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SCOUR IN SUPERCRITICAL FLOW

Final Report

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16. ABSTRACT

Scour in supercritical flow is one extreme aspect of the effects of velocity on scour. Analysis of the case of scour in a long contraction shows that if all other independent variables are kept constant (1) some finite velocity is necessary to have any scour, (2) as the velocity is increased, the scour increases as long as there is no sediment movement in the wide, approach reach, and (3) as sediment movement in the approach increases with further increase in the velocity, the scour decreases a modest amount. The analysis does not indicate that there should necessarily be a change in behavior in supercritical flow -- although the definition of scour needs to consider velocity head changes and energy losses. Rather than velocity, the variable of interest should be the ratio of the particle shear to the critical tractive force.

Adaption of the long-contraction solution to the case of the pier or abutment indicates that the scour at a pier or abutment should display the same behavior: scour increasing with velocity for the clear-water condition and decreasing slightly for sediment-transporting flow. Experiments agree with the analysis for both geometries. No instability of flow or other "strange" behavior was noted in the supercritical flow, possibly because of the simplicity of the geometries, or because the equipment could not achieve high enough Froude numbers.

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SCOUR IN SUPERCRITICAL FLOW

TABLE OF CONTENTS

SCOUR IN SUPERCRITICAL FLOW

LIST OF TABLES

SCOUR IN SUPERCRITICAL FLOW

LIST OF FIGURES

LIST OF FIGURES (Continued)

LIST OF FIGURES (Continued)

SI UNIT CONVERSION FACTORS

The material contained in this report is presented in terms of English units. The following factors may be used to convert between measures used in this report and the International System of Units (SI):

1 foot = 0.3048 meter

1 meter = 3.2808 feet

1 foot per second (fps) = 0.3048 meters per second

1 meter per second = 3.2808 feet per second

1 cubic foot per second (cfs) = 0.0283 cubic meters per second

1 cubic meter per second = 35.31 cubic feet per second

SCOUR IN SUPERCRITICAL FLOW

LIST OF SYMBOLS

a — exponent in power law approximation of function in Laursen sediment transport relation

A — coefficient in power law approximation of function in Laursen sediment transport relation

α — angle of attack for a pier ($^{\circ}$)

b — width of pier (ft) (unusual geometry can give difficulty in averaging to obtain a representative width)

B — width of rectangular channel between banks (ft) (If channel is not rectangular the width should be that of an equivalent rectangular channel; i.e., one which will carry the Q and Q_s at the same slope)

c — sediment load concentration in percent by weight

C — coefficient for shear factor (see Fig. 3 and Eq. (16), Page 16)

d — diameter of sediment particles (ft) (in this study a single size sediment is assumed)

D — diameter of circular pier (ft)

d_s — depth of scour measured from the stream bed (ft) (other definitions of depth of scour can be appropriate for various problems)

LIST OF SYMBOLS (Continued)

F Froude number of the flow, $V/\sqrt{gy}$

F_c threshold Froude number used in Eq. (2)

g acceleration of gravity (fps^2)

K coefficient for hydraulic radius factor, R/y

K_L loss coefficient , $h_L = K_L(V_2{}^2/2g - V_1{}^2/2g)$

ℓ effective length of an abutment (ft), $\ell = Q_o/V_o y_o$
where Q_o is the overbank flow obstructed
by the embankment/abutment, and V_o and y_o
are characteristic of the flow approaching
the abutment scour hole

L length of pier (ft)

n resistance coefficient in Manning Equation

Q water discharge (cfs)

Q_c water discharge within channel (ft)

Q_o water discharge on overbank(s) as appropriate

Q_s sediment load (tons per day)

Q_t total water discharge (cfs), the sum of $Q_c + Q_o$

Q_w water discharge approaching abutment scour hole
in a width, $w = 2.75\ d_s$ (cfs)

R hydraulic radius (ft)

S stream slope (ft/ft)

τ_c critical tractive force or boundary shear
often assumed to be 4d (psf)

τ_o boundary shear at streambed for a wide channel (psf)

LIST OF SYMBOLS (Continued)

τ_o' boundary shear at streambed due to particle shear evaluated as $V^2\ d^{1/3}/30\ y^{1/3}$ (psf)

θ inclination of embankment/abutment (^{o})

V velocity of flow (fps)

w fall velocity of quartz sphere in large quiescent tank of water (fps)

or

width (approximately 2.75 d_s) of approach flow of Q_w associated with abutment scour hole (ft)

The only superscript is a prime (') used to differentiate particle shear from total boundary shear.

Several subscripts are used as follows:

o for a reference velocity or depth of flow, or a reference boundary shear, or the appropriate overland flow

1 for the velocity, depth or boundary shear in a wide, approach reach

2 for the velocity, depth or boundary shear in the narrow, contracted reach

c for a critical tractive force (boundary shear) or threshold Froude number

and a few others as defined in this listing.

Final Report

SCOUR IN SUPERCRITICAL FLOW

ABSTRACT

Scour in supercritical flow is one extreme aspect of the effect of velocity on scour. Analysis of the case of scour in a long contraction shows that if all other independent variables are kept constant, (1) some finite velocity is necessary to have any scour, (2) as the velocity is increased, the scour increases as long as there is no sediment movement in the wide, approach reach, and (3) as sediment movement in the approach increases with further increase in the velocity, the scour decreases a modest amount. The analysis does not indicate that there should necessarily be a change in behavior in supercritical flow --although the definition of scour needs to consider velocity head changes and energy losses. Rather than velocity, the variable of interest should be the ratio of the particle shear to the critical tractive force.

Adaptation of the long-contraction solution to the case of the pier or abutment indicates that the scour at a pier or abutment should display the same behavior: scour increasing with velocity for the clear-water condition and decreasing slightly for sediment-transporting flow. Experiments agree with the analysis for both geometries. No instability of flow

or other "strange" behavior was noted in the supercritical flow, possibly because of the simplicity of the geometries, or because the equipment could not achieve high enough Froude numbers.

BACKGROUND OF THE PROBLEM

One of the findings of the Iowa investigation of scour around bridge piers and abutments (1, 2, 3) was that if the flow was transporting sediment, the scour depth, as a first approximation, was a function of geometry only. The notion that velocity and sediment size have little effect of scour depth has been difficult, if not impossible, for many people to understand or accept. "Everyone" knows that bridges may fail in floods, although some speak of "liquifaction" rather than scour holes, and others believe that the stream bed lowers as much as the water surface rises. Those who are aware of scour holes around the piers and abutments are often more impressed by the velocity of the flood water than the depth of the flood water; therefore, they naturally attribute the scour which occurs to the increase in the velocity of flow.

Scour occurs because of an imbalance between the capacity of the flow to remove sediment from an area and the supply of sediment to that area by the flow (4). If the capacity to remove sediment exceeds the supply, there will be scour. If the supply exceeds the capacity, there will be deposition. The limit to the scour or deposition is a geometry such that the capacity equals the supply. When there is sediment transport by the stream, it does not matter much what the rate of sediment transport is (and, therefore, what the velocity and sediment size are) just so long as there is a balance between the amount of material coming into the area in question and the amount going out of the area. In the laboratory, the depth and other geometry can be kept constant and the velocity of flow increased (or the sediment size changed). The rate of transport will change but the scour depth will not measurably change if the boundary shear is well above the critical tractive force. In a real river in flood, both depth and velocity of flow increase, making it difficult to sort out what is doing what to what. In addition, it is possible for the flow pattern of the river to change with stage during the course of the flood, and the pier geometry can change with the accumulation of debris.

When the flow is not transporting material as large as the bed material which must be removed in the scour process, the condition is essentially that of clear-water flow and the sediment supply to the area in question is zero. The limit of scour is then a boundary shear equal to the critical tractive force of the material which could be scoured. The boundary shear is certainly a function of the velocity of flow and the critical tractive force is certainly a function of the sediment size. Both then matter (as well as geometry) in the depth of clear-water scour. Indeed, they matter together in a parameter which is the ratio of the reference particle boundary shear to the reference critical tractive force; thus, if both velocity and sediment size increase, but the parameter stays the same, the scour depth does not change.

The controversy over the effect of velocity -- and/or sediment size -- has persisted since the publications resulting from the Iowa experiments. Several of the discussions of the ASCE paper (3) cited clear-water scour studies in disagreeing with the conclusion that in sediment-transporting flow there was little effect of velocity and sediment size on scour. The closing discussion tried to make the distinction clear in a qualitative argument which led to a subsequent paper on the clear-water scour case (5). Years later the small effect of velocity and sediment size was investigated (6, 7, 8) and it was found that the scour depth decreased with an increase in velocity or the particle shear/critical tractive force ratio. Straub, of course, had found the effect years earlier (9, 10) when he presented the first analytical long-contraction scour solution.

Over the years a number of investigators have proposed scour-prediction formulae which include the velocity in some way. Typical of these are those presented in a FHWA Training and Design Manual prepared by several of the Colorado State University Group (11). Their expression for the scour at a rectangular pier aligned with the flow can be written as

$$\frac{d_s}{b} = 2.2\left(\frac{y_o}{b}\right)^{0.35} F^{0.43} \qquad (1)$$

where d_s is the depth of scour measured from the stream bed,

V_o is the depth of the approach flow,

y_o is the depth of the approach flow,

b is the width of the pier, and

F is the Froude number of the approach flow, $V_o / \sqrt{g y_o}$.

Several comments serve to increase this prediction of the "equilibrium" scour depth, "...maximum scour depth at piers could be as large as 30 percent greater than equilibrium scour depth" and "...y_o would normally be measured from some level closer to the tops of dunes. Scour depths on the other hand should be referenced nearer the trough of the dunes." Elsewhere it is implied that the fluctuations above the average or equilibrium is due to the dunes, and these comments would seem to correct twice for the same phenomenon. However, this is a matter of the absolute value of the predicted depth of scour, not the question of the effect of velocity on scour. For two identical piers in identical rivers (except for the velocity) if the one river is in the Midwest with a Froude number of 0.2 and the other river is in the Southwest with a Froude number of 1.0, Eq. (1) would predict twice the scour depth in the Southwest as in the Midwest -- e.g., 20 feet compared to 10 feet. In general, the depths of scour predicted by Eq. (1), especially if increased as suggested for conditions in the Southwest are so large that if the predictions were correct, very few bridges over alluvial streams should be still standing in Arizona -- or lands like it. It is of some interest to note that the sediment size does not affect the depth of scour predicted by Eq. (1).

A FHWA-sponsored project performed by the Iowa Institute of Hydraulic Research (12) on scour at Froude numbers up to 1.2 and 1.5 suggested a similar relationship which included sediment size in a threshold Froude number. The envelope curve included both pier scour and bed-form scour and was for a circular pier (a rectangular pier would experience 10% more scour).

$$\frac{d_s}{D} = 2.0 \left(\frac{y_o}{D}\right)^{0.5} (F - F_c)^{0.25} \tag{2}$$

where d_s is the total depth of scour measured from the stream bed,

D is the diameter of the circular pier,

y_o is the depth of flow,

F is the Froude number of the flow, and

F_c is the threshold Froude number based on a threshold velocity obtained from the Shields diagram and the logarithmic velocity distribution. (This term involves the sediment size.)

Even if the coefficient in Eq. (2) is increased because of the shape factor, it will usually predict less scour than Eq. (1), especially if the Eq. (1) suggestions for increasing the predicted scour are followed. Because Eq. (2) includes the combined effect of pier scour and bed-form scour, it should embody (approximately) the suggestions for predicting scour by

Eq. (1). In general, Eq. (2) will predict about 50 percent more scour at high Froude numbers than would be predicted by the relationships proposed in References 2 and 3.

There is no theoretical basis for Eq. (2). It is similar to Eq. (1) which has no theoretical basis either, the difference being in the coefficient, the exponents, and the inclusion of a critical term. Both equations can best be described as power curve fitting to limited experimental data with parameters obtained from dimensional analysis. As usual, dimensional analysis doesn't get one very far; dimensional analysis requires, first, that one knows what variables are important (and independent) and, second, that one knows what dimensionless combinations are meaningful.

Having a suspicion of what laboratory equipment was used in these Iowa experiments, there is a chance that the scour measured is not necessarily the scour associated with the flow characteristics measured.

In recent years, the State of Arizona experienced several large floods and a number of bridges were lost or damaged. As a consequence, the Arizona Department of Transportation (and some cities and counties) have been trying to identify possible vulnerable bridges and then taking some action in the way of remedial works to make them less vulnerable. It makes a difference if the predicted scour is ten, fifteen or twenty feet. Streams in Arizona are relatively steep -- one-half of one percent or more, rather than a foot per mile -- and the

Froude number of streams in flood can approach or even exceed unity. Therefore, the need for this research is obvious and readily apparent.

THE LONG-CONTRACTION SOLUTION

Straub in his original analytic solution (9, 10) showed that in the long contraction the depth ratio y_2/y_1 decreased as the ratio of boundary shear to critical tractive force $1/\ c$ increased from slightly greater than unity. (At a ratio of unity his solution broke down.) His full equation was derived using the DuBoys sediment-transport equation and the Manning equation and is

$$\frac{y_2}{y_1} = \left(\frac{B_1}{B_2}\right)^{3/7} \left\{ \frac{-\frac{\tau_c}{\tau_1} + \left[\left(\frac{\tau_c}{\tau_1}\right)^2 + 4\left(1 - \frac{\tau_c}{\tau_1}\right)\frac{B_1}{B_2}\right]^{1/2}}{2\left(1 - \frac{\tau_c}{\tau_1}\right)} \right\}^{3/7} \qquad (3)$$

where τ_c is the critical tractive force and the subscripts 1 and 2 refer to the wide approach, and narrow, contracted reaches, respectively.

When τ_1/τ_c is large (∞), the full solution reduces to

$$\frac{y_2}{y_1} = \left(\frac{B_1}{B_2}\right)^{9/14} \qquad (4)$$

wherein the velocity, sediment size, Froude number and shear ratio, all have no effect on the depth ratio.

In Straub's full solution there is about a 15-percent decrease in the depth ratio as the shear ratio increases from 1.01 to ∞; 10 percent occurring as the shear ratio increases from 1.01 to 2, and 5 percent occurring as the shear ratio increases from 2 to ∞.

Note that Straub's solution is for the sediment-transporting flow case only, not the clear-water case, that the full solution suffers from the use of the total boundary shear instead of the particle boundary shear in the Duboys sediment-load equation and probably should use Straub's evaluation of critical tractive force, but that for a river in flood the reduced equation (Eq. 4) is sufficient because the critical tractive force should then be small compared to the boundary shear.

A more general solution of long-contraction scour can be performed which will illustrate more fully the effect of velocity on scour. Figure 1 is a definition sketch of a general long contraction in which the total flow Q_t is divided between a portion Q_c in the approach channel of width B_1 and a portion Q_o on the overbank, or floodplain. (The division of Q_o into two equal parts is immaterial.) If there is some additional overbank flow outside of the contracted channel of width B_2, it can simply be ignored; it is not doing anything of importance in this problem.

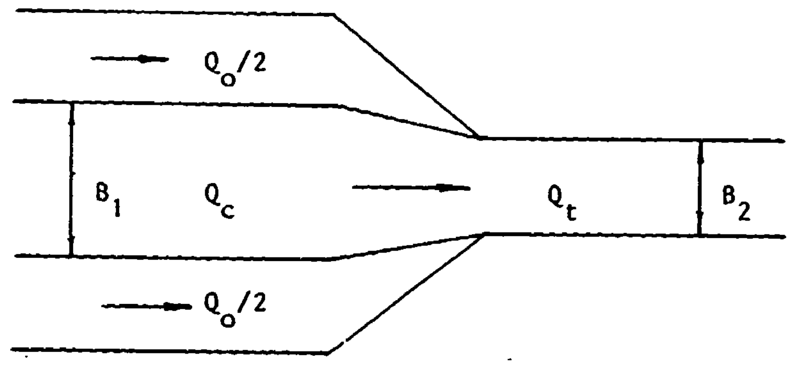


Figure 1. Definition Sketch of the General Long Contraction.

The solution of the clear-water case proceeds from the realization that if there is active scour in the long contraction, it will continue (slower and slower) until the particle boundary shear is equal to the critical tractive force. In the approach reach, the shear ratio will be less than unity. There will be some flow (some particle boundary shear in the approach) greater than zero when the shear ratio in the contraction is unity without any scour having taken place. If there is overbank flow in the approach, the width B_1 should be increased so that the total flow will exist at the velocity and depth of the channel flow. The following evaluations and approximations are used:

$$\tau_o' = \frac{V^2 d^{1/3}}{30y^{1/3}} \tag{5}$$

where τ_o' is the particle shear; i.e., the boundary shear for a wide channel with a "smooth" bed with a texture of the sand grains solved by the Manning equation and Strickler's n .

$$\tau_c = 4d \tag{6}$$

$$R = y \tag{7}$$

where the channel is wide so the hydraulic radius R equals the depth y .

With these, an expression is obtained for the depth ratio

$$\frac{y_2}{y_1} = \left(\frac{\tau_1'}{\tau_c}\right)^{3/7} \left(\frac{B_1}{B_2}\right)^{6/7} \tag{8}$$

The solution proceeds by writing

$$\tau_2' = \frac{V_2^2\ d^{1/3}}{30Y_2^{\ 1/3}} = \quad c = 4d$$

$$\tau_1' = \frac{V_1^2\ d^{1/3}}{30Y_1^{\ 1/3}}$$

and

$$Q_1 = V_1 Y_1 B_1 = Q_2 = U_1 Y_2 B_2$$

and, then, equating

$$\frac{\tau_1'}{\tau_c} = \frac{\tau_1'}{\tau_2'}$$

This expression is shown graphically in Figure 2. Note that the depth ratio or the relative depth of scour (for many problems the depth of scour can be taken as $y_2 - y_1$) is a function of the geometry B_1/B_2 and the shear ratio which includes both velocity and sediment size (and depth). Other evaluations of the particle boundary shear and the critical tractive force could conceivably give a somewhat different expression or result in a slightly different answer in an application. The solution is of limited, but occasionally important, use in real river situations but, as will be seen, it can be adapted to the riprap problem. Its importance here is the light it sheds on the scour problem in general.

For the sediment-transporting flow case, more insight into the effect of velocity (or velocity-related parameters) on scour in the long contraction can be gained by a solution similar to Straub's but using several approximations of the Laursen sediment-transport relation (instead of the DuBoys equations), the Manning formula, and the other following statements, approximations and evaluations (6, 7, 8)

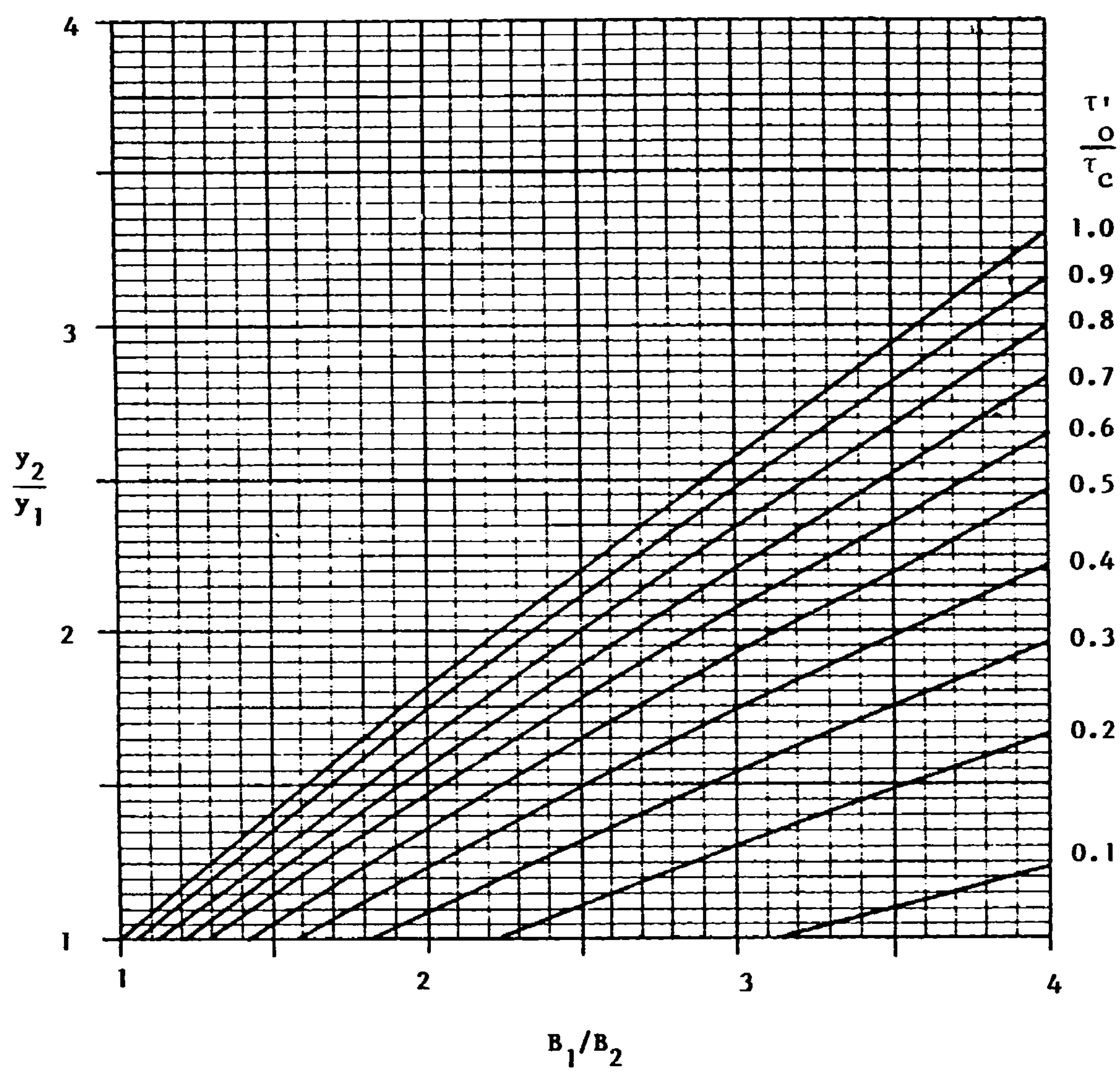


Figure 2. Clear-Water Scour in a Long Contraction.

$$Q_c = B_1 y_1 \frac{1.49}{n_1} R_1^{2/3} S_1^{1/2} \tag{9}$$

$$Q_t = B_2 y_2 \frac{1.49}{n_2} R_2^{2/3} S_2^{1/2} \tag{10}$$

$$Q_t = Q_c + Q_o \tag{11}$$

$$\bar{c}_1 Q_c = \bar{c}_2 Q_t \tag{12}$$

$$\bar{c}_1 = (\frac{d}{y_2})^{7/6} (\frac{\tau_1'}{\tau_c} - 1) A (\frac{\sqrt{\tau_1/\rho}}{w})^a \tag{13}$$

$$\bar{c}_2 = (\frac{d}{y_2})^{7/6} (\frac{\tau_2'}{\tau_c} - 1) A (\frac{\sqrt{\tau_2/\rho}}{w})^a \tag{14}$$

where Q_c is the between-banks discharge in the approach channel of width B_1 , depth y_1 , and slope S_1 ,and having a resistance coefficient n_1 .

Q_t is the total discharge confined within the long contraction of width B_2 , depth y_2 , and slope S_2 , and having a resistance coefficient n_2 .

Q_o is the overbank (or floodplain) discharge which is here arbitrarily divided equally between the right and left sides.

The concentration of the sediment load $\bar{c}$ is in percent by weight of a single size sediment of diameter d , critical tractive force τ_c , and fall velocity w . A mixture, or natural sediment, can be considered on a case-by-case basis, using the proper version of the Laursen total load relation but has not been generalized -- generalization may not be possible. The term τ_o' is the "particle shear" as mentioned previously and τ_o is the "total shear" (γyS). An alternate formulation of the shear velocity $\sqrt{\tau_o/\rho}$ is $\sqrt{gyS}$. The subscripts 1 and 2 refer to the approach and contracted reaches, respectively, and the subscript o is dropped for simpler topography.

The function of $\sqrt{\tau_o/\rho}/w$ in the Laursen sediment-transport relation is approximated by power functions over three ranges. One would expect that the approach and contracted reaches would be in the same range; further refinement would seldom be of interest. The intercept (A) values drop out of consideration and the exponent (a) values are

$$\sqrt{\tau_o/\rho}/w < 1/2 \;, \quad a = 1/4$$

$$\sqrt{\tau_o/\rho}/w = 1 \;, \quad a = 1$$

$$\sqrt{\tau_o/\rho}/w > 2 \;, \quad a = 9/4$$

The fall velocity w is that of a quartz sphere of diameter d falling in large quiescent container, just as was done in the development of the original relationship. A different, better fall velocity for grains of sand and gravel would require revision of the function $f(\sqrt{\tau_o/\rho}\ w)$.

A rectangular cross section is assumed but not a very wide channel, so

$$R = Ky \tag{15}$$

The shear ratio term is written as

$$\frac{\tau_o'}{\tau_c} - 1 = C\,\frac{\tau_o'}{\tau_c} \tag{16}$$

This equation is shown graphically in Figure 3.

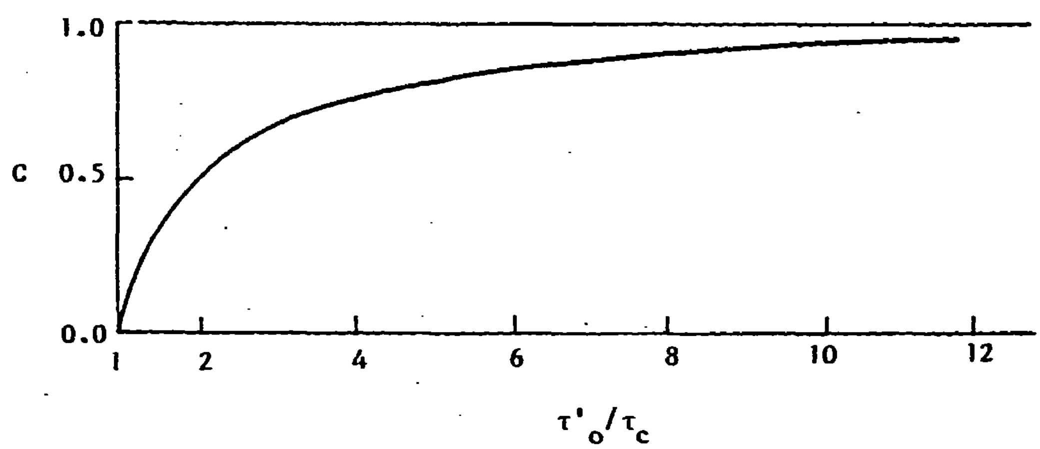


Figure 3. Critical-Tractive Force Term.

Equating the discharge and sediment load in the two reaches and manipulating algebraically to eliminate either the slopes or the depths, results in the following equations for the depth or slope ratios:

For bed load ($\sqrt{\tau_o/\rho}\ /w < 1/2$)

$$\frac{y_2}{y_1} = \left(\frac{Q_t}{Q_c}\right)^{0.86} \left(\frac{B_1}{B_2}\right)^{0.59} \left(\frac{n_2}{n_1}\right)^{0.07} \left(\frac{C_2}{C_1}\right)^{0.26} \left(\frac{K_1}{K_2}\right)^{0.01} \tag{17}$$

$$\frac{S_1}{S_2} = \left(\frac{Q_t}{Q_c}\right)^{0.86} \left(\frac{B_1}{B_2}\right)^{-0.02} \left(\frac{n_1}{n_2}\right)^{1.78} \left(\frac{C_2}{C_1}\right)^{0.88} \left(\frac{K_2}{K_1}\right)^{1.30} \tag{18}$$

For some suspended load ($\sqrt{\tau_o/\rho}\ /w = 1$)

$$\frac{y_2}{y_1} = \left(\frac{Q_t}{Q_c}\right)^{0.86} \left(\frac{B_1}{B_2}\right)^{0.64} \left(\frac{n_2}{n_1}\right)^{0.21} \left(\frac{C_2}{C_1}\right)^{0.21} \left(\frac{K_1}{K_2}\right)^{0.04} \tag{19}$$

$$\frac{S_1}{S_2} = \left(\frac{Q_t}{Q_c}\right)^{0.86} \left(\frac{B_1}{B_2}\right)^{0.14} \left(\frac{n_1}{n_2}\right)^{1.29} \left(\frac{C_2}{C_1}\right)^{0.71} \left(\frac{K_2}{K_1}\right)^{1.21} \tag{20}$$

For mostly suspended load ($\sqrt{\tau_o/\rho}\ /w > 2$)

$$\frac{y_2}{y_1} = \left(\frac{Q_t}{Q_c}\right)^{0.86} \left(\frac{B_1}{B_2}\right)^{0.69} \left(\frac{n_2}{n_1}\right)^{0.37} \left(\frac{C_2}{C_1}\right)^{0.16} \left(\frac{K_1}{K_2}\right)^{0.06} \tag{21}$$

$$\frac{S_1}{S_2} = \left(\frac{Q_t}{Q_c}\right)^{0.86} \left(\frac{B_1}{B_2}\right)^{0.31} \left(\frac{n_1}{n_2}\right)^{0.78} \left(\frac{C_2}{C_1}\right)^{0.54} \left(\frac{K_2}{K_1}\right)^{1.13} \tag{22}$$

The depth ratio for these three modes of movement are shown graphically in Figures 4, 5 and 6 as a function of width ratio and shear factor.

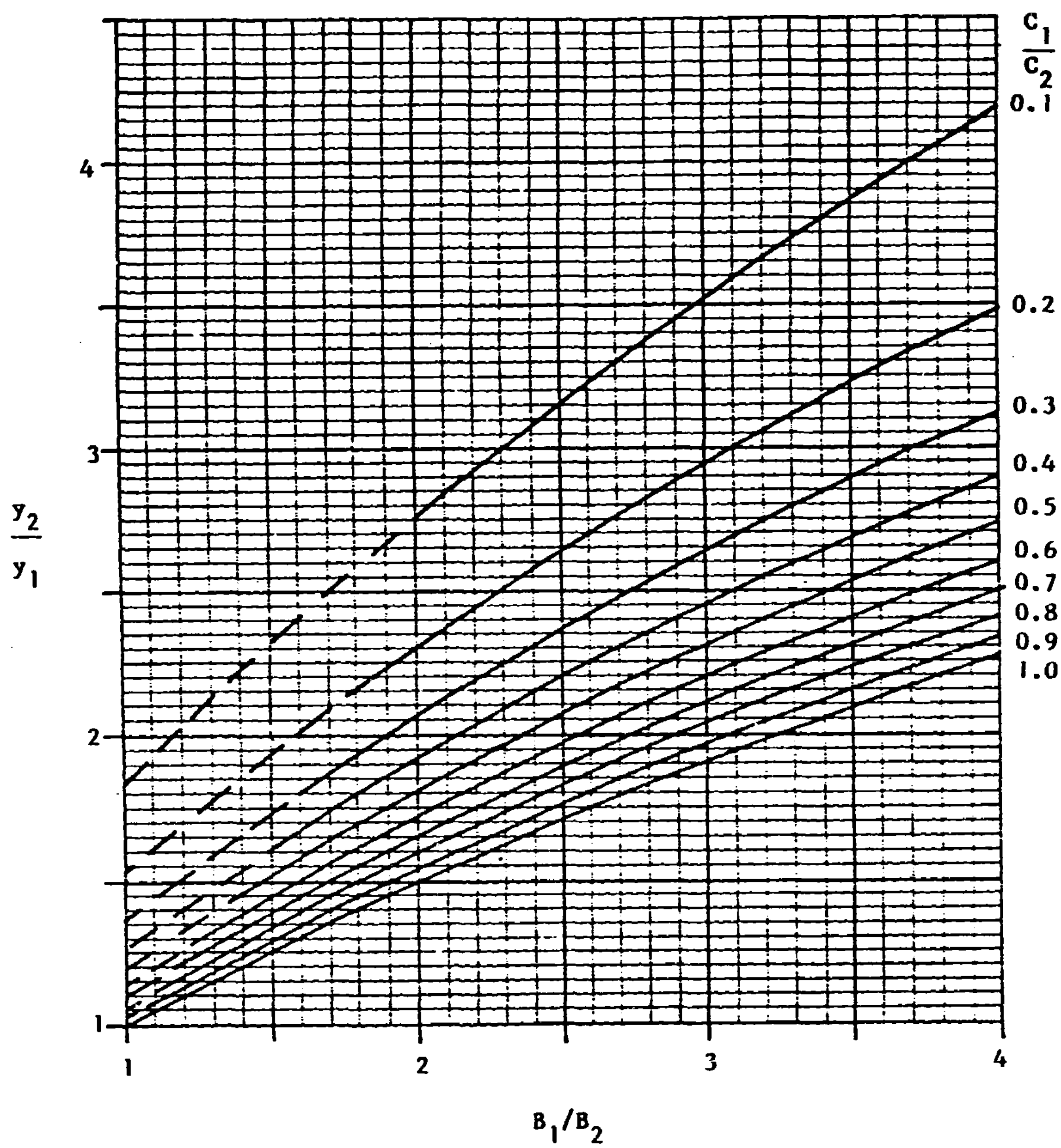


Figure 4. Depth Ratio vs Width Ratio and Shear Factor for Bed Load in a Long Contraction.

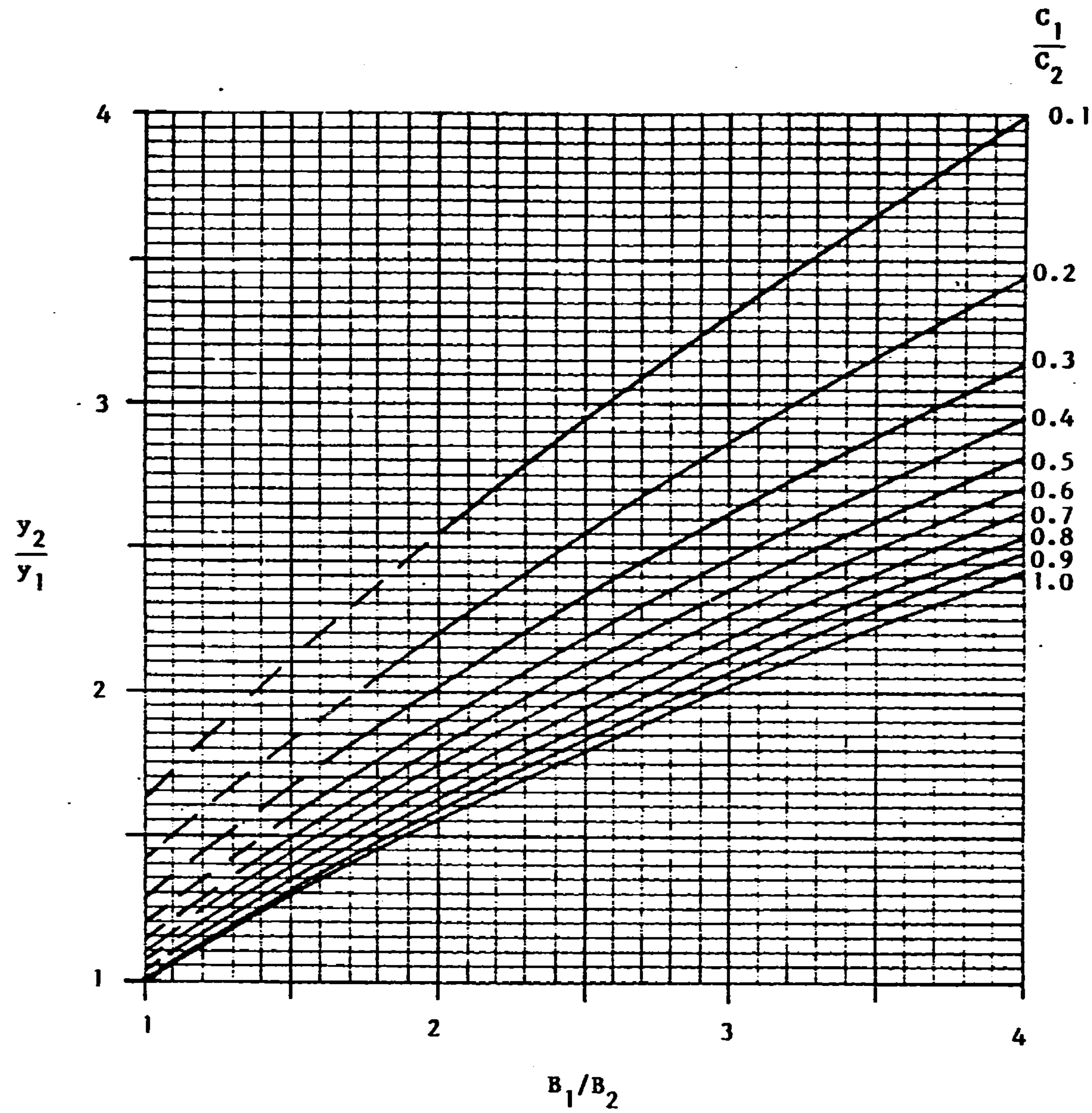


Figure 5. Depth Ratio vs Width Ratio and Shear Factor for Some Suspended Load in a Long Contraction.

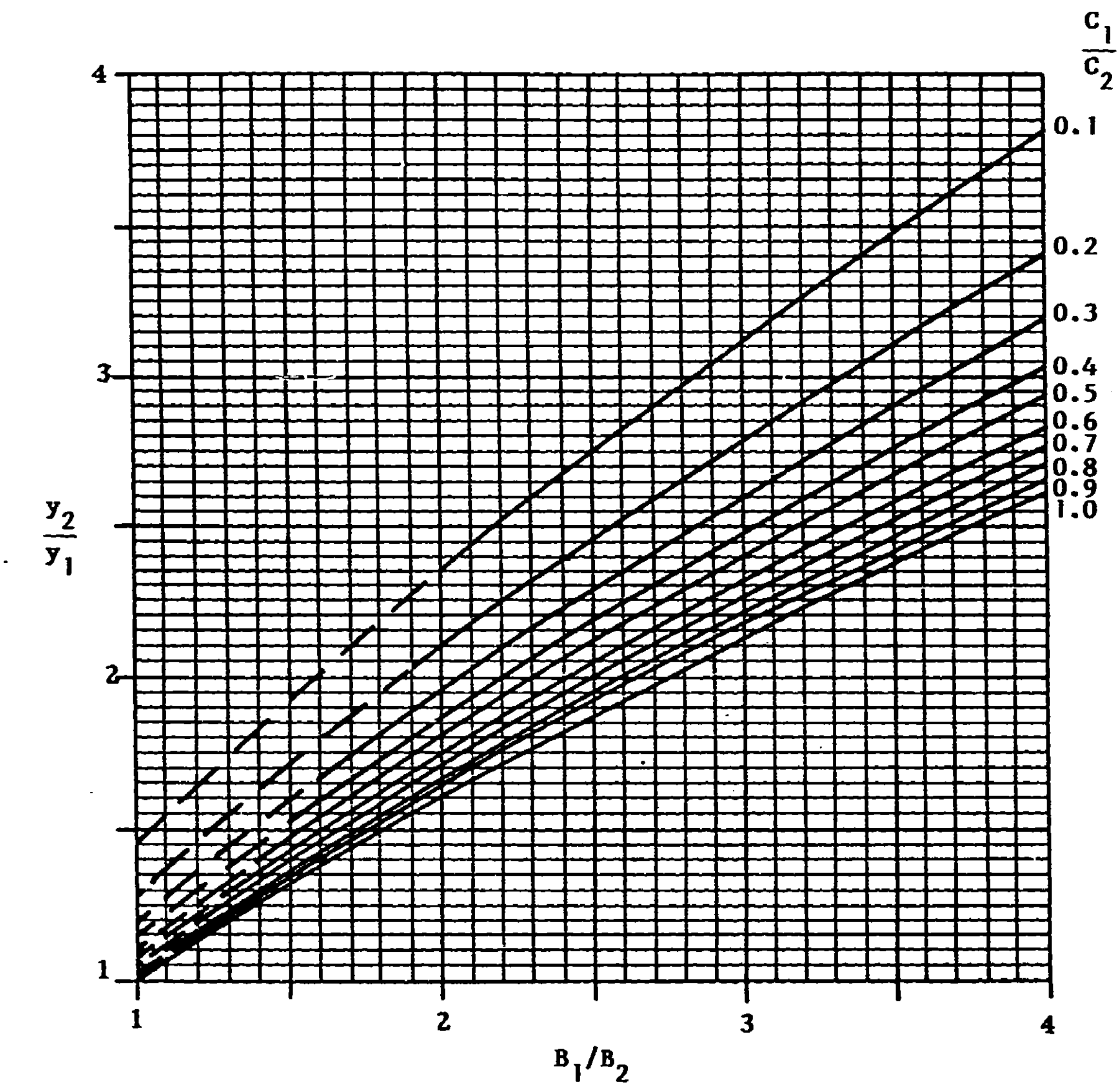


Figure 6. Depth Ratio vs Width Ratio and Shear Factor for Mostly Suspended Load in a Long Contraction.

To put this velocity effect in perspective, consider four streams all with depth of five feet and bed material of 0.02 feet (1/4 inch), and a contracted reach just half of the width of the approach reach. The first stream has a very, very high velocity and a depth in the contraction of 7.53 feet. The second has a shear ratio of 10, a velocity of 12.3 fps, a Froude number of 0.79, and depth in the contraction of 7.60 feet --only one percent more. The third has a shear ratio of 1.1, a velocity of 4.1 fps, a Froude number of 0.32 and a depth in the contraction of 8.75 feet -- still only 16 percent more. The fourth has a shear ratio of 1.0, a velocity of 3.88 fps, a Froude number of 0.31, and a depth in the contraction 9.43 feet -- 25 percent more than the limiting sediment-transporting case.

This example illustrates the point that with everything except the velocity kept constant, the depth (and scour) in the contraction increases with an increase in velocity until the sediment starts moving in the approach reach, and then the depth (and scour) decreases with further increase of the velocity, asymptotically approaching a limiting value. The example is a little contrived because the four streams probably cannot be found. If these are all streams in flood, it would be found that the sediment size in the slow-moving streams would be much finer than that of the fast-moving streams. For streams in flood, the shear ratio will almost assuredly be so high that the limiting solution is sufficient for practical purposes -- especially if discharge and depth, etc. have been evaluated a trifle conservatively. Moreover, the ratio of the total shear velocity to fall velocity also has an effect on the depth of scour as do the velocity heads and losses.

The mode of movement changes from bed load only to some suspended load to mostly suspended load, as the shear velocity/fall velocity ratio changes from less than 1/2 to unity to greater than 2 (according to the Laursen sediment-transport relation). The sediment-transport dependence on the shear velocity/fall velocity ratio also changes, and the exponents of the independent parameters determining the depth ratio change slightly (Eqs. 17, 19 and 21). The extreme of mostly suspended load will result in a depth of flow in the contraction a little over seven percent more than the condition of bed load only. This effect tends to compensate for the previous effect that was associated with velocity and sediment transport. Note, however, the fall velocity is that of the material being scoured out, not the fall velocity of the fine fraction of the suspended load which would be sampled.

In all of these examples of how velocity can seemingly affect the scour process, it is not the velocity in itself which affects the scour, but rather something else which can be shown to be related to the velocity (such as particle shear or total shear velocity). Moreover, the velocity or velocity-related variable is contained in a dimensionless parameter or ratio such that if other things vary together with the velocity in such a way that the parameter does not change, there is no apparent velocity effect on the scour.

There is one way, however, in which the velocity can have a more direct effect on the scour in a long contraction. This comes about in the definition of the depth of scour. The depth of scour needs to be defined differently, depending on the problem involved. A common definition in the case of the long contraction when the question is, "How much will the bed scour for some given rate of flow in a 'short' long contraction," is simply

$$d_s = y_2 - y_1 \tag{23}$$

If the Froude number is high (approaching or exceeding unity) this is not an adequate definition of the depth of scour. The difference in velocity heads and the loss in energy should also be considered in defining the depth of scour as shown in Figure 7. If the long contraction is long enough for the difference in slopes in the wide approach and the narrow contraction to be significant, this contribution to bed lowering should also be included.

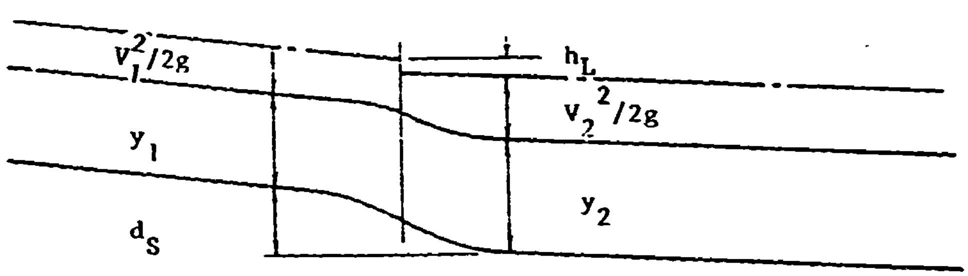


Figure 7. Definition of Depth of Scour for High Froude Number.

Extrapolating the conditions in the two reaches to the midpoint of the transition between the two, it is evident that

$$\frac{d_s}{y_1} + 1 + \frac{V_1^2}{2gy_1} = \frac{y_2}{y_1} + \frac{V_2^2}{2gy_1} + K_L \left(\frac{V_2^2}{2gy_1} - \frac{V_1^2}{2gy_1}\right) \tag{24}$$

If the head loss is taken as $h_L = K_L \, (V_2{}^2/2g - V_1{}^2/2g)$, a little algebraic manipulation will result in

$$\frac{d_s}{y_1} = \left(\frac{y_2}{y_1} - 1\right) + \left[\left(\frac{1 + K_L}{2}\right) \left(\left(\frac{V_2}{V_1}\right)^2 - 1\right)\right] F_1^2 \tag{25}$$

For the case of bed load, a contraction to half the original width, and no energy loss, the inclusion of this velocity effect increases the scour by 75 percent at a Froude number of unity. With a loss of half the difference in the velocity heads, the increase in scour is over 100 percent. At a Froude number of 0.2, however, the increase in scour depth is only a few percent. It is interesting to note that at the downstream end of the long contraction the difference in elevation of the bed would decrease instead of increase with the inclusion of a loss term.

For other problems, and consequently other definitions of scour, the energy losses and velocity heads should similarly be included when the Froude number is high so that the "velocity effect" is significant. Note, however, that this analysis is for the long contraction, and that it is not necesarily transferable to the pier and abutment. The flow pattern in the pier or abutment case is three dimensional, the velocity in the scour hole is about the same as in the approach (the "pressure" or piezometric gradient determining the scour hole velocity is due to the stagnation rideup of the approach velocity), the scour results from the boundary shear in the nonuniform flow, and any energy losses occur largely downstream of the scour hole as the horseshoe vortex mixes with the general flow.

ADAPTATION TO PIER AND ABUTMENT SCOUR

The solution of the scour in a long contraction was possible because the Manning equation and a sediment-transport relation along with the usual expressions involving continuity, boundary shear, critical tractive force, and Manning's n were sufficient to obtain equations for depth, slope, etc., in the contracted reach. The Laursen sediment-transport relationship was used, but other equations would give similar results; most of them very similar (13). Those which do not result in very similar expressions, predict behavior that does not seem to be quite reasonable. The reason most sediment-transport equations result in almost the same predicted scour in a long contraction is that relative, rather than absolute, rates of sediment transport are involved in the solution. Therefore, only the general form of the equation has to be approximately correct.

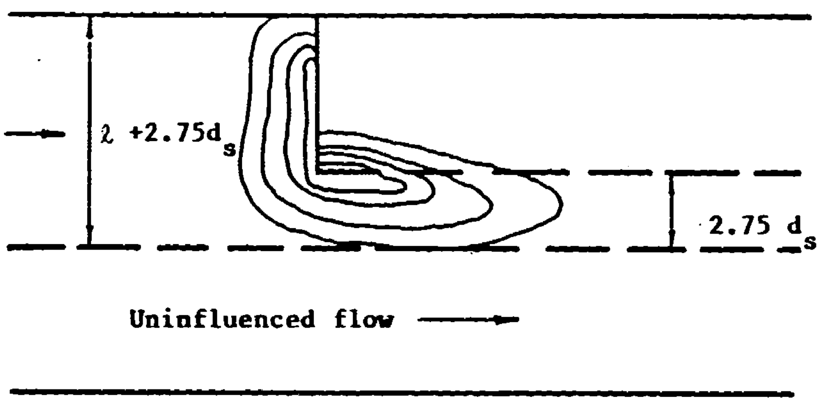
In order to obtain the solution for the scour at a pier or abutment in the same manner as for the long contraction, it would be necessary to be able to describe the flow pattern, the boundary shear pattern, and the sediment-transport pattern equally as well as it is possible to describe these characteristics of the total phenomenon in uniform flow. Fortunately, what cannot be solved in a straightforward manner can sometimes be solved with the aid of a trick or two -- or, more palatably, an assumption or two.

The observations that are needed in order to make some assumptions which serve that purpose are:

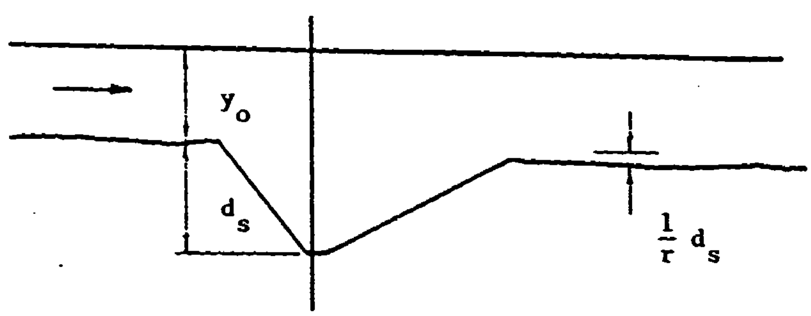
1. The flow and sediment being transported which are beyond the lateral extent of the scour hole behave as if the obstruction and scour hole were not there. (When the scour holes of adjacent obstructions -- either piers and/or abutments -- overlap, there will be some mutual interference; otherwise each obstruction and its scour hole is independent.)
2. The flow over the scour hole, but not obstructed by the pier or abutment, is virtually unchanged.
3. The flow obstructed by the pier or abutment dives into the scour hole, becomes a horseshoe vortex wrapped around the obstruction, and exits in a tail(s) downstream of the obstruction as it gradually mixes with the mainstream flow.
4. The front of the scour hole can be approximated as half of a truncated cone at the angle of repose. A pier at an angle to the flow can distort the cone, as can other geometry of the pier or abutment.

5. The sediment being transported moves straight ahead with the flow approaching the obstruction and scour hole, and falls into the scour hole.

The two key assumptions that can be made on the basis of these observations are that thin walls could be placed in the vicinity of the pier or abutment, as shown in Figure 8 to create a long contraction. In the case of the abutment, one



(a) Plan view



(b) Profile through scour hole

Figure 8. The Fictitious Long Contraction.

wall would be at the outside edge of the scour hole, and another wall would be downstream from the abutment at the end of the embankment. In the case of the pier, a third wall is needed through the centerline of the pier. The width of the approach reach is then

$$B_1 = \ell + 2.75\ d_s$$

and the width of the contracted reach is

$$B_2 = 2.75\ d_s$$

where $2.75\ d_s$ is the lateral extent of the top of the scour hole measured out from the side, or end, of the pier or abutment, and   is the half-width of the pier (b/2), or the effective length of the embankment-abutment

$$\ell = \frac{Q_o}{V_o y_o}$$

where Q_o is the discharge being obstructed on the appropriate floodplain or in the portion of the channel being encroached upon, and V_o and y_o are the velocity and depth of flow in the channel approaching the river side of the scour hole. For very large actual embankment lengths, the pattern of the obstructed flow may not be well described by this simple notion of effective length. A better evaluation of the overbank flow as it approaches the bridge opening requires a two-dimensional flow analysis and detailed knowledge of the geometry and vegetation of the floodplain. At this extreme situation, the flow might return to the channel well upstream of the opening, or it might return to the channel as a confined stream flowing parallel to and upstream of the embankment (depending on the path of least resistance to the flow).

The coefficient 2.75 was obtained from measurements taken in the Iowa experiments. If the bed material has an angle of repose steeper than those sands, the coefficient would be smaller; if flatter, larger. The angle of repose would have to change considerably to make a significant change in the depth of scour that would be predicted. A steeper angle of repose results in a deeper scour because less sediment is supplied to the scour hole.

The depth of scour in the fictitious long contraction can be obtained by using these two widths and the definition of scour depth as the difference in the flow depths of the two reaches. However, this is not the scour depth desired and a second assumption is needed. The scour depth desired is the scour at the pier or abutment, and the assumption is made that this scour is a factor r times the scour in the fictitious long contraction. A little algebraic manipulation will result in the following expression for the bed load case with r = 11.5

$$\frac{\ell}{y_o} = 2.75 \frac{d_s}{y_o} \left[\left(\frac{C_1}{C_2}\right)^{0.44} \left(\frac{1}{11.5} \frac{d_s}{y_o} + 1\right)^{1.69} - 1 \right] \qquad (26)$$

For a river in flood, C_1/C_2 should be close to unity and this term can be dropped from Eq. (25). The coefficient 11.5 is the ratio of the scour at the pier or abutment to that in the ficticious long contraction for the condition that the velocity of the flow being obstructed is about equal to the velocity of the flow approaching the scour hole and supplying sediment to the scour hole. Figures 9 and 10 display Eq. (26) for $C_1/C_2 = 1$ for the embankment-abutment which encroaches into the channel and for the pier, respectively. This solution is for a rectangular pier aligned with the flow or a vertical wall embankment-abutment. For other shapes, multiplying factors to reduce the predicted scour for different geometry are given in Tables 1 and 2.

Table 1: Multiplying Factors for Piers Aligned with the Flow

Nose Form	Length/Width Ratio	K_s
Rectangular		1.00
Semicircular		0.90
Elliptic	2:1	0.80
	3:1	0.75
Lenticular	2:1	0.80
	3:1	0.70

Table 2: Multiplying Factors for Abutment Type For Small Encroachment Length

Abutment Type	K_s
Vertical Wall	1.00
45° Wing Wall	0.90
Spill-Through	0.80

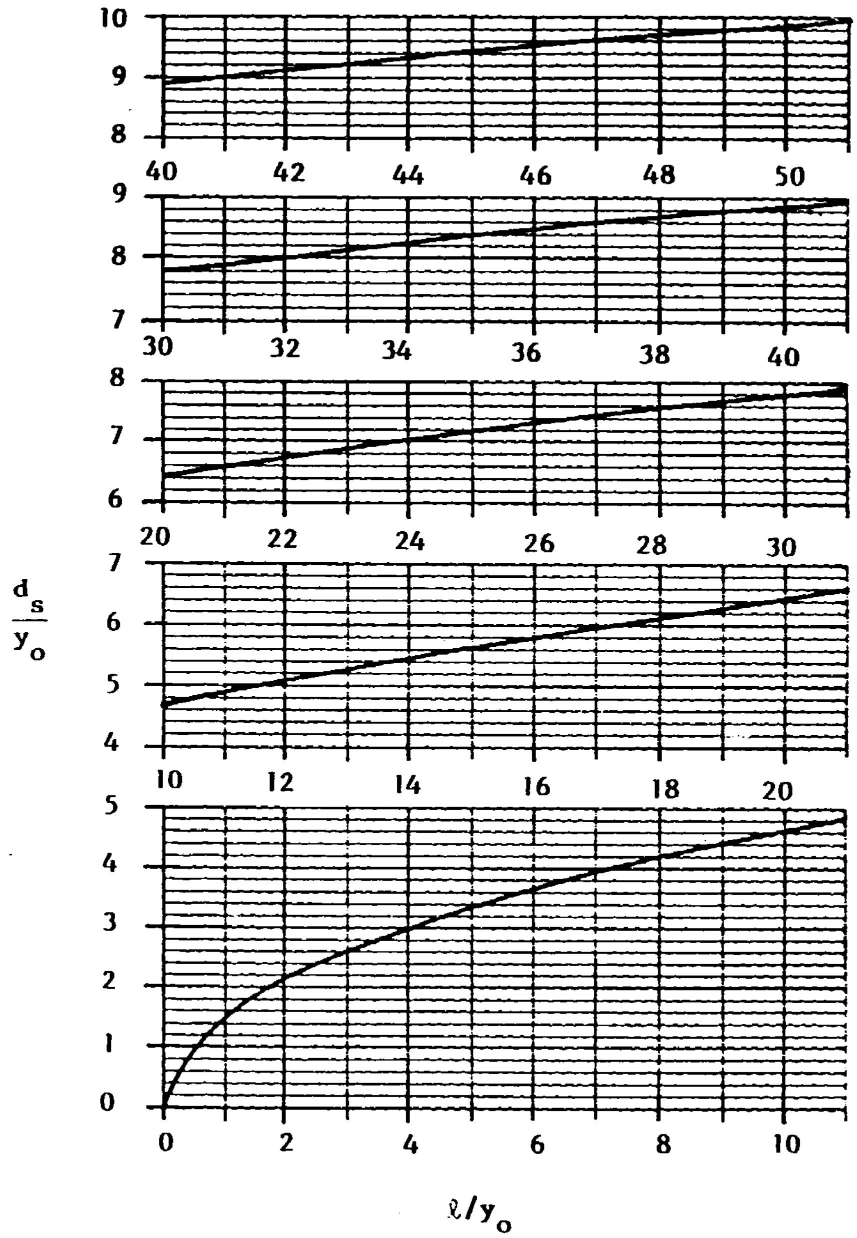


Figure 9. Scour Ratio for Encroaching Embankment-Abutment.

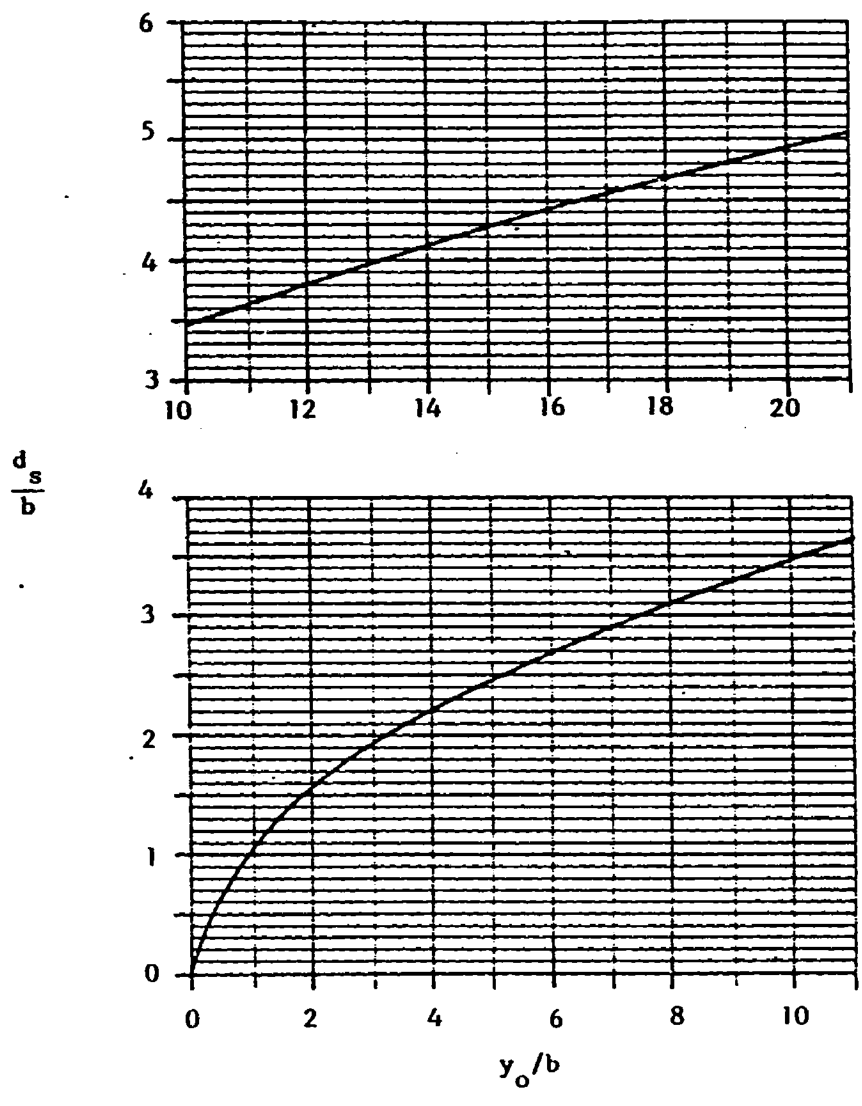


Figure 10. Scour Ratio for a Rectangular Pier Aligned with the Flow.

If the pier is not aligned with the flow, a multiplying factor greater than unity from Figure 11 should be used and the shape factor from Table 1 should <u>NOT</u> be used. Although a round pier does not lose its shape effect as the flow direction becomes misaligned, even a short 1:1-1/2 ellipse loses most (but not quite all) of its shape effect. Two questions that do not (and can not) have completely satisfactory answers are, "What might be the angle of attack during the life of the bridge?" and "How much debris might accumulate during a large flood thereby changing the geometry?" The two questions can be combined if the pier is a line of caissons with a spacing which is not large.

In a like manner, if the bridge does not cross the river at right angles, the scour can be greater or less, depending on the angle of incidence being greater or less than 90 degrees as shown in Figure 12. Finally, there is another multiplying factor to be used if the mode of sediment movement is not bed load, but either some suspended load or mostly suspended load; the amount of suspension being a function of the shear velocity/fall velocity ratio as shown in Figure 13.

A similar adaptation will permit the solution of the embankment-abutment that obstructs low-velocity flow on a floodplain which is not carrying a sediment load of the size of the material which must be scoured. The predicting equation is

$$\frac{Q_o w}{Q_w y_o} = 2.75 \frac{d_s}{y_o} \left[\left(\frac{1}{4.1} \frac{d_s}{y_o} + 1 \right)^{7/6} - 1 \right] \qquad (27)$$

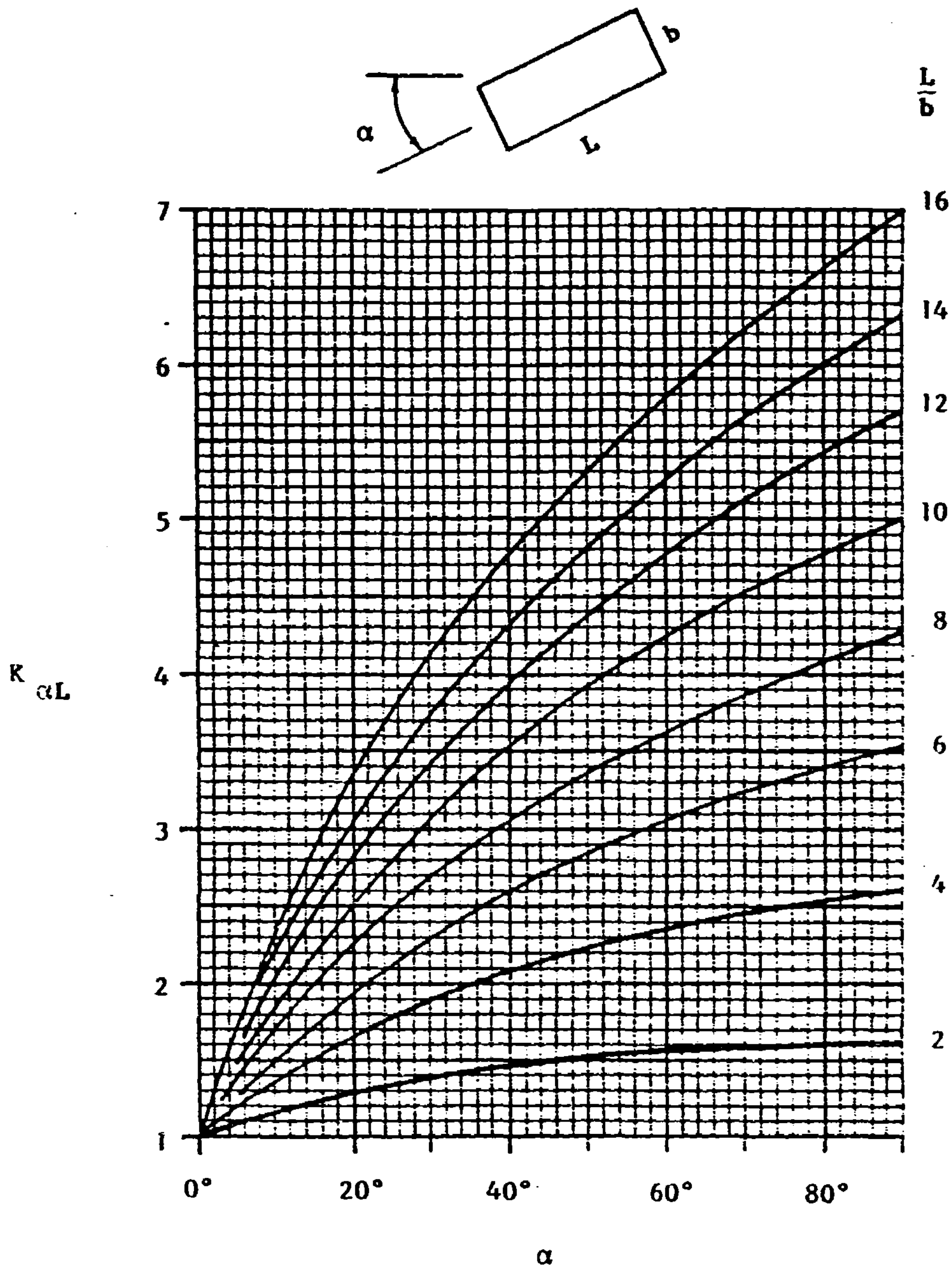


Figure 11. Multiplying Factor for Angle of Attack on Pier.

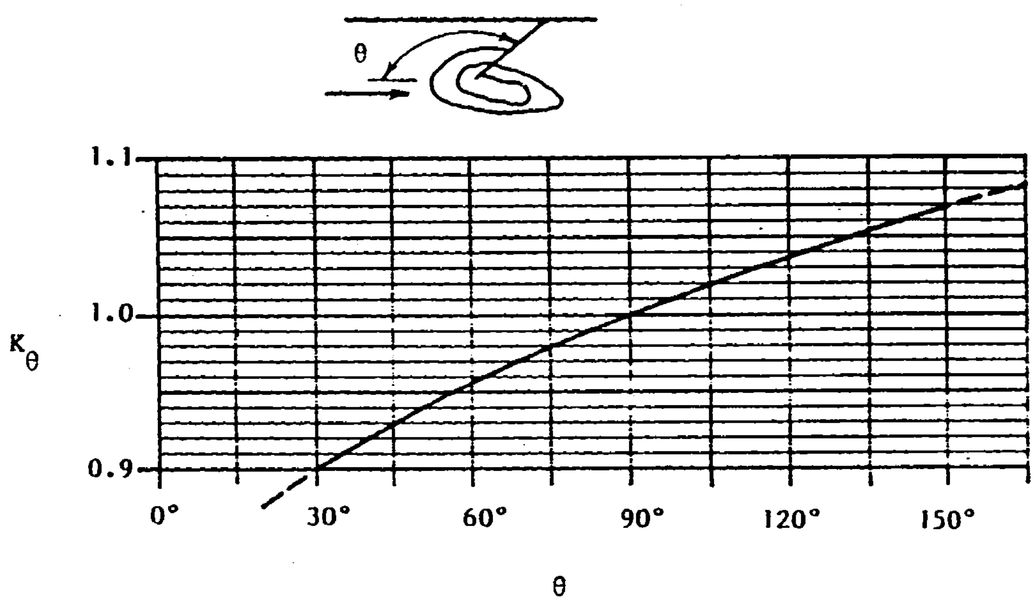


Figure 12. Multiplying Factor for Angle of Incidence of Encroaching Embankment-Abutment.

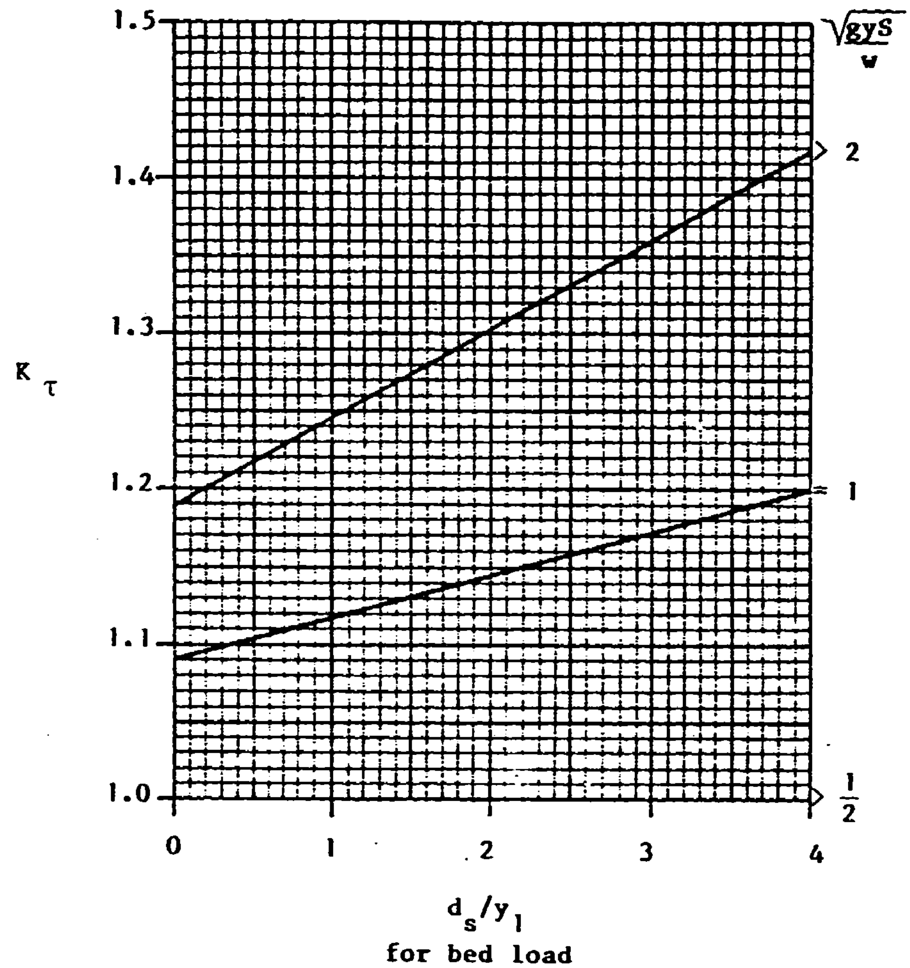


Figure 13. Multiplying Factor for Mode of Movement.

where the coefficient 11.5 is replaced by the value 4.1 (the obstructed flow is relatively low velocity), Q_o is the overbank flow being obstructed, w here is a width about equal to 2.75 d_s and Q_w is the channel flow in the width w approaching the scour hole.

Equation (27) is shown grapically in Figure 14. The dashed line is meant to be a reminder that when Q_o is first evaluated as being small, it is a good idea to check again.

The case of clear-water scour can also be adapted to piers and abutments with the basic equation being

$$\frac{l}{y_o} = 2.75\,\frac{d_s}{y_o}\left\{\frac{\left[\frac{1}{11.5}\,\frac{d_s}{y_o} + 1\right]^{7/6}}{\left[\frac{\tau_o'}{\tau_c}\right]^{1/2}} - 1\right\} \qquad (28)$$

The clear-water relationships for piers and abutments are shown in Figures 15 and 16. In real life this case is of little interest because in floods, rivers generally have a bed load. The relationships, however, can be used to size the riprap needed to stop the scour at some predetermined level that can be tolerated. This case is of greater interest for old existing bridges than for bridges being designed.

The critical assumption in using Eq. (28) for sizing riprap is that riprap of some some size placed at some level below the steambed will stay in a flood and limit the scour depth to that level, and that the size of riprap and the placement level can be predicted as if the entire streambed was

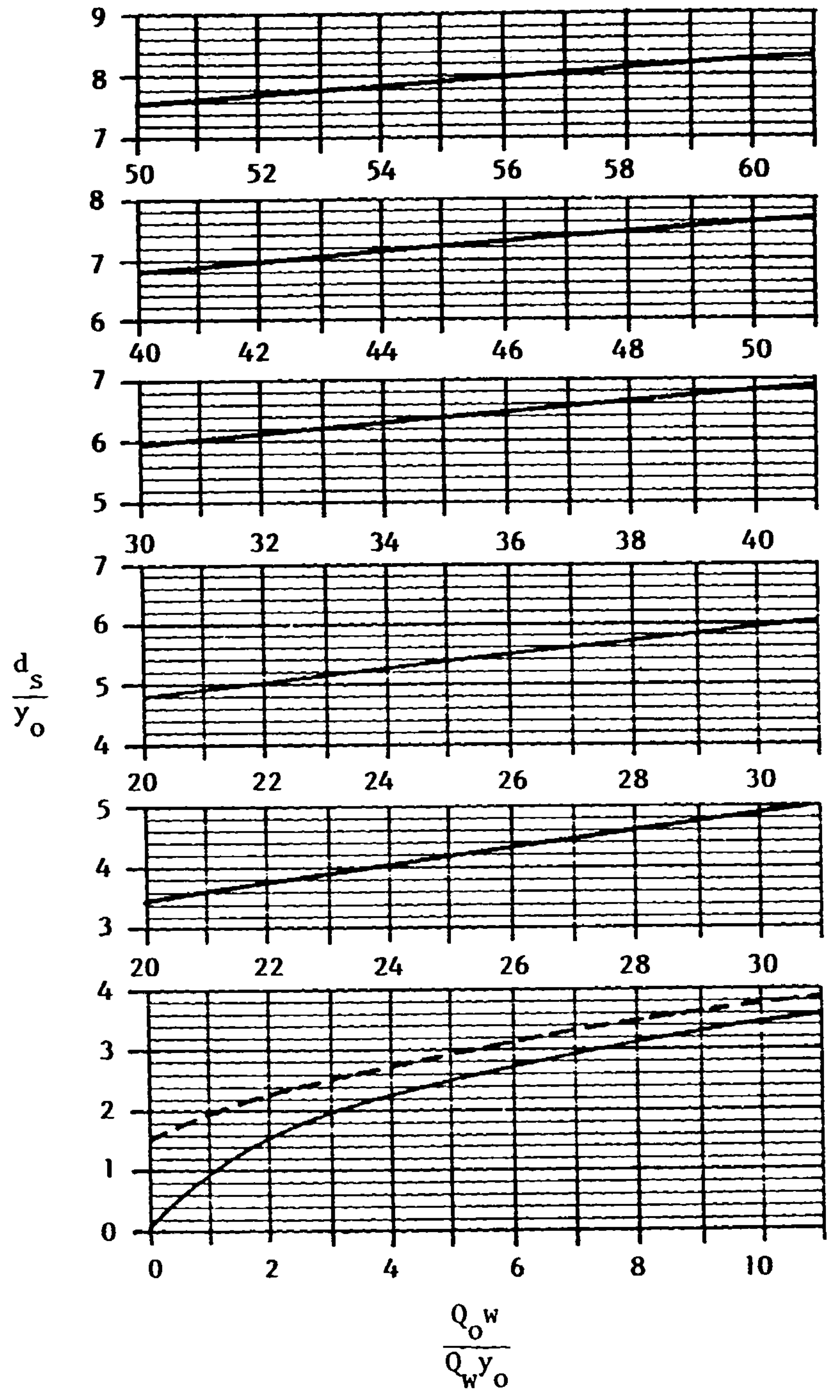


Figure 14. Scour Ratio for Floodplain Constriction.

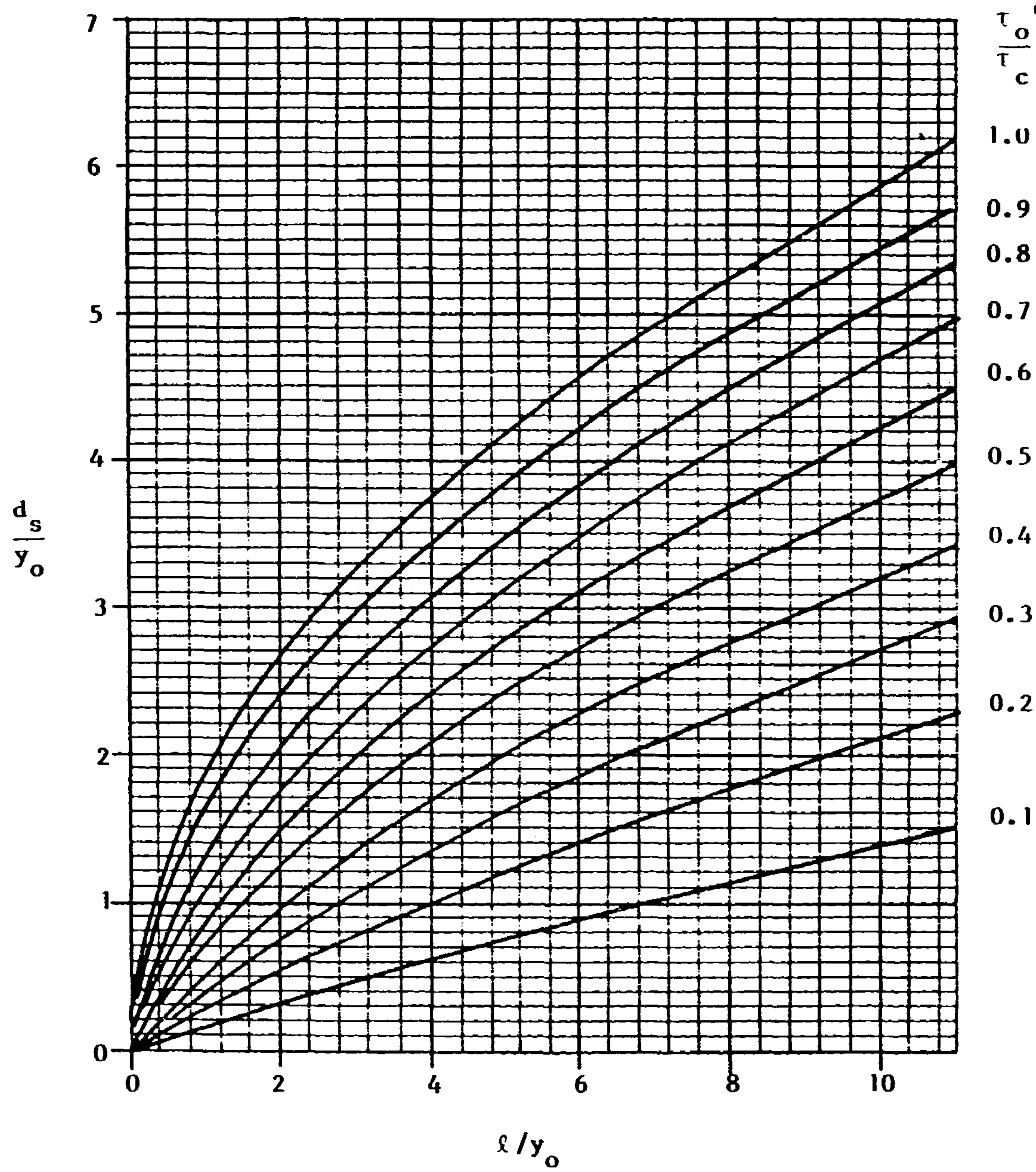


Figure 15. Scour Ratio for Encroaching Embankment-Abutment for Clear-Water Scour.

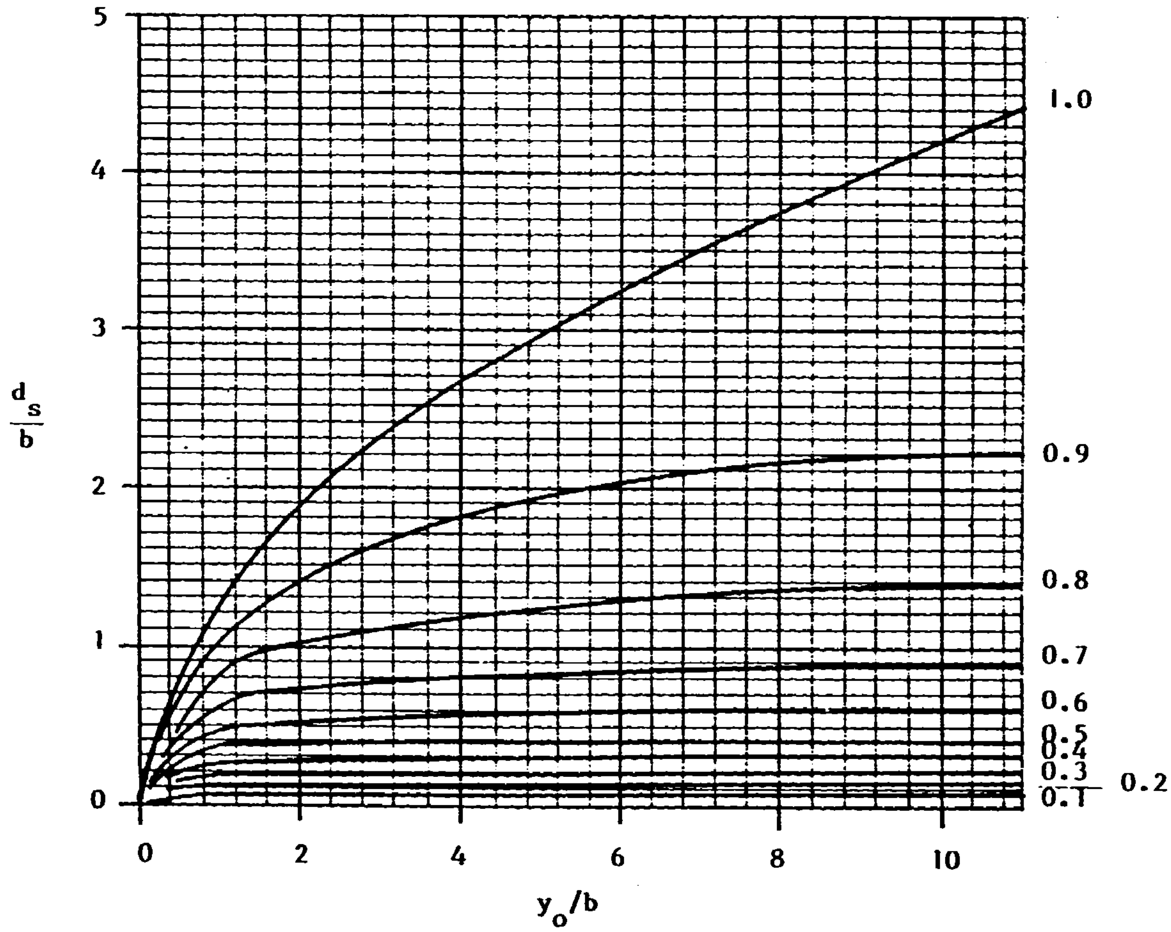


Figure 16. Scour Ratio for Pier for Clear-Water Scour.

composed of the riprap. Preliminary tests indicate this to be so -- surprisingly even without tampering with the coefficients for τ'_0 or τ_c .

THE EXPERIMENTS

Two flumes were used in the investigation. For the investigation of the long contraction, the flume used was 100 feet long and 3 feet wide in the test section -- although a few feet at each end of the flume were discounted because of end effects. The narrow contracted reach was 30 feet long and 1.5 feet wide and centered slightly upstream of the midpoint of the flume. Transition sections at each width change were 10 feet long and composed of two circular arcs. Figure 17 is a photograph of the 100-foot flume.

The flume could be tilted to various slopes, but not during operation. At the head end, sand was supplied from a hopper through a flexible tube which traversed back and forth across the width of the flume. At very low rates of sand feed, the orifice in the bottom of the hopper was so small it would clog and hand feeding at the prescribed rate was necessary. At the highest rates of sand feed, the capacity of the flexible tube was exceeded even though a larger diameter tube was installed and hand feeding by bucket directly into the flume was resorted to. At the highest rate of sand feed, the trap at the tail end of the flume was filled in the time it took to get the readings for the water-surface and bed profiles at two-foot intervals.

Elevations were taken with a point gage fastened to a carriage riding on rails on the wall behind the flume. In supercritical flow, the "sinusoidal" waves over antidunes made

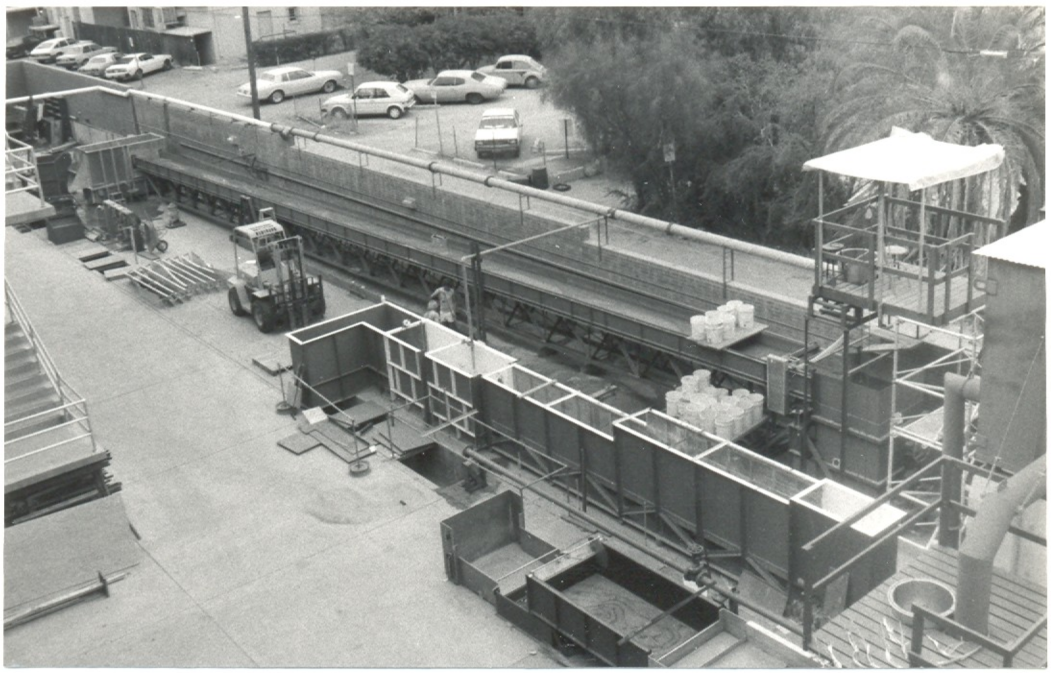


Figure 17. One-Hundred Foot, Long-Contraction Flume.

the water surface very difficult to measure. These waves were especially strong in the transition from the narrow, contracted reach to the normal, wide reach downstream. Elsewhere the train of waves tended to shift from side to side and to come and go. For this condition, the profiles were taken on a line just inside of the wall of the contracted reach in order to minimize the height of the waves.

The point of the point gage was replaced with a half-inch cylinder for the measurement of bed elevation. Positioning of the gage was a matter of feel as much as sight. Especially at higher velocities, a small scour hole could develop as the cylinder approached the bed. Therefore, it was necessary to set the gage on the bed quickly, but not so quickly as to drive the cylinder into the bed. There was probably a systematic error with the measured bed elevations slightly low. However, it is not believed that this error is significant. The possible errors due to the wavy water surface and the ripples, dunes and anti-dunes are larger, but by averaging, the values of depth and slope are sufficiently accurate for the purpose of the study. The values of depth, which are of primary interest, are more reliable than the values of slope.

The smaller flume was only 10 feet long but was 4 feet wide, and could accommodate a vertical wall abutment on one side and a rectangular half-pier on the other side. The flume had the same arrangement for sediment supply at the head end, a trap at the tail end, and a weir for measuring the discharge. This flume is shown in Figure 18.

The abutment model was a vertical wall nominally 6 inches by 12 inches which actually encroached into the flow 6-1/4 inches. The pier half-model was 1 inch wide by 12 inches long. The scour holes around the pier were small and the approach conditions were not sufficiently uniform to be able to obtain meaningful measurements. However, qualitatively the pier exhibited the same scour behavior as the abutment.

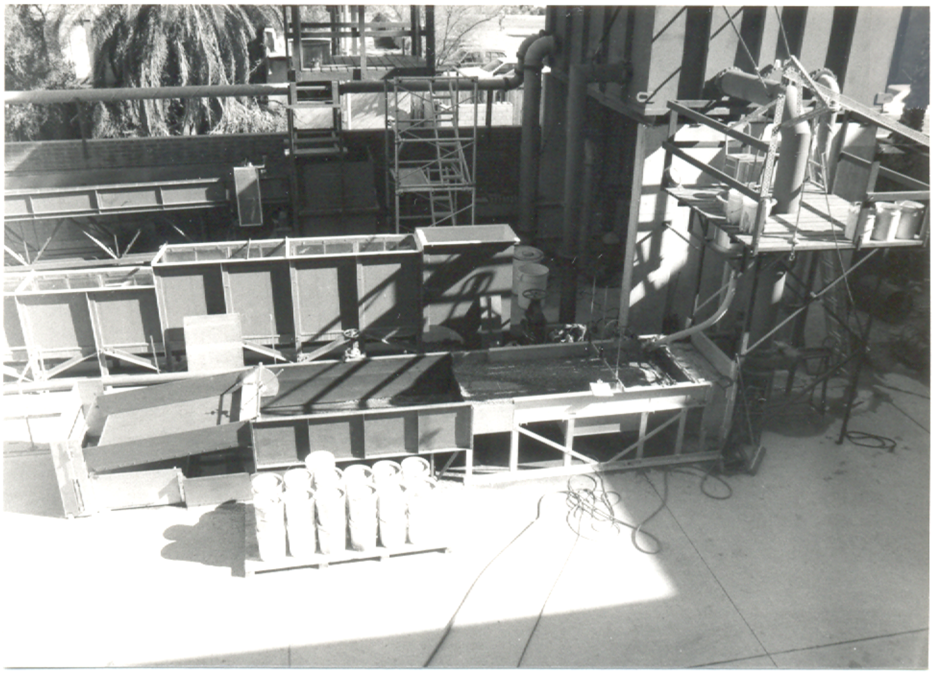


Figure 18. Pier-Abutment Flume.

Because the flume was so short, profiles were not taken, and the measurements simply established depth of flow, depth of scour, width of scour and, of course, discharge. The point gages, like those used with the 100-foot flume, were attached in this case to an angle which rested on the flume walls.

Two sediments were used in the experiments: a pea gravel with a mean size of 5.6mm and a sand with a mean size of 1.35mm. The size distribution of the two sediments are shown in Figure 19.

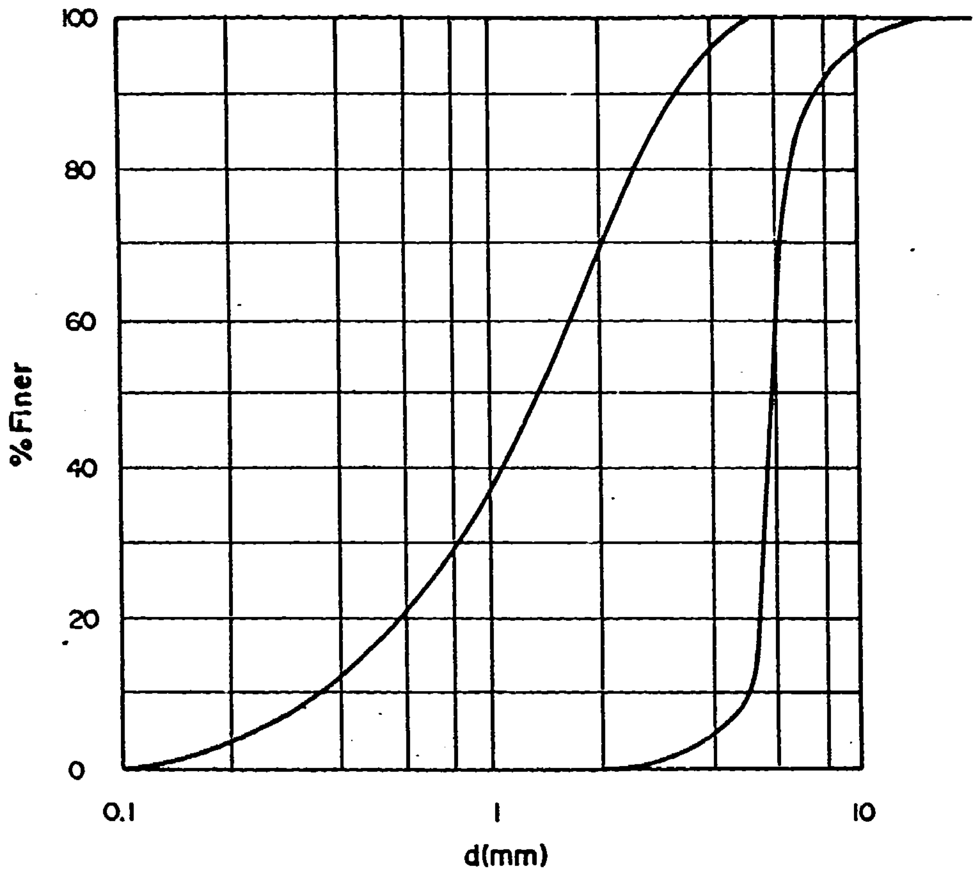


Figure 19. Size Distribution of Sand and Gravel.