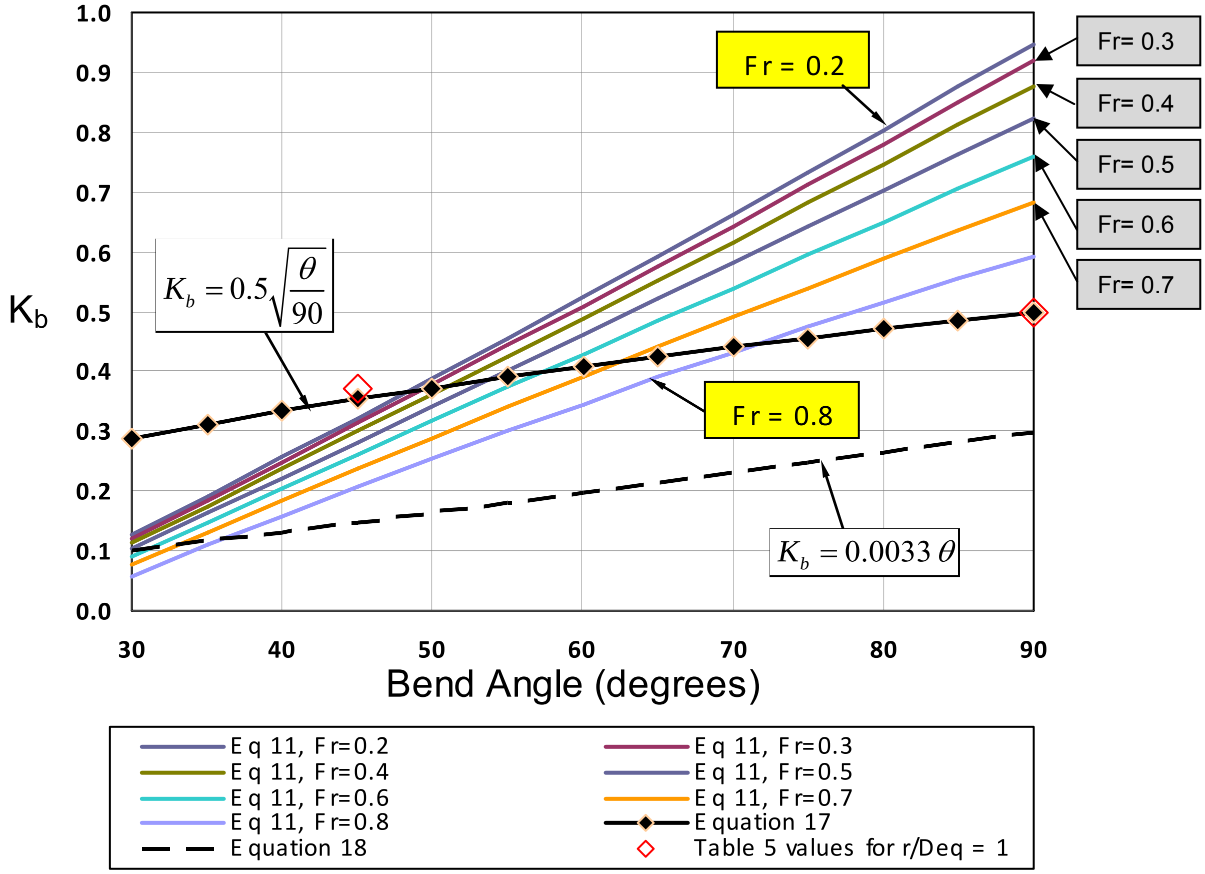

Figure 5.1: Comparison of Abrupt Bend Loss Coefficients from Eq. 4.3 with Values from Other Sources

A method for determining the head loss for gradual bends was developed that incorporated an adjustment factor β. The β factor was dependent on r/D_{eq} where r is the radius of curvature of the bend and D_{eq} is the equivalent diameter of the culvert ($D_{eq}=(4A/\pi)^{1/2}$). The β factor is equal to the average value of $K_{gb}/K_{gb,1}$ for the bend angles of 22.5, 45 and 90 degrees where $K_{gb,1}$ is the bend loss coefficient for r/D_{eq} =1. It was assumed herein that $K_{gb,1}$ equals K_b for an abrupt bend as defined by Eq. 3.5 and Table 3.4. The head loss for a gradual bend is, therefore, given by the equation.

$$h_b = K_{gb}\frac{V_{ds}^2}{2g} = K_b\left[K_{gb}/K_{gb,1}\right]\frac{V_{ds}^2}{2g} = \beta K_b\frac{V_{ds}^2}{2g} \qquad \text{Equation 5.2}$$

Where K_{gb} is the gradual bend loss coefficient. The Shock waves are created for supercritical flow and the flow can be analyze systematically. (2, 3) Moreover, if the culvert flow is supercritical it is likely that it is under inlet control. Thus, the effects of the bend on the supercritical flow profile will not affect the flow upstream from the culvert inlet unless it ultimately creates a hydraulic jump within the culvert and, in turn, the supercritical flow is “flooded out.”

This study produced results that can be used by practicing engineers to compute head loses for abrupt and gradual culvert bends. Tables 2.4 and 3.1 and Fig. 3.9 are repeated in the Design Tables and Charts section of APPENDIX A. Culvert bends will likely become more commonplace in the face of continued restrictions on construction at stream crossings. The bend-loss coefficients can be input into HEC-RAS via the ***Steady Flow Data editor*** option as shown by these screen captures. The ***K Loss*** button allows the user to input the bend loss coefficient when a culvert is modeled as an open channel with a lid. (1)

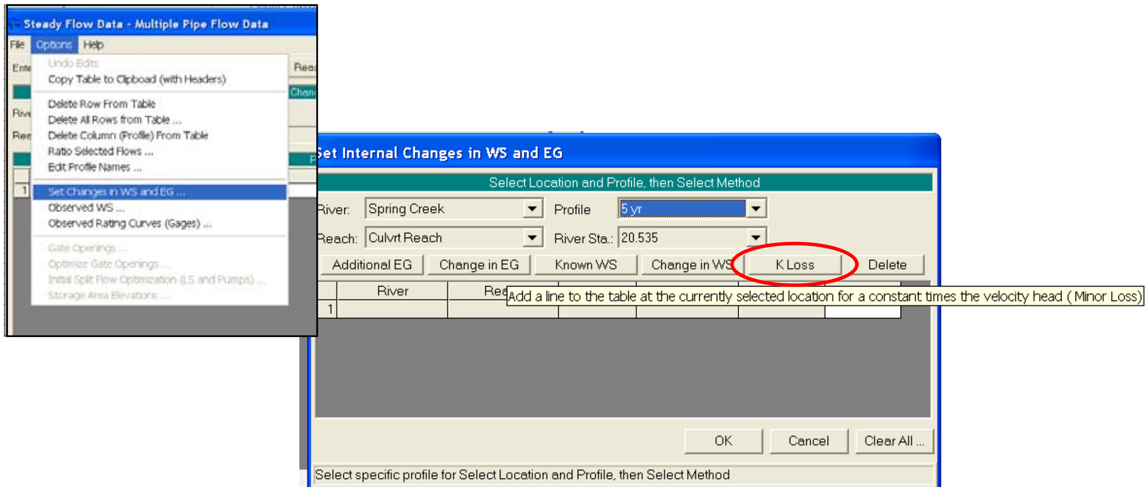

Figure 5.2: Screen Capture of HEC-RAS Input Location for of Bend Loss Coefficient

It is recommended that the results of this study be used with a good degree of engineering judgement. It is also recommended that experiments be undertaken to test the validity of the β factor developed herein to determine losses for gradual bends in rectangular channels.

CHAPTER 6 - REFERENCES

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APPENDIX A

Experimental Data

Table A1: Data for 30 Degree Experiments

Exp No.	Angle (deg)	Q (cfs)	$y_{us} = y_2$ (ft)	$y_{ds} = y_{16}$ (ft)	$z_2 - z_{16}$ (ft)	V_{ave} (fps)	h_{loss} (ft)	h_f (ft)
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
1	30	0.714	0.783	0.776	0.0156	1.83	0.0220	0.0138
2	30	0.756	0.789	0.779	0.0156	1.93	0.0245	0.0153
3	30	0.398	0.496	0.495	0.0156	1.61	0.0168	0.0126
4	30	0.714	0.727	0.715	0.0156	1.98	0.0259	0.0165
5	30	0.299	0.396	0.395	0.0156	1.52	0.0168	0.0125
6	30	0.586	0.627	0.617	0.0156	1.89	0.0242	0.0158
7	30	0.756	0.732	0.719	0.0156	2.08	0.0265	0.0183
8	30	0.656	0.659	0.645	0.0156	2.01	0.0272	0.0177
9	30	0.477	0.526	0.517	0.0156	1.83	0.0232	0.0160
10	30	0.535	0.548	0.536	0.0156	1.97	0.0253	0.0183
11	30	0.714	0.659	0.640	0.0156	2.20	0.0305	0.0212
12	30	0.398	0.438	0.425	0.0156	1.84	0.0257	0.0177
13	30	0.586	0.564	0.547	0.0156	2.11	0.0287	0.0208
14	30	0.756	0.667	0.644	0.0156	2.31	0.0331	0.0232
15	30	0.656	0.604	0.584	0.0156	2.21	0.0308	0.0221
16	30	0.477	0.469	0.447	0.0156	2.08	0.0314	0.0220
17	30	0.586	0.527	0.505	0.0156	2.27	0.0310	0.0248
18	30	0.714	0.592	0.559	0.0156	2.48	0.0379	0.0283
19	30	0.756	0.612	0.575	0.0156	2.55	0.0403	0.0294
20	30	0.586	0.509	0.478	0.0156	2.38	0.0358	0.0277
21	30	0.656	0.549	0.509	0.0156	2.48	0.0414	0.0292
22	30	0.398	0.384	0.356	0.0156	2.15	0.0330	0.0261
23	30	0.299	0.306	0.280	0.0156	2.05	0.0303	0.0270
24	30	0.535	0.454	0.408	0.0156	2.48	0.0413	0.0322
25	30	0.756	0.562	0.497	0.0156	2.86	0.0495	0.0387
26	30	0.477	0.409	0.361	0.0156	2.48	0.0399	0.0339
27	30	0.398	0.362	0.317	0.0156	2.34	0.0380	0.0325
28	30	0.299	0.299	0.262	0.0156	2.14	0.0339	0.0303
29	30	0.586	0.473	0.409	0.0156	2.66	0.0476	0.0366
30	30	0.535	0.443	0.380	0.0156	2.60	0.0463	0.0361
31	30	0.477	0.405	0.347	0.0156	2.54	0.0426	0.0360
32	30	0.656	0.509	0.428	0.0156	2.80	0.0541	0.0393

Table A1: Data for 30 Degree Experiments (Continued)

Exp No. 1	$F_{r,ds}$ 11	$h^*_b=h_b/y_{ds}$ 12	V_{ds} (fps) 13	$V^2_{ds}/2g$ (ft) 14	$F^2_{r,ds}$ 15	K_{bend} $[h_b/(V^2_{ds}/2g)]$ 16	y_{ds}/W 17
1	0.368	0.0106	1.84	0.0526	0.136	0.157	1.57
2	0.388	0.0119	1.94	0.0585	0.150	0.158	1.58
3	0.403	0.0085	1.61	0.0401	0.162	0.104	0.99
4	0.416	0.0131	2.00	0.0619	0.173	0.151	1.45
5	0.426	0.0108	1.52	0.0358	0.181	0.119	0.79
6	0.427	0.0135	1.90	0.0562	0.182	0.148	1.25
7	0.437	0.0115	2.10	0.0687	0.191	0.120	1.46
8	0.446	0.0148	2.03	0.0643	0.199	0.149	1.32
9	0.452	0.0139	1.84	0.0529	0.205	0.136	1.05
10	0.481	0.0130	2.00	0.0619	0.231	0.112	1.10
11	0.492	0.0147	2.23	0.0773	0.242	0.121	1.32
12	0.506	0.0189	1.87	0.0545	0.256	0.147	0.88
13	0.511	0.0145	2.15	0.0715	0.262	0.111	1.13
14	0.516	0.0154	2.35	0.0856	0.266	0.116	1.33
15	0.518	0.0149	2.25	0.0784	0.269	0.111	1.21
16	0.563	0.0211	2.13	0.0707	0.317	0.134	0.94
17	0.577	0.0123	2.32	0.0839	0.332	0.074	1.05
18	0.602	0.0173	2.56	0.1014	0.363	0.095	1.18
19	0.611	0.0189	2.63	0.1074	0.374	0.101	1.22
20	0.626	0.0170	2.46	0.0936	0.392	0.087	1.02
21	0.637	0.0239	2.58	0.1032	0.406	0.118	1.10
22	0.661	0.0193	2.24	0.0776	0.437	0.088	0.77
23	0.714	0.0115	2.14	0.0713	0.510	0.045	0.61
24	0.724	0.0224	2.62	0.1068	0.524	0.086	0.91
25	0.761	0.0217	3.04	0.1438	0.579	0.075	1.12
26	0.776	0.0165	2.64	0.1085	0.602	0.055	0.82
27	0.787	0.0173	2.51	0.0979	0.619	0.056	0.72
28	0.789	0.0138	2.29	0.0814	0.622	0.044	0.60
29	0.791	0.0269	2.87	0.1279	0.626	0.086	0.95
30	0.806	0.0269	2.82	0.1232	0.649	0.083	0.89
31	0.823	0.0190	2.75	0.1174	0.678	0.056	0.81
32	0.826	0.0344	3.07	0.1460	0.683	0.101	1.02

Table A2: Data for 45.2 Degree Experiments

Exp No.	Angle (deg)	Q (cfs)	$y_{us} = y_2$ (ft)	$y_{ds} = y_{16}$ (ft)	$z_2 - z_{16}$ (ft)	V_{ave} (fps)	h_{loss} (ft)	h_f (ft)	h_b (ft)
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
33	45.2	0.514	0.847	0.820	-0.0130	1.23	0.0125	0.0061	0.00635
34	45.2	0.565	0.833	0.801	-0.0130	1.38	0.0167	0.0078	0.00892
35	45.2	0.514	0.766	0.733	-0.0130	1.37	0.0174	0.0078	0.00960
36	45.2	0.642	0.855	0.820	-0.0130	1.53	0.0189	0.0094	0.00951
37	45.2	0.693	0.859	0.819	-0.0130	1.65	0.0230	0.0110	0.01200
38	45.2	0.565	0.747	0.710	-0.0130	1.55	0.0202	0.0101	0.01008
39	45.2	0.514	0.686	0.646	-0.0130	1.54	0.0225	0.0103	0.01222
40	45.2	0.749	0.872	0.828	-0.0130	1.76	0.0260	0.0124	0.01357
41	45.2	0.693	0.809	0.765	-0.0130	1.76	0.0256	0.0127	0.01291
42	45.2	0.642	0.766	0.724	-0.0130	1.72	0.0238	0.0124	0.01141
43	45.2	0.565	0.675	0.628	-0.0130	1.73	0.0272	0.0132	0.01407
44	45.2	0.749	0.803	0.752	-0.0130	1.93	0.0304	0.0153	0.01515
45	45.2	0.514	0.626	0.579	-0.0130	1.71	0.0269	0.0131	0.01382
46	45.2	0.693	0.744	0.692	-0.0130	1.93	0.0306	0.0157	0.01487
47	45.2	0.642	0.702	0.651	-0.0130	1.90	0.0295	0.0155	0.01402
48	45.2	0.749	0.734	0.673	-0.0130	2.13	0.0357	0.0193	0.01645
49	45.2	0.693	0.678	0.612	-0.0130	2.15	0.0383	0.0203	0.01800
50	45.2	0.565	0.598	0.529	-0.0130	2.01	0.0406	0.0186	0.02197
51	45.2	0.642	0.642	0.573	-0.0130	2.11	0.0402	0.0200	0.02011
52	45.2	0.514	0.553	0.492	-0.0130	1.97	0.0339	0.0185	0.01540
53	45.2	0.749	0.669	0.592	-0.0130	2.38	0.0424	0.0250	0.01744
54	45.2	0.642	0.591	0.511	-0.0130	2.33	0.0423	0.0253	0.01694
55	45.2	0.565	0.549	0.466	-0.0130	2.23	0.0445	0.0240	0.02048
56	45.2	0.693	0.617	0.528	-0.0130	2.42	0.0474	0.0269	0.02046
57	45.2	0.514	0.512	0.411	-0.0130	2.23	0.0535	0.0250	0.02845
58	45.2	0.565	0.522	0.418	-0.0130	2.40	0.0503	0.0290	0.02132
59	45.2	0.642	0.559	0.451	-0.0130	2.54	0.0511	0.0313	0.01981
60	45.2	0.514	0.490	0.388	-0.0130	2.34	0.0484	0.0283	0.02005

Table A2: Data for 45.2 Degree Experiments (Continued)

Exp No. *1*	$F_{r,ds}$ *11*	$h^*_b=h_b/y_{ds}$ *12*	V_{ds} (fps) *13*	$V^2_{ds}/2g$ (ft) *14*	$F^2_{r,ds}$ *15*	K_{bend} $[h_b/(V^2_{ds}/2g)]$ *16*	y_{ds}/W *17*
33	0.244	0.0077	1.25	0.0244	0.059	0.260	1.69
34	0.278	0.0111	1.41	0.0309	0.077	0.288	1.67
35	0.289	0.0131	1.40	0.0305	0.083	0.314	1.53
36	0.305	0.0116	1.57	0.0380	0.093	0.250	1.71
37	0.329	0.0147	1.69	0.0444	0.108	0.270	1.72
38	0.333	0.0142	1.59	0.0394	0.111	0.256	1.49
39	0.349	0.0189	1.59	0.0393	0.122	0.311	1.37
40	0.350	0.0164	1.81	0.0508	0.123	0.267	1.74
41	0.365	0.0169	1.81	0.0509	0.133	0.253	1.62
42	0.367	0.0158	1.77	0.0488	0.135	0.234	1.53
43	0.400	0.0224	1.80	0.0503	0.160	0.280	1.35
44	0.405	0.0201	1.99	0.0616	0.164	0.246	1.61
45	0.411	0.0239	1.77	0.0489	0.169	0.283	1.25
46	0.424	0.0215	2.00	0.0622	0.180	0.239	1.49
47	0.431	0.0215	1.97	0.0603	0.185	0.232	1.40
48	0.478	0.0244	2.23	0.0769	0.229	0.214	1.47
49	0.510	0.0294	2.26	0.0796	0.260	0.226	1.36
50	0.518	0.0415	2.14	0.0709	0.268	0.310	1.20
51	0.521	0.0351	2.24	0.0779	0.272	0.258	1.28
52	0.525	0.0313	2.09	0.0677	0.275	0.227	1.11
53	0.579	0.0295	2.53	0.0994	0.336	0.175	1.34
54	0.619	0.0331	2.51	0.0979	0.383	0.173	1.18
55	0.626	0.0439	2.43	0.0913	0.392	0.224	1.10
56	0.636	0.0387	2.62	0.1069	0.405	0.191	1.23
57	0.687	0.0692	2.50	0.0970	0.472	0.293	1.02
58	0.737	0.0510	2.70	0.1135	0.543	0.188	1.04
59	0.746	0.0439	2.85	0.1257	0.557	0.158	1.12
60	0.749	0.0517	2.65	0.1089	0.561	0.184	0.98

Table A3: Data for 58.5 Degree Experiments

Exp No.	Angle (deg)	Q (cfs)	$y_{us} = y_2$ (ft)	$y_{ds} = y_{16}$ (ft)	$z_2 - z_{16}$ (ft)	V_{ave} (fps)	h_{loss} (ft)	h_f (ft)	h_b (ft)
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
61	58.5	0.493	0.753	0.723	-0.0052	1.34	0.0227	0.0075	0.01527
62	58.5	0.287	0.514	0.489	-0.0052	1.15	0.0179	0.0064	0.01155
63	58.5	0.537	0.764	0.728	-0.0052	1.44	0.0279	0.0087	0.01920
64	58.5	0.589	0.791	0.752	-0.0052	1.53	0.0303	0.0096	0.02069
65	58.5	0.493	0.700	0.663	-0.0052	1.45	0.0284	0.0090	0.01945
66	58.5	0.537	0.712	0.671	-0.0052	1.55	0.0315	0.0104	0.02117
67	58.5	0.691	0.836	0.791	-0.0052	1.70	0.0350	0.0117	0.02332
68	58.5	0.662	0.802	0.750	-0.0052	1.71	0.0409	0.0120	0.02892
69	58.5	0.589	0.739	0.694	-0.0052	1.64	0.0347	0.0114	0.02324
70	58.5	0.625	0.763	0.716	-0.0052	1.69	0.0363	0.0120	0.02435
71	58.5	0.287	0.454	0.424	-0.0052	1.31	0.0213	0.0089	0.01245
72	58.5	0.493	0.632	0.586	-0.0052	1.62	0.0348	0.0118	0.02306
73	58.5	0.691	0.787	0.733	-0.0052	1.82	0.0417	0.0137	0.02797
74	58.5	0.747	0.825	0.766	-0.0052	1.88	0.0458	0.0144	0.03141
75	58.5	0.537	0.650	0.595	-0.0052	1.73	0.0417	0.0133	0.02847
76	58.5	0.662	0.740	0.681	-0.0052	1.86	0.0450	0.0147	0.03026
77	58.5	0.625	0.710	0.650	-0.0052	1.84	0.0457	0.0146	0.03109
78	58.5	0.589	0.682	0.624	-0.0052	1.80	0.0439	0.0142	0.02970
79	58.5	0.691	0.725	0.653	-0.0052	2.00	0.0538	0.0172	0.03661
80	58.5	0.747	0.758	0.683	-0.0052	2.07	0.0560	0.0182	0.03782
81	58.5	0.493	0.575	0.514	-0.0052	1.81	0.0445	0.0154	0.02911
82	58.5	0.287	0.401	0.356	-0.0052	1.52	0.0314	0.0129	0.01851
83	58.5	0.662	0.683	0.606	-0.0052	2.05	0.0562	0.0185	0.03765
84	58.5	0.537	0.595	0.524	-0.0052	1.92	0.0513	0.0171	0.03413
85	58.5	0.625	0.654	0.578	-0.0052	2.03	0.0550	0.0184	0.03661
86	58.5	0.589	0.623	0.546	-0.0052	2.02	0.0552	0.0185	0.03662
87	58.5	0.691	0.673	0.580	-0.0052	2.20	0.0653	0.0216	0.04369
88	58.5	0.747	0.702	0.599	-0.0052	2.30	0.0717	0.0231	0.04856
89	58.5	0.493	0.525	0.446	-0.0052	2.03	0.0528	0.0203	0.03250
90	58.5	0.662	0.634	0.535	-0.0052	2.26	0.0666	0.0234	0.04316
91	58.5	0.625	0.604	0.507	-0.0052	2.25	0.0640	0.0236	0.04040
92	58.5	0.589	0.579	0.480	-0.0052	2.23	0.0646	0.0235	0.04110
93	58.5	0.537	0.544	0.451	-0.0052	2.16	0.0603	0.0228	0.03752
94	58.5	0.691	0.631	0.513	-0.0052	2.42	0.0747	0.0268	0.04790
95	58.5	0.747	0.655	0.525	-0.0052	2.53	0.0799	0.0292	0.05078
96	58.5	0.662	0.597	0.456	-0.0052	2.51	0.0814	0.0301	0.05125
97	58.5	0.493	0.493	0.371	-0.0052	2.28	0.0694	0.0271	0.04223
98	58.5	0.625	0.572	0.430	-0.0052	2.50	0.0798	0.0303	0.04941
99	58.5	0.691	0.614	0.458	-0.0052	2.58	0.0882	0.0314	0.05682
100	58.5	0.589	0.550	0.410	-0.0052	2.46	0.0779	0.0299	0.04795
101	58.5	0.537	0.519	0.384	-0.0052	2.38	0.0748	0.0290	0.04582
102	58.5	0.747	0.647	0.477	-0.0052	2.66	0.0953	0.0328	0.06256
103	58.5	0.493	0.490	0.357	-0.0052	2.33	0.0724	0.0285	0.04384
104	58.5	0.537	0.517	0.377	-0.0052	2.41	0.0757	0.0297	0.04604

Table A3: Data for 58.5 Degree Experiments (Continued)

Exp No. *1*	$F_{r,ds}$ *11*	$h^*_b=h_b/y_{ds}$ *12*	V_{ds} (fps) *13*	$V^2_{ds}/2g$ (ft) *14*	$F^2_{r,ds}$ *15*	K_{bend} $[h_b/(V^2_{ds}/2g)]$ *16*	y_{ds}/W *17*
61	0.283	0.0211	1.36	0.0289	0.080	0.529	1.51
62	0.296	0.0236	1.18	0.0215	0.088	0.538	1.03
63	0.305	0.0264	1.48	0.0339	0.093	0.567	1.53
64	0.319	0.0275	1.57	0.0381	0.101	0.542	1.58
65	0.322	0.0293	1.49	0.0343	0.104	0.567	1.40
66	0.345	0.0316	1.60	0.0399	0.119	0.531	1.42
67	0.346	0.0295	1.75	0.0474	0.120	0.492	1.67
68	0.359	0.0386	1.77	0.0484	0.129	0.598	1.60
69	0.359	0.0335	1.70	0.0448	0.129	0.519	1.48
70	0.364	0.0340	1.75	0.0474	0.132	0.514	1.53
71	0.367	0.0294	1.36	0.0286	0.135	0.436	0.91
72	0.387	0.0394	1.68	0.0439	0.150	0.525	1.26
73	0.388	0.0382	1.88	0.0552	0.151	0.507	1.57
74	0.393	0.0410	1.95	0.0591	0.154	0.531	1.65
75	0.413	0.0479	1.81	0.0507	0.171	0.561	1.30
76	0.415	0.0444	1.94	0.0587	0.172	0.516	1.48
77	0.421	0.0478	1.92	0.0575	0.177	0.541	1.42
78	0.422	0.0476	1.89	0.0554	0.178	0.536	1.36
79	0.462	0.0561	2.12	0.0695	0.213	0.527	1.45
80	0.467	0.0554	2.19	0.0743	0.218	0.509	1.52
81	0.471	0.0567	1.92	0.0571	0.222	0.510	1.15
82	0.477	0.0520	1.62	0.0405	0.228	0.457	0.80
83	0.495	0.0622	2.18	0.0741	0.245	0.508	1.37
84	0.500	0.0652	2.05	0.0654	0.250	0.522	1.19
85	0.502	0.0634	2.16	0.0727	0.252	0.503	1.31
86	0.515	0.0671	2.16	0.0724	0.265	0.506	1.25
87	0.551	0.0754	2.38	0.0881	0.304	0.496	1.35
88	0.568	0.0811	2.50	0.0967	0.323	0.502	1.40
89	0.583	0.0729	2.21	0.0759	0.340	0.428	1.05
90	0.596	0.0807	2.48	0.0951	0.356	0.454	1.27
91	0.611	0.0797	2.47	0.0945	0.373	0.427	1.21
92	0.625	0.0857	2.46	0.0937	0.390	0.439	1.16
93	0.626	0.0832	2.38	0.0883	0.392	0.425	1.09
94	0.663	0.0934	2.69	0.1127	0.439	0.425	1.26
95	0.693	0.0968	2.85	0.1259	0.480	0.403	1.31
96	0.758	0.1125	2.90	0.1310	0.575	0.391	1.19
97	0.769	0.1139	2.66	0.1096	0.591	0.385	0.99
98	0.782	0.1150	2.91	0.1315	0.612	0.376	1.14
99	0.786	0.1241	3.02	0.1414	0.618	0.402	1.23
100	0.792	0.1170	2.88	0.1284	0.627	0.373	1.10
101	0.797	0.1194	2.80	0.1218	0.635	0.376	1.04
102	0.800	0.1312	3.13	0.1525	0.640	0.410	1.29
103	0.815	0.1229	2.76	0.1184	0.664	0.370	0.98
104	0.819	0.1222	2.85	0.1264	0.671	0.364	1.03

Table A4: Data for 76 Degree Experiments

Exp No.	Angle (deg)	Q (cfs)	$y_{us} = y_2$ (ft)	$y_{ds} = y_{16}$ (ft)	$z_2 - z_{16}$ (ft)	V_{ave} (fps)	h_{loss} (ft)	h_f (ft)	h_b (ft)
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
105	76.2	0.228	0.581	0.582	0.0130	0.78	0.0120	0.0028	0.00922
106	76.2	0.228	0.507	0.504	0.0130	0.90	0.0159	0.0039	0.01191
107	76.2	0.228	0.426	0.413	0.0130	1.09	0.0249	0.0062	0.01862
108	76.2	0.228	0.370	0.350	0.0130	1.27	0.0302	0.0092	0.02104
109	76.2	0.228	0.324	0.288	0.0130	1.49	0.0408	0.0140	0.02688
110	76.2	0.364	0.759	0.760	0.0130	0.96	0.0120	0.0038	0.00823
111	76.2	0.364	0.700	0.694	0.0130	1.04	0.0187	0.0047	0.01405
112	76.2	0.364	0.640	0.632	0.0130	1.15	0.0205	0.0058	0.01470
113	76.2	0.364	0.581	0.566	0.0130	1.27	0.0267	0.0074	0.01928
114	76.2	0.364	0.521	0.499	0.0130	1.43	0.0323	0.0098	0.02242
115	76.2	0.364	0.468	0.426	0.0130	1.63	0.0472	0.0136	0.03360
116	76.2	0.364	0.426	0.360	0.0130	1.85	0.0608	0.0188	0.04205
117	76.2	0.364	0.401	0.299	0.0130	2.08	0.0741	0.0252	0.04889
118	76.2	0.479	0.807	0.798	0.0130	1.19	0.0215	0.0058	0.01569
119	76.2	0.479	0.750	0.736	0.0130	1.29	0.0260	0.0070	0.01907
120	76.2	0.479	0.689	0.668	0.0130	1.41	0.0321	0.0086	0.02347
121	76.2	0.479	0.629	0.600	0.0130	1.56	0.0384	0.0109	0.02754
122	76.2	0.479	0.572	0.530	0.0130	1.74	0.0478	0.0142	0.03366
123	76.2	0.479	0.527	0.463	0.0130	1.94	0.0618	0.0184	0.04345
124	76.2	0.479	0.489	0.392	0.0130	2.18	0.0768	0.0245	0.05232
125	76.2	0.311	0.645	0.630	0.0000	0.97	0.0143	0.0042	0.01012
126	76.2	0.436	0.845	0.825	0.0000	1.04	0.0192	0.0044	0.01480
127	76.2	0.629	0.890	0.860	0.0000	1.44	0.0278	0.0082	0.01960
128	76.2	0.436	0.670	0.640	0.0000	1.33	0.0275	0.0077	0.01973
129	76.2	0.749	0.860	0.810	0.0000	1.79	0.0440	0.0130	0.03105
130	76.2	0.629	0.705	0.645	0.0000	1.86	0.0504	0.0150	0.03537
131	76.2	0.749	0.740	0.660	0.0000	2.14	0.0636	0.0195	0.04410
132	76.2	0.436	0.505	0.440	0.0000	1.85	0.0503	0.0170	0.03328
133	76.2	0.311	0.410	0.355	0.0000	1.62	0.0431	0.0146	0.02846
134	76.2	0.629	0.600	0.505	0.0000	2.28	0.0669	0.0242	0.04269
135	76.2	0.749	0.650	0.480	0.0000	2.65	0.1012	0.0325	0.06870
136	76.2	0.629	0.575	0.405	0.0000	2.57	0.0945	0.0324	0.06207
137	76.2	0.311	0.365	0.260	0.0000	1.99	0.0613	0.0246	0.03678

Table A4: Data for 76 Degree Experiments (Continued)

Exp No. 1	$F_{r,ds}$ 11	$h^*_b=h_b/y_{ds}$ 12	V_{ds} (fps) 13	$V^2_{ds}/2g$ (ft) 14	$F^2_{r,ds}$ 15	K_{bend} $[h_b/(V^2_{ds}/2g)]$ 16	y_{ds}/W 17
105	0.181	0.0158	0.78	0.0095	0.033	0.968	1.16
106	0.225	0.0236	0.90	0.0127	0.050	0.937	1.01
107	0.303	0.0451	1.10	0.0189	0.092	0.984	0.85
108	0.388	0.0601	1.30	0.0263	0.151	0.799	0.74
109	0.520	0.0933	1.58	0.0389	0.270	0.691	0.65
110	0.194	0.0108	0.96	0.0143	0.038	0.577	1.52
111	0.222	0.0202	1.05	0.0171	0.049	0.821	1.40
112	0.255	0.0233	1.15	0.0206	0.065	0.713	1.28
113	0.301	0.0341	1.29	0.0257	0.091	0.750	1.16
114	0.364	0.0449	1.46	0.0331	0.133	0.678	1.04
115	0.462	0.0789	1.71	0.0454	0.213	0.740	0.94
116	0.594	0.1168	2.02	0.0636	0.353	0.662	0.85
117	0.785	0.1635	2.44	0.0921	0.616	0.531	0.80
118	0.237	0.0197	1.20	0.0224	0.056	0.700	1.61
119	0.268	0.0259	1.30	0.0264	0.072	0.724	1.50
120	0.310	0.0351	1.44	0.0320	0.096	0.734	1.38
121	0.364	0.0459	1.60	0.0397	0.132	0.694	1.26
122	0.438	0.0635	1.81	0.0508	0.192	0.662	1.14
123	0.536	0.0938	2.07	0.0666	0.288	0.652	1.05
124	0.688	0.1335	2.45	0.0929	0.474	0.563	0.98
125	0.219	0.0161	0.99	0.0151	0.048	0.670	1.29
126	0.205	0.0179	1.06	0.0174	0.042	0.853	1.69
127	0.278	0.0228	1.46	0.0332	0.077	0.590	1.78
128	0.300	0.0308	1.36	0.0288	0.090	0.684	1.34
129	0.362	0.0383	1.85	0.0531	0.131	0.584	1.72
130	0.428	0.0548	1.95	0.0591	0.183	0.599	1.41
131	0.492	0.0668	2.27	0.0800	0.242	0.551	1.48
132	0.527	0.0756	1.98	0.0610	0.277	0.545	1.01
133	0.518	0.0802	1.75	0.0476	0.268	0.599	0.82
134	0.618	0.0845	2.49	0.0964	0.382	0.443	1.20
135	0.794	0.1431	3.12	0.1513	0.630	0.454	1.30
136	0.860	0.1533	3.11	0.1499	0.740	0.414	1.15
137	0.826	0.1415	2.39	0.0887	0.682	0.415	0.73

Table A5: Data for 90.5 Degree Experiments

Exp No.	Angle (deg)	Q (cfs)	$y_{us} = y_2$ (ft)	$y_{ds} = y_{16}$ (ft)	$z_2 - z_{16}$ (ft)	V_{ave} (fps)	h_{loss} (ft)	h_f (ft)	h_b (ft)
1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
138	90.5	0.278	0.608	0.572	-0.0182	0.94	0.0163	0.0040	0.01227
139	90.5	0.561	0.935	0.884	-0.0182	1.23	0.0303	0.0060	0.02436
140	90.5	0.473	0.830	0.785	-0.0182	1.17	0.0246	0.0056	0.01902
141	90.5	0.278	0.557	0.515	-0.0182	1.04	0.0214	0.0051	0.01627
142	90.5	0.380	0.680	0.634	-0.0182	1.16	0.0251	0.0058	0.01924
143	90.5	0.380	0.680	0.634	-0.0182	1.16	0.0251	0.0058	0.01924
144	90.5	0.561	0.866	0.808	-0.0182	1.34	0.0361	0.0072	0.02887
145	90.5	0.582	0.888	0.825	-0.0182	1.36	0.0408	0.0074	0.03337
146	90.5	0.473	0.771	0.717	-0.0182	1.27	0.0323	0.0068	0.02556
147	90.5	0.616	0.917	0.853	-0.0182	1.39	0.0416	0.0077	0.03396
148	90.5	0.380	0.627	0.575	-0.0182	1.26	0.0297	0.0072	0.02246
149	90.5	0.667	0.901	0.831	-0.0182	1.54	0.0460	0.0094	0.03658
150	90.5	0.667	0.901	0.831	-0.0182	1.54	0.0460	0.0094	0.03658
151	90.5	0.473	0.721	0.661	-0.0182	1.37	0.0369	0.0080	0.02886
152	90.5	0.616	0.858	0.786	-0.0182	1.50	0.0478	0.0091	0.03876
153	90.5	0.561	0.802	0.736	-0.0182	1.46	0.0423	0.0088	0.03348
154	90.5	0.582	0.821	0.751	-0.0182	1.48	0.0459	0.0090	0.03688
155	90.5	0.278	0.496	0.445	-0.0182	1.18	0.0282	0.0070	0.02125
156	90.5	0.473	0.680	0.614	-0.0182	1.46	0.0412	0.0094	0.03177
157	90.5	0.702	0.880	0.795	-0.0182	1.68	0.0581	0.0113	0.04676
158	90.5	0.380	0.587	0.525	-0.0182	1.37	0.0375	0.0087	0.02878
159	90.5	0.667	0.848	0.763	-0.0182	1.66	0.0579	0.0111	0.04680
160	90.5	0.561	0.753	0.677	-0.0182	1.57	0.0498	0.0104	0.03936
161	90.5	0.616	0.800	0.715	-0.0182	1.63	0.0577	0.0110	0.04671
162	90.5	0.582	0.770	0.686	-0.0182	1.60	0.0567	0.0108	0.04598
163	90.5	0.473	0.640	0.563	-0.0182	1.57	0.0490	0.0112	0.03787
164	90.5	0.702	0.827	0.725	-0.0182	1.81	0.0705	0.0135	0.05699
165	90.5	0.667	0.788	0.692	-0.0182	1.80	0.0648	0.0136	0.05120
166	90.5	0.380	0.540	0.473	-0.0182	1.50	0.0396	0.0109	0.02875
167	90.5	0.616	0.750	0.650	-0.0182	1.76	0.0681	0.0132	0.05487
168	90.5	0.561	0.700	0.610	-0.0182	1.71	0.0593	0.0128	0.04648
169	90.5	0.278	0.441	0.378	-0.0182	1.36	0.0360	0.0099	0.02617
170	90.5	0.582	0.711	0.612	-0.0182	1.76	0.0664	0.0135	0.05291
171	90.5	0.702	0.780	0.663	-0.0182	1.95	0.0796	0.0160	0.06364
172	90.5	0.473	0.598	0.503	-0.0182	1.72	0.0609	0.0138	0.04704
173	90.5	0.667	0.738	0.620	-0.0182	1.96	0.0788	0.0166	0.06221
174	90.5	0.561	0.648	0.540	-0.0182	1.89	0.0694	0.0162	0.05326
175	90.5	0.582	0.666	0.543	-0.0182	1.93	0.0810	0.0167	0.06430
176	90.5	0.380	0.494	0.402	-0.0182	1.70	0.0552	0.0147	0.04048
177	90.5	0.702	0.733	0.593	-0.0182	2.12	0.0919	0.0195	0.07234
178	90.5	0.278	0.399	0.313	-0.0182	1.56	0.0491	0.0141	0.03502
179	90.5	0.473	0.557	0.443	-0.0182	1.89	0.0699	0.0175	0.05241
180	90.5	0.667	0.697	0.551	-0.0182	2.14	0.0938	0.0203	0.07353
181	90.5	0.561	0.608	0.475	-0.0182	2.07	0.0811	0.0202	0.06091
182	90.5	0.616	0.651	0.502	-0.0182	2.14	0.0930	0.0210	0.07205
183	90.5	0.582	0.626	0.474	-0.0182	2.12	0.0939	0.0210	0.07297
184	90.5	0.702	0.701	0.533	-0.0182	2.28	0.1044	0.0231	0.08130
185	90.5	0.667	0.665	0.493	-0.0182	2.30	0.1028	0.0243	0.07848
186	90.5	0.616	0.632	0.456	-0.0182	2.27	0.1036	0.0241	0.07945
187	90.5	0.702	0.683	0.493	-0.0182	2.39	0.1115	0.0260	0.08559
188	90.5	0.582	0.609	0.435	-0.0182	2.23	0.1014	0.0238	0.07761
189	90.5	0.667	0.658	0.472	-0.0182	2.36	0.1078	0.0258	0.08200
190	90.5	0.702	0.679	0.482	-0.0182	2.42	0.1135	0.0268	0.08673

Table A5: Data for 90.5 Degree Experiments (Continued)

Exp No. 1	$F_{r,ds}$ 11	$h^*_b=h_b/y_{ds}$ 12	V_{ds} (fps) 13	$V^2_{ds}/2g$ (ft) 14	$F^2_{r,ds}$ 15	K_{bend} $[h_b/(V^2_{ds}/2g)]$ 16	y_{ds}/W 17
138	0.227	0.0214	0.97	0.0147	0.051	0.835	1.22
139	0.238	0.0276	1.27	0.0250	0.057	0.972	1.87
140	0.240	0.0242	1.21	0.0226	0.058	0.842	1.66
141	0.265	0.0316	1.08	0.0181	0.070	0.898	1.11
142	0.265	0.0304	1.20	0.0223	0.070	0.863	1.36
143	0.265	0.0304	1.20	0.0223	0.070	0.863	1.36
144	0.272	0.0357	1.39	0.0300	0.074	0.963	1.73
145	0.274	0.0405	1.41	0.0309	0.075	1.078	1.78
146	0.275	0.0357	1.32	0.0271	0.076	0.944	1.54
147	0.276	0.0398	1.44	0.0324	0.076	1.048	1.83
148	0.307	0.0391	1.32	0.0271	0.094	0.828	1.25
149	0.310	0.0440	1.60	0.0400	0.096	0.915	1.80
150	0.310	0.0440	1.60	0.0400	0.096	0.915	1.80
151	0.311	0.0437	1.43	0.0319	0.096	0.906	1.44
152	0.312	0.0493	1.57	0.0382	0.097	1.015	1.72
153	0.313	0.0455	1.53	0.0361	0.098	0.926	1.60
154	0.315	0.0491	1.55	0.0373	0.099	0.988	1.64
155	0.330	0.0478	1.25	0.0243	0.109	0.876	0.99
156	0.347	0.0517	1.54	0.0369	0.120	0.860	1.36
157	0.349	0.0588	1.77	0.0485	0.122	0.965	1.76
158	0.352	0.0548	1.45	0.0325	0.124	0.885	1.17
159	0.353	0.0613	1.75	0.0474	0.124	0.987	1.70
160	0.355	0.0581	1.66	0.0427	0.126	0.921	1.51
161	0.359	0.0653	1.72	0.0461	0.129	1.013	1.60
162	0.361	0.0670	1.70	0.0448	0.131	1.027	1.54
163	0.395	0.0673	1.68	0.0439	0.156	0.862	1.28
164	0.401	0.0786	1.94	0.0583	0.161	0.978	1.65
165	0.408	0.0740	1.93	0.0577	0.167	0.888	1.58
166	0.412	0.0608	1.61	0.0401	0.170	0.717	1.08
167	0.415	0.0844	1.90	0.0558	0.172	0.983	1.50
168	0.415	0.0762	1.84	0.0526	0.173	0.884	1.40
169	0.422	0.0693	1.47	0.0336	0.178	0.778	0.88
170	0.429	0.0865	1.90	0.0562	0.184	0.941	1.42
171	0.459	0.0960	2.12	0.0697	0.210	0.913	1.56
172	0.468	0.0936	1.88	0.0550	0.219	0.855	1.20
173	0.481	0.1004	2.15	0.0718	0.232	0.866	1.48
174	0.499	0.0987	2.08	0.0671	0.249	0.793	1.30
175	0.513	0.1185	2.15	0.0715	0.263	0.900	1.33
176	0.526	0.1007	1.89	0.0555	0.276	0.730	0.99
177	0.542	0.1220	2.37	0.0871	0.294	0.830	1.47
178	0.560	0.1119	1.78	0.0491	0.314	0.714	0.80
179	0.566	0.1184	2.14	0.0710	0.320	0.739	1.11
180	0.575	0.1335	2.42	0.0910	0.330	0.808	1.39
181	0.605	0.1283	2.36	0.0868	0.366	0.702	1.22
182	0.611	0.1436	2.46	0.0936	0.373	0.770	1.30
183	0.629	0.1540	2.46	0.0938	0.396	0.778	1.25
184	0.636	0.1526	2.64	0.1079	0.405	0.754	1.40
185	0.679	0.1592	2.71	0.1136	0.461	0.691	1.33
186	0.706	0.1743	2.70	0.1135	0.498	0.700	1.26
187	0.715	0.1737	2.85	0.1261	0.512	0.679	1.37
188	0.716	0.1785	2.68	0.1114	0.512	0.697	1.22
189	0.725	0.1738	2.83	0.1240	0.526	0.661	1.32
190	0.740	0.1800	2.91	0.1319	0.548	0.657	1.36

Design Tables and Charts

Table A6: Loss Coefficients, K_b, for Abrupt Bends in Rectangular Channels (Report Table 3.4)

Fr	Abrupt Bend Angle, θ, in degrees												
	30	35	40	45	50	55	60	65	70	75	80	85	90
0.20	0.127	0.191	0.256	0.322	0.389	0.457	0.525	0.593	0.663	0.733	0.804	0.875	0.948
0.21	0.126	0.191	0.256	0.322	0.388	0.455	0.523	0.592	0.661	0.731	0.802	0.873	0.945
0.22	0.126	0.190	0.255	0.321	0.387	0.454	0.522	0.591	0.660	0.729	0.800	0.871	0.943
0.23	0.125	0.189	0.254	0.320	0.386	0.453	0.521	0.589	0.658	0.727	0.798	0.869	0.940
0.24	0.125	0.189	0.254	0.319	0.385	0.452	0.519	0.587	0.656	0.725	0.795	0.866	0.937
0.25	0.124	0.188	0.253	0.318	0.384	0.451	0.518	0.586	0.654	0.723	0.793	0.863	0.934
0.26	0.123	0.187	0.252	0.317	0.383	0.449	0.516	0.584	0.652	0.721	0.791	0.861	0.931
0.27	0.123	0.187	0.251	0.316	0.382	0.448	0.515	0.582	0.650	0.719	0.788	0.858	0.928
0.28	0.122	0.186	0.250	0.315	0.381	0.447	0.513	0.580	0.648	0.716	0.785	0.855	0.925
0.29	0.122	0.185	0.249	0.314	0.379	0.445	0.511	0.578	0.646	0.714	0.783	0.852	0.922
0.30	0.121	0.184	0.248	0.313	0.378	0.444	0.510	0.576	0.644	0.711	0.780	0.849	0.918
0.31	0.120	0.184	0.247	0.312	0.377	0.442	0.508	0.574	0.641	0.709	0.777	0.845	0.914
0.32	0.120	0.183	0.246	0.310	0.375	0.440	0.506	0.572	0.639	0.706	0.774	0.842	0.911
0.33	0.119	0.182	0.245	0.309	0.374	0.439	0.504	0.570	0.636	0.703	0.771	0.839	0.907
0.34	0.118	0.181	0.244	0.308	0.372	0.437	0.502	0.568	0.634	0.700	0.767	0.835	0.903
0.35	0.117	0.180	0.243	0.307	0.371	0.435	0.500	0.565	0.631	0.697	0.764	0.831	0.899
0.36	0.116	0.179	0.242	0.305	0.369	0.433	0.498	0.563	0.628	0.694	0.761	0.827	0.895
0.37	0.116	0.178	0.241	0.304	0.367	0.431	0.496	0.560	0.626	0.691	0.757	0.824	0.890
0.38	0.115	0.177	0.239	0.302	0.366	0.429	0.493	0.558	0.623	0.688	0.754	0.820	0.886
0.39	0.114	0.176	0.238	0.301	0.364	0.427	0.491	0.555	0.620	0.685	0.750	0.815	0.881
0.40	0.113	0.175	0.237	0.299	0.362	0.425	0.489	0.553	0.617	0.681	0.746	0.811	0.877
0.41	0.112	0.174	0.236	0.298	0.360	0.423	0.486	0.550	0.614	0.678	0.742	0.807	0.872
0.42	0.111	0.173	0.234	0.296	0.359	0.421	0.484	0.547	0.611	0.674	0.738	0.802	0.867
0.43	0.110	0.171	0.233	0.295	0.357	0.419	0.481	0.544	0.607	0.671	0.734	0.798	0.862
0.44	0.109	0.170	0.231	0.293	0.355	0.417	0.479	0.541	0.604	0.667	0.730	0.793	0.857
0.45	0.108	0.169	0.230	0.291	0.353	0.414	0.476	0.538	0.600	0.663	0.726	0.788	0.851
0.46	0.107	0.168	0.229	0.290	0.351	0.412	0.473	0.535	0.597	0.659	0.721	0.784	0.846
0.47	0.106	0.166	0.227	0.288	0.349	0.410	0.471	0.532	0.593	0.655	0.717	0.779	0.841
0.48	0.105	0.165	0.225	0.286	0.346	0.407	0.468	0.529	0.590	0.651	0.712	0.773	0.835
0.49	0.104	0.164	0.224	0.284	0.344	0.405	0.465	0.525	0.586	0.647	0.707	0.768	0.829
0.50	0.103	0.162	0.222	0.282	0.342	0.402	0.462	0.522	0.582	0.642	0.703	0.763	0.823
0.51	0.102	0.161	0.221	0.280	0.340	0.399	0.459	0.519	0.578	0.638	0.698	0.758	0.817
0.52	0.100	0.160	0.219	0.278	0.337	0.397	0.456	0.515	0.574	0.634	0.693	0.752	0.811
0.53	0.099	0.158	0.217	0.276	0.335	0.394	0.453	0.512	0.570	0.629	0.688	0.746	0.805
0.54	0.098	0.157	0.216	0.274	0.333	0.391	0.450	0.508	0.566	0.625	0.683	0.741	0.799
0.55	0.097	0.155	0.214	0.272	0.330	0.388	0.446	0.504	0.562	0.620	0.677	0.735	0.792
0.56	0.095	0.154	0.212	0.270	0.328	0.386	0.443	0.501	0.558	0.615	0.672	0.729	0.786
0.57	0.094	0.152	0.210	0.268	0.325	0.383	0.440	0.497	0.554	0.610	0.667	0.723	0.779
0.58	0.093	0.151	0.208	0.266	0.323	0.380	0.436	0.493	0.549	0.605	0.661	0.717	0.772
0.59	0.092	0.149	0.206	0.263	0.320	0.377	0.433	0.489	0.545	0.600	0.655	0.710	0.765

$$h_L = K_b \frac{V_{ds}^2}{2g}$$

Table A6: (Cont.) Loss Coefficients, K_b, for Abrupt Bends in Rectangular Channels (Report Table 3.4)

Fr	Abrupt Bend Angle, θ, in degrees												
	30	35	40	45	50	55	60	65	70	75	80	85	90
0.59	0.092	0.149	0.206	0.263	0.320	0.377	0.433	0.489	0.545	0.600	0.655	0.710	0.765
0.60	0.090	0.147	0.204	0.261	0.317	0.374	0.429	0.485	0.540	0.595	0.650	0.704	0.758
0.61	0.089	0.146	0.202	0.259	0.315	0.370	0.426	0.481	0.535	0.590	0.644	0.698	0.751
0.62	0.087	0.144	0.200	0.256	0.312	0.367	0.422	0.477	0.531	0.585	0.638	0.691	0.744
0.63	0.086	0.142	0.198	0.254	0.309	0.364	0.418	0.472	0.526	0.579	0.632	0.684	0.736
0.64	0.085	0.141	0.196	0.252	0.306	0.361	0.415	0.468	0.521	0.574	0.626	0.678	0.729
0.65	0.083	0.139	0.194	0.249	0.303	0.357	0.411	0.464	0.516	0.568	0.620	0.671	0.721
0.66	0.082	0.137	0.192	0.247	0.300	0.354	0.407	0.459	0.511	0.562	0.613	0.664	0.713
0.67	0.080	0.135	0.190	0.244	0.297	0.350	0.403	0.455	0.506	0.557	0.607	0.656	0.706
0.68	0.079	0.133	0.188	0.241	0.294	0.347	0.399	0.450	0.501	0.551	0.600	0.649	0.698
0.69	0.077	0.132	0.185	0.239	0.291	0.343	0.395	0.445	0.496	0.545	0.594	0.642	0.689
0.70	0.075	0.130	0.183	0.236	0.288	0.340	0.391	0.441	0.490	0.539	0.587	0.634	0.681
0.71	0.074	0.128	0.181	0.233	0.285	0.336	0.386	0.436	0.485	0.533	0.580	0.627	0.673
0.72	0.072	0.126	0.179	0.231	0.282	0.332	0.382	0.431	0.479	0.527	0.573	0.619	0.664
0.73	0.071	0.124	0.176	0.228	0.279	0.329	0.378	0.426	0.474	0.520	0.566	0.612	0.656
0.74	0.069	0.122	0.174	0.225	0.275	0.325	0.373	0.421	0.468	0.514	0.559	0.604	0.647
0.75	0.067	0.120	0.171	0.222	0.272	0.321	0.369	0.416	0.462	0.508	0.552	0.596	0.638
0.76	0.065	0.118	0.169	0.219	0.269	0.317	0.365	0.411	0.457	0.501	0.545	0.588	0.629
0.77	0.064	0.116	0.166	0.216	0.265	0.313	0.360	0.406	0.451	0.495	0.537	0.579	0.620
0.78	0.062	0.113	0.164	0.213	0.262	0.309	0.355	0.401	0.445	0.488	0.530	0.571	0.611
0.79	0.060	0.111	0.161	0.210	0.258	0.305	0.351	0.395	0.439	0.481	0.522	0.563	0.602
0.80	0.058	0.109	0.159	0.207	0.255	0.301	0.346	0.390	0.433	0.474	0.515	0.554	0.592
0.81	0.057	0.107	0.156	0.204	0.251	0.297	0.341	0.384	0.426	0.467	0.507	0.546	0.583
0.82	0.055	0.105	0.154	0.201	0.247	0.292	0.336	0.379	0.420	0.460	0.499	0.537	0.573
0.83	0.053	0.102	0.151	0.198	0.244	0.288	0.331	0.373	0.414	0.453	0.491	0.528	0.563
0.84	0.051	0.100	0.148	0.195	0.240	0.284	0.326	0.368	0.408	0.446	0.483	0.519	0.554
0.85	0.049	0.098	0.145	0.191	0.236	0.279	0.321	0.362	0.401	0.439	0.475	0.510	0.544
0.86	0.047	0.096	0.143	0.188	0.232	0.275	0.316	0.356	0.394	0.431	0.467	0.501	0.533
0.87	0.046	0.094	0.141	0.186	0.230	0.272	0.313	0.353	0.390	0.427	0.462	0.495	0.527

$$h_L = K_b \frac{V_{ds}^2}{2g}$$

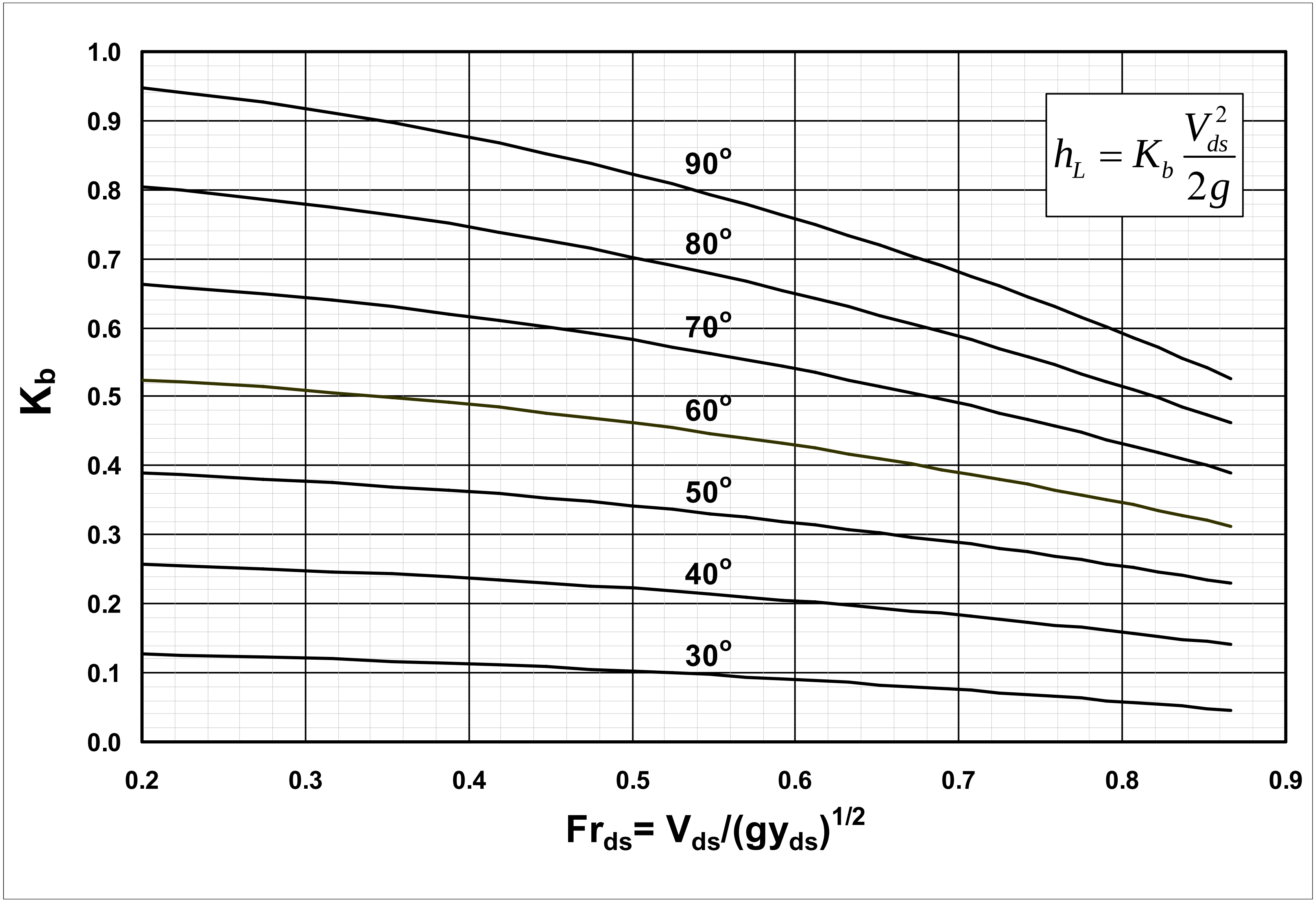

Figure A.1: K_b versus Downstream Froude Number (Report Fig. 3.10)

Table A7: β Factor versus r/D_{eq} for Gradual Bends (Report Table 7)

r/D_{eq}	$\beta = K_{gb}/K_{gb,1}$									
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	1.000	0.960	0.920	0.879	0.839	0.799	0.759	0.719	0.679	0.638
2	0.598	0.593	0.588	0.583	0.578	0.573	0.568	0.563	0.558	0.553
3	0.548	0.543	0.538	0.533	0.528	0.523	0.518	0.513	0.508	0.503
4	0.498	0.488	0.479	0.469	0.459	0.450	0.440	0.431	0.421	0.411
5	0.402	0.392	0.383	0.373	0.363	0.354	0.344	0.335	0.325	0.315
6	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306
7	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306
8	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306	0.306

$$h_L = K_{gb}\frac{V_{ds}^2}{2g} = (\beta K_b)\frac{V_{ds}^2}{2g}$$

where

K_{gb} = gradual bend loss coefficient

K_b = abrupt bend loss coefficient

K - TRAN

**KANSAS TRANSPORTATION RESEARCH
AND
NEW - DEVELOPMENTS PROGRAM**

A COOPERATIVE TRANSPORTATION RESEARCH PROGRAM BETWEEN:

KANSAS DEPARTMENT OF TRANSPORTATION

THE UNIVERSITY OF KANSAS

KANSAS STATE UNIVERSITY