## DRIVER BEHAVIOR AT

## RAIL-HIGHWAY GRADE CROSSINGS:

## A SIGNAL DETECTION THEORY ANALYSIS

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## INTRODUCTION

Signal Detection Theory (SDT) is often used in studies of sensory psychology and perception to describe laboratory experiments in which subjects are asked to detect small changes in very wellcontrolled, precisely defined stimuli such as the intensity of a monochromatic light or the frequency of a pure tone. Consequently, it may seem odd that such a theory can be of any practical use in describing the situation that occurs when the driver of an automobile approaches a grade crossing and must decide whether it is safe to drive across the railroad track(s). Locomotives and trains are not well-controlled and precisely defined stimuli like those used in the sensory laboratory. By comparison with the stimulus changes used in the laboratory, a locomotive surely represents an enormous potential change in the sensory environment of the automobile driver. Why then is this theory applicable to the driver at the grade crossing? The answer to this question lies in an examination of the types of accidents that occur at grade crossings which suggest that motorists have difficulty with the tasks of detecting trains and related decision-making at grade crossings. For instance, motorists regularly drive into the side of passing trains at grade crossings and drive directly in front of approaching trains at close range. These accidents suggest that an examination of the grade crossing from the perspective of SDT and human information processing may provide a useful model for analysis, research, and the development of new strategies for grade crossing accident prevention.

The plan of this analysis is as follows. In Section I, the basic model of SDT is described with reference to a driver approaching a grade crossing with a train also approaching. The driver's task is to decide if he can cross the tracks safely or if he must stop. The treatment employs some mathematics, which can be omitted without losing the sense of the model. In describing the basic model, it becomes apparent that accident rates for different types of grade crossings are predicted by the SDT model to vary with train frequency. Section II examines accident rates at grade crossings and develops a Poisson process model of accident probability with reference to the frequency of trains and cars at grade crossings. The Poisson model predicts maximal accident rates and is useful for evaluating the effectiveness of different grade crossing devices in preventing accidents. The maximal accident rate concept is also used in Section III in applying SDT to a quantitative analysis of grade crossing devices. Section IV examines the implications of the SDT analysis for various schemes to improve grade crossing safety, contrasts the SDT model with existing models of accident prediction, and suggests areas of research which can be implemented to achieve Goal \# 2 of the RDV Action Plan for Grade Crossings:

Improve our understanding and knowledge of motorist behavior at grade crossings in causing collisions between trains and motor vehicles - including: 1) detection, recognition, perception and comprehension of warning devices and trains; and 2) decision making, perception of collision risk, and motivation involved in circumvention of active warning devices - in order to improve upon design, deployment and operation of grade crossing protection devices.

Section V models the performance of an ideal motorist who uses information concerning the
distances of his vehicle and the train from the intersection to determine whether to cross the intersection or to stop. Visual search with and without auditory localization (train horn) is incorporated into the model.

## I. SIGNAL DETECTION THEORY

## Detection of a Signal in a Background of Noise

The point of view to be developed here is that a motorist at a grade crossing with an approaching train is in an analogous situation to an observer attempting to detect a signal in a background of noise (for a more detailed description of SDT than is provided in this section consult Green and Swets, 1974 and Egan, 1975). In both instances, it is often difficult to distinguish signal from noise, and a decision is made which is not solely dependent upon the sensory information alone. From this point of view, the locomotive is a multi-sensory signal, and the same is true of the background noise. The train or locomotive has auditory, visual, tactile (vibration), and olfactory components which contribute to its "signalness". The background noise also consists of a variety of auditory, visual, tactile and olfactory components. In the SDT model both the signal and the noise are represented as a single perceptual continuum which varies in magnitude. Signals, such as the locomotive, are capable of producing perceptual magnitudes which vary between encounters, even when all of the sensory components are identical. Consequently, there is a probability distribution of perceptual magnitudes which are associated with a particular locomotive configuration (e.g., size, loudness, color, brightness, etc.). This distribution of perceptual magnitudes has a mean and variance which can be used to specify the perceptual magnitude of the locomotive as a signal. Similarly, the background noise also has a distribution of perceptual magnitudes which can also be specified by a mean and a variance. For the sake of simplicity it is often assumed that the distribution of perceptual magnitudes for noise and signal are gaussian or normal. Additionally, the basic SDT model assumes that the variances of signal and noise distributions are equal. Neither assumption is critical to the theory.

Figure 1 is a typical representation of noise and signal-plus-noise distributions in SDT. It should immediately be noted that the distributions overlap. The chief difference, from this point of view, between a signal and noise is that, on the average, signals have a larger mean percept magnitude than noises. The perceiver (the motorist in our case) can only distinguish between a signal and noise on the basis of the magnitude of the perceptual event. Given a perceptual event, the perceiver must decide if the event represents a signal or noise. The perceiver does this by adopting a criterion. In Fig. 1, a criterion line has been drawn to illustrate. If a perceptual event has a magnitude which falls to the right of the criterion, the perceiver decides that the event is a signal. If the event has a magnitude which falls to the left of the criterion, the perceiver decides that the event is not a signal. Hence, we have the following four-fold table, Table 1. ${ }^{1}$ There are two response categories: "Yes, Stop (the train is too close)." and "No, Don't stop (the train is not too close).", and there are two possible events: a train is close to the crossing and a train is not too close to the crossing (or not present).


Figure 1. Noise and signal-plus-noise distributions. A criterion line is drawn to show how the probabilities of Table 1 are determined.

TABLE 1. STIMULUS AND RESPONSE MATRIX FOR A MOTORIST AT A GRADE CROSSING.

|  | Yes, Stop. | No, don't stop. |
| :--- | :--- | :--- |
| Train is close | VALID STOP <br> (motorist stops at crossing) | ACCIDENT <br> (motorist doesn't stop) |
| Train is not close, or <br> No train in vicinity | FALSE STOP <br> (motorist stops unnecessarily) | CORRECT CROSSING <br> (motorist crosses tracks <br> safely) |

If a train is close and the motorist decides not to stop, an ACCIDENT (AC) occurs. The decision to stop when a train is close is termed a VALID STOP (VS). The decision criterion divides the distribution of "train is close" percepts (signal distribution in Fig. 1) into VALID STOPs and ACCIDENTs. The criterion also divides the distribution of "train is not close" percepts (noise distribution in Fig. 1) into two parts: FALSE STOPs (FSs) and CORRECT CROSSINGs (CCs).

Since the two distributions are probability distributions, the probability of a VS $(\mathrm{P}(\mathrm{VS})$ ) is the complement of the probability of an $\mathrm{AC}(\mathrm{P}(\mathrm{AC})$ ), etc. (i.e., $\mathrm{P}(\mathrm{AC})=1-\mathrm{P}(\mathrm{VS})$ ). Moreover, since both distributions are divided by the criterion, only two probabilities are needed to totally describe the effect of changes in the criterion. In SDT these are usually $\mathrm{P}(\mathrm{VS})$ and $\mathrm{P}(\mathrm{FS})$.

The first point to note is that changes in the criterion do not change the detectability of the proximity of the train. The only aspect of this model which is capable of altering detectability is the separation of the signal and noise distributions. In this regard, there are three options: decrease the level of background noise, increase the level of the signal, and change the variance of one or both distributions. In a later section we will address the nature of the signal and the nature of the noise with a view to understanding safety issues. Changes in the criterion only change the probabilities of the outcomes, while changes in the distributions can effect a change in both detectability and the probabilities of the outcomes. Factors which affect the criterion are very important, especially if it is not possible to achieve an increase in detectability. These factors will also be addressed in a subsequent section.

To illustrate basic features of the SDT model, consider the criterion in Fig. 1 which is set at a percept magnitude of 1.65 . As noted above, detectability is not influenced by the setting of the criterion, although the specific location of the criterion will determine the probability of accidents $(\mathrm{P}(\mathrm{AC})$. For example, the values of $\mathrm{P}(\mathrm{VS})$ and $\mathrm{P}(\mathrm{FS})$ in Fig. 1 at this value of the criterion are 0.055 and 0.0047 , respectively. (It should be noted that $\mathrm{P}(\mathrm{VS})$ is the area under the signal curve to the right of the criterion, and that $\mathrm{P}(\mathrm{FS})$ is the area under the noise curve to the right of the criterion.) Because of the complimentary relationship between $\mathrm{P}(\mathrm{VS})$ and $\mathrm{P}(\mathrm{AC})$, the probability of an ACCIDENT is quite high with the criterion set at $1.65: \mathrm{P}(\mathrm{AC})=0.945$. Leftward shifts in the criterion would increase $\mathrm{P}(\mathrm{VS})$ and decrease $\mathrm{P}(\mathrm{AC})$. For instance, if the criterion is set at a value of 1.35 , then the values obtained for $\mathrm{P}(\mathrm{VS})$ and $\mathrm{P}(\mathrm{FS})$ are 0.34 and 0.08 . A criterion set at 1.05 would cause $\mathrm{P}(\mathrm{VS})$ and $\mathrm{P}(\mathrm{FS})$ to have values of 0.79 and 0.42 . Consequently, in these three examples the probability of an ACCIDENT (P(AC)) would change from 0.945 to 0.66 to 0.21 . Note that these changes in the probability of an ACCIDENT have not involved changes in the detectability of the locomotive or train.

In SDT detectability is independent of the setting of the criterion. Mathematically, detectability (sometimes referred to as sensitivity) is defined as the difference between the means of the signal and noise distributions divided by their common standard deviation:

$$
\begin{equation*}
d^{\prime}=\frac{\mu_{s}-\mu_{n}}{\sigma} \tag{1}
\end{equation*}
$$

In the example illustrated in Fig. 1, the mean of the noise distribution is 1.0 and that of the signal distribution is 1.25 . Each distribution has been created to be normally distributed with a standard deviation of 0.25 . As a result, the value of $\mathrm{d}^{\prime}$ for the example in Fig. 1. will always be 1.0. In most practical situations, however, the means and standard deviations are usually not known. Under these
circumstances detectability is often derived from outcome information, namely $\mathrm{P}(\mathrm{VS})$ and $\mathrm{P}(\mathrm{FS})$. Note that if the distributions are normal and of equal variance, then the formula for $\mathrm{d}^{\prime}$ can be rewritten as the difference of two standardized (z-) scores. For instance, if a criterion c is selected, $z$-scores for the noise and signal distributions can be defined as:

$$
\begin{equation*}
z_{n}=\frac{c-\mu_{n}}{\sigma} \tag{2}
\end{equation*}
$$

and

$$
\begin{equation*}
z_{s}=\frac{c-\mu_{s}}{\sigma} \tag{3}
\end{equation*}
$$

By definition, $\mathrm{z}_{\mathrm{s}}$ is the standardized score for HITS and $\mathrm{z}_{\mathrm{n}}$ is the standardized score for FAs:

$$
\begin{equation*}
d^{\prime}=z_{n}-z_{s}=\frac{\left(c-\mu_{n}\right)-\left(c-\mu_{s}\right)}{\sigma}=\frac{\mu_{s}-\mu_{n}}{\sigma} . \tag{4}
\end{equation*}
$$

Because they are observable and can indicate the separation of the signal and noise distributions as well as the location of the criterion, $\mathrm{P}(\mathrm{VS})$ is often plotted as a function of $\mathrm{P}(\mathrm{FS})$ in a plot which is called a Receiver-Operator Characteristic (ROC) curve. Figure 2 illustrates this for the distributions shown in Fig. 1. Each point on a ROC curve corresponds to a particular criterion line. The line which connects the origin $(0,0)$ with the upper right corner $(1,1)$ corresponds to the ROC curve for identical signal and noise distributions (i.e., $\mathrm{d}^{\prime}=0$ ). The ROC curve which is labeled " d ' $=1$ " was generated from Fig. 1. The ROC curve labeled " d " $=2$ " was generated from the same noise distribution as that in Fig. 1, but with a signal distribution with a mean of 1.5 and a standard deviation of 0.25 .

The ROC curve for $\mathrm{d}^{\prime}=2$ illustrates the effect of an increase in detectability on the outcome probabilities. For a criterion value of $1.35, \mathrm{P}(\mathrm{VS})=0.725$ and $\mathrm{P}(\mathrm{FS})=0.08$. Recall that for $\mathrm{d}^{\prime}=$ 1 , for the same criterion $\mathrm{P}(\mathrm{VS})=0.34$ and $\mathrm{P}(\mathrm{FS})=0.08$. Thus, an increase in detectability reduces the accident rate from 0.66 to 0.275 at a constant criterion.

## Decision-Making: Setting the Criterion

Setting the criterion involves the process of decision-making. To this point we have not discussed how the criterion is set, or how a criterion can be changed. From the discussion above, it should be clear that the accident rate is directly influenced by changes in discriminability of the
locomotive and by the setting of the criterion. Consequently, decision-making is an important aspect of the SDT model.


Figure 2. Receiver-operator characteric curves for Figure 1 and other conditions described in the text. The value, $c=1.35$, indicates two points with a constant criterion.

As was noted above, the distributions of signal and noise are assumed to overlap. Suppose a perceptual event of magnitude $x$ occurs which falls into the region of overlap. The probability that the event, x , came from the noise distribution is the conditional probability, $\mathrm{P}(\mathrm{x} \mid \mathrm{n})$. Similarly, the probability that x came from the signal distribution is $\mathrm{P}(\mathrm{x} \mid \mathrm{s})$. A rational decision concerning which distribution $x$ came from can be made on the basis of these two conditional probabilities. The likelihood ratio ( L ), which is defined as $\mathrm{P}(\mathrm{x} \mid \mathrm{s}) / \mathrm{P}(\mathrm{x} \mid \mathrm{n})$, indicates the likelihood that x arose from the signal distribution. Put differently, L indicates the strength of the evidence that the event was, in our example, a "train is close" percept. L is not a probability and can range from zero to infinity.

Signals and noise, however, do not always occur with equal probability. This is particularly true of trains at grade crossings. The probability of a signal $\mathrm{P}(\mathrm{s})$ and the probability of noise $\mathrm{P}(\mathrm{n})$ during any observation interval are important to the observer. In SDT these probabilities are called the prior probabilities. The observer has no control over the prior probabilities, but has knowledge of them based on experience, etc. If the prior probabilities are equal, then $L$ provides a direct estimate of the odds that x arose from the signal distribution. The likelihood that the evidence was a signal or a noise is equal when $\mathrm{L}=1$. This occurs in Fig. 1 where the two distributions crossover. $\mathrm{L}>1$ for all perceptual magnitudes to the right of the crossover. Thus, the likelihood that the evidence was a signal is greater as the perceptual magnitude increases. If the prior probabilities are not equal, then the likelihood ratio does not provide an estimate of the odds that x arose from the signal distribution, and the posterior probabilities must be considered.

The posterior probabilities are the conditional probabilities of signals and noises given the sensory evidence, $\mathrm{x}: \mathrm{P}(\mathrm{s} \mid \mathrm{x})$ and $\mathrm{P}(\mathrm{n} \mid \mathrm{x})$, respectively. Note first that since there are only two categories, s and $\mathrm{n}, \mathrm{P}(\mathrm{s} \mid \mathrm{x})+\mathrm{P}(\mathrm{n} \mid \mathrm{x})=1$. By definition the joint probability, $\mathrm{P}(\mathrm{s}, \mathrm{x})=\mathrm{P}(\mathrm{x} \mid \mathrm{s}) \cdot \mathrm{P}(\mathrm{s})=$ $\mathrm{P}(\mathrm{s} \mid \mathrm{x}) \cdot \mathrm{P}(\mathrm{x})$. Consequently,

$$
\begin{equation*}
P(s \mid x)=\frac{P(x \mid s) \cdot P(s)}{P(x)}=\frac{P(x \mid s) \cdot P(s)}{P(x \mid s) \cdot P(s)+P(x \mid n) \cdot P(n)} \tag{5}
\end{equation*}
$$

The posterior probability, $\mathrm{P}(\mathrm{n} \mid \mathrm{x})$, can be similarly defined:

$$
\begin{equation*}
P(n \mid x)=\frac{P(x \mid n) \cdot P(n)}{P(x)}=\frac{P(x \mid n) \cdot P(n)}{P(x \mid s) \cdot P(s)+P(x \mid n) \cdot P(n)} . \tag{6}
\end{equation*}
$$

The ratio of the posterior probabilities, $\mathrm{P}(\mathrm{s} \mid \mathrm{x}) / \mathrm{P}(\mathrm{n} \mid \mathrm{x})$, is called the posterior odds and it indicates the likelihood that a signal was present given the evidence, $x$. The posterior odds are

$$
\begin{equation*}
\frac{P(s \mid x)}{P(n \mid x)}=\frac{P(s)}{P(n)} \cdot \frac{P(x \mid s)}{P(x \mid n)}=\frac{P(s)}{P(n)} \cdot L . \tag{7}
\end{equation*}
$$

This last equation indicates that two sources of information are contained in the posterior odds: the relative frequency of occurrence of the two events, $s$ and $n$, and the likelihood ratio. In this way the observer's expectations about the frequency of the events and the sensory information provided by the evidence are combined in the posterior odds. Decisions in SDT are made on the basis of the magnitude of the likelihood ratio, L , relative to some decision criterion. It is easily seen that L is a monotone function of the posterior odds.

Decision strategies come in many forms and may not even be rationally based. We will consider the most common strategy only. Decision strategies are usually the result of decision goals. A common goal in forming decisions is to maximize the expected value. Assume that the observer has a value (positive or negative) for each of the outcome cells in Table 1. Table 2 illustrates this.

TABLE 2. PAYOFF MATRIX.

|  | Yes, stop | No, don't stop |
| :--- | :--- | :--- |
| Train is close | $\mathrm{V}(\mathrm{s}, \mathrm{Y})$ | $\mathrm{V}(\mathrm{s}, \mathrm{N})$ |
| Train is not close | $\mathrm{V}(\mathrm{n}, \mathrm{Y})$ | $\mathrm{V}(\mathrm{n}, \mathrm{N})$ |

In Table 2 each of the outcomes has a probability of occurrence as well as a value. The expected value of an outcome, by definition, is its probability multiplied by its value. The probabilities of concern here are the joint probabilities of signal and a "Yes" response $[\mathrm{P}(\mathrm{Y}, \mathrm{s})$ ], a noise and a "Yes" response $[\mathrm{P}(\mathrm{Y}, \mathrm{n})]$, etc. By definition, $\mathrm{P}(\mathrm{Y}, \mathrm{s})=\mathrm{P}(\mathrm{Y} \mid \mathrm{s}) \cdot \mathrm{P}(\mathrm{s}), \mathrm{P}(\mathrm{Y}, \mathrm{n})=\mathrm{P}(\mathrm{Y} \mid \mathrm{n}) \cdot \mathrm{P}(\mathrm{n})$, etc. The expected value of the decision is the sum of all of the expected values for the outcomes. Hence the expected value, $E(V)$, for Table 2 is
$\mathrm{E}(\mathrm{V})=\mathrm{P}(\mathrm{Y} \mid \mathrm{s}) \cdot \mathrm{P}(\mathrm{s}) \cdot \mathrm{V}(\mathrm{s}, \mathrm{Y})+\mathrm{P}(\mathrm{Y} \mid \mathrm{n}) \cdot \mathrm{P}(\mathrm{n}) \cdot \mathrm{V}(\mathrm{n}, \mathrm{Y})+\mathrm{P}(\mathrm{N} \mid \mathrm{s}) \cdot \mathrm{P}(\mathrm{s}) \cdot \mathrm{V}(\mathrm{s}, \mathrm{N})+\mathrm{P}(\mathrm{N} \mid \mathrm{n}) \cdot \mathrm{P}(\mathrm{n}) \cdot \mathrm{V}(\mathrm{n}, \mathrm{N})$.
The goal is to maximize $\mathrm{E}(\mathrm{V})$ which is accomplished by drawing the criterion line so as to achieve this. This is done by maximizing: $\mathrm{P}(\mathrm{Y} \mid \mathrm{s})-\beta \mathrm{P}(\mathrm{Y} \mid \mathrm{n})$. Green and Swets (1974) show that $\beta$ is given by

$$
\begin{equation*}
\beta=\frac{V(n, N)+V(n, Y)}{V(s, Y)+V(s, N)} \cdot \frac{P(n)}{P(s)} . \tag{8}
\end{equation*}
$$

The expected value is maximized by saying "Yes" whenever the likelihood ratio, L , is equal to or exceeds $\beta$. In short, $\beta$ defines the location of the criterion line in Fig. 1.

If $\mathrm{P}(\mathrm{s})=\mathrm{P}(\mathrm{n})$, then $\beta$ is only determined by the values of the outcomes. If all of the values of the outcomes are equal, $\beta$ is only determined by the prior probabilities. When the values of the outcomes are all equal and the prior probabilities are also equal, $\beta=1$. As was noted above, this is the value of L at the crossover of the signal and noise distributions in Fig. 1.

We are now in a position to examine the effects of bias on the decision-making of our motorist. In SDT bias is defined as the tendency of an observer to place his or her criterion anywhere except at the intersection of the noise and signal distributions (i.e., $\beta \neq 1$ ). Bias is independent of detectability (also called sensitivity or discriminability and measured by d' as noted
above) and is determined by the observer's expectations (probability of signal, probability of noise), motivation (values of each of the decision outcomes), and other cognitive functions (e.g., memory, attention, decision strategy). For instance, a driver who is familiar with a particular grade crossing has an expectation regarding the frequency of trains at that crossing. We will use as an example a crossing where the frequency of trains varies markedly with time of day: on a railroad which carries only heavy morning and evening commuter trains. Drivers who use the crossing at different times of day will have markedly different expectations regarding the frequency of trains at the crossing. This is captured by the prior probabilities. Suppose trains are more frequent between 7 AM and 9 AM than they are between 1 PM and 3 PM, and that Driver \#1 (the morning driver) uses the crossing to go to work between 7:30 AM and 8 AM and Driver \#2 (the afternoon driver) uses the same crossing between 1 PM and 3 PM to visit a relative in a nursing home. If $\mathrm{P}(\mathrm{s})=0.62(62$ out of 100 times the driver encounters a train at the crossing) for the morning period and $\mathrm{P}(\mathrm{s})=0.26$ (26 out of 100 times the driver encounters a train at the crossing) for the afternoon period, the ratio, $\mathrm{P}(\mathrm{n}) / \mathrm{P}(\mathrm{s})$ for the two periods are $0.38 / 0.62=0.61$ and $0.74 / 0.26=2.8$, respectively. Hence, for the morning $\beta=0.61$ and for the afternoon $\beta=2.8$. Both drivers have a bias, because $\beta \neq 1$. For the morning driver, there is a bias to say "Yes, the train is close, stop" given the identical sensory information that the afternoon driver gets. This can also be viewed in terms of the perceptual magnitudes that each driver would require to indicate that he or she detects a close train (i.e., the "threshold" for detection). Referring to the distributions of Fig. 1, one can find the perceptual magnitudes which correspond to the values of $\beta$. Expected value is maximized by saying "Yes" whenever L is equal to or exceeds $\beta$. L is the ratio of the probability densities at each percept magnitude in Fig. 1. Thus, values of L map directly onto percept magnitudes in Fig. 1. Given the distributions in Fig. 1, a percept magnitude of 1.15 would be the "threshold" for the morning driver to say a train was close, and a percept magnitude of 1.45 would be the "threshold" for the afternoon driver. For the morning driver $\mathrm{P}(\mathrm{VS})=0.65$ and $\mathrm{P}(\mathrm{FS})=0.27$, while for the afternoon driver $\mathrm{P}(\mathrm{VS})=0.21$ and $\mathrm{P}(\mathrm{FS})=$ 0.035. Referring to Table 1, the probability of an ACCIDENT ( $\mathrm{P}(\mathrm{AC})$ ) is 0.35 for the morning driver and 0.79 for the afternoon driver, even though all other conditions are identical. It should be also kept in mind that for both drivers the train is assumed to be equally discriminable. This is shown in Fig. 2.

This observation may appear surprising, but it has been made previously with regard to the rail-highway grade crossing by Lerner et al. (1990, p. 3-12):

A related principle from the area of signal detection theory is that the higher the perceived probability of an event, the higher is the likelihood that an observer will report having detected the event. If the driver assigns a low probability to the presence of a train at a railhighway crossing, he will adopt a higher criterion for detecting the train, and this will increase his chances of missing the train. It is important to note that the criterion for detection is not consciously set, but rather corresponds to the amount of visual "evidence" required for detection.

SDT predicts that expectations play a major role in accidents at rail-highway grade crossings. All other things being equal, this analysis suggests that crossings with a lower frequency of trains
should have a higher accident rate. Thus, for a particular type of crossing (active vs. passive protection, etc.), the frequency of trains should vary inversely with the accident rate at the crossing. This prediction will be explored in Section II.

Expectations also play a role with regard to signage at grade crossings. From the point of view of SDT, a role of signage is to inform the motorist that trains are frequent at the crossing. Personal experience with a crossing, however, is likely to be more important since a sign does not indicate the actual frequency of trains. Motorists who are unfamiliar with a grade crossing which has signage posted should assume that trains are highly frequent and exhibit a high degree of caution relative to motorists who are familiar with the crossing. This prediction of SDT is supported in the literature. Lerner et al (1990, p. 3-61) state that

There is no question that familiar and unfamiliar drivers often behave differently at crossings, and that traffic is sensitive to the schedule of train operations. Sanders et al. (1973) found that driver looking and speed reductions were inversely correlated with the frequency of using the crossing. Expectancies based on familiarity have been implicated in accident causation research (Knoblauch et al., 1982). Sanders et al. (1973) also found that drivers were sensitive to the actual frequencies of trains. The correlation of looking with train frequency at the crossing was $r=0.66$, and the correlation of speed at the crossing with train frequency was $r=-0.85$. Others have reported similar findings (e.g., Aberg, 1988).

Values associated with decision outcomes are also predicted to play a role in driver behavior at grade crossings. Again, the analysis assumes that all other aspects of the situation are the same, including the detectability of the train. Consequently, the distributions of Fig. 1 will again be used.

Recall that the morning motorist was driving to work and that the afternoon motorist was driving to visit a relative in a nursing home. For both drivers this is a daily trip. However, there are different values associated with the outcomes of decisions at the crossing for each driver. Moreover, the values are not necessarily monetary or even linear with dollar value. Thus, for the purposes of illustration, numbers indicating relative subjective value will be assigned to the outcomes in the payoff matrix so as to allow $\beta$ to be calculated.

TABLE 3. PAYOFF MATRIX FOR MORNING DRIVER.

|  | Yes, stop | No, don't stop |
| :--- | :--- | :--- |
| Train is close | 0.5 | -20 |
| Train is not close | -10 | 1 |

TABLE 4. PAYOFF MATRIX FOR AFTERNOON DRIVER.

|  | Yes, stop | No, don't stop |
| :--- | :--- | :--- |
| Train is close | 1 | -20 |
| Train is not close | -1 | 1 |

Tables 3 and 4 present payoff matrices for the morning and afternoon drivers, respectively. For both drivers it is assumed that a very large negative value is associated with the error of saying "No" when in fact a train is close. Also, for both drivers there is a relatively small positive value associated with correctly saying "No". The drivers differ with regard to the values of the "Yes" responses. The morning driver is going to work, and saying "Yes" means that he will delay his arrival at work because of a necessary or unnecessary stop at the crossing. Consequently, a moderately high negative value is associated with stopping unnecessarily, and a very low positive value is associated with stopping for a train. By contrast, the afternoon driver has equally low values associated with the consequences of a "Yes" response.

If we assume that $\mathrm{P}(\mathrm{s})=\mathrm{P}(\mathrm{n})$ for each driver, the value of $\beta$ for the morning driver is 0.54 , while for the afternoon driver it is 0.095 . In terms of thresholds, the morning driver requires a perceptual magnitude of 1.03 to cause a stop; the afternoon driver requires only a perceptual magnitude of 0.58 . Because of the perceived negative consequences associated with stopping, the morning driver is much more willing to risk an accident.

It was previously noted that the frequency of trains differed substantially for the morning and afternoon drivers. When this is also included in the calculation of $\beta$, it is found that the morning driver now requires a perceptual magnitude of $0.8(\beta=0.33)$ and the afternoon driver requires a perceptual magnitude of $0.85(\beta=0.27)$. Thus the higher frequency of trains in the morning causes the morning driver to become more conservative, while the lower frequency of trains in the afternoon causes the afternoon driver to become less conservative.

## II. ACCIDENT RATES AT GRADE CROSSINGS



Figure 3. Accidents per crossing per year for the year 1986. See text for details.

SDT predicts an inverse relationship between train frequency at a grade crossing and the accident rate at that crossing. This prediction appears to be counterintuitive, since one would expect the highest accident rate to occur where the exposure is the highest. This section discusses accident rates and exposure.

## Accident Rates and Exposure

Accident rates are usually reported so as to equalize differences in exposure. For example, the Rail-Highway Crossing Accident/Incident and Inventory Bulletin reports accidents as a rate (accidents per crossing per year) for each of the grade crossing protection device categories ${ }^{2}$ rather than as accidents per year. This is because there are different numbers of crossings which are
protected by the various devices, and the most common device will have more opportunities for accidents. However, this is not sufficient to equalize the accident exposure of different devices. For instance, as can be seen in Figure 3, the accident rate is much higher for categories 8 (gates), 7 (flashing lights) and 6 (highway signals, wigwags, or bells) than for any other device category. However, since device categories differ with regard to the number of trains per day and the number of cars per day that traverse the crossings at which they are placed, it is obvious that the device category with a higher amount of train and/or car traffic will have a higher accident rate. To reflect the true (equal exposure) accident rate for each device category the rate should be reported as the number of accidents per crossing per train per car per year (i.e., divide the reported accident rate by the average number of cars and by the average number of trains). The analysis of train frequency which follows uses equal exposure accident rates.


Figure 4. Accidents per crossing per year per train per car as a function of train frequency.

## Train Frequency and Accident Rate

Based on data provided in the Rail-Highway Crossing Accident/Incident and Inventory Bulletin, Figure 4 plots accidents per crossing per train per year as a function of train frequency for 1986. From Fig. 4 it is clear that the prediction of SDT is correct. Per train, accident rates are higher for crossings with the lowest frequency of trains. This is a direct effect of the decision-making process at the crossing because $\mathrm{P}(\mathrm{S}) / \mathrm{P}(\mathrm{N})$ determines the setting of the criterion, as described in Section I.

Figure 4 plots accident rate as a function of train frequency averaged across device types. Protection devices are placed at grade crossings on the basis of the number of cars and trains at that crossing. In fact, the device categories noted above constitute a rank ordering of devices with respect to train and car frequency. Consequently, SDT also predicts that that rank ordering, to the extent that it reflects train frequency (which it only does partially), should be inversely related to equal exposure accident rate.


Figure 5. Equal exposure accident rates (accidents per crossing per train per car per minute) for the various device categories. See text for details.

Information on the frequency of trains and cars for different device categories is also provided in the Rail-Highway Crossing Accident/Incident and Inventory Bulletin. Tables 53 and 58 in the 1986 bulletin are typical. For easy reference, they are presented as Tables A1 and A2 in the Appendix.

There are two problems with the information that is presented in Tables A1 and A2. First, the frequencies are reported per day rather than per year. Obviously, all the units of frequency should be the same, and for reasons which will soon be apparent, all rates in this report will henceforth be expressed as the number of observations per minute. Second, a frequency distribution is provided for each category rather than a single measure of central tendency such as a mean. With regard to this problem, since the frequency distributions include bins for $<1$ and $>25$, a mean cannot be calculated from the information provided. A reasonable alternative, the median (which is also a robust measure of central tendency), can be easily calculated and is used to estimate the average number of trains and cars per min for each device category. This information is presented in Table $5 .{ }^{3}$

Figure 5 shows the equal exposure accident rate (accidents/crossing/train/car/min) for the various device categories. Given the normalization for traffic (cars and trains) exposure through different crossings, it can be seen that crossings with only crossbucks have the highest accident rate and crossings equipped with gates have the lowest rate. With the exception of crossbucks and special warning devices, the inverse relationship predicted by SDT holds well. As noted above this discrepancy could be due to the fact that the rank ordering of the devices also includes the frequency of cars.

TABLE 5. Median Train and Car Frequencies As a Function of
Grade Crossing Warning Device Category. Grade Crossing Warning Device Category.

| Device Category | Median Trains/min | Median Cars/min |
| ---: | ---: | ---: |
| Gates | 0.009 | 2.08 |
| Flashing Lights | 0.0028 | 2.08 |
| Highway Signals | 0.0028 | 0.52 |
| Special Warnings | 0.001 | 2.08 |
| Crossbucks | 0.0028 | 0.09 |
| Stop Signs | 0.0028 | 0.09 |
| Other Signs | 0.001 | 0.26 |
| No Signs or Signals | 0.001 | 0.09 |

The above analysis suggests that, in part at least, the accident rate at grade crossings is determined by decision-making processes based on the frequency of trains at the crossing. This aspect of the decision-making process (i.e., the aspect that relates to train frequency) is probably independent of the grade crossing device, unless the device also conveys information to the motorist concerning train frequency. In the absence of objective information concerning the frequency of trains at a crossing, motorists must be assumed to rely on the perceived and remembered frequency of trains. This could be problematic, because the heuristics that people use to estimate the probability of an event can lead to severe and systematic errors (Tversky and Kahneman, 1974). For instance, a motorist who normally travels over a grade crossing when there is light train traffic could wrongly conclude that the same is true at all times of day. Accurate information concerning the frequency of trains at a grade crossing could help to ameliorate the contribution of these effects to accidents.

## Grade Crossing Protection Device Effectiveness

The above analysis begs the question of how effective different grade crossing devices actually are. At first glance one might suggest that the equal exposure accident rate speaks to this point, but careful consideration of Fig. 5 indicates that there is a problem with accident rate data. For instance, crossings protected by crossbucks have an even higher accident rate than crossings which have no signs at all. Since we would expect any sign to be more effective than no sign, this illustrates the problem, noted above, of using accident rate data as an indicant of device effectiveness. The accident rate confounds the reduction in accidents with the risk of accidents. Adjusting the accident rate for exposure does not unconfound these elements because we do not know how many accidents might have occurred if no device was in place. Two elements are required to determine the effectiveness of a device to prevent accidents: the accident rate (observed frequency of accidents) and the accident risk (how many accidents would have occurred if the device was not in place). In the previous subsection an equal exposure accident rate was developed. In this subsection we developed a metric for accident risk. In the following section (Section III), we use that information to ask a more complex question: do grade crossing devices achieve their effectiveness by enhancing the signal-to-noise ratio ( $\mathrm{S} / \mathrm{N}$ ) or by changing the location of the criterion?

## Accident risk

Accident risk is defined here as the probability that both a train and a car will be observed at a grade crossing during any one minute observation period. As noted above, the Rail-Highway Crossing Accident/Incident and Inventory Bulletin provides information on the frequency of trains and cars for the various device categories. The average (i.e., median) frequency of trains and cars for each device category was used to equalize exposure for the accident rate data. The same information can also be used to determine accident risk.

Two probabilities are required to determine accident risk for a particular grade crossing or a grade crossing category: the probability that in a one minute period one or more trains will be observed at the grade crossing and the probability that in a one minute period one or more


Figure 6. Maximum probability of an accident for various devices. See text for details.
cars will be observed at the grade crossing. If we assume that trains and cars are random events, equally likely to occur throughout the day, and that each occurrence of a train or a car is independent of the occurrence of other trains or cars, then the Poisson probability distribution can be used to model the situation (see Feller (1957), Parzen (1960), and Daniel (1974) for more detail on the Poisson distribution and its uses).

If $x$ is the number of occurrences of a train (car) in a one minute ${ }^{4}$ period of time, the probability that x will occur is

$$
\begin{equation*}
p(x)=\frac{e^{-\lambda} \lambda^{x}}{x!} \tag{9}
\end{equation*}
$$

The parameter $\lambda$ is the mean rate of occurrence, and can be estimated by the average frequency of trains (cars) as described above ${ }^{5}$. The probability that one or more trains (cars) will occur in a one minute period of time is the cumulative Poisson probability distribution for $1 \leq \mathrm{x} \leq \infty$, or $1-\mathrm{p}(0)$. This can be written as

$$
\begin{equation*}
\mathrm{p}(1 \leq \mathrm{x} \leq \infty)=1-\mathrm{e}^{-\lambda} . \tag{10}
\end{equation*}
$$

The product of the probability of one or more trains being observed in a one minute period
and of the probability of one or more cars being observed in a one minute period at the crossing provides an estimate of the maximum probability of an accident (i.e., risk). This is presented in Figure 6 for the various categories of protection devices. Since the risk of an accident is based on the probability that one or more cars and trains will be observed at a grade crossing within a specified one minute period, and these probabilities are based on the frequency of trains and cars at grade crossings, it is not surprising that the greatest risk exists for those devices which have the greatest aggregate train and car traffic. This supports the validity of the procedure for estimating accident risk.

## Device Effectiveness

Given an estimate of risk (accident probability) and an observed rate of accidents for various crossing devices, device effectiveness is easy to estimate. It should first be noted that since risk is defined as a probability, it is not necessarily directly comparable to the observed accident rate. However, at the low rates of occurrence observed in Fig. 5, the accident rate is the same as the probability of observing one or more accidents in a one minute period at a grade


Figure 7. Device effectiveness for various grade crossing devices. See text for details.
crossing. ${ }^{6}$ It should be noted that the abscissa in Fig. 5 is already labeled as a probability domain.
Device effectiveness is determined by comparing the risk of an accident (maximum probability) with the observed probability of an accident. Figure 7 shows the ratio of the risk to the observed probability for each device category on a logarithmic scale. If the ratio has a value of 1 , the device has no effectiveness since the observed probability of an accident is the same as the risk. Ratios greater than 1 indicate increasing levels of effectiveness. Gates are the most effective devices, followed by flashing lights, special warnings, and highway signals. Passive devices are less effective than active devices by an order of magnitude. Finally, grade crossings without any protection (no signage) have a higher probability of accidents than is expected on the basis of train and car traffic.

## III. SDT ANALYSIS OF GRADE CROSSING DEVICES



Figure 8. Hypothetical ROC plot demonstrating the possibility that grade crossing devices differ in effectiveness because of sensitivity differences. The major diagonal (lower left corner to upper right corner) shows zero sensitivity $\left(d^{\prime}=0\right)$. The minor diagonal (upper left corner to origin) shows zero bias. Points which fall above the minor diagonal have a bias to stop, while points which fall below the minor diagonal have a bias to cross. The devices all have points along the minor diagonal (no bias) but differ in sensitivity. Gates have the highest sensitivity ( $d^{\prime}$ ~ 6 ), and no signage has the lowest sensitivity ( $d^{\prime} \approx 0.2$ ). See footnote 7 for details concerning the use of different axes in this figure and in Fig. 2.

In the previous section we determined the relative effectiveness of the various categories of grade crossing protection devices. In this section SDT is applied to grade crossing devices to determine the source of that effectiveness. From the point of view of SDT, there are two major, independent classes of variables which can influence effectiveness. One is the separation of the signal and noise distributions. For a constant decision criterion, as the $\mathrm{S} / \mathrm{N}$ ratio increases, the probability of an ACCIDENT ( $\mathrm{P}(\mathrm{AC}$ )) decreases. This factor involves the relative magnitudes of the signal (train) and the noise (everything else in the immediate vicinity of the grade crossing). Both signal and noise are multisensory stimuli, but SDT considers that each can be represented as a single perceptual magnitude. All other factors remaining constant, the "detectability" of a signal increases as the $\mathrm{S} / \mathrm{N}$ ratio increases. This means that increasing


Figure 9. Hypothetical ROC plot demonstrating the possibility that grade crossing devices differ in effectiveness because of bias differences. The devices all have the same sensitivity. The line drawn through the points is an isosensitivity contour for $d^{\prime}=2$. The devices all differ in bias. Gates have the highest bias to stop, while no signage has the highest bias to cross.
locomotive conspicuity, increasing the audibility of train horns, decreasing visual obstructions at grade crossings, etc., all increase the detectability of trains. The second factor involves human decision-making processes and the setting of the criterion. Expectations, attention, motivation, and decision goals constitute a short list of potentially important variables. In SDT these variables are independent of the $\mathrm{S} / \mathrm{N}$ ratio, but affect whether the observer acts on a signal (i.e., stops at the grade crossing) or fails to act on a signal (does not stop). Since these variables are independent of the $\mathrm{S} / \mathrm{N}$ ratio, they do not affect detectability. They do affect the tendency of the observer to report or to not report a signal, and therefore they are said to affect "bias". In SDT, bias and detectability are independent. Examples of what could alter bias at the grade crossing includes: train frequency (expectancy of signal), signage (expectancy of signal?), time of day (motivation; factory workers would have a higher cost associated with a delay at the crossing in the morning on their way to work, than they would at the end of their work day). The question asked in this section is: Do grade crossing protection devices achieve effectiveness because they increase the $\mathrm{S} / \mathrm{N}$ ratio, or because they influence the setting of the criterion?

## Estimating Valid Stop and False Stop Rates

One can determine the source of effectiveness of grade crossings by plotting the probability of a VALID STOP ( $\mathrm{P}(\mathrm{VS})$ ) vs. the probability of a FALSE STOP ( $\mathrm{P}(\mathrm{FS})$ ) for each device type in a Receiver-Operator Characteristic (ROC) plot similar to Fig. 2. If grade crossings differ in effectiveness because they increase the $\mathrm{S} / \mathrm{N}$ ratio, then the most effective device (gates) should have the highest value of $\mathrm{d}^{\prime}$ and the least effective device (no signage) should have the lowest value of $\mathrm{d}^{\prime}$. This possibility is illustrated in Fig. $8^{7}$. On the other hand, if grade crossings differ in effectiveness because they influence the criterion (bias to stop), then the most effective device should have the highest bias to stop $(\beta \lll 1)$ and the least effective device should have the highest bias not to stop ( $\beta \ggg 1$ ). This possibility is illustrated in Fig. 9. Note that in Fig. $8 \beta$ is constant, while if Fig. $9 \mathrm{~d}^{\prime}$ is constant. The third possibility is that both $\mathrm{d}^{\prime}$ and $\beta$ will vary with effectiveness.

The primary problem in performing a quantitative SDT analysis of grade crossings is obtaining estimates of $\mathrm{P}(\mathrm{VS})$ and $\mathrm{P}(\mathrm{FS}) . \mathrm{P}(\mathrm{VS})$ can be estimated from accident statistics. Recall that $\mathrm{P}(\mathrm{VS})=1-\mathrm{P}(\mathrm{AC}) . \mathrm{P}(\mathrm{AC})$ is the equal exposure accident rate which was developed in the previous section, and $\mathrm{P}(\mathrm{VS})$ is easily calculated.

Accident risk was defined as the probability that a car and a train are simultaneously in the crossing. It was assumed that the car and the train cannot stop. Note that the situation in which a car and a train are at a crossing and the car does not stop also defines an ACCIDENT in SDT. Hence, accident risk defines maximum $\mathrm{P}(\mathrm{AC})\left[\mathrm{P}(\mathrm{AC})_{\max }\right]$. If the car cannot stop, then the probability that the car won't have an accident is $1-\mathrm{P}(\mathrm{AC})_{\max }$. This can be taken as an estimate of $\mathrm{P}(\mathrm{CC})$. By definition $\mathrm{P}(\mathrm{FS})=1-\mathrm{P}(\mathrm{CC})=1-\left[1-\mathrm{P}(\mathrm{AC})_{\max }\right]=\mathrm{P}(\mathrm{AC})_{\max }$. Consequently, in the ROC analysis that follows, $\mathrm{P}(\mathrm{FS})$ is estimated from the accident risk associated with each device type.

Figure 10 is the ROC plot for the seven grade crossing protection device categories listed in the Rail-Highway Crossing Accident/Incident and Inventory Bulletin for 1986. The dashed line drawn through the seven points is the mean d' value. The points all fall in close proximity to the mean (dashed line), which indicates that there are very small differences in the $\mathrm{S} / \mathrm{N}$ ratio between the different devices. The mean $\mathrm{d}^{\prime}$ value is 7.13 , which indicates that a train at a grade crossing represents an enormous signal relative to the background noise.

In Fig. 10, the solid line which has been drawn from the origin $(0,0)$ to the upper left-hand corner is the equal bias $(\beta=1)$ line. Points which fall to the right of that line indicate a bias to stop ( $\beta<1$ ), and points to the left of the line indicate a bias not to stop $(\beta>1)$. The most effective device (gates) has the highest bias to stop, while the least effective device (none) has the highest bias not to stop. In addition, the bias to stop is higher for active devices than it is for passive devices. A statistically reliable correlation was found between device effectiveness (log(risk/probability)) and $\beta$ across devices ( $\mathrm{r}=-0.77, \mathrm{p}<.05$ ). This indicates that bias


Figure 10. ROC plot for grade crossing devices (filled square, gates; filled circle, flashing lights; filled up triangle, special warnings; filled down triangle, highway signal; open square, cross bucks; open down triangle, stop signs; open circle, other; X, no signage). The line drawn through the points is an isosensitivity contour for $d^{\prime}=7.13$. See text for details.
accounts for almost $60 \%$ of the variations in effectiveness in the devices. By contrast, the correlation of effectiveness and $\mathrm{d}^{\prime}$ was not statistically reliable ( $\mathrm{r}=-0.63, \mathrm{p}>.05$ ).

Based on the correlations and visual analysis of the ROC plot in Fig. 10, it can be concluded that grade crossing devices achieve their effectiveness primarily because they affect the decisionmaking process. There is no strong evidence in this analysis that grade crossing devices enhance the $\mathrm{S} / \mathrm{N}$ ratio. On the other hand, the correlation between d' and effectiveness, although not reliable, was negative which indicates a possibility that the devices actually degrade the $\mathrm{S} / \mathrm{N}$ ratio. Since the grade crossing and its protective devices are not a part of the train per se, this makes sense: the auditory and visual stimulation produced by the devices must be adding to the noise, thereby degrading the $\mathrm{S} / \mathrm{N}$ ratio.

## IV. IMPLICATIONS

There are two classes of variables which can be manipulated to prevent accidents. First, there are those variables which increase the $\mathrm{S} / \mathrm{N}$ ratio. Second, there are those variables which increase the bias to stop.

## Measures to increase the $\mathrm{S} / \mathrm{N}$ ratio

The analysis in section III indicates that the $\mathrm{S} / \mathrm{N}$ ratio is already very large since the value of $\mathrm{d}^{\prime}$ is approximately 7 . This means that trains are highly detectable, which makes sense considering their visual and auditory properties. However, there are numerous strategies available to further increase the detectability of trains. This includes enhancing locomotive conspicuity, reflectorization of freight cars, altering the train horn, and improving line of sight (reduction in noise).

Since detectability is already high, how big a reduction in accidents could be expected by further increases in $\mathrm{S} / \mathrm{N}$ ? Conversely, what would happen to accidents if $\mathrm{S} / \mathrm{N}$ were decreased? Because $\mathrm{P}(\mathrm{VS})=1-\mathrm{P}(\mathrm{AC})$, and $\mathrm{P}(\mathrm{AC})$ is the accident rate, these questions can be answered. From the theory of the ideal observer we know that the relationship between $\mathrm{d}^{\prime}$ and $\mathrm{S} / \mathrm{N}$ is

$$
\begin{equation*}
\mathrm{d}^{\prime}=\eta(\mathrm{S} / \mathrm{N}) \tag{11}
\end{equation*}
$$

where $\eta$ is the efficiency of a human observer relative to an ideal observer. The value of $\eta$ is often assumed to be 0.4 (Potter et al., 1977). From equation (4) we also know that $\mathrm{d}^{\prime}=\mathrm{z}(\mathrm{VS})-\mathrm{z}(\mathrm{FS})$. Consequently, if bias is held constant, then we can relate changes in $\mathrm{S} / \mathrm{N}$ to changes in $\mathrm{d}^{\prime}$ and to changes in accidents.

Figure 11 shows the predicted number of accidents as a function of changes in d' for crossings protected by gates using 1986 data. The base value of $d^{\prime}$ is $6.86, \beta$ is held constant at .000927 , and each change in d' of 0.25 units changes $\mathrm{S} / \mathrm{N}$ by 0.625 units. In 1986 there were approximately 1000 accidents at crossings protected by gates. This corresponds to the 0 change in d' point in Fig. 11. Changes of one d' unit cause accidents to increase or decrease by almost an order of magnitude. On the basis of this analysis, it must be concluded that even small changes in the $\mathrm{S} / \mathrm{N}$ ratio can result in dramatic changes in the number of grade crossing accidents. As a case in point, when Florida imposed a ban on night-time use of train horns a three-fold increase in accidents resulted. Figure 11 indicates that a d' change of about 0.75 units ( 1.875 units change in $\mathrm{S} / \mathrm{N}$ ) would produce a change in accidents of this magnitude for crossings protected by gates, under conditions of constant bias. If bias is not held constant (which would occur, for example, if P(FS) is held constant), then the same three-fold increase in accidents would result from a d' change of about 0.25 units (. 625 units change in $\mathrm{S} / \mathrm{N}$ ). Obviously, eliminating the train horn reduces $\mathrm{S} / \mathrm{N}$, so this "natural experiment" is consistent with the prediction of SDT.

This analysis also has implications for the placement of horns at grade crossings instead of on the train. It was noted above that grade crossings with active devices had lower d' values


Figure 11. Predicted accidents as a function of changes in d' (sensitivity). See text for details.
than crossings with passive devices or no devices (see Fig. 10). This could be because the crossing is not part of the train, and consequently increases in light and sound at the crossing increase noise and decrease S/N. SDT accordingly predicts that automated horns should increase the accident rate at grade crossings, regardless of whether they sound like train horns or not.

The same argument can be applied to the illumination of grade crossings. To the extent that such illumination enhances train visibility (i.e., the train is in the crossing and the train rather than the pavement is illuminated), $\mathrm{S} / \mathrm{N}$ will be increased, and accidents (particularly accidents in which the car hits the side of the train) should decrease. If illumination enhances the contrast between the train and its background relative to daylight conditions, accident rates should be lower than during the day (all other factors being equal). However, if the crossing is illuminated prior to the train entering the crossing, the noise level will be increased and more accidents (particularly those in which the car is struck by the train) should result.

Another obvious method to increase $\mathrm{S} / \mathrm{N}$, is to improve the line of sight of the motorist at the crossing and during the approach. In the absence of visual cues to the location of the


Figure 12. Predicted accients as a function of changes in log $\beta$. See text for details
train, the motorist must rely on a smaller signal which only consists of auditory and other non-visual cues. Improvements in the line of sight would increase the signal by adding visual cues and increase S/N.

Visual clutter (other traffic, traffic signs and signals, street lights, etc.) at crossings would tend to increase the noise and thereby reduce $\mathrm{S} / \mathrm{N}$. A reduction in visual clutter would increase $\mathrm{S} / \mathrm{N}$ and reduce accidents. A recent FRA examination of 56 grade crossing with an average of more than one accident per year supports this conclusion. It was found that $97 \%$ of these crossings had visual obstructions, $95 \%$ had a large number of driveways and intersecting roadways, and $80 \%$ had visual clutter on the approach.

## Measures To Increase the Bias to Stop

In Fig. 10, the changes in bias ( $\beta$ ) between crossings without signage and those with gates were not specified. To correct that situation, it is here indicated that for no signage $\beta=1.64$ and for gates $\beta=0.000927$. $\beta$ is calculated as the ratio of the ordinates of the standard normal curve corresponding to $\mathrm{z}(\mathrm{VS})$ and $\mathrm{z}(\mathrm{FS})$ :

$$
\begin{equation*}
\beta=\frac{y_{V S}}{y_{F S}} \tag{12}
\end{equation*}
$$

where $y_{v s}$ is

$$
\begin{equation*}
y_{V S}=\frac{1}{\sqrt{2 \pi}} e^{\frac{-z(V S)^{2}}{2}} \tag{13}
\end{equation*}
$$

and $y_{F S}$ is similarly defined.
Recall that there is no bias when $\beta=1$. Values of $\beta<1$ indicate a bias to stop and values of $\beta>1$ indicate a bias to not stop. The data in Fig. 10, therefore, indicates that there are very large differences in bias between crossings with no signage and crossings protected by gates. Since gates produce such a large increase in the bias to stop, can a further change in accidents be expected for a change in bias with d' held constant? For the sake of comparison with Fig. 11, the 1986 data for gates is used as an example. Figure 12 shows predicted accidents as a function of the change in log $\beta$. To allow comparability, $\mathrm{d}^{\prime}$ has a constant value of 6.86 , the base value of $\beta$ is .000927 , and accidents range across the same values as in Fig. 11. $\log \beta$ is plotted instead of $\beta$ to allow a direct comparison with changes in d' in Fig. 11. From Fig. 12 it can be seen that an increase in $\log \beta$ of approximately .75 units results in approximately a three-fold increase in accidents. Accidents, therefore, are almost equally affected by changes in $\mathrm{d}^{\prime}$ and $\log \beta$. Moreover, just as was concluded for $\mathrm{d}^{\prime}$, even modest changes in $\beta$ are capable of producing large changes in the number of accidents.

There are several variables identifiable in SDT which can be manipulated to change bias. Recall the definition of $\beta$ given in Equation 8:

$$
\begin{equation*}
\beta=\frac{V(n, N)+V(n, Y)}{V(s, Y)+V(s, N)} \cdot \frac{P(n)}{P(s)} . \tag{8}
\end{equation*}
$$

The ratio $\mathrm{P}(\mathrm{n}) / \mathrm{P}(\mathrm{s})$ relates to the expectation of the motorist that a train will be encountered in the crossing. The ratio $\mathrm{V}(\mathrm{n}, \mathrm{N})+\mathrm{V}(\mathrm{n}, \mathrm{Y}) / \mathrm{V}(\mathrm{s}, \mathrm{Y})+\mathrm{V}(\mathrm{s}, \mathrm{N})$ relates to the motivation of the motorist with regard to the value of Valid Stops, Accidents, Correct Crossings and False Stops. Expectation and motivation are "psychological" variables, and in this context it should be emphasized that although the terms of Equation (8) are all capable of measurement at a physical level (e.g., P(s) as a Poisson probability based on the frequency of trains per minute at a crossing; $\mathrm{V}(\mathrm{s}, \mathrm{N})$ as a dollar loss associated with an accident), perceived or subjective probabilities and values would be more
appropriate. People tend to overestimate the probability of low frequency events and to underestimate the probability of high frequency events. Moreover, the subjective value of gains and losses is not a linear function of dollar value. In the discussion which follows, when estimates of probabilities ( P ) are available these are transformed to subjective probabilities using the relationship: $\Psi_{\mathrm{P}}=\mathrm{P}^{0.35}$, where $\Psi_{\mathrm{P}}$ is the subjective probability of P. Similarly, the subjective value of money $\left(\Psi_{\mathrm{S}}\right)$ is related to the true value of money (\$) by: $\Psi_{\$}=\$^{0.5}$ (Stevens, 1975).

Section II has already confirmed the prediction of SDT that accident rates vary inversely with train frequency. It should be noted from Equation (8), however, that expectation is multiplied by the motivational factor to determine the value of $\beta$. In most situations, it is probably the case that both motivation and expectation are influential in determining $\beta$. This can be easily appreciated by assuming that $\mathrm{V}(\mathrm{n}, \mathrm{N})+\mathrm{V}(\mathrm{n}, \mathrm{Y}) / \mathrm{V}(\mathrm{s}, \mathrm{Y})+\mathrm{V}(\mathrm{s}, \mathrm{N})=1$ in Equation (8) and calculating $\beta$ using the Poisson probability of a train that was developed in Section II.

In the case of gates, Table 5 shows that the Poisson probability of a train in the crossing is 0.009. The subjective probability of a train is then $0.19\left(0.19=0.009^{0.35}\right)$. This defines $\mathrm{P}(\mathrm{s})$ and $\mathrm{P}(\mathrm{n})$ $=1-\mathrm{P}(\mathrm{s})$, so that $\mathrm{P}(\mathrm{n}) / \mathrm{P}(\mathrm{s})=4.2=\beta$. Recall that if $\beta>1$, there is a bias to not stop. Consequently, in the absence of motivation to stop, low train frequency predisposes motorists not to expect trains and biases them not to stop. In Section III, however, the value of $\beta$ for gates was found to be 0.000927 , which indicates a large bias to stop. Therefore, there must be a large motivational factor which is counteracting the bias not to stop. The motivation to stop can be calculated from Equation (8) given the value of $\beta=.000927$ and with $\mathrm{P}(\mathrm{n}) / \mathrm{P}(\mathrm{s})=4.2$. The calculation indicates that the motivation to stop is 4534.59 times the motivation not to stop (i.e., the motivation ratio is $1 / 4534.59$ ). In terms of actual costs and benefits, if the subjective value of not stopping is 1 and subjective value of stopping is 4534.59 , then the equivalent dollar amounts are $\$ 1$ and $\$ 20,562,506.47$ (because $\Psi_{\$}=\$^{0.5}, 4534.59=[\$ 20,562,506.47]^{0.5}$ ). It should be kept in mind that the $\$ 20,562,506$ includes the perceived cost of death, dismemberment, loss of property and grief due to an accident, so perhaps this dollar ratio is not unrealistic.

One method of increasing the bias to stop is enforcement of the law which requires motorists to stop when gates are lowered and lights are flashing. Considering that there is already considerable motivation to stop at lowered gates $(\$ 20,562,506)$, it seems questionable that a $\$ 50$ or $\$ 100$ fine would be effective in further increasing that motivation. However, there are other costs associated with fines which do not have a directly known dollar value. For instance, there is inconvenience and loss of time, especially if a court appearance is necessary. Embarrassment caused by publicly receiving a fine constitutes a social cost. If the act of non-compliance is considered a moving violation, points can be added to the driver's license and the license might be lost, which can have tremendous economic and personal consequences. Enforcement programs, such as the photo enforcement program in Los Angeles (Meadow, 1994), have been shown to decrease violations (which means that there is an increase in compliance), so the dollar value of the fine must not be the only perceived cost of receiving a fine.

The Los Angeles program found that photo enforcement decreased violations by $84 \%$
(Meadow, 1994). In the SDT model this means that $\mathrm{P}(\mathrm{FS})$, which is compliance, has increased by $84 \%$. If we assume that $\mathrm{d}^{\prime}$ remains unchanged and that only $\beta$ is changed, it should be possible to determine the change in the motivation ratio which a fine causes for the average gate-protected crossing. For a $84 \%$ increase in $P(F S)$, the bias to stop increases $(\beta=0.000138$ rather than 0.000927 ) and the motivation ratio becomes $1 / 30,532$. The corresponding dollar value, which includes the dollar value of the fine, is $\$ 932,203,024$. From this example it should be clear that human motivation is not limited to dollar-valued costs and benefits. A better understanding of the motivation for stopping and not stopping has the potential for generating innovative, cost-effective strategies for enhancing grade crossing safety.

Attention and Memory. Attention and memory have the capability to alter both $\beta$ and $\mathrm{d}^{\prime}$. A primary function of crossing devices is probably attentional, and the variation in $\beta$ with device type is consistent with a link between attention and $\beta$. In the psychophysical literature it is often found that attention to a signal does not affect $\mathrm{d}^{\prime}$. Instead, attention is found to enhance performance by causing a shift in the criterion. Recall that Fig. 10 showed that different devices differed in bias, but not in $\mathrm{d}^{\prime}$. Differences in $\beta$ as a function of device type are probably, in part, the result of enhanced expectation of a train (i.e., the expectation ratio, $\mathrm{P}(\mathrm{n}) / \mathrm{P}(\mathrm{s})$, has been decreased). In this regard, the role that accurate information concerning train frequency could play in the further reduction of $\mathrm{P}(\mathrm{n}) / \mathrm{P}(\mathrm{s})$ remains unexplored.

Because attention also involves orientation towards a source of stimulation, attention may also serve to enhance the $\mathrm{S} / \mathrm{N}$ ratio. Signage which indicates where motorists should look for trains would strengthen this function of attention, especially if active devices were used to indicate train direction. Note that knowledge of train direction assumes that the probability of a train is close to one. Signage which actively indicates train direction could function to enhance both d' and $\beta$. Signals and other changes in the sensory stimulation provided by grade crossing devices should be more focused on causing motorists to orient toward the train. This should enhance the bias to stop and ameliorate the previously noted decrement in $\mathrm{S} / \mathrm{N}$ caused by the active devices (p. 26).

Memory has important functions for responding at the decision point and for stimulus recognition. Motorists at a grade crossing must remember what responses are appropriate given a particular device, the proximity of a train, and the consequences of the various outcomes. Regardless of whether the motorist is stopped or in motion, imperfect memory at the decision point can only bias the motorist to continue to remain stopped or in motion. Signage advising motorists of the appropriate actions at the crossing could relieve the motorist of this human limitation and enhance safety.

Memory can also affect $\mathrm{S} / \mathrm{N}$ through the process of stimulus recognition. Imperfect memory in this instance degrades $\mathrm{S} / \mathrm{N}$ primarily by enhancing the noise. Driver education and public service announcements that show motorists the appearance of locomotives and trains under different lighting conditions, angles, distances, etc. could improve stimulus memory and enhance $\mathrm{S} / \mathrm{N}$. Greater consistency in the pattern of stimulation which locomotives provide to motorists (position and number of lights, frequency and intensity of horns, etc.) would also aid to improve recognition
memory, and thereby $\mathrm{S} / \mathrm{N}$.

## Accident Prediction

The SDT model can be used for accident prediction in situations for which specific changes in $S / N$ or $\beta$ are under consideration. For instance, in the example where photo enforcement was discussed relative to motivation, the model indicates that a $84 \%$ increase in compliance at the average gate-protected crossing would also result in a reduction in accidents. The model predicts a $74 \%$ reduction in accidents at the average gate-protected crossing through photo enforcement based on the 1986 data. Crossing accidents were observed to decrease by $60 \%$ at the photo enforcement crossings in Los Angeles (Federal Railroad Administration, 1994).

As a perceptual and sensory information processing model SDT is particularly suited to evaluate a variety of sensory manipulations (e.g., improvements in line of sight) and psychological manipulations (e.g., dollar amount of fines) which are not captured by any other accident prediction model. Moreover, unlike other accident prediction models, SDT is based on clearly stated assumptions concerning underlying processes which have been systematically studied over a 30 year period. As a theoretical model, rather than an empirical model, SDT has the flexibility to incorporate new variables and can be used to extrapolate predictions beyond the empirical inputs. The unified view of accident causation at grade crossings provided by SDT allows an understanding of trade-offs between sensory and decision-making variables which is unavailable in other models. As a model of human behavior it can be used in both a descriptive (how do people actually perform) and a prescriptive (how would an ideal observer perform) mode. An SDT analysis of a grade crossing allows essential engineering data to be used within the context of human decision making. No other model of accident prediction has this capability.

However, for SDT to be fully useful as a model of decision-making at the grade crossing, there are a number of areas in which more information is required. Recall that it was necessary to estimate $\mathrm{P}(\mathrm{FS})$ from the maximum probability of an accident. $\mathrm{P}(\mathrm{FS})$ is an aspect of compliance and this had to be estimated because there is little good information on the average rate of compliance or of P(FS) for various grade crossing devices. Compliance rates have most frequently been studied in the past when particular crossings are noted to have an unusually high accident rate. Compliance at "normal" crossings is unknown. The accurate determination of $d^{\prime}$ and $\beta$ requires knowledge of both $\mathrm{P}(\mathrm{VS})$ and $\mathrm{P}(\mathrm{FS})$.

Laboratory and/or field studies are also needed to determine basic relationships between sensory aspects of the train and d'. For instance, recent basic and applied research on perception of time-to-collision (e.g., Berthelon and Mestre, 1993; Bootsma and Oudejans, 1993; Kaiser and Mowafy, 1993; Wannm, Edgar and Blair, 1993) has not considered the special problems of the railhighway intersection. While it is possible that some of this information is already available in the psychological literature, the sensory magnitudes which trains present are not ordinarily encountered in a laboratory setting. Consequently, verification of published relationships may be necessary. Of particular concern is the most appropriate rule for combining multi-sensory information into a single percept. Prediction of the detectability and/or proximity of trains having various combinations of
lights and audible warning devices requires information on how people integrate sensory information.

Virtually no information exists concerning the motivation of motorists for stopping or not stopping, how this is affected by the type of device at the crossing, and effect of various enforcement programs. No information exists concerning the perceived frequency of trains at crossings and how that is affected by protection devices and signage. No information exists on the perceived risk present at crossings protected by different types of devices.

In the absence of quantitative information to specify the variables in the model, SDT will remain a useful heuristic model but will not achieve its full potential as an analytic and predictive tool. The basic model which has been presented here can be modified and refined to meet a variety of demands and needs. However, to do so it is necessary to have quantitiative information available to determine which aspects of the basic model are unsuitable. For instance, if it is determined that the assumption concerning gaussian distributions of signals and noise is not applicable, the theory can be adapted to other probability distributions such as the Gamma, Rayleigh, Chi-square, Poisson, and Binomial distributions (Egan, 1975). The theory has been adapted to the analysis of attention, conceptual judgment, learning, medical diagnosis, memory, personality, reaction time, recognition and vigilance (Green and Swets, 1974). Consequently, the current application of the theory has a wealth of resources on which to draw in order to improve our understanding of driver behavior at the rail-highway grade crossing.

## IV. THEORY OF THE IDEAL OBSERVER: TIME TO COLLISION,VISUAL SEARCH, AND ACCIDENT PREDICTION.

The theory of the ideal observer is used in SDT to model the performance of the perfect, or ideal, observer. Such an observer uses all of the available information in a maximally effective fashion to reach a rational decision within the bounds of the model limits. Actual performance of less than ideal observers can then be compared with that of the ideal observer to determine if the model has validity in the search for underlying processes, or as a means to improve observer performance (Swets, Tanner and Birdsall, 1964/1988). In the present instance, we explore the possibility that the ideal driver bases decisions at grade crossings on subjective estimates of the arrival time of his/her own vehicle and of the train at the grade crossing. Visual search for and localization of the train consumes time during which the decision to stop can be safely made and directly affects accident probability. A quantitative description of these processes is presented below.

## Sight Distance and Time to Collision

An important aspect of driving behavior at grade crossings is the visual search for a train, the localization of that train, and a decision to stop or not stop at the crossing given the location of the highway vehicle and train relative to the crossing. Figure 13 diagramatically presents the situation. The speed of the train and of the vehicle each determine the physical amount of time required to arrive at the crossing. The Railroad-Highway Grade Crossing Handbook (Tustin et al., 1986) defines two distances which are important for our analysis. The first, $\mathrm{d}_{\mathrm{H}}$, is the "Sight distance measured along the highway from the nearest rail to the driver of the vehicle which allows the vehicle to be safely stopped without encroachment of the crossing area..." (p. 132). The formula for $\mathrm{d}_{\mathrm{H}}$ is:

$$
\begin{equation*}
d_{H}=1.47 V_{v} t+\frac{V_{v}^{2}}{30 f}+D+d_{e^{\prime}} \tag{14}
\end{equation*}
$$

where $\mathrm{V}_{\mathrm{v}}$ is the vehicle velocity in $\mathrm{mph}, \mathrm{t}$ is the perception reaction time in seconds, f is the coefficient of friction, D is the distance in feet from the stop line or front of the vehicle to the nearest rail, and $\mathrm{d}_{\mathrm{e}}$ is the distance from the driver to the front of the vehicle in feet. ${ }^{8}$ The second, $\mathrm{d}_{\mathrm{T}}$, is the "Sight distance along the railroad tracks to permit the vehicle to cross and be clear of the crossing upon arrival of the train..." (p. 132). The formula for $\mathrm{d}_{\mathrm{T}}$ is:

$$
\begin{equation*}
d_{T}=\frac{V_{T}}{V_{V}}\left(1.47 V_{V} t+\frac{V_{v}^{2}}{30 f}+2 D+L+W\right), \tag{15}
\end{equation*}
$$

where $\mathrm{V}_{\mathrm{T}}$ is the train velocity in $\mathrm{mph}, \mathrm{L}$ is the length of the vehicle in feet, and W is the distance in feet between the outer rails. ${ }^{9}$


Figure 13. Definition of distances used in equations 14 and 15.
It is assumed that the highway vehicle and train are initially located at $\mathrm{d}_{\mathrm{H}}$ and $\mathrm{d}_{\mathrm{T}}$, respectively. Consequently, the vehicle can either stop or cross without an accident if the driver knows at that instant the exact location of the train. However, the driver of the vehicle, as a human information processor, must first locate the train, calculate the distance and time to the intersection of both the train and the vehicle, and decide whether to cross or stop. Assume that this ideal driver has all the information required to accurately solve the equations for the two distances, once the train is located. Since he knows the distances and the velocities of both the train and the vehicle, he also knows the the amount of time he has to cross (Tc) and the amount of time to stop (Ts). His perception of these times, however, is not veridical. Estimates of the relationship between judged time to passage (TTP*) and actual time to passage (TTP) were obtained from Kaiser and Mowafy (1993, Figs. 7 and 8) and were used to adjust the values of Tc and Ts to reflect this aspect of human time perception:

$$
\begin{equation*}
\mathrm{TTP}^{*}=0.84375 \mathrm{TTP}+0.84375 . \tag{16}
\end{equation*}
$$

It is typically found in the human time perception literature that short durations are overestimated and that long durations are underestimated. For instance, using the relationship above, a 4 s duration
would be judged to take 4.22 s and an 8 s duration would be judged to take 7.59 s .
The driver decides whether or not to cross on the basis of the perceived difference between judged Tc (Tc*) and judged Ts (Ts*). The perceived difference between the two durations can be modeled as follows:

$$
\begin{equation*}
d^{\prime}=\frac{\mu\left(T C^{*}-T s^{*}\right)}{\sigma \sqrt{\left(T C^{*}\right)^{2}+\left(T s^{*}\right)^{2}}}, \tag{17}
\end{equation*}
$$

where $\mu / \sigma=\gamma=$ a constant (Raslear, 1988). $\gamma$ is the inverse of the Weber constant for time. An estimate of $\gamma=57.47$ was obtained from a study of perceived time to collision by Bootsma and Oudejans (1993, experiment 1) for use in equation 17.

A value of $\mathrm{d}^{\prime}$ can be determined for various vehicle speeds from equation 17. Train speed is not a factor in determining d' because of the use of equations 14 and 15 to determine sight distances. Once a value of $\mathrm{d}^{\prime}$ is obtained, accident probabilities can be estimated for particular types of grade crossings. As an example, grade crossings with crossbucks are considered.

The previous analysis of grade crossing warning devices provides an estimate for each warning device of the probability of a false stop (P(FS)). For crossbucks $\mathrm{P}(\mathrm{FS})=0.000231$. SDT defines $\mathrm{d}^{\prime}=\mathrm{Z}(\mathrm{VS})-\mathrm{Z}(\mathrm{FS})$. Converting $\mathrm{P}(\mathrm{FS})$ into $\mathrm{Z}(\mathrm{FS})$ and adding $\mathrm{d}^{\prime}$ from equation 17 to $\mathrm{Z}(\mathrm{FS})$ yields $\mathrm{Z}(\mathrm{VS})$, and the corresponding probability, $\mathrm{P}(\mathrm{VS})$, is easily obtained. By definition $\mathrm{P}(\mathrm{AC})=$ $1-\mathrm{P}(\mathrm{VS})$, so the probabiliity of an accident is obtained.

Figure 14 shows the probability of an accident as a function of highway vehicle speed. Note that the probability of an accident is low between 10 and 20 mph . Beyond 20 mph , the probability of an accident rises steeply and begins to asymptote at very high levels above 50 mph . For comparison, fatal crash data from Klein, Morgan and Weiner (1994) is also plotted in Fig. 14. The comparison should be considered tentative for several reasons: the Klein et al. data is for fatalities, for a 10 year period, for all crossings, and is aggregated differently; while the prediction from the SDT model uses accident data from one year for crossbucks. Nevertheless, there is a surprising degree of agreement between the model and the data. In particular, both the model and the data suggest that highway vehicle speed has a functional role in accident probability at grade crossings.

## Visual Search

As was noted in a preceeding section, crossings with a higher than expected accident rate also tend to have a considerable amount of visual clutter. Visual clutter can be modeled by assuming that the prediction in Fig. 14 is for 0 items of visual clutter and that the $180^{\circ}$ visual search requires no time. Thus, the driver scans the visual field over a $180^{\circ}$ range,


Figure 14. Accident probability as a function of highway vehicle speed. The predicted function is based on the model in equation 17. The observed data are from Klein et al. (1994).
instantaneously localizes the train and makes a decision. The process of visual search, however, requires time. Moreover, the average search time for a specific item (the train) increases with the number of items in the visual field. In addition, because visual search is a variable process (i.e., sometimes the target is found after examining one or two non-target items, and sometimes after examining all non-target items), visual search adds variance to the time-based decision-making process (i.e., equation 17). Visual search time has been extensively studied and an excellent summary of that work can be found in Luce (1986, p. 428). The average time required to search for an item is given by

$$
\begin{equation*}
S=1_{2} k M+r_{0}, \tag{18}
\end{equation*}
$$

where k is the mean time per item, M is the number of items and $\mathrm{r}_{0}$ is the residual time. The variance for visual search time is given by

$$
\begin{equation*}
\sigma^{2}=\sigma_{m}^{2} M+\sigma_{r}^{2}, \tag{19}
\end{equation*}
$$

## No Train Horn



$$
\rightarrow 0 \quad-4 \quad-\quad-8 \quad-16 \quad \% 32
$$

Figure 15. Accident probability as a function of visual clutter and highway vehicle speed without a train horn to indicate train location. See text for details.
where $\sigma_{\mathrm{m}}{ }^{2}$ is the per item variance and $\sigma_{\mathrm{r}}{ }^{2}$ is the residual variance. The modeling of visual clutter in the search for a train consists of reducing the values of $\mathrm{Tc}^{*}$ and $\mathrm{Ts}^{*}$ in equation 17 by the appropriate value of S in equation 18 and adding the variance obtained from equation 19 to the variance (demoninator) of equation 17. ${ }^{10}$

Figure 15 shows accident probability (based on crossbuck data) as a function of highway vehicle speed and amount of visual clutter ( $0,4,8,16$ and 32 items). Note that as the amount of visual clutter increases, there is a corresponding increase in accident probability. The model clearly predicts that accidents should decrease as visual clutter is removed from a grade crossing.

## Train Horns and Visual Search

If we assume that a horn has been placed on the train to aid in the localization of the train, we can model the change in accidents that results. The sound localization literature indicates that there is approximately a $10^{\circ}$ error in localization of a sound source for pure tones (Licklider, 1951, p. 1026-1030). Since the motorist is searching a $180^{\circ}$ field for the train, the inclusion of a horn on the train can be assumed to reduce the field of search to $10^{\circ}$. This means

## With Train Horn


$\rightarrow-0 \quad-4 \quad \rightarrow-8 \quad \rightarrow 16 \quad \rightarrow 32$

Figure 16. Accident probability as a function of visual clutter and highway vehicle speed with a train horn to indicate train location. See text for details.
that visual search time and variance have been reduced by a factor of $1 / 18$. Figure 16 shows the effect of this change in the train by plotting accident probability as a function of vehicle speed and visual clutter. The train horn, by decreasing visual search time, also decreases the probability of an accident. Figure 17 provides a clearer picture of this by plotting the vehicle speed at which there is a 0.5 probability of an accident as a function of the number of visual distractors for a train with and without a horn. In all instances, if no train horn is sounded, the same level of accident probability occurs at a lower speed. A train horn, it must be concluded, enhances safety, and this conclusion is supported by the results of the Florida Train Whistle Ban study as noted previously.



Figure 17. Highway vehicle speed at which there is a 0.5 probability of an accident as a function of visual clutter, with and without train horns.

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## APPENDIX

TABLE A1. Total Crossings by Number of Trains Per Day
And Warning Device Category (From Table 53 of the 1986 Rail-Highway Crossing Accident/Incident and Inventory Bulletin).

| Device | $<1$ | $1-2$ | $3-5$ | $6-10$ | $11-15$ | $16-20$ | $21-25$ | $>25$ | Total |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Gates | 807 | 2063 | 2231 | 4082 | 2664 | 3224 | 2308 | 4687 | 22066 |
| Flashing lights | 3491 | 7806 | 6181 | 7354 | 2880 | 2279 | 1068 | 1719 | 32778 |
| Hwy. signals, etc. | 291 | 689 | 376 | 425 | 181 | 128 | 71 | 110 | 2271 |
| Special | 2607 | 2300 | 781 | 465 | 234 | 189 | 49 | 137 | 6762 |
| Crossbucks | 19160 | 40967 | 20185 | 18697 | 6346 | 4741 | 2381 | 3621 | 116098 |
| Stop signs | 136 | 304 | 239 | 131 | 46 | 43 | 23 | 40 | 962 |
| Other signs | 210 | 214 | 89 | 88 | 29 | 20 | 2 | 29 | 681 |
| No signs | 3965 | 3692 | 1292 | 983 | 380 | 245 | 89 | 190 | 10836 |
| Total | 30667 | 58035 | 31374 | 32225 | 12760 | 10869 | 5991 | 10533 | 192454 |

NOTE: Cells which are emphasized contain the median for the device. The bin midpoint is the median. For instance, for Gates the median is 13 trains/day. This means that $50 \%$ of these crossings had fewer than 13 trains/day and $50 \%$ had more than 13 trains/day.

TABLE A2. Total Crossings by Annual Average Daily Traffic And Warning Device Category (From Table 58 of the 1986 Rail-Highway Crossing Accident/Incident and Inventory Bulletin).

ANNUAL AVERAGE DAILY TRAFFIC

| Device | $1-250$ | $251-$ <br> 500 | $501-$ <br> 1000 | $1001-$ <br> 5000 | $5001-$ <br> 10000 | $>$ <br> 10000 | Total |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Gates | 2940 | 2167 | 2852 | 8025 | 3367 | 2715 | 22066 |
| Flashing lights | 4544 | 3671 | 4961 | 12386 | 4310 | 2906 | 32778 |
| Hwy. signals, etc. | 659 | 271 | 315 | 598 | 220 | 208 | 2271 |
| Special | 1508 | 844 | 871 | 2167 | 828 | 544 | 6762 |
| Crossbucks | 76318 | 13864 | 10182 | 12435 | 2254 | 1045 | 116098 |
| Stop signs | 489 | 134 | 122 | 172 | 30 | 15 | 962 |
| Other signs | 288 | 148 | 96 | 102 | 37 | 10 | 681 |
| No signs | 5836 | 1195 | 1122 | 1882 | 520 | 281 | 10836 |
| Total | 92582 | 22294 | 20521 | 37767 | 11566 | 7727 | 192454 |

NOTE: Cells which are emphasized contain the median for the device. The bin midpoint is the median. For instance, for Gates the median is 3000 cars/day. This means that $50 \%$ of these crossings had fewer than 3000 cars/day and $50 \%$ had more than 3000 cars/day.

## FOOTNOTES

1. SDT terminology differs from what is presented here. In SDT, the names of the cells in Table 1 are as follows:

|  | Yes, Stop. | No, don't stop. |
| :--- | :--- | :--- |
| Train is close | HIT <br> (motorist stops at crossing) | MISS <br> (accident) |
| Train is not close, or <br> No train in vicinity | FALSE Alarm <br> (motorist stops unnecessarily) | CORRECT REJECTION <br> (motorist crosses tracks <br> safely) |

In the terminology of SDT, an accident would be called a MISS, and the avoidance of an accident would be called a HIT. The use of alternative terminology seems advisable to avoid confusion.
2. Device Categories listed in the Rail-Highway Crossing Accident/Incident and Inventory Bulletin are: gates (category 8), flashing lights (category 7), highway signals, wigwags, or bells (category 6), special warning devices (category 5), crossbucks (category 4), stop signs (category 3 ), other signs (category 2 ), and no signs or signals (category 1 ).
3. It should be noted that several of the medians in Table 5 are identical. This is a result of the use of bins (ranges of values) in Tables A1 and A2. The median is located in the bin which cumulatively contains $50 \%$ of the observations. Since this is a range of values in Tables A1 and A2, the midpoint of the bin is used to represent the median. The inaccuracy which is introduced by this calculation is easily avoided by determining a mean based on train and car frequencies reported at each crossing.
4. A one minute observation period is suggested by the fact that the average freight train is approximately 67 cars long (AAR, 1993) and the average train speed through a crossing is 30 mph (Table 55, 1986 Rail-Highway Crossing Accident/Incident and Inventory Bulletin). At an approximate car length of 50 feet, such the average train would take approximately one minute to go through the average crossing.
5. It should be noted that the mean is the recommended estimator of $\lambda$ in the present case. The median is used of necessity and with full knowledge that it is not the optimal estimator of the rate parameter of the Poisson distribution.
6. In Equation 10, as $\lambda \rightarrow 0, \mathrm{e}^{-\lambda} \rightarrow 1$, and $\mathrm{p}(1 \leq \mathrm{x} \leq \infty) \rightarrow \lambda$.
7. Fig. 8,9 and 10 plot $z(V S)$ vs. $z(F S)$ rather than $P(V S)$ vs. $P(F S)$. This has several advantages for analytic purposes. Because it is assumed that the underlying probability
distributions are normal and of equal variance, ROC curves which are plotted as normal deviates (z-scores) are linear rather than curvilinear (as in Fig. 2). As a result, isosensitivity contours (ROC curves for which $\mathrm{d}^{\prime}$ values are equal) are all parallel to the major diagonal ( $\mathrm{d}^{\prime}=0$ contour) and have a slope of 1 . Moreover, because $z$-scores, unlike probabilities, have no upper limit, high levels of sensitivity can be plotted and distinguished. This characteristic of z-scores also allows the effect of bias to seen at high levels of sensitivity.
8. In solving equation $14, \mathrm{D}=15, \mathrm{~d}_{\mathrm{e}}=10, \mathrm{t}=2.5$, and f was obtained from Table 35 in Tustin et al.
9. In solving equation $15, \mathrm{D}=15, \mathrm{~L}=19, \mathrm{~W}=5, \mathrm{t}=2.5$, and f was obtained from Table 35 in Tustin et al.
10. The following values of the parameters in equations 18 and 19 were used to apply the model: $\mathrm{k}=0.02, \mathrm{r}_{0}=0.4$ (Sternberg, 1966), $\sigma_{\mathrm{m}}{ }^{2} \mathrm{M}=\mathrm{M}^{2} / 12$ (variance of a rectangular distribution of M items), and $\sigma_{\mathrm{r}}^{2}=8.2944$ (Luce, 1986, p. 428).

