A Volume Warrant
For Urban Stop Signs

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PREFACE

This paper represents another step in the course of adding to the scientific foundation for traffic engineering. Naturally, early work in the field was directed more toward practical results than toward developing an integrated theory, but continued progress requires a basic understanding of the patterns of traffic movements. In controls for intersectional traffic, where much of the practice is based on rules of thumb, it is particularly desirable that numerical methods supersede intuitive ones.

This study was suggested by the pioneering work of Greenshields, Schapiro, and Ericksen in the *Traffic Performance at Urban Street Intersections.* In the year following the publication of that Traffic Performance study, both authors of the following monograph, "A Volume Warrant for Urban Stop Signs," were students in the graduate course at the Bureau of Highway Traffic. The Greenshields, Shapiro, and Ericksen work influenced them to choose thesis subjects having to do with intersectional traffic performance. Greenshields' applications of the mathematical theory of probability to traffic behavior were of particular interest to Mr. Raff, who had worked as a mathematician before coming to Yale.

This project was undertaken through a grant from the Eno Foundation, and the work was done jointly by the Bureau of Highway Traffic and the Eno Foundation. The opportunity to use the facilities of both organizations has been helpful.

The authors wish to acknowledge valuable assistance from staff members of both the Yale Bureau and the Eno Foundation; from Mr. Edward H. Holmes and the U.S. Bureau of Public Roads for use of the graphic time recorder; from Mr. Donald Schapiro of the Yale Law School, and Mr. David F. Votaw, Jr. of the Mathematics Department; Mr. Edmund R. Ricker, the New Haven City Traffic Engineer, for his careful reading of the manuscript and for assistance in facilitating the collection of field data, and from Mrs. Laurence Walker, whose months of analyzing field data permitted the authors to spend more hours gathering data than would otherwise have been possible.

M.S.R. J.W.H.

CHAPTER I

INTRODUCTION

The Stop sign is one of the most widely used of all traffic control devices. Its message is brief and to the point. The sign is inexpensive in comparison with traffic signals, and it serves an important purpose when used *properly.* The object of this report is to develop numerical standards for some of the conditions under which Stop signs should be used.

Of the three classes into which traffic signs are commonly divided — regulatory, warning, and guide — the Stop sign is regulatory. It imposes a definite legal obligation on every driver who approaches it. The exact legal meaning of the sign varies from one jurisdiction to another, but the general idea is always that each driver approaching the sign must bring his car to a complete stop, and must not proceed until to do so is safe.

From a traffic engineering point of view, the purposes served by Stop signs fall into two categories: (1) reduction of accident hazards, and (2) facilitation of traffic movement. At appropriate locations, Stop signs which are obeyed can be of great value in achieving both goals.

Observance of Stop Signs

While a good enforcement program can greatly improve the observance of traffic control devices, it is physically impossible to police more than a small fraction of the intersections in a city at one time. Consequently, the regulatory signs and signals must be largely self-enforcing, if they are to serve efficiently the purposes for which they were installed.

In comparison with speed limits and parking restrictions — two common types of regulatory signs — the Stop regulation is fairly well observed. By and large, Stop signs seem more reasonable to most drivers than do the majority of speed and parking signs. To the traffic official this respect is a valuable asset, and should not be dissipated by careless or unreasonable policies regarding their use.

In a traffic survey of Decatur, Illinois, for example, seven per cent of the cars approaching Stop signs *when there was traffic interference* failed to slow down even to fifteen miles an hour, while only two per cent of all traffic at signalized intersections went through red lights.[1] Similar results have been obtained in many other cities.

It is apparent from these figures that Stop sign observance, even when defined in an extremely liberal way, is much poorer than the observance of red lights.[2] One naturally wonders why this is so.

One reason, certainly, is the more dynamic appearance of the signal, with its strong illumination and its changing lights. Another is that the driver who approaches a red light has less reason to be aggressive, since he can be sure of getting across by simply waiting for his turn. A third reason is that Stop sign violations, being harder to recognize than signal violations, are less likely to attract the attention of enforcement officers. Finally, warrants for traffic signals have been more carefully developed and are more conscientiously applied than those for Stop signs. It is a common experience to drive up to a Stop sign and find no apparent need for stopping. At signalized locations, on the other hand, the reason for the signal is usually obvious. All too often Stop signs are erected hastily, in response to an ill-considered public demand.

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[1] *The Traffic Survey of Decatur* (Ill.), WPA, 1936-37. Pp. 25, 28-32. The Stop-sign observance figure is based on observation of 91,977 cars at 299 Stop signs. Of 54,704 cars observed at these Stop signs when there was no traffic interference, 9 percent failed to slow down to fifteen miles per hour. The figure of observance of red lights is based on 177,875 cars at eighteen intersections.

[2] The differing attitudes of motorists and enforcement officers toward Stop-sign observance deserve some attention here. The motorist usually feels that he has done his duty if he has slowed down sufficiently to enable him to see what traffic is approaching on the main street, because he regards the Stop-sign as a device to assure main-street traffic of the right-of-way. The policeman, on the other hand, wants the motorist to make a full stop, because he wants his testimony in court to involve as little personal judgment as possible. In this difference of attitude it is the policeman, of course, who has the letter of the law on his side.

In the last analysis, observance of Stop signs depends on the amount of respect motorists have for them. When motorists realize that signs have been erected in accordance with a consistent, intelligent policy, their respect will naturally follow. The policy must be consistent, in order that the driver approaching a Stop sign will know what sort of physical or traffic conditions to expect. It must also be intelligent, so there will be neither too few signs nor too many.

The objection to using Stop signs too sparingly is simply that full advantage is not taken of their usefulness. The excessive use of signs, on the other hand, breeds disrespect for the important as well as the unimportant ones, since the driving public is unable to distinguish the one from the other.

WARRANTS FOR STOP SIGNS

There is need for a set of warrants, to prescribe the conditions under which Stop signs should or should not be used. The present research was undertaken because of a belief that the current practices in the use of Stop signs are haphazard and that the warrants now in use are inadequate. An authoritative statement of these warrants is given in the *Manual on Uniform Traffic Control Devices,* which lists the following conditions warranting Stop signs:[3]

1. Intersection of a less important road with a main road where application of the normal right-of-way rule is unduly hazardous.
2. Intersection of a county road, city street, or township road with a State route.
3. Intersection of two main highways where no traffic signal is present.
4. Street entering a through highway or street.
5. Unsignalized intersection in a signalized area.
6. Railroad crossing where a stop is required by law or by order of the authority having jurisdiction over the highway or street.
7. Other intersection where high speed, restricted view, or serious accident record indicates a need for control by the Stop sign.

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[3] *Manual on Uniform Traffic Control Devices for Streets and Highways,* prepared by a joint committee of the American Association of State Highway Officials, the Institute of Traffic Engineers, and the National Conference on Street and Highway Safety. Published by the Public Roads Administration, 1948. Pp. 19-20.

Only the last of these conditions refers to traffic, and that reference is vague, qualitative, and oriented toward safety rather than toward the expediting of traffic. Since present knowledge seems insufficient for more specific recommendations, a *clear need for more knowledge* about traffic behavior at intersections presents itself.

Considerable work was done before the war on the development of formulas for the maximum safe approach speed at intersections having sight obstructions.[4] The writers of these reports agreed that a Stop sign should be used at locations where the critical speed is less than eight miles an hour. Here we have an example of a numerical Stop-sign warrant, based on the accident hazard at blind intersections. The work on safe-approach speeds is subject to criticism, however, on the ground that some of the underlying assumptions are highly artificial.

Scope of This Study

There seems to be no published material to point toward a numerical warrant based on the delaying effect of Stop signs, and the task of the present report is to make a start in this direction.[5] As limited time has restricted the scope of the research, the study has been devoted to developing a Stop-sign warrant for right-angled crossings that are (1) in urban areas, and (2) isolated, in the sense of not being part of a through-street system. The warrant is for the conventional pair of Stop signs on the minor street, not for the more controversial four-way stop.

It should be pointed out that the warrant developed here is in-

[4] Examples are the two progress reports by the subcommittee of the Traffic Department of the Highway Research Board, entitled *Maximum Safe Approach Speeds at Intersections;* the National Safety Council's Public Safety Memo #73, entitled *Critical Speeds at Blind Intersections;* the report of the Philadelphia Office of Traffic Engineering, entitled *Safe Approach Speeds at Intersections;* and the American Automobile Association's booklet, *Normal Safe Approach Speeds at Intersections,* which was based on the Philadelphia report.

[5] A volume warrant for *rural* intersections is given by the American Association of State Highway Officials in *A Policy on Intersections at Grade* (1940), p. 96.

tended to supplement other warrants, not to replace them. The use of Stop signs to protect through routes, for example, is outside the scope of this study. Similarly, the treatment of blind intersections at which the primary need is to reduce the likelihood of accidents is not dealt with in this report. The subject of interest here is the isolated urban intersection at which Stop signs can help to make the traffic move more easily, with proper consideration of the delaying effect of the signs on the side-street traffic.

Warrant Criteria

In order to decide what criterion should be used for a Stop-sign warrant based on traffic flow, it is necessary to examine what a Stop sign does. Legally, it requires every side-street car to make a full stop before entering the intersection. In practice, however, it has the effect of assigning the right-of-way steadily to the main street (except in occasional situations when side-street traffic is very heavy) instead of permitting the give-and-take which goes on at an uncontrolled intersection. In other words, the Stop sign imposes a delay on every side-street car in order to allow the main-street traffic to pass through the intersection without interruption.

This immediately suggests one way of approaching a warrant criterion. If there are no Stop signs to halt the side-street cars, what proportion of them can be expected to interfere with the free flow of traffic on the main street? If the principal purpose of the signs is to facilitate traffic, it is certainly wrong to erect Stop signs at an intersection where only a small fraction of the side-street cars interferes with the main-street traffic, for this means delaying the other large fraction by making them stop when there is no need to do so. Or more realistically, the sign is likely to be ignored and to lead to disrespect for other Stop signs, because drivers familiar with the location will know that it is safe most of the time to cross without stopping.

It is clear, therefore, that if a Stop sign is to be fair and is to have some chance of commanding the respect of the motorists, the traffic conditions must be such that a substantial proportion of the side-street cars will conflict, in the absence of some kind of control, with the main-street traffic. It is an arbitrary decision as to just what constitutes a "substantial proportion", but the following criterion is a reasonable one: *a Stop sign is warranted at any intersection where assigning the right-of-way to the main street at all times would delay more than half the side-street cars.*[6] What this means in terms of traffic volumes is the main subject of Chapter III.

We have seen how the percentage of side-street cars which are delayed can be used as the basis for a volume warrant. It is also possible, however, to look at the delay to side-street cars in another way. Instead of counting the *number* of cars which are delayed, one can base a warrant on the *amount of time* contained in the delays. In one respect this approach is more equitable than the other, since it attaches more importance to a long delay than to a short one. Unfortunately, there are both logical and practical difficulties in this kind of warrant criterion. Logically, there is no basis for deciding how long the average delay should be before a Stop sign is warranted;[7] and practically, the average delay does not correlate well with traffic volumes, as we shall see in Chapter III.

To look ahead, a volume warrant for Stop signs at isolated urban intersections is developed in Chapter III, and several examples of its application are discussed in Chapter V. It should be emphasized that the warrant is based not on field observations alone, nor on mathematical theory alone, but on a combination of the two. The mathematics permits generalization to a wide range of conditions, while the use of empirical data insures the practical usefulness of the warrant.

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[6] This warrant is for the lower end of the range, that is, for deciding whether to use Stop signs or some lesser form of control. For the upper end, where the choice is between Stop signs and a traffic signal, one would have to use a warrant of the sort which the Uniform Manual recommends for traffic signals. *Op. cit.*, pp. 127 ff.

[7] Lest it be thought that this objection applies with equal force to the warrant based on the percentage of delayed side-street cars, it should be pointed out that the percentage of delayed cars has to be somewhere between zero and 100%, whereas there is no limit to how long the average delay time can become under some circumstances.

CHAPTER II

METHOD OF FIELD OBSERVATION

In order to examine the factors on which a volume warrant must depend, it was necessary to find a field method that would permit measurements of the following traffic characteristics:

(1) Main-street volume.
(2) Side-street volume.
(3) Times of all arrivals and departures of cars on both streets.
(4) Number of delayed cars on the side street.

In addition, it was essential that the observers and the apparatus be located where they would not attract attention and thereby distort the normal traffic behavior. Also it was desirable to have a method of observation which could be used at night, in order to obtain a wide range of traffic volumes at each location.

CHOICE OF APPARATUS

The listed requirements, especially the third one, suggested the use of some kind of apparatus that would make a continuous record. There are basically two kinds of apparatus that do this: (1) a motion picture camera, and (2) a graphic time-recorder in which one or more pens write on a moving chart. Both types of apparatus were considered.

Greenshields[1] and others have had considerable success in studying intersection traffic movements with a motion picture camera. The technique has been to mount the camera at some high vantage point overlooking the intersection, and to use a special timing device to slow down the camera to a rate of the order of one picture a second. After the pictures are taken in this way they are analyzed by projecting them one picture at a time on a perspective

[1] Bruce D. Greenshields, Donald Schapiro, and Elroy L. Ericksen, *Traffic Performance at Urban Street Intersections*. Technical Report No. 1, Yale Bureau of Highway Traffic, 1947. The equipment is described on pp. 1-7, 119-121 of this work.

drawing of the intersection, from which the positions of the various cars can be measured.

The advantages of the camera are that the record is free from errors due to the observer's reaction time, and that a large number of cars can be simultaneously observed with no difficulty. Also, the camera makes it possible to get the whole behavior pattern of each car on the record *before* any analysis is made of the movements. This eliminates the need to judge the exact instants at which a car stops and starts until after the maneuver has been completed.

The camera has serious disadvantages, however. The need for taking pictures from a high building imposes a severe limitation on the number of intersections which can be studied.[2] In addition, the camera can be used only when light conditions are favorable and where the view is unobstructed. All in all, the camera is not well suited to a type of research which requires round-the-clock observations at locations which have to be carefully selected for their traffic conditions.

The Esterline-Angus Graphic Time-Recorder is a machine in which a roll of graph paper moves past a bank of twenty pens at a constant speed. Each pen is connected with a different telegraph key so that when the key is depressed the corresponding pen makes a characteristic mark on the moving paper. With this machine it is possible to record simultaneously as many as twenty different operations. A detailed description of the apparatus and its use in this study will be found in Appendix A.

The graphic time-recorder has none of the previously listed disadvantages of the camera. It can be operated from inside a parked car (see Fig. 1) at any intersection, in any kind of weather or light. The machine is so inconspicuous that one has to look carefully to see it in the photograph. Two observers sat in the car while using the machine.

[2] An attempt was made to develop a mirror attachment which would make it possible to photograph more than one approach to an intersection from a ground level position. There were many difficulties, of which the most important were (1) the impossibility of photographing more than two approaches at the same time, and (2) the conspicuousness of the apparatus.

This instrument was used in all the field observations for this report, primarily because of the flexibility which it made possible in choosing locations and times for observation. The data were all collected by the same two observers.

DESCRIPTION OF LOCATIONS

Since the intersections studied are analyzed separately, it might be well to discuss them in detail. Four urban intersections were used, all of them in built-up areas of New Haven, Connecticut. There were a number of reasons for taking all the data in the same city: (1) driving habits were more consistent than if different cities had been used, since many of the same drivers used more than one of the intersections; (2) the policies of enforcement officials were more consistent, since the same police force was in charge of all the intersections; and (3) it was convenient for the researchers to do all their work in the city where most of the office and laboratory facilities were located.

All four locations were intersections of a minor side street with a through street.[3] Intersections of this type were chosen in order to study the delaying effect of Stop signs on the traffic movement, and thereby to determine the volume conditions under which this amount of delay would be justified.

Intersection A: *Wallace and Chapel Streets.* A photograph and a drawing of this intersection are shown in Figure 2. It is located in an industrial section of New Haven, about a mile east of the central business district. Chapel Street is an arterial street leading directly to the downtown area. At this intersection it is thirty-four feet wide, with solid parking on both sides, so that it carries only one lane of traffic in each direction. Wallace Street is a minor side street, twenty-four feet wide; it carries one-way traffic northbound.

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[3] At two of them the through street was protected by Stop signs throughout the study, while at the other two there was no control at first but Stop signs were added during the course of the study. The addition of the Stop signs had no observable effect on the traffic behavior, the through street being treated with respect even when there were no signs.

FIGURE 1. The Graphic Time-Recorder. The photograph shows how the machine was used inside an automobile.

A considerable part of its traffic consists of heavy trucks,[4] which is to be expected in a neighborhood of this character. Most of its traffic moves in a single lane, but an occasional car approached the intersection alongside another vehicle that was already waiting to cross.

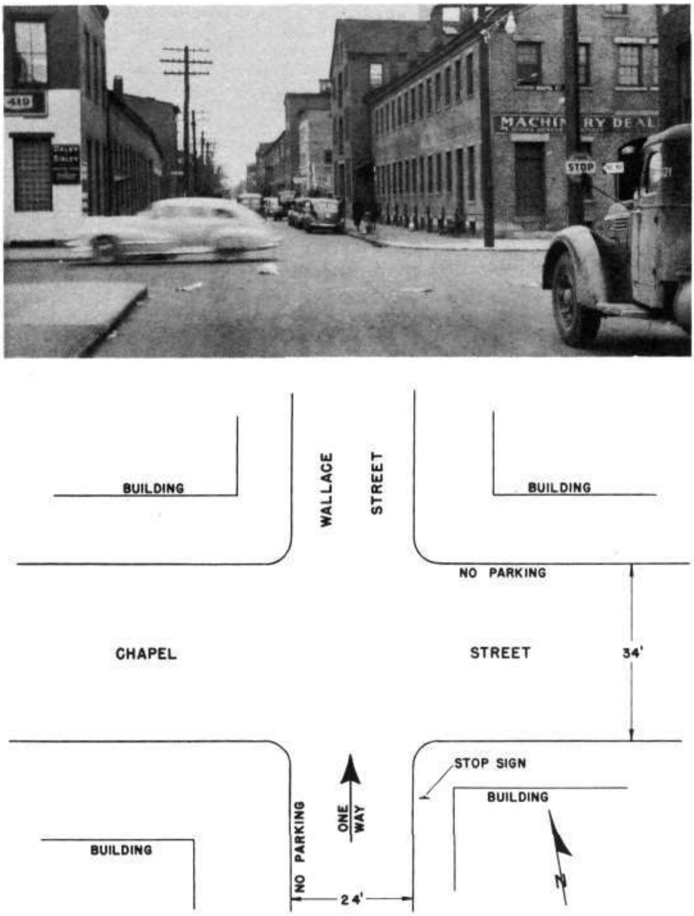

FIGURE 2. Picture and Plan of Intersection A.

[4] The amount of truck traffic did not seem to have any effect on the quantities that were measured in this study, inasmuch as the data taken at Intersection A show the same pattern as those taken two blocks away at Intersection B, where the proportion of trucks was much less.

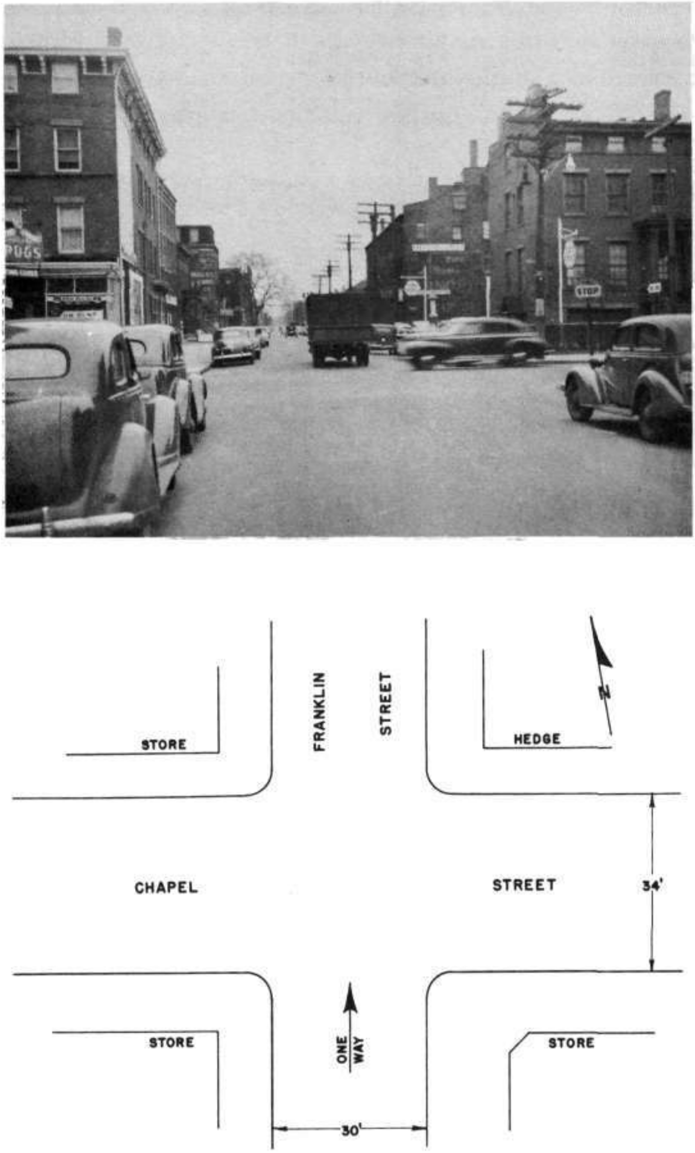

FIGURE 3. Picture and Plan of Intersection B.

Intersection B: Franklin and Chapel Streets. This intersection (see Fig. 3) is two blocks west of Intersection A. The neighborhood is similar, except that there is less heavy industry near this intersection. Chapel Street has the same width and traffic pattern. Franklin Street, like Wallace, is one-way northbound and carries strictly local traffic. It is thirty feet wide, with parking on both sides. Like Wallace, its traffic generally moves in one lane, but the cars waiting to cross Chapel Street occasionally use two lanes.

Intersection C: Orange and Willow Streets. This intersection (see Fig. 4) is in a residential section about two miles north of the center of the city. Orange Street, forty-one feet wide, is an arterial street leading to the downtown area; there is scattered parking along both sides of the street. The pattern of lane operation varies considerably, depending on the volume of traffic. Willow Street, thirty feet wide, carries two-way traffic and has scattered parking on both sides. Its traffic moves almost always in a single lane in each direction. A few blocks east on Willow Street is a medium-sized factory whose workers create a heavy surge of traffic on Willow Street at certain times of day. The wide range of traffic volumes on both streets makes this intersection particularly useful for the purposes of this study.

Intersection D: Whalley and Winthrop Avenues. Whalley Avenue, a state route, is a broad thoroughfare connecting the downtown district with the Westville shopping center. At Winthrop Avenue, about two miles from the center of the city, it is lined with small neighborhood stores. Its width of sixty-three feet easily permits four lanes of traffic in addition to the parking which the stores attract. Winthrop Avenue is purely residential in character, carrying two-way traffic. Its width is thirty feet on the north side of the intersection and forty-two feet on the south side, and its traffic is so light that it almost never moves in more than one lane in each direction. The intersection is illustrated in Figure 5.

Altogether, about fifty-two hours of observations were made at the four intersections.

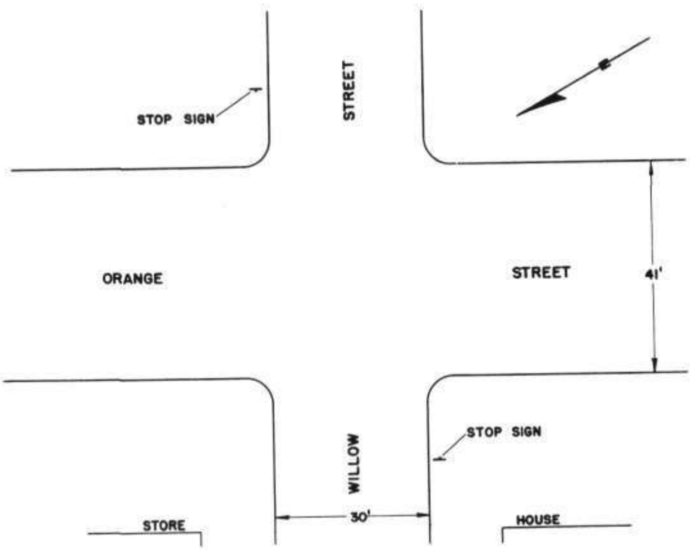

FIGURE 4. Picture and Plan of Intersection C.

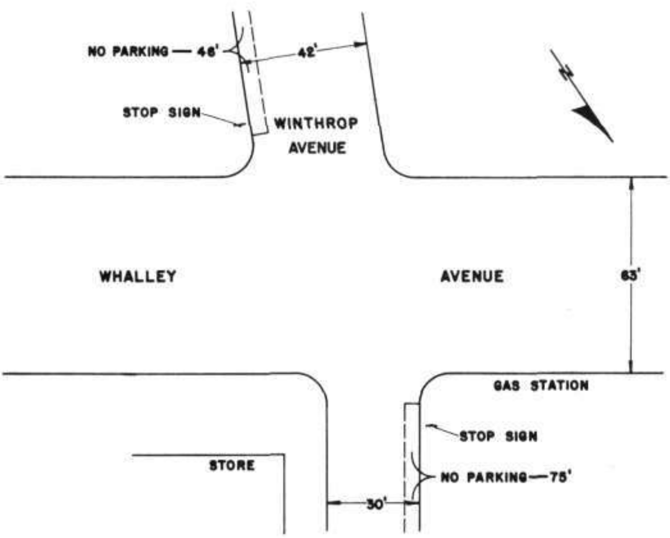

FIGURE 5. Picture and Plan of Intersection D.

CHAPTER III

EXPERIMENTAL RESULTS

In this chapter the authors discuss the observed behavior of traffic under a wide range of volume conditions. They analyze behavior from several different points of view, and derive a volume warrant for Stop signs from one part of their analysis.

In analyzing traffic behavior at an intersection, it is convenient to have special names for certain kinds of time-intervals that recur in the discussion. Two such terms have been defined as follows:

> A *gap* is the interval from the arrival of one *main*-street car at the intersection to the arrival of the next main-street car.
>
> A *lag* is the interval from the arrival of a *side*-street car at the intersection to the arrival of the next main-street car.

To make these definitions completely clear, it is necessary to explain exactly what is meant by the *arrival* of a car at the intersection. In the case of a main-street car, its arrival is the time at which the car enters the area bounded by the extensions of the curb lines.

For a side-street car, the definition is twofold, depending on whether or not there are other side-street cars waiting to enter the intersection when the car under consideration gets there. If no cars are waiting, the arrival is defined as the time when the side-street car stops, or reaches its slowest speed.[1] If the car under consideration has to stop behind another side-street car waiting to enter the intersection, its arrival is defined *not* as the time when it stops but as the time when the car immediately ahead of it enters the intersection. In either case, the arrival of the side-street car is defined in such a way as to make the lag the interval during which the side-street car has a choice of entering the intersection immediately or waiting until a main-street car has passed.

[1] In these observations, there were so few cars that entered the intersection in complete disregard of the Stop sign that it was almost unnecessary to define arrival for them. For the sake of completeness, however, their arrival was defined in the same way as for a main-street car.

The drawing in Fig. 6 illustrates these definitions. It shows a sample section of the recording chart and indicates how the lags and gaps were measured from the markings on the chart.[2] In this drawing, the pen that traced out the top line was assigned to eastbound traffic on the main street; the next pen to westbound traffic on the main street; the next two pens to southbound traffic on the side street; and the next two pens to northbound traffic on the side street. Since the drawing is merely symbolic, six pens were regarded as sufficient to permit an explanation of the recording technique.

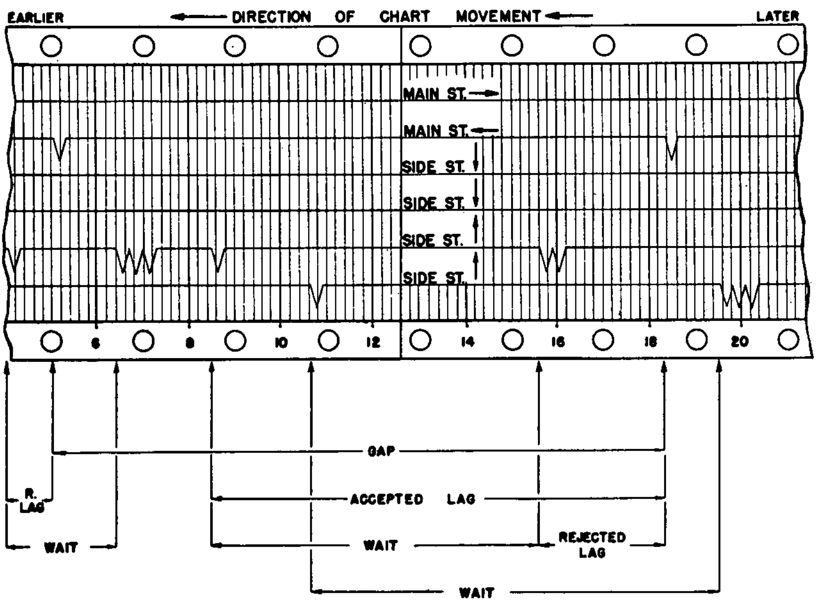

FIGURE 6. Sample Section of Recording Chart, showing measurement of lags, gaps, and waits.

Looking at Fig. 6, we see that the first event shown is the arrival of a northbound side-street car at time 4.1, followed by the passage of a westbound main-street car at time 5.1. After letting

[2] An illustration of the arrangement of cars to which this piece of chart corresponds may be found in Appendix A, where the use of the machine is described in detail.

the main-street car go by, the side-street car proceeded to cross at time 6.4. (With the side-street pens, a single notch means an arrival, while a multiple notch or an extended notch means a departure.) Let us analyze this sequence of events. The side-street driver faced a lag of 1.0 seconds (5.1 minus 4.1). He had a choice between *accepting* this lag (that is, entering the intersection ahead of the main-street car) or *rejecting* it (that is, permitting the main-street car to pass first). In this particular example, the lag of 1.0 seconds was rejected.

Another concept (discussed later in detail) is the *wait* of a side-street car, by which is meant the interval from the time the car first reaches the intersection (or the end of a line of cars waiting at the intersection) until it actually begins to enter the intersection. The wait for the car under discussion was 2.3 seconds (6.4 minus 4.1).

Continuing with Figure 6, we find a northbound side-street car arriving at time 8.5, followed by another car pulling up behind it at time 10.6. There was no main-street car at the intersection until 18.4. This time the first of the two side-street cars accepted its lag of 9.9 seconds (18.4 minus 8.5), but the second side-street car waited until after the main-street car had passed. Since there may be some confusion about what interval constitutes the lag for this second car, let us review the meaning of the term. The lag faced by a side-street driver is the time interval in which he is free to enter the intersection if he wants to, and which he can either accept or reject in accordance with his judgment. Now clearly, the second car was not free to do anything until the first car was out of his way; consequently, its lag did not begin until 15.6, the departure time of the first car. The lag of the second car, therefore, was 2.8 seconds (18.4 minus 15.6), and it was rejected.

The waits of these two cars were 7.1 seconds and 9.0 seconds, respectively.

Finally, Fig. 6 shows one gap, the interval between the two main-street cars. Its length is 13.3 seconds. This chapter contains comparatively little discussion of gaps, but the concept is important in the mathematical developments of Chapter IV.

It is apparent that the distribution of gaps is a property of the main-street traffic alone, while the occurrence of lags involves an interaction between the two streams of traffic.

Accepted and Rejected Lags

As we have seen, every side-street driver is faced with a lag, which is the time-interval from his arrival until the arrival of the next main-street car. If the side-street driver enters the intersection before the main-street car reaches it, he is said to *accept* his lag. On the other hand, if he waits until the main-street car has passed before entering the intersection, he is said to *reject* his lag.

It is reasonable to expect that more drivers facing long lags will accept them than drivers facing short lags. This suggests the idea of arranging the lags according to their size and noting the relative numbers of each size accepted and rejected. This has been done in Tables I - III and IX. The first three of these tables also list the percentage of the lags of each size that are accepted, since this is a quantity of considerable interest. The material in these tables is also presented graphically in Figures 7-10 and 29.[3]

Except at Intersection D, no attempt was made to separate the side-street cars which proceeded straight across the intersection from those which turned left or right. It appeared to the observers that most of the side-street cars watched traffic coming from both directions on the main street (except on the wide Whalley Ave-

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[3] The reader may wonder why no attention has been paid to the gaps which occur during the time that a delayed side-street car is waiting to enter the intersection. For instance, if a side-street car reaches the intersection, rejects its lag, and waits for *two* cars to go by before it enters the intersection, the rejected gap between the first car and the second one might be counted along with the rejected lags, and the accepted gap between the second car and the next one to come along might be counted in the same way as if it were an accepted lag.

There are two reasons why it would be illogical to do this. In the first place, it is obvious that each driver can accept only one lag or gap, while he can reject several of them. This means that if all lags and gaps are counted equally in tabulating the numbers of accepted and rejected intervals, then the percentage of *intervals* accepted, nue), so there appeared to be no good reason for treating the turning cars separately. It is obvious that fine distinctions of this sort must be kept to a minimum if the Stop-sign warrant is to be simple enough to have practical value.

At Intersection D, however, where it was evident that the right-turning side-street cars were influenced only by the main-street cars approaching from their left, the lags for these side-street cars were computed with reference only to the main-street traffic in this one direction. For side-street cars which proceeded straight across or turned left, the lags were computed according to the original definition. Separate treatment of the right-turning cars is recommended only at intersections where the main-street traffic moves in two or more lanes in each direction.

Definition of Critical Lag

Just as the center of gravity of a solid object is a single point which for some purposes represents the whole object, it is desirable to have a single quantity which can be used to summarize the whole pattern of acceptance and rejection of lags. The critical lag serves this purpose, and is defined as follows:

> The *critical lag L* is the size lag which has the property that the number of accepted lags shorter than L is the same as the number of rejected lags longer than L.

An important finding of the present study is that the critical lag size varies from one location to another, even when the intersec-

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for a particular size, will not be a true measure of the percentage of *drivers* who find such an interval acceptable. For example, if fifty per cent of the drivers are willing to accept a lag or gap of five seconds while the other fifty per cent are not, the percentage of five-second lags and gaps which are accepted will be less than fifty per cent; this is because each driver who accepts an interval of this size will accept only one of them, while the driver who rejects this interval may reject a number of them. If the percentage of *intervals* accepted is to be used to determine the percentage of *drivers* who are willing to accept them, then the same number of intervals must be counted for each driver. This is accomplished by counting only the lags and ignoring the gaps.

There is still another reason why it is incorrect to throw lags and gaps together as if there were no distinction between them. A lag and a gap of the same size are not really comparable, because the intersection is clear for the entire duration of a lag, while there is a certain interval at the beginning of a gap during which a main street car blocks the intersection while crossing from one side to the other.

tions are in the same city. In order to find out why this is so, the four intersections have been analyzed separately.

The possibility was also considered that the value of L might change with changes of volume at the same location. This does not appear to be the case. Detailed evidence on this point will be presented in a later section of this chapter.

Observed Lags at the Four Intersections

At Intersection A, the number of lags of each size accepted and rejected, and the per-cent accepted for each size, are listed in Table I. The table shows that there were more short lags than long ones; in fact, the number of lags of each size decreases steadily with increasing lag size.[4] The critical lag is determined by plotting

TABLE I

ACCEPTED AND REJECTED LAGS AT INTERSECTION A

Length of Lag (seconds)	*Number Accepted*	*Number Rejected*	*Total Number*	*Per Cent Accepted*
0 — 0.9	3	156	159	2
1 — 1.9	9	125	134	7
2 — 2.9	22	95	117	19
3 — 3.9	22	74	96	23
4 — 4.9	30	42	72	42
5 — 5.9	45	25	70	64
6 — 6.9	39	17	56	70
7 — 7.9	41	9	50	82
8 — 8.9	33	4	37	89
9 — 9.9	38	2	40	95
10 — 10.9	24	3	27	89
11 — 11.9	35	1	36	97
12 — 12.9	24	1	25	96
13 — 13.9	20	1	21	95
14 — 14.9	21	0	21	100
Over 15	111	0	111	100
	517	555	1072	

[4] This is to be expected, if the cars are distributed at random on both streets. According to the mathematical theory of probability, whose application to problems of this kind is developed in a methodical fashion in Chapter IV, the number of lags of each size should fall off according to an exponential curve.

two cumulative distribution curves on the same graph: the number of accepted lags shorter than t and the number of rejected lags longer than t. The value of t for which these two curves intersect is the critical lag L.

Figure 7 presents this information in graphical form. The upper graph is a bar chart in which the number of lags accepted is indicated by a ruled bar above the line, while the number rejected is indicated by a solid bar below the line. (The total number of lags of any particular size is the combined length of the ruled and solid bars for that size.) The lower graph shows the curves which are used to determine the critical lag. At Intersection A, the critical lag was found to be 4.6 seconds.

The same information at Intersection B, listed in Table II, is shown graphically in Figure 8. At this location the critical lag was found to be 4.7 seconds, almost the same as at Intersection A. The

TABLE II

ACCEPTED AND REJECTED LAGS AT INTERSECTION B

Length of Lag (seconds)	*Number Accepted*	*Number Rejected*	*Total Number*	*Per Cent Accepted*
0 — 0.9	3	119	122	2
1 — 1.9	11	139	150	7
2 — 2.9	22	116	138	16
3 — 3.9	23	90	113	20
4 — 4.9	44	66	110	40
5 — 5.9	58	34	92	63
6 — 6.9	53	22	75	71
7 — 7.9	41	14	55	75
8 — 8.9	44	1	45	98
9 — 9.9	46	3	49	94
10 — 10.9	42	3	45	93
11 — 11.9	29	0	29	100
12 — 12.9	34	0	34	100
13 — 13.9	27	0	27	100
14 — 14.9	17	0	17	100
Over 15	161	1	162	99
	655	608	1263	

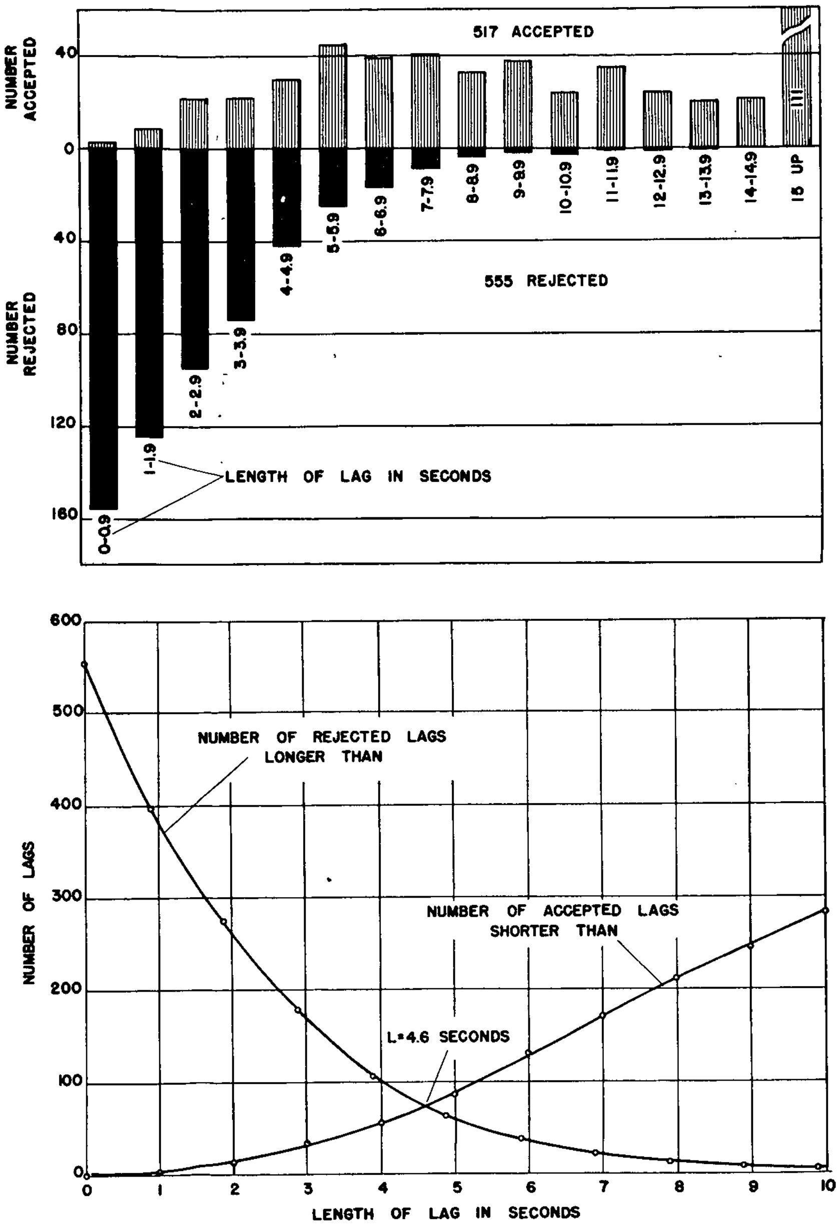


FIGURE 7. Distribution of Accepted and Rejected Lags at Intersection A

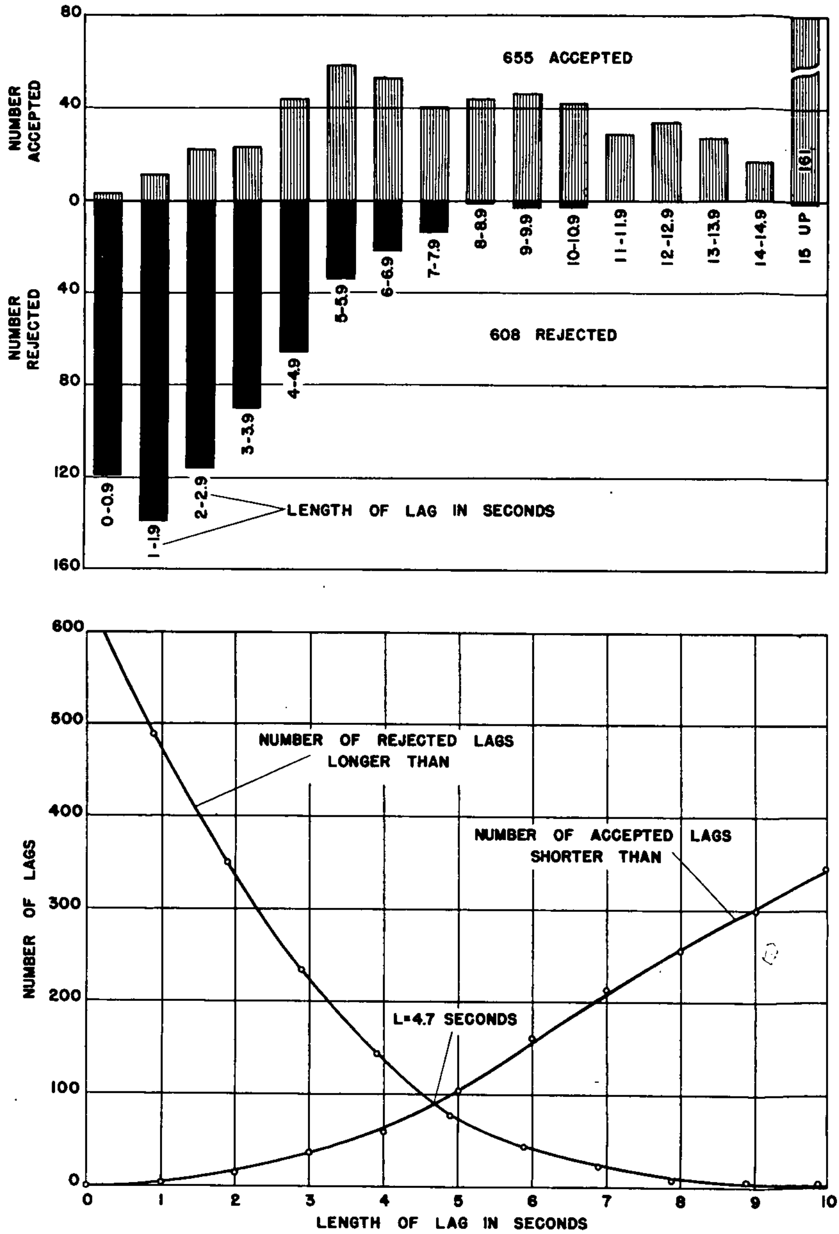


FIGURE 8. Distribution of Accepted and Rejected Lags at Intersection B

close agreement between these two values lends support to the idea, expressed earlier, that the amount of truck traffic has little effect on the values that were measured in this study. The two intersections are alike in most other respects: (1) the side streets carry one-way traffic at both intersections; (2) the traveled width of the side street is about the same; (3) the main street is identical in width and travel pattern; and (4) there are severe sight restrictions at both intersections.

The results were quite different at Intersection C, as Table III and Figure 9 illustrate. The general character of the graphs is the same, but the critical lag at this location was 5.9 seconds, appreciably longer than at the first two locations.

TABLE III

ACCEPTED AND REJECTED LAGS AT INTERSECTION C

Length of Lag (seconds)	*Number Accepted*	*Number Rejected*	*Total Number*	*Per Cent Accepted*
0 — 0.9	4	220	224	2
1 — 1.9	6	259	265	2
2 — 2.9	22	232	254	9
3 — 3.9	37	214	251	15
4 — 4.9	59	172	231	26
5 — 5.9	81	139	220	36
6 — 6.9	111	63	174	64
7 — 7.9	126	56	182	69
8 — 8.9	142	35	177	80
9 — 9.9	144	17	161	90
10 — 10.9	110	15	125	88
11 — 11.9	122	1	123	99
12 — 12.9	93	2	95	98
13 — 13.9	94	1	95	99
14 — 14.9	101	2	103	98
Over 15	957	4	961	100
	2209	1432	3641	

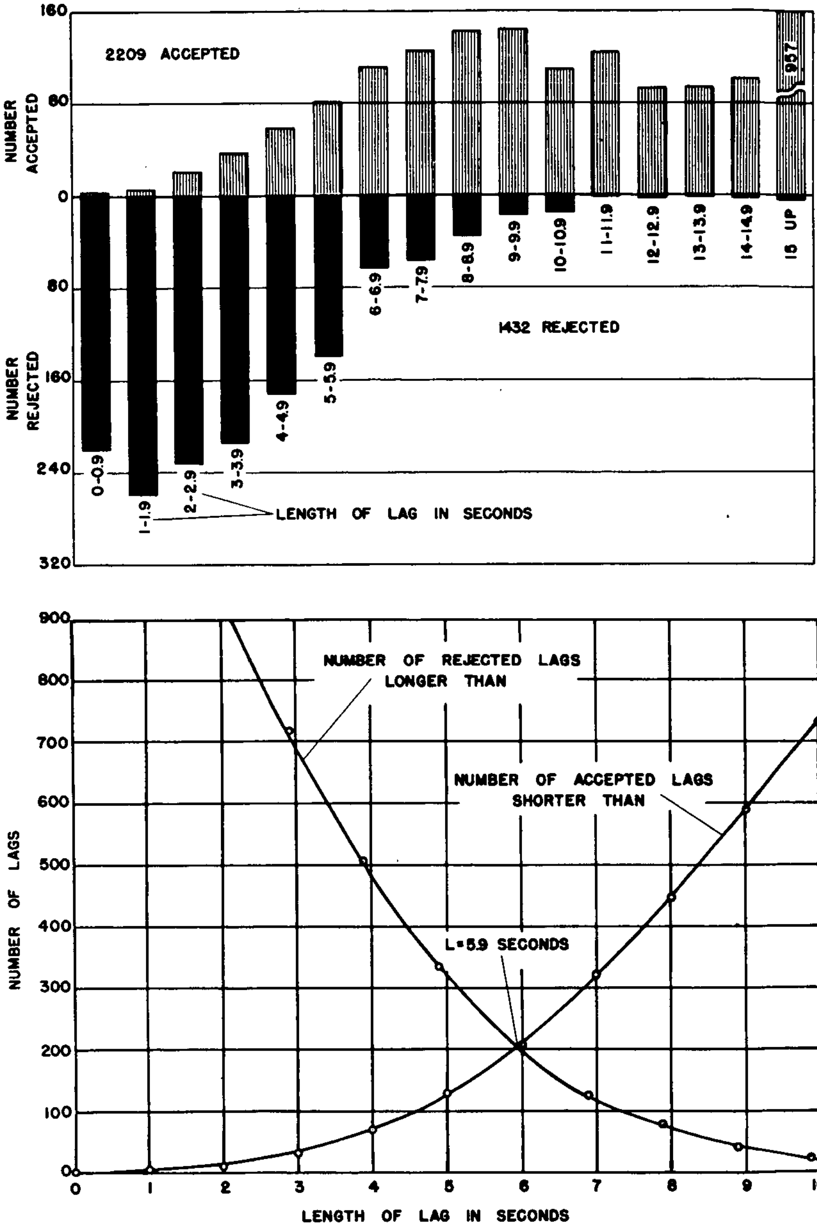


FIGURE 9. Distribution of Accepted and Rejected Lags at Intersection C

Intersection D showed a similar result (see Table IX and Figures 10 and 29). The critical lag at this location was found to be 6.0 seconds, very nearly the same as at Intersection C.

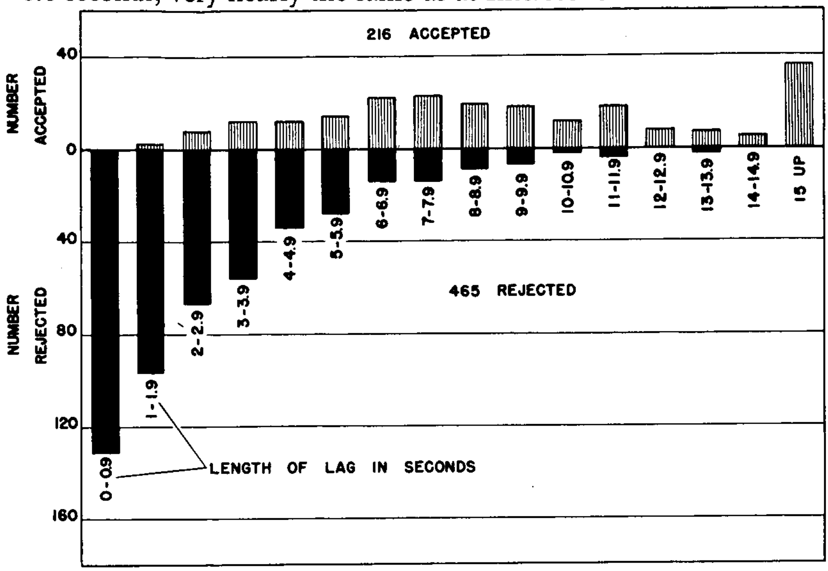


FIGURE 10. Distribution of Accepted and Rejected Lags at Intersection D

Critical Lag Compared with Greenshields's 'Time Gap'

Readers who are familiar with the Greenshields study will have noticed a similarity between the critical lag and a quantity which Greenshields called the "accepted average-minimum time gap" or the "minimum-acceptable time gap".[5] While intended for the same purpose as the critical lag, Greenshields's term was defined somewhat differently, and the difference merits discussion here.

In the terminology of the present report, Greenshields's "acceptable average-minimum time gap" is a lag of a size accepted by more than 50 per cent of the drivers. "More than 50 per cent" is somewhat vague; but what was probably meant is that if the percentage of lags accepted — the last column in Tables I - III — is plotted against lag size (the first column in these tables), and a smooth curve is drawn to fit these points as well as possible, then

[5] *Op. cit.*, pp. 68 ff.

the "acceptable average-minimum time gap" is the time-value for which the ordinate of the curve equals 50 per cent. For the data presented in Tables I - III and IX, the Greenshields quantity is 0.2 seconds longer, on the average, than the critical lag.

The reason for using the critical lag in preference to the Greenshields quantity is that the critical lag is defined in a way that relates directly to the manner in which it is used, while the Greenshields quantity is not. The principal use of the quantity, in both Greenshields's study and the present one, is to simplify the computation of the number of delayed cars by permitting the assumption that all lags shorter than a certain size are rejected while all lags longer than that same size are accepted. From its very definition, the critical lag is the proper quantity to use for this purpose, since the accepted lags shorter than L are exactly balanced out by the rejected lags longer than L.

Reasons for Variation in Critical Lag

The four intersections seem to fall into two groups, as far as their critical lags are concerned: Intersections A and B on the one hand, and Intersections C and D on the other. The critical lags at the latter two places were found to be about twenty-five per cent longer than at the former. To account for this considerable difference, it is necessary to seek out the features that were the same at each of the two intersections in each pair, but different between the one pair and the other. Several types of differences suggest themselves, and they will be considered in turn. They are differences in (1) traffic volume, (2) speeds of main-street cars, (3) sight obstructions, (4) directional traffic pattern on the side street, and (5) width of the main street.

Critical Lag and Volume

The main-street volume at Intersections A and B averaged substantially higher than at Intersection C.[6] Hence, it might be thought

[6] See Table IV.

that there is an inverse relationship between critical lag and main-street volume. The best way of testing such a hypothesis seemed to be to examine the data for one single intersection, to see whether the critical lags computed from different parts of the data would show a high degree of correlation with the main-street volumes for those parts. This procedure was tried first for Intersection C because this was the intersection which exhibited the widest range of main-street volume. The critical lag and the main-street volume were listed for each roll of the recording chart — representing about half an hour of observations — and a coefficient of correlation was computed. No significant correlation was found.[7]

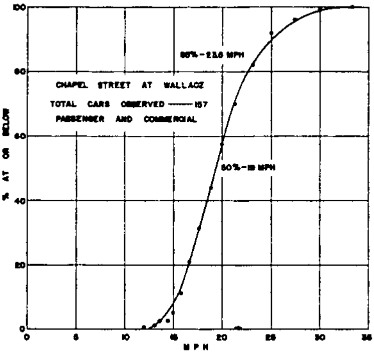
The same computations were made for the other intersections, with similar results. In some cases the correlation between critical lag and main-street volume was *positive,* showing that there was no agreement on the direction of the relationship. It is safe to say that the main-street volume does not have an appreciable effect on the critical lag.

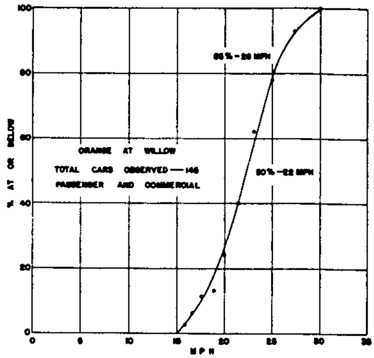
Critical Lag and Speed

It is reasonable to suppose that side-street drivers will be more cautious, that is, have a longer critical lag, when the main-street traffic flows at high speeds than when it flows at moderate speeds. The possibility was considered that the difference in main-street speeds might account for the variation in critical lag.

Observations of main-street speeds were taken in order to explore this possibility. Figure 11 shows the cumulative distribution curves. It will be seen that the critical lags and the main-street speeds vary in the same direction, that is, the intersections with the longer critical lags are also the ones with the higher speeds. On the other hand, the range of speeds is much narrower than the range of critical lags, the smallest eighty-five percentile speed being

7 The correlation coefficient was found to be —.27, a value which could occur more than one time in five from pure chance. By contrast with this, the coefficient of correlation between the percentage of side-street cars delayed and the main-street volume was found to be .94, which is certainly significant.





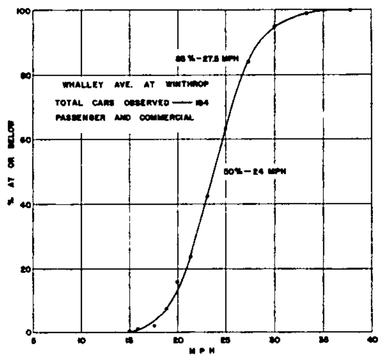


FIGURE 11. Cumulative Distribution of Main-Street Speeds at Intersections A and B (*upper left*), C (*upper right*), and D (*lower graph*)

23.5 miles per hour and the largest 27.5. It hardly seems reasonable that such a small variation in speed could account for a range of critical lags which runs from 4.6 seconds to 6.0. Speed may be one factor which affects the critical lag, but it is certainly not the only one.

Critical Lag and Sight Obstructions

If sight obstructions have any effect on the critical lag, one might expect the longer critical lags (i.e., the more cautious driving) to occur at the intersections where the sight obstructions are bad. Yet more careful consideration leads to the opposite conclusion, that the *shortest* critical lags ought to be found at the blind intersections. The reasoning is as follows: at an open intersection, where the side-street driver can see a considerable distance up and down the main street, he makes his decision in accordance with his own normal critical lag. At a blind intersection, however, where it is hard to see around the corner, the same side-street driver will not be able to see as far down the main street from a comfortable stopping position as his normal critical lag would require. If the sight obstruction is severe, he may have to move part way into the intersection in order to see. Consequently, the side-street driver at an intersection of this character is likely to accept shorter lags than he would want at an open type of intersection, because of a reluctance to encroach on the main street any more than is absolutely necessary.

The difference in sight conditions may well be the principal explanation for the variation in critical lag between the four intersections, inasmuch as visibility is poor at Intersections A and B while it is fairly good at Intersections C and D. It would be desirable, however, to have information from several additional intersections before drawing definite conclusions about this matter.

Critical Lag and Side-Street Traffic Pattern

The most obvious difference between Intersections A and B on the one hand and Intersections C and D on the other is that the side streets carry one-way traffic at the former and two-way traffic at the latter. Here there is a perfect correlation — based on only four examples, to be sure, but perfect nonetheless. Yet it is difficult to see why the directional characteristics of the side-street traffic should determine the critical lag.

One conceivable reason, which does not seem important enough, is the following: a side-street driver on a one-way street has more security in crossing than he would have on a two-way street. The direction pattern on the side street may possibly affect the critical lag, but it will require further confirmation to prove such an unlikely conclusion.

Critical Lag and Main-Street Width

Here, too, the evidence is confusing, although it does point steadily in one direction. In every case where one street is wider than another, the critical lag is greater: but the relationship is not a uniform one, since the critical lag goes all the way from 4.7 to 5.9 seconds when the main-street width changes from 34 to 41 feet, while it rises only to 6.0 seconds when the street width jumps to 63 feet. In mathematical language, the coefficient of correlation between critical lag and main-street width is .79, which is good but scarcely good enough.

It should be fairly clear why the critical lag goes up (rather than down) with increasing main-street width, if indeed there is any validity to the relationship. Obviously, it takes longer to cross a wide street than a narrow one, so it is reasonable to suppose that side-street drivers will want longer lags in which to cross.

Summary

The critical lag is a single value which indicates how large a car-free time interval is required for the typical side-street driver to enter an intersection. It represents the behavior of the typical driver, because it is defined in such a way that the drivers who are more cautious than the average are exactly counterbalanced by the drivers who are bolder than the average. The use of a single typical figure, rather than the whole range of observed human behavior, makes possible a great simplification in the more advanced stages of the analysis of intersection behavior.

It has been clearly established that each intersection has its own characteristic value of the critical lag, and that these values are not the same at all intersections. While there is need for additional research into the factors which affect the value of the critical lag at any particular intersection, the present study suggests that sight conditions at the intersection probably play a more important part than any of the other factors examined.

Table IV presents a convenient summary, for the four intersections which have been studied, of the variations in critical lag and in the factors which may be related to it.

TABLE IV

RELATIONSHIP BETWEEN CRITICAL LAG AND OTHER INTERSECTION CHARACTERISTICS

Intersection	*A*	*B*	*C*	*D*
Critical Lag (seconds)	4.6	4.7	5.9	6.0
Average Main-Street Volume During Period of Observation (cars per hour)	631	596	235	828
Main-Street Speed (miles per hour) Median	19	19	22	24
85 Percentile	23.5	23.5	26	27.5
Main-Street Width (feet)	34	34	41	63
Side-Street Width (feet)	24	30	30	30 & 42
Sight Conditions	Poor	Poor	Fair	Fair
Traffic Pattern on Side Street	1-way	1-way	2-way	2-way

NUMBER OF SIDE-STREET CARS DELAYED

One of the criteria suggested in Chapter I for a Stop-sign warrant is that the warrant should be based on the number of side-street cars which must wait if the main-street traffic is to have the right-of-way at all times. This section is concerned with exploring the relationship between the number of delayed side-street cars and the volumes on the two streets.

The simplest way to define a delayed car would be to consider a car delayed if it is prevented from entering the intersection by any other car. The preventing car might be a car on the main street which is too near the intersection to permit the side-street car to cross ahead of it, or it might be a car on the side street which has previously arrived at the intersection and is waiting for its opportunity to cross. This definition can be stated in a neater form by using the idea of position. A side-street car will be called an nth-position car if there are (n —1) cars ahead of it when it first gets to the intersection. For example, a first-position car is one which reaches the intersection at a time when no other cars are waiting ahead of it in its lane, while a third-position car is one which has to get in line behind two other waiting cars.

The above definition of a delayed car may be rephrased as follows: a car is said to be delayed if (1) it rejects its lag, or (2) it is not a first-position car.

This definition of a delayed car is clear and easy to use, but it leads to results that vary more than if a slightly different definition is used. The unnecessary variation in results arises from the fact that some cars accept lags as short as two seconds, while others reject lags as long as ten seconds. This variation can be eliminated by using the definition of the critical lag, which has the property that the number of accepted lags shorter than L is exactly equal to the number of rejected lags longer than L. In fact, the term critical lag was invented for precisely this reason, to make it possible to iron out chance variations when it is desired to count the number of delayed cars in a relatively small sample of data. Since the critical lag is a characteristic of the intersection as such, it is proper to use the entire set of observations at the intersection in calculating the critical lag, and then to apply this lag to short periods of observation which are part of the larger set. This makes possible a more useful definition of a delayed car, as follows: a side-street car is considered delayed if (1) its lag is shorter than the critical lag, or (2) it is not a first-position car. This is the definition which has been used in analyzing the observed data. It is obvious that this definition gives the same number of delayed cars as the earlier definition when applied to the complete set of observations from which the critical lag is computed.

Tabulation by Fifteen-Minute Periods

To explore the relationship between the number of delayed side-street cars and other factors, the total observation time at each location was subdivided into fifteen-minute periods. A number of quantities have been tabulated for each of these periods, among them the volume on each street, the percentage of side-street cars which are delayed, and the average wait per side-street car.[8] For the pur-

[8] The complete table is given in Appendix B.

pose of this section, the relevant quantities are the two volumes and the percentage of side-street cars delayed. The volumes are total volumes, including both directions of travel on two-way streets.

At Intersection A, the data have been divided into three groups on the basis of side-street volume. One group contains the fifteen-minute periods in which the side-street volume was less than 100 cars an hour, the second group contains the periods in which the side-street volume was between 100 and 199 vehicles an hour, and the third group contains the remaining fifteen-minute periods. For each group the percentage of side-street cars delayed has been plotted against the main-street volume; each point in Fig. 12 represents

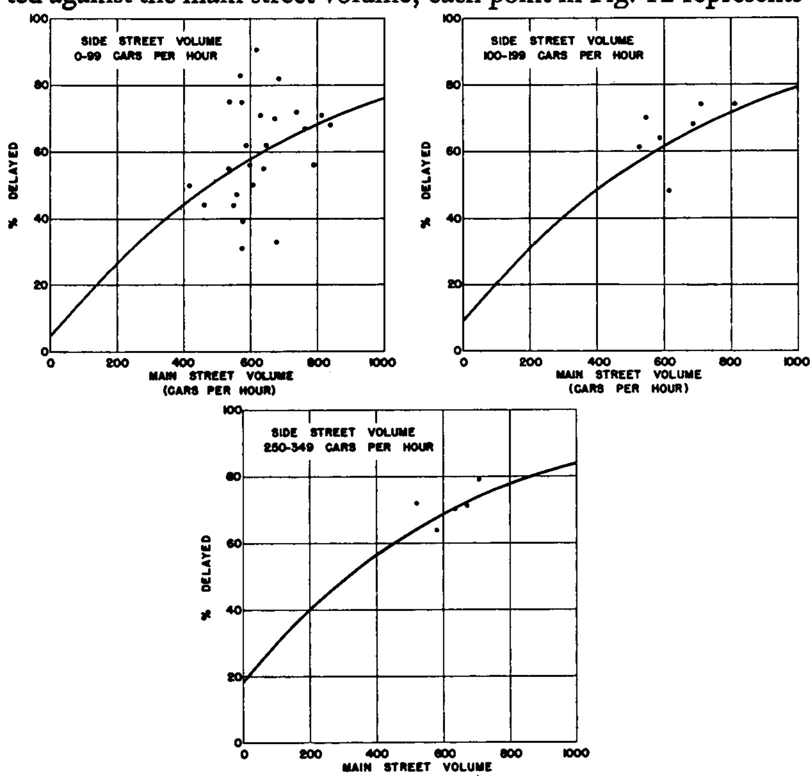


FIGURE 12. Percentage of Side-Street Cars Delayed at Intersection A. Each Point Represents One Fifteen-Minute Period of Observation. The Curves are Drawn from a Formula Given in the Text.

one fifteen-minute period.

While these points show a certain amount of scatter, there is clearly a general tendency for the percentage of side-street cars delayed to increase as the main-street volume increases. The curves which are drawn on these graphs (and also in Figs. 13-15) are not drawn free-hand for each particular set of points, but are all taken from a single formula which involves the main-street volume, the side-street volume, and the critical lag. The basis of the formula, and some of its properties, will be discussed later.

The percent delayed at Intersection B is presented in the same way in Figure 13. One graph is used for the entire set of observations at this location, inasmuch as the side-street volumes stayed within a fairly narrow range. As before, each point on the graph represents a fifteen-minute period.

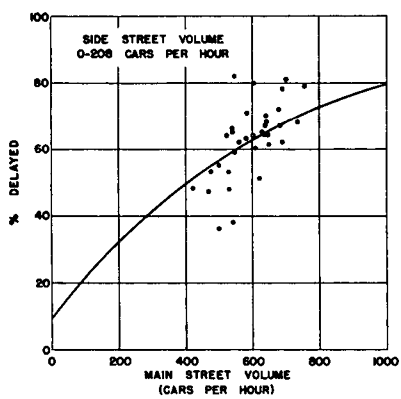


FIGURE 13. Percentage of Side-Street Cars Delayed at Intersection B. Each Point Represents One Fifteen-Minute Period of Observation. The Curve is Drawn from a Formula Given in the Text.

The same information at Intersection C is plotted in Fig. 14. Because of the wide range of volumes at this intersection, the graph is divided into four sections on the basis of side-street volume. The ranges of side-street volume, from the first graph to the last one, are 0 - 149, 150 - 249, 250 - 349, and 350 - 508 vehicles per hour.

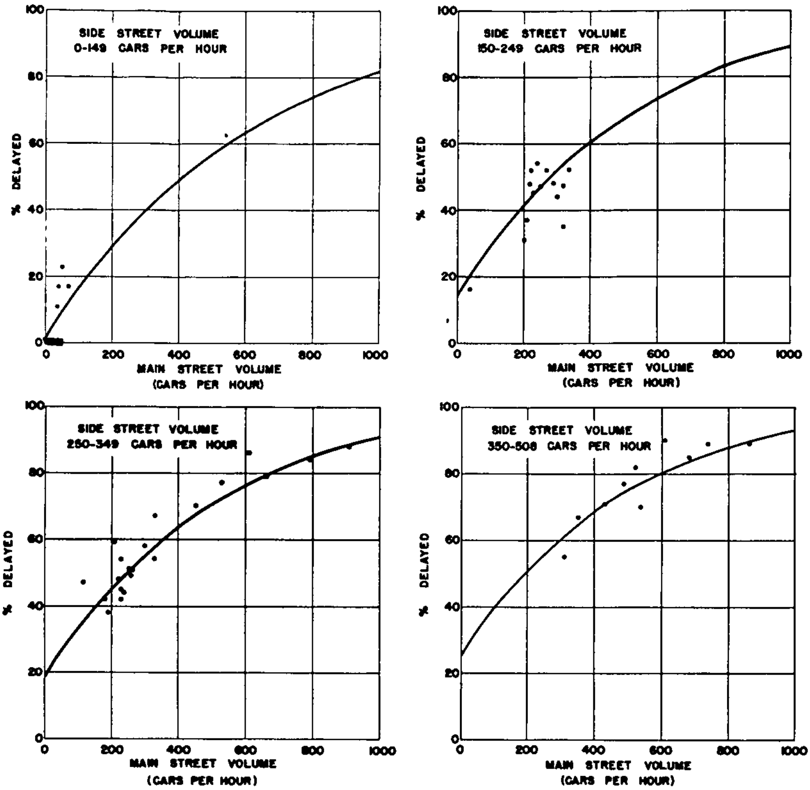


FIGURE 14. Percentage of Side-Street Cars Delayed at Intersection C. Each Point Represents One Fifteen-Minute Period of Observation. The Curves are Drawn from a Formula Given in the Text.

Finally, the data from Intersection D are plotted in a single graph in Figure 15.

Formula for Percentage of Side-Street Cars Delayed

It is of particular importance that the curves on these graphs are all taken from a single formula, making it possible to generalize about volume combinations other than the ones that have been observed. The use of the formula in drawing warrant curves will be explained after the formula itself is discussed.

The formula is a hybrid, in that it is based partly on pure probability theory and partly on the empirical data. To avoid excessive digression at this point, the derivation of the theoretical portion of the formula is reserved for Chapter IV.[9] The reasoning is sketched briefly in the following paragraphs.

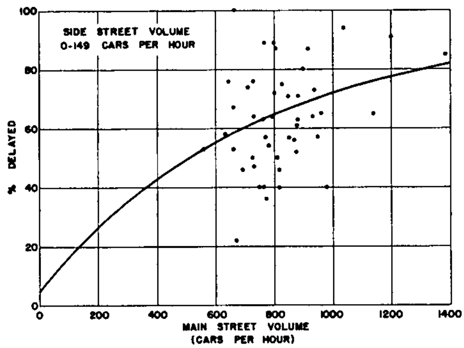


FIGURE 15. Percentage of Side-Street Cars Delayed at Intersection D. Each Point Represents One Fifteen-Minute Period of Observation. The Curve is Drawn from a Formula Given in the Text, with the Right-Turning Cars Treated Separately from the Others.

The mathematical theory of probability can be used to develop a formula for the percentage of side-street cars delayed, provided certain assumptions are made, viz. (1) that the cars on both streets arrive at the intersection in a completely random fashion, and (2) that every side-street car is a first-position car.[10] From the assumption of randomness on the *main* street one can develop a formula for the distribution of gap sizes, that is, a formula from which the number of gaps in any desired range of sizes can be determined. For example, this formula would tell what proportion of the gaps are between three and four seconds in length when the main-street volume is 600 cars per hour.

[9] See pp. 64 ff.

[10] The first assumption is pretty nearly correct in practice, but the second is not, except at very low volumes. The empirical part of the formula serves to compensate for the error which is introduced by this assumption.

From the formula for the distribution of gap sizes one can compute the percentage of the time which is not available for crossing, that is, the percentage of the time during which the interval before the arrival of the next main-street car is less than the critical lag. The assumption of randomness on the *side* street, plus the assumption that all side-street cars are first-position cars, means that a side-street car is just as likely to want to cross at one time as at any other. This being so, the percentage of side-street cars which are delayed is the same as the percentage of the time which is not available for crossing. The formula for this percentage is $100 (1 - e^{-NL})$ where e is the base of natural logarithms (about 2.718), N is the main-street volume, and L is the critical lag.

The values predicted from this formula are too small, because of the sluggishness with which cars get into motion from a stopped position. The analogous situation at a traffic signal may help to make the point clear. The simple probability theory says in effect that the number of delayed cars is the number of cars which arrive at the intersection when the light is red. These cars *are* delayed, to be sure, but it is common knowledge that some of the cars which arrive when the light is green are also delayed, because of the time which is used by the cars stopped by the previous red light in getting started.

In the same way, a side street car which arrives at a Stop sign in the second or higher position (that is, behind one or more stopped cars) is certain to be delayed—though it might not have been delayed if it could have been a first-position car. It is clear that the deviation from the theoretical formula should increase as the side-street volume increases; for the more side-street cars there are in a given length of time, with a given main-street volume, the more piling up there will be. In the limit, as the side-street volume approaches zero, the accurate formula should approach the theoretical formula.

The question arises whether the accurate formula should approach the theoretical formula when the *main*-street volume approaches zero. In other words, should the per cent delayed be equal to zero when there is no traffic on the main street? The answer is no, because every side-street driver who obeys the spirit of the Stop regulation must take a certain amount of time to judge the situation before entering the intersection. There is the possibility that another side-street car will approach before this first-position car has moved; the second car in such a situation would be delayed, according to part two of the definition of a delayed car. Thus when the main-street volume is zero, the formula should give for the per-cent delayed an expression which increases as the side-street volume increases.

A formula which has all these properties, and which in addition fits the empirical data, is

$$P = 100 \left\{ 1 - \frac{e^{-2.5N_s} \; e^{-2NL}}{1 - e^{-2.5N_s} (1 - e^{-NL})} \right\},$$

where P = Percentage of side-street cars delayed
N = Main-street volume, in cars per second
N_s = Side-street volume, in cars per second
L = Critical lag, in seconds
e = Base of natural logarithms, about 2.71828.

Examining the formula, we find:

1. The limit of P, as N_s approaches zero, is $100\,(1-e^{-NL})$, which is the theoretical formula. In other words, if there are no side-street cars, there is no sluggishness effect.
2. P always exceeds $100\,(1-e^{-NL})$, except when N_s equals zero. In other words, the sluggishness effect delays more cars than would be delayed if it did not exist.
3. P is always less than 100 per cent, for any finite volumes.
4. The partial derivatives of P with respect to N, N_s, and L are all positive. This means that an increase in either of the two volumes or the critical lag causes an increase in the percentage of cars delayed, as given by this formula.

These properties are discussed at length because they are all essential to a formula which is intended to have general validity. No formula lacking any of these properties would warrant being used for the generalizations which are to be made.

A question may arise as to why the number of lanes on each street does not enter into the formula. There are two reasons for this, one applying to the main street and the other to the side street. In the case of the main street, the gaps are distributed at random irrespective of the number of lanes in which the cars are flowing; the only effect of a large number of lanes is to increase the capacity of the street and thereby to increase the value of main-street volume for which the random theory ceases to be valid because of congestion.[11]

On the side street, the number of lanes does make a difference, but it is a second-order effect which can safely be disregarded. It will be recalled that if there were no cars in the second and higher position, the side-street volume would not enter into the formula at all; the formula would be simply $P=100\ (1 - e^{-NL})$. The effect of the side-street volume on P, therefore, is to make a correction in the simple formula which takes the higher-position cars into account. Increasing the number of side-street lanes will reduce the number of higher-position cars in a way which is by no means simple to compute. Since this would be a correction of another correction which itself is not very large, the additional complexity which would be introduced into the formula does not seem justified.

The place where the empirical results enter into this formula is in the coefficient of Ns. Any non-negative number might be used; the larger the number, the greater the sluggishness effect.[12] The

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[11] Widening the main street beyond a certain point does, in addition, make it necessary to deal separately with the side-street cars which make right turns. However, this is not the sort of continuous effect which can be taken account of in a formula; one must decide what to do with the right-turning cars before using the formula at all. The method of using the formula to treat these cars separately is discussed on page 50.

[12] If the coefficient of Ns is set equal to zero, one gets the theoretical formula $100\ (1 - e^{-NL})$, which involves no sluggishness at all.

number 2.5 was chosen, as against 2.0 or 3.0, because it came closer to fitting the experimental points in Figs. 12-15.

Something should be said here about the effect of right turns, when the main street carries two or more lanes of traffic in each direction. The percentage of right-turning cars which are delayed will be less than the corresponding percentage for cars proceeding straight through or making left turns, for two reasons: (1) the right-turning cars are affected by only half the main-street traffic, and (2) the critical lag is about 20 per cent less for the right-turning cars than for the others, because of the merging character of the maneuver.[13] These two factors, in effect, reduce the exponent NL in the formula to $(.5N)\ (.8L) = .4NL$.

In predicting the percentage of side-street cars delayed at an intersection where the right turns must be counted separately, it is necessary to make two separate calculations (both of which can be done graphically by using the warrant curves which are discussed in the following section). First, the percentage of delayed left-turning and straight-through cars is obtained from the formula or the warrant curve, using the observed values for the critical lag and the volumes on both streets. Secondly, the percentage of delayed right-turning cars is obtained from the formula or the curve, using the observed values for the critical lag and the side-street volume but substituting for the main-street volume four-tenths of its actual value. Finally, each of these percentage figures is weighted according to the proportion of the side-street traffic which it represents, and the two are added.

The above procedure was used in computing the curve which appears in Figure 15, with results which are highly satisfactory. For the data depicted in this graph, about 41 per cent of the side-street cars made right turns.

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[13] The figure of 20 per cent was obtained experimentally at Intersection D.

A VOLUME WARRANT

If the criterion of percentage of side-street cars delayed is to be used as a Stop-sign warrant, the formula for P provides a basis for drawing warrant curves. This is done as follows:

1. The value of L must be determined for the particular intersection under consideration.
2. The value of P must be selected as the warrant criterion. For example, it might be decided that a Stop sign is needed if fifty per-cent of the side-street cars are delayed; in this case the value fifty would be chosen for P.
3. With L and P already selected, the formula is used to find what values of N and Ns go together to give these values of L and P. A curve of Ns against N is called a *warrant curve,* because any combination of volumes on one side of the curve warrants a Stop sign, while any point on the other side of the line does not.

Although the criterion of fifty per-cent delayed seems reasonable, it is recognized that the figure is arbitrary and that some may prefer a different figure. For this reason the graphs have been drawn in such a way that any percentage figure can be used, with three separate curves for twenty-five per-cent delayed, fifty per cent delayed, and seventy-five per cent delayed. Other percentages can be estimated by interpolation between these curves.

Five different warrant graphs have been drawn, based on different values of L ranging from 4.6 seconds to 5.9 seconds. They are given in Chapter V (Figures 30-34), where the method of using them is discussed in detail. The shaded portion of each graph covers the area where the side-street volume exceeds the main-street volume, since a traffic engineer would probably not be interested in designating the more lightly traveled street as the main street.[14]

The Volume Warrant. There is a further question before the warrant can be used. Suppose that a certain intersection has been selected for study, and that values of L and P have been determined. What volume figures should be used in these warrant graphs?

[14] This is not an ironclad rule, however, for the relative volumes on the two streets might fluctuate from one part of the day to another.

Should it be the average hourly volume for the whole day, the volume in the peak hour, or what? It seems wise to follow the provisions of the Manual on Uniform Traffic Control Devices in its recommendation of a volume warrant for fixed-time signals.[15]

A Stop sign is warranted, under the criterion of per cent delayed, if an average day contains eight hours during which the volumes are such as to delay at least fifty per cent of the side-street cars. The method of using this warrant is illustrated by several examples in Chapter V.

Application of Warrants to Intersections

As a check on the reasonableness of the numerical Stop-sign warrants, it is instructive to apply them to the four intersections which were studied in this project. Both the safe approach speed warrant[16] and the volume warrant will be used for this purpose.

Let us first apply the safe approach speed criterion, by which a Stop sign is held to be warranted if the critical speed is less than eight miles an hour.[17] At Intersections A and B, where there are buildings close to the corners, the critical speed is five miles per hour; therefore, Stop signs are warranted at these intersections. The critical speed at Intersection C is eleven miles per hour, which is not so low as to require a Stop sign. At Intersection D, the store on the northeast corner reduces the critical speed to seven miles per hour on the north approach to the intersection, requiring a Stop sign on that side; however, traffic approaching from the south does not have to stop because of sight restrictions. To summarize these results, Intersections A and B require Stop signs, Intersection C does not, while Intersection D needs a sign on one approach, according to the safe approach speed criterion.

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[15] *Op. cit.*, p. 127. The manual states that a signal is warranted under the volume criterion if the average volumes exceed certain figures for any eight hours of an average day.

[16] See Chapter I, page 12.

[17] The critical speeds have been obtained from pages 65-67 of *A Policy on Intersections at Grade* (1940), published by the American Association of State Highway Officials.

According to the volume warrant developed in this study, Stop signs are warranted at all four intersections. This is evident for Intersections A, B, and D, where the main-street volumes exceed 550 vehicles an hour for the entire period between the morning and afternoon peaks. It is clear from Figs. 30 and 34 that with main-street volumes of this magnitude, Stop signs are warranted irrespective of the amount of traffic on the side street. Intersection C is a border-line case in which the volumes are barely sufficient to call for Stop signs.[18]

Both numerical warrants agree, therefore, that Stop signs are needed at Intersections A and B. At Intersections C and D, the sight conditions do not warrant Stop signs on all approaches, but the volume conditions do. It is of interest to compare these results of using arbitrary rules with the considered opinions of the observers who spent many hours watching traffic at these intersections.

The opinion of the authors is that Intersection C is the only intersection of the four at which Stop signs are really necessary. Without Stop signs at this intersection, there would be great uncertainty as to which drivers should have the right-of-way, and it seems likely that there would be frequent collisions between drivers who would each expect the other to yield the right-of-way. The Stop signs prevent this confusion. At the other three intersections the Stop signs do no harm, since it is most assuredly necessary for the side-street cars to stop. This necessity is so apparent, however, that no signs are really needed to apprise the drivers of this fact. It was noted earlier[19] that the presence or absence of Stop signs had no observable effect on the traffic behavior at Intersections A and B. This conclusion is borne out by the accident records at these intersections before and after the installation of the Stop signs. It is suggested that the need for Stop signs is greater at intersections where the traffic volumes on the two streets are about equal than at intersections where one of the two streets is much more heavily traveled than the other.

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[18] See pp. 100-101. [19] Footnote on page 18.

TIME INVOLVED IN DELAYS

In the preceding section a Stop-sign warrant has been developed on the basis of the *number* of delayed side-street cars. Since a car which is delayed three seconds has been considered of equal importance with a car which is delayed thirty seconds, it has been suggested that a warrant should be developed on the basis of the total amount of time consumed by the intersection delays. This approach has the merit that the monetary value of the time saved or lost by drivers can be weighed against the costs of alternative traffic control devices. It has not been possible to establish the needed relationship between delay time and traffic volumes. The chance variations from one driver to another and from one situation to another are so extreme that no usable relationships could be found.

Definition of Wait

The *wait* of a side-street car has been defined as the time interval from the time the car stopped (or reached its slowest speed) until it began to accelerate and enter the intersection. Decisions as to the times when these events occurred were, of course, made by the field observers and may have been influenced by a personal judgment or reaction time. Yet these errors, however large, should have been consistent inasmuch as the same two observers took all the field data. It is consistency, above all else, that is lacking in the results.

In the tabulation of waits, the total observation time was divided into fifteen-minute periods, the same periods that were used in counting delayed cars. For each fifteen-minute period, two delay time quantities were computed:

(1) The average wait for first-position cars; that is, the sum of all the waits of first-position cars, divided by the number of first-position cars.

(2) The average wait of all cars; that is, the sum of the waits of all the side-street cars, divided by the total number of side-street cars.

Average Wait of First-Position Cars

For each of the four locations, the average wait of first-position cars has been plotted against main-street volume. Since there is no reason why the side-street volume should make any difference, all the fifteen-minute periods at each location have been plotted on a single graph. Figure 16 shows the average wait of first-position cars at Intersection A, Fig. 17 at Intersection B, Fig. 18 at Intersection C, and Fig. 19 at Intersection D. There is a good deal of scatter in the points, which in itself is not very serious since an *ad hoc* curve can nevertheless be drawn on each graph. What *is* serious is that these *ad hoc* curves for the different graphs do not bear any sensible relationship to one another, so that there is no basis for the kind of general formula which was developed in the discussion of the number of delayed cars.

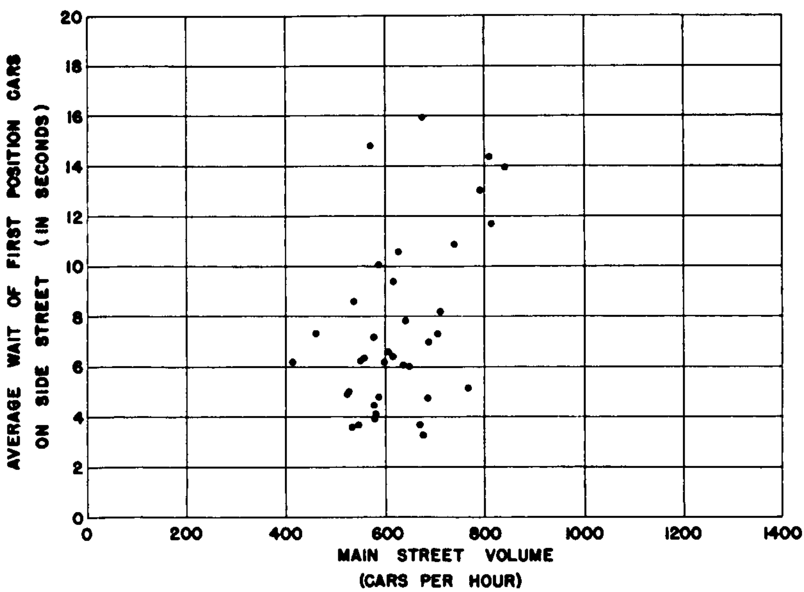

FIGURE 16. Average Wait of First-Position Cars at Intersection A. Each Point Represents One Fifteen-Minute Period of Observation.

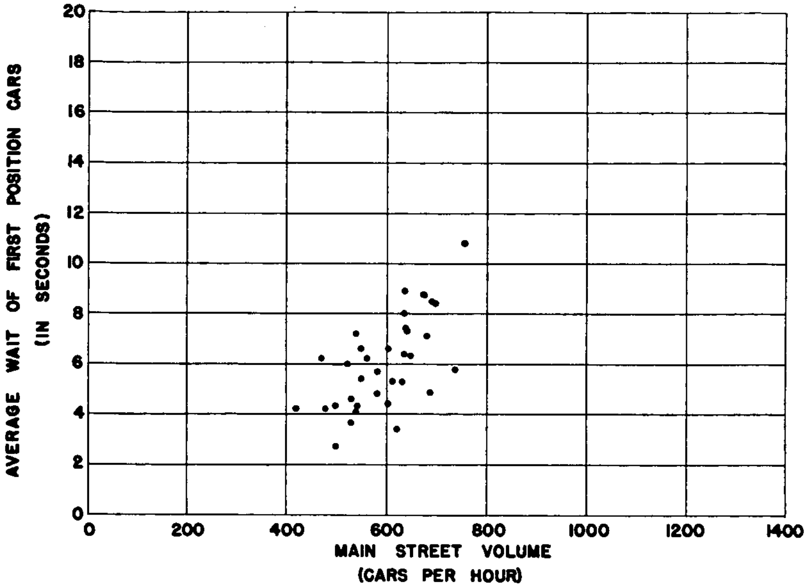

FIGURE 17. Average Wait of First-Position Cars at Intersection B. Each Point Represents One Fifteen-Minute Period of Observation.

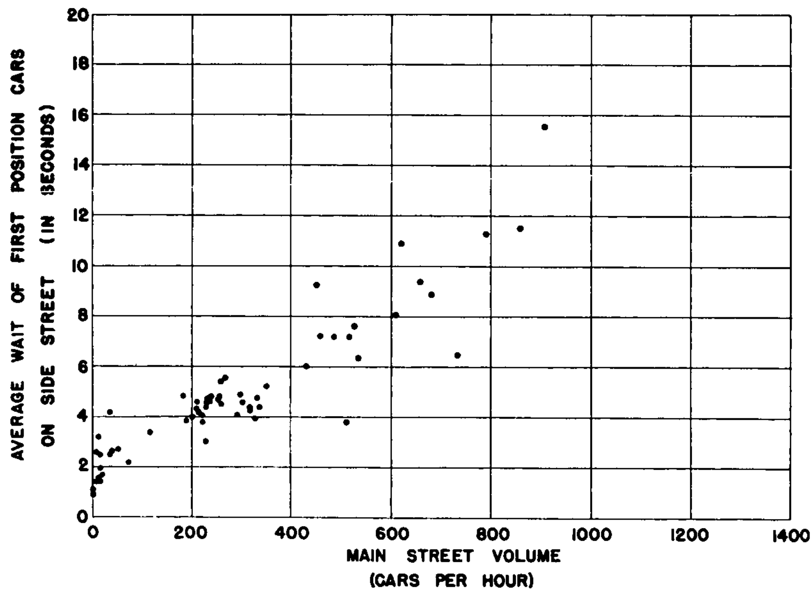

FIGURE 18. Average Wait of First-Position Cars at Intersection C. Each Point Represents One Fifteen-Minute Period of Observation.

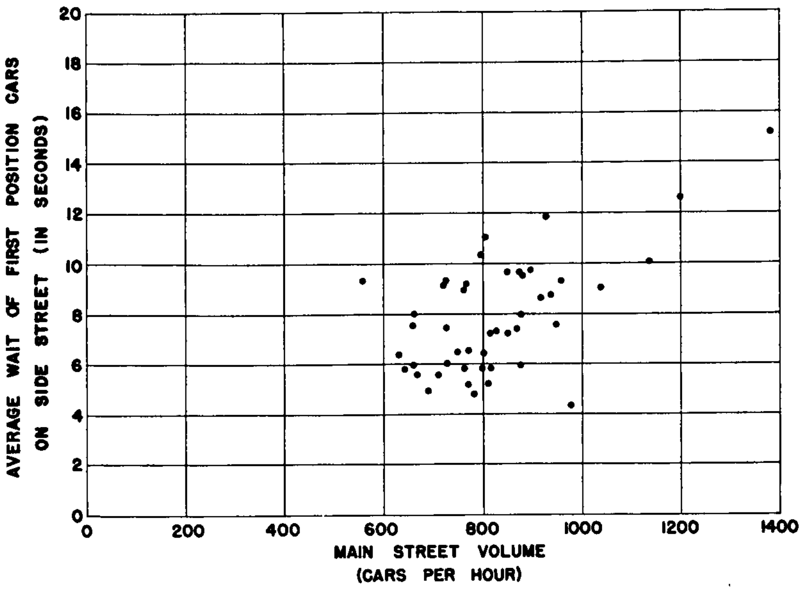

FIGURE 19. Average Wait of First-Position Cars at Intersection D. Each Point Represents One Fifteen-Minute Period of Observation.

A theoretical formula for the average wait of first-position cars can be worked out on the basis of probability theory — this is done in Chapter IV, formula (34) — but it deviates so greatly from the experimental results that it does not merit discussion in this chapter.

Average Wait of All Cars

The average wait of all side-street cars is plotted against main-street volume in the same way in Figs. 20-23. These graphs exhibit the same erratic behavior as the graphs discussed above. It might be thought that a pattern would appear when the fifteen-minute periods are grouped according to the range of side-street volume, but the facts do not bear this out, as Table V demonstrates:

TABLE V

Intersection	*Range of Side-Street Volume*	*Average Main-Street Volume*	*Average Wait of All Cars*	*Number of Side-Street Cars in Sample*
	0-99	629	9.1	407
A	100-199	642	8.8	235
	200-320	625	7.2	366
	0-149	21	2.4	96
C	150-249	247	5.1	767
	250-349	364	9.0	1600
	350-508	280	10.1	568

The results at Intersection C make sense: as the side-street volume goes up, so does the average wait. At Intersection A, however, the average wait goes *down* as the side-street volume increases, with the main-street volume remaining about the same.

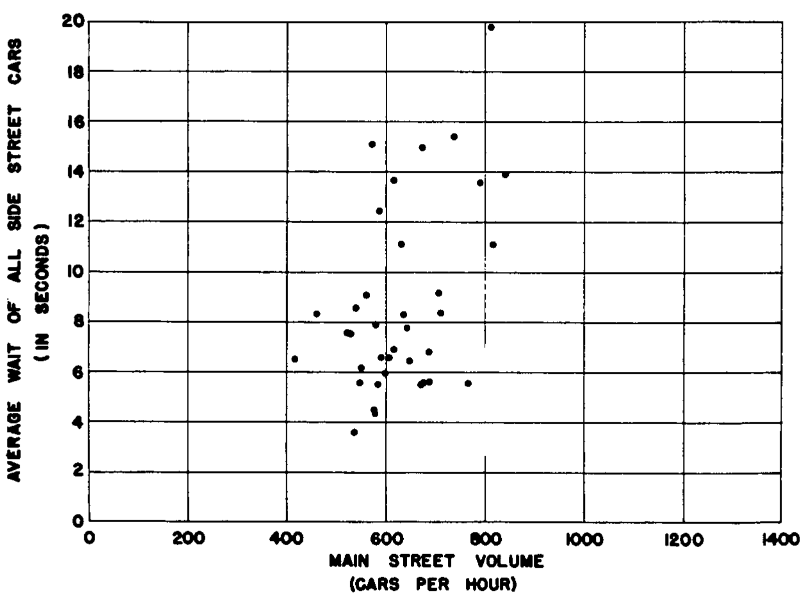

FIGURE 20. Average Wait of All Cars at Intersection A. Each Point Represents One Fifteen-Minute Period of Observation.

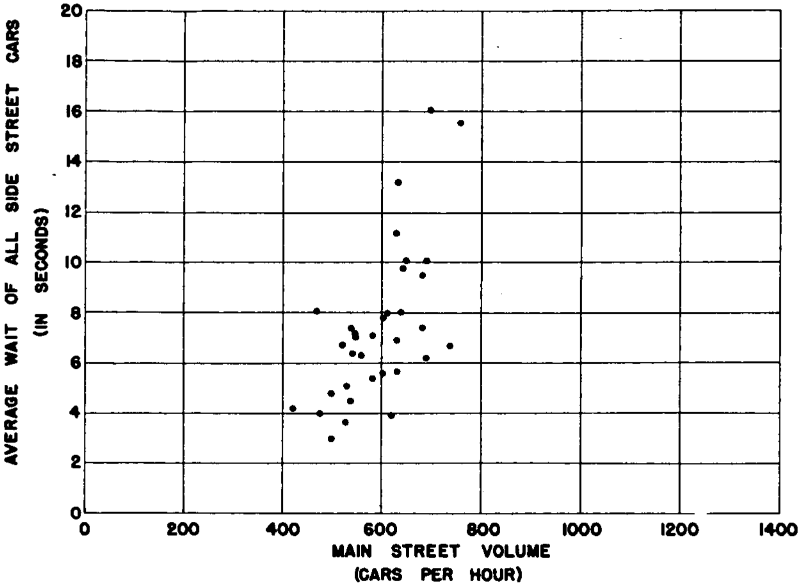

FIGURE 21. Average Wait of All Cars at Intersection B. Each Point Represents One Fifteen-Minute Period of Observation.

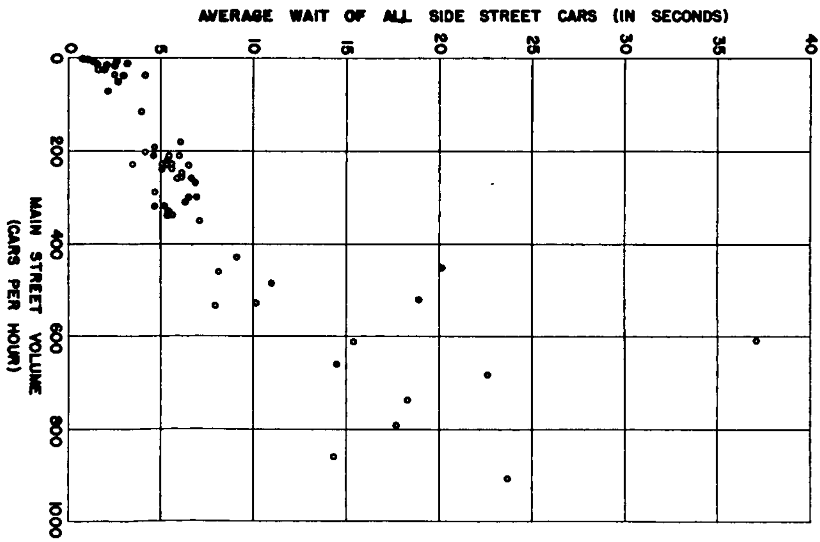

FIGURE 22. Average Wait of All Cars at Intersection C. Each Point Represents One Fifteen-Minute Period of Observation.

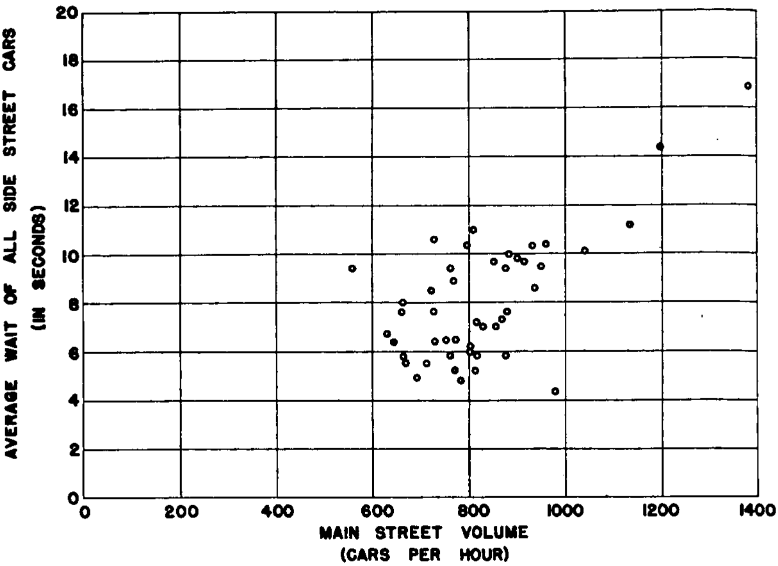

FIGURE 23. Average Wait of All Cars at Intersection D. Each Point Represents One Fifteen-Minute Period of Observaion.

Clearly, no sensible formula would predict such a result. This erratic result may be due to an unusual combination of circumstances. The chief purpose of this discussion is to show the unreliability of using delay time as a basis for a Stop-sign warrant.

PROBLEMS CALLING FOR FURTHER STUDY

In any investigation of this type there are bound to be certain problems which are not completely solved and call for additional field research. In this particular study, there seem to be four such problems: (1) the arbitrary nature of the warrant criterion, (2) the variation in critical lag from one location to another, (3) the scatter in experimental results, and (4) the effect of turning movements on intersection traffic behavior.

The recommended warrant is based on the assumption that a Stop sign is needed if more than half the side-street cars are delayed by traffic on the main street. The choice of the fifty per-cent figure is wholly arbitrary, because there is nothing in the shape of the curves to indicate a preference for any particular figure.

Critical Lag

That the value of the critical lag is different at different locations has been proved beyond any doubt, but the cause of the variation needs to be explored much more carefully than has been possible in this study. The critical lag seems to be connected with sight obstructions, main-street speeds, main-street width, and the pattern of traffic flow on the side street. The relationships are not simple, and additional research is definitely needed.

Scatter in Experimental Results

Scatter in experimental results can be traced either to inadequate amounts of data or else to a real variability in the quantities being measured. The first explanation would seem to apply to the tabulations of the number of delayed cars, but the second appears necessary in order to explain the average wait per car. It would be interesting to know what conclusions would emerge from a set of observations many times as extensive as the data used here.

Turning Movements

It has been pointed out that turning movements received little attention in this study, primarily because of a desire to keep the analysis as simple as possible. It does seem, however, that the number of turning movements at an intersection ought to affect the amount of delay which occurs, and a future investigator might well decide to take this factor into account.

CHAPTER IV

PROBABILITY THEORY

Applications of the mathematical theory of random distributions to highway traffic problems have been discussed by several writers, and it might be well to review their work before going on to the new developments which have been made in the course of the present study.

To begin with, let us define a random distribution and indicate the two different ways in which these distributions occur in traffic. A set of points on a line is said to be distributed at random, provided:

(a) the location of each point is independent of the location of any other point; and

(b) any two equal segments of the line have the same likelihood of containing any particular number of points.[1]

Uncongested traffic is distributed at random in two different ways: (1) in space, where the set of points are the positions of the cars on a road at a particular instant; (2) in time, where the set of points are the instants at which cars pass a particular location. To see these instants as a set of points, one need only plot them on a time scale.

EARLIER WORK

The earliest systematic discussion of the use of random distribution theory in traffic problems appeared in an article in a British technical journal by William F. Adams.[2] Adams pointed out that vehicles in traffic could be compared to a set of points on a line in

[1] Some writers give separate names to these two properties. Fry, for example, calls a distribution *individually at random* if it has the first property, and *collectively at random* if it has the second. The two properties are independent, as Fry shows by means of examples. Thornton C. Fry, *Probability and Its Engineering Uses.* New York: D. Van Nostrand Company, Inc., 1928, pp. 218-219.

[2] William Frederick Adams, *Road Traffic Considered as a Random Series.* The Institution of Civil Engineers Journal, Volume 4, pp. 121-130.

both the space sense and the time sense, and he developed the theory to a considerable extent in the latter application. He stated the Poisson law[3] for the probability of having any particular number of cars in a given interval of time, and also a formula giving the number of gaps in any particular range of sizes.[4] He verified that traffic conforms closely to these formulas for moderate traffic volumes.

Adams also gave another set of formulas — all stated without proof — for certain quantities which arise in the problem of determining the average delay experienced by pedestrians who require a certain minimum lag in order to cross a street carrying randomly distributed traffic. The numerical results from these formulas were likewise verified by actual observation. It is apparent that Adams's pedestrian behavior has much in common with the performance of side-street cars at a Stop sign.

The other principal precursor of the present work is the report referred to, by Greenshields, Schapiro, and Ericksen.[5] Where Adams dealt with only a single stream of traffic, Greenshields devoted considerable attention to the interaction of two intersecting traffic streams. This interaction was found to be extremely complicated at unsignalized intersections, and it is not surprising that Greenshields[6] was unable to apply the probability theory to this problem.

At signalized intersections, Greenshields had more success with the mathematical theory. He was fully aware of the importance of sluggish starting, and he developed a method of estimating the number of cars delayed by a red light, based on a combination of the random distribution theory and a detailed empirical study of the starting performance of a line of stopped cars. Some success was

[3] Formula (1) on page 64.

[4] Formula (2) on page 65.

[5] *Op. cit.*

[6] The name of Greenshields, as the senior author of the report, has been used throughout the present study when referring to *Traffic Performance at Urban Street Intersections.* The present authors have no desire, however, to slight the important contributions made by the other authors of that study.

achieved, in addition, in estimating the average amount of delay caused by a red light.

THEORY OF A SINGLE TRAFFIC STREAM

The basic law describing a random series of events (i.e., a random distribution in the time sense) is the Poisson formula, of which Fry[7] has given an excellent derivation. The Poisson formula states that when a set of randomly distributed events occur at an average rate N, the probability that k of these events will occur during an interval t is

$$e^{-Nt} \frac{(Nt)^k}{k!}, \tag{1}$$

where e = the base of natural logarithms, about 2.71828

N = average rate of occurrence (e.g., traffic volume)

t = length of time interval

k = number of occurrences whose probability is desired

k! = k factorial, the product of all the integers from one up through k.

Both Adams and Greenshields applied this law to the distribution of gaps in a traffic stream. If we think of a traffic stream as a flow of cars past a particular place, the times at which the cars pass this place can be plotted as a set of points on a time axis (see Fig. 24). A gap is defined as the interval from one of these points to the next

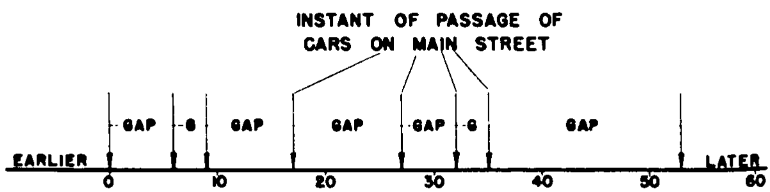

FIGURE 24. Gaps in a Single Traffic Stream.

[7] *Op. cit.*, pp. 220-227.

one, or in other words, the time interval from the passage of one car to the passage of the next car after it. Thus we may think of the traffic stream as a succession of gaps. These gaps will be of all different sizes in an uncongested traffic stream, but the average gap size will always be the reciprocal of the volume. For example, if the volume is 600 cars an hour, the average gap will be 1/600 of an hour, or 6 seconds.

If the gaps are arranged not in their chronological order but in order of increasing size, they fall into a neat pattern, which is a consequence of the Poisson law. Greenshields has proved,[8] and Adams has stated without giving the proof, that if a gap is selected at random from the whole set of gaps, the probability of its being greater than t is

$$e^{-Nt}. \qquad (2)$$

Since it is often easier to think of actual numbers than of probabilities, we restate this formula differently. If we select a period of time containing A cars, passing by at an average rate N, the number of gaps greater than t is equal to[9]

$$Ae^{-Nt}. \qquad (2')$$

Greenshields[10] has verified that the distribution of gaps in actual traffic conforms closely to this theory, except for a slight deficiency of gaps shorter than two seconds. Since the observations of the

[8] *Op. cit.*, pp. 75-76.

[9] Expressed in this way, the statement is not strictly correct, for two reasons: first, because the number of gaps must be an integer, and expression (2′) is not generally an integer; secondly, because expression (2′) gives only the *most probable* number of gaps, from which an actual case may deviate. However, one may regard these provisos as merely sharpening the meaning of statement (2′), which is correct provided its meaning is properly understood. Another way of looking at the statement is to observe that it becomes nearer and nearer to being strictly correct as A increases without limit.

[10] *Ibid.*, p. 78.

present study merely confirm this result, it does not seem necessary to present further evidence on this point.

Equation (2′) can be used to find the total time contained in the gaps. While the derivation that follows may seem a roundabout way of getting at this simple quantity, it illustrates a method used later in the chapter when blocks and antiblocks are discussed.

First the easy way: the total number of gaps is A, the same as the the total number of cars. The average gap size is 1/N. Therefore the total time is the total number of gaps multiplied by the average gap size, i.e., $A \times 1/N = A/N$.

Now the interesting way: from equation (2′), the number of gaps greater than t is

$$(\; Ae^{-Nt}. \;) \qquad (2')$$

The number of gaps whose length is between t and t + dt is the negative differential of this expression, i.e.,

$$ANe^{-Nt}\,dt. \qquad (3)$$

The time contained in the gaps whose length is between t and t + dt is t times expression (3), i.e.,

$$ANte^{-Nt}\,dt; \qquad (4)$$

and the total time in the gaps is the integral of (4) over the whole range of gap sizes from zero to infinity, i.e.

$$AN \int_0^\infty te^{-Nt}\,dt = A/N, \qquad (5)$$

the same as before.

Difficulty of Synthesizing Two Streams

The theory developed in the preceding section has been given here, not because it is new, but because it is the basis of the derivations that follow. It is limited, as we have seen, to the description of *a single stream* of traffic. The problem at an intersection, however, is to analyze the interaction of *two* traffic streams. It is in this direction that the theory must be developed further.

It is easy to see that intersections controlled by fixed-time signals are the simplest to treat mathematically, because of the regularity with which the right-of-way is assigned back and forth by the signal. Greenshields has dealt with this situation by analyzing what happens to the traffic stream facing the red light. In this way he has been able to treat the streams one at a time.

No such simplification is possible at an unsignalized intersection, where a real synthesis of the two traffic streams must be handled. Detailed study has shown this to be an exceedingly difficult problem. The work of the remainder of this chapter makes only a modest contribution to the solution of this problem, but it does accomplish a real synthesis of two independent random patterns. It is hoped it may point the way to a more definitive treatment.

A New Concept: the Block

In order to develop a mathematical theory of intersection performance, a new concept is needed: the block. We have already seen how a stream of traffic can be thought of as a succession of gaps. A different kind of breakdown, however, is desirable in dealing with intersection behavior. From the point of view of a driver on side-street B who wants to cross main-street A, it is useful to divide stream A alternately into intervals during which crossing is *impossible* and intervals in which crossing is *possible.*

These intervals will be called *blocks* and *antiblocks,* respectively, in stream A. If the intersection is signalized, the blocks are the periods when street B faces a red light, and the antiblocks are the periods when street B faces a green light. Looked at in this way, the distinguishing feature of an intersection controlled by a fixed-time signal is that the sequence of blocks and antiblocks is periodic.

In a free-flowing traffic stream, the arrangement of blocks is not periodic, of course, but there is a pattern to it. The general approach of the rest of this chapter will be, first, to find out just what this pattern is on the main street, and then to see what happens when the randomly distributed traffic on the side street faces it.

Let us begin with a precise definition of a block. Consider some definite arrangement of gaps. Time preceding the passage of any car by L (the critical lag) or less is contained in blocks, while all the time which is more than L before the passage of the next car is in antiblocks. Thus, every antiblock is a part of some gap greater than L. When each gap greater than L is divided into two parts in such a way that the second part is of length L, then the first part is an antiblock. A block is defined as the interval separating two successive antiblocks. This definition will be made clearer by means of a numerical example.

Consider the arrangement of gaps depicted in Figure 25. In this illustration the critical lag is five seconds, and the main-street cars go by at times 0, 6, 9, 17, 27, 32, 35, and 53. Therefore the gaps,

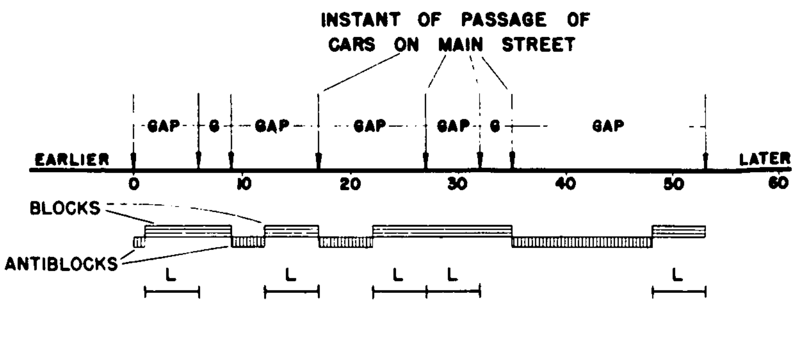

FIGURE 25. Gaps, Blocks, and Antiblocks in a Traffic Stream.

which are the intervals between these arrival times, are 6, 3, 8, 10, 5, 3, and 18 seconds, in that order. To determine the antiblocks, we must first locate the gaps which are longer than five seconds. There are four of these — the first, third, fourth, and seventh ones — with lengths of 6, 8, 10, and 18 seconds respectively.

In each one of the gaps, the antiblock is the part which precedes the last five seconds. Thus the antiblocks are the first one second of the first gap, the first three seconds of the third gap, the first five seconds of the fourth gap, and the first thirteen seconds of the seventh gap. The intervals separating these antiblocks from one another are the blocks, whose lengths are 8, 5, 13, and 5 seconds respectively.

Antiblock Distribution

It will be shown in the next few sections that there are formulas for the distribution of block and antiblock sizes, analogous to formula (2′) for gap sizes.

Consider first the antiblocks, because they are simpler. Every antiblock is a part of exactly one gap greater than L. In fact, every antiblock of length t corresponds to a gap of length L + t. Therefore the number of antiblocks greater than t is equal to the number of gaps greater than L + t, i.e.,

$$Ae^{-N(L+t)}. \qquad (6)$$

The total number of antiblocks is, of course, the number of antiblocks greater than zero, and can be obtained from (6) by setting t equal to zero. The total number of antiblocks is

$$Ae^{-NL}. \qquad (7)$$

To get the total time contained in antiblocks, we use the reasoning employed in the derivation involving equations (3) - (5). The number of antiblocks whose length is between t and t + dt is the negative differential of (6), i.e.,

$$ANe^{-N(L+t)}dt. \qquad (8)$$

The time contained in these antiblocks is

$$ANte^{-N(L+t)}dt; \qquad (9)$$

and the total time contained in all antiblocks is the integral of this expression over the whole range of antiblock sizes, i.e.,

$$ANe^{-NL} \int_0^\infty te^{-Nt}dt = \frac{A}{N} e^{-NL}. \qquad (10)$$

The average length of antiblocks can be obtained by dividing the total time contained in them (10) by the total number of them (7). When this is done, the average antiblock length comes out 1/N, the same as the average gap length.

On first thought one might expect the average length of antiblocks to be less than the average gap length, inasmuch as each antiblock is only a part of some gap longer than the antiblock. True, the average antiblock length is less — by an amount L, to be exact — than the average length of the selected group of gaps from which the antiblocks are taken. But this selected group of gaps is only part of the entire set of gaps, which includes many gaps shorter than L in addition to the ones which are longer.

A numerical example may help to clear up the difficulty. Suppose we have a main street whose volume is 360 cars an hour, with a value of L equal to five seconds. If we consider a one-hour period, the constants in the formulas will have the values $A = 360$, $N = 0.1$, and $L = 5$. The total number of gaps (of all sizes) is 360, the total time in the gaps is one hour, so the average gap length is ten seconds. The total number of antiblocks, which is the same as the number of gaps longer than five seconds, is 218; the total time in these antiblocks is thirty-six minutes and twenty-four seconds, so the average length of the antiblocks is also ten seconds.

Block Distribution

The distribution of block sizes is a more difficult problem than the distribution of antiblock sizes, and it has not been possible to solve the problem with complete mathematical rigor. However, a solution believed correct has been found. The authors would welcome either a more adequate proof or a demonstration that the formula is invalid.

What are the conditions a block-distribution formula must satisfy? In the first place, the total number of blocks must equal the number of antiblocks, which is given by expression (7). Secondly, the total time contained in blocks must be the difference between the total time and the time contained in antiblocks, i.e., expression (5) minus expression (10). Thirdly, there can be no blocks shorter than L, for every block begins with an interval L preceding the passage of a car. In the fourth place, there are a certain number of blocks exactly equal to L; this number is the same as the number of pairs of successive gaps greater than L, because each pair of this kind contains a pair of antiblocks separated by a block of length L.[11] Finally, the formula must permit the existence of blocks of any size greater than L, no matter how large.

A formula that satisfies all these conditions can be found. Indeed, we shall produce a whole family of them. Only the experimental data make it possible to decide which formula is correct.[12]

Before setting up the equations, let us look more closely at the fourth condition. How many pairs of successive gaps greater than L are there? According to formula (2), if a gap is selected at random, the probability of its being greater than L is e^{-NL}. Hence, the total number of such gaps is Ae^{-NL}. If we now consider the set of

[11] This is illustrated by the third and fourth gaps in Fig. 25.

[12] The reader who wishes to proceed directly to the solution will find it in equation (19).

gaps consisting of the successors of *these* gaps, the probability that one of *them* is greater than L is also e^{-NL}, because these successors are just as truly a random set of gaps as the original set. Accordingly, the number of pairs of successive gaps greater than L is

$$Ae^{-NL} \times e^{-NL} = Ae^{-2NL}, \qquad (11)$$

which is the desired expression for the number of blocks equal to L.[13]

Now we are ready to set up equations that the block-distribution formula must satisfy. Let us write the formula in three parts:

a) the number of blocks less than L is zero

b) the number of blocks equal to L is Ae^{-2NL}

c) the number of blocks greater than t, for t equal to or greater than L, is $H(t)$,

where $H(t)$ is the function to be determined. As before, the number of blocks between t and $t + dt$, for any value of t equal to or greater than L, is $-H'(t)dt$. Let us define $G(t) \equiv -H'(t)$.

The first condition states that the total number of blocks is equal to Ae^{-NL}, i.e.,

$$\begin{array}{ccccccc} & & \text{No. of blocks} < L & & \text{No. of blocks} = L & & \text{No. of blocks} > L \\ 0 & + & Ae^{-2NL} & + & \int_L^\infty G(t)dt & = & Ae^{-NL}, \end{array}$$

which may be written

$$\int_L^\infty G(t)dt = Ae^{-NL}(1 - e^{-NL}). \qquad (12)$$

---

[13] By a more sophisticated argument it can be proved that even when A is small, the expected number of pairs of successive gaps greater than L is $(A - 1)\, e^{-NL}$. For large values of A, the difference between using this expression and using formula (11) is negligible.

If we introduce a new variable $y = t - L$, some of the later equations will be simplified; in terms of y, equation (12) becomes

$$\int_0^\infty G(y)dy = Ae^{-NL}(1-e^{-NL}). \qquad (12')$$

The second condition states that the total time in blocks must equal

$$\frac{A}{N}(1-e^{-NL}), \text{ i.e.,}$$

Time in blocks $< L$ Time in blocks $= L$ Time in blocks $> L$

$$0 \quad + \quad ALe^{-2NL} \quad + \quad \int_L^\infty tG(t)dt = \frac{A}{N}(1-e^{-NL}), \qquad (13)$$

which may be written

$$\int_0^\infty yG(y)dy = \frac{A}{F} e^{-NL}(1-e^{-NL}), \qquad (13')$$

where F is defined to equal

$$\frac{Ne^{-NL}(1-e^{-NL})}{1-e^{-NL}-NLe^{-NL}}. \qquad (14)$$

The third and fourth conditions have already been taken care of, and the fifth condition merely imposes the general restriction that $G(y)$ must be positive for all positive values of y.

To summarize at this stage, we are looking for a function $G(y)$ which is positive for all positive values of y and which satisfies the two integral equations (12′) and (13′). Even without an explanation of the method by which the equations were solved, the reader can verify that the functions in the following family do in fact satisfy all the conditions:

$$G_n(y) = \frac{n^n}{(n-1)!} AF^n e^{-NL}(1-e^{-NL}) y^{n-1} e^{-nFy}, \qquad (15)$$

where n is any positive integer. The corresponding H-functions are

$$H_n(y) = Ae^{-NL}(1-e^{-NL})\,e^{-nFy}\sum_{k=0}^{n-1}\frac{(nFy)^k}{k!}. \qquad (16)$$

This looks rather complicated, but fortunately the function which fits the data is the simplest one of the family, namely the function for which n = 1. Thus we have

$$G(y) = AFe^{-NL}(1-e^{-NL})\,e^{-Fy} \qquad (17)$$

$$H(y) = Ae^{-NL}(1-e^{-NL})\,e^{-Fy}. \qquad (18)$$

Therefore, the formula for block distribution is as follows:

a) the number of blocks less than L is zero

b) the number of blocks equal to L is Ae^{-2NL} (19)

c) the number of blocks equal to or greater than t, for t equal to or greater than L, is $Ae^{-NL}(1-e^{-NL})\,e^{-F(t-L)}$.

$$\text{Total number of blocks} = Ae^{-NL}. \qquad (20)$$

$$\text{Total time in blocks} = \frac{A}{N}(1-e^{-NL}). \qquad (21)$$

$$\text{Average block length} = \frac{1-e^{-NL}}{Ne^{-NL}}. \qquad (22)$$

The experimental verification of formula (19) is based on observations of 1536 cars on the Merritt Parkway at a time when traffic appeared to be flowing freely without congestion.

The distribution of gap sizes in this traffic conforms very closely to expression (2′), which gives added confirmation to the belief that the traffic flow was essentially random.

Block-lengths were computed from this set of observations, using five different values of L ranging from two seconds to twelve seconds; this is equivalent to using sets of data for five different values of the traffic volume. In each of the five cases the distribution of block sizes was plotted and compared with formula (19), and all five showed satisfactory agreement.[14] Two of the graphs are shown in Figure 26.

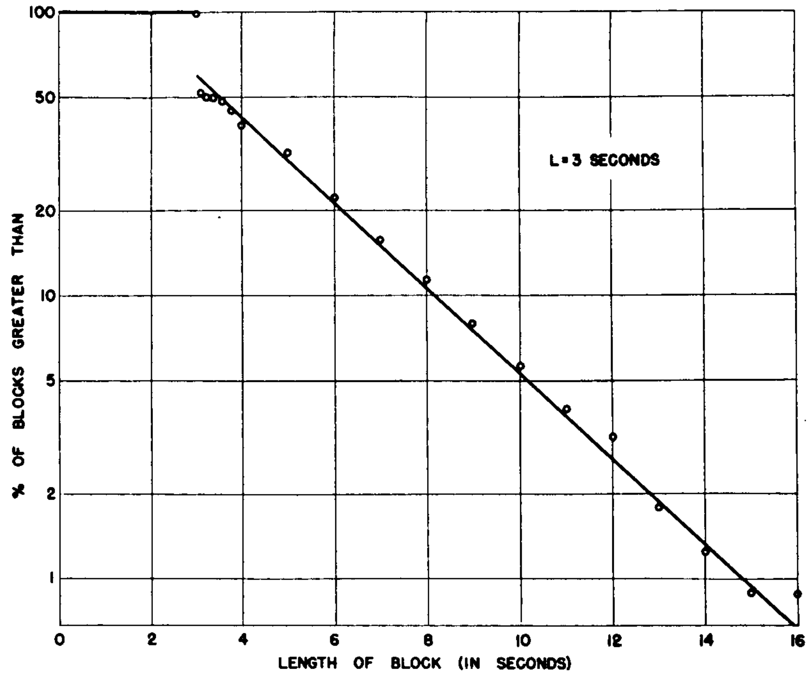

FIGURE 26a. Cumulative Distribution of Block Sizes. The Points Represent Observations on the Merritt Parkway, while the Lines are Taken from Formula (19) of the Text.

[14] It should be noted that none of the other formulas of th family (18) would give a straight line when plotted on semi-log paper. $H_2(y)$ was plotted on one of the graphs for the sake of comparison and was clearly incorrect.

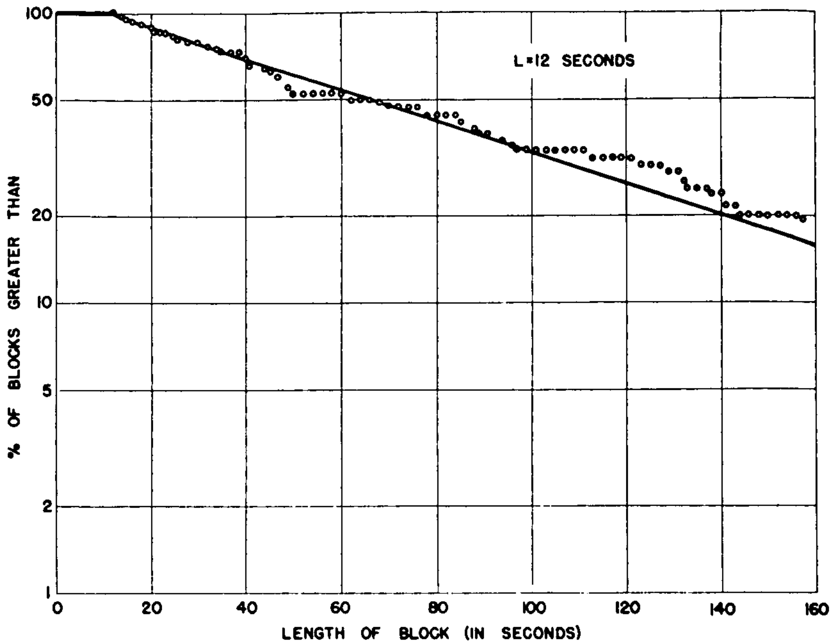

FIGURE 26b. Cumulative Distribution of Block Sizes. The Points Represent Observations on the Merritt Parkway, while the Lines are Taken from Formula (19) of the Text.

THE THEORY OF THE INTERACTION BETWEEN TWO INTERSECTING TRAFFIC STREAMS

We now have the mathematical tools with which to examine the interaction between the side-street stream and the main-street stream at an intersection controlled by a Stop sign. It is assumed that both streams have random distributions, with each distribution based on the volume of its own stream. It is further assumed that a side-street car will enter the intersection immediately if it arrives during an antiblock in the main stream, or that it will enter as soon as the block is over, if it happens to arrive during a block.

This latter assumption, which may be called the assumption of instantaneous clearing, is of course not true in practice, for it ignores the sluggish starting which is such an important feature of intersection performance. However, one cannot run until one has learned to walk, and it would have been impossible to make any headway with this theory had the assumption not been made.

Number of Side-Street Cars Delayed

How many side-street cars are delayed on account of the main-street traffic? An empirical answer was given in Chapter III, but it is desirable to see if a theoretical answer can be found. Since we are assuming random traffic distributions and instantaneous clearing, the proportion of cars delayed is the same as the proportion of time contained in blocks, which is given by expression (21). That is, the proportion of cars delayed is

$$1 - e^{-NL}. \qquad (23)$$

Note that this result is independent of the volume on the side street. Now actual experience contradicts this, for we have seen in Chapter III that the proportion of cars delayed goes up as the side-street volume increases.[15]

One of the mistakes in formula (23) is that it assumes that a car which is second in line — or third or fourth or fifth — will enter the intersection in the first antiblock that appears after it arrives. Since this is not always true in practice, it may be useful to study the frequency with which various numbers of cars can be expected to accumulate at the intersection.

Two additional terms will help to clarify this discussion. A *pile of size n* is defined as an accumulation of n side-street cars in a single lane before the first one is able to move into the intersection. An *nth-position car* is one which arrives at the intersection at a time when (n — 1) cars are already waiting ahead of it. Thus if three cars accumulate at an intersection before the first one can move, there is a pile of size 3, and the cars are a first-position car,

[15] The *ad hoc* formula that was used in Chapter III reduces to (23) when the side-street volume is zero. Its values increase steadily as the side-street volume increases.

a second-position car, and a third-position car, in the order of their arrival.

Pile Distribution

With the assumption of instantaneous clearing, the theory permits us to predict the number of piles of each size.[16] Even though the results are vitiated by this false assumption, it is worthwhile to go through the derivation both for the limited value which the results do have and for the insight which the method provides into one way of approaching these problems.

The question to be answered is this: in a period of time containing A cars on the main street, how many piles of size n will occur? In other words, during how many of the blocks will exactly n side-street cars accumulate in each side-street lane? We have used the symbol N to represent the main-street volume; let us introduce the symbols N_2 for the side-street volume in one lane and for the number of piles of size n in the lane. The ensuing discussion applies to a single side-street lane. The total number of piles of size n is the sum of the separate α_n 's computed for each approach-lane.

In a block of length t, the probability of having exactly n side-street cars in the lane under consideration is, according to (1),

$$e^{-N_2 t} \frac{(N_2 t)^n}{n!} \qquad (24)$$

The number of such blocks is, according to (19),

$$\begin{array}{ll} 0 & \text{if } t < L \\ Ae^{-2NL} & \text{if } t = L \qquad (25) \\ AFe^{-NL}(1-e^{-NL})\, e^{-F(t-L)}\, dt \text{ in a range of width } dt, & \\ & \text{if } t > L. \end{array}$$

Therefore, the number of such blocks containing exactly n side-street cars in the lane is the product of (24) and (25) i.e.,

[16] The analogous problem for an intersection controlled by a fixed-time signal has been solved by Greenshields, and in fact consists merely in substituting the appropriate values in formula (1). The present problem is more difficult because the blocks, instead of being all of the same size, have the distribution of sizes represented by formula (19).

$$
\left.\begin{array}{ll}
0 & \text{if } t < L \\
e^{-N_2 L} \dfrac{(N_2 L)^n}{n!} Ae^{-2NL} & \text{if } t = L \\
e^{-N_2 t} \dfrac{(N_2 t)^n}{n!} AFe^{-NL} (1 - e^{-NL})\, e^{-F(t-L)}\, dt \text{ in a range} & \\
\qquad\qquad\qquad \text{of width } dt, \quad \text{if } t > L. &
\end{array}\right\} \quad (26)
$$

To get the total number of piles of size n in the lane, we must sum (26) over the whole range of t, i.e.,

$$
\begin{aligned}
\alpha_n &= Ae^{-2NL} e^{-N_2 L} \frac{(N_2 L)^n}{n!} \\
&\quad + AFe^{-NL} (1 - e^{-NL})\, e^{FL} \frac{N_2{}^n}{n!} \int_L^{\infty} t^n e^{-(N_2 + F)t} dt \\
&= Ae^{-2NL} e^{-N_2 L} \frac{(N_2 L)^n}{n!} + Ae^{-NL} (1 - e^{-NL})\, e^{-N_2 L} \times \\
&\quad \frac{F}{N_2 + F} \left(\frac{N_2}{N_2 + F} \right)^n \sum_{k=0}^{n} \frac{[(N_2 + F)L]^k}{k!}.
\end{aligned} \quad (27)
$$

It can be proved that

$$
\sum_{n=0}^{\infty} \alpha_n = Ae^{-NL}, \qquad (28)
$$

which simply means that the total number of *piles* in the lane, of all sizes from zero on up, is equal to the number of blocks. It can also be proved that

$$
\sum_{n=0}^{\infty} n\,\alpha_n = A \frac{N_2}{N} (1 - e^{-NL}), \qquad (29)
$$

which means that the total number of *cars* in all the piles in any lane is equal to the number of side-street cars which arrive during blocks in that lane.

Values of expression (27) have been computed for a range of volumes on both streets, and are listed in Table VI.

TABLE VI

Theoretical Number of Piles Per Hour, for a Critical Lag of 6.0 Seconds

Main Street Volume (cars per hr.)	*Side Street Lane Volume (cars per hr.)*	*Size of Pile* 0	1	2	3	4	5
200	0	143	0	0	0	0	0
	100	118	23	2	0	0	0
	200	97	37	8	1	0	0
	300	81	45	14	3	1	0
	400	67	49	19	6	2	0
	500	56	51	24	8	3	1
400	0	205	0	0	0	0	0
	100	163	36	5	1	0	0
	200	131	56	14	3	1	0
	300	106	66	24	7	2	1
	400	87	70	32	11	4	1
	500	71	70	38	16	6	2
600	0	221	0	0	0	0	0
	100	168	44	8	1	0	0
	200	131	64	19	5	1	0
	300	103	71	30	11	4	1
	400	82	73	38	16	7	3
	500	66	70	43	21	10	5
800	0	211	0	0	0	0	0
	100	152	46	10	2	0	0
	200	114	63	23	8	2	1
	300	88	67	32	14	6	2
	400	69	66	38	19	10	5
	500	54	62	42	24	13	7
1000	0	189	0	0	0	0	0
	100	128	45	12	4	1	0
	200	92	57	24	10	4	2
	300	69	58	32	16	8	4
	400	53	54	35	19	10	6
	500	41	50	37	24	14	9

Position Distribution

The number of nth-position cars in one lane can be obtained from expression (27), since the number of nth-position cars in blocks is equal to the number of piles of size n or greater. This can also be looked at as the total number of blocks minus the number of blocks containing fewer than n cars in the lane under consideration. Let us use the symbol β_n for the number of nth-position cars in blocks in a lane having a volume N_2.

$$\beta_n = Ae^{-NL} - \sum_{m=0}^{n-1} \alpha_m$$

$$= Ae^{-NL} - Ae^{-2NL} e^{-N_2L} \sum_{m=0}^{n-1} \frac{N_2^m L^m}{m!} \tag{30}$$

$$- \frac{AFe^{-NL}(1-e^{-NL})e^{-N_2L}}{N_2+F} \sum_{m=0}^{n-1} \sum_{k=0}^{m} \frac{N_2^m}{(N_2+F)^m} \frac{(N_2+F)^k L^k}{k!}$$

Consider the double summation in the last term. As the following steps show, the double summation can be transformed into a single summation on k. If we define a new summation index $i = n - m$,

$$\sum_{m=0}^{n-1} \sum_{k=0}^{m} = \sum_{i=1}^{n} \sum_{k=0}^{n-i} = \sum_{k=0}^{n-1} \sum_{i=1}^{n-k},$$

$$\sum_{m=0}^{n-1} \sum_{k=0}^{m} \frac{N_2^m (N_2+F)^{k-m} L^k}{k!} = \sum_{k=0}^{n-1} \sum_{i=1}^{n-k} \frac{N_2^{n-i}(N_2+F)^{k-n+i} L^k}{k!}$$

$$= \frac{N_2^n}{(N_2+F)^n} \sum_{k=0}^{n-1} \frac{(N_2+F)^k L^k}{k!} \sum_{i=1}^{n-k} \frac{(N_2+F)^i}{N_2^i}$$

$$= \frac{N_2^n}{(N_2+F)^n} \sum_{k=0}^{n-1} \frac{(N_2+F)^k L^k}{k!} \left\{ \frac{N_2+F}{F} \left[\frac{(N_2+F)^{n-k}}{N_2^{n-k}} - 1 \right] \right\}$$

$$= \frac{N_2+F}{F} \sum_{k=0}^{n-1} \frac{N_2^k L^k}{k!} - \frac{N_2^n}{(N_2+F)^{n-1} F} \sum_{k=0}^{n-1} \frac{(N_2+F)^k L^k}{k!}.$$

Putting this into (30), we get

$$\beta_n = Ae^{-NL} - Ae^{-2NL} e^{-N_2 L} \sum_{k=0}^{n-1} \frac{N_2^k L^k}{k!}$$

$$- Ae^{-NL}(1-e^{-NL}) e^{-N_2 L} \sum_{k=0}^{n-1} \frac{N_2^k L^k}{k!}$$

$$+ Ae^{-NL}(1-e^{-NL}) e^{-N_2 L} \left(\frac{N_2}{N_2+F} \right)^n \sum_{k=0}^{n-1} \frac{(N_2+F)^k L^k}{k!}$$

$$= Ae^{-NL} - Ae^{-NL} e^{-N_2 L} \sum_{k=0}^{n-1} \frac{(N_2 L)^k}{k!}$$

$$+ Ae^{-NL}(1-e^{-NL}) e^{-N_2 L} \left(\frac{N_2}{N_2+F} \right)^n \sum_{k=0}^{n-1} \frac{[(N_2+F)L]^k}{k!}.$$

This is the number of nth-position cars *in blocks* in the lane being considered. The total number of nth-position cars, in blocks and antiblocks both, is the same as β_n except when $n = 1$; for under the assumption of instantaneous clearing, all cars arriving in antiblocks are first-position cars.

Computations of the numbers of cars in various positions have been made for several combinations of volumes and are listed in

Table VII. In comparison with actual observations, this theory assigns too many cars to the low positions, as Figure 27 demonstrates. In this graph the percentages of first-position cars actually observed at Intersection A — where the side street carries only a single lane of one-way traffic — are indicated by the points, while the curve shows the values to be expected from the theory which has been developed.

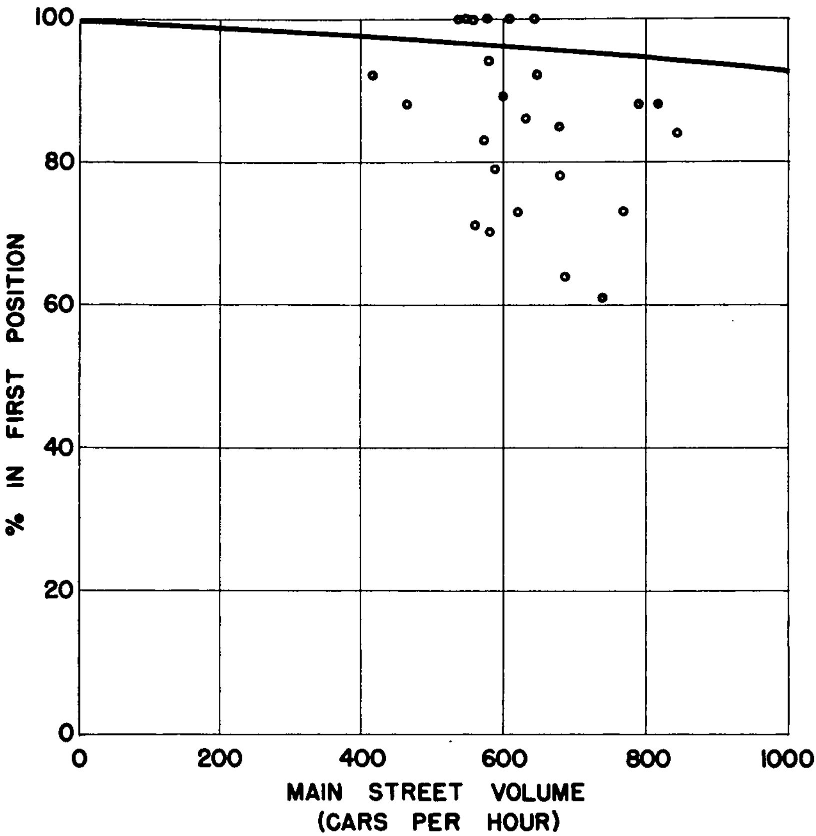

FIGURE 27. Percentage of Side-Street Cars in First Position, Theory vs. Observation. Each Point Represents One Fifteen-Minute Period of Observation at Intersection A. The Range of Side-Street Volumes Is from 0 to 99 Cars per Hour. The Curve is Taken from Formula (31) of the Text.

TABLE VII

THEORETICAL NUMBER OF CARS PER HOUR IN EACH POSITION, FOR A CRITICAL LAG OF 6.0 SECONDS

Main Street Volume (Cars per hour)	*Side Street Lane Volume (Cars per hour)*	*Position of Cars* 1	2	3	4	5
0	100	100	0	0	0	0
200		97	2	0	0	0
400		93	6	1	0	0
600		90	9	1	0	0
800		84	12	2	0	0
1000		80	15	4	0	0
0	200	200	0	0	0	0
200		189	9	2	0	0
400		177	18	4	1	0
600		164	26	7	2	0
800		150	34	11	4	1
1000		135	40	16	6	2
0	300	300	0	0	0	0
200		278	18	4	1	0
400		253	33	10	3	1
600		228	46	16	6	2
800		202	56	24	10	4
1000		177	63	31	15	7
0	400	400	0	0	0	0
200		363	27	8	2	1
400		324	49	18	6	2
600		285	66	28	12	5
800		247	77	38	19	9
1000		212	82	46	28	18
0	500	500	0	0	0	0
200		445	37	12	4	1
400		391	65	27	11	4
600		338	84	41	19	9
800		289	95	53	29	16
1000		242	98	61	38	23

Removing Assumption of Instantaneous Clearing

Unfortunately, it is difficult to remove this assumption of instantaneous clearing, not only because the sluggish starting of a pile increases the effective length of each block but also because these increases cause blocks to coalesce with each other. For example, if three cars accumulate during a certain block, and the antiblock which follows it is only long enough for two of the cars to clear, then the third car is held over during the next block as well. From the point of view of this third car, the two blocks and the antiblock between them have coalesced into one long block, and there might just as well have not been any antiblock. The idea which this example illustrates is that without the assumption of instantaneous clearing, the pattern of blocks and antiblocks is not the same for all cars. The complexity of the situation is apparent.

The attempt to take sluggish starting into account by assuming that each stopped car adds a certain length of time to the time in blocks[17] has proved fruitless, in the sense that the curves which result from this process come no closer to describing actual performance than do the curves which are based on instantaneous clearing.

Time Involved in Delays

The theory based on instantaneous clearing can be used to derive a formula for the *amount of time* involved in delays. The total wait of blocked cars is the integral, over the whole range of t, of t times the number of cars which arrive during a block at a time which is t before the end of the block. For example, every car which arrives during a block three seconds before the end of the block has a wait of three seconds.

If Ns is the volume on the side street, the number of cars which arrive during blocks between t and $t + dt$ before the end of the

[17] Greenshields did this, with fair success, in his discussion of the number of cars retarded by a red signal. *Op cit.*, pp. 95-97. His success was possible because the likelihood of coalescence—signal failure, in his terminology—is sufficiently small with a fixed-time signal that it can be ignored a good part of the time.

block in which they arrive is Nsdt times the number of blocks greater than t, which is

$$
\begin{aligned}
&AN_se^{-NL}\,dt && \text{if } t < L, \text{ or} \\
&AN_se^{-NL}(1-e^{-NL})\,e^{-F(t-L)}dt && \text{if } t \geqslant L.
\end{aligned}
\qquad (32)
$$

The total wait, therefore, is

$$
AN_se^{-NL} \int_0^L tdt + AN_se^{-NL}(1-e^{-NL})\,e^{FL} \int_L^\infty te^{-Ft}\,dt
$$

$$
= AN_se^{-NL} \left\{ \tfrac{1}{2}L^2 + \frac{1}{F^2}(1-e^{-NL})(1+FL) \right\}. \quad (33)
$$

Since the total number of side-street cars is ANs/N, the average wait per side-street car — all cars, not just the blocked ones — is

$$
Ne^{-NL} \left\{ \tfrac{1}{2}L^2 + \frac{1}{F^2}(1-e^{-NL})(1+FL) \right\}. \qquad (34)
$$

Two observations are in order concerning expression (34). In the first place, the theoretical average wait is independent of the side-street volume; this is a consequence of assuming instantaneous clearing. In the second place, Adams[18] gives a formula for the same thing, and the comparison of expression (34) with Adams's formula is interesting. Altogether different in mathematical form, the two formulas give similar numerical values, as Table VIII illustrates.

It is also of interest to compare the theoretical average wait with the observed values, which were plotted in Figs. 22-25. It will be recalled that there was considerable scatter in these observed values, in fact so much scatter that it was impossible to develop an empirical formula. Nevertheless, a curve drawn from formula (34) has been superimposed on the points representing the average wait of all cars at Intersection C (the dashed line in Fig. 28).

---

[18] *Op. cit.*, p. 127. In the mathematical symbols of the present report, Adams's formula for the average wait is $(1 - e^{-NL})/F$. The derivation of this formula is not explained in Adam's article.

Clearly this theoretical curve is too low, i.e., it underestimates the actual waits. This was to be expected, since the effect of sluggish starting is to increase the waits of all cars which are affected by it.

TABLE VIII

THEORETICAL AVERAGE WAIT OF ALL CARS, FOR A CRITICAL LAG OF 6.0 SECONDS

Main Street Volume	*Average Wait of All Cars (seconds)*	
(cars per hour)	*Authors Formula*	*Adams Formula*
0	0	0
200	1.16	1.12
400	2.64	2.53
600	4.49	4.31
800	6.80	6.57
1000	9.71	9.46

An attempt has been made to take the sluggish starting into account in computing theoretical average waits, in the following way. It was observed that cars which were actual first-position cars,

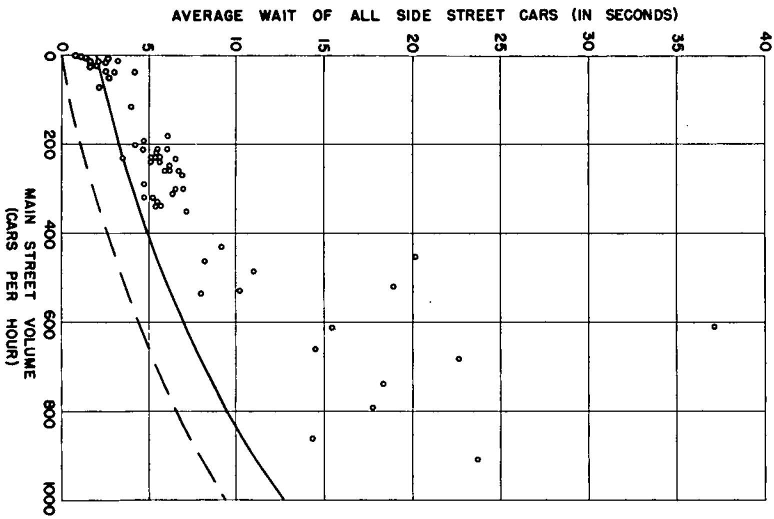

FIGURE 28. Average Wait of All Cars, Theory vs. Observation. Each Point Represents One Fifteen-Minute Period of Observation at Intersection C. The Dashed Curve is Taken from the Crude Theory and the Solid Curve from a More Refined Theory, as Described in the Text.

and which were not blocked by through traffic, took about two seconds to start moving, on the average. In other words, the first-position cars actually waited about two seconds longer than the time suggested by the theory. Using this idea, one can modify expression (33) by adding to it two seconds for each first-position car (the number predicted by the theory, that is), four seconds for each second-position car, six seconds for each third-position car, and so on. This procedure gives an expression[19] for the average wait, which has been plotted as the solid line in Fig. 28. This fits the observed points much better than expression (34), but it is still too low at the higher volumes. The most likely explanation of the discrepancy is that the solid line still leaves out of account the coalescence of blocks, which was referred to in the previous section.

THE UNSOLVED PROBLEM

This, then, is the big unsolved problem: how to deal mathematically with the situation which arises when a pile fails to clear during a single antiblock. The essential mathematical difficulty lies in the discontinuity which is involved, that is, in the fact that a small difference in the length of an anti-block can cause a large difference in the number of cars delayed and in the total delay time.

To illustrate this by a concrete example, suppose that six seconds is the smallest antiblock which can clear a pile of size three. If the antiblock is made a little longer than six seconds, there will be a small decrease in the number of cars delayed and in the average wait, and these decreases will approach zero as the change in the antiblock size approaches zero. This is mathematical continuity.

Suppose, on the other hand, that the antiblock is made a little *shorter* than six seconds: in this case there is an abrupt increase in the number of cars delayed and an abrupt increase in the average wait (since the wait of the third car is increased by the length of the next block, which must be at least equal to L), no matter how little the antiblock has been shortened. This is mathematical dis-

[19] Because it is rather complicated, the expression has not been included here.

continuity, for an infinitesimal change in one variable, the length of the antiblock, causes a finite change in the variables which are dependent on it.

The mathematical complexities arising out of this discontinuity are enormous. When they are surmounted, it will become possible to make a highly important contribution to the theory of traffic behavior. For the present, the authors have had to content themselves with the assumption of instantaneous clearing, which avoids these difficulties.

CHAPTER V

APPLICATIONS OF THE VOLUME-WARRANT

The method of using the volume-warrant for Stop signs will be illustrated in this chapter by several examples. With these examples and the detailed description of the procedure given here, the traffic engineer should have no difficulty in applying the warrant to the particular intersections in which he is interested.

WARRANT CRITERION

The warrant criterion which has been used is the following: *a Stop sign is warranted if an average day contains eight or more hours during which the volumes are such as to delay at least fifty per-cent of the side-street cars.* When the warrant is stated in this form, its use requires hourly volume counts on both streets for a substantial part of the day. A simplified — and less accurate — form of the warrant, which uses shorter counts, is given in a later section of this chapter.

STEPS IN APPLYING THE WARRANT

There are four steps in applying this warrant to a particular intersection: (1) it must be decided which street is to be the main street and which is to be the side street; (2) the critical lag at the intersection must be determined; (3) the volume counts must be made; and (4) the figures must be applied to the appropriate warrant graph. The four steps are discussed in detail in the sections which follow.

Choice of Main and Side Streets

No hard-and-fast rule can be laid down for deciding which street should be treated as the main street, but the following criteria are suggested. They are listed in order of decreasing importance.

1. *Volumes.* If the hourly traffic volume is substantially greater for most of the day on one of the two streets, the main street should be the one with the larger volume.

2. *Street Widths.* All other things being equal, the preference should go to the wider street.
3. *Sight Conditions.* If the sight conditions are such that the approach to the intersection is noticeably more blind for traffic on street A than for traffic on street B, the main street should be street B.
4. *Location of Other Stop Signs.* When there are Stop signs already installed at nearby intersections, their location will sometimes be helpful in choosing which of the two streets should get the preference. If one of the two streets is preferred at several other intersections near the one under consideration, it should probably be made the main street.
5. *Warrant Curves.* In cases where none of the above considerations are of much help, it may be necessary to use the warrant curves twice, trying first one street and then the other as the main street. It will sometimes happen that Stop signs will be warranted with one arrangement, while they are not warranted with the other. This situation is illustrated in Example III.

Determination of the Critical Lag

A driver who approaches the intersection on the side street may proceed immediately into the intersection, or he may wait for one or more main-street cars to go by. Which choice he will make depends principally on how far away (*in time,* rather than in distance) the nearest main-street car is from the intersection when the side-street driver reaches the intersection. If the nearest main-street car is only two seconds away, he will almost certainly decide to wait; in technical language, one would say that he has faced a *lag* of two seconds and has *rejected* it.

On the other hand, if the interval from the arrival of the side-street car until the arrival of the next main-street car were fifteen seconds, the side-street driver would in all probability *accept* this lag, that is, he would enter the intersection ahead of the main-street car. The *critical lag* is the interval which is just barely acceptable to the average driver. Since it does not have the same value at all intersections, it has to be determined at each intersection where the volume-warrant for Stop signs is to be used.

The field observations needed in determining the critical lag can easily be made by a single observer with a stopwatch, and can be made, if desired, at the same time the volumes are being counted.

It is necessary to measure the lags of a representative group of side-street cars — at least 200 cars should be used — and to note for each lag whether it is accepted or rejected. In measuring a lag, the stopwatch should be started when the side-street car stops (or reaches his slowest speed). The watch is to be stopped when the next *main*-street car enters the intersection. Side-street cars which arrive behind other stopped cars or which fail to slow down appreciably should not be used for these lag measurements.

After these measurements of accepted and rejected lags are made, a table like Table IX should be prepared. The range of lengths, the number of accepted lags in that range, and the number of rejected lags in that range are listed in the first three columns. The fourth column shows the number of accepted lags which are in that range of lengths *or shorter,* while the fifth column shows the number of rejected lags which are in that range of lengths *or longer.*

TABLE IX

ACCEPTED AND REJECTED LAGS AT INTERSECTION D

Length of Lag (seconds)	*Number Accepted*	*Number Rejected*	*Cumulative Number Accepted*	*Cumulative Number Rejected*
0 — 0.9	0	131	0	465
1 — 1.9	2	97	2	334
2 — 2.9	8	67	10	237
3 — 3.9	12	56	22	170
4 — 4.9	12	34	34	114
5 — 5.9	14	28	48	80
6 — 6.9	22	14	70	52
7 — 7.9	23	14	93	38
8 — 8.9	19	9	112	24
9 — 9.9	18	7	130	15
10 — 10.9	12	2	142	8
11 — 11.9	18	4	160	6
12 — 12.9	8	0	168	2
13 — 13.9	7	2	175	2
14 — 14.9	5	0	180	0
Over 15	36	0	216	0

Finally, these last two columns are plotted on a single graph, as shown in Figure 29. The critical lag is the value of time for which the two curves intersect. It is denoted by the letter L in Figure 29 and in Chapters III and IV.

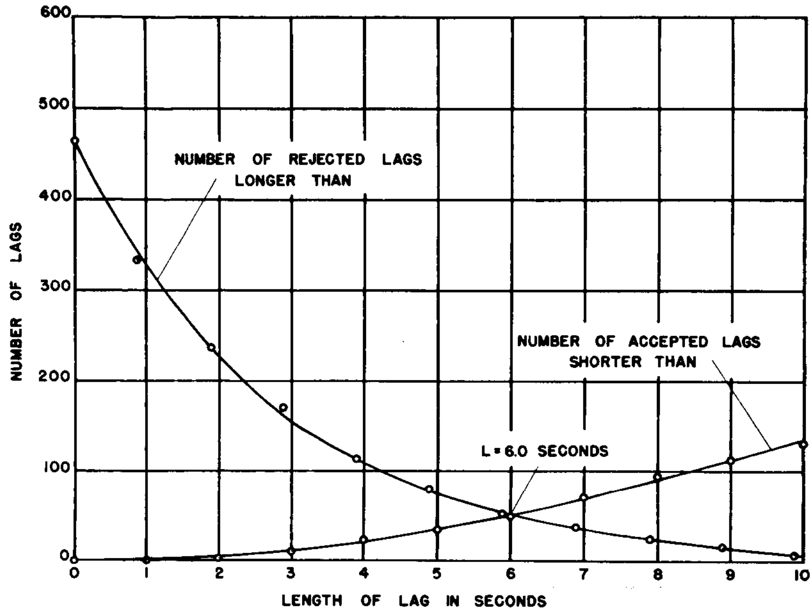

FIGURE 29. Cumulative Distribution of Accepted and Rejected Lags at Intersection D.

Volume Counts and Warrant Graphs

Hourly volumes on the two streets should be counted for a long enough period to be sure of getting the eight busiest hours of the day. A normal weekday should be used. The volume figures for each street should include all cars approaching the intersection from both directions on that street.

The final step is to apply the volume figures to the appropriate warrant graph. Five such graphs are given (Figures 30-34), for critical lags of 4.6, 4.9, 5.2, 5.5, and 5.9 seconds respectively.

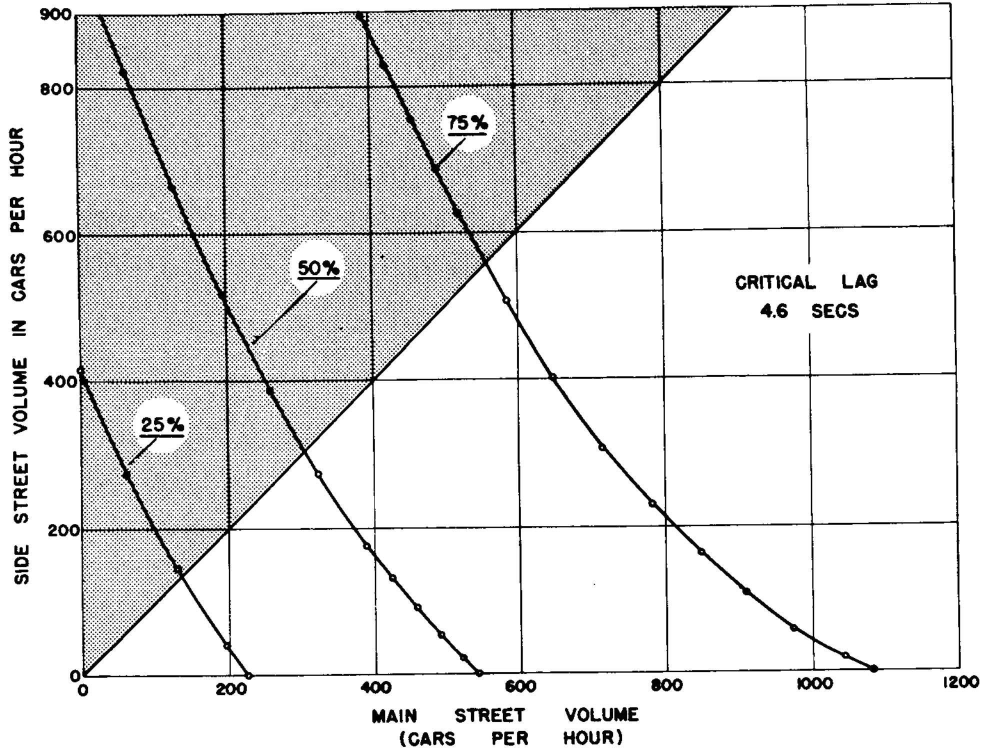

FIGURE 30. Warrant Graph, for a Critical Lag of 4.6 Seconds. The Curves Show the Volumes for which 25%, 50%, and 75% of the Side-Street Cars are Delayed. The Shading Indicates the Portion of the Graph in which the Side-Street Volume Exceeds the Main-Street Volume.

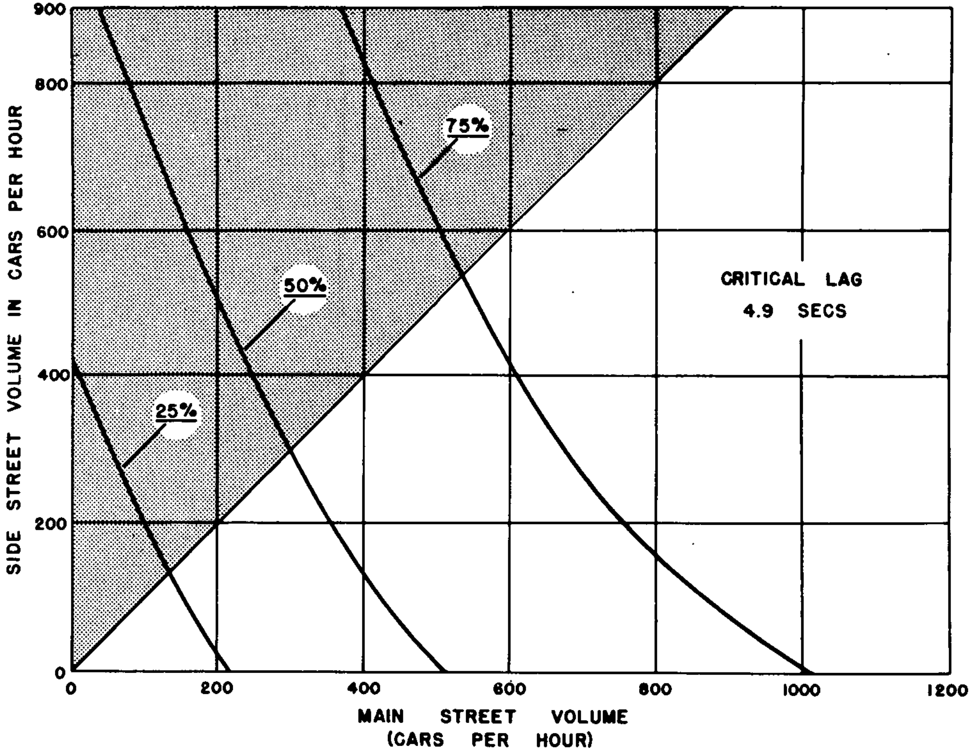

FIGURE 31. Warrant Graph, for a Critical Lag of 4.9 Seconds.

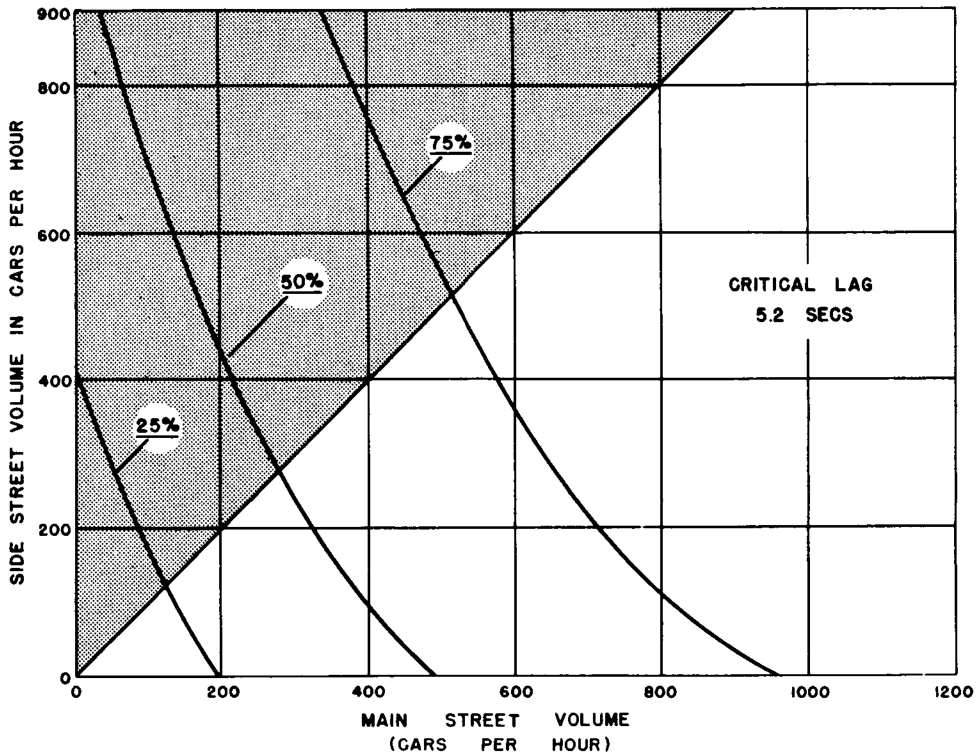

FIGURE 32. Warrant Graph, for a Critical Lag of 5.2 Seconds.

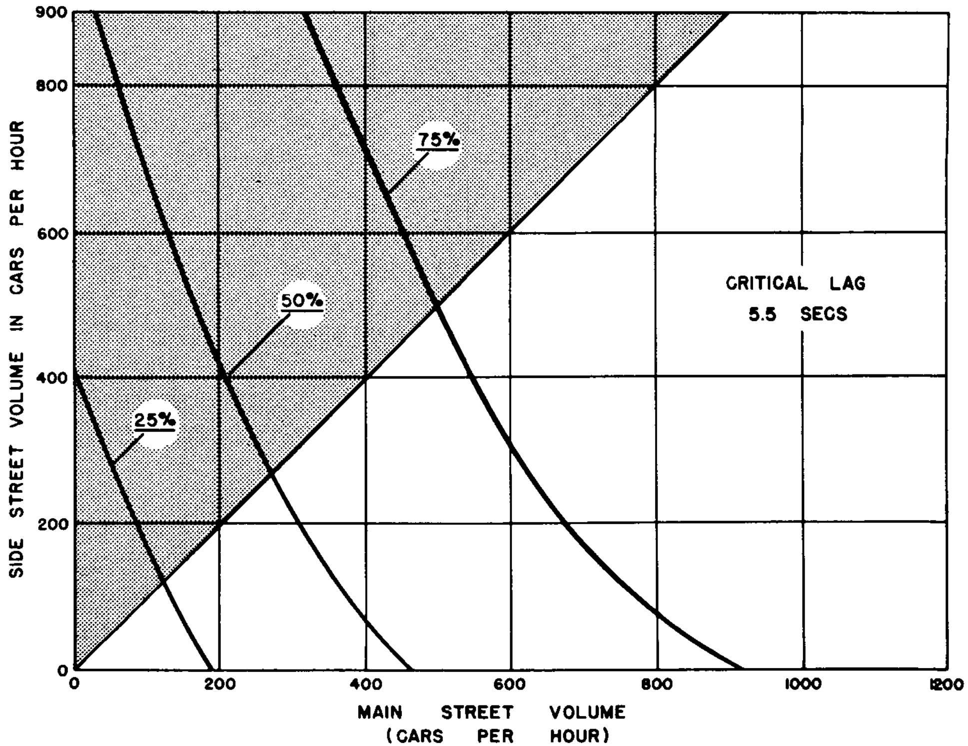

FIGURE 33. Warrant Graph, for a Critical Lag of 5.5 Seconds.

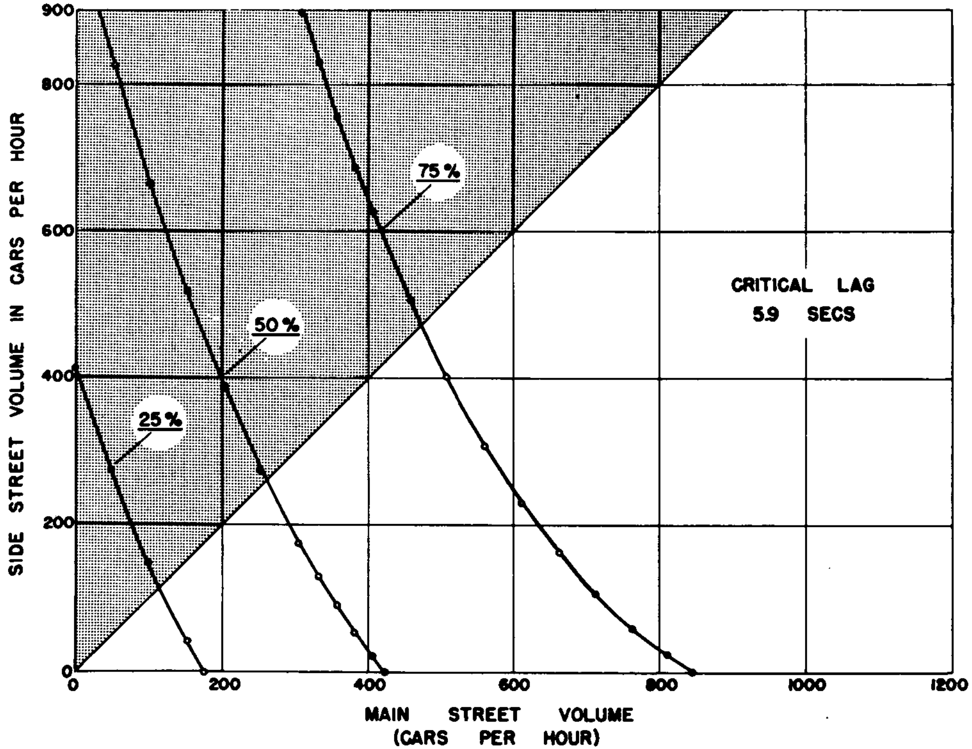

FIGURE 34. Warrant Graph, for a Critical Lag of 5.9 Seconds.

In each warrant graph the main-street volume (in cars per hour) is plotted along the horizontal axis, while the side-street volume is plotted along the vertical axis. The three curves on each graph show the volumes for which 25 per cent, 50 per cent, and 75 per cent of the side-street cars can expect to be delayed. For a Stop sign to be warranted under the volume warrant, there must be at least eight hours during the day for which the volumes, when plotted on the proper warrant graph, fall to the right of the 50 per cent curve. It is immaterial whether or not the points are in the shaded portion of the graph, because the shading has no connection with the percentage of delayed cars.

It will be seen from the warrant graphs that the percentage of delayed side-street cars is large when both the main- and side-street volumes are large. Similarly, the per cent delayed is small when both volume figures are small. It will also be noted that Stop signs may sometimes be warranted when the side-street volume is very low, provided the main-street volume is large enough. For example, a volume of 600 cars per hour on the main street will warrant Stop signs *even if there are as few as 2 cars per hour on the side street.*

EXAMPLES

The four examples that follow are taken from the intersections which were studied for this report. They are identified and described in detail in Chapter II.

Example I: Intersection A. At Intersection A, the critical lag was found to be 4.6 seconds, and the hourly traffic volumes were as follows:

Hour	*Main-St. Volume (cars per hour)*	*Side-St. Volume (cars per hour)*	*More than 50% Delayed?*
10 — 11 A.M.	590	20	Yes
11 — 12 NOON	640	210	Yes
12 — 1 P.M.	630	70	Yes
1 — 2	580	100	Yes
2 — 3	660	120	Yes
3 — 4	810	120	Yes
4 — 5	820	70	Yes
5 — 6	570	60	Yes

It is clear that Stop signs are warranted at this intersection, since all eight of the hours from 10 A.M. to 6 P.M. give "yes" answers in the last column. These "yes" answers mean that the points fell to the right of the 50 per cent curve in Figure 30.

Example II: *Intersection B.* The critical lag at Intersection B is 4.7 seconds, which is closer to 4.6 than to 4.9, so Figure 30 is used again for this intersection. The hourly volumes were as follows:

Hour	*Main-St. Volume (cars per hour)*	*Side-St. Volume (cars per hour)*	*More than 50% Delayed?*
10 — 11 A.M.	580	160	*Yes*
11 — 12 NOON	590	160	Yes
12 — 1 P.M.	600	130	Yes
1 — 2	600	140	Yes
2 — 3	600	150	Yes
3 — 4	790	170	Yes
4 — 5	810	140	Yes
5 — 6	580	130	Yes

Here again the Stop signs are warranted, because there are eight "yes" answers in the last column.

In both of the preceding examples, there were eight consecutive hours during which the volumes were sufficiently high to give "yes" answers. For this reason, it was not necessary to count volumes for a longer part of the day. If hourly counts for the entire day had been used, there would very likely have been twelve or more "yes" answers, indicating additional need for the signs.

Example III: *Intersection C.* This example differs from the preceding ones in two respects. In the first place, it is by no means obvious which street should be the main street, since the average volumes are about the same on the two streets. In addition, with many of the hours having volumes which are close to the 50 per cent curve, the graph needs to be used with care.

The critical lag at this intersection was found to be 5.9 seconds, on the assumption that Orange Street was to be the main street. Figure 34, therefore, is the proper warrant graph to use for this example.

Hour	*Orange Street Volume (cars per hr.)*	*Willow Street Volume (cars per hr.)*	*—More than 50% delayed?— (Orange as Main Street)*	*(Willow as Main Street)*
6 — 7 A.M.	40	170	No	No
7 — 8	120	350	No	Yes
8 — 9	360	260	Yes	Yes
9 — 10	230	240	No	No
10 — 11	210	200	No	No
11 — 12 NOON	220	240	No	No
12 — 1 P.M.	200	180	No	No
1 — 2	300	220	Yes	No
2 — 3	300	240	Yes	No
3 — 4	260	340	Yes	Yes
4 — 5	740	460	Yes	Yes
5 — 6	460	260	Yes	Yes
6 — 7	260	280	Yes	Yes
7 — 8	320	190	Yes	No
8 — 9	230	280	No	Yes
9 — 10	240	220	No	No

With Orange Street considered as the main street, there are a total of eight "yes" answers in the entire day, which is the minimum for a Stop sign. Stop signs on Willow Street, therefore, *are* warranted.

If, on the other hand, we think of Willow Street as the main street, we find that Stop signs on Orange Street *are not* warranted, because we get only seven "yes" answers this way. This example illustrates how the warrant curves themselves can sometimes be used to decide which street should become the main street, in the absence of other decisive considerations.

Example IV: Intersection D. Intersection D has a critical lag of 6.0 seconds, so the graph in Figure 34 is the one to use. The following hourly volumes were found:

Hour	*Main-St. Volume (cars per hour)*	*Side-St. Volume (cars per hour)*	*More than 50% Delayed?*
8 — 9 A.M.	980	100	Yes
9 — 10	820	50	Yes
10 — 11	710	60	Yes
11 — 12 NOON	740	60	Yes
12 — 1 P.M.	700	70	Yes
1 — 2	830	80	Yes
2 — 3	850	70	Yes
3 — 4	850	70	Yes
4 — 5	970	90	Yes
5 — 6	1290	100	Yes

Clearly, Stop signs are warranted at this intersection.

SHORT-COUNT WARRANT

Some traffic engineers may feel that they cannot afford to take hourly counts all day long at each intersection where Stop signs are being considered. For their benefit, a simplified version of the same warrant is given, which permits as little as a single one-hour count during the middle of the day to be used in applying the volume warrant for Stop signs.

A warrant based on a one-hour count is not, of course, very satisfactory. When a sign is up for twenty-four hours every day, there ought to be a real need for the sign during a substantial part of the twenty-four hours; otherwise, it would be better to have an officer direct traffic during the short periods when traffic control is required. Nevertheless, a warrant based on a short count is better than no warrant at all, and the short-count warrant is given here for that reason.

In developing a warrant based on a one-hour count, it is necessary to assume that hourly variations in traffic-volume during the day follow a typical pattern. It has been found that the volume exceeded during eight and *only* eight hours of the day is often about the same as the average volume during the period between the morning and afternoon peaks. Therefore a typical hour taken from the mid-day period — or even better, the average of several such hours — may be used for obtaining the volume figures which are applied to the warrant graphs of Figures 30-34.

The short-count warrant may therefore be stated as follows: *a Stop sign is warranted if a typical hour during the mid-day period of a typical day shows volumes which can be expected to delay at least fifty per cent of the side-street cars.*

The short-count warrant is used in almost exactly the same way as the long-count method. First the main street is decided upon; then the critical lag is determined; then a volume count of one or more hours is made in the proper part of the day; and finally the main- and side-street volume figures are applied to whichever warrant graph has its critical lag closest to the observed value. If the point representing the volumes is to the right of the 50 per cent curve, then Stop signs are warranted; otherwise, they are not.

It should be emphasized that the only saving in this method is in the duration of the volume counts. There is no short-cut method of determining the critical lag.

Applying the short-count warrant to the examples which have been discussed, we would pick out one typical hour (say, 2-3 P.M.) and base our conclusions on the entries in the last column for that hour alone. A "yes" answer means the signs are warranted; a "no" answer means they are not. It will be seen that this procedure gives the right answers for all four examples. This is partly the result of a lucky choice of the hour to use, however. If the hour 10-11 A.M. had been used instead, the conclusion about Example III would have been erroneous.

Ratio of Volumes

The question has frequently been raised whether it is possible to avoid the procedure of determining critical lags and using the warrant graphs by considering the ratio of the volumes on the two streets. The answer is no, because the ratio of volumes has no connection with the percentage of cars which are delayed. If there are 1000 cars per hour on the main street and 100 cars per hour on the side street, the per cent delayed will exceed 75 for any reasonable critical lag. On the other hand, volumes of 100 on the main street and 10 on the side street — the same ten-to-one ratio — will delay less than 25 per cent.

Importance of Critical Lag

The importance of using the correct value of the critical lag can best be demonstrated by considering the effect of using incorrect values in the four examples. In Examples I, II, and IV, the volumes are so far from the critical range of values that the results would have been the same no matter which of the warrant graphs had been used. In Example III, however, which is a "close case," the critical lag makes a significant difference. If the incorrect value of 4.6 seconds had been used instead of the correct 5.9, the number of "yes" answers in the last two columns would have been three in each column, instead of seven and eight respectively.

CHAPTER VI

SUMMARY AND CONCLUSIONS

The purpose of this research has been to develop a volume-warrant for installing two-way Stop signs at urban intersections. No warrant of this type is suggested in the standard reference works on traffic engineering, and it is hoped that the present report will meet a real need. It is only by means of warrants based on factual studies that the haphazard use of Stop signs can be brought to an end.

The volume warrant has been based on the percentage of side-street cars that meet interference from traffic on the main street. In applying the warrant to a specific intersection, only three quantities need to be measured: (1) the main-street volume, (2) the side-street volume, and (3) a quantity called the critical lag, which is a constant for each particular intersection but varies from one intersection to another.

The recommended warrant is based on the principle that a Stop sign is needed if the percentage of side-street traffic which can be expected to be delayed exceeds fifty per cent. The per cent delayed is computed from the following formula:

$$P = 100 \left\{ 1 - \frac{e^{-2.5N_s}\, e^{-2NL}}{1 - e^{-2.5N_s}(1 - e^{-NL})} \right\},$$

where P = percentage of side-street cars delayed,
e = base of natural logarithms (about 2.718),
N_s = side-street volume, in cars per second,
N = main-street volume, in cars per second, and
L = critical lag, in seconds.

The application of the mathematical theory of probability to the movements of traffic has been extended beyond the results achieved in previous work. In particular, a start has been made toward a theoretical explanation of the interaction between two intersecting streams of traffic, when both streams are distributed at random.

In addition to the development of the volume warrant, which is discussed in detail in Chapters III and V, the principal conclusions from this study are briefly summarized as follows:

1. The critical lag has been found to vary from one intersection to another, but there is a need for additional research to discover what other intersections characteristics are most closely related to the critical lag.
2. The average length of delays for side-street cars does not correlate well with traffic volumes, and is therefore not a good basis for a sign warrant.
3. The usefulness of the mathematical theory has been extended, but there remains the problem of taking adequate account of the sluggish starting of a line of stopped cars.

GLOSSARY

Some of the less familiar technical terms used in this report are defined as follows:

Lag. A lag is the interval from the time a side-street car reaches the intersection (or the head of the line, if there is a line of waiting cars) until the passage of the next main-street car. A lag is said to be *accepted* if the side-street car enters the intersection before the main-street car; it is said to be *rejected* if the side-street car waits until the main-street car has passed.

Critical Lag. At a particular intersection, the critical lag is the value of t for which the total number of accepted lags shorter than t is equal to the total number of rejected lags longer than t.

Gap. A gap is the time interval between the passage of two successive main-street cars.

Block. A block is a period of time during which it is *impossible* for a typical side-street car to cross the main street. When a side-street car reaches the intersection during a block, its lag is shorter than the critical lag.

Antiblock. An antiblock is a period of time during which it is *possible* for a typical side-street car to cross the main street. When a side-street car reaches the intersection during an antiblock, its lag is longer than the critical lag.

Pile. A pile of size n is an accumulation of n side-street cars in one lane before the first car is able to enter the intersection.

Position. An nth-position car is a side-street car which reaches the intersection at a time when (n—1) cars are waiting in line ahead of it.

Delayed Car. A delayed car is a side-street car which (1) has a lag shorter than the critical lag, or (2) is not a first-position car.

Wait. The wait of a side-street car is the interval from the time it reaches the intersection (or the end of the line, if there is a line of waiting cars) until it actually starts to enter the intersection.

The mathematical symbols used in this report are defined as follows:

A The total number of main-street cars in the time period under consideration.

α_n The number of piles of size n in a single lane.

β_n The number of nth-position cars in one lane.

F A shorthand symbol for the expression $\dfrac{Ne^{-NL}(1-e^{-NL})}{1-e^{-NL}-NLe^{-NL}}$

$G(t)$ The density function for the distribution of blocks longer than t. $G(t) = -H'(t)$.

$H(t)$ The number of blocks longer than t, for $t \geq L$.

$$H(t) = \int_t^{\infty} G(t)dt.$$

i An index of summation.

k An index of summation, except in formula (1).

L The critical lag, in seconds.

m An index of summation.

N The main-street volume, in cars per second.

N_s The side-street volume, in cars per second.

N_2 The volume in one lane of the side street, in cars per second.

n The number of cars in a pile.

P The percentage of side-street cars delayed, as expressed in the formula of Chapter III.

t Time, in seconds. This is the variable used in functions of time, such as the number of gaps or blocks of a particular size.

y $t-L$, a time variable used in deriving the formula for the distribution of block sizes.

APPENDIX A

THE GRAPHIC TIME-RECORDER

The Esterline-Angus graphic time-recorder, which was used for the field observations of this study, consists of four basic parts.

(1) A paper chart, 103 feet long and six inches wide, which is driven past the recording pens at a rate of three feet per minute. At this speed, the smallest division of the chart corresponds to one-tenth of a second.

(2) An electric motor, operated by two 6-volt storage batteries, which drives the moving chart. The speed of the motor can be adjusted by means of a rheostat.

(3) Twenty pens, resting in a common inkwell. Each pen is connected mechanically to a separate solenoid in such a way that the pen is moved out of its normal position when its solenoid is energized.

(4) A set of telegraph keys to energize the solenoids. It will be seen from the photograph (Figure 35) that there are only fifteen keys mounted on the board, but this number was more than sufficient for the purposes of the present study. (The alert reader will also notice that only ten of the pens were in place when the picture was taken.)

In collecting the data, the machine was operated by two men sitting in the back seat of a car (see Fig. 1). By working in this way, the observers attracted almost no attention, even from passing pedestrians.

The code for recording traffic behavior was as follows: for the traffic on the main street, one pen was assigned to each direction of flow. Whenever a main-street car entered the intersection, the appropriate key was given one quick tap. For the side-street traffic, a set of from two to five pens were assigned to each approach; several pens were required in order to keep track of the individual cars when they piled up in line waiting to enter the intersection.

With each car, the appropriate key was given one quick tap when the car stopped or reached its slowest speed. When the car started up to enter the intersection, the same key was tapped according to a code (two shorts if it was not delayed by through traffic, one long for a right turn, etc.). In addition to the keys which were used for car movements, one key was used to indicate the time shown by a stopwatch, as a check on the speed of the chart drive.

The accompanying drawing should help to clarify this description of the recording technique. Figure 36 shows a sample section of the chart, in which the legend indicates the role of each pen. The intersection drawing above the chart illustrates the positions of the various cars at the time when the center of the chart passed the bank of pens, that is, at time 12.6.

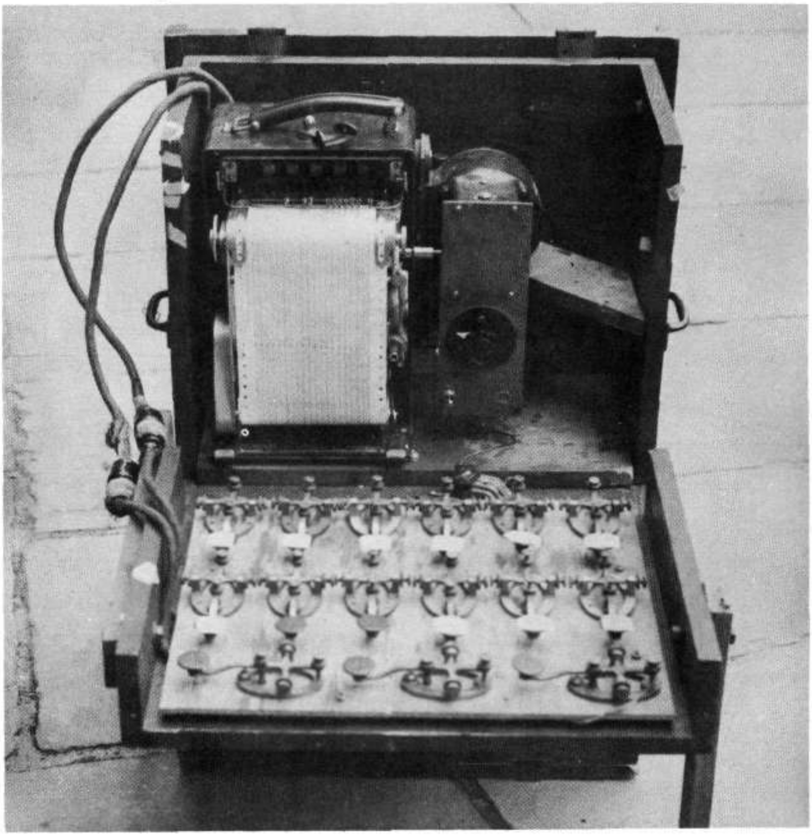

FIGURE 35. The Graphic Time-Recorder (courtesy Bureau of Public Roads).

This is the same sample of chart whose analysis is illustrated in Figure 6 and discussed in Chapter III.

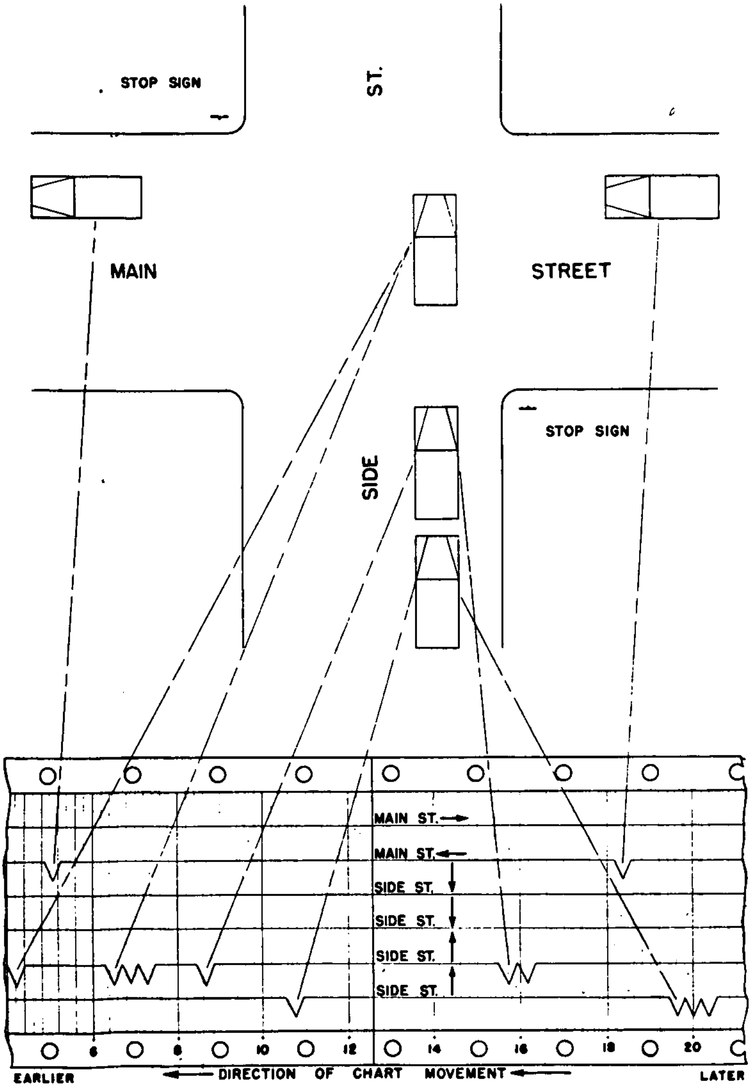

FIGURE 36. Sample Section of Recording Chart, Showing Positions of Vehicles at Intersection.

APPENDIX B

TABLE X

FIELD DATA FROM INTERSECTION A, BY FIFTEEN-MINUTE PERIODS

Side-St. Vol. (cars per hr.)	*Main-St. Vol. (cars per hr.)*	*% in 1st Position*	*% Delayed*	*Ave. Wait of 1st-Pos. Cars (sec.)*	*Ave. Wait of All Cars (sec.)*	*Time of Day*
48	416	92	50	6.2	6.5	2 P.M.
64	464	88	44	7.3	8.3	7 P.M.
44	536	100	55	3.6	3.6	7 P.M.
16	540	100	75	8.6	8.6	11 A.M.
72	552	100	44	6.2	6.2	2 P.M.
68	560	71	47	6.3	9.1	6 P.M.
48	572	83	83	14.8	15.1	6 P.M.
64	576	100	31	4.5	4.5	2 P.M.
80	580	70	75	7.2	7.9	2 P.M.
72	580	94	39	3.9	4.4	2 P.M.
96	588	79	62	10.1	12.5	3 P.M.
72	600	89	56	6.2	6.0	1 P.M.
56	608	100	50	6.6	6.6	1 P.M.
44	620	73	91	9.4	13.7	2 P.M.
28	632	86	71	10.6	11.1	11 A.M.
88	644	100	55	7.8	7.8	3 P.M.
96	648	92	62	6.0	6.5	3 P.M.
80	676	85	70	15.9	15.0	NOON
40	680	78	33	3.3	5.6	1 P.M.
88	688	64	82	7.0	6.8	NOON
72	740	61	72	10.9	15.4	3 P.M.
60	768	73	67	5.1	5.6	2 P.M.
64	792	88	56	13.0	13.5	5 P.M.
96	816	88	71	11.7	11.1	4 P.M.
76	844	84	68	13.9	13.9	5 P.M.
176	528	68	61	5.0	7.5	2 P.M.
188	548	72	70	3.7	5.6	NOON
100	592	92	64	4.8	6.6	2 P.M.
100	616	96	48	6.4	6.9	1 P.M.
112	688	82	68	4.7	5.6	3 P.M.

Side-St. Vol. (cars per hr.)	Main-St. Vol. (cars per hr.)	% in 1st Position	% Delayed	Ave. Wait of 1st-Pos. Cars (sec.)	Ave. Wait of All Cars (sec.)	Time of Day
108	712	78	74	8.2	8.4	3 P.M.
152	812	61	74	14.3	19.8	4 P.M.
260	524	58	72	4.9	7.6	NOON
304	584	71	64	4.1	5.5	2 P.M.
280	636	63	70	6.1	8.3	3 P.M.
320	672	69	71	3.7	5.5	NOON
300	708	53	79	7.3	9.2	NOON

TABLE XI

Field Data From Intersection B, by Fifteen-Minute Periods

Side-St. Vol. (cars per hr.)	Main-St. Vol. (cars per hr.)	% in 1st Position	% Delayed	Ave. Wait of 1st-Pos. Cars (sec.)	Ave. Wait of All Cars (sec.)	Time of Day
124	424	94	48	4.2	4.2	3 P.M.
136	468	82	47	6.2	8.1	NOON
152	476	97	53	4.2	4.0	3 P.M.
132	500	85	55	2.7	3.0	11 A.M.
112	500	93	36	4.3	4.8	NOON
188	524	74	64	6.0	6.7	11 A.M.
144	528	97	53	3.7	3.7	11 A.M.
100	532	88	48	4.6	5.1	1 P.M.
124	540	87	65	4.1	4.5	3 P.M.
140	540	83	66	7.2	7.4	2 P.M.
168	544	81	38	4.3	6.4	3 P.M.
152	548	63	82	5.4	7.2	2 P.M.
136	548	91	59	6.6	7.0	2 P.M.
116	560	83	62	6.2	6.3	2 P.M.
160	584	82	63	4.8	5.4	1 P.M.

Side-St. Vol. (cars per hr.)	*Main-St. Vol. (cars per hr.)*	*% in 1st Position*	*% Delayed*	*Ave. Wait of 1st-Pos. Cars (sec.)*	*Ave. Wait of All Cars (sec.)*	*Time of Day*
208	584	65	71	5.7	7.1	11 A.M.
112	604	86	64	4.4	5.6	1 P.M.
120	604	73	80	6.6	7.8	2 P.M.
140	612	71	60	5.3	8.0	2 P.M.
156	624	95	51	3.4	3.9	2 P.M.
160	632	75	65	5.3	5.7	1 P.M.
188	636	70	64	6.4	6.9	2 P.M.
132	636	70	64	8.0	11.2	11 A.M.
144	636	67	67	8.9	13.2	2 P.M.
120	640	77	70	7.4	8.1	1 P.M.
136	644	71	68	7.3	9.8	2 P.M.
176	648	80	61	6.3	10.1	NOON
156	680	87	72	8.7	9.5	2 P.M.
120	684	77	67	7.1	7.4	3 P.M.
148	688	73	62	4.9	6.2	11 A.M.
180	692	58	78	8.5	10.1	3 P.M.
148	700	49	81	8.4	16.1	3 P.M.
200	736	80	68	5.8	6.7	NOON
152	756	55	79	10.8	15.6	3 P.M.

TABLE XII

Field Data From Intersection C, by Fifteen-Minute Periods

Side-St. Vol. (cars per hr.)	*Main-St. Vol. (cars per hr.)*	*% in 1st Position*	*% Delayed*	*Ave. Wait of 1st-Pos. Cars (sec.)*	*Ave. Wait of All Cars (sec.)*	*Time of Day*
4	4	100	0	1.1	1.1	4 A.M.
4	4	100	0	1.0	1.0	4 A.M.
0	8	100	0	....	....	3 A.M.
16	8	100	0	2.6	2.6	4 A.M.
0	8	100	0	....	....	4 A.M.
24	8	100	0	1.4	1.4	5 A.M.
40	12	100	0	3.2	3.2	5 A.M.
20	16	100	0	1.6	1.6	2 A.M.
20	16	100	0	1.6	1.6	2 A.M.
24	16	100	0	2.5	2.5	5 A.M.
8	20	100	0	1.9	1.9	3 A.M.
36	24	100	0	1.7	1.7	1 A.M.
36	36	100	11	4.2	4.2	1 A.M.
48	36	100	17	2.5	2.5	5 A.M.
52	52	92	23	2.7	2.7	1 A.M.
48	72	100	17	2.2	2.2	1 A.M.
172	40	93	16	2.6	2.9	6 A.M.
184	204	87	31	4.0	4.2	11 A.M.
196	212	84	37	4.3	4.6	11 A.M.
244	216	75	48	4.2	5.4	11 A.M.
240	224	70	52	4.1	5.3	7 P.M.
240	228	82	45	3.0	3.4	11 A.M.
224	240	89	54	4.7	5.1	1 P.M.
228	252	74	47	4.7	6.2	2 P.M.
232	268	71	52	5.5	6.8	2 P.M.
216	292	85	48	4.1	4.7	1 P.M.
248	304	76	44	4.6	6.5	2 P.M.
192	320	98	35	4.4	4.7	7 P.M.
228	320	84	47	4.4	5.2	6 P.M.
224	336	79	52	4.4	5.6	6 P.M.

Side-St. Vol. (cars per hr.)	*Main-St. Vol. (cars per hr.)*	*% in 1st Position*	*% Delayed*	*Ave. Wait of 1st-Pos. Cars (sec.)*	*Ave. Wait of All Cars (sec.)*	*Time of Day*
348	116	82	47	3.4	3.9	6 A.M.
264	184	77	42	4.8	6.1	2 P.M.
276	192	83	38	3.8	4.7	1 P.M.
292	212	66	59	4.6	6.0	2 P.M.
252	224	79	48	3.8	5.3	NOON
252	232	75	54	4.6	5.6	NOON
260	232	83	42	4.4	5.2	1 P.M.
276	232	70	45	4.6	6.5	7 P.M.
272	240	79	44	4.8	5.6	2 P.M.
276	256	75	51	4.8	6.1	7 P.M.
288	260	74	49	5.4	6.7	3 P.M.
340	260	84	51	4.5	5.9	3 P.M.
264	300	70	58	4.9	6.9	1 P.M.
348	328	71	54	3.9	5.3	1 P.M.
312	332	73	67	4.7	5.3	1 P.M.
348	452	51	70	9.3	20.1	4 P.M.
260	460	72	60	7.2	8.2	5 P.M.
280	528	64	77	7.6	10.2	5 P.M.
308	612	49	86	10.9	15.4	5 P.M.
268	660	54	79	9.4	14.5	5 P.M.
276	792	54	84	11.3	17.7	5 P.M.
336	908	38	88	15.5	23.7	5 P.M.
392	312	68	55	3.8	6.3	3 P.M.
384	352	61	67	5.2	7.2	4 P.M.
392	432	53	71	6.0	9.1	4 P.M.
424	488	52	77	7.2	10.9	4 P.M.
404	520	46	82	7.2	18.8	4 P.M.
352	536	72	70	6.3	7.9	4 P.M.
508	608	23	90	8.1	37.2	4 P.M.
464	684	40	85	8.9	22.6	4 P.M.
464	736	35	89	6.5	18.3	5 P.M.
440	860	45	89	11.5	14.3	5 P.M.

TABLE XIII

FIELD DATA FROM INTERSECTION D, BY FIFTEEN-MINUTE PERIODS

Side-St. Vol. (cars per hr.)	*Main-St. Vol. (cars per hr.)*	*% in 1st Position*	*% Delayed*	*Ave. Wait of 1st-Pos. Cars (sec.)*	*Ave. Wait of All Cars (sec.)*	*Time of Day*
76	560	89	53	9.3	9.4	10 A.M.
76	632	95	58	6.3	6.7	NOON
68	644	82	76	5.8	6.4	NOON
76	660	95	53	7.5	7.6	11 A.M.
72	664	94	67	5.9	5.8	11 A.M.
32	664	75	100	8.0	8.0	11 A.M.
36	668	100	22	5.5	5.5	10 A.M.
52	692	100	46	4.9	4.9	NOON
76	712	100	74	5.5	5.5	NOON
48	724	92	50	9.1	8.5	10 A.M.
84	728	90	76	7.4	7.6	2 P.M.
44	728	91	64	9.3	10.6	11 A.M.
76	732	89	47	6.0	6.4	10 A.M.
60	752	100	40	6.5	6.5	11 A.M.
60	764	100	40	5.8	5.8	10 A.M.
64	764	81	63	8.9	9.4	10 A.M.
72	768	89	89	9.1	8.9	NOON
92	772	96	57	5.1	5.2	1 P.M.
100	772	60	36	6.5	6.5	NOON
52	784	100	54	4.8	4.8	10 A.M.
56	796	100	64	10.3	10.3	3 P.M.
108	800	85	89	5.8	6.0	1 P.M.
72	804	94	72	6.4	6.2	2 P.M.
60	808	100	87	11.0	11.0	1 P.M.
56	812	100	50	5.2	5.2	1 P.M.
60	816	100	40	7.2	7.2	2 P.M.
52	816	100	46	5.8	5.8	9 A.M.
64	828	94	75	7.3	7.0	2 P.M.
84	852	95	57	7.2	7.1	3 P.M.
68	852	100	71	9.7	9.7	3 P.M.

Side-St. Vol. (cars per hr.)	*Main-St. Vol. (cars per hr.)*	*% in 1st Position*	*% Delayed*	*Ave. Wait of 1st-Pos. Cars (sec.)*	*Ave. Wait of All Cars (sec.)*	*Time of Day*
72	868	94	56	7.4	7.3	1 P.M.
72	876	94	61	9.6	9.4	2 P.M.
84	876	100	52	5.9	5.9	8 A.M.
64	880	94	63	7.9	7.6	2 P.M.
56	884	86	71	9.5	10.0	2 P.M.
100	900	96	80	9.7	9.8	4 P.M.
60	916	93	87	8.6	9.7	3 P.M.
112	932	75	64	11.9	10.3	8 A.M.
120	936	77	73	8.7	8.6	1 P.M.
84	948	100	57	7.5	7.5	2 P.M.
92	960	96	65	9.3	10.4	8 A.M.
80	980	100	40	4.3	4.3	11 A.M.
72	1040	89	94	9.0	10.1	4 P.M.
92	1136	87	65	10.1	11.2	8 A.M.
92	1200	91	91	14.6	14.4	5 P.M.
104	1384	73	85	15.2	16.9	5 P.M.

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