TURN CONTROLS IN URBAN TRAFFIC

PREPARED BY THE STAFF OF THE ENO FOUNDATION

THE ENO FOUNDATION FOR HIGHWAY TRAFFIC CONTROLSAUGATUCK• 1951• CONNECTICUT

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PREFACE

The need for a factual basis on which to establish traffic turning regulations has long been apparent. Recognizing this need, the Eno Foundation initiated a study early in 1949 aimed at determining whether a mathematical relationship could be established between conflicting vehicle turns and crossing pedestrians. Included in the study are the results of information collected from urban traffic officials as to practices and experiences in the use of various types of turn controls.

The importance of the study is borne out by these results and the wide criteria employed in establishing turning regulations. Only rough, empirical warrants appear to exist and these are used in only a few cities. The value of turn restrictions is well proved by their extensive use and by marked results in accident reduction and relief of congestion.

The mathematical results of this study are not set forth as representative of all conditions under which turn regulations may be applied. They are based on limited samples, and accepted assumptions of traffic behavior. Definite mathematical relations have been established in Chapter III between the volumes of turning vehicles and volumes of pedestrians conflicting at crosswalks. These were developed from the initial survey data and were later substantiated by empirical data collected at other locations. The results are encouraging and indicate that mathematical approaches are possible, and can provide uniform and reliable solutions to many common traffic situations.

It is hoped they will be useful to practicing traffic officials and to others in developing uniform factual warrants for establishing turn prohibitions and controls and that they will arouse additional interest in the mathematical approach to this and other traffic engineering problems. It is a field in which much additional research and study are needed.

Mr. Jack Hart, Research Engineer on the Eno Foundation staff, collected the major portion of the field data used in the

TURN CONTROL

study. He assisted in the overall planning of the study and analysis of the data.

Mr. Morton Raff, who made the mathematical deductions, was employed by the Eno Foundation for that purpose. Formulas and mathematical results contained in this study are largely his work. Mr. Raff wrote Chapters I, III, and IV, and the Appendices. He also prepared the tables in Chapter II and the drawings on which the graphs are based in Chapter II.

Professor Herman Betz, of the Department of Mathematics at the University of Missouri, kindly reviewed Chapters III and IV and the Appendices. His comments were invaluable. Especial acknowledgment is due him for suggesting a briefer mathematical technique for arriving at the results previously developed. Appendix B has been revised by Mr. Raff to incorporate the mathematical methods derived by Professor Betz and the original calculations were thereby greatly reduced.

Editing and revision have been done by the Editorial Staff of the Eno Foundation.

Appreciation is extended to the many city officials who cooperated in making the field observations possible. Traffic engineers and other traffic authorities were most helpful in Waterbury, New Haven, and Bridgeport, Connecticut, and in New York City, in facilitating conduct of the surveys. Acknowledgment is made to the Eno Foundation staff for its helpful discussions and comments.

It is sincerely hoped that the data and results presented will be useful to many, that further interest in this important area of traffic control will be suggested, and additional research pursued.

> ROBERT C. F. GOETZ Colonel, U.S.A. Ret. President

CHAPTER I

GENERAL INTRODUCTION

The intersection is the critical element in an urban traffic plan. Most city accidents occur at intersections, and most delays to traffic can be traced to conflicts at intersections. It is important to fully understand these conflicts and their effect if the intersection is to perform properly. Under the best conditions, a busy intersection is bound to be a focal point of congestion, for the simple reason that an area that is common to two streets must of necessity handle more traffic. For this reason they deserve the utmost attention to assure their efficient use.

As the frequency and severity of intersection conflicts increase, it becomes necessary to use traffic controls to maintain an orderly movement of the traffic. The least stringent type of traffic control is a pair of STOP signs which require all traffic approaching the intersection on one of the streets to make a full stop before entering the intersection. When this degree of regulation appears insufficient, it is the practice in some localities to install STOP signs on *all* approaches to the intersection. There is considerable controversy over the value of such four-way stop regulations.

Next in the scale of increasing restrictiveness is the widely used traffic signal, which assigns the use of the intersection first to one stream of traffic and then to the other. Added to any of these and further restricting traffic is a regulation prohibiting one or more turning movements.

Most types of traffic control are employed solely on the basis of observation and judgment. This method, without a scientific or technical basis, is at best a random procedure and leads to the excessive and inefficient use of traffic control. It frequently creates confusion and delay where logical and better solutions are available. Its promiscuous use clearly indicates the need for further research in the field of traffic regulation warrants.

TURN CONTROL

Conflicts at Intersections

Conflicts at an intersection are of two types: conflicts between one vehicle and another, or conflicts between vehicles and pedestrians. The inter-vehicle conflicts, again, are of two basic types: conflicts between vehicles approaching the intersection on different streets, and those involving left urns. When warranted, the first type of inter-vehicle conflict is normally controlled by traffic signals which assign the right-of-way alternately between the one street and the other.

Assuming that the intersection under consideration is equipped with traffic signals, and further that all vehicles and pedestrians obey the signals, we are left with two kinds of conflicts: (1) intervehicle conflicts between cars approaching the intersection from opposite directions when one car makes a left turn; and (2) vehicle-pedestrian conflicts between pedestrians using a crosswalk on the green light and vehicles making right or left turns across their path. These conflicts are illustrated for an intersection of two-lane streets, in Figure 1. The first drawing shows the conflicts that occur at a completely uncontrolled intersection, while the second shows the conflicts at a signalized intersection. It will be seen that the use of signals greatly reduces the number of points of conflict, eliminating 14 of the 16 inter-vehicle conflict points and 12 of the 16 points of vehicle-pedestrian conflict. It is obvious that the number of conflict points would increase rapidly with additions to the number of traffic lanes. This does not mean that turn problems are always to be corrected by signals. Turning movements constitute only a small part of the overall warrant for signals.

The conflicts that remain after signalization involve turning vehicles and can be eliminated by controlling turns. Methods of controlling turns will be discussed in a later section, but something should first be said about the effects of *not* controlling them.

Delay to a motorist is more than a strictly personal problem. When a motorist must delay his turning maneuver on account of a conflict with a stream of other vehicles or pedestrians, all following vehicles desiring to turn—in the case of a narrow street, the

GENERAL INTRODUCTION

CONFLICTS AT AN INTERSECTION OF TWO TWO-LANE. TWO-WAY STREETS

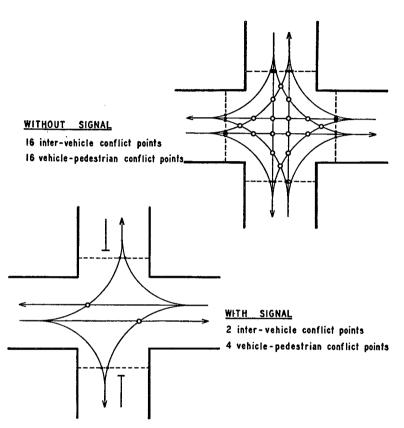


FIGURE 1. Conflicts at an Intersection of Two Two-Lane, Two-Way Streets.

entire traffic stream behind him—are delayed by the conflict. A small number of conflicts or unwarranted signalization can seriously hamper efficiency of the intersection as a vehicular clearing house. Advantages and disadvantages must be seriously and accurately weighed.

There is danger of accidents when two vehicles, or a vehicle and a pedestrian, have the right to be in the same place at the

TURN CONTROL

same time. Accidents are spectacular and are a serious matter to those involved, but fortunately with good driving and pedestrian care, most drivers and pedestrians manage to avoid collisions most of the time.

In the present study emphasis has been placed on vehicle-pedestrian conflicts. While nothing more will be said in this report about inter-vehicle conflicts involved in left turns, much of the discussion is applicable to such inter-vehicle conflicts.

Ways of Eliminating Vehicle-Pedestrian Conflicts

Basically, there are three ways to eliminate the conflict between a stream of turning vehicles and a pedestrian stream at a signalized intersection. The streams may be separated physically by means of a bridge or tunnel; one of the conflicting movements may be prohibited by regulation; or there may be special signal intervals which permit the two streams to move at different times. Less drastic measures which have considerable value include the construction of channelizing islands which separate the points of conflict and provide places of refuge for pedestrians, and the use of one-way streets which reduces the number of permissible turns to one direction. Of the basic methods of eliminating vehiclepedestrian conflicts, grade separations are the most expensive and hence least widely used. Many improvements which reduce vehicle-pedestrian conflicts also reduce vehicle-vehicle conflicts at the intersection.

The prohibition of one of the conflicting movements offers broad possibilities, both because no important physical changes are required and because the regulation is easily rescinded if it proves unsatisfactory or if a better control is found. The question arises as to whether it is more desirable to favor the pedestrian by prohibiting the vehicular turning movement, or to favor the motorist by preventing the pedestrian from using a particular crosswalk (eliminating certain crosswalks). Both methods have been used, but it is more general to favor the pedestrian because (1) there are usually more of them, (2) they are more exposed to

the weather, and (3) the extra inconvenience and delay in using an indirect route are greater for the pedestrian.

Since the rerouting of vehicles prohibited from turning at a particular intersection may sometimes create problems at other intersections, it is necessary that each location be analyzed in detail to determine whether or not the benefits of a turn prohibition outweigh the drawbacks.

Special signal intervals for turning movements are widely used. Compared with the prohibition of certain turns, this method has the advantage of not requiring re-routing; but it frequently necessitates a lengthening of the signal cycle, which means added delay for other motorists and pedestrians.

Very little is known about turn control regulations, beyond the fact that they are in common use. Their value is generally recognized but there is little precise knowledge of their exact value or of conditions that warrant their use.

The present study undertakes to provide this information. It has been divided into two separate parts: (1) a report on a study of current practices in controlling turns; and (2) a study of vehiclepedestrian conflicts wherein empirical data were analyzed in terms of the effect of conflicts on the operating efficiency of intersections.

CHAPTER II

USE OF TURN CONTROLS

To collect information that would make possible the analysis of practices now in use in controlling turns of all types, a comprehensive questionnaire was mailed to all cities of over 50,000 population. One hundred and five cities replied, but nine of them failed to furnish usable information. It will be noted in Table I that 35 of the cities were in the population range from 50,000 to 100,000. Almost an equal number of cities, 37, were in the next population group, ranging from 100,000 to 200,000. The remaining 24 cities had populations of 200,000 and over. These cities were well spread geographically and represented typical cities in each population group.

Table I

POPULATION DISTRIBUTION OF CITIES REPORTING TURN DATA

Population	Number of Cities	Per Cent
50,000 to 100,000	35	36
100,000 to 200,000	37	39
200,000 and over	24	25
Total	<u></u> 96	100

Most of the information was supplied by officials directly concerned with traffic. Approximately ten per cent were not. See Table II.

It is significant that as the size of the cities increased, traffic engineers collected and provided a proportionately greater part of the information. For example, in the group of smaller cities, threefourths of the questionnaires were turned in by police, whereas in the largest cities they prepared only 11 per cent. In the large cities, traffic engineers provided most of the information.

Table II

Sources of Data

		-		
Source	50,000 to 100,000	100,000 to 200,000	200,000 and Up	All Cities
Police Official	28	23	3	54
Traffic Engineer	3	12	17	32
Other Traffic Official	0	1	4	5
Other	1	1	0	2
Unknown	3	0	0	3
No Usable Information	3	3	3	9
	<u> </u>			
Total	<u></u> 38	40	27	105

Number of Cities by Population Groups

General Use of Turn Controls

Seven of the cities furnishing information do not use any type of turn control. Five of these are in the 50,000 to 100,000 population group.

The extent to which cities of different populations use fulltime, part-time, and signal turn controls is indicated in Table III. About half of the cities use two different types of controls; these range from combinations of full-time and part-time prohibitions, to full-time and signal regulations, to part-time and signal controls. Few cities use only a single type control.

It is interesting to note that only two of the cities control turns exclusively through signal indications. With modern flexible signal equipment one would have expected this number to be considerably greater.

Variations in practices in the use of turn controls by different population groups are shown in Figure 2. The use of all three types of controls is much greater in the largest cities. While multiple controls are more numerous in the larger cities, it is surprising to find that more than half the cities in the smaller population groups use two or more types.

Table III

Use of Various Types of Turn Controls

		Population Ranges								
	50,000 i	to 100,000	100,000	to 200,000	200,0	oo and Up	All Cities			
Type Controls Used	Number of Cities	Per Cent	Number of Cities	Per Cent	Number of Cities	Per Cent	Number of Cities	Per Cent	н	
No Controls	5	14	1	3	1	4	7	7	UR	
Full-Time Controls Only	8		4		1		13		Ż	
Part-Time Controls Only	1	29	2	16	0	9	3	19	8	
Signal Provision for Turns Only	1		0		1		2		NTRO	
Full-Time and Part-Time	5	,	8	•	5	,	18		Ĕ	
Full-Time and Signal	ő	34	10	54	2	. 29	18	41		
Part-Time and Signal	1		2		0		3			
Full-Time, Part-Time and Signal	. 8	23	10	27	14	58	32	33	۰.	
Total	35	100	37	100	24	100	96	100		

EXTENT TO WHICH VARIOUS TURN CONTROLS USED IN CITIES FULL-TIME, PART-TIME OR SIGNAL

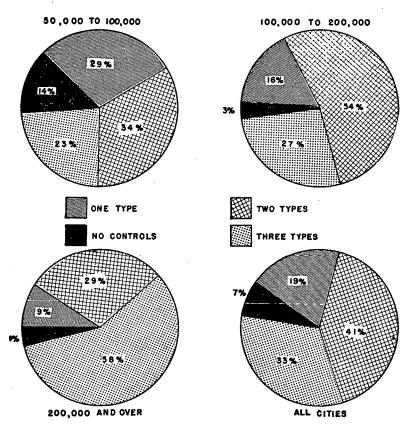


FIGURE 2. Use of Turning Regulations in Cities of Different Populations.

The frequency with which full-time, part-time, and signal controls are applied in the various population groupings, and also a summary of left-turn, right-turn, and all-turn prohibitions is given in Table IV. Eighty-one cities have applied full-time controls to 1,090 locations, an average of 13.5 full-time controls per

Table IV

FREQUENCY WITH WHICH TURN CONTROLS USED

Average Use and Number of Locations by Population Groups

				-		•				-			
	50,	000 to 10	00,000	100	,000 to 20	00,000	20	0,000 an	d Up		All Cities		
Type Control	Number of Cities	Average*	Number of Locations		Average*	Number of Locations		Average*	Number of Locations	Number of Cities	Average*	Number of Locations	
Full-Time Controls	27	7.0	189	32	7:3	234	22	30.3	667	81	13.5	1,000	
Part-Time Controls Signal Provision	15	7.9	118	22	7.5	164	19	32.1	610	56	15.9	892	
for Turns	16	3.6	58	22	7.9	173	17	15.2	258	55	8.9	489	
Left Turn Controls	25	7.2	179	33	10.2	336	23	54.1	1,243	81	21.7	1,758	
Right Turn Controls Number of Cities	13	6.1	79	16	6.1	97	15	14.4	216	44	8.9	392	
Using Controls	30			36			23			89			

* Average number of locations per city.

city. The average goes up considerably as the population increases. For example, there are only seven full-time controls, on the average, in cities from 50,000 to 100,000; 7.3 in the cities from 100,000 to 200,000; and in the largest cities the average is 30.3. Part-time controls are not as widely used as the full-time. Fiftysix of the cities reported part-time controls at 892 locations, for an average of almost 16 part-time controls per city. Fifty-five of the 96 cities furnishing information use traffic signal indications

APPLICATION OF FULL-TIME, PART-TIME AND SIGNAL CONTROLS

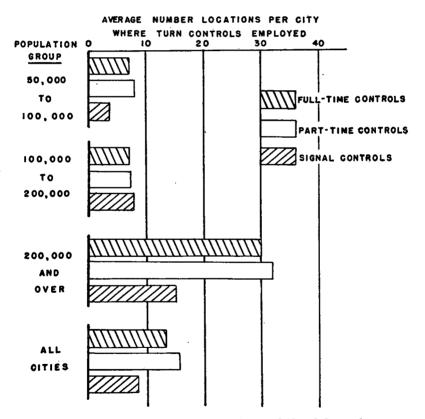
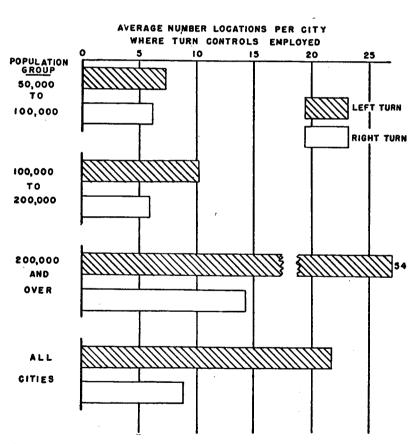


FIGURE 3. Use of Full-Time, Part-Time and Signal Controls.

to prohibit turns at 489 locations, for an average of 8.9 applications per city.

Left-turn prohibitions are by far more generally used. Eightyone cities reported their use at 1,758 locations, for an average of 21.7 locations per city. Again the average increased as the populations increased.



APPLICATION OF RIGHT AND LEFT TURN PROHIBITIONS

FIGURE 4. Use of Right and Left Turn Controls by Population Groupings.

The control of right turns is limited in comparison with the control of left turns. Forty-four cities reported the prohibition of right turns, and in these cities 392 intersections were involved.

Accurate information on the prohibition of *all* turns is not available, except that prohibitions of this type are even less widely used than the right-turn prohibitions.

The extent to which full-time, part-time, and signal controls are used in cities of different populations is shown graphically in Figure 3. The application of left-turn and right-turn prohibitions is illustrated in Figure 4. These figures clearly demonstrate the trends by population groupings which have been discussed and shown in tabulations.

Districts Where Turn Controls Are Applied

The cities were asked to indicate the common types of urban districts in which turning regulations are used. These were generally classified as (a) downtown; (b) outlying shopping areas; (c) industrial areas; (d) school zones; and (e) other districts. The results are shown in Table V.

More cities use turn prohibitions in downtown districts than in other areas. All cities of over 100,000 population which use turn controls use them downtown. Several cities from 50,000 to 100,000 population which use controls do not use them in downtown districts.

It is apparent that the use of turning regulations in outlying shopping centers is becoming increasingly popular. About onehalf of all the cities with turn controls use one or more types in outlying shopping areas. Almost three-fourths of the cities of 200,000 and up indicated the use of turning regulations in outlying business districts. Traffic conditions apparently are not affected sufficiently by industrial areas to justify the extensive application of turning regulations therein. A number of instances were reported, however, in which part-time prohibitions are being used in the vicinity of large industrial establishments. Seven cities reported the use of turning regulations in the vicinity of

Table V

DISTRICTS IN WHICH TURN CONTROLS ARE USED

				Populatio	n Range				
	50,000 t	0 100,000	100,000 t	to 200,000	200,000	and Up	All	Cities	Т
Type District	Number of Cities	Per Cent	Number of Cities	Per Cent	Number of Cities	Per Cent	Number of Cities	Per Cent	URN
Downtown	26	90	36	100	23	100	85	97	S
Outlying Shopping Areas	13	45	12	33	16	70	41	47 ·	z
Industrial Section	5 -	17	4	11	6	26	15	17	TROL
School Zones	2	7	1	3	4	17	7	8	5 L
Other	3	10	7	19	9	39	19	22	
Number of Cities Giving							·		
Positive Answers	29		36		23		88		

schools. It was not possible to ascertain whether or not these were needed primarily for school children, for traffic generated by the schools, or whether the schools happened to be so situated that the regulations were required by other traffic generators. The use of turning regulations is not limited to the four areas just described; nineteen cities use them in other types of areas.

Benefits of Full-Time Controls

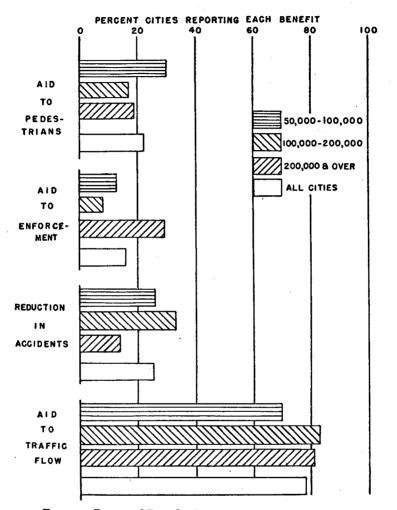
Sixty-eight of the cities reported benefits from the application of full-time turning controls. No distinction was made as to whether or not the full-time controls involved left turns, right turns, or both. It was possible to classify these benefits under four general headings: (a) aid to traffic flow; (b) aid to pedestrians; (c) aid to enforcement; (d) accident reduction. More than half the cities stated that full-time prohibitions are used to benefit traffic flow, or to alleviate congestion. One-fourth of the cities felt that the prohibitions were of material benefit to pedestrians. Others find that they help enforcement authorities; and some reported substantial accident reductions following application of turn prohibitions.

Table VI

BENEFITS OF FULL-TIME TURN CONTROLS

	Benefits by Population Ranges						
Benefits	50,000 to 100,000	100,000 to 200,000	200,000 and Up	All Cities			
Aid to Traffic Flow	16	20	17	53			
Aid to Pedestrians	7	4	4	15			
Aid to Enforcement	3	2	6	11			
Reduction in Accident	s 6	8	3	17			
Total	32	34	30	96			
Number of Cities							
Reporting Benefits	23	24	21	68			

Variations in the reported benefits can be observed, by population groupings, in Table VI and Figure 5. The largest cities apparently feel that the greatest benefits are derived through better



BENEFITS OF FULL-TIME TURN CONTROLS

FIGURE 5. Reported Benefits from Various Type Turn Controls.

traffic flow, whereas the small cities show more interest in benefits to pedestrians and accident reduction; but there was no significant variation in benefits in the different city sizes.

In reporting the benefits of full-time turning prohibitions, sev-

eral urban officials pointed out that part-time controls might be more in keeping with traffic demand, but that the over-all observance by the public was considerably better if the regulations were applied at all times.

Some authorities feel that there is greater fairness in part-time controls and are taking steps to replace as many as possible of the full-time prohibitions with special peak-hour or other part-time regulations. Here again, doubt was raised as to whether or not the observance would be satisfactory and whether a high accident rate might develop during the hours when the turns are permitted, particularly at locations where most citizens have already become accustomed to full-time regulations. Factual data showing the improvements achieved with turning prohibitions are scarce. Most replies to the question concerning survey data proving relief of congestion were indefinite. Several stated in effect that "It is a known fact that where left turns are eliminated, the traffic moves by these intersections with greater ease and safety, with less chance of accidents." Others said, "Congestion in general has been reduced and delays are less, but exact figures are not at hand." A few before-and-after volume studies, made in connection with the application of turning prohibitions, showed marked improvements. Increases of from 10 to 50 per cent were found in the traffic flow through intersections following establishment of turning prohibitions.

Several cities cited figures showing that when turning prohibitions were put into effect, it was possible to re-time signals for faster progressive movements along a main artery. In one case, signal timing speeds were increased by 25 per cent, and in another they were advanced 20 per cent. In one city, which had made studies of transit vehicle speeds along a street where turning prohibitions were applied for a ten-block distance, it was reported that the running time was reduced by two minutes. Before the average transit run required $71/_2$ minutes, whereas afterward it was reduced to $51/_2$ minutes.

It can be concluded that while the number of cities making factual measurements were few, the overall results of the effect of turning prohibitions on traffic movements were very favorable. They were so favorable in many instances, that persons providing information pointed out that "Studies are unnecessary, as anyone familiar with local conditions can readily see that snarls are greatly reduced by the turning regulations."

Almost every city having experience with turning prohibitions indicated that there had been a favorable effect on accident rates at intersections. Reductions of one-fourth were commonly noted. In a few cases the reductions were as great as 70 per cent, and two cities reported that a bad accident situation had been practically eliminated since turning prohibitions were put into effect.

It was impossible to ascertain from data available the relative effect of the regulations on various types of accidents. It was also impossible to relate accident frequency to accident severity. Some cities reported that approximately the same number of accidents occurred after application of the turning regulation, but that the severity was reduced. Other cities had marked increases in the total number of accidents, but a decrease was noted in severity. Still other cities found that even though the total number of accidents was smaller, there was no notable change in the severity of accidents. Even where the over-all severity of accidents increased, turn prohibitions greatly reduced congestion and increased movement; damages caused by conflicts were greater than they could have been with slow movements prevailing before application of the regulations. In those cases where no changes were found in over-all accident severity, it was noted that the areas covered were in downtown districts where speeds were normally low.

The effect of turning prohibitions upon complaint rates is another measure of benefits. Almost one-half of the cities in the population grouping 50,000 to 100,000 stated that the number of complaints regarding intersections prior to turning prohibitions, were reduced by the application of the regulation. Three-fourths of the cities in the largest population grouping, 200,000 and over, stated that where turning prohibitions were used complaints were materially reduced. Most complaints prior to the application of the regulations involved either chronic congestion, interferences between vehicles making turns and pedestrians crossing with signals, or high accident frequencies. These are the conditions which are usually improved when left-turn, right-turn, or all-turn prohibitions are used. Complaints from pedestrians were greatly reduced. Numerous cities reported that pedestrian complaints were entirely eliminated when turning restrictions were applied.

In a few instances, complaints increased, or new type complaints arose. One city indicated that local residents complained because of difficulty in getting to certain destinations after the turning restrictions were applied. In another, local residents complained because many were arrested for violation of the turning prohibition.

It is generally apparent that the proper application of turning prohibitions can be effective in reducing congestion along a principal traffic artery, and in reducing the frequency and severity of accidents. Also, the regulations can bring about a marked reduction in the number of complaints about common traffic ills. Two cities indicated that the use of turning prohibitions tended to make pedestrians less cautious.

Basis for Application of Turn Controls

It was possible to classify all of the reasons for applying turning regulations under the same categories used for reported benefits. Most cities indicated that they applied turning regulations on the basis of: (a) volume; (b) all types of intersection conflicts; (c) high accident experience; or (d) a congestion and delay problem along a particular section of roadway. In some cases, other secondary reasons were given. These included: (a) unusual physical conditions; (b) adjunct to signal timing; and (c) other miscellaneous traffic problems.

In Table VII, seventy-two cities reported reasons for applying turning controls. Many of these cities reported several. In all population groupings, the most predominant reason was that of

						-			00
			Populatio	n Groups					
	50,000 ta	100,000	100,000 t	0 200,000	200,000	and Up	All C	lities	
Reasons for Using Controls	Number of Bases Reported	Per Cent	Number of Bases Reported	Per Cent	Number of Bases Reported	- Per Cent	Number of Bases Reported	Per Cent	
Volumes	-	32	•	34	-	30		32	
Vehicle	3	0	6	JT	5	50	14	54	
Pedestrian	3		11		5 4		18		
Turning Vehicle	4		1		- -		6		
Conflicts	-	10		2	-	15	Ū	8	
Vehicles Only	1		0	-	2	-9	9	Ū	
Vehicles and Pedestrians	2		1		3		3 6		TU
Accidents		7		13	5	9	Ũ	10	TURN
Vehicle	0	•	0	5	0	5	0	10	-
Pedestrian	0		3		0		3		CONTROL
Turning Vehicle	0		2		2		5 4		E
Just "Accidents"	2		2		1		5		RO
Congestion and Delay		45		42		40	5	42	Н
General	7	10	16	-	4	-	27	1-	
Due to Turning Vehicles	7		6		9		22		
Other	•	6		9	0	6		8	
Physical Considerations	1		3	•	0		4		
Signal Timing Considerations	0		1		1		2		
Other	1		1		1		3		
Total	31	100	53	100	33	100	117	100	
Number of Cities Answering This Question	22		32		18		72		

Table VII: BASES FOR TURN CONTROLS

eliminating congestion and delay. Almost half of all reasons were classified in this category.

The next most commonly used basis was one very closely allied with that of eliminating congestion and delay, namely volumes, both vehicular and pedestrian. Approximately one-third of the reasons for turning prohibitions in all population groupings were related directly to volumes of either vehicles or pedestrians, or both. Combining volume and congestion, it will be noted that three-fourths of the reasons for applying turning regulations relate to one or the other. About ten per cent of the reasons were based on accident hazards.

Miscellaneous reasons for controls constituted less than ten per cent of the total. They are shown in relation to others in Table VII.

Many cities consider the above in combination. One city stated that "congestion at intersections is studied, so is our spot map as to accident frequencies.—Study is also made of accident-prone intersections through accident and collision diagrams, width of streets and sidewalks, traffic and pedestrian volume—all this is necessary before we resort to a ban of left-turns, or all turns, whether partor full-time."

Whether cities consider the above factors singly or in combination, it is apparent that in the final analysis most decisions to prohibit turns are usually based on opinions and experiences of an individual city official.

Fixed standards are rare. The following cover most of the attempts at factual bases, reported for establishing turning regulations:

A. When the left turn volume exceeds 20 per cent of the total traffic.

- B. When left turns constitute 10 per cent of the total movement on a given street.
- C. Where left-turn movements interfere with straight-through movements of 15,000 vehicles for 24 hours, regardless of number of lanes and at signalized 4-way intersections.
- D. Where a left turn or a right turn movement interferes with pedestrian cross-walk volumes in excess of 2,000 persons per hour.

TURN CONTROL

- E. 600 cars (total) with 1,000 pedestrians per hour at an intersection.
- F. With turn vehicles to the extent of 7 per green interval for several successive signal changes.
- G. Where more than 3 intersection accidents involving turning vehicles occur within a 12-month period.
- H. When the number of traffic lanes available at the intersection will accommodate only a single movement in each direction and there is an appreciable demand for left turns.

Most of the cities use even more general bases for applying turn controls. These include such considerations as:

- A. When intersection capacity is reduced beyond tolerable limits by turns.
- B. Appreciable conflicts in vehicular traffic flow and between pedestrians and vehicular traffic in central business district.
- C. Where turns create excessive congestion.
- D. Undue delays to traffic and accidents caused by turning vehicles.
- E. To ease congestion.
- F. When congestion causes complaints.
- G. High collision frequency from left-turn movements.
- H. When at a signalized intersection, turning movements result in blocking flow to the extent that no vehicles can move through on a green signal interval.
- I. Congestion coupled with unavailability of an alternate route.
- J. Inability to provide for progressive signal operation because of turning interferences.
- K. When a few turns from a minor traffic stream seriously impede a major movement.

Several cities made it clear that even though they have no factual bases for applying turning regulations, they are hampered in the use of such regulations by totally inadequate nearby parallel streets to accommodate the movements that would be created as a consequence of turn prohibitions.

It is clear from information collected that traffic authorities are almost totally without factual warrants for applying various types of turning controls. As a general rule they are not only without

such warrants in their practical day-to-day operations, but few of the officials were able to suggest any factual bases which in their opinion would be possible of development into warrants. This is obviously another area of traffic control in which research is needed to establish factual warrants for the uniform application of an important regulation. It is towards this end that the studies described later in this report are aimed.

Traffic Volume as a Warrant for Full-Time Turn Prohibitions

Feeling that volumes of vehicles would constitute the most common basis for establishing full-time turning prohibitions, a special effort was made to acquire information on this particular phase of the subject. As has been pointed out, great emphasis is placed on "congestion" as the basis for the regulations, but few cities have developed specific volume figures as a measure of congestion. Answers to the question concerning traffic volume as a warrant were vague. Some stated "we don't use traffic volumes, we use congestion." Others pointed out that "volumes are used, but we have no set volumes." Others indicated that "volumes are the principal base, but they vary greatly depending on 'local conditions.""

The following data were reported:

- 1. Left turns are prohibited at most intersections carrying more than 1,000 cars per hour. (The duration of this requirement was not indicated.)
- 2. On streets carrying over 900 cars per hour in four lanes.
- 3. 250 vehicles per lane per hour of green in opposing through movements.
- 4. 800 to 1,000 cars per hour.
- 5. 1,500 vehicles per hour.
- 6. Over 1,000 vehicles per hour at an intersection if there is apparent congestion.

Few of the cities from 50,000 to 100,000 population use volume data alone as a basis for establishing full-time controls. In the

TURN CONTROL

population groupings over 100,000, about half the cities use volumes alone. Where volumes were used alone or in combination with other factors, there was not an instance in which they were broken down according to turning movements at particular type intersections. Only the gross volumes through the intersection are considered.

Pedestrian Volumes as a Warrant for Turning Regulations

Very few cities use pedestrian counts in establishing turning controls. A few consider pedestrian safety and general pedestrian protection, but the actual volumes and movements of pedestrians are rarely taken into account. In a number of instances it was indicated that accurate pedestrian counts were not necessary where turning controls are used. There appears to be a general feeling that at most intersections, pedestrian traffic is a secondary consideration in deciding whether or not to prohibit turns and that the first thought should be vehicular traffic.

Some bases for turn controls involving pedestrians have been suggested, such as:

- 1. Prohibit all turns at downtown intersections where 90,000 pedestrians cross the roadways in a 12-hour period.
- 2. Turning prohibitions are considered where volumes of pedestrians in cross-walks limit turns to one or two autos per signal cycle.
- 3. When pedestrian traffic in a single cross-walk exceeds 300 per hour.
- 4. Where right turns are 1,250 and pedestrian cross-walk volumes are 25,000 in the same 12-hour period, from 7 A.M. to 7 P.M.
- 5. 500 pedestrians per hour for 4 or more consecutive hours.
- 6. Where a turning movement interferes with pedestrian volumes of 2,000 or more per hour in a single cross-walk.
- 7. "No-turn" prohibitions installed at intersections carrying over 3,000 pedestrians per hour.
- 8. Pedestrian volumes of 600 per hour in a given cross-walk warrant the prohibition of a turn.

- 9. Desirable to prohibit turns which conflict with cross-walks carrying 250 pedestrians per hour.
- 10. When pedestrian traffic out-numbers vehicular traffic in the ratio of 20 to 1.

Another very interesting approach is that in which pedestrians are counted in terms of the average occupancy of vehicles. In other words, right-of-way is assigned where vehicles carry more persons across a pedestrian cross-walk than there are pedestrians using the cross-walk. If the average occupancy of vehicles is two persons, then the problem is considered in terms of two pedestrians per vehicle and the decision to allow or prohibit turns depends on whether or not the pedestrians outnumber the persons using vehicles which interfere with the pedestrian movement.

In the final analysis, it must be concluded that the warrants based on pedestrian volumes are little if any better than those based on vehicular volumes alone. They are all rule-of-thumb in character and do not suggest a pattern of common practices.

Use of Accident Data in Establishing Turning Prohibitions

Accident data alone are rarely used to support recommendations for prohibiting turns. Several instances were reported in which accident facts are related to other available data. Most cities do not consider accident frequency, type, or severity in any manner in connection with turn controls. A few cities did indicate that where analyses of high accident locations show the involvement of a high percentage of turning movements in accidents, this information is used as a basis for further investigating the possibility of turn controls.

It was apparent that most of the controls are aimed primarily at the relief of congestion and that accident factors are a minor consideration. Accident experience does play an important part in a few cities but in most cases it is unimportant.

The nearest factual information regarding application of turning regulations based on accidents was as follows:

- 1. Turning prohibitions are imposed when five or more personal injury accidents involving turning vehicles occur at an intersection within a 12-month period.
- 2. Five or more accidents per year of the type that could be eliminated by turn restrictions.
- 3. A special left-turn signal interval is installed in one city where there are ten or more accidents at the intersection per year involving left turns.

Many cities use accident experience to measure the before-andafter efficiency of turning regulations. In other words, accidents do not provide a basis for establishing the turn controls, but they are employed to ascertain whether or not an improvement has been achieved.

Formulas and Equations for Determining Whether to Use Turn Prohibitions

City authorities have not reduced turn controls to a strict factual basis, permitting development of formulas, equations, or curves. This is further indication of the need for research and technical investigations into the whole area of turning regulations.

Even though tentative and experimental, several rather interesting factual approaches were indicated. Oakland, California, for example, applies no-turning prohibitions under the following conditions:

- 1. Vehicular traffic—where a left-turn movement opposes a straight through movement of 15,000 vehicles per 24 hours regardless of number of lanes and at signalized square intersections.
- 2. Pedestrian volume—where a left turn or right turn movement interferes with cross-walk volumes in excess of 2,000 per hour.
- 3. Accident record-indices in excess of twice normal.
- 4. The accident experience index of 52 intersections (including both signalized, channelized and non-signalized intersections) for a seven-year period and having a vehicular-entering range from 4,000 to 67,000 cars per 24-hour day gives a hyperbolic curve which apparently approaches both axes as asymptotes.

One city allows a value of three for each left-turn movement added to opposing through movement. The ability or inability to develop signal timing, taking into account these adjusted volumes, determines whether or not turning regulations are desirable.

One city has a fixed policy of recommending parking restrictions so as to develop additional lanes for moving traffic where turning movements make up 20 per cent of the total intersection traffic.

These values are limited to local application and do not suggest a pattern of thought with reference to the development of formulas and equations. They do reflect, however, a sincere desire on the part of traffic authorities to have a uniform basis of factual warrants which would make it possible for them to approach the whole problem of regulating turns in a uniform and technical manner.

Why Full-Time Controls Not Employed

To gain further knowledge of reasons why turning regulations may not be feasible, traffic authorities were asked to indicate whether or not they had considered the adoption of turning prohibitions, but for some reason had not been able to use them. They were asked to explain the reasons why the prohibitions had not been applied.

As expected, most of the cities did not indicate that they had had undue difficulty in developing controls which were considered necessary. There were, however, a few significant comments in answer to this query: In most instances it was stated that street patterns or physical conditions made it impossible or awkard to apply turning prohibitions at particular locations. In one instance, the traffic authority stated that support of the city council had not been obtained and that the controls could not be effected without a traffic ordinance. In a few instances, the regulations met such strong opposition from merchant groups that officials had been unable to have them adopted. Only one city reported that turn controls were opposed by the highway department which

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maintains and controls operations over streets used as state highways.

One city has not applied the controls because when advocated there was strong public objection. Several city officials feel that the best approach to turn controls is through special signal indications and that the regulations should not be applied until the necessary signal equipment could be purchased.

Inadequacy of street patterns is the most common hurdle to turn controls. If certain turns are prohibited, it might become a physical impossibility to accommodate certain traffic flows because of no adequate alternate routes. One police official indicated that in his city the prohibition of turns would do nothing more than shift the problem from one location to another and that they would not provide an overall corrective. In some cases, the decision to withhold the application of turn restrictions is based upon experiences of other cities. Certain important city groups decide that the regulations in neighboring towns are not doing a job and promote objections which carry over in decisions to apply similar controls.

Of particular significance were the reports that inadequate criteria to be used as a guide in applying turning regulations is the principal reason why such regulations have not been more common. It is possible in these cases that the traffic and other public officials are seeking a factual approach to regulating and controlling turns and are not willing to enter the problem on a hit-ormiss basis such as is necessary under the available warrants and formulas.

Why Turn Controls Discontinued

Information was sought as to why cities have discontinued turning restrictions. Apparently turn controls which have been used have been rather long-lived, inasmuch as only fourteen of all the cities gave reasons for discontinuance.

The reasons cited for the discontinuance of the full-time turning controls are shown in Table VIII. It will be noted that in almost half of the cases, the prohibitions were no longer considered

necessary after one-way traffic was instituted. In approximately one-fourth of the cases, the turns had to be changed, or it was decided to change them, because of the objections raised by merchant groups. In another case the merchants' criticisms, coupled with those of citizen groups, provided the basis for discontinuance. Only one city discontinued turning regulations because of citizen complaints. Merchants appear to oppose turning restrictions more frequently in the larger cities than in smaller ones.

Table VIII

REASONS FOR DISCONTINUING	FULL-TIME	TURN	CONTROLS
---------------------------	-----------	------	----------

Nu	mber Reported by Population Ranges			
Reasons	50,000 to 100,000	100,000 to 200,000	200,000 and Up	All Cities
Objections from Merchants	0	0	3	3
Unpopular with Citizens	1	0	0	1
Criticized by Merchants and				
Citizens	0	1	0	1
Changed to One-Way Traffic	2	2	2	6
Changed to Part-Time Control	ol 1	0	0	1
Other	0	2	0	2
	<u> </u>	<u></u>	·····	
Total	4	5	5	14
Number of Cities Reporting				
Discontinuance	4	5	5	14

The validity of turning regulations is reflected by the fact that so few cities have had to discontinue them because of merchants', citizens' and others' objections. Where discontinuances have occurred, they are frequently associated with other traffic regulations; sometimes more permanent correctives allow abolishment of turn restrictions. One city stated, for example, that all turns had formerly been prohibited in the central district for 25 years and that it was only with the establishment of one-way streets that they were able to rescind the regulations.

Several officials reported that bus routings caused difficulties in establishing turning regulations. In only one case, however, was there evidence that a regulation which had been applied had to be removed because of a change in bus routings. In the other instances, it is assumed that bus routes are adequately considered prior to the application of turning regulations and not after they have been put in effect.

Those cases in which the motorists raised complaints seem to stem from situations where the peak traffic volume varied greatly from the normal. In a number of these locations the turn controls would likely have been acceptable on a part-time basis.

New Problems Developed by Turning Prohibitions

In some instances the prohibition of turns at an intersection will create new problems, at the same location or at adjacent intersections. Twenty-four cities reported 27 negative conditions developing when turn prohibitions were applied. In more than three-fourths of these, the only problem of importance was the over-loading or shifting problems to other intersections.

As will be noted in Table IX, the matter of shifting the problem from one intersection to another was almost the only reason given by the largest cities. In smaller cities, the problems developed in the form of complaints about route changes and from merchants' objections. These are also summarized in Table IX.

Table IX

PROBLEMS CREATED BY FULL-TIME TURN PROHIBITIONS

	-	• •	-	0
Problems	50,000 to 100,000	100,000 to 200,000	200,000 and Up	All Cities
Other Intersections Adversely Affected Complaints about Change	3 es	7	11	. 21
in Routes	2	2	о	4
Merchant Objections	0	1	1	2
Total	5	10	12	27
Number of Cities Reporting Problems	5	8	11	24

Number Reported by Population Groupings

One city reported that turning prohibitions forced approximately 50 per cent of the turns from a major street intersection to extremely minor intersections where the physical conditions are totally inadequate to accommodate the movements.

Problems of through highway routing are common in some cities where turning prohibitions are applied. An instance was cited where the prohibition of all turns at a major intersection within a city makes it necessary to have an important throughtraffic movement routed into three additional intersections, requiring two right-turns and a left-turn, whereas the movement could have been made by a single right-turn at the controlled intersections.

No factual evidence was given to support the statement that merchants have lost business as a result of turn restrictions. The merchants were, however, able to muster enough political support or to create enough pressure on traffic authorities to have the turn regulations removed in a number of instances.

Several cases were reported in which strong complaints were received from citizen groups because the prohibition of turns meant routing transit vehicles over residential streets.

One city has found certain turning prohibitions to be very effective in reducing or eliminating "block circling," or cruising while looking for curb parking spaces.

It is generally found that prohibition of turns, when properly determined, has brought praise from public and civic groups rather than complaints. Most of the transit, merchant, and street user groups favor turning prohibitions at busy downtown intersections where volumes of vehicular and pedestrian traffic are great. Based on the returns for this study, it is apparent that favorable comments outweigh criticism at about the ratio of ten to one.

Police Viewpoints

Generally the police are favorable to turning prohibitions. They usually make the work of police easier in developing the proper segregation of vehicular and pedestrian movements and in elimi-

TURN CONTROL

nating movements which are the principal causes of delay and congestion, if not also the principal causes of traffic accidents. Several cases were found where chronic traffic difficulties which had prevailed for as long as 25 years were almost immediately corrected by turning prohibitions. It is only natural in such conditions that the police will attempt to enforce turn controls.

The most popular ban in the eyes of police is the "no-left-turn" control. It seems to be the regulation which creates more relief in terms of traffic congestion and accidents and therefore relieves the police of considerable problems of manual control in peak hours. Some police reported it their greatest aid in keeping major roadways fluid.

The police generally recognize these benefits and acknowledge the lessening of complaints when turn controls are applied.

A few cities reported that the police have some additional difficulties a few weeks after turn regulations are put into effect. This continues until motorists become educated and accustomed to their use. In every one of these cases the regulation was strongly supported and liked by the enforcement authorities after the motorists became familiar with them.

One or two cities stated that the police are not favorable to continuous or full-time controls, but that they much prefer the parttime controls which necessitate restricting traffic only during the period of the day when traffic volumes are heavy enough to create acute congestion.

The police like turning prohibitions so well in some areas that they advocate the prohibition of all turns in the business district. They have reacted so favorably that they would like to have turning regulations extended wherever possible. Often they suggest their application to locations where it is not feasible and where conditions do not warrant. One city reported that many of its uniformed officers actually favor "no-turns" at every intersection throughout the entire business area. This would be practically impossible with most street plans.

The police reported in only one city that they look with disfavor on all types of turning prohibitions. There are many in-

stances, however, in which the police reported that they liked restricted turns, provided the regulations are not over-applied, so that the motoring public will be penalized as little as possible. It was common to get the reply from police: "Highly desirable where warranted."

Inquiry was made as to whether or not any special enforcement problems have resulted from the application of turning regulations. About the only problem reported was that additional personnel was required immediately after the regulations had been installed and until the motoring public had become acquainted with the new regulation. Motorists must have an opportunity to fit travel plans and origins and destinations to the routing necessitated by the regulations.

Most police agencies have found it desirable to utilize advance publicity in order to ease their problem of educating the public when new turning prohibitions are installed.

A few towns feel that the violation rate becomes so high in offpeak hours when it is not feasible to assign traffic officers to all intersections with turning prohibitions that a serious accident hazard might develop. This contention was not supported by accident facts.

While turning prohibitions might relieve demands for officer assignments at some intersections, it is generally believed by police executives that they have little if any effect upon manpower requirements for the overall urban enforcement program. This means that intersections with turn controls are important in the overall traffic enforcement picture, and require continued police attention after the regulations are applied.

A few police departments have found it almost impossible to effectively enforce no-turn prohibitions. They cite the doubt in each instance that such prohibitions were justified by traffic conditions and that therefore it was difficult to enforce them.

It is interesting to note that from 80 to 90 per cent of all cities furnishing information reported no unusual enforcement problems developed by turning prohibitions.

While collecting information regarding the attitudes of police

and the creation of special enforcement problems related to turn controls, an effort was also made to determine whether or not any special difficulties have arisen regarding convictions for violations. There was not a single indication that the courts have shown a proneness to dismiss cases or to fail in their responsibilities to levy penalties when turn regulations are violated. This indicates complete court support of turning regulations in the cities surveyed. It was pointed out, however, that in many cases convictions for violations of turning regulations are made complicated by motorists' complaints that they did not see the sign, or special signal indication. Such complaints naturally develop an attitude of leniency on the part of the courts when the regulations are first applied, but later such leniency is usually extended only to strangers.

It would appear that in some cities the signs advising of parttime controls or the prohibition of turns only during certain hours are difficult to understand by strangers. This is undoubtedly traceable to the message on the sign, the size of letters, or to other conditions which make it difficult for motorists to read and comprehend the message in the short time available under most downtown driving conditions. One city reported that the courts are very much inclined not to convict for turning violations at night because the signs are not illuminated. It was explained that this condition, however, is no worse than that related to enforcement of other traffic regulations within the city.

One police department reported difficulty in the enforcement of turning regulations because most of the violations occurred during the peak traffic period when it is difficult to keep traffic officers stationed at every intersection where the regulations are in effect. Also it is recognized by the police that at certain intersections a hopeless problem of congestion is apt to be developed if violators of the turning prohibitions are apprehended during peak hours. Cognizant of this situation, some local motorists take advantage of the police and persistently violate the regulations during peak traffic periods.

The attitude of police toward turn controls is summarized in

USE OF TURN CONTROLS

Table X. It will be noted that almost all the departments favor the regulations. Only three per cent of the cities reporting, and these were in towns of less than 200,000 population, were generally opposed.

Turn Signs

Information was collected as to the general design, size, color, wording and location of signs indicating turning prohibitions. Fifty-four cities gave information regarding sign sizes. This is summarized in Table XI. Fifty-four cities use 61 different sizes of signs for indicating turning prohibitions. Two cases were found in which the signs are over 1,000 square inches in area. The most commonly used signs are 432 square inches in area and have outside dimensions of 18" by 24." Almost as popular are the 12" x 18" signs with a total area of only 216 square inches. Nine sizes with areas of only 108 to 200 square inches were found. It will be noted that these are used almost entirely in the population grouping from 100,000 to 200,000. In the largest cities, one-third of the signs are of the 18" x 24" size.

White backgrounds with black letters are by far the most popular colors for turn prohibition signs. As will be noted in Fig. 6, and in Table XII, about three-fourths of the color combinations reported were of this type. Black messages on yellow backgrounds constituted 15 per cent of the total; 9 cases were cited in which other combinations of colors were used, mostly red letters on white background; three cities employ neon turn signs. As the size of the cities increases, the trend towards black letters on white background increased markedly.

It should be remembered that where the neon type signs are in use, not more than three or four intersections within the city are affected by turning regulations. It was reported that the expense of providing and maintaining such electrically operated signs would make their common use prohibitive in most of the cities where turning regulations are applied at numerous intersections.

Rectangular shaped signs, as indicated by data given in Table XI, are predominant. Only about 15 per cent of the signs indicat-

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POLICE ATTITUDES TOWARD FULL-TIME TURN CONTROLS

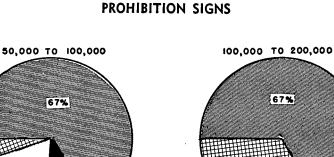
		Number and Per Cent of Cities in Each Population Group						
	50,000 to 100,000		100,000 to 200,000		200,000 and Up		All Cities	
Attitude	Number	Per Cent	Number	Per Cent	Number	Per Cent	Number	Per Cent
In Favor	21	92	24	96	17	94	62	94
Opposed	1	4	1	4	0	• O	2	3
Prefer Part-Time Controls	1	4	0	0	0	. 0	1	2
Concerned about Adequate Signing	0	0	0	0	1	6	1	2
Total	23	100	25	100	18	100	66	100

Table XI

SIZES OF TURN PROHIBITION SIGNS

	Number and Per Cent of Cities in Each Population Group							
	50,000 te	0 100,000	100,000 t	0 200,000	200,000	and Up	All C	lities
Sign Size	Number	Per Cent	Number	Per Cent	Number	Per Cent	Number	Per Cent
Over 1,000 square inches	1	5	0	0	1	7	2	3
700–800 square inches	1	5	1	4	1	7	3	5
500–600 square inches	4	19	5	20	2	13	11	18
432 square inches (18x24)	6	28	6	24	5	33	i7	28
300–400 square inches	1	5	3	12	1	7	5	8
216 square inches (12 x 18)	6	28	4	16	4	26	14	23
108-200 square inches	2	10	6	24	1	7	9	15
Total	21	100	25	100	15	100	61	100
Number of Cities Reporting	19		23		12		54	

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COLOR COMBINATIONS OF TURN

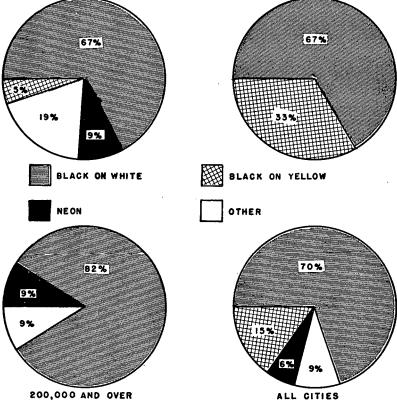


FIGURE 6. Sign Color Combinations by Population Groupings of Cities.

ing turning regulations are square. No unusual shapes were reported.

The most common location for no-turn, or other turn control signs, is on traffic signals. Some cities locate the sign immediately under the signal heads, some locate them to the right or left of the

Table XII

COLORS OF TURN PROHIBITION SIGNS

	Number and Per Cent of Colors Reported by Population Groups							
	50,000 te	0 100,000		0 200,000		and Up		Cities
Color Combinations	Number	Per Cent	Number	Per Cent	Number	Per Cent	Number	Per Cent
Black on White	14	67	14	67	9	82	37	70 ·
Black on Yellow	1	5	7	33	0	0	8	15
Neon	2	9	Ō	0	`1	9	3	6
*Other	4	19	ο	0	1	9	5	9
Total	21	100	21	100	11	100	53	100
Number of Cities Reporting	21		20		11		52	

* "Other" includes white on black, white on red, red on white, and green on white.

signal heads, and in a few instances the signs are placed on posts or stanchions which support the signals. Two cities follow the practice of placing signs only on islands, five locate them on special stanchions. In all except one of these latter cases, however, the signs are for part-time controls and are used on stanchions primarily for ease in moving from the roadway when not needed.

In general, the cities are standardizing the designs of their signs for turn regulations in accord with the standards prescribed in the Manual on Uniform Traffic Control Devices.¹ Standards prescribed for turn control signs in the Uniform Manual are as follows:²

Turn Prohibition signs shall be used at intersections to indicate regulations prohibiting the types of turns specified. The NO U-TURN sign may also be used between intersections on a wide roadway where dangerous U turns might be made. Where U turns are prohibited by statute throughout a given area, it is unnecessary to erect prohibitory signs at or between intersections as prescribed above.

Turn Prohibition signs shall be 18 inches by 24 inches in size except that when mounted below traffic signals where vertical clearance is limited they may be 14 inches by 9 inches in size. They shall have black lettering on a white background.

Where required at unsignalized intersections, the NO RIGHT TURN sign shall be placed on the near right-hand corner. Where NO LEFT TURN, NO U-TURN or NO TURNS signs are required, two shall be used, one at the near right-hand and one at the far left-hand corner. Such signs shall be mounted, facing traffic approaching the intersection, so that the bottom part of the sign will not be less than 7 feet nor more than 10 feet above the top of the curb and so that no part of the sign will be less than 1 foot back from the face of the curb. At signalized intersections the signs shall be mounted just below the signal faces governing the traffic to which they apply. These are minimum requirements, and additional signs should be placed as necessary, at or in advance of the intersections. If advance signs are used, care should be taken that no alley or driveway exists between them and the intersection where the turning movement is prohibited. At an intersection with a one-way street, whether signal-

¹ Manual on Uniform Traffic Control Devices for Streets and Highways, U. S. Public Roads Administration, August 1948.

² Ibid. Section 37, p. 24.

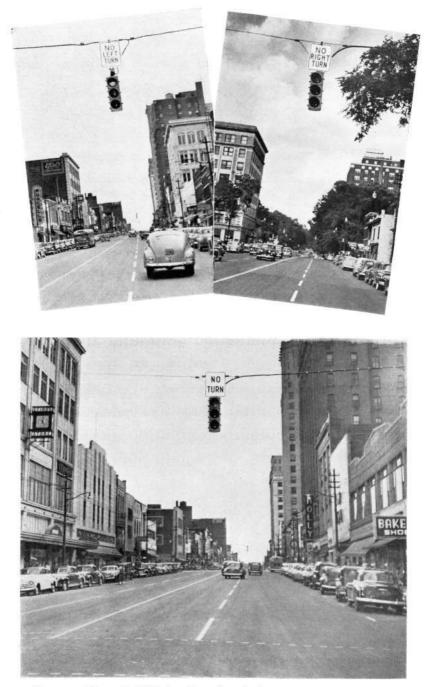


FIGURE 7. Turn Prohibition Signs Attached to Traffic Signal Heads. (Courtesy Charlotte, North Carolina)









FIGURE 8. Methods of Designating Turn Prohibitions. (Photos 1 and 2, Courtesy San Jose, California) (Photo 3, Courtesy Wichita, Kansas) (Photo 4, Courtesy Duluth, Minnesota) ized or not, the ONE WAY sign should be used, rather than the TURN PROHIBITION sign.

A TURN PROHIBITION sign mounted on a traffic signal installed directly over any roadway shall have a clearance of at least $14\frac{1}{2}$ feet above the roadway.

When the movement restriction applies during brief periods only, the TURN PROHIBITION sign shall be mounted on a movable pedestal and placed in the roadway adjacent to the curb or in the middle of the intersection during such periods only.

Some of the signs employed in various cities for indicating turning controls are shown in Figures 7 and 8.

CHAPTER III

THE TURN STUDY

It is clear from the questionnaire study that many traffic officials regard excessive vehicle-pedestrian conflict as an important reason for using turn controls. It is equally clear that most of these officials have no well-considered basis for deciding *how much* conflict justifies a turn control regulation. What is needed is a *warrant* for prohibiting turns on the basis of the conflicts between pedestrians and turning vehicles.

General Discussion of Warrant

The development of a warrant for any traffic control device consists of two basic stages. First, there must be a warrant criterion, which is a general but definite statement of the conditions under which the device is considered to be useful. For example, one possible warrant criterion in the present study might be that a turning movement should be prohibited if more than fifty per cent of the cars making the turn are delayed as a result of conflicts with pedestrians. The merit of this particular criterion can be criticized, but it illustrates the meaning of the term.

The second stage in developing a warrant consists in discovering, by analysis of empirical data, the relationship between the warrant criterion and the empirical quantities which can easily be observed. With the criterion suggested in the above example, this second stage would involve a study to determine how the percentage of delayed turns is related to the volume of pedestrians, the volume of turning vehicles, the width of the street, and other similar factors.

The Field Data

At the time the empirical data for this study were obtained, it was believed that the warrant criterion should involve either (1)

the absolute number of turning cars which are delayed as a result of conflicts with pedestrians, or (2) the proportion of all the turning cars which are delayed. The traffic facts which were observed, therefore, were the following:

- (1) Pedestrian volume on a crosswalk
- (2) Volume of vehicular turns across that crosswalk
- (3) Number of delayed turns.

Before discussing the data in detail, it is in order to describe the locations that were studied. Three "typical" crosswalks were used, all of them at right-angled intersections with one lane of moving traffic in each direction. The locations were all in downtown areas, with heavy vehicular and pedestrian traffic in the busiest periods. Physically, the intersections were also similar— 30 to 40 foot pavement widths, asphalt surfaces, no channelization or safety islands, and no special signal control for pedestrians or turning vehicles. All turning movements from the parallel vehicular streams were permitted through each crosswalk, and the pedestrian and vehicular streams both moved on the same green light.

Data were taken for one crosswalk at each intersection. From all parts of the day 15-minute counts were made of the three quantities: (1) the number of pedestrians using the crosswalk, (2) the number of cars starting parallel to the crosswalk and making turns across it, and (3) the number of these turning cars which were delayed as a result of conflicts with pedestrians. The counts of pedestrians include those who crossed when the light was against them; these constituted about 20 per cent of all pedestrians. No jaywalkers were counted, since the study was made at marked crosswalks. Observance of the signals *by motorists* was entirely satisfactory.

Definition of a Delayed Turn

A very important aspect of the data collection was the determination of what was and what was not a delayed turning maneuver. A car making the turn without opposition of any sort is obviously

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not delayed. But a car may be delayed by a car turning ahead of him, by a car turning across his path from the opposite direction, or for a number of other reasons not related to pedestrian interference. A driver turning a corner may even be forced by a small group of pedestrians to reduce his speed slightly without having to wait for any pedestrians to get out of his way. Such a turn was not considered delayed. The same judgment was applied to the driver who was able to take advantage of a gap in the pedestrian stream created by a preceding driver.

The turns that *were* considered delayed were of two types, those delayed directly by having to wait for pedestrians or having to thread their way carefully through the pedestrian stream, and those delayed indirectly by having to slow down or stop behind other cars which were delayed directly. A car delayed by another car which was delayed indirectly was also considered a victim of indirect delay. In cases where two cars coming from opposite directions attempted simultaneously to turn across the crosswalk being studied, the delay was attributed to pedestrians only in those situations where pedestrians appeared to be the chief cause of the delay.

These determinations, which necessarily involved considerable personal judgment, were made "on the spot" by the observer who collected the data. While there might be some difference of opinion among careful observers as to whether certain turns were or were not delayed, the person who collected all the data for this study found no particular difficulty in distinguishing between delayed and undelayed turns according to the foregoing criteria. There is every reason to believe that his judgments were consistent throughout the study.

Much of the data turned out to be unusable, because the best techniques of field observation could only be developed by trying various methods of collecting data. In noting the amount of material in the appendix, where the full set of usable data is given, it should be kept in mind that these figures represent only a fraction of the total time that was spent watching traffic.

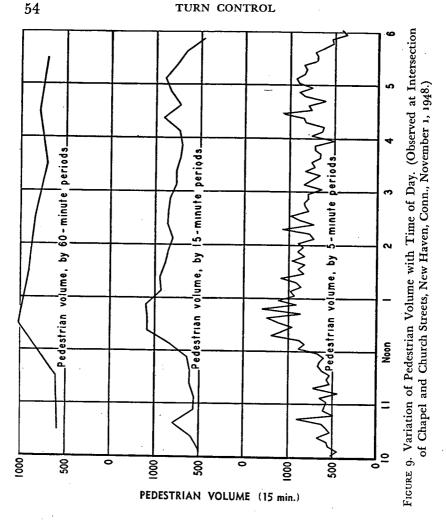
The usable data represent about equal amounts of observation time at the three locations in New Haven and Bridgeport, Con-

necticut. It may be of interest to note that a total of 89,344 pedestrians and 3884 turning vehicles were observed in the turn study. While the field observations did not cover the complete range of possible values of the various quantities, the pedestrian volumes ranged from 364 to 4392 per hour, and the vehicular turning rates ran from 12 to 184 per hour. The average rates from all the data were 2192 pedestrians per hour and 95 turning movements per hour.

Analysis of Data

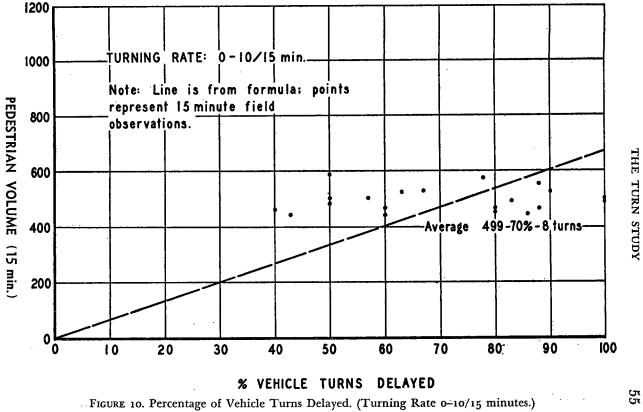
In analyzing the data, one of the first questions which had to be answered concerned the size of the interval to be used in counting volumes. Should one use five-minute counts, fifteen-minute counts, hourly counts, or what? Too short an interval will cause the results to show a lot of meaningless fluctuations, while too long an interval will mask some of the variations which are really significant. To explore this question, the pedestrian volumes on one of the observation days were plotted by 5-minute periods, by 15-minute periods, and by hour-long periods. These graphs are shown in Figure 9. The 5-minute periods are clearly too short, with three-fourths of the points being either peaks or valleys. This amount of fluctuation in an eight-hour period is meaningless. The hourly counts, on the other hand, are too broad to give an adequate picture of either the lunch-hour peak or the late afternoon peak, both of which show up clearly in the 15-minute counts. The 15-minute counts appear to provide the best compromise, and therefore all volume figures will be stated in terms of the number of vehicles or pedestrians per fifteen minutes.

A related question concerns the constancy of pedestrian values from one day to another. It is widely known that vehicular volumes show fairly regular patterns of variation which tend to repeat themselves from one day to another, from one week to another, and from one year to the next. It is natural to assume that pedestrian volumes exhibit the same kind of regularity. This assumption could not be either confirmed or denied by the data of this study, since there were not enough cases where the same intersection was observed at the same time on different days.



The full set of data is given in Appendix A. In analyzing these figures it was decided that the three most useful quantities were the pedestrian volume, the turning volume, and the percentage of turns delayed as a result of interference between vehicles and pedestrians. These are denoted by the letters V, T, and P respectively.

Figures 10-13 show the results of plotting the pedestrian



THE TURN STUDY

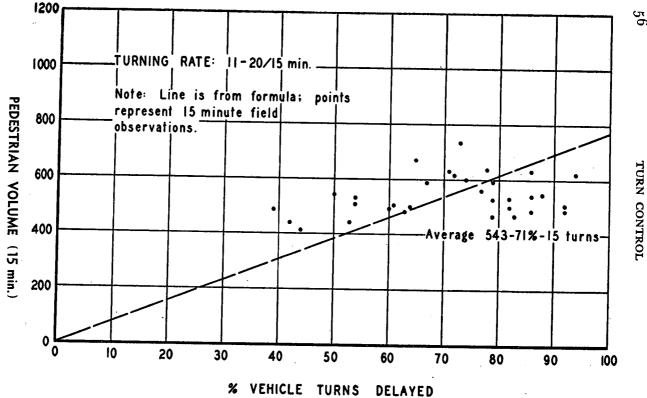
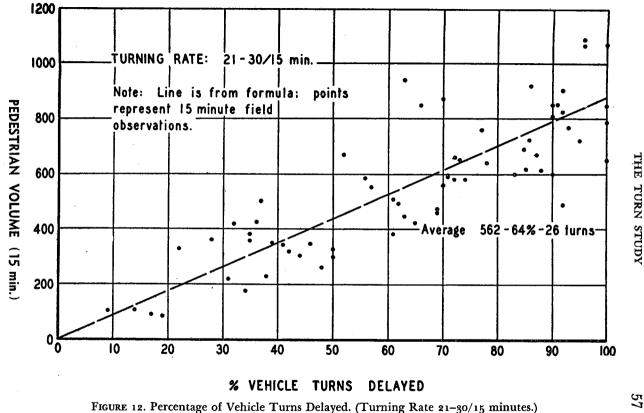


FIGURE 11. Percentage of Vehicle Turns Delayed. (Turning Rate 11-20/15 minutes.)



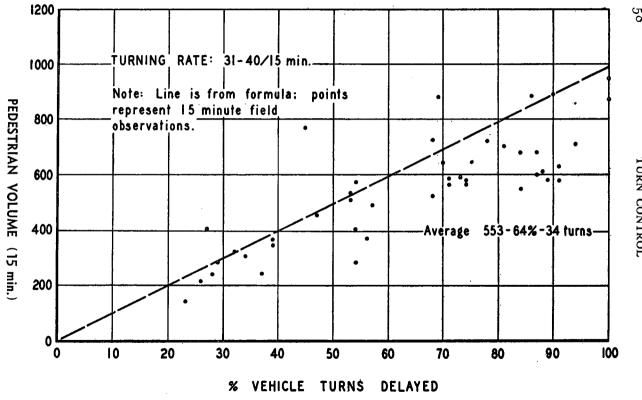


FIGURE 13. Percentage of Vehicle Turns Delayed. (Turning Rate 31-40/15 minutes.)

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volume (V) against the percentage of delayed turns (P), with each graph confined to a small range of turning rates (T). Each point represents one fifteen-minute period, in accordance with the finding that this was the most suitable interval for volume counts. Data from the three intersections are all thrown together in each graph, inasmuch as no difference was found in the figures between one intersection and another.

It is hard to see from these graphs just what the relationship is between V and P, because there is so much scatter in the data points. The relationship becomes clearer when all the points from each graph are combined into a single average point. These "average points" are plotted in Figure 14 from which the time relationship between V, P, and T begins to emerge. Two things will be noticed especially: (1) P is proportional to V, approximately, when T doesn't vary too much; (2) the ratio of P to V goes *down* as T increases. The significance of these points, especially the second one, will be discussed a little further on.

Because of the scatter in the raw data it was considered desirable, in the search for a mathematical relationship, to group the data by pedestrian volumes, by turning volumes, and by percentages of delayed turns.

Table XIII shows the results of grouping the data by pedestrian volumes. The first row in the table represents the average of all the 15-minute periods in which the number of pedestrians ranged from 0 to 99, the second row is the average of the periods in which V was between 100 and 199, and so on. The columns of the table list the pedestrian volume, the turning rate, the number of pedestrians per turning vehicle, the number of turns delayed, and the percentage of turns delayed.

Since the turning rates show no steady trend from the top of Table XIII to the bottom, the table illustrates nicely the relationship between V (pedestrian volume) and P (percentage of turns delayed). As V increases, so does P, and it looks almost like a simple proportionality relationship. V goes up a little faster than P, to be sure, but there is no doubt of the fact that they vary in the same direction.

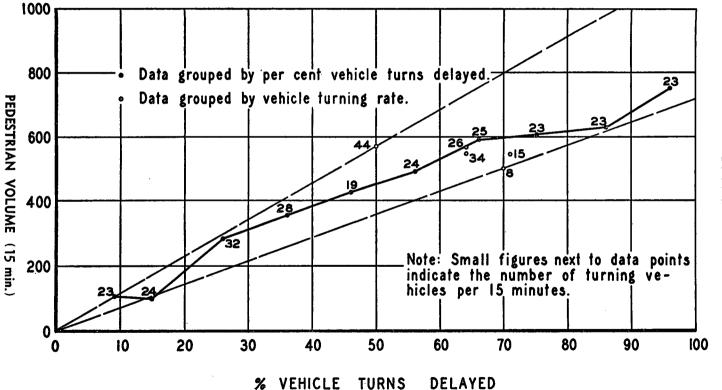


FIGURE 14. Percentage of Vehicle Turns Delayed as a Function of Pedestrian Volume and Vehicle Turning Rate.

TURN CONTROL

Table XIII

Pedestrians per 15 Minutes	Turns per 15 Minutes	Pedestrians per Turn	Delayed Turns per 15 Minutes	Per Cent of Turns Delayed
93	25	3.7	4.5	18
135	26	5.2	5.5	21
250	32	7.8	11.6	36
345	28	12.3	11.5	41
462	18	25.6	10.3	57
548	21	26.1	14.3	68
540 643	26	24.7	20.8	8 o
744	27	27.6	22.2	82
863	29	29.8	24.6	85
932	30	31.1	25.5	85
1,085	24	45.2	23.0	96

GROUPED PEDESTRIAN VOLUMES

Table XIV, in which the data are grouped on the basis of P, confirms the direct relationship between P and V. From this table the two quantities appear to vary at the same rate. It is reasonable to suppose, therefore, that the correct relationship is a straight line starting at the origin and rising to the point where P equals 100. For volumes larger than that, P must remain equal to 100, since it is impossible to have more than 100 per cent of the turns delayed.

Table XIV

GROUPED PER CENT OF TURNS DELAYED

Pedestrians per 15 Minutes	Turns per 15 Minutes	Pedestrians per Turn	Delayed Turns per 15 Minutes	Per Cent of Turns Delayed
109	23	4.7	2.0	9
99	24	4.1	4.0	17
284	32	<u>8</u> .9	8.3	26
360	28	12.9	10.2	36
431	19	22.7	8.7	46
496	24	20.7	13.1	55
597	25	23.9	16.2	65
602	23	26.2	16.9	73
630	23	27.4	19.8	86
75 ²	23	32.7	21.9	95

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Having established the fact that P is proportional to V, the remaining question is how the proportionality factor varies with T. Table XV, in which the data are grouped according to their values of T, throws light on this question. The table shows that the ratio of P to V gets *smaller* as T increases; that is, for a given pedestrian volume, the percentage of turns delayed goes down as the turning rate increases.

Table XV

Pedestrians per 15 Minutes	Turns per 15 Minutes	Pedestrians per Turn	Delayed Turns per 15 Minutes	Per Cent of Turns Delayed
499	8	62.4	5.4	68
543	15	36.2	10.3	69
562	26	21.6	16.6	64
553	34	16.3	21.8	64
572	44	13.0	22.0	50

GROUPED TURNING RATES

This may at first seem a surprising result, even though the *number* of delayed turns does increase as the turning rate goes up, for a fixed volume of pedestrians. The reasonableness of the result is apparent, however, when one realizes how the turning cars pave the way for one another. Assume, for the moment, that the pedestrian volume is fixed and has a large value. If the turning volume is small, nearly every one of the turning cars will have to clear its own path through the heavy pedestrian stream, and almost all the turning cars will be delayed. As the turning volume increases, more and more of the turning cars will be able to proceed right through the pedestrian stream in the wake of other turning cars, with the result that a smaller percentage of the turning cars is greater.

The Formula

These relationships are all combined in the formula

$$P = \frac{V}{.117 \text{ T} + 5.80} \text{ or 100, whichever is smaller}$$
(1)

where P is the percentage of turns which are delayed as a result of vehicle-pedestrian conflicts, V is the pedestrian volume (in persons per fifteen minutes), and T is the turning volume (in vehicles per fifteen minutes).

The lines in Figures 10–13 have been drawn from this formula. The meaning of the formula is made clearer by Figures 15–18. Since these graphs are all drawn from the formula, no inconsistencies will be found among them. The values in Tables XIII–XV have been found to fit satisfactorily in all of them.

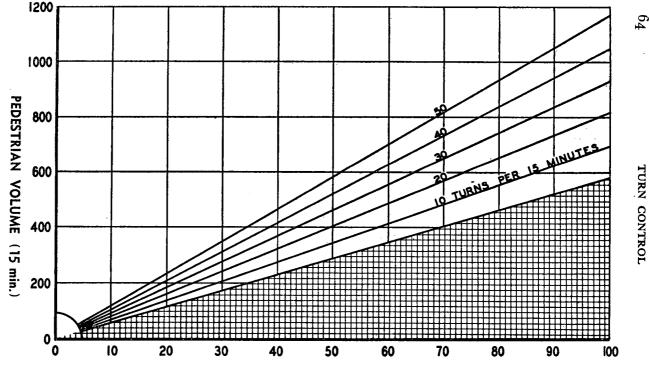
Figure 15 shows the relationship between the pedestrian volume (V) and the percentage of turns delayed (P) for several values of the turning volume (T). As suggested in the discussion of the tables, the relationship is a straight line whose slope depends upon the turning rate. It will be noted that the slopes (V/P) get steeper as the turning rate increases.

In Figure 16 the pedestrian volume (V) is plotted against the *number* of turns delayed (.01PT) for several values of the turning rate (T). These relationships are also straight lines starting at the origin, but this time the slopes become less steep as the turning rate increases. The dashed lines indicate the values for which P is constant.

Figure 17 gives the most useful type of presentation for a warrant criterion involving the percentage of delayed turns. This graph shows the relationship between the pedestrian volume (V) and the turning rate (T), for several values of the percentage of delayed turns (P). If a turning movement is to be prohibited whenever the percentage of delayed turns exceeds a certain value, the warrant curve will be one of the lines in Figure 17. All values above this line will warrant the prohibition, while all values below it will be insufficient to justify the prohibition.

Critique of Warrant Criteria

But is this really a reasonable kind of warrant criterion? The use of this kind of criterion implies that there is more reason to prohibit a turn if there are 2 turning cars per hour and 100 per



% VEHICLE TURNS DELAYED

FIGURE 15. Percentage of Vehicle Turns Delayed vs. Pedestrian Volume, for Various Turning Rates. (Plotted from Formula.)

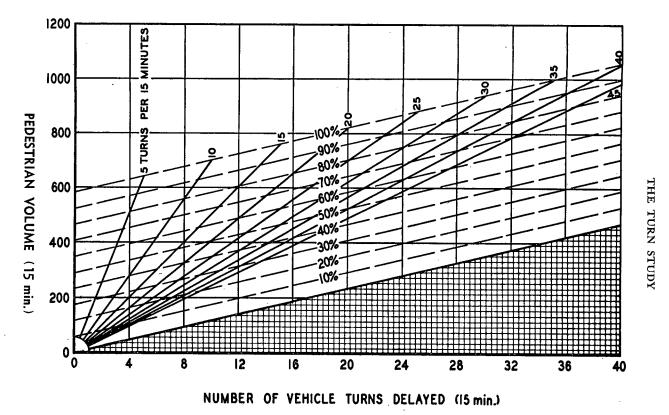
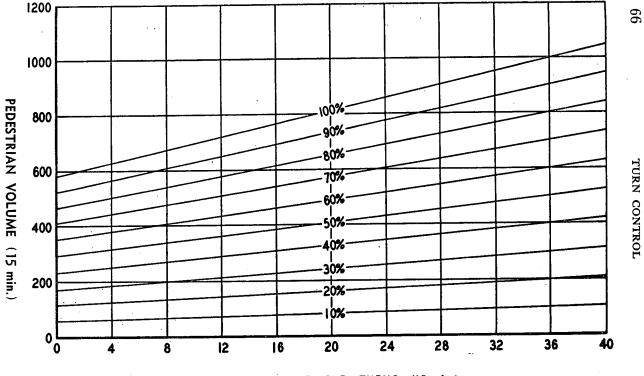


FIGURE 16. Number of Vehicle Turns Delayed vs. Pedestrian Volume, for Various Turning Rates and Percentages of Delayed Turns. (Plotted from Formula.)



NUMBER OF VEHICLE TURNS (15 min.)

FIGURE 17. Percentage of Vehicle Turns Delayed as a Function of Pedestrian Volume and Vehicle Turning Rate. (Plotted from Formula.)

cent of them are delayed than if there are 150 turning cars per hour and only 30 per cent of them are delayed. This absurd conclusion suggests the need for finding a better warrant criterion.

One alternative criterion might be that a turn prohibition is warranted whenever the *number* of delayed turns exceeds a certain rate. This information can be obtained from Figure 16, but it is more clearly presented in Figure 18, where the relationship between the pedestrian volume and the turning rate is shown for several values of the number of delayed turns per fifteen minutes.

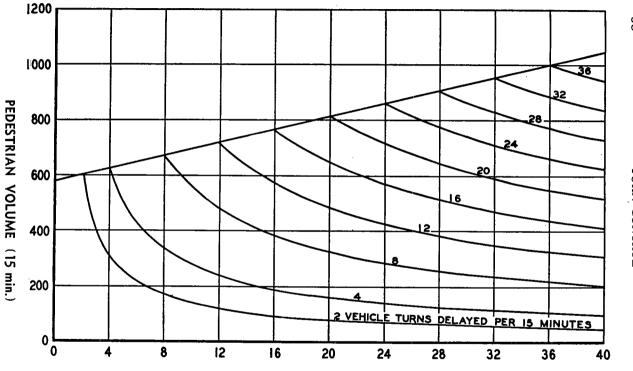
Figure 18 is a much more sensible warrant graph than Figure 17, because the warrant curves have a negative instead of a positive slope. This means that an increase in the pedestrian volume can be compensated by a decrease in the turning rate, and vice versa. This conclusion is in accord with common sense.

Even this warrant criterion is not fully satisfactory, however, since it leaves out of account the effect of delays to turning vehicles on the vehicles which desire to proceed straight across the intersection. It is well-known that delays to a handful of turning cars can hold up a long line of non-turning cars unless the street is wide enough to provide separate lanes for the turning vehicles. A proper warrant for turn prohibitions would have to take into account the volumes of non-turning cars, the number of *them* which are delayed, and the number of lanes in the roadway.

Inasmuch as the original field observations failed to include any counts of non-turning cars, it was decided to apply the mathematical theory of probability to show how the proportion of delayed non-turning cars is related to the pedestrian volume and the various vehicular volumes. The following section explains the underlying ideas and the results of this analysis, while the mathematical details will be found in Appendix B.

Delayed Non-Turning Cars

On the basis of certain reasonable assumptions one can develop a mathematical formula for the proportion of the non-turning cars which are delayed from being held up by delayed turning



NUMBER OF VEHICLE TURNS (15 min.)

FIGURE 18. Number of Vehicle Turns Delayed as a Function of Pedestrian Volume and Vehicle Turning Rate. (Plotted from Formula.)

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TURN CONTROL

cars ahead of them. The analysis applies to a single lane. Since each lane is to be treated separately, this type of analysis automatically takes account of the differences in width between one street and another.

Underlying the analysis are the following three assumptions:

- (a) The arrival times of all cars are distributed at random.
- (b) Every delayed turn blocks the traffic stream behind it for a length of time D following its entry into the intersection.
- (c) When a group of non-delayed cars begin to move after being stopped by a red light, they enter the intersection with a constant headway H.

These assumptions are all quite sensible. It will be noted that they involve two constants, D and H, which probably do not vary appreciably from one intersection to another. (The value of D may vary to some extent with the area available for shadowing the turning movement, which in turn depends upon the presence or absence of parked cars near the corner, the radius of the curb return, and the exact location of the crosswalk.) Since D and H are constants, their values can be measured once and for all and then incorporated into the numerical coefficients in the final formula.

Four additional quantities are needed, in order to use the formula: the pedestrian volume, the volume of turning vehicles, the volume of non-turning vehicles, and the length of the signal cycle. The first two do not enter directly into the formula but are used in formula (1) to compute the average number of delayed turns per cycle.

The procedure in computing the number of delayed non-turning cars per cycle is to figure out how many non-turning cars will be delayed for any particular number of delayed turns during the cycle (0, 1, 2, 3, etc.), multiply this number of delayed non-turning cars by the probability of having that number of delayed turns during the cycle, and add up these products for all the different numbers of delayed turns. The resulting infinite series turns out to be summable, the sum being the expected number of delayed non-turning cars per cycle. Dividing this by the average number of non-turning cars per cycle gives the proportion of the non-turning cars which are delayed, which is

$$\frac{\mathrm{mD}}{\mathrm{L}} + \mathrm{R} - \frac{\mathrm{1}}{\mathrm{m}} \left[1 - \mathrm{e}^{-(\mathrm{1 + NH}) \mathrm{Rm}} \right] + \frac{\mathrm{NH}}{\mathrm{m}} \left(1 - \mathrm{e}^{-\mathrm{Rm}} \right), (2)$$

where m = average number of delayed turns per cycle

- D = duration of delay for each delayed turn, in seconds
- L = cycle length, in seconds
- R = fraction of the signal cycle during which the light is red
- N = volume of non-turning cars, in cars per second
- H = starting headway, in seconds.

From formula (1) the value of m is as follows:

m =
$$\frac{VL}{10,530 + \frac{522,000}{T}}$$
 or $\frac{LT}{900}$, whichever is smaller. (3)

In using formula (2) it is best to think of it in two parts, the part which depends on the value of D and the part which doesn't. Thus the first part is the first term, which is a simple expression involving m, D, and L. The second part, consisting of the rest of expression (2) looks complicated but really isn't because it is a product of R times a function of only two variables (the products Rm and NH) and hence is easily represented in tables and graphs. With this simplification the percentage of non-turning cars which are delayed is

$$\frac{100 \text{ mD}}{\text{L}} + 100 \text{ R f (Rm, NH)}, \qquad (4)$$

where f (Rm, NH) has the values given in Table XVI and shown graphically in Figure 19.

It may be asked why formulas (2) and (4) are not equal to zero for all values of m when D and H are both zero. One would think that if the process of making a delayed turn caused no delay to the cars behind the turning car (i.e., if D = 0) and if there were no delay caused by sluggish starting (i.e., if H = 0), there would be no delayed non-turning cars. For that matter, why shouldn't

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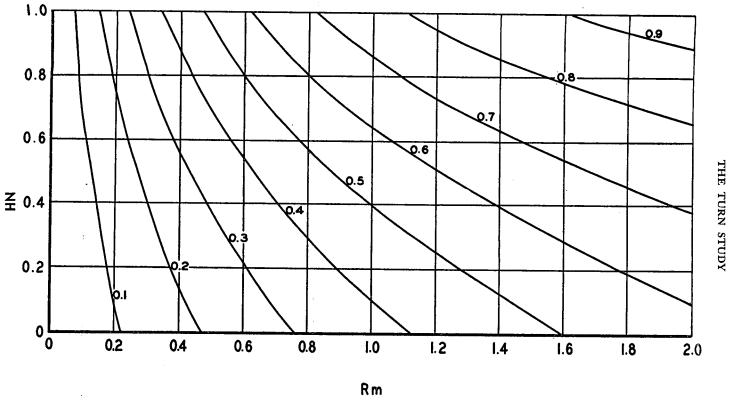


FIGURE 19. Graph of f (Rm, NH).

formulas (2) and (4) equal zero when D is zero, regardless of what H is?

Actually they should. These formulas are not correct for D = 0. But they are correct for any non-zero value of D, no matter how small it is. The reason for this discontinuity is that whenever D is positive, all the cars which arrive during the red light behind a car whose turn is destined to be delayed by pedestrians will be delayed on account of the delayed turn. The larger H is, the longer it will take for the car making the delayed turn to reach the intersection after the light turns green, and hence the more non-turning cars will be delayed by it. This is why the values in Table XVI increase from left to right in each row (other than the top row, of course: if there are no delayed turns, there can be no cars delayed by them).

The reason the values increase from top to bottom of each column in Table XVI is that the larger the number of delayed turns per cycle, the sooner the first one to arrive during the red light is likely to get there, and hence the more cars he is likely to catch behind him.

	~	11201		, ,		
Rm	NH 0	.2	•4	.6	.8	1
о	0	0	О	0	о	о
.2	.09	.11	.14	.17	.21	.26
•4	.17	.21	.26	.31	•37	•44
·4 .6	.25	.29	.35	.42	.50	.59
.8	.31	·37	•43	.51	.60	.69
1	·37	•43	.50	.58	.67	•77
1.2	.42	.48	.56	.64	•73	.82
1.4	.46	•53	.60	.68	•77	.87
1.6	.50	•57	.64	.72	.81	.90
1.8	.54	.60	.67	·75	.84	.92
2	•57	.63	.70	.78	.86	.94
8	1	1	1	1	1	1

Table XVI VALUES OF f (Rm. NH)

Rm = average number of delayed turns per red interval.

NH = average number of non-turning cars per headway interval

CHAPTER IV

EMPIRICAL TEST OF FORMULAS DEVELOPED

After the formulas in Chapter III were developed, additional field work was undertaken in order to test the formulas and determine the values of D and H. Of the two intersections at which these data were collected, one was the Bridgeport location which had been studied earlier while the other was in midtown Manhattan.

One observer kept track of the vehicle movements in the righthand lane with an Esterline-Angus twenty-pen recorder, while a second observer counted pedestrians. This time both non-turning cars and turning cars were counted, and the counts in both groups were subdivided according to whether or not the cars were delayed by conflicts between pedestrians and turning vehicles. The results of these observations are given in columns 1, 2, 3, 5, and 7 of Table XVII.

In addition the recorder data were used to determine the values of D and H. The time of entry of each car into the intersection was noted, along with a code symbol indicating whether or not this entry time was dependent on the behavior of the preceding car. Only those cases where a car entered immediately behind the preceding car were used in determining headway values.

The average headway interval H was computed from the intervals following undelayed non-turning cars. The average length of these intervals was 2.9 seconds. A second computation was made for the first-in-line cars, where the interval used was from the beginning of the green light to the crossing of the curb line by the first of the stopped cars. Since these intervals also had the same average, the two have been combined in Table XVIII, which shows the frequency distribution of headway intervals:

Pedestrians	Turns	Non-turning Cars	Per Cent of Cars Del		Per Cent of No Cars Dela	iyed
per 15 minutes	per 15 minutes	per 15 minutes	Theoretical	Actual	Theoretical	Actual
FEB. 20, 1950 (1	MAIN AND GOLDEN	HILL STS., BRIDGEPO	RT, CONN.; 70-SEC	OND CYCLE, W	ITH 35-SEC. GREEN O	n main st.)
126	10	60	19	30	2	2
122	9	65	19	22	2	2
142	4	56	20	0	I	0
143	8	62	21	12	2	2
144	10	57	21	30	2	5
151	9	56	22	67	2	14
138	9	55	20	22	2	7
143	7	54	22	57	2	9
110	19	65	13	11	4	3
123	11	62	19	36	2	10
102	16	60	13	31	- 2	
90	5	90	14	0	2	O .
118	14	92	16	43	4	· 1
100	6	83	16	17	1	0
FEB. 21, 1950 ((SIXTH AVE. AND	42D ST., NEW YORK, I	N.Y.; 90-SECOND	CYCLE, WITH	59-SEC. GREEN ON	SIXTH AVE.)
391	19	129	49	58	9	3
465	11	125	66	82	7	3
556	14	141	75	86	11	4
54 9	11	119	78	91	9	4
521	16	132	68	88	11	8
505	12	110	70	92	8	10
448	18	111	57	78	10	9
365	19	117	45	63	8	7
326	16	148	42	44	7	4
328	18	- 97	41	39	7	3

Table XVII: EMPIRICAL TEST OF FORMULAS

TURN CONTROL

Table XVIII

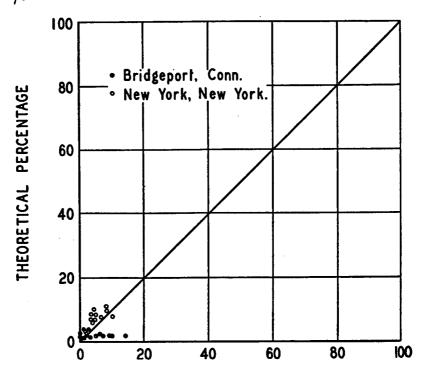
Length of Interval	Number of Headways	Length of Interval	Number of Headways
o0.4 sec.	ο	5.5–5.9 sec.	9
o.5–o.9 "	19	6.0-6.4 "	3
1.0-1.4 "	48	6.5-6.9"	5
1.5–1.9 "	77	7.0-7.4"	1
2.0-2.4 "	128	7.5-7.9 "	1
2.5-2.9 "	147	8.0-8.4 "	1
3.0-3.4 "	106	8.5-8.9 "	0
3.5-3.9 "	65	9.0-9.4 "	0
4.0-4.4 "	37	9.5-9.9 "	1
4.5-4.9 "	18		
5.0-5.4 "	27	Total	693
		Average	2.9 seconds

FREQUENCY DISTRIBUTION OF HEADWAY INTERVALS

Therefore H = 2.9 seconds.

To find the value of D it was necessary to make similar tabulations for (a) the intervals following undelayed turns and (b) the intervals following delayed turns; in each case the only intervals used were those where the car following the turning vehicle entered the intersection as soon as possible. The average of 52 such intervals following *undelayed* turns was 2.9 seconds, the same as H. This is as it should be. The average of 22 intervals following *delayed* turns was 4.8 seconds, indicating that a delayed turn held up the cars behind it for 1.9 seconds (4.8 minus 2.9) on the average. Thus D = 1.9 seconds.

The fourth column of Table XVII was computed from formula (1), which gives the theoretical percentage of delayed turning cars. The comparison between theory and observation can be seen in columns 4 and 5 of this table or in Figure 20, where each fifteen-minute period of observation is represented by a point whose ordinate is the theoretical percentage while the abscissa is the actual percentage. If the theory were perfect the points would all fall on the 45-degree line, which they clearly do not; nevertheless the fit is fairly good. The coefficient of correlation between the two sets of percentages is 0.80.



ACTUAL PERCENTAGE

FIGURE 20. Percentage of Delayed Vehicle Turns. Theoretical vs. Observed. (15-minute observation periods in Bridgeport, Conn. and New York, N. Y.)

The sixth column of Table XVII does the same thing for the non-turning cars, where the theoretical percentage was computed from formulas (2) and (3), using the values D = 1.9 seconds and H = 2.9 seconds. The graphical presentation of theory vs. observation in this case is shown in Figure 21.

Examples

To illustrate the practical application of the formulas two examples are shown in detail.

Example I. For the first example, consider the last row in the

EMPIRICAL TEST OF FORMULAS DEVELOPED

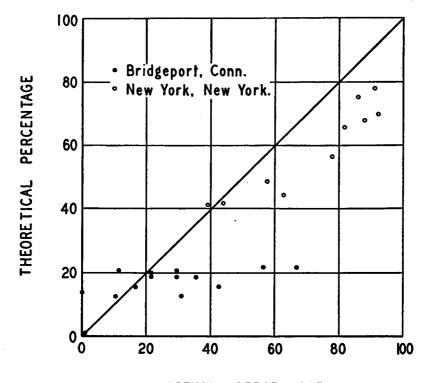




FIGURE 21. Percentage of Delayed, Non-turning Vehicles. Theoretical vs. Observed. (15-minute observation periods in Bridgeport, Conn. and New York, N. Y.)

upper section of Table XVII. V = 100, T = 6, N = 83/900 = .092, L = 70, R = .50. And, of course, D = 1.9 and H = 2.9.

The first step is to find the percentage of delayed turning cars, which can be obtained either from formula (1) or from Fig. 17. The figure immediately gives the value 16, while the formula more accurately gives $P = 100 \div (.117 \times 6 + 5.80) = 15.4$. We shall use the latter value in carrying through this example.

The second step is to determine m, the average number of delayed turns per fifteen minutes divided by the number of cycles per fifteen minutes, or $.01PT \div (900/L) = PTL/90,000$. Ac-

tually, however, what we want is not m itself but the two combinations m/L and Rm which occur in formula (2).

Consider first m/L, which is the average number of delayed turns per second. This can be obtained from Fig. 18, where the number of delayed turns per fifteen minutes is to be divided by 900; or we can use the formula m/L = PT/90,000 = (15.4)(6)/90,000 = .0010. The second cominbation involving m is Rm, which is as m/L times RL; that is, it is the product of the quantity we have just found and RL, which is the duration of the red light in seconds. Thus Rm = .0010 x 35 = .035.

NH = (.092) (2.9) = .27. So f (NH, Rm) = f (.27, .035) = .02 from Figure 19.

Thus the percentage of non-turning cars delayed, given by formula (4), is (100) (.0010) (1.9) + (100) (0.50) (.02) = .19 + 1.00 = 1.19 per cent.

Summary of Procedure. To recapitulate briefly, the percentage of delayed turning cars is most easily found by applying the values of V and T to Fig. 17. The steps in finding the percentage of delayed non-turning cars are as follows:

- (a) Get the average number of delayed cars per fifteen minutes by applying V and T to Fig. 18.
- (b) Divide this number by 900, which gives m/L.
- (c) Multiply this number by the duration in seconds of the red light, which gives Rm.
- (d) Divide the number of non-turning cars per fifteen minutes by 311, which gives NH.
- (e) Find f (Rm, NH) by applying (c) and (d) to Fig. 19.
- (f) Multiply (b) by 190. Multiply (e) by 100R. Add these two numbers. This is the percentage of delayed non-turning cars.

Example II. Let us apply the procedure of the above paragraph to the last row of Table XVII:

For the percentage of delayed turning cars, put V = 328 and T = 18 in Fig. 17. The result is 42 per cent.

For the percentage of delayed non-turning cars, carry out the steps (a) - (f):

EMPIRICAL TEST OF FORMULAS DEVELOPED

- (a) Put V = 328 and T = 18 in Fig. 11. The result is 7.5.
- (b) $7.5 \div 900 = .0083$.
- (c) $.0083 \times 31 = .26$.
- (d) $97 \div 311 = .31$.
- (e) From Fig. 19 f(.26, .31) = .17.
- (f) (190)(.0083) + (100)(.34)(.17) = 1.6 + 5.8 = 7.4 per cent.

APPENDIX A

FIELD DATA

		Delayed	Per Cent of Vehicle Turns
Pedestrians	Vehicle Turns	Vehicle Turns	Delayed
AUGUST 25,	1948 (TEMPLE AND	CHAPEL STS., NEW H	aven, conn.)
242	32	9	28
222	29	9	31
266	27	13	48
283	35	19	54
347	33	13	· 39
332	30	15	50
388	26	9	35
361	23	8	35
420	28	9	32
494	13	5	38
500	26	16	62
595	24	17	71
539	32	17	53
447	17	9	53
366	36	14	39
	AUGUST 27 (TEMP	LE AND CHAPEL STS.)	
310	23	10	43
323	37	12	32
336	27	6	22
357	28	11	39
450	24	15	63
437	22	8	36
508	30	11	37
473	46	16	35

Pedestrians	Vehicle Turns	Delayed Vehicle Turns	Per Cent of Vehicle Turns Delayed
459	32	15	47
345	27	11	41
362	25	7	28
400	34	9	26
323	24	10	42
301	24	12	50
353	24	11	46
	SEPTEMBER 1 (TEMP	LE AND CHAPEL STS.	_
727	31	21	68
658	26	19	73
661	29	21	72 72
726	32	25	78
588	27	20	74
645	37	26	70
652	36	27	75
619	24	21	88
615	33	29	88
581	35	32	91
590	31	22	5 71
589 589	25 25	18	72
515	28	17	61
478	29	20	69
604	19	14	59 74
566	31	-1	71
500 636	33	30	91
601	55 31	30 27	91 87
	з- September 3 (темр	-	•
100	• •		
109	23	2	9
91 96	21	4	19
90 110	30 21	5	17
		3	14
142	31	7	23
180	² 9	10	34
236	26	10	38
218	38	10	26
287	34	10	29
246	35	13	37

FIELD DATA

Pedestrians	Vehicle Turns	Delayed Vehicle Turns	Per Cent of Vehicle Turns Delayed
309	32	11 ·	34
371	32	18	56
388	28	17	61
426	23	15	65
567	39	29	74
687	32	28	87
706	37	30	81
580	31	23	74
670	42	28	Ĝ7
596	25	14	56
615	26	22	85
561	30	17	57
406	37	20	54
524	37	25	68
598	40	29	73
498	24	22	92
SEPTEMBER	30 (CHURCH AND C	HAPEL STS., NEW HA	ven, conn.)
513	34	18	53
578	35	19	54
676	23	12	52
858	29	19	66
943	30	19	63
880	27	19	70
881	32	22	69
773	31	14	45
765	26	20	77
	OCTOBER 1 (CHURC	CH AND CHAPEL STS.)	
495	37	21	57
583	35	31	89
697	26	22	85
562	27	19	70
552	37	31	84
55- 604	30	27	- 90
60 <u>4</u>	30 23	19	83
639	² 3 18	14	78 78
		-	90 90
858	ģo	27	100
1,080	24	24	100

Pedestrians	Vehicle Turns	Delayed Vehicle Turns	Per Cent of Vehicle Turns Delayed
1,077	23	22	96
1,098	24	23	9 6
9 ² 4	28	24	86
951	36	36	100
888	35	30	86
860	23	21	91
808	30	27	90
878	32	32	100
857	24	24	100
794	23	23	100
729	28	24	86
729	21	20	95
678	30	26	87
649	23	18	78
681	3.2	27	84
90 9	25	23	92
717	31	29	94
831	24	22	92
895	31	28	90
775	29	27	93
659	22	22	100
465	26	18	69
OCTOBER	26 (MAIN AND GOLDE	N HILL STS., BRIDGEPC	RT, CONN.)
548	17	15	88
542	14	12	86
528	14	11	79
517	13	7	54
534	11	9	82
471	12	10	83
495	9	9	100
617	18	13	72
467	14	11	79
	OCTOBER 27 (MAIN	AND GOLDEN HILL STS.)
445	10	6	6o
467	5	4	80
504	8	4	50
531	6	4	67

FIELD DATA

Pedestrians	Vehicle Turns	Delayed Vehicle Turns	Per Cent of Vehicle Turns Delayed
470	8	7	88
500	3	3	100
458	10	8	80
528	10	9	90
535	13	7	54
503	11	9	82
484	6	3	50
442	12	5	42
497	6	5	83
489	13	12	92
509	12	11	92
502	7	4	57
445	7		43
448	7	3 6	86
465	10	4	40
	OCTOBER 28 (MAIN A	AND GOLDEN HILL STS	3.)
542	12	6	50
511	18	11	61
525	8	5	63
594	19	15	79
739	15	11	73
621	16	15	94
671	17	11	65
<u>590</u>	6	3	50
634	14	12	86
561	13	10	77
591	12	8	67
559	8	7	88
579	9	7	78
	APRIL 22 (MAIN AN	D GOLDEN HILL STS.)	
630	14	10	71
495	15	9	60
489	14	12	86
482	19	12	63
420	16	7	44
504	11	7 6	64
470	10	6	60

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TURN CONTROL

APPENDIX B

NUMBER OF NON-TURNING CARS DELAYED BY BEING IN BACK OF DELAYED TURNING CARS

[EDITOR'S NOTE: The mathematical reasoning and results contained in this Appendix are the work of Mr. Raff. As pointed out in the Preface, however, Professor Betz of the University of Missouri suggested a much briefer mathematical technique for reaching the same results which had been developed by Mr. Raff. The present Appendix incorporates the suggestions and mathematical techniques of Professor Betz. Mr. Raff and the Eno Foundation are grateful to Professor Betz for his criticisms of the original manuscript and for improvements which he made in this portion of the report.]

Expected number of delayed non-turning cars per cycle is

 $\sum_{n=0}^{\infty} (\text{probability of having } n \text{ delayed turns in a cycle}) \quad x \qquad (1)$

(number of delayed non-turning cars when there are n delayed turns in one cycle).

Expressions will be developed for both factors, subject to the following assumptions:

- a. The arrival times of both non-turning and delayed turning cars are distributed at random.
- b. Every delayed turn blocks the traffic stream behind it for a constant length of time D following its entry into the intersection.
- c. When a group of non-delayed cars starts up at the beginning of the green period, they enter the intersection with a constant headway H.

The letters used in the formulas are defined as follows:

- T = turning cars per 15 miutes
- V = pedestrians per 15 minutes
- P = percentage of turns delayed $\left(=\frac{V}{.117T+5.80}$ or 100, whichever is smaller)

- L = cycle length, in seconds
- m = average number of delayed turns per cycle

$$\left(=\frac{VL}{10,530+\frac{5^{22},000}{T}} \text{ or } \frac{LT}{900}, \text{ whichever is smaller}\right)$$

- $\mathbf{D} =$ duration of delay for each delayed turn, in seconds
- H = starting headway, in seconds
- N = volume of non-turning cars, in cars per second
- R = Fraction of the signal cycle during which the light is red for the street under consideration.

Consider first the second factor in each term of the summation (1).

If there are no delayed turns in a given cycle, then there are no delayed turning cars.

If there is one delayed turn in a given cycle, how many non-turning cars will be delayed?

Let time o be the beginning of the red period, RL the beginning of the green, and L the beginning of the next red.

If the delayed turn arrives during the first (red) part of the cycle, it will delay all the cars which arrive during the remainder of the red, plus those which arrive during an interval D + kH of the following green, where k is the number of cars in line *ahead of* the delayed turn.

If the delayed turn arrives during the second (green) part of the cycle, the cars which arrived during the first part will have partly (or perhaps entirely) cleared the intersection. If only part of them have cleared the intersection, the delayed turn will delay the non-turning cars which arrive during a period D + k' H, where k' is the number of cars still stopped at the intersection when the delayed turns arrives. If these stopped cars are all out of the way, then the number of delayed non-turning cars will be only those

which arrive during a period D. What are the values of k and k'? If t is the arrival time of the delayed turn.

$$k = Nt if OLt \leq RL (2)$$

$$k' = NRL - \frac{1}{H}(t - RL) if RL \leq t \leq RL(1 + NH)$$

$$= \frac{RL(1 + NH) - t}{H} (3)$$

Therefore the number of non-turning cars delayed, when a delayed turn arrives at time t, is

 $N [D + RL - (1 - NH) t] \text{ if } Ot \leq RL$ (4)

$$N[D + RL(1 + NH) - t] \text{ if } RL \leq RL(1 + NH)$$
(5)

ND if
$$RL(1 + NH) \leq t < L$$
, (6)
or
N(A - Bt) if $O < t \leq l_1$ (4')

$$N(A + C - t) \qquad \text{if } l_1 \leq t \leq l_2 \qquad (5')$$

ND if
$$l_2 \leq t < L$$
, (6')

where A = D + RL, B = 1 - NH, C = RLNH, $l_1 = RL$, and $l_2 = RL(1 + NH)$.

Since the probability that the delayed turn will arrive during any interval of length dt is $\frac{1}{L}$ dt, the expected number of delayed non-turning cars in a cycle containing exactly one delayed turn is

$$\frac{N}{L} \left[\int_{0}^{l_{1}} (A - Bt) dt + \int_{l_{1}}^{l_{2}} (A + C - t) dt + \int_{l_{1}}^{L} D dt \right]$$

= $\frac{N}{L} \left[Al_{1} - \frac{B}{2} l_{1}^{2} + (A + C) C - \frac{1}{2} (l_{2}^{2} - l_{1}^{2}) + D(L - l_{2}) \right].$ (7)

Now consider the case where there are two delayed turns during the cycle. Call their times of arrival t_1 and t_2 .

The number of cars delayed by the first delayed turn is given by expression (4'), (5'), or (6'), depending upon the value of t_1 . The probability distribution of t_1 will be discussed shortly.

The number of additional cars delayed by the second delayed turn is ND if it arrives during the red period, because its presence adds an amount D to the blocking period. If it arrives during the green, it also delays ND of the cars behind it, as in the earlier case (see expression (6')). Thus the second delayed turn delays no non-turning cars, irrespective of when it arrives at the intersection. This fact simplifies the results considerably.

The probability distribution of t_1 is given by Votaw's theorem* of December 4, 1948 which was worked out with another purpose in mind, but happens to be exactly suited to the present problem. The probability that t_1 is between t and t + dt is

$$\frac{2}{L}\left(1-\frac{t}{L}\right)dt$$
(8)

Let us introduce the variable $x = 1 - \frac{t}{L}$.

Then for t = 0, x = 1 $t = l_1$, x = g $t = l_2$, x = h t = L, x = owhere g = 1 - Rh = 1 - R - NH.

Thus the number of cars delayed by the first delayed turn is

$$-2N\left[\int_{1}^{g} (A - BL + BLx) x dx + \int_{g}^{h} (A + C - L + Lx) x dx + \int_{h}^{o} Dx dx\right] + \int_{h}^{o} Dx dx \left[\cdot \right]$$
(9)

Since the second delayed turn delays ND cars, the total number of cars delayed by the two delayed turns is

$$ND + expression (9).$$
 (10)

* David F. Votaw, Jr., Department of Mathematics, Yale University.

With three or more delayed turns per cycle—in general, with n turns per cycle—the reasoning is similar. The number of cars delayed by the first delayed turn is given by (4'), (5'), and (6'), where the probability distribution of t_1 is, by Votaw's theorem,

$$p(t_1)dt = \frac{n}{L} \left(1 - \frac{t}{L}\right)^{n-1} dt = -hx^{n-1} dx.$$
 (11)

Each of the other delayed turns delays ND cars, regardless of when it arrives. Thus the number of cars delayed by the first delayed turn is

$$-nN\left[\int_{1}^{g} (A - BL + BLx) x^{n-1} dx + \int_{g}^{h} (A + C - L + Lx) x^{n-1} dx + D \int_{h}^{o} x^{n-1} dx\right]$$
$$= nN\left[D\int_{o}^{h} x^{n-1} dx + \int_{h}^{g} (E + Lx) x^{n-1} dx + \int_{g}^{1} (F + BLx) x^{n-1} dx\right], \quad (12)$$

where E = A + C - L and F = A - BL.

Performing the integrations we get

$$nN\left[\frac{D}{n}h^{n} + \frac{E}{n}(g^{n} - h^{n}) + \frac{L}{n+1}(g^{n+1} - h^{n+1}) + \frac{F}{n}(1 - g^{n}) + \frac{BL}{n+1}(1 - g^{n+1})\right]$$
$$= N\left[Dh^{n} + Eg^{n} - Eh^{n} + \frac{nL}{n+1}g^{n+1} - \frac{nL}{n+1}h^{n+1} + F - Fg^{n} + \frac{nBL}{n+1} - \frac{nBL}{n+1}g^{n+1}\right]$$

DERIVATION OF FORMULA

$$= N \left[F - NHLg^{n+1} + Lh^{n+1} + \frac{nL(1 - NH)}{n+1} + \frac{nNHL}{n+1}g^{n+1} - \frac{nL}{n+1}h^{n+1} \right]$$
$$= ND + NL \left[R - \frac{1 - NH}{n+1} - \frac{NH}{n+1}g^{n+1} + \frac{1}{n+1}h^{n+1} \right].$$
(13)

This is the number of non-turning cars delayed by the first delayed turn. Since each of the subsequent delayed turns delays ND non-turning cars, the total number of non-turning cars delayed in a cycle containing n delayed cars is

$$nND + NL \left[R - \frac{1 - NH}{n+1} - \frac{NH}{n+1} g^{n+1} + \frac{1}{n+1} h^{n+1} \right].$$
(14)

This expression is correct for all values of n, including 0 and 1. It can be verified that expressions (7) and (10) are special cases of (14), for n = 1 and n = 2. Returning now to expression (1), we see that the second factor in each term of the summation is given by expression (14). The first factor, the probability of having n delayed turns in a cycle is, from the Poisson law for random distributions,

$$e^{-m}\frac{m^{n}}{n!},\qquad(15)$$

where m is the average number of delayed turns per cycle. Thus the expected number of delayed non-turning cars per cycle is

$$\sum_{n=0}^{\infty} e^{-m} \frac{m^n}{n!} \left[nND + NL \left(R - \frac{1 - NH}{n+1} - \frac{NH}{n+1} g^{n+1} + \frac{1}{n+1} h^{n+1} \right) \right]$$

TURN CONTROL

$$= mND + NL \left[R - \frac{1}{m} + \frac{NH}{m} - \frac{NHe^{-Rm}}{m} + \frac{e^{-(1+NH)Rm}}{m} \right]$$

$$= mND + RNL \left[1 - \frac{1}{Rm} \left[1 - e^{-(1+NH)Rm} \right] + \frac{NH}{Rm} (1 - e^{-Rm}) \right]$$
(16)
$$= mND + RNL f(Rm, NH).$$
(17)

This is the desired result. The proportion of non-turning cars which are delayed is equal to

 $\frac{\text{MD}}{\text{L}} + \text{Rf}(\text{Rm, NH}).$

(18)