PREDICTION OF RESILIENT MODULUS FROM SOIL INDEX PROPERTIES

FINAL REPORT

By

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16. Abstract

Subgrade soil characterization in terms of Resilient Modulus (M_R) has become crucial for pavement design. For a new design, M_R values are generally obtained by conducting repeated load triaxial tests on reconstituted/undisturbed cylindrical specimens. Because the test is complex and time-consuming, in-situ tests would be desirable if reliable correlation equations could be established. Alternately, M_R can be obtained from correlation equations involving stress state and soil physical properties. Several empirical equations have been suggested to estimate the resilient modulus. The main focus of this study is to substantiate the predictability of the existing equations and evaluate the feasibility of using one or more of those equations in predicting resilient modulus of Mississippi soils. This study also documents different soil index properties that influence resilient modulus.

Correlation equations developed by the Long Term Pavement Performance (LTPP), Minnesota Road Research Project, Georgia DOT, Carmichael and Stuart Drumm et al., Wyoming DOT, and Mississippi DOT are studied/analyzed in detail. Eight road (subgrade) sections from different districts are selected and soils tested (TP 46 Protocol) for M_R in the laboratory. Other routine laboratory tests are conducted to determine physical properties of the soil. Validity of the correlation equations are addressed by comparing measured M_R to predicted M_R. In addition, variations expected in the predicted M_R due to inherent variability in soil properties is studied by the method of point estimates. The results suggest that LTPP equations are suited for purposes of predicting resilient modulus of Mississippi subgrade soils. For fine-grain soils, even better predictions are realized with the Mississippi equation.

A sensitivity study of those equations suggests that the top five soil index properties influencing M_R include moisture content, degree of saturation, material passing #200 sieve, plasticity index and density.

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PREDICTION OF RESILIENT MODULUS FROM SOIL INDEX PROPERTIES: A CRITICAL REVIEW

ABSTRACT

Subgrade soil characterization in terms of Resilient Modulus (M_R) has become crucial for pavement design. For a new design, M_R values are generally obtained by conducting repeated load triaxial tests on reconstituted/undisturbed cylindrical specimens. Because the test is complex and time-consuming, in-situ tests would be desirable if reliable correlation equations could be established. Alternately, M_R can be obtained from correlation equations involving stress state and soil physical properties. Several empirical equations have been suggested to estimate the resilient modulus. The main focus of this study is to substantiate the predictability of the existing equations and evaluate the feasibility of using one or more of those equations in predicting resilient modulus of Mississippi soils. This study also documents different soil index properties that influence resilient modulus.

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INTRODUCTION

1.1 Background

Characterizing subgrade soils in terms of resilient modulus (M_R) is essential for pavement design of both flexible and rigid pavements. The 1986 AASHTO guide for design of flexible pavements (1) replaces soil support value (SSV) and recommends the use of M_R for characterizing the subgrade soil as it indicates a basic material property which can be used in mechanistic analysis of multi-layered systems. M_R attribute has been recognized widely for characterizing materials in pavement design and evaluation. Resilient modulus is a measure or estimate of the elastic modulus of the material at a given stress or temperature. Mathematically it is expressed as the ratio of applied deviator stress to recoverable strain.

$$M_R = \sigma_d / \varepsilon_r \tag{1.1}$$

where, σ_d = Applied deviator stress

ε_r = Resilient strain.

M_R is generally estimated directly in the laboratory using repeated load triaxial testing, indirectly through correlation with other standard tests, or by back calculating from deflection tests results. For a new design, M_R is generally obtained by conducting repeated load triaxial tests on reconstituted/undisturbed samples, according to harmonized test protocol, NCHRP1-28A (2). Because tests are complex and time consuming, in-situ tests would be desirable if reliable correlation could be established. Alternately, resilient modulus can be obtained from the correlation equations involving stress state and soil physical properties.

The 1993 AASHTO Guide for Design of Pavement Structures: Appendix L (3), lists four different approaches to determine a design resilient modulus value. The first approach is laboratory testing, another approach is by Non-Destructive Testing (NDT) backcalculation, the third approach consists of estimating resilient modulus from correlations with other properties, and the last is from original design and construction data. In 1995, Darter, et al.(4), reported that about 75 percent of the State Highway Agencies (SHAs) in the United States use either 1986 or 1993 versions of the AASHTO design guide. However, most of the agencies do not routinely measure the M_R in the laboratory, but estimate from experience or from other material or soil properties; i.e., CBR, R-value or physical properties.

1.2 Why this Study?

Two types of correlation equations have been developed in predicting resilient modulus from soil physical properties; they will be described in detail in the next chapter. Since several equations are available from past studies, there is a need to substantiate the predictability of these equations. Those equations, if proved to be valid, could serve a vital role in proposing a preliminary pavement design for budgeting purposes. Final design can await completion of the grading contract, followed by additional in-situ tests.

1.3 Objective and Scope:

As suggested in the 1993 AASHTO Guide, resilient modulus can also be predicted directly from correlation equations involving soil index properties. Long Term Pavement Performance (LTPP) (5), Minnesota Road Research project (6), Santha (7), Carmichael and Stuart (8), Drumm (9), Farrar and Truner (10) and the Mississippi equation derived by Ashraf and George (11) are of special interest in this study. A few other researchers also have developed correlation equations to predict the resilient modulus from soil physical properties.

The primary objective of this study is to validate the existing equations cited in the literature (5,6,7,8,9,10 and 11) and evaluate the feasibility of using one or more equations for predicting the design resilient modulus of Mississippi soils. To achieve this objective, eight road (subgrade) sections representing typical Mississippi soils were selected and tested in the laboratory for M_R in accordance with the AASHTO TP46 test protocol. In addition, routine laboratory tests were conducted on the soils to determine the physical properties. Validation of the equations is accomplished by comparing measured M_R with the predicted M_R. In addition, expected variation in predicted values owing to inherent soil variability is investigated, employing the Point Estimate (PE) method. Model sensitivity is examined by evaluating to what extent each independent (response) variable effects the predicted M_R value.

CHAPTER 2

REVIEW OF LITERATURE

2.1 Introduction

The objective of pavement design is to provide a structural and economical combination of materials such that it serves the intended traffic volume in a given climate over the existing soil conditions for a specified time interval. Traffic volume, environmental loads, and soil strength determine the structural requirements of a pavement, and failure to characterize any of them adversely affects the pavement performance. Traffic is estimated from present traffic and traffic growth projections. Climatic conditions are incorporated in the design by accounting for their effects on material properties. The subgrade may be characterized in the laboratory or by field tests or both. It is essential that methods adopted to characterize reflect the actual subgrade's role in the pavement structure, and the frequency of the sampling should account for spatial variation in the field. As noted by Yoder and Witczak (12) "all pavements derive their ultimate support from the underlying subgrade: therefore, knowledge of basic soil mechanics is essential."

The AASHTO guide for the design of pavement structures, which was proposed in 1961 and then revised in 1972, characterized the subgrade in terms of soil support value (SSV). SSV has a scale ranging from 1 to 10, with a value of 3 representing the natural soil at the Road Test. In the revised 1986 AASHTO guide, the road bed resilient modulus, M_R, was selected to replace the SSV, used in the previous editions of the Guide, for the following reasons:

1. It indicates a basic material property, which can be used in mechanistic analysis of multi-layered systems for predicting roughness, cracking, rutting, faulting, etc.

2. M_R has been recognized internationally as a method for characterizing materials for use in pavement design and evaluation.
3. Methods for determination of M_R are described in AASHTO Test Method T274-82 and others, the latest being Harmonized Test, NCHRP 1-28A.
4. Techniques are available for estimating the resilient properties of various materials in-place by non-destructive tests.

2.2 Why Repeated Load Triaxial Test for Determination of M_R

Since the pavement materials are subjected to a series of distinct load pulses, a laboratory test duplicating this condition is desirable. The repeated load type test has been used for many years to simulate vehicle loading. In this test, cylindrical specimens of soil are subjected to a series of load pulses applied with a distinct rest period, simulating the stresses caused by multiple wheels moving over the pavement. A constant all-around confining pressure applied on the specimen simulates the lateral stresses caused by the overburden pressure and applied wheel load. The total resilient (recoverable) axial deformation response of the specimen to the stress pulses measured is used to calculate the resilient modulus of the material. Cited below are two reasons favoring the use of repeated load triaxial test for determination of M_R (13).

- In the triaxial test, predetermined principal stresses σ_1 and σ_3 are applied to the specimen; therefore, the stress conditions within the specimen on any plane are defined throughout the test. The stress conditions applied are, in fact, those which occur when an isolated wheel load is applied to the pavement directly above the element of material simulated in the test.
- Axial, radial, and volumetric strains can all be measured in the triaxial test.

For about the last 35 years the repeated load triaxial compression test has been the basic test procedure to evaluate resilient modulus of cohesive and granular materials for pavement design applications.

2.3 Laboratory Test to Determine Resilient Modulus

The 1986 AASHTO Guide has stipulated and the 2002 Guide reaffirmed, that the M_R be the parameter for characterizing the subgrade. Responding to the need, AASHTO T278-82 laboratory test was proposed to describe the behavior of pavement materials subjected to moving traffic. In 1991, AASHTO modified the T278-82 testing procedure in terms of sample conditioning, load magnitude, and load application. With the revised test designation it changed to TP292-92I. Later, TP46-94, a "harmonized" M_R test protocol, was proposed in the NCHRP 1-28A study; and the latest is the P46 proposed by LTPP. In this study, samples, undisturbed/reconstituted, are subjected to the repeated load triaxial test in accordance with the AASHTO (TP-46) protocol to determine the resilient modulus. For undisturbed samples, Shelby tube sampling is relied upon. Reconstituted samples are molded in the laboratory to obtain desired density and moisture content representative of the field.

The sample in the repeated load triaxial test is subjected to a combination of three confining stresses and five deviator stresses, thus yielding 15 resilient modulus values for each sample. Now, a constitutive model comprising M_R-stress relation is chosen, describing the resilient property of the material. This model is then fitted to the data of each sample so the M_R for a desired stress state can be obtained.

Despite several improvements made over the years, Seed et al. (14) cited the following uncertainties as well as limitations associated with the test procedure.

1. The laboratory resilient modulus sample is not completely representative of in-situ conditions because of sample disturbance and differences in aggregate orientation, moisture content and level of compaction.
2. Inherent equipment flaws make it difficult to simulate the state of stress of material in-situ.
3. Inherent instrumentation flaws create uncertainty in the measurement of sample deformation.
4. Lack of uniform equipment, calibration, and verification procedures may lead to differences between the laboratories and within a given laboratory.

Overall, these issues have kept M_R testing from achieving general acceptance by the researchers as well as user agencies.

2.4 Factors Affecting Resilient Modulus

The resilient modulus of fine-grain soils is not a constant stiffness property (15) but depends upon various factors like load state or stress state, which includes the deviator and confining stress, soil type and its structure, which primarily depends on compaction method and compaction effort of a new subgrade. Previous studies show that deviator stress is more significant than confining stress for fine-grain soils. Resilient modulus is found to increase with a decrease in moisture content and an increase in density, and decrease with an increase in deviator stress.

For coarse-grain soils, M_R is primarily influenced by the stress state, degree of saturation and compactive effort (density). Research (10, 16) has shown that M_R increases with increasing confining stress. Studies have also indicated that there is a critical degree of saturation near 80-85 percent, above which granular material becomes unstable and undergoes degradation rapidly under repeated loading. Lekarp et al. (17) noted, and other researchers concur that M_R of granular materials increases with increasing confining stress and sum of principal stresses, otherwise known as bulk stress (θ), and slightly increases with deviator stress.

2.5 Resilient Modulus Based on Single Soil Parameter

Simple correlation equations have been reported to predict M_R from standard California Bearing Ratio (CBR), R value, and soil physical properties. Heukelom and Klomp (18) reported correlation between the Corps of Engineers CBR value using dynamic compaction and the in-situ resilient modulus of soil.

$$M_R \text{ (psi)} = 1500 \text{ CBR} \tag{2.1}$$

The data used for developing this equation ranged from 750 to 3000 times CBR. Equation (2.1) has been extensively used by design agencies and researchers for fine grained soils with a soaked CBR of 10 or less.

The Georgia Department of Transportation tested a number of cohesionless soils in repeated load triaxial test following the AASHTO procedure (19). The objective was to create a database so that the resilient modulus can be predicted. A typical equation for medium clay sand follows:

$$M_R \text{ (psi)} = 3116 \text{ (CBR)}^a \tag{2.2}$$

where $a = 0.4779707$

The results showed no significant change in resilient modulus as long as soils are within ± 1.5 % of optimum.

Though CBR is widely used to characterize subgrade soils, it is a measure of shear strength, which is not necessarily expected to correlate with modulus or stiffness. Thompson and Robnett (20) reported weak correlation between CBR and resilient modulus of Illinois soils. Besides, CBR-based relationships do not recognize the stress dependence on modulus (21) and are, therefore, critiqued to be inadequate.

Similar relationships were developed by the Asphalt Institute (22) which related R value to resilient modulus. Their equation is,

$$M_R \text{ (psi)} = A + B \text{ (R value)} \tag{2.3}$$

where, A = 772 to 1155;

B = 369 to 555; and

R value = Stabilometer value, lbs

Yeh and Su (23) of the Colorado Department of Highways tested the resilient properties of Colorado soils with the objective of establishing a correlation between resilient modulus and stabilometer R value. Triaxial modulus was determined adopting a procedure different from AASHTO T274. The equation finally derived between the M_R and R value is as follows:

$$M_R \text{ (psi)} = 3500 + 125 \text{ (R value)} \tag{2.4}$$

The fundamental problem with empirical relationships developed to correlate resilient modulus to soil parameters such as CBR or R value is that those tests themselves are pretty much empirical. Whereas resilient modulus is a mechanistic parameter and dependent on a host of soil index properties and stress state.

2.6 Regression Equations for Resilient Modulus Based on Soil Properties and Stress State

Carmichael and Stuart in 1985 (8) studied the resilient properties of soils with the objective of developing correlation equations for predicting subgrade modulus from basic soil tests. The Highway Research Information Service (HRIS) database was searched to compile the necessary data for correlation analysis. Regression studies were made for individual soil types according to the Unified Soil Classification system. Two models were developed, one for fine-grain soils and another for coarse-grain soils. Equation 2.5 presents the model for coarse-grain soils.

$$\text{Log } M_R = 0.523 – 0.025(w_c) + 0.544(\log \theta) + 0.173(SM) + 0.197(GR) \quad (2.5)$$

where, M_R = Resilient Modulus, ksi;

w_c = moisture content, %;

θ = bulk stress ($\sigma_1 + \sigma_2 + \sigma_3$), psi;

SM = 1 for SM soils (Unified Soil Classification)

= 0 otherwise; and

GR = 1 for GR soils (GM, GW, GC or GP)

= 0 otherwise.

A different equation was derived for fine-grain soils:

$$M_R = 37.431 – 0.4566(PI) – 0.6179(w_c) – 0.1424(P_{200}) + 0.1791(\sigma_3) – 0.3248(\sigma_d) + 36.722 (CH) + 17.097 (MH). \quad (2.6)$$

where, PI = plasticity index, %;

P_{200} = percentage passing #200 sieve;

σ_3 = confining stress, psi;

σ_d = deviator stress, psi;

CH = 1 for CH soil

= 0 otherwise (for MH, ML or CL soil); and

MH = 1 for MH soil

= 0 otherwise (for CH, ML or CL soil).

Drumm et al. (9) conducted a resilient modulus study on cohesive soils, employing AASHTO test specifications. The authors tried to establish a simple procedure for modulus testing. Their result showed that unconfined compressive strength, q_u, is a better property to predict M_R. Accordingly, they correlated the soil index properties and the initial tangent modulus obtained from unconfined compression test to the resilient modulus. A statistical model developed with a nonlinear relationship between resilient modulus and the deviator stress follows:

$$M_R \text{ (ksi)} = \frac{a' + b' \sigma_d}{\sigma_d} \quad \text{for } \sigma_d > 0 \tag{2.7}$$

where, $a' = 318.2 + 0.337 (q_u) + 0.73(\%Clay) + 2.26(PI) - 0.915(\gamma_s) - 2.19(S) - 0.304(P_{200})$; (2.8)

$b' = 2.10 + 0.00039(1/a) + 0.104(q_u) + 0.09(LL) - 0.10 (P_{200})$; (2.9)

q_u = unconfined compressive strength, psi;

$1/a$ = initial tangent modulus, psi, obtained from unconfined compression tests;

%Clay = percent clay;

LL = liquid limit, %;

S = degree of saturation; and

γ_s = dry density, pcf.

It was concluded that a similar relationship could be established for soils other than those investigated and might be helpful to agencies that lack the capability for complex repeated load testing.

Two regression models, to predict resilient modulus, were developed, employing thirteen fine-grain Wyoming soils (10). In the first model, R value is the primary response variable, and in the other soils index properties and stress state, namely, σ_3. The latter equation, investigated in this study, follows:

$$MR = 34280 - 359 \, S\% - 325 \, \sigma_d + 236 \, \sigma_3 + 86 \, PI + 107 \, P_{200} \tag{2.10}$$

Note that the authors of the Wyoming study observed that resilient modulus is negatively correlated with degree of saturation and positively associated with PI and P_{200}.

Ashraf and George (11) investigated the relevance of soil index properties in predicting resilient modulus of Mississippi soils. Two equations, herein referred to as Mississippi equations, were proposed, one for fine-grain soil and another for coarse-grain soil. The former equation developed using 12 soils from Mississippi, had been substantiated with eight other soils, also from Mississippi. The two models are presented here:

Fine-grain soil:

$$M_R\ (\mathrm{MPa}) = 16.75((LL/w_c\ \gamma_{dr})^{2.06} + (P_{200}/100)^{-0.59}) \tag{2.11}$$

Coarse-grain soil:

$$M_R\ (\mathrm{MPa}) = 307.4\ (\gamma_{dr}/\ w_c)^{0.86}\ (P_{200}/\log c_u)^{-0.46} \tag{2.12}$$

where, γ_{dr} = dry density/maximum dry density; and

c_u = uniformity coefficient

2.7 Resilient Modulus Constitutive Models

The concept of resilient modulus has been used to explain the nonlinear stress-strain characteristics of subgrade soils. During the past two decades, several constitutive models have been proposed by many researchers for modeling resilient moduli of soils and aggregates. No stress or deformation analysis can be meaningful unless a correct constitutive equation describing the actual behavior of the material has been used in the analysis. In 1963, Dunlap (24) suggested the following relationship for presenting resilient modulus data:

$$M_R = k_1\ (\sigma_3/P_a)^{k2} \tag{2.13}$$

where, k_1, k_2 = regression coefficients obtained from regression analysis,

P_a = reference pressure (atmospheric pressure); and

σ_3 = confining stress.

However, this relationship does not consider the effect of deviator stress on resilient modulus.

Seed et al. (25) suggested a relation where resilient modulus is a function of bulk stress (θ), also known as the K–θ model. This model, generally adopted for granular soils, uses θ as the main attribute in the model.

$$M_R = k_1 \, (\theta/P_a)^{k2} \tag{2.14}$$

Where, θ = bulk stress ($\sigma_1 + \sigma_2 + \sigma_3$).

The main drawback of this model is that it does not account for shear stresses and shear strains developed during loading and is, therefore, applicable only in the range of low-strain values (26). Brown and Pappin (27) noted that this model does not properly handle volumetric strains or dilative behavior of soils. Moreover, this model potentially provides the same resilient properties for the same bulk stress input. This shows that the model does not incorporate the realistic responses of confining and deviator stresses on resilient properties.

Moossazadeh and Witczak (28) proposed a relation known as the deviator stress model recommended for reporting cohesive soil results, known as K-σ_d model. It uses deviator stress as the main and only attribute of the model.

$$M_R = k_1 \, (\sigma_d / P_a)^{k2} \tag{2.15}$$

where, σ_d = deviator stress (σ_1 - σ_3).

Though this model does not consider the effect of confining stress on resilient modulus, for clay soils, this aspect is still considered insignificant since cohesive soils derive their overall strength mainly from cohesion rather than from frictional characteristics. This modeling approach is perhaps adequate for cohesive soils found at shallow depths, but for soils found at greater depth under high traffic loads, it is necessary to include confining stress in addition to deviator stress (23).

May and Witczak (29) and LTPP (30) proposed a model to describe the nonlinear behavior revealed in the repeated load triaxial test. This model considers the effects of shear stress, confining stress and the deviator stress with the model formulated in terms of bulk and deviator stress.

$$M_R = k_1 P_a (\theta / P_a)^{k2} (\sigma_d / P_a)^{k3} \tag{2.16}$$

Uzan in 1992 introduced octahedral shear stress in place of deviator stress in Equation (2.16), which provided a better explanation for the stress state of the material, in which the normal and shear stress change during loading. The proposed model is known as the k_1- k_3 model or universal model. The universality of this model stems from its ability to conceptually represent all types of soils from pure cohesive soils to non-cohesive soils.

$$M_R = k_1 P_a (\theta / P_a)^{k2} (\tau_{oct}/P_a)^{k3} \tag{2.17}$$

where, $\tau_{oct} = \frac{1}{3}\left((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right)^{1/2}$

The coefficients k_1, k_2, and k_3 are constants, dependent on material type and physical properties, and are obtained from regression analysis. Since coefficient k_1 is proportional to Young's modulus, it should always be positive as M_R can never be negative. The coefficient k_2 should be positive, because increasing the volumetric stress produces stiffening or hardening of the material, yielding higher modulus. The coefficient k_3 should be negative because an increase in the shear stress softens the material, thereby yielding lower modulus. If nonlinear property coefficients k_2 and k_3 are set to zero, then the model can be simplified as linear elastic. If k_3 is zero, the behavior could be non-linear hardening and if k_2 is zero, the behavior is non-linear softening.

Various modified versions of the universal equation are currently used to predict/calculate M_R. A modified version, proposed in Long Term Pavement Performance research (31), has been adopted in this study. The model takes the following form:

$$Log(M_R/P_a) = k_1 + k_2 Log(\theta/P_a) + k_3 Log(\tau_{oct}/P_a) + k_4 [Log(\tau_{oct}/P_a)]^2 \quad (2.18)$$

In the above expression, an additional second order term of octahedral shear stress has been introduced, since there is a reasonably strong trend for TP46 results to be more nonlinear in octahedral shear stress (31).

2.8 Prediction Models of M_R Based on Constitutive Equation

Most of the State Highway Agencies in the United States do not routinely measure M_R in the laboratory but estimate the design M_R either from experience or from other material properties. The potential benefit of estimating M_R from soil physical properties is that the seasonal variations in the M_R can be determined from seasonal changes in the material's properties; however, the effect of stress sensitivity is not captured. In order to capture the effects of stress sensitivity and physical properties on design M_R, Von Quintos and Killingsworth (32), Dai et al.(6), Santha (7) and Mohammad et al. (33), among others, have developed prediction equations for M_R by regressing the coefficients of selected constitutive equations relating them to soil physical properties.

Researchers in the past have developed relationships between the soil properties and the regression coefficients (k_1, k_2, k_3) of the constitutive equation. Those relations that have good statistics were generally confined to specific soil types (7). Other studies that used a wide range of soils and conditions resulted in poor correlations (32). Included is a list of studies to predict M_R based on the constitutive relations and physical properties:

1. Long Term Pavement Performance Study (5);
2. Georgia Department of Transportation Research study (7);
3. Minnesota Road Research Project (6); and

4. Louisiana Study (33).

The primary soil properties which influence resilient modulus are moisture content, density and percent passing # 200 sieve, and liquid limit and plasticity index. It was observed that M_R increases with an increase in density and decreases with an increase in moisture content above optimum. Hence, these soil index properties were invariably used to frame correlation equations. Other properties like compressive strength, degree of saturation, percent clay, percent silt and CBR were also used in a few equations.

The LTPP-FHWA study program (5) is a comprehensive review of the resilient modulus test data measured on pavement materials and soils recovered from the LTPP test sections. The M_R data was reviewed in detail to identify anomalies or potential errors in the database, and the effect of test variables such as the test and sampling procedures on the resilient modulus. The resilient modulus data was further investigated to evaluate relationships between M_R and the physical properties of the unbound materials and soils. Equations for each base and soil type were developed to calculate M_R at a specific stress state from physical properties of the base materials and soils using nonlinear regression optimization techniques.

The constitutive equation used is of the form:

$$M_R = k_1 P_a (\theta / P_a)^{k2} [(\tau_{oct}/P_a) + 1]^{k3} \quad (2.19)$$

The regression constants k_1, k_2 and k_3 are material-specific, as listed in the following equations, 2.20-2.28.

- For coarse-grained sand soils, the $k_1 - k_3$ constants are described as follows:

$$k_1 = 3.2868 - 0.0412\, P_{3/8} + 0.0267\, P_4 + 0.0137\, (\%Clay) + 0.0083\, LL - 0.0379\, w_{opt} - 0.0004\, \gamma_s \quad (2.20)$$

$k_2 = 0.5670 + 0.0045\, P_{3/8} - 2.98x10^{-5}\, P_4 - 0.0043\, (\%Silt) - 0.0102\, (\%Clay) - 0.0041$

$$LL + 0.0014\ w_{opt} - 3.41\text{x}10^{-5}\ \gamma_s - 0.4582\ (\gamma_s / \gamma_{opt}) + 0.1779\ (w_c/w_{opt}) \quad (2.21)$$

$$k_3 = -3.5677 + 0.1142\ P_{3/8} - 0.0839\ P_4 - 0.1249\ P_{200} + 0.1030\ (\%Silt) + 0.1191\ (\%Clay) - 0.0069LL - 0.0103\ w_{opt} - 0.0017\ \gamma_s + 4.3177(\gamma_s\ /\ \gamma_{opt}) - 1.1095\ (w_c/w_{opt}). \quad (2.22)$$

- Fine-grain silt soils:

$$k_1 = 1.0480 + 0.0177\ (\%Clay) + 0.0279\ PI - 0.0370\ w_c \quad (2.23)$$

$$k_2 = 0.5097 - 0.0286\ PI \quad (2.24)$$

$$k_3 = -0.2218 + 0.0047\ (\%Silt) + 0.0849\ PI - 0.1399\ w_c \quad (2.25)$$

- Fine-grain clay soils:

$$k_1 = 1.3577 + 0.0106\ (\%Clay) - 0.0437\ w_c \quad (2.26)$$

$$k_2 = 0.5193 - 0.0073\ P_4 + 0.0095\ P_{40} - 0.0027\ P_{200} - 0.003\ LL - 0.0049\ w_{opt} \quad (2.27)$$

$$k_3 = 1.4258 - 0.0288\ P_4 + 0.0303\ P_{40} - 0.0521\ P_{200} + 0.0251\ (\%Silt) + 0.0535\ LL - 0.0672\ w_{opt} - 0.0026\ \gamma_{opt} + 0.0025\ \gamma_s - 0.6055\ (w_c / w_{opt}) \quad (2.28)$$

where, M_R = Resilient Modulus, MPa;

$P_{3/8}$ = percentage passing sieve #3/8;

P_4 = percentage passing #4 sieve;

P_{40} = percentage passing #40 sieve;

w_c = moisture content of the specimen, %;

w_{opt} = optimum moisture content of the soil, %;

γ_s = dry density of the sample, kg/m^3; and

γ_{opt} = optimum dry density, kg/m^3.

The resilient modulus test results of the laboratory reconstituted samples were exclusively used in developing the correlation equations, because the samples taken for measuring the soil physical properties by Shelby tube were at different depths. The primary result from these studies is that the resilient modulus can be reasonably predicted from the physical properties included in the LTPP database.

Santha (7) of the Georgia Department of Transportation compared two well known constitutive models (bulk stress model and universal model) in modeling granular subgrade soils, concluding that M_R of granular soils is better described by the universal model. Also studied were the effects of material and physical properties of subgrade soils on the resilient moduli. Subgrade soil samples were reconstituted in the laboratory and tested for M_R according to the AASHTO T 274-82 procedure. Results showed that the k-parameters in the constitutive equation can be estimated using the soil physical properties, and the values of the k-parameter vary over a wide range of cohesive and granular soils. From the study of 14 cohesive soils and 15 granular soils correlation equations were developed. A multiple correlation analysis approach was used to obtain the relationships among k-parameters (dependent variable) and soil properties such as percent passing #40 sieve (P_{40}), percent passing #60 sieve (P_{60}), percent clay (%Clay), percent silt (%Silt), percent swell (%SW), percent shrinkage (SH), maximum dry density (γ_d), optimum moisture content (w_{opt}), California Bearing Ratio (CBR), sample moisture content (w_c), sample compaction (COMP) and percent saturation (SATU). Two correlation equations were developed, one for granular soils and the other for cohesive soils, and they are of the following form.

For granular soils:

$$M_R = k_1 P_a (\theta / P_a)^{k2} (\sigma_d / P_a)^{k3} \tag{2.29}$$

where, $\text{Log } k_1 = 3.479 - 0.07 w_c + 0.24 w_{c\ ratio} + 3.681 COMP + 0.011\ \%Silt + 0.006\ \%Clay -$

$$0.025 SW - 0.039\ \gamma_s + 0.004(SW^2 / \%Clay) + 0.003(\gamma_s^{\ 2} / P_{40}); \tag{2.30}$$

$$k_2 = 6.044 - 0.053 w_{opt} - 2.076 COMP + 0.0053 SATU - 0.0056\%Clay +$$

$$0.0088SW – 0.0069SH – 0.027\,\gamma_s + 0.012\,CBR + 0.003\,(SW^2 / \%Clay)$$

$$– 0.31(SW+SH) / \%Clay; \text{ and} \tag{2.31}$$

$$k_3 = 3.752 – 0.068w_c + 0.309w_{c\,ratio} – 0.006\,\%Silt + 0.0053\,\%Clay$$

$$+0.026SH – 0.033\gamma_s – 0.0009(SW^2/\%Clay) + 0.00004(SATU^2/SH) –$$

$$0.0026(CBR*SH). \tag{2.32}$$

For cohesive soils:

$$M_R = k_1\,P_a\,(\sigma_d / P_a)^{k3} \tag{2.33}$$

$$\text{where, } \operatorname{Log} k_1 = 19.813 – 0.045\,w_{opt} – 0.131w_c – 9.171\,COMP + 00337\,\%Silt + 0.015\,LL$$

$$– 0.016\,PI – 0.021\,SW – 0.052\,\gamma_s + 0.00001\,(P_{40}*SATU); \tag{2.34}$$

$$k_3 = 10.274 – 0.097\,w_{opt} – 1.06\,w_{c\,ratio} – 3.471\,COMP + 0.0088\,P_{40} –$$

$$0.0087PI + 0.014\,SH – 0.046\,\gamma_s; \text{ and} \tag{2.35}$$

$w_{c\,ratio}$ = moisture content of specimen / optimum moisture.

Dai et al. (6), in an attempt to compare the two well known constitutive models (Universal model and deviator stress model) in describing subgrade soil resilient behavior, and to study the effects of material properties on the M_R, selected Shelby tube samples from six different pavement sections of the Minnesota Road Research project. Repeated load triaxial tests were conducted on the soil specimens to determine M_R at Minnesota DOT laboratories along with some other soil property tests. Resilient modulus test data is used in the regression analysis to develop correlation equations between the model constants (k_1, k_2, k_3) and the soil physical properties. The constitutive equation developed to predict the resilient modulus for a stress state with soil physical properties takes the following form:

$$M_R = k_1\,\theta^{k2}\,\sigma_d^{k3} \tag{2.36}$$

$$\text{where, } k_1 = 5770.8 – 520.98\,(\gamma_s)^{0.5} – 3941.8(w_c)^{0.5} + 33.1PI – 36.62\,LL – 17.93\,P_{200} \tag{2.37}$$

$$k_2 = -5.334+0.000316(\gamma_s)^3+9.686(w_c)–0.054PI+0.046LL+0.022\ P_{200} \quad (2.38)$$

$$k_3 = 409.9 –306.18\ (\gamma_s)^{0.1}–82.63(w_c)+0.033\ PI+0.138S–0.041LL \quad (2.39)$$

γ_s = dry density of soil specimen, kN/m^3 ; and

S = saturation, %.

However, the relationships presented in this study were based on soils with physical index properties of a narrow range. Therefore, the predictability of the model is suspect.

In order to validate the octahedral stress state model in characterizing resilient modulus, Mohammad et al. (33), selected eight different soils representing major soil types in Louisiana and tested for M_R in the laboratory. Additional analysis was performed to develop correlations between the model parameters and soil properties. Multiple linear regression analysis was performed between the model constants of the constitutive equation and the basic soil properties. The correlation equations developed are as follows:

$$M_R = k_1\ P_a\ (\sigma_{oct} / P_a)^{k2}\ (\tau_{oct} / P_a)^{k3} \quad (2.40)$$

where, k_1, k_2, k_3 are the regression constants listed in the following equations 2.41-2.43.

$$\text{Log } k_1 = - 0.679+ 0.0922\ w_c+0.00559\ \gamma_s+3.54\ (\gamma_s / \gamma_{opt})+2.47\ w_{c\ ratio}+0.00676\ LL + 0.0116\ PL+0.022\ (\%sand)+0.0182\ (\%silt) \quad (2.41)$$

$$\text{Log } k_2 = - 0.887+0.0044w_c+0.00934\ \gamma_s+0.264(\gamma_s / \gamma_{opt})+ 0.305\ w_{c\ ratio}+0.00877\ LL+ 0.00665\ PL+0.0116\ (\%sand)+0.00429\ (\%silt) \quad (2.42)$$

$$\text{Log } k_3 = - 0.638+0.00252w_c+0.00207\gamma_s +0.61\ (\gamma_s / \gamma_{opt})+0.152\ w_{c\ ratio}+0.00049\ LL+ 0.00416\ PL+0.00311\ (\%sand)+0.00143\ (\%silt). \quad (2.43)$$

where, P_a = atmospheric pressure, psi;

σ_{oct}= octahedral normal stress, $(\sigma_1 + \sigma_2 + \sigma_3)/3$, psi;

τ_{oct} = octahedral shear stresses, psi;

γ_s = dry density, kN/m3; and

PL = plastic limit, %;

It appeared from the analysis that model constants for resilient modulus were mainly governed by density, moisture content, liquid limit, and plastic limit, the same soil attributes widely employed in several other models. Based on the study, it was recommended that the models be used for the prediction of resilient properties of Louisiana subgrade soils.

Preliminary analysis of the model revealed that the above equations were unsuitable in predicting resilient modulus of Mississippi soils. Upon contacting the authors with the result, however, they referred to certain ongoing work to improve the model; which was not available to the researcher in time for this report.

2.9 Comparison of Predictive Equations for Determination of M_R

With numerous equations proposed over the years, a comparison of their predictability was undertaken in a recent study by Kyatham et al. (34). They compared primarily three equations: the bilinear model by Thompson and Robnett (20) and Drum et al. (9), and Farrar and Turner (10). The former two equations predict the breakpoint resilient modulus whereas the latter predicts M_R directly for a given stress state. Breakpoint modulus refers to the modulus at which the slope of M_R versus deviator stress changes. Soil test results from four states – Illinois, Wyoming, Tennessee and New Jersey –have been discussed and analyzed in detail to determine if any of those predictive equations are universally applicable. Based on the analysis it was concluded that there is no universally available predictive equation to estimate resilient modulus. The study suggested that Universal Model (Eq. 2.16) is suitable for determining resilient modulus as a function of confining pressure and deviator stress, but the constants should be determined at a stress range of interest.

2.10 Critique of Explanatory Variables for M_R Prediction

Soil index properties commonly used in developing the correlation equations in the order of importance are material passing # 200 sieve, Atterberg limits (LL, PI), moisture content, and dry density. Though used in several models, no definite trend can be seen between resilient modulus and material passing # 200 sieve. A cursory examination of LTPP equations suggests, however, that M_R attains a peak value in the range of 40 to 60 percent material passing # 200 sieve. From a majority of the equations it can be seen that, M_R increases with an increase in PI. Though PI is an important factor, its effect on M_R is inconsistent with the general soil mechanics principles namely, the higher the PI the less stable the soil is. A soil with a PI value in the range of 10 to 20 percent is considered satisfactory. Intuitively, M_R should decrease with an increase in moisture above optimum, however, different equations show different trends. In equations, for example 2.32-2.38, M_R increases with an increase in the moisture. M_R increases with the percent clay, in the range of 10 to 40 percent, beyond which it decreases. Regarding the effects of density on M_R, the results are inconclusive because in several equations, (for example, 2.21, 2.22, 2.31, 2.32 and 2.34) M_R decreases with increase in density. From a physical point of view, one would expect M_R to increase with the density.

In general, LTPP study (5), proposes the following broad conclusions. Liquid limit, plasticity index, and material passing #200 sieve are important for the lower strength materials, while a measure of moisture content and density are important for the higher strength materials. Percent silt is important for all soil groups, excluding gravel soils.

2.11 Summary

Resilient modulus of subgrade soil is an important material property, a requisite parameter to input in the pavement design equation, generally determined in the laboratory by performing a repeated load triaxial test (AASHTO TP 46) procedure. Because the test is complex and time consuming several user agencies in the United States and abroad now estimate design resilient modulus from correlation equations developed from soil physical properties. This chapter presents various forms of correlation equations including constitutive models and the importance of soil properties in their formulation. A cursory study of the equations suggest that soil index properties such as material passing #200 sieve, Atterberg limits, moisture and dry density significantly affect M_R. Due in part to nonlinear behavior of soil, stress state becomes an important parameter as well.

CHAPTER 3

EXPERIMENTAL WORK

3.1 Introduction

By comparing predicted M_R with laboratory M_R only, validity of equations is appraised. With the objective of compiling laboratory $M_{R,}$ subgrade soil samples collected from different locations in Mississippi are classified and tested for resilient modulus in the MDOT laboratory, in accordance with the AASHTO TP46 protocol. Employing the soil index properties and a realistic stress state, resilient modulus is predicted using the correlation equations cited in the previous chapter and compared with measured resilient modulus. A summary of the tests conducted along with results of each soil is presented in the ensuing sections.

3.2 Laboratory Tests

The soils tested in this study were selected to provide a general representation of typical subgrade soils in Mississippi. Eight different subgrade soils from nine different sections were tested. All of the eight soil materials have been used recently in subgrade construction. These test sections were selected in connection with a study investigating the use of a Falling Weight Deflectometer for subgrade characterization (35).

Composite bag samples were collected from each section for routine laboratory tests and resilient modulus tests as well. A summary of section locations is presented in column 2 of Table 3.1. Also listed in Table 3.1 are the Standard Proctor test results.

3.2.1 Routine Laboratory Tests

The eight subgrade soil samples were classified into fine-grain and coarse-grain soils according to AASHTO classification. Laboratory tests performed to classify the soils are the

Particle size distribution test (AASHTO T88-90), Liquid limit test (T89-90), Plastic limit test (T90-87), and Standard Proctor test (T99-90). An unconfined compressive strength test was performed on all the fine-grain soils in accordance with AASHTO T208-90. Soil index properties are listed in Table 3.2. Since Drumm's equation required unconfined compressive strength and initial tangent modulus as inputs they were determined as presented in Table 3.3.

3.2.2 Laboratory Resilient Modulus Test

Making use of the bulk material from each section, three cylindrical samples 2.8 inch (71 mm) diameter by 5.8 inch (147 mm) length were molded at the target density (i.e. the maximum dry density) and optimum moisture content, as listed in Table 3.1. These samples were prepared in three layers in a split mold, each layer receiving 25 blows with a tamping rod 5/8 inches (16 mm) diameter. The final compaction was accomplished by a compressive load of the order of 5000 lbs. Wrapped with cellophane wrap, they were stored in a humidity room for 5 days and then tested in the Repeated Load Triaxial machine in accordance with the AASTHO TP46 test protocol. The tests were conducted using the MDOT repeated load triaxial machine, supplied by Industrial Process Control (IPC), Borona, Australia. The load sequence and the combinations are presented in Appendix A. Axial deformation of the specimen is recorded by two externally mounted Linear Variable Differential Transducers (LVDT). The average of the resilient modulus values of the last five loading cycles of the 100 cycle sequence yields the requisite resilient modulus. Typical plots of laboratory M_R test results of reconstituted samples related to deviator stress are presented in Figure 3.1 to 3.4, the former two figures for a fine-grain and the latter two for a coarse grain soil.

3.3 Summary

A detailed discussion of the laboratory tests performed on the bulk samples is presented.

Summary of the physical properties of the samples are presented as well. Detailed discussion and analysis of the test results will be the topic of the following chapter.

Table 3.1 Test section locations and Proctor test results of Bag samples

Section #	County / Road	Section Length (ft)	Optimum Moisture Content (%)	Dry Density (lb/ft^3)
1	Montgomery /US 82 W	200	13.8	115.2
2	Coahama / US 61 N	500	14.1	113.7
3	Coahama / US 61 N	200	12.9	116.2
4	Montgomery / US 82 W	200	13.8	115.5
6	Hinds / Norell W.	200	17.8	105.6
7	Wayne / US 45 N	200	11.0	118.0
8/9	Wayne / US 45 N	200	12.0	118.9
10	Madison county/Nissan west parkway	200	18.6	106.1

1 ft = 0.305m; 1 lb/ft^3 = 0.157 kN/m^3;

Table 3.2 Soil index properties of bulk samples from various sections

Section #	Liquid Limit (%)	Plasticity Index (%)	Passing # 200 sieve (%)	Passing # 40 sieve (%)	Clay (%)	Silt (%)	Classification	
							AASH-TO	U S C
1	22.3	6.1	55.0	NA	10.6	44.5	A4	CL
2	27.0	8.0	56.0	NA	14.2	41.8	A4	CL-ML
3	25.0	7.0	40.0	NA	10.8	45.2	A4	SM-SC
4	28.1	12.4	60.0	90	12.3	48.1	A6	CL
6	37.2	13.1	96.0	99	19.3	78.7	A6	CL
7	20.5	1.0	28.0	NA	3.2	25.4	A2-4	SM
8/9	24.4	4.9	42.0	NA	9.0	33.1	A4	CL-ML
10	35.8	13.3	98.0	99	18.9	79.1	A6	CL

Table 3.3 Unconfined compressive strength results for fine-grain soils

Section #	Unconfined compressive strength (psi)	Initial tangent modulus, (psi)
1	15.4	2300
2	26.6	2500
3	18.5	1400
4	17.4	2400
6	26.9	3350
8/9	20.7	2400
10	25.4	2300

1 psi = 6.89 kPa

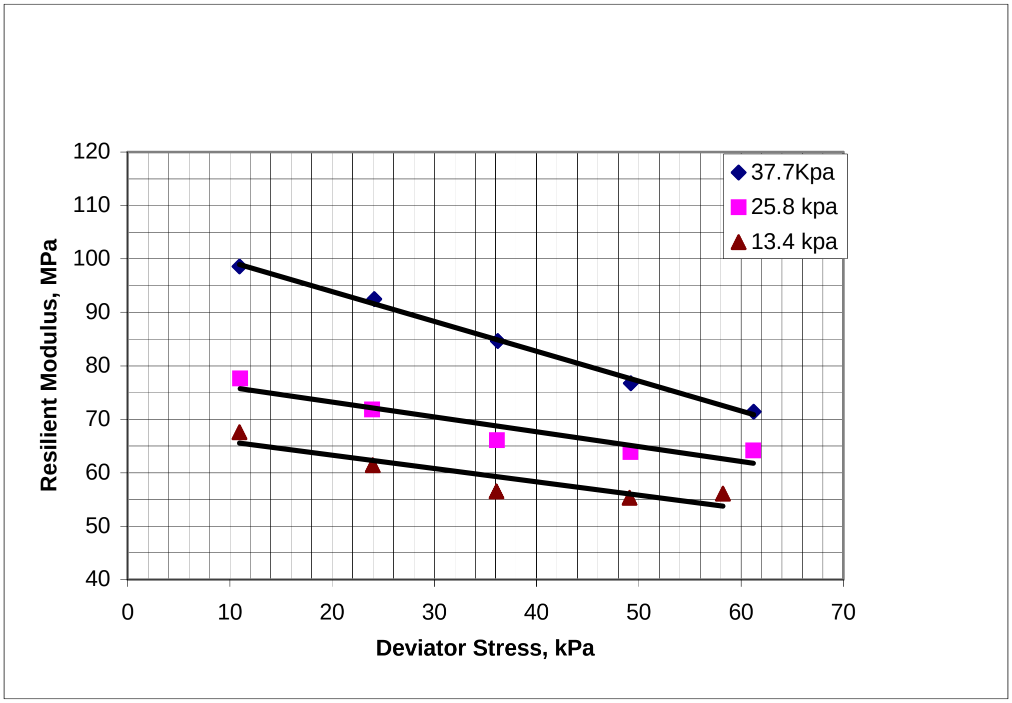

Figure 3.1 Resilient Modulus vs. Deviator Stress at Three Confining Pressures, Section #1, Sample #1; 1 kPa = 0.15 psi

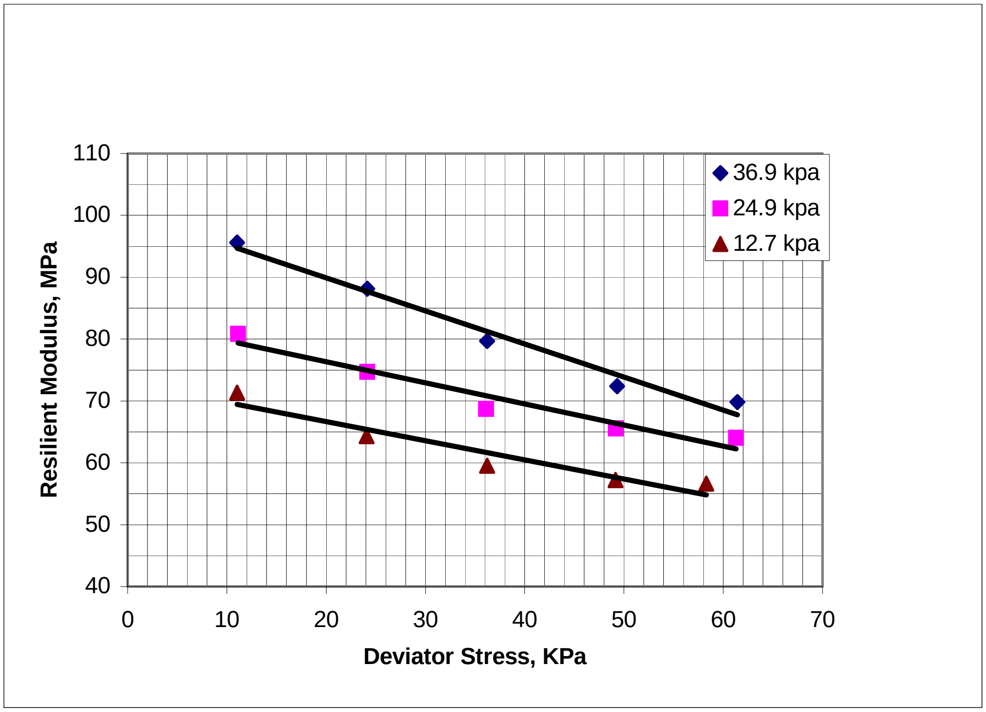

Figure 3.2 Resilient Modulus vs. Deviator Stress at Three Confining Pressures, Section #1, Sample #2; 1 kPa = 0.15 psi

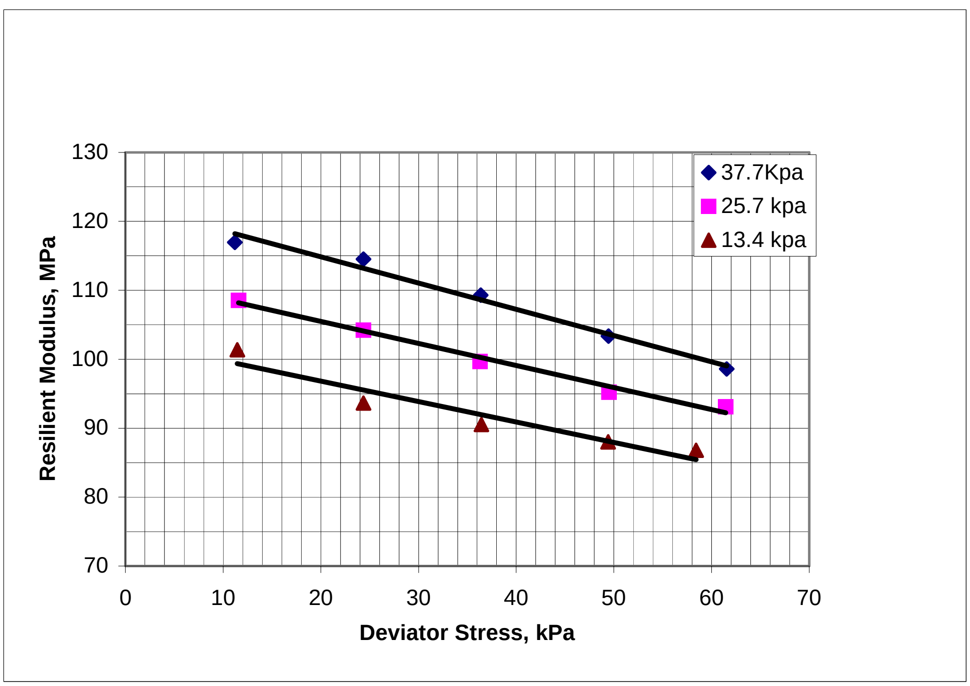

Figure 3.3 Resilient Modulus vs. Deviator Stress at Three Confining Pressures, Section #6, Sample #31; kPa = 0.15 psi

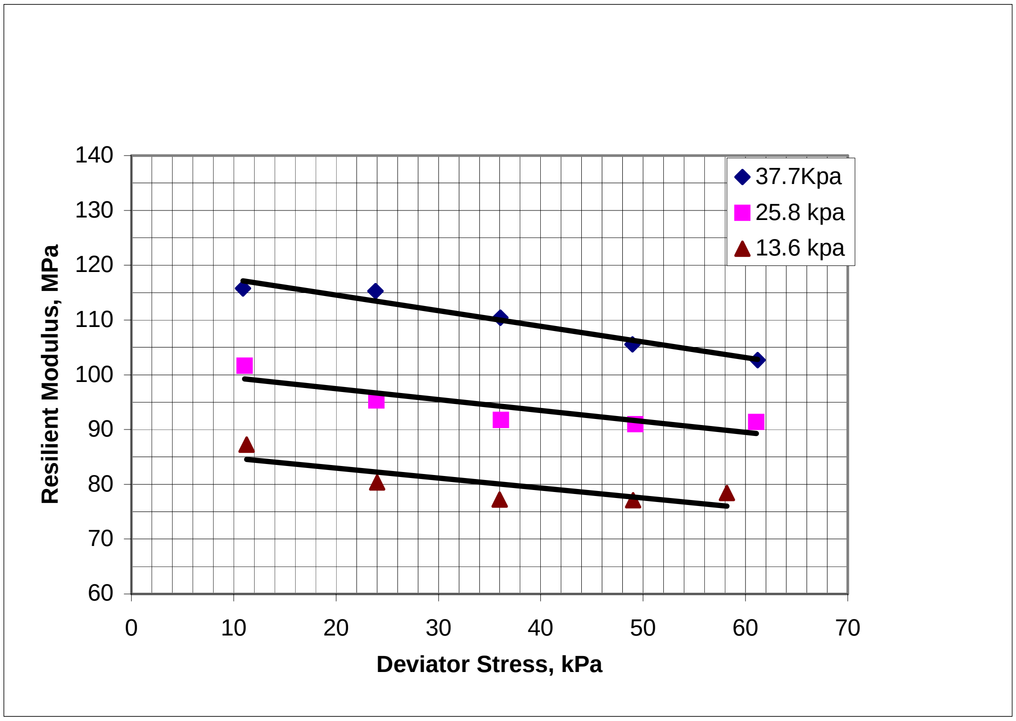

Figure 3.4 Resilient Modulus vs. Deviator Stress at Three Confining Pressures, Section #7, Sample #1; 1 kPa = 0.15 psi

CHAPTER 4

ANALYSIS AND DISCUSSION

4.1 Introduction

The primary objective of this study is to explore the predictability of the correlation equations reported in previous research studies, for example, LTPP program, Georgia DOT, Minnesota Road Research project, Carmichael and Stuart, Drumm et al., Wyoming and Ashraf and George. This chapter presents the data and analysis comparing measured resilient modulus to predicted resilient modulus from the aforementioned correlation equations. Also presented in this chapter is the expected variability in predicting resilient modulus owing to inherent soil property variations. Model sensitivity results are also presented seeking the significance of response variables in predicting M_R. A discussion on the feasibility of using existing correlation equations is also presented at the end of this chapter.

4.2 Resilient Modulus Determination

Three reconstituted samples of 2.8 inch (71mm) diameter and 5.8 inch (147 mm) height from the bag samples collected from each section are molded and tested for M_R in accordance with TP46 protocol. The cylindrical samples are subjected to 15 stress combinations (three confining stresses and five deviator stresses) yielding 15 M_R values. Principal stresses, bulk stress (θ), and octahedral shear stress (τ) are calculated for each stress combination. The equation (2.18) is then fitted for each set of data expressing M_R as a function of θ and τ_{oct}. Regression constants (k_1 to k_4) of the reconstituted samples are listed in Table 4.1.

Regression constants (k_1 to k_4 values) indicate that M_R increases with an increase in bulk stress as suggested by positive values of k_2, and decreases with an increase in octahedral shear stress as k_3 is negative. This relation holds well as long as one of the terms is kept constant and the other is changed, but whether a soil exhibits stress hardening or softening under simultaneous change in bulk and shear stress state would depend on the nature of the stress state and the resulting change in bulk and octahedral stresses. Hence, it is difficult to predict how a soil behaves under changing stress states, but can only be analyzed by substituting the corresponding values of shear and bulk stresses.

Since M_R is a function of the stress state, for calculating a representative M_R for correlation, an appropriate stress state has to be selected because the AASHTO Guide does not mandate one. A stress state pertaining to the actual condition (overburden stress) of soil in its final location, including the stress generated by a standard axle load could be an appropriate scenario. Relying on the results of Thompson and Robnett (20), Elliot (36) suggested using a zero confining pressure and a 6 psi (41.6 kPa) deviator stress when selecting an M_R value from laboratory test data. In the field, however, subgrade has to sustain the overburden of pavement layers, in addition to the standard 18-kip axle load. In-situ stress in a typical subgrade includes stresses due to a 4500 lb (20 kN) wheel load at a tire pressure of 100psi (690 kPa) and overburden pressure as well. Stress analysis by KENLAYER (37) yielded a stress state of 7.4 psi (51 kPa) vertical stress and 2 psi (14 kPa) lateral compressive stress. This stress combination is used to calculate the representative M_R of each sample, and the average of the three samples of a section is taken as a representative M_R of the section for the given stress state. The individual M_R values of the samples of each section and their average are presented in Table 4.2.

4.3 Prediction of M_R Employing Correlation Equations

Resilient moduli of eight subgrade soils are predicted employing the correlation equations cited in the previous chapter.

4.3.1 Prediction of M_R from LTPP Equations

LTPP correlation equations (2.20-2.28) were employed to predict the resilient modulus of eight subgrade soils based on soil index properties. Regression coefficients calculated from LTPP equations and the resulting M_R are presented in Table 4.3. Note that stress state of $\sigma_1 = 7.4$ psi (51 kPa) and $\sigma_2 = \sigma_3 = 2$ psi (14 kPa) is the input in calculating M_R values.

Since coefficient k_1 in the equation 2.19 is proportional to the Young's modulus, it must always be positive as M_R cannot be negative. Increasing the bulk stress should produce stiffening or hardening of the material, resulting in higher modulus. So the constant k_2 of the constitutive equation should be positive. Since increasing shear stress should produce a softening of the material, values of k_3 should be negative. From Table 4.3, it can be observed that coefficient k_1 and k_2 are positive and k_3 negative, just what is expected of typical soils. A simple comparison of predicted M_R (column 3 of Table 4) and laboratory values (column 2 of Table 4) suggests fair agreement in five out of eight soils. Treating soil 3 laboratory M_R with reservation, soils 6 and 10 are the ones showing large discrepancy in prediction. A cursory examination of the Universal model, having two exponential coefficients, suggests that significant errors could be introduced in predicted values of M_R even for moderate changes in k_2 and k_3.

4.3.2 Prediction of M_R from Georgia DOT Equations

Equations 2.33, 2.34 and 2.35 for fine-grain soil were solved inputting the soil properties of seven Mississippi soils, and the results are presented in Table 4.5. Comparing the laboratory and predicted values, it is clear that the model severely over predicts M_R values. Difficulties encountered in using the equation are, first, a suspected typographical error in the equation could not be verified by the authors, and second, percent swell (SW) for the seven soils had to be estimated from PI data.

The question persisting is why the equation over predicts M_R values (compare columns 2 and 5 of Table 4.4). A cursory examination of k_1 and k_3 values suggest that k_3 fluctuates significantly from soil to soil causing corresponding large variations in M_R values (see Table 4-5). Second, the Georgia DOT equation was developed by relying on 14 fine-grain soils of average clay content and PI of 38% and 18% respectively, both significantly larger than those for which predictions are made. Average clay content and PI of Mississippi soils are 14% and 9%, respectively. Attempting to predict beyond the inference space of the model seems to be the primary reason for the over prediction. Third, the Georgia equation denies "deductive result". For example, M_R decreases with compaction ratio (note the large regression coefficient) as well as the effect of density contradictory to an intuitive result that M_R should increase with density. Though this requirement is not a requisite condition, a physically intuitive model stands a better chance of being "robust". In other words, the validity of the equation is limited only to Georgia soils.

With the necessity to estimate even more variables in the coarse-grain equations, the predicted value failed to match the laboratory M_R value, therefore, those results are not presented in this report for brevity.

4.3.3 Prediction of M_R from Minnesota Equations

Equations (2.36-2.39) were employed to predict the resilient modulus of the eight Mississippi soils. Regression coefficients (k-values) derived and M_R predicted from the correlation equations are presented in Table 4.6.

It can be observed from Table 4.6, that negative resilient modulus is predicted for the sections 6 and 10 because the constant k_1 predicted was negative, due in part to excessive material passing the #200 sieve. Equation 2.37 could not predict positive k_1 values for soils 6 and

10, as the equation was developed employing soils having a narrow range of material passing the #200 sieve (57 to 68 percent). Since inference space of the model is limited, soils 6 and 10, with more than 95 percent material passing the #200 sieve, could not predict reasonable values. It was mentioned earlier that constant k_2 should be positive and k_3 negative. But for sections 1, 2, 3, 4, 7 and 8/9 the k_2-value predicted was negative, which could be attributed to low moisture content and low liquid limit of Mississippi soils relative to Minnesota soils. The positive value of k_3 for sections 1, 2 and 4 is attributed to the low liquid limit of the soils. With no valid estimated k-values, resilient modulus predictions for the eight subgrade soils are highly unsatisfactory. In other words, the correlation equation developed in the Minnesota Road project could not reasonably predict the resilient moduli of Mississippi subgrade soils, implying that the model does not satisfy the validation criterion.

4.3.4 Prediction of M_R from Carmichael and Stuart Equations

Carmichael and Stuart equations (2.5-2.6), which heavily depend on soil index properties, are used in predicting the resilient modulus of eight subgrade soils. Table 4.4 lists the M_R values. In-order to compute M_R, a deviator stress of 5.4 psi (37 kPa) and lateral stress of 2.0 psi (14 kPa) were employed. As can be seen from the Table 4.4 (Column 9), resilient moduli for sections 6 and10 were predicted low because of the large amount of material passing the #200 sieve. An observation is in order here that moisture content and percent passing the #200 sieve are more significant than the deviator stress and the confining stress.

4.3.5 Prediction of M_R from Drumm's Equation

Resilient modulus of eight subgrade soils has been predicted with regression models (equations 2.7-2.9) developed by Drumm et al. They are reported in column 11 of Table 4.4.

The constant a' estimated from the prediction equation for all the soils is within the range of the values used in developing the model. For sections 1, 4, 6 &10, predicted b' values, however, are out of range compared to the values used in developing the equation. Low compressive strength and low clay content of the samples could have resulted in this outcome.

Note that resilient modulus is 2 to 3 times larger than the initial tangent modulus obtained from unconfined compression tests. Though no conclusive proof can be offered, this result seems quantitatively reasonable.

4.3.6 Prediction of M_R from Wyoming Equations

Resilient modulus of 7 soils predicted by Eq. 2.10 can be seen in Table 4.4 (Column 13). The predicted values are substantially lower than the TP 46 values except in two soils namely, #6 and #10, both with nearly 98% passing the #200 sieve. Note that the predicted M_R values are relatively small in soils #3 and #8/9, coincidentally with small amounts of material passing the #200 sieve, namely 40% and 42%, respectively. Accordingly, it is presumed that P_{200} plays a major role in the prediction process. The 13 Wyoming soils employed in developing the model had an average P_{200} value of 73% (P_{200} range from 43% to 89%), suggesting that the inference space of the Wyoming equation is limited, curtailing its predictability in Mississippi soils of widely varying fines content. In other words, much like for other equations, the validity of the Wyoming equation is also suspect.

4.3.7 Prediction of M_R Employing Mississippi Equations

Despite being simple and concise, the equation predicts moduli values close to the laboratory values in 6 out of 7 soils. Coarse-grain soil prediction is not as good, perhaps affected by the explanatory variable, uniformity coefficient ($_{Cu}$). The uniformity coefficient, the ratio of

D_{60} over D_{10}, is considered less precise than the other soil index properties. Note also that only one sample is available to test the validity of the equation. It would appear that an additional explanatory variable, for example, confining stress could well improve the predictability of the equation.

4.3.8 Comparison of Laboratory M_R and Predicted M_R from Various Models

Table 4.4 presents a comparison of the laboratory M_R with resilient modulus predicted from various models. Assuming laboratory resilient modulus represents the true modulus of the material, it is compared and critiqued with the predicted values.

From Table 4.4, it can be seen that the M_R predicted by LTPP equations for fine-grain silt soils (sections 1, 2 and 8/9) is comparable, though somewhat smaller in section 2 and 8/9. Section 3 is an exception. For sections 4, 6 and 10, which are fine-grain clay soils, very low modulus is predicted primarily due to excessive silt (≈80%), and relatively small amounts of clay. And for section 7, which is a coarse-grain sand soil, again M_R is under predicted, which can be attributed to low clay content (3%). It can be seen from Table 4.4 that an average deviation of M_R of seven soils (excluding section 3, which is an outlier) from LTPP equations is within 19% percent of the measured M_R, which is encouraging.

Georgia DOT equations severely over predict M_R values of Mississippi soils. As cited in a previous section, over prediction could be attributed to large variations of k_3. Also, the 14 Georgia soils, from which the equation was developed, are uniformly high plasticity clay soils where as Mississippi soils are predominantly silty.

Minnesota Road research equations could not satisfactorily predict the constants (k-values) for the Mississippi subgrade soils. Resilient modulus is over predicted for some sections and under predicted for others. Since soils from one project, with a very narrow range of properties were used in developing the prediction equations, they are not versatile enough to satisfactorily predict the resilient modulus of Mississippi soils. Simply put, the validity of this equation in predicting M_R of soils of other texture is questionable.

Resilient modulus predictions from Carmichael and Stuart-equations yielded high modulus for some sections and low modulus for others. Again very low modulus predicted for sections 6 and 10 is primarily due to excessive fines (≈96%). Note that material passing the #200 sieve of sample data in developing the equation ranged from 60 to 90 percent only. And, the high modulus predicted for section 1,2,3, & 8/9 could be attributed to (i) material passing the #200 sieve being relatively small and (ii) the plasticity index also low relative to the PI range of soils employed in developing the equations. Only the coarse soil M_R (section 7) is predicted satisfactorily.

Drumm's equation satisfactorily predicts the resilient modulus of fine-grain soils (sections 1,2, and 6). On average, all of the M_R predictions are within 15% of the measured M_R. One drawback of the Drumm's model is that it relies on variables such as initial tangent modulus and unconfined compressive strength. The fact that the initial tangent modulus is relatively difficult to estimate could affect the predictions. Note that Drumm's equation is especially suited for fine-grain soils; no such equation is available for coarse-grain soils.

A majority of the predictions by the Wyoming equation is substantially lower than the laboratory M_R, except in soils #6 and #10. As discussed in a previous section, the inference space of the equation comprised of A-7-6 soils with large amount of P_{200} (73%). This P_{200} is relatively larger than those found in most Mississippi soils. Also, note that the contribution of P_{200} to M_R value in the equation is substantial. In other words, using this equation for Mississippi soils entails extrapolation of the equation. The Wyoming equation, therefore, is judged to be invalid for Mississippi soils.

The Mississippi equation predicts six out of seven fine soils within 10 Mpa, which is considered a satisfactory match. It should be pointed out that the equation was developed from an independent set of soils different from those whose M_R values are predicted, and compared.

4.4 Predictability of Equations under Uncertainties

In all of the correlation equations identified in the literature M_R turns out to be a function of soil index properties and stress states. Once a grading job is completed, the subgrade soil could show significant spatial variation, resulting in randomness and uncertainty. Therefore, it is important to estimate how this variability would affect the predicted M_R values. The problem then boils down to estimating the variability in predicted M_R as the independent variables change over a reasonable range. This problem can be studied either by method of point estimates (38) or Taylor's series expansion method.

4.4.1 Method of Point Estimates

The method of Point Estimates (PE) facilitates computations for the first two moments (i.e. mean and variance) of a dependant variable in terms of the first two moments of the independent variables. Approximate formulas for the moment's calculation can be obtained from a Taylor series expansion of the function about the first moment of the random variables. However, due to excessive restrictions imposed on the function (existence and continuity of the first few derivatives) and the requirement to compute the derivatives, the moment's calculation turns out to be difficult. These difficulties can be overcome through the use of the method of point estimates. Equations required to perform the required calculations are presented in Appendix B.

In the correlation equations 2.19, 2.29, 2.36 and 2.40, M_R is expressed as a function of stress state. The constants (k values), which are exponents of the stress state are, in turn, functions of soil index properties. The first two moments of the k values are calculated from the mean and variance of the independent variables (soil physical properties). Subsequently, mean and variance of M_R are, calculated based on the first two moments of the k-values. Thus, the variations expected in predicting M_R due to inherent variability in soil index properties are quantified. Soil properties for each section were determined by performing routine laboratory tests. With mean values obtained from laboratory tests, coefficient of variation is adapted from a list of values in reference 39. The suggested coefficient of variation of each independent variable is listed in Table 4.7. Since the complexity of PE analysis increases exponentially, the number of input properties with inherent variability is limited to four in each case.

4.4.2 Variance in Model Prediction

By introducing variation in soil physical properties, expected variation in resilient modulus prediction is calculated employing the method of point estimates. From a list of independent variables included in column 3 of Table 4.8, three or four variables are chosen, which are assumed to vary +/- one standard deviation (SD). As can be seen from the table a variety of soil properties appear in those seven models investigated. Table 4.9 is prepared to rank the response variables in the order of their importance. For example, P_{200} is shown to be the most-often adopted variable, in six out of seven models. In addition to mean M_R values, Table 4.4 lists the PE-based mean and coefficient of variation (CV) that are computed with each equation.

Assuming inherent variability in four variables, the mean values calculated from LTPP equations are reported in Table 4.4. As expected, the PE mean values (column 4 of Table 4.4), are practically the same, and rightly so, as the mean values (computed by direct substitution of the independent variables in the equation, column 3 of Table 4.4). Note that the predicted mean M_R values are smaller than the laboratory values, excluding section #3 which is deemed to be an outlier. In general, a relatively small coefficient of variation is an indication that inherent variations in the independent variables, i.e. soil index variables, would have minimal effect on the predicted values. The fact that the average CV of 16% for 9 soils, therefore, reinforces the previous observation that LTPP equations are capable of predicting M_R from soil physical properties.

PE mean values of Georgia DOT equations, when introducing variation in four soil properties are listed in column 6 of Table 4.4. Not only the predicted mean values, but also the CVs are relatively large. Accordingly, the validity of the Georgia DOT equation, in predicting M_R of Mississippi soils, is suspect.

PE analysis of Minnesota equations, assuming variability in four variables resulted in unacceptably large mean values and coefficients by variation. Those results are not reported here for brevity ruling out its use for predicting M_R of Mississippi Soils.

The Georgia DOT and Minnesota equations both resulting in unrealistic mean and coefficient of variation suggest an apparent weakness of the Universal model, i.e., expressing M_R as power functions of θ (volumetric stress) and τ, σ_d or σ_3. The modeling entails deriving first k_1, k_2 …., followed by another regression analysis where by those constants are expressed as function of soil properties. The constant k_1 and powers k_2 and k_3 could cause the equations to result in unrealistic M_R values. For example, k_1 had become negative for soils #6 and #10, in the case of the Minnesota equation. Therefore, PE mean values become negative. Another scenario would be that the mean and standard deviations could blow up or even become negative should k_2 and/or k_3 equations are not robust. Since k_2 and k_3 are exponents (typically k_2 is positive and

k_3 negative), small change in k_2 and/or k_3 could make large difference in predicted M_R, which is believed to be the reason for "wild" predictions of Minnesota and Georgia DOT equations. Note that LTPP equations are not vulnerable to such large swings owing primarily to the robustness of k_1-, k_2- and k_3- equations.

Table 4.4 (Column 10) presents the mean and coefficient of variations when inherent variability in index properties are introduced in the Carmichael equation. The PE mean values are again very close to the predicted mean values with average soil properties, and coefficients of variation of all of the soils are reasonably low, except in soils #6 and #10. Despite relatively small CVs the agreement between measured (TP 46) and predicted M_R values is less than satisfactory. The one coarse soil (soil 7) investigated shows satisfactory agreement, however.

The means and coefficients of variation obtained in predicting M_R from Drumm's equation are presented in column 12 of Table 4.4. Not only are the M_R values under predicted (excluding sections 2 and 3), but the CVs are relatively large as well. Large CV means large swings in the predicted value with relatively minor variations in the basic soil property. Put differently, when using Drumm's equation, there is a good chance of predicting very high or very low M_R depending on the accuracy of input values.

Including variability in three independent variables and making PE calculations with the Wyoming equation, not only are the mean M_R predictions unsatisfactory, but the CVs are also substantially large. As pointed out in previous sections, because the Wyoming equation had its root in a database of fine-grain soils of nearly identical texture, it can hardly be valid for other soils, in this case, Mississippi soils.

PE mean values of the Mississippi equation with inherent variability introduced in three input parameters agree well with the mean values and TP 46 values as well. The CV of the predicted M_R values, however, are relatively large. Note that the predicted M_R values in six out of seven soils agree with the laboratory values.

4.5 Model Validation

The question now arises; which model(s) satisfies validation criteria? Validation refers to the process to confirm that the proposed model can produce robust and accurate predictions for cases other than those used in model development and/or calibration. Two criteria are employed to accomplish this task. First, the predicted value shall agree with the laboratory measured value within +/- 20% range. This 20% range is chosen with due consideration to the typical variability observed in the laboratory M_R values. Second, the inherent variability in input variables shall not generate a large enough coefficient of variation, (not exceeding 20%). Table 4.10 lists to what extent each criterion is satisfied for all of the six models. Respectively, one coarse- and seven fine-soils are included.

Though none of the equations entirely meet the stated criteria, two equations stand out, in regard to satisfying the two criteria. The LTPP equations present a strong showing in so far as variability in predictions, and Mississippi equations satisfying the equality criterion (see Table 4.10). Specifically, the variability is satisfied by LTPP equations in one coarse soil and seven fine soils. That is, in all of the soils the variability does not exceed 20%. And, Mississippi equation predictions agree in six out of seven fine soils, and the only one coarse soil prediction fails the 20% criterion. LTPP equation is successful due in part to them being derived from a large database of soils collected and tested nationwide. The specialty of Mississippi equation is that it is derived from Mississippi soils.

4.6 Sensitivity Analysis of Models

The sensitivity analysis examines the effect of each independent (response) variable on the predicted M_R value, in contrast to the PE analysis that evaluates the inherent variability collectively of all of the independent variables. Independent variables include soil index properties and compaction attributes, and stress state variables. Since soil properties are likely to vary temporally and spatially, with them beyond designer's control, sensitivity study seeks how each of them effects M_R prediction.

The methodology of a sensitivity study entails changing the mean value of each independent variable by +/- one standard deviation and calculating the corresponding change in the predicted M_R value. The mean value of each independent variable is nothing but its measured value, for example, material passing the #200 sieve determined by sieve analysis. Typical values listed in Table 4.7 are the coefficient of variation adopted for each soil index/compaction/strength property. One silty soil (soil 1), as well as a clay soil (soil 4) is studied by changing each independent variable by plus or minus one standard deviation and predicting M_R. Tables 4.11 and 4.12 list those M_R values for soils 1 and 4, respectively. Having determined that the prediction of Minnesota and Georgia models are not satisfactory, a sensitivity study of those models is not attempted. Accordingly, the results of only the remaining five models are reported here.

Two criteria have been adopted to rank the sensitivity of response variables; namely, "significance", and "acceptable trend". Explanatory variable is said to be significant when predicted change (M_R in this case) exceeds 10% with a change of one standard deviation of the variable in question. The second criterion is that the predicted M_R change shall be intuitively satisfying, meaning that the trend of predicted change is physically meaningful. Based on those two criteria, water content or its surrogate percent saturation appears to be the most significant soil property, followed by material passing the #200 sieve and plasticity index in that order (see

Table 4.13). Density explicitly appears only in two equations: LTPP and Drumm. Its significance in the LTPP fine grain soil equations is negligible, where as it turns out to be significant in the Drumm equation. The trend of increasing density giving rise to decreasing resilient modulus, however, is suspect, negating its role in those two equations. A density ratio, appearing in the Mississippi equation, moderately effects M_R prediction in fine soil. In coarse soil, density, however, has a major role in the prediction. The sensitivity study and ensuing results should guide the design engineer in paying special attention to those soil properties that are shown to be significant.

Concluding, the sensitivity study not only singles out significant variables, but also reveals the trend of M_R accompanying the variation of each variable. For example, resilient modulus is shown to be negatively correlated with water content/ degree of saturation, positively associated with percent passing the #200 sieve, and a mixed trend with PI. Though only two equations include soil density explicitly, the result that M_R decreasing with increase in density is suspect.

4.7 Summary

Resilient modulus is predicted for eight Mississippi subgrade soils employing selected correlation equations, and compared with the laboratory resilient modulus. Comparisons reveal that the Mississippi equation reasonably predicts M_R of typical Mississippi fine-grain soils. A statistical study of variance expected in calculating M_R confirms that LTPP model predictions are less prone to uncertainties that may arise from inherent variations in soil properties. A sensitivity study of the soil properties in predicting M_R has been conducted, choosing three soil properties. Moisture content, percent passing the #200 sieve and PI are the most significant in M_R prediction.

Table 4.1 Constants (k-values) from Regression Analysis of Resilient Modulus Expressed by Equation 2.18

Section #	Sample #	k_1	k_2	k_3	k_4
1	1	2.448	0.616	-0.630	-0.073
	2	2.496	0.471	-0.580	-0.064
	3	2.433	0.606	-0.712	-0.140
2	1	2.655	0.258	-0.515	-0.151
	2	2.607	0.230	-0.741	-0.323
	3	2.536	0.381	-0.897	-0.413
3	1	2.551	0.470	-0.250	0.025
	2	2.476	0.495	-0.431	-0.074
	3	2.502	0.463	-0.363	-0.040
4	1	2.604	0.383	-0.734	-0.255
	2	2.648	0.277	-0.823	-0.358
	3	2.649	0.229	-0.730	-0.297
6	1	2.610	0.279	-0.670	-0.275
	2	2.723	0.256	-0.494	-0.199
	3	2.752	0.252	-0.469	-0.169
7	1	2.736	0.491	-0.331	-0.045
	2	2.726	0.459	-0.384	-0.081
	3	2.725	0.436	-0.398	-0.084
8/9	1	2.594	0.448	-0.831	-0.319
	2	2.761	0.351	-0.521	-0.159
	3	2.734	0.355	-0.657	-0.237
10	1	2.521	0.290	-0.734	-0.289
	2	2.565	0.278	-0.707	-0.287
	3	2.530	0.290	-0.778	-0.302

Table 4.2 M_R Values Calculated for Stress State, σ_1=7.4 psi and σ_3= 2 psi

Section #	Sample #	M_R, MPa	Average M_R, MPa
1	1	64.0	66.5
	2	68.6	
	3	65.5	
2	1	82.8	85.8
	2	88.2	
	3	84.5	
3	1	49.5	49.8
	2	49.7	
	3	49.4	
4	1	91.3	99.2
	2	105.8	
	3	98.6	
6	1	82.8	88.7
	2	88.0	
	3	93.7	
7	1	79.3	82.0
	2	81.7	
	3	83.5	
8/9	1	95.8	104.3
	2	103.4	
	3	111.4	
10	1	73.7	77.9
	2	78.2	
	3	79.9	

1 MPa = 0.15 ksi

Table 4.3 Prediction of Constants (k-values) and M_R from LTPP Equations

Section #	Sample #	k_1	k_2	k_3	Predicted M_R (MPa)	Average M_R (MPa)
1	1	0.903	0.335	-1.398	64.8	65.6
	2	0.911	0.335	-1.370	65.6	
	3	0.907	0.335	-1.384	65.2	
2	1	1.019	0.281	-1.249	75.8	75.8
	2	1.015	0.281	-1.263	75.4	
	3	1.011	0.281	-1.277	74.9	
3	1	0.968	0.310	-1.178	73.0	72.9
	2	0.965	0.310	-1.192	72.0	
	3	0.972	0.310	-1.164	72.9	
4	1	0.880	0.377	-0.874	68.3	70.1
	2	0.885	0.376	-0.879	68.6	
	3	0.893	0.376	-0.864	69.4	
6	1	0.780	0.344	-1.478	55.1	57.4
	2	0.798	0.344	-1.444	56.7	
	3	0.798	0.344	-1.442	56.7	
7	1	0.877	0.456	-1.286	62.5	64.7
	2	0.877	0.454	-1.275	62.6	
	3	0.876	0.452	-1.263	62.7	
8/9	1	0.989	0.338	-1.082	76.6	75.9
	2	0.997	0.338	-1.054	77.5	
	3	1.000	0.338	-1.040	78.0	
10	1	0.710	0.335	-1.704	48.5	51.9
	2	0.723	0.334	-1.702	49.4	
	3	0.719	0.334	-1.704	49.1	

1 MPa= 0.15 ksi

Table 4.4 Comparison of Average M_R: (i) Laboratory M_R vs. Predicted M_R from Various Models, (ii) Variability in Prediction Employing Point Estimates method.

Soil/ Section #	Resilient Modulus														
		LTPP		Georgia		Minnesota		Carmichael		Drumm		Wyoming		Mississippi	
	TP 46 Mean, MPa	Mean, MPa	PE Mean, Mpa / CV	Mean, MPa	PE Mean, Mpa / CV	Mean, MPa	PE Mean, Mpa / CV	Mean, MPa	PE Mean, Mpa / CV	Mean, MPa	PE Mean, Mpa / CV	Mean, MPa	PE Mean, Mpa / CV	Mean, MPa	PE Mean, Mpa / CV
			65.7		1037.9		191324.0		117.0		56.8		40.6		70.7
1	66.0	65.6	12%	653.3	97%	260.8	182%	116.2	12%	56.8	40%	40.6	52%	68.9	24%
			75.9		956.1		1854562.0		109.2		85.2		46.5		90.0
2	85.2	75.8	13%	625.3	95%	233.0	20%	109.9	14%	85.2	28%	46.5	45%	87.4	27%
			73.0		2193.6		26633.5		132.5		84.4		37.9		96.9
3	49.6	72.9	12%	917.6	68%	66.3	169%	132.7	10%	84.4	26%	37.9	53%	94.2	25%
			70.1		1628.2		336846.0		91.0		81.4		46.3		97.9
4	98.6	70.1	18%	961.4	101%	252.3	210%	90.7	20%	73.2	31%	46.3	47%	95.1	28%
			58.7		26280.3		-390000000.0		36.4		75.2		71.7		96.5
6	88.2	57.4	26%	10693.9	110%	-34878.0	-199%	37.1	65%	75.1	37%	75.5	30%	93.6	30%
			63.0						75.3						63.2
7	81.5	64.8	30%					75.8	13%					62.9	20%
			70.7		575.3		103260.2		144.0		76.2		37.1		103.1
8/9	103.5	75.9	7%	428.8	83%	35.4	168%	145.0	8%	76.2	29%	37.1	55%	100.2	26%
			53.6		24235.0		-490000000.0		30.4		59.8		69.2		84.0
10	77.3	51.9	27%	7954.4	121%	-28167.0	-197%	27.9	79%	59.8	46%	73.9	33%	81.5	29%

1 MPa = 0.15 ksi

Table 4.5 Prediction of Constants and M_R from Georgia DOT Equations

Section #	k_1	k_3	Mean M_R (MPa)	TP 46 Mean (MPa)
1	3.836	0.0123	653.3	66.0
2	3.818	0.0165	625.3	85.2
3	3.986	0.327	917.6	49.6
4	3.936	-0.151	961.4	98.6
6	5.05	0.005	10693.9	88.2
8/9	3.655	0.325	428.8	103.5
10	4.875	-0.099	1954.4	77.3

1 MPa= 0.15 ksi

Table 4.6 Prediction of Constants (k-values) and M_R from Minnesota Road Equations

Section #	Sample #	k_1	k_2	k_3	Predicted M_R (MPa)	Average M_R (MPa)
1	1	502.9	-0.252	0.130	267.7	260.8
	2	511.7	-0.262	0.122	254.0	
	3	507.4	-0.258	0.127	260.8	
2	1	385.8	-0.167	0.056	228.3	233.0
	2	381.5	-0.162	0.059	232.8	
	3	377.9	-0.161	0.066	237.7	
3	1	744.8	-0.540	-0.018	66.2	66.3
	2	739.8	-0.533	-0.013	68.9	
	3	749.2	-0.543	-0.026	63.6	
4	1	387.1	-0.154	0.074	257.9	252.3
	2	389.7	-0.150	0.065	256.2	
	3	398.8	-0.162	0.058	242.9	
6	1	-665.4	0.945	-0.045	-34918	-34878
	2	-651.9	0.927	-0.017	-35021	
	3	-652.5	0.930	-0.023	-34693	
7	1	1005.4	-0.753	-0.216	17.2	16.1
	2	1010.6	-0.759	-0.228	16.1	
	3	1015.0	-0.760	-0.247	15.1	
8/9	1	733.5	-0.437	-0.292	37.8	35.4
	2	743.2	-0.444	-0.324	33.2	
	3	737.7	-0.438	-0.311	35.4	
10	1	-708.9	1.047	-0.254	-27288	-28167
	2	-696.2	1.021	-0.199	-29130	
	3	-702.3	1.037	-0.231	-28080	

1 MPa= 0.15 ksi

Table 4.7 Coefficient of Variation for Soil Engineering Tests (adapted from reference 39)

Sl.No	Soil Property / Variable	Coefficient of Variation
1	Clay %	25
2	Silt %	25
3	P_{40} %	10
4	P_{200} %	15
5	W_s %	15
6	LL %	10
7	PI %	30
8	Density	5
9	Saturation	10
10	$P_{3/8}$	25
11	P_4	25
12	C_u	25
13	Unconfined Comp Strength	40

Table 4.8 List of Soil Properties Employed in Model Building. (The Last Column Lists the Variables Which are Assumed to Vary).

Model	Soil Texture	List of Variables	Selected Variables
LTTP	Coarse-grain	$P_{3/8}$, P_4, %Clay, LL, w_{opt}, γ_s, %Silt, γ_{opt}, w_c, P_{200}	%Clay, LL, w_{opt}, %Silt, γ_{opt}, P_{200}
	Fine-grain	% Clay, PI, w_c, % Silt	% Clay, PI, w_c, % Silt
	Fine-grain clay	% Clay, w_c, P_4, P_{40}, P_{200}, LL, w_{opt}, % Silt, LL, γ_{opt}, γ_s	% Clay, w_c, P_{40}, P_{200}, LL, w_{op}t, % Silt, LL,
Georgia	Coarse-grain	w_{opt}, w_c, COMP, %Silt, LL, PI, SW, SH, P_{40}, γ_s , S	w_c, %Silt, PI, S
Minnesota	Fine-grain	γ_s, w_c, PI, LL, P_{200},S	γ_s, w_c, PI, LL, P_{200},S
Carmichael	Coarse-grain	w_c, Soil Class	w_c,
	Fine-grain	PI, w_c, P_{200}, Soil Class	PI, w_c, P_{200}
Drumm	Fine-grain	q_u, % Clay, PI, γ_s, S, w_c, P_{200}, LL	q_u, % Clay, PI, S, w_c, P_{200}, LL
Wyoming	Fine-grain	PI, S, P_{200}	PI, S, P_{200}
Mississippi	Coarse-grain	w_c, γ_d, P_{200},c_u	w_c, γ_d, P_{200}
	Fine-grain	LL, w_c, γ_d, γ_{opt}, P_{200}	LL, w_c, γ_d, γ_{opt}, P_{200}

Table 4.9. Rank Order (By Count) of Important Variables

Sl. No.	Soil Variable	Rank Order
1	Passing # 200	(6/7)
2	PI	(6/7)
3	Moisture	(5/7)
4	LL	(5/7)
5	Density	(4/7)

Table 4.10 Model Validation Based on Two Criteria

Sl.No	Model		Agreement with TP46 (+/- 20%)	Coefficient of Variation (20 %)	Comments
1	LTTP	(C)	(1/1)[a]	1/1	Satisfactory
		(F)	2/7	7/7	Satisfactory
2	Georgia	(F)	0/7	0/7	
3	Carmichael	(C)	1/1	1/1	
		(F)	1/7	5/7	
4	Drumm	(F)	3/7	0/7	
5	Wyoming	(F)	2/7	0/7	
6	Mississippi	(C)	0/1	1/1	Satisfactory
		(F)	6/7	1/7	Satisfactory

C = Coarse-grain soil

F = Fine-grain soil

a = Number of Soils satisfying each criterion out of total number tested

Table 4.11 Sensitivity Analysis (Effect of Response Variables on M_R Prediction). Silt Soil #1

Response Variable	Resilient Modulus, MPa									
	LTPP		Carmichael		Drumm		Wyoming		Mississippi	
	M_R^+	M_R^-	M_R^+	M_R^-	M_R^+	M_R^-	M_R^+	M_R^-	M_R^+	M_R^-
%Clay	69.0	62.2	-	-	58.9	54.5	-	-	-	-
%Silt	66.2	65.2	-	-	-	-	-	-	-	-
P_{200}	-	-	108.4	124.6	48.2	65.3	47.0	34.0	66.9	71.3
w_c	57.5	74.4	107.8	125.2	-	-	-	-	57.6	86.7
LL	-	-	-	-	58.1	55.4	-	-	78.6	60.1
PI	71.9	59.7	110.7	122.2	61.5	52.0	42.0	39.0	-	-
%Saturation	-	-	-	-	36.1	77.5	20.0	61.0	-	-

$M_R^+ = M_R$ calculated with response variable increased by one standard deviation

$M_R^- = M_R$ calculated with response variable decreased by one standard deviation

1 MPa= 0.15 ksi

Table 4.12 Sensitivity Analysis (Effect of Response Variables on M_R Prediction) Clay Soils # 4

Response Variable	LTPP		Carmichael		Drumm		Wyoming		Mississippi	
	M_R^+	M_R^-	M_R^+	M_R^-	M_R^+	M_R^-	M_R^+	M_R^-	M_R^+	M_R^-
%Clay	73.5	68.2	-	-	75.8	70.7	-	-	-	-
%Silt	74.6	67.2	-	-	-	-	-	-	-	-
P_{40}	72.7	68.9	-	-	-	-	-	-	-	-
P_{200}	65.9	76.1	81.3	99.1	63.9	82.6	53.0	40.0	93.3	97.4
w_c	63.8	77.8	81.3	99.1	-	-	-	-	76.9	123.9
LL	72.6	69.1	-	-	74.9	71.5	-	-	110.8	80.9
PI	70.9	70.7	78.5	101.9	82.9	63.6	48.0	44.0	-	-
Saturation	-	-	-	-	52.4	94.1	26	67	-	-

$M_R^+ = M_R$ calculated with response variable increased by one standard deviation
$M_R^- = M_R$ calculated with response variable decreased by one standard deviation
1 MPa= 0.15 ksi

Table 4.13 Ranking of Response Variables Based on Sensitivity

Sl. No.	Explanatory Factor	Significance*	Acceptable Trend
1	Water content, w_c	6/6	MR decreases with increase in Wc (5/6)
2	Percent saturation, S	4/4	MR decreases with increase in S (4/4)
3	Passing #200 sieve, P_{200}	4/9	MR decreases with increase in P_{200} (7/9)
4	Plasticity Index, PI	3/8	MR decreases with decrease in PI? (5/8)

* If the change in M_R exceeds 10% for a variation of one standard deviation

CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1 Summary

The primary objective of this study is to validate and select a model for estimating resilient modulus of soils for pavement design. The first category of models is heavily weighted with soil index properties, and the second relies on stress state and soil index properties in tandem. Eight Mississippi subgrade soils were tested in the laboratory for M_R in accordance with TP 46 protocol. Routine laboratory tests were also performed on the soils to determine the soil index properties.

The predictability of the equations is sought by comparing predicted resilient modulus to measured value and agreement thereof. Uncertainty in predicting resilient modulus arising from inherent variability of soil physical properties is also studied, employing the point estimate method. The significance of independent variables is investigated by conducting a sensitivity study.

A critique of various equations investigated in this study reveals the following: Though earlier equations emphasized soil index properties in predicting the resilient modulus, the current trend is to start with a pseudo constitutive equation and then expand it to take into account soil properties. Frequently used variables in developing the equations are moisture content, degree of saturation, plasticity index, material passing the #200 sieve, and dry density. However, other variables such as liquid limit, percent clay, percent silt, material passing the #40 sieve etc., are also employed in a few equations.

5.2 Conclusions

The major conclusions resulting from the analysis validating the models are:

- Simple strength correlations, for example, the CBR test to estimate resilient modulus should be used with caution.
- M_R values predicted by the Georgia and Minnesota equations do not agree with the laboratory values. Wyoming equation predictions are considered unsatisfactory as well.
- Carmichael and Drumm equations are not recommended, for the reason that estimation of a few of the input parameters could be subjective and / or complex.
- Mississippi equations for fine-grain soil has resulted in close predictions in six out of seven soils; and therefore, considered to be acceptable for predicting M_R of Mississippi soils. The coarse-grain soil equations, however, need to be revised.
- For having developed from an extensive materials database, LTPP equations have shown potential to predict M_R of a range of soils with a wide geographical coverage. In addition, the models accommodate both stress variables and soil index properties in tandem. LTPP models–coarse-grain, fine-grain silt and clay soils–therefore, deserve strong consideration in level II Mechanistic Empirical pavement design.

Based on a sensitivity study of seven equations, investigating the significance of soil index properties in predicting M_R, the most important input variable is judged to be sample moisture followed by material passing the #200 sieve, PI and sample density in that order.

5.3 Recommendation/Implementation of Results

As MDOT is in the process of implementing the Mechanistic Empirical Pavement Design Guide, (ME PDG), subgrade characterization in terms of resilient modulus becomes a prerequisite. Two sets of prediction equations deserve consideration for this purpose: LTPP equations and Mississippi equations. Both models, in turn, present separate equations, one for coarse soil and another for fine soil. For coarse soil (A-2 and A-3) LTPP equation 2.19 in conjunction with 2.20 to 2.22 is the sole choice. For fine soil (A-4, A-5, A-6 and A-7), however, the recommendation is to use both LTPP equations (2.23 to 2.28) and Mississippi equation 2.11, and adopt an average of the two for design. Those values computed for both coarse- and fine-grain soil could be used for a level II pavement design category or in preliminary design while pursuing level I design. In the latter case, preliminary design could be revised when in-situ resilient modulus becomes available, which can be ascertained only upon completion of the grading project.

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APPENDIX A

Testing Sequence for Subgrade Soil Materials, TP 46 Protocol

Sequence No.	Confining Pressure, σ_3	Seating Stress, 0.1 σ_{max}	Cyclic Stress, σ_{cyclic}	Max. Axial Stress, σ_{max}	No. of Load Applications
	psi	psi	psi	psi	
0	6	0.4	3.6	4	500-1000
1	6	0.2	1.8	2	100
2	6	0.4	3.6	4	100
3	6	0.6	5.4	6	100
4	6	0.8	7.2	8	100
5	6	1.0	9.0	10	100
6	4	0.2	1.8	2	100
7	4	0.4	3.6	4	100
8	4	0.6	5.4	6	100
9	4	0.8	7.2	8	100
10	4	1.0	9.0	10	100
11	2	0.2	1.8	2	100
12	2	0.4	3.6	4	100
13	2	0.6	5.4	6	100
14	2	0.8	7.2	8	100
15	2	1.0	9.0	10	100

APPENDIX B

Method of Point Estimates: Illustration

Let Y be a function of two random variables, $Y= f(x_1, x_2)$

Approximate values of the first two moments of a function (Y) from the first two moments of the random variables (x_1, x_2) can be obtained from the method of point estimates.

The mean and variance of Y are given by,

Mean, $\mu_Y = P_{++}Y_{++} + P_{+-}Y_{+-} + P_{-+}Y_{-+} + P_{--}Y_{--}$

Variance, $\sigma^2_Y = P_{++}Y^2_{++} + P_{+-}Y^2_{+-} + P_{-+}Y^2_{-+} + P_{--}Y^2_{--} - \mu^2_Y$

where, $Y_{++} = Y(x_{1+}, x_{2+})$;

$Y_{+-} = Y(x_{1+}, x_{2-})$;

$Y_{-+} = Y(x_{1-}, x_{2+})$;

$Y_{--} = Y(x_{1-}, x_{2-})$;

$x_+ = \mu_x + \sigma_{x;}$

$x_- = \mu_x - \sigma_{x;}$

$P_{++} = P_{--} = 0.25\,(1 + r_{x1,\,x2})$;

$P_{+-} = P_{-+} = 0.25\,(1 - r_{x1,\,x2})$; and

$r_{x1,x2}$ is the correlation coefficient between x_1, x_2. If x_1, x_2 are independent, then $r_{x1,x2} = 0$, an assumption adopted in all of the calculations.