FINAL REPORT

EFFECT OF DESIGN PARAMETERS ON THE DYNAMIC RESPONSE OF BRIDGES

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ABSTRACT

Engineers and researchers have long recognized the importance of vehicle-induced vibration with regard to the response and service life of bridges. In spite of the recognition of the importance and role of dynamic response in deterioration and fatigue damage, the use of current design practices can still result in bridges with undesirable dynamic response characteristics.

The primary objective of this study was to develop a procedure for representing moving loads on a bridge model within the context of a commercially available computer code. Such an evaluation might reveal ways to improve design procedures to mitigate unacceptable dynamic response. To achieve this goal, it was necessary to employ a moving load, and, thus, a secondary objective was to develop a procedure for representing moving loads on a bridge model within the context of a commercially available computer code. A third objective was to evaluate the relative effects of various parameters that have a substantial effect on dynamic response.

Finite element models of typical bridge structures were developed using ANSYS, a commercially available computer code. A feature of this program, the ANSYS Parametric Design Language (APDL), was employed to represent moving loads with various characteristics. The algorithm developed to represent transient loads in the finite element beam model solution provided results essentially identical with those determined from theory. The relative influence of various design and load parameters was investigated using a finite element model of a section of an actual bridge. Midspan displacements of the bridge were calculated and normalized with respect to the static displacement. Changes in displacement attributable to modifications of the bridge characteristics and to loading parameters were determined.

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INTRODUCTION

The goal of bridge engineers has always been to design economical structures that are safe, serviceable, and durable. Accomplishing this task requires an understanding of the behavior of bridge structures, a knowledge of parameters affecting their response, and an ability to predict their response to all types of loading. Engineers and researchers have long recognized the importance of vehicle-induced vibration in the response and service life of bridges (Green et al., 1995). In spite of the recognition of the importance and role of dynamic response in the deterioration and fatigue damage of bridges, it is evident that using the current design practice can still result in bridges with undesirable dynamic response characteristics. Current methods of analysis and design frequently underestimate the effects of dynamic loads on structures (Cantieni, 1992; Huang et al., 1992).

In fact, design criteria, such as those established by the American Association of State Highway and Transportation Officials (AASHTO), provide for safe and serviceable structures, but these criteria are based primarily on static loadings. In most cases, the response of bridge structures to static loadings can be determined quite satisfactorily by the use of any of a number of approximate analysis techniques, including relatively simple finite element models. It has become increasingly evident, however, that dynamic response is not so easy to predict and yet can play a major role in bridge response. In fact, dynamic response may be the major factor influencing long-term behavior (Humar & Kashif, 1995).

Under current design practice, dynamic effects are taken into account by increasing the normal static design loads by an impact factor. In the AASHTO specifications, the impact factor is a function of span length only. However, dynamic response depends on a number of factors, including vehicle properties, bridge characteristics, and pavement roughness (Cantieni, 1983; Chan & O'Connor, 1990a; Kou & DeWolf, 1997; O'Connor & Pritchard, 1985). Although designs that comply with current codes may satisfy safety and strength requirements, they may also greatly underestimate actual bridge response in many cases (Chang & Lee, 1994; Huang et al., 1992; Inbanathan & Wieland, 1987; Yang et al., 1995). Consequently, some bridge structures may suffer distress as a result of unexpected dynamic response. For example, stresses generated by heavy vehicles moving at high speeds over a rough bridge deck may greatly exceed those predicted by incrementing static live loads by a dynamic impact factor prescribed in bridge codes. Existing analysis and design procedures do not always predict these unexpected and undesirable results.

Much research has been done on bridge-vehicle interaction and resulting dynamic response. During the past several years, several studies, employing a variety of numerical models, have specifically studied the influence of different factors on the dynamic behavior of bridges (Chan & O'Connor, 1990b; Fafard et al., 1998; Huang et al., 1992; Wang et al., 1992). In these studies, different computational models were employed to represent the bridge and the vehicle. Bridge models ranged from simple beam models to complex finite element models (Chang & Lee, 1994; Fafard et al., 1998; Yang & Lin, 1995). Vehicle models included everything from a constant moving force to a multidegree of freedom system with multiple springs and dashpots (Cao et al., 1996; Huang et al., 1995; Humar & Kashif, 1992). The results of these studies indicated that a number of parameters can have a great impact on bridge response, including vehicle characteristics, bridge characteristics, vehicle speed, deck profile and roughness, number of vehicles, and braking. Because of the complexity of bridge dynamics, establishing a clear correlation between these parameters and bridge response has not been possible, even when sophisticated models were used in the analytical studies. In fact, the use of simplified models may be more effective in identifying such correlation (Humar & Kashif, 1995).

PROBLEM STATEMENT

The cited studies have made valuable contributions to a better understanding of those parameters that affect dynamic bridge response the most. However, the analyses used in these studies all relied on specialized, locally developed computer programs that were intended primarily for research use. Because of their limited application, such analysis techniques have limited utility for most bridge engineers. For this reason, a research program that focused on developing a capability for investigating the effect of design parameters on dynamic response in the context of a commercially available finite element program should offer important advantages to practicing engineers for several reasons. First, it would permit engineers to study the dynamic response of bridges using computer codes that are both available and familiar. Second, such a program would provide a procedure to represent moving loads within a commercially available finite element code. Third, the results from such research would indicate the relative significance of the various parameters on dynamic response. Since a number of design variables can substantially affect dynamic response, bridge engineers should be able to identify those parameters and to understand their relative influence on the overall response of the structure.

PURPOSE AND SCOPE

This study was one phase of a continuing broad, long-range study on the dynamic response of highway bridges. The long-term objectives of the ongoing study are (1) to develop a better understanding of the dynamic response of highway bridges, (2) to identify those factors that contribute to dynamic response and their effect, and (3) to be able to better predict the response of bridge structures to a variety of loadings.

The primary objective of this phase of the study was to develop a procedure for representing moving loads on a bridge model within the context of a commercially available computer code. Such an evaluation might reveal ways to improve design procedures to mitigate unacceptable dynamic response. To achieve this goal, it was necessary to employ a moving load, and, thus, a secondary objective was to develop a procedure for representing moving loads on a bridge model within the context of a commercially available computer code. A third objective was to evaluate the relative effects of various parameters that have a substantial effect on dynamic response.

METHODOLOGY

Fulfilling the objectives of this study required three steps:

1. Develop or adopt a modeling and analysis capability for bridge structures subjected to dynamic loads.

2. Develop a methodology for easily and accurately representing a variety of stationary and moving loads that can be applied to finite element models.

3. Employ the analysis capability and loading methodology to evaluate the relative effects of various parameters that have a substantial effect on dynamic response.

Development of a Modeling and Analysis Capability

For the modeling and analysis of the bridge structures studied, it was desirable to have a finite element code that would be able not only to predict dynamic response but also to represent a variety of bridge structures. The commercial code ANSYS was adopted for this purpose. In addition, analytical models that could represent moving load configurations were utilized to provide theoretically exact response information for comparison.

A number of analytical and computational bridge models were employed to provide a framework in which various static and dynamic loads could be applied and in which the effect of various parameters could be evaluated. As a benchmark for comparison in selected cases, an analytical model of a beam for which exact solutions were available was selected. This analytical model consisted of a two-dimensional, homogeneous, isotropic Bernoulli-Euler beam with a prismatic cross section that was supported by simple pin supports at the ends. Such a beam can be described by specifying the elastic modulus E, moment of inertia I, span length L, cross-sectional area A, and mass density ρ, together with appropriate boundary conditions and loads. The mathematical model describing the behavior of this model is the familiar fourth-order, partial differential equation given in any number of references (e.g., Meirovitch, 1997). For the purposes of this study, the effect of damping was assumed negligible. For arbitrary dynamic loadings, solutions to this equation were obtained using modal superposition. The methodology employed follows that outlined by Baber and Massarelli (1994). For prescribed values of load, beam geometry, and mass, a solution for transverse displacement, $v(x,t)$, may be easily obtained. In this study, Mathcad was used to calculate the solution.

Finite element beam models having the same dimensions and properties as the analytical models were next constructed using ANSYS. These simple models, consisting of three-dimensional beam elements, were used to develop and evaluate procedures for representing various loading conditions, either static loads or constant magnitude loads moving across the span at a specified velocity. Load representations included a single static concentrated load, a constant load moving at a constant velocity, a harmonic load applied at a fixed point, and a harmonic load moving at a constant velocity. This latter case also included the option for specifying the percentage of the load that varied harmonically. The manner in which the moving loads were implemented in ANSYS is discussed later. Static and dynamic responses from the finite element beam models were compared with the exact results from the analytical model. This comparison was used to validate the method of load representation in ANSYS and to determine the effect of model parameters, such as number of elements, on the accuracy of predicted response.

Finite element models of typical and actual bridge structures were developed next. An interactive modeling program using the ANSYS Parametric Design Language (APDL) within ANSYS, which was developed in an earlier phase of the investigation (Barefoot, 1995), was employed for construction of finite element bridge models. This interactive program provides a capability for modeling any steel girder–concrete slab bridge and requires input of only basic geometric and material parameters of the slab and girders. This program is easy to use and allows the quick creation of accurate finite element bridge models.

Finite element models of both complete bridges and segments of bridges consisting of single girder lines were developed. The girder line model was a longitudinal segment of the bridge consisting of a single girder and the adjacent portion of the slab. In these models, the girder was modeled as a beam using three-dimensional beam elements and the slab was represented by three-dimensional shell elements. The composite action was implemented by connecting the beam and shell elements by rigid beam elements. The girder line model was evaluated by comparing static and dynamic response from this model, resulting from typical loadings, with corresponding response from the complete bridge model having the same properties. Critical response values, such as displacement and fundamental frequency, were found to be essentially the same for both models. Accordingly, in the interest of computational efficiency, most of the effort in subsequent parameter evaluation was done using the girder line model rather than the model of the entire bridge.

Development of Procedure for Representing Moving Loads

The development of a procedure for representing moving loads, especially in the context of finite element bridge models, was less straightforward. A technique was finally devised that involved employing a programming capability in ANSYS called a parametric design language. Using this language, a programming loop was constructed that controlled the sequential loading and unloading of contiguous nodes in the model as a way of simulating a moving load. Comparing the exact response of a simple beam subjected to a moving load with the computer solutions validated the results obtained using this procedure. Exact solutions were obtained with the aid of Mathcad.

Although the development of finite element models is well established, the manner in which loadings are applied can be more complex. This is especially true in the case of dynamic loads and even more so when the loading involves moving dynamic loads. The way in which moving loads were implemented in all of the models developed is briefly described here.

For the analytical beam models, exact formulations and solutions were available for a constant load applied statically, for a harmonically varying load of constant amplitude applied at any point, or for either of these loads moving at a constant velocity. When finite element models are developed using ANSYS, procedures exist within the program to permit the application of either a constant load applied at a specified node within the model or a harmonically varying load at a specified point. If applying either of these loads as a moving load is desired, special techniques must be developed.

Constant Load

Within ANSYS, the powerful and versatile feature ANSYS Parametric Design Language (APDL) offers a wide range of options to the user including the capability of simulating moving loads. In particular, APDL includes features such as repeating commands, if-then-else branching, do-loops, and matrix operations. To represent a constant load moving at a constant velocity, a macro was written using APDL in which the constant load was placed at successive locations along the span length using do-loops and a specified distance increment. A preselected time step and a prescribed velocity determined the distance the load was moved along at each step of the analysis. If the distance increment placed the load between nodes, the load was allocated statically to the nodes on either side. A dynamic analysis was carried out, and the process repeated through use of the do-loop strategy. At each time step within the loop, a new load was placed at a location on the structure defined by the velocity and time step chosen and, at the same time, the previous load was removed. This had the effect of moving the constant load at a constant velocity but in discrete steps.

This unique approach for representation of moving loads was validated by applying a constant load moving at constant velocity to a finite element model of a simply supported beam and comparing the numerical results from the model with the exact solution.

Harmonic Load

The same approach was used for the case of a harmonic load moving at a constant velocity. The only modification was to recalculate the amplitude of the load at each time step to correspond to a harmonic time variation. Results from using this approach to apply a constant or harmonic force moving at a constant velocity were compared with the exact solution for a simply supported beam with a moving load and were found to compare favorably. Accordingly, this technique was implemented in the solution process and was used not only for beam finite element models but also for girder line models and full bridge models. In the case of these latter two configurations, the load was moved along a longitudinal line of nodes.

Three variations of harmonically varying loads were considered and implemented in the ANSYS computer code. The first was simply a harmonically varying load placed at a fixed location along the span. The second consisted of a harmonically varying load moving at a constant velocity along the span. The final, and most realistic, load representation consisted of a load moving at a constant velocity along the span in which only a portion of the total load was permitted to vary harmonically.

The finite element model of the structure used in this portion of the investigation dealing with harmonically varying loads consisted of the standard or benchmark girder-slab model with the corresponding values of stiffness and mass. The variables considered and evaluated in this phase of the study included the vehicle speed, the percentage of total load that varies harmonically, and the frequency at which the load varies. The capability for having only a portion of the load vary harmonically required developing a different macro program within ANSYS. The reliability of the strategy for implementing this loading was validated in two ways. First, the variable percentage of the load was set to 0%, and the results were compared with those obtained for a constant moving load. Second, the variable load percentage was set to 100%, and the results were compared with those of an exact harmonic analysis obtained earlier. In both cases, the ANSYS results for the harmonically varying load corresponded almost identically with those obtained by exact analysis.

Evaluation of Relative Effects of Parameters Having a Substantial Effect on Dynamic Response

With a modeling and analysis capability and a loading procedure developed and validated, parameter studies were then conducted to evaluate the relative effects of the design parameters having a substantial effect on the dynamic response of bridge models. Bridge characteristics, such as stiffness and mass, and loading parameters, including magnitude, frequency, and vehicle speed, were varied, and the resulting response characteristics determined. Comparisons were made with comparable loads applied statically to ascertain the relative effects of these parameters on dynamic response.

To evaluate the effects of various parameters on response, a reference model was selected and used as a benchmark in subsequent analyses where selected parameters were varied. The model chosen represents a typical interior, longitudinal section of a bridge consisting of the girder and associated slab. The slab was assumed to be 2.4 m (8 ft) wide, a consequence of the 2.4-m (8-ft) girder spacing, and 216 mm (8 1/2 in) thick. The plate girder has a nominal depth of 1 m (40 in), a web thickness of 12.7 mm (0.5 in), a top flange width of 406 mm (16 in), and a bottom flange width of 508 mm (20 in). The thickness of the top flange is 31.8 mm (1.25 in), and that of the bottom flange is 41.3 mm (1.625 in). These girder dimensions correspond, in an average sense, to those of the girders on the Meherrin River Bridge on Route 58 in Southside Virginia. The span length for this bridge was taken to be 30.5 m (100 ft). In studies where the load was a moving load, the reference velocity was chosen to be 64.4 km/h (40 mph). In subsequent discussions, this model is referred to as the reference, or benchmark, model.

For this particular model, detailed dimensions of the girder may be specified and the program calculates the corresponding moment of inertia and area of the girder. These values are then used to define the properties of the equivalent beam model used to represent the girder as described earlier. A straightforward static analysis, with a concentrated load at midspan, and a modal analysis were conducted to determine the fundamental natural frequency of the benchmark model and to provide a value for midspan displacement that would be used subsequently as a normalization factor. For future reference, the fundamental frequency was found to be 2.83 Hz, a value very close to the actual measured value for the Meherrin River Bridge.

RESULTS AND DISCUSSION

This section contains response information from the analysis of the analytical and finite element models of the various configurations employed in this study. The response data are based on three models: an analytical beam model, a finite element beam model, and a finite element girder-line model. The results also represent four loading conditions, a concentrated load at midspan (static), a constant force moving at a constant velocity, a harmonic load at midspan, and a harmonic load moving at a constant velocity. The initial portion of this section presents results from the various models to indicate the nature of the response of beams and bridge segments to moving loads and to illustrate the accuracy of the numerical approach employed. The second part focuses on the effect of the various design parameters on response and an evaluation of those parameters that appear to affect response the most.

Load Verification and Response Characteristics

The character of response resulting from the various loading conditions can be illustrated most conveniently by examining the response of a simply supported beam subjected to these loads. In addition, evaluation of the exact analytical beam response is a convenient means of validating the procedure for load representation in a corresponding finite element code. A reference or benchmark beam was defined by selecting the parameters describing both the analytical model and the finite element model of the beam to be an approximation of the characteristics of a longitudinal segment of an actual bridge. The span length *L* was chosen to be 30.48 m (100 ft), the moment of inertia *I* was selected to be 14.15 x 10^9 mm^4 (34,000 in^4), the modulus of elasticity was defined as 206.8 GPa (30 x 10^6 psi), the cross-sectional area was defined as 142 x 10^3 mm^2 (220 in^2), and the ends were simply supported. This resulted in a structure with a fundamental natural frequency of 2.75 Hz, a value closely approximating the fundamental frequencies measured in field tests of the Meherrin River Bridge.

The static response of the reference beam subjected to a concentrated load at midspan was calculated using the analytical and finite element models. As would be expected, the midspan deflection predicted by the finite element model was identical with that determined analytically. These results are not shown, but the static midspan deflection is used for normalizing response data calculated subsequently for other load cases. Similarly, the midspan response of the reference beam subjected to a harmonically varying load at midspan was calculated both analytically and with the finite element model. Again, the responses determined from the two procedures were essentially identical. The responses for the two previous load cases, calculated both analytically and with ANSYS, should compare favorably since these types of loading are part of the standard loading capability in ANSYS. A capability for defining moving loads, however, is not a standard procedure in ANSYS, and this capability had to be implemented by programming in APDL. Accordingly, verification of the accuracy of the response of finite element models subjected to moving loads was essential.

Validation was accomplished using the response of a prismatic, simply supported beam subjected to a constant load moving across the span at a constant velocity. The beam model used was the reference beam described earlier that had a fundamental natural frequency of 2.75 Hz. This type of response data were obtained both analytically using an exact formulation and numerically from the finite element beam model.

The characteristics of the transient response of a simply supported beam to a moving load are shown in Figure 1. In this plot, the displacement at midspan, normalized with respect to the midspan static displacement and calculated both analytically and with the finite element model, is plotted as a function of load location on the span. For this particular case, the velocity of the load was 64.4 km/h (40 mph). As indicated on the plot, the exact analytical formulation and the finite element model produced essentially identical results, which indicates the procedure developed for representing a moving load on a finite element model is reliable and yields accurate results. The displacement is seen to oscillate slightly about the pseudo-static displacement, i.e., the displacement at midspan attributable to a moving load but with all dynamic effects neglected. For the particular conditions of this problem, i.e., natural frequency

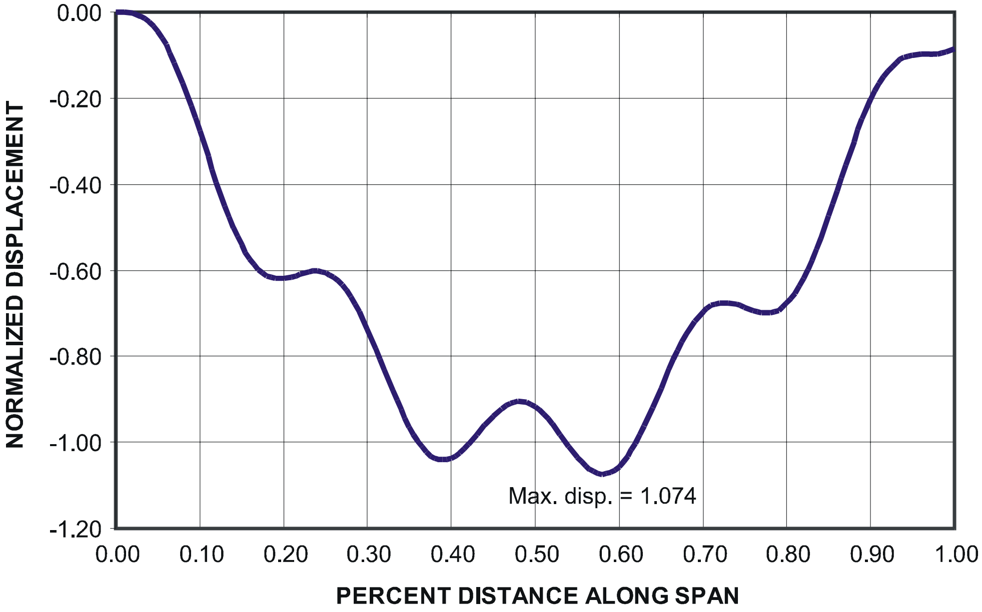

Figure 1. Midspan Displacement of Benchmark Beam Model, Constant Moving Load, Load Velocity = 64.4 km/h (40 mph)

of the structure and load velocity, the maximum displacement was only approximately 8% larger than the static displacement.

Although not illustrated in plots, several other characteristics of the midspan displacement of a beam attributable to a moving load are worth noting. First, the normalized response attributable to a load moving at a constant velocity will be the same, regardless of the values of the moment of inertia and cross-sectional area of a beam, as long as the ratio of moment of inertia to area is constant. This means that as long as the load velocity remains constant, the only beam parameter affecting the normalized dynamic response is the fundamental frequency of the beam. In general, the more flexible the beam, i.e., the lower the natural frequency, the larger the normalized displacement. For frequencies in the range of typical bridge structures, the maximum normalized displacement may vary between 5% and 20% larger than the maximum static displacement. The oscillatory behavior of the dynamic displacement is a function only of vehicle speed and fundamental frequency of the structure. The three curves in Figure 2 represent the normalized midspan displacement of the reference beam resulting from a constant load traveling at speeds of 32.2 km/h (20 mph), 64.4 km/h (40 mph), and 96.6 km/h (60 mph). The higher the vehicle speed, the shorter the time the load is on the bridge, and, consequently, fewer oscillations of the bridge will occur during passage. The response data shown in Figure 2 also indicate that the higher the vehicle speed, the larger the normalized displacement, although the increase is not large.

Next, the response of the same simply supported beam subjected to a harmonically varying load moving along the span at a constant velocity was considered. For this type of actual loading, the variables should include forcing frequency and damping as well as load velocity. In addition, only a portion of the total load should be considered to act harmonically. However, the

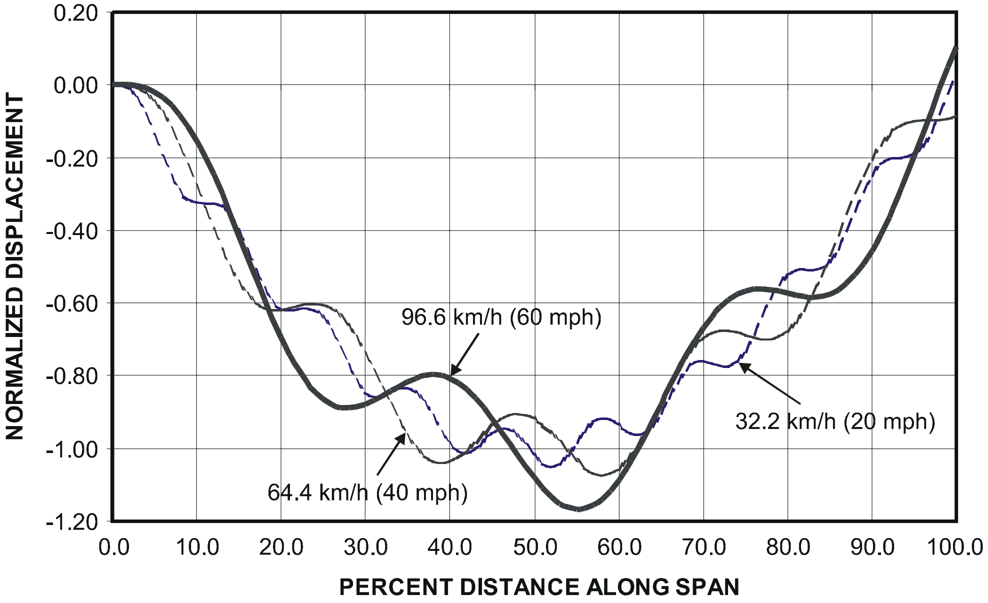

Figure 2. Midspan Response of Benchmark Beam Model to Constant Moving Load, Load Velocities = 32.2, 64.4, 96.6 km/h (20, 40, 60 mph)

basic response characteristics of this type of loading can be illustrated by examining harmonic loads that vary only in velocity and forcing frequency.

Figure 3 is a plot of midspan response of the benchmark beam model to a harmonic load, with a forcing frequency of approximately 1.4 Hz, moving across the span at a velocity of 301 km/h (187 mph). Although this velocity is certainly unrealistic, it was selected in order to compare the response from the computer model with that obtained from an earlier report (Baber & Massarelli, 1994) in which the response to these conditions was obtained using an exact analytical formulation and using Mathcad to obtain the solution. The comparison was excellent, again confirming the reliability and accuracy of the representation of the moving load in ANSYS and of the harmonic nature of the load in the computer solution. Figure 4 presents the midspan response to the same harmonic load moving at a velocity of 64.4 km/h (40 mph). The response to a harmonic load traveling at 64.4 km/h (40 mph) but with a forcing frequency of approximately 4 Hz is shown in Figure 5. From these latter three plots, it may be observed that the displacement response of the beam to this particular type of loading is harmonic about the undeflected position of the beam and that the frequency of response is a function of both forcing frequency and load velocity. In addition, the magnitude of the dynamic displacement response appears to be directly proportional to load velocity and inversely proportional to forcing frequency.

Although these plots represent the general characteristics of a moving harmonic load, they are somewhat unrealistic in the sense that these particular loads act vertically on the beam in both directions, a loading feature that would not be imparted by an actual vehicle. The difference between the idealized loadings, reflected in the previous plots, and the actual vehicle loads is that in actual vehicle loads, only a portion of the weight of the actual vehicle acts dynamically while

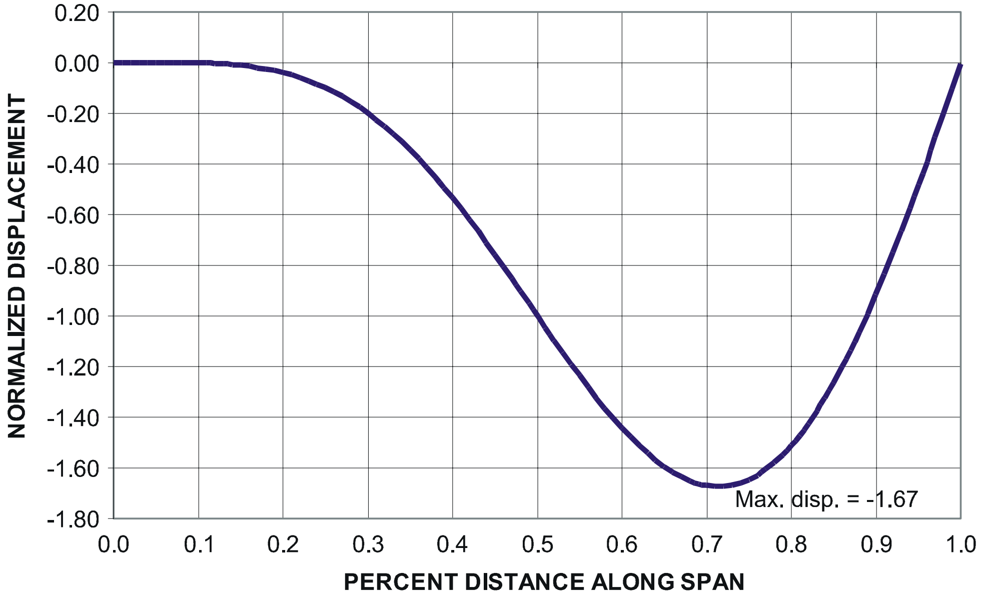

Figure 3. Midspan Response of Benchmark Beam Model to Harmonic Moving Load, Load Velocity = 301 km/h (187.4 mph), Load Frequency = 1.37 Hz

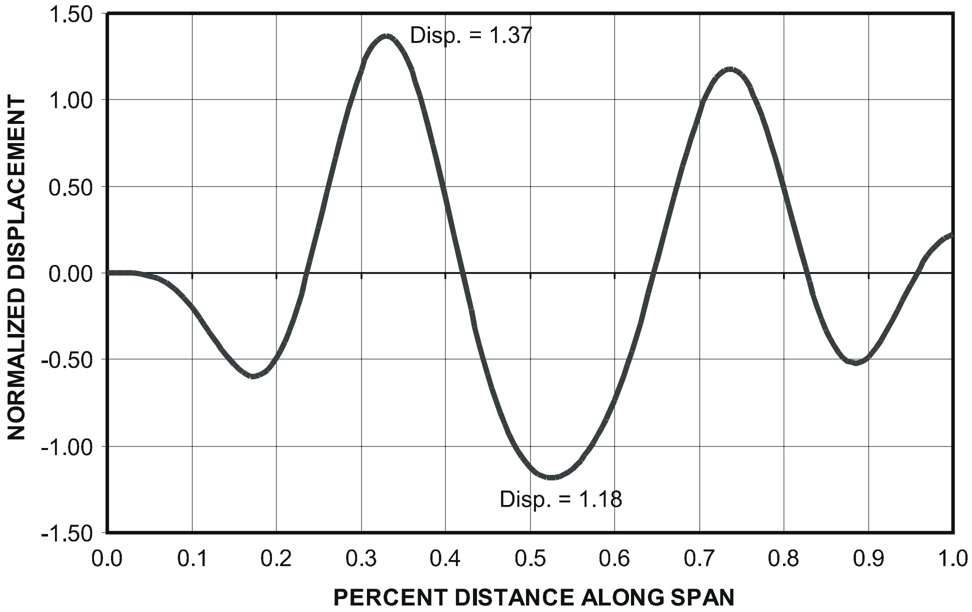

Figure 4. Midspan Response of Benchmark Beam Model to Harmonic Moving Load, Velocity = 64.4 km/h (40 mph), Frequency = 1.374 Hz

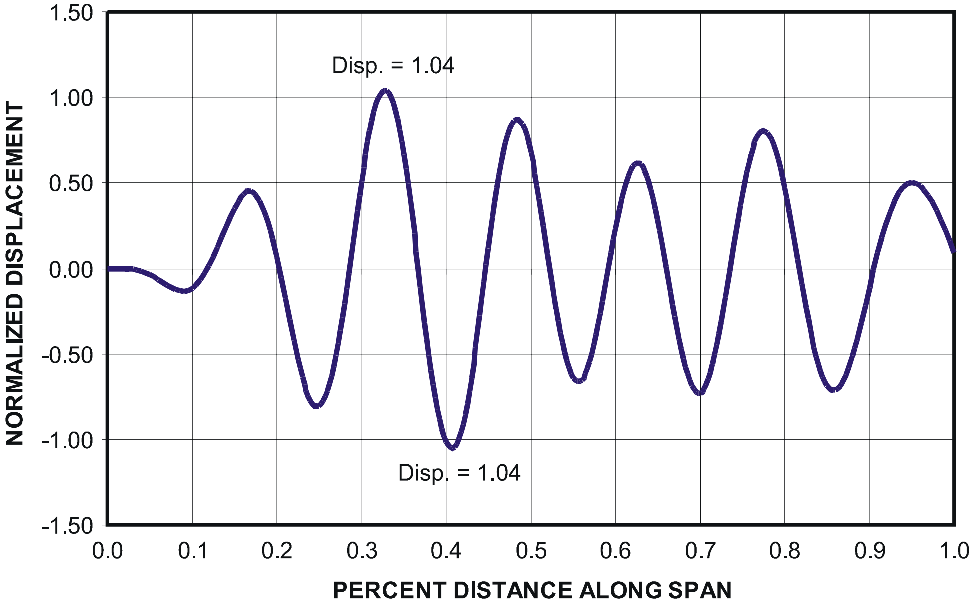

Figure 5. Midspan Response of Benchmark Beam Model to Harmonic Moving Load, Velocity = 64.4 km/h (40 mph), Forcing Frequency = 4 Hz

most of the weight continues to act as a gravity load. This difference is accounted for in the subsequent discussion on the effect of the various parameters on the response of an actual structure.

The previous plots of response were obtained for loadings applied to an idealized beam model. These plots illustrated the basic characteristics of response of a beam model to moving loads and, more important, demonstrated that the procedure for representing moving loads in the computer program were valid and yielded correct response information. The next section discusses the evaluation of the effect of various bridge parameters on dynamic response and the use of a more realistic model of a bridge structure, specifically a girder-slab model representing a portion of an actual bridge structure.

Evaluation of Parameter Effects

The effect of vehicle velocity on dynamic response was evaluated by considering the benchmark model subjected to a constant load moving across the span at various velocities. Displacements at midspan were selected as the measure of response and were calculated as a function of vehicle position on the bridge. Displacement response was calculated for velocities of 32.2 km/h (20 mph), 64.4 km/h (40 mph), and 96.6 km/h (60 mph). Response plots for midspan displacement, normalized with respect to static displacement, at these three velocities are given in Figure 6. Maximum displacement appears to increase slightly with increasing velocity. For load velocities of 32.2 km/h and 64.4 km/h (20 and 40 mph), dynamic displacements are 5% to 10% larger than the static displacement, whereas for a load velocity of

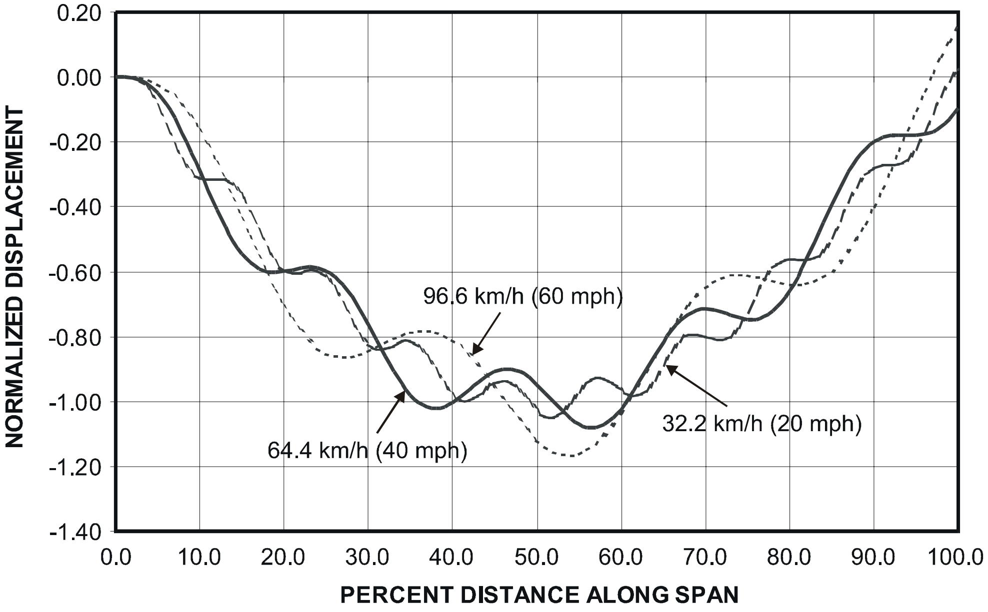

Figure 6. Midspan Response of Benchmark Girder-Slab Model to Constant Moving Load, Load Velocities = 32.2, 64.4, 96.6 km/h (20, 40, 60 mph)

96.6 km/h (60 mph), the maximum dynamic displacement exceeds the static displacement by approximately 18%.

Another set of variables considered were stiffness and mass, which are the usual factors determining the fundamental natural frequency. In this evaluation, stiffness and mass were changed simultaneously in such a way that the natural frequency remained constant. Dynamic midspan displacements, resulting from a constant load moving at 64.4 km/h (40 mph), were calculated for the values of stiffness and mass in the benchmark model and for variations in the coupled combination of stiffness and mass corresponding to a simultaneous reduction and increase of 40% in each. In each of these cases, as long as the same percentage changes were made in both stiffness and mass, the fundamental frequency remained constant. Figure 7 provides plots of normalized midspan displacement as a function of load location for three sets of values of stiffness and mass. The results were as might be expected. For a 40% increase in stiffness and mass, the maximum normalized displacement was reduced approximately 20%, whereas for a 40% reduction in stiffness and mass, the maximum displacement was increased by approximately 80%. The displacements were normalized with respect to the static midspan displacement for the benchmark model. Had the dynamic displacements for the three models been normalized with respect to static displacements for the corresponding models, the responses would have been identical. Thus, the greatest changes observed in the model response resulted primarily from the changes in the stiffness of the structure.

The individual effects of stiffness and mass variation were examined next. Figure 8 presents plots of normalized dynamic displacements as a function of load location for three values of structure stiffness with the mass held constant. Changes in displacement magnitude are seen to be similar to those shown in Figure 7, attributable primarily to the change in stiffness.

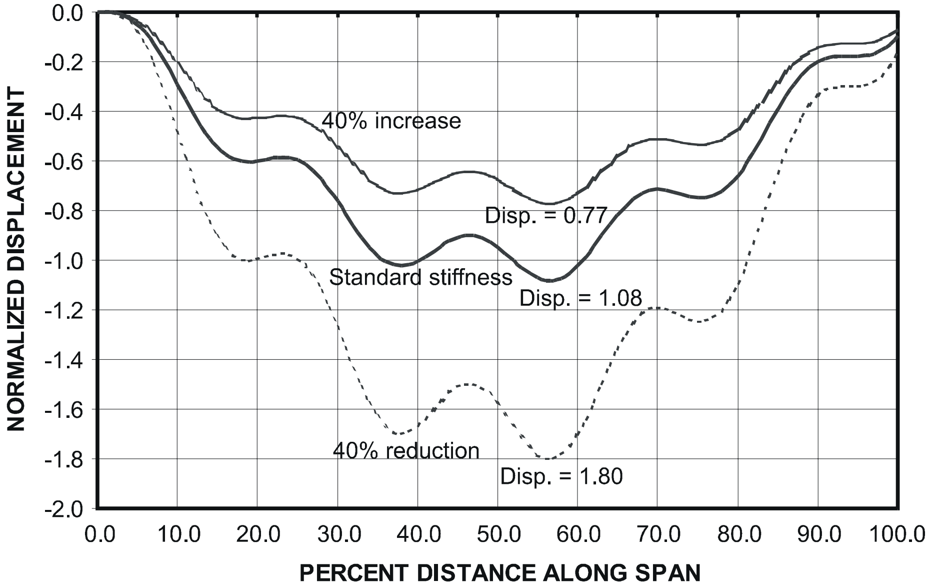

Figure 7. Midspan Response of Benchmark Girder-Slab Model to Constant Moving Load, Load Velocity = 64.4 km/h (40 mph), Variable Stiffness and Mass, Natural Frequency = 2.83 Hz

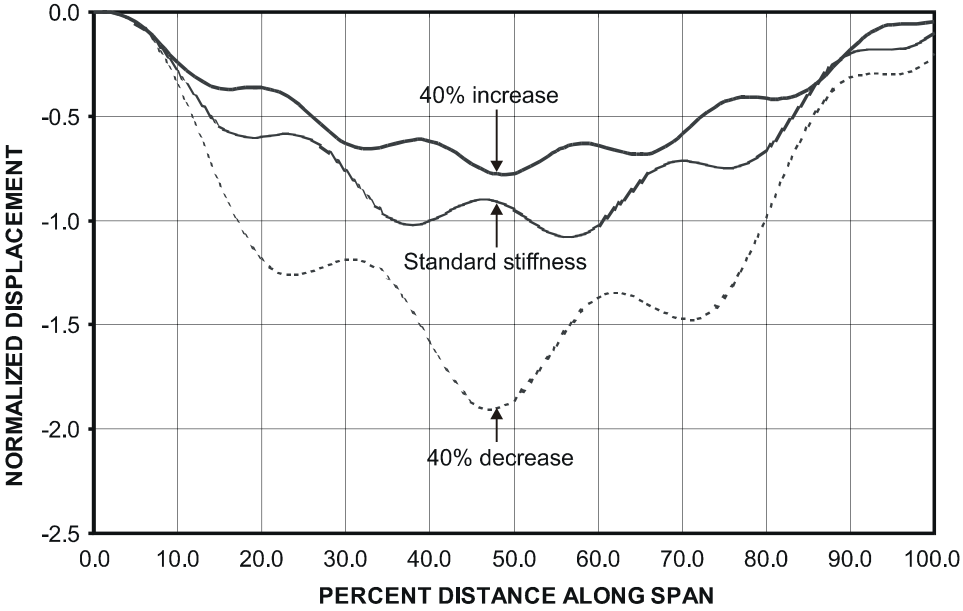

Figure 8. Midspan Response of Benchmark Girder-Slab Model to Constant Moving Load, Load Velocity = 64.4 km/h (40 mph), Variable Stiffness, Constant Mass

The difference in shape of the response curves of Figures 7 and 8 result from the fact that the natural frequencies are different for the three cases plotted in Figure 8. As before, an increase in stiffness results in a decrease in displacement whereas a reduction in stiffness produces an increase in displacement as indicated in the figures.

A similar analysis was carried out in which the stiffness was held constant while the values of mass were varied. Normalized dynamic displacements at midspan, attributable to a constant load moving at 64.4 km/h (40 mph), were calculated using the reference values of mass and stiffness and for mass values increased and decreased by 40%. These responses are plotted in Figure 9. The variation in mass resulted in obvious changes in natural frequency, as evidenced by the different time variations in the response plots. However, the differences in response magnitude were not very great, with the largest increase in displacement on the order of 10% larger than the static displacement.

From the preceding analyses of dynamic displacement attributable to a constant moving load, several observations seem warranted. First, since the dynamic displacements were normalized with respect to static displacements of the benchmark model, the largest increases in displacement magnitude were a result of a reduction in stiffness. Thus, one parameter that obviously has a substantial effect on dynamic response under a constant moving load, and thus should be seriously evaluated in the design, is the flexibility of the structure. Changes in mass or weight of the structure in the absence of a corresponding change in stiffness had relatively little effect on dynamic response other than to modify the natural frequency.

Although these results are of interest in terms of comparative effects of various parameters, they are all attributable to a constant moving load, which is not the most realistic representation of actual vehicle loads. It is not feasible, with this bridge-vehicle model combination, to represent an actual vehicle traversing the bridge, which would include interaction effects between bridge and vehicle. However, it is possible to represent the vehicle as a moving load in which a portion of the weight varies harmonically. This is a more realistic representation of the loading imparted to the bridge by a vehicle moving at a constant velocity and has been used by previous investigators (e.g., Chan & O’Connor, 1990a).

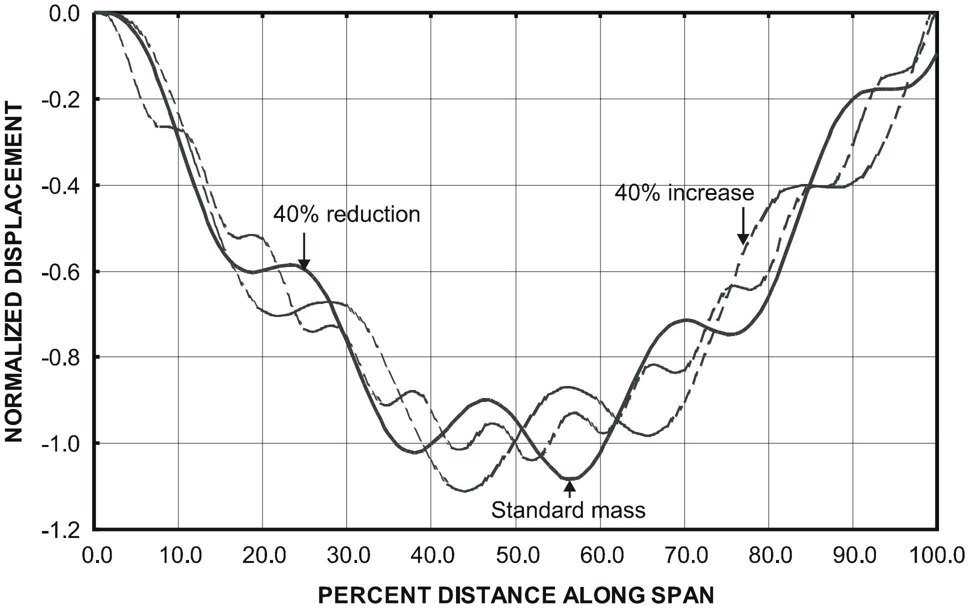

Figure 9. Midspan Response of Benchmark Girder-Slab Model to Constant Moving Load, Load Velocity = 64.4 km/h (40 mph), Variable Mass with Constant Stiffness

Several analyses were conducted to determine the effect of the load percentage that was varied harmonically. Midspan response for a harmonically varying load, with a forcing frequency of 2 Hz., moving at 64.4 km/h (40 mph) is shown in Figure 10. In this figure, percentages of the total load that varied harmonically of 10%, 20%, and 30% are depicted. As expected, the larger the load percentage, the larger the response. Based on results from experimental tests and previous research (Chan & O'Connor, 1990a), it appeared that a reasonable percentage to use in representing that portion of the load that varied dynamically was 10% to 20%. Accordingly, to reduce the number of variables studied, the percentage of load that varied harmonically was maintained at 20% and only the variables of load velocity and load frequency are presented and discussed.

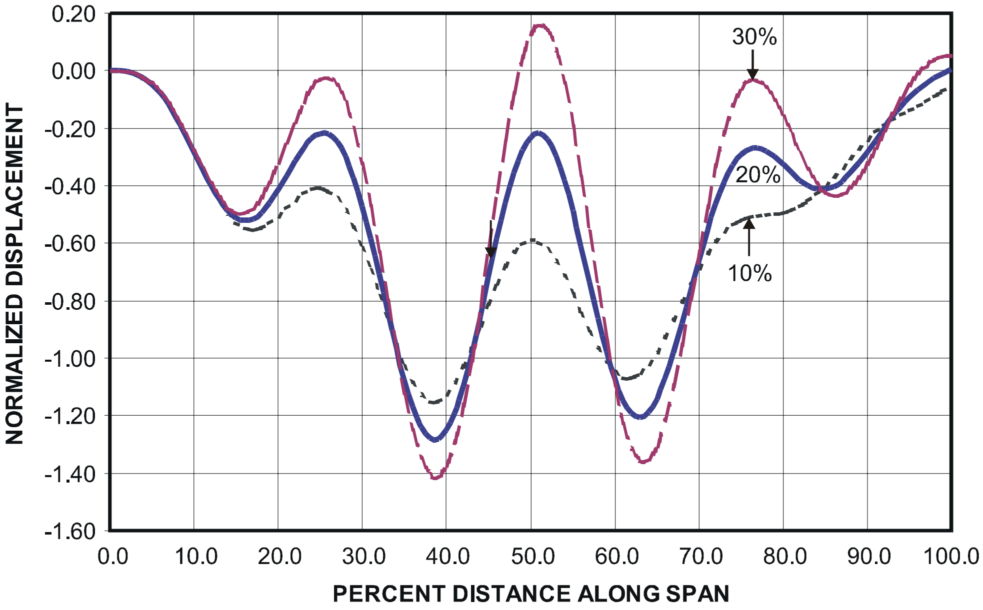

Figure 10. Midspan Response of Benchmark Girder-Slab Model, Load Velocity = 64.4 km/h (40 mph), Load Frequency = 2 Hz, Harmonic Increments = 10%, 20%, 30%

As in previous evaluation, the results are presented in terms of normalized displacements in which the midspan displacements were normalized with respect to the static midspan displacement of the structure under the same load magnitude. Figure 11 shows the midspan response of the bridge produced by a load moving at 64.4 km/h (40 mph) in which 20% of the load varies harmonically. Included are response values for load frequencies from 2 Hz to 5 Hz. The fundamental frequency of this structure was 2.83 Hz. The figures show that the maximum normalized displacement varied from approximately 0.93 at a forcing frequency of 5 Hz to a high of approximately 1.9 at a load frequency of 3 Hz. As expected, the displacements were substantially magnified when the forcing frequency was in the neighborhood of the fundamental frequency. The second natural frequency was approximately 11 Hz, and it is unlikely that the forcing frequency of a vehicle would be that high. When the forcing frequency was in the vicinity of the fundamental frequency, the displacement response grew with time since no damping was provided in these models.

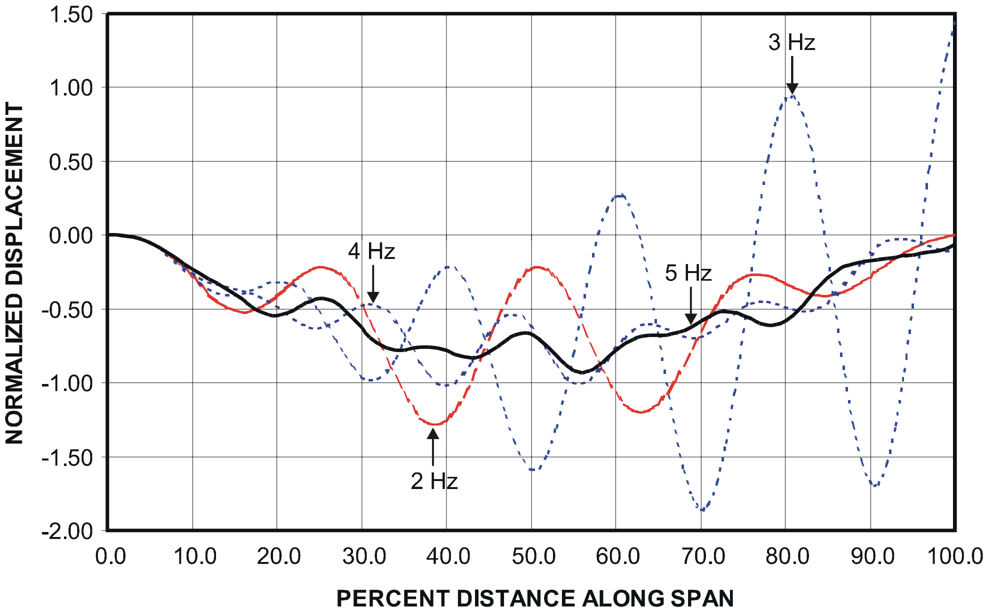

Figure 11. Midspan Response of Benchmark Girder-Slab Model to Harmonic Moving Load, Harmonic Increment = 20%, Load Velocity = 64.4 km/h (40 mph), Load Frequencies = 2, 3, 4, 5 Hz

Figure 12 gives the same variation of displacement with load frequency for vehicle velocities of 96.6 km/h (60 mph). Similar trends in the variation of magnitude of midspan displacement as a function of load frequency may be observed. However, the maximum normalized displacement was reduced from a value of approximately 1.9 to a value of 1.5 as a result of the load traversing the span in a shorter time. For higher load velocities, the displacements were reduced even further.

The effect of load velocity on displacement magnitude is of interest. The same data presented previously to illustrate the effect of load frequency are now presented in a different form to show the effect of load velocity. In Figure 13, the midspan response attributable to a load frequency of 2.83 Hz for different load velocities is presented. As would be expected, since the forcing frequency is at the fundamental frequency of the structure, the maximum response is larger for lower velocities since the load is applied for a longer time. Although the figure shows that the normalized response can be 2 to 3 times the static response for the lower velocities, it is unlikely that the harmonic excitation would be as large at the lower speeds.

Although the effects of other combinations of variables could be investigated, the research team thought that the ones considered provide sufficient insight into the effect of the primary design parameters on dynamic response. More important, the results shown clearly validate the procedure employed for the representation of moving and transient loads within the commercial finite element code ANSYS. As might have been expected, those design parameters having the most effect on dynamic response are the flexibility of the structure and the relationship between the forcing frequency of the load and the natural frequency of the structure. Of particular interest is the confirmation that natural frequency of the structure is, by itself, relatively meaningless in terms of its effect on dynamic response.

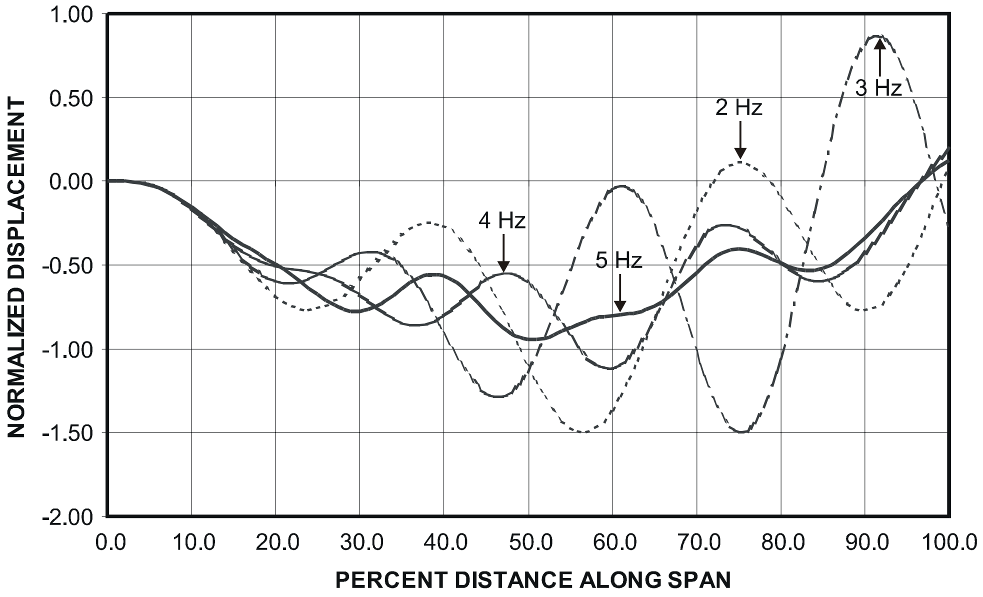

Figure 12. Midspan Response of Benchmark Girder-Slab Model to Harmonic Moving Load, Harmonic Increment = 20%, Load Velocity = 96.6 km/h (60 mph), Load Frequencies = 2, 3, 4, 5 Hz

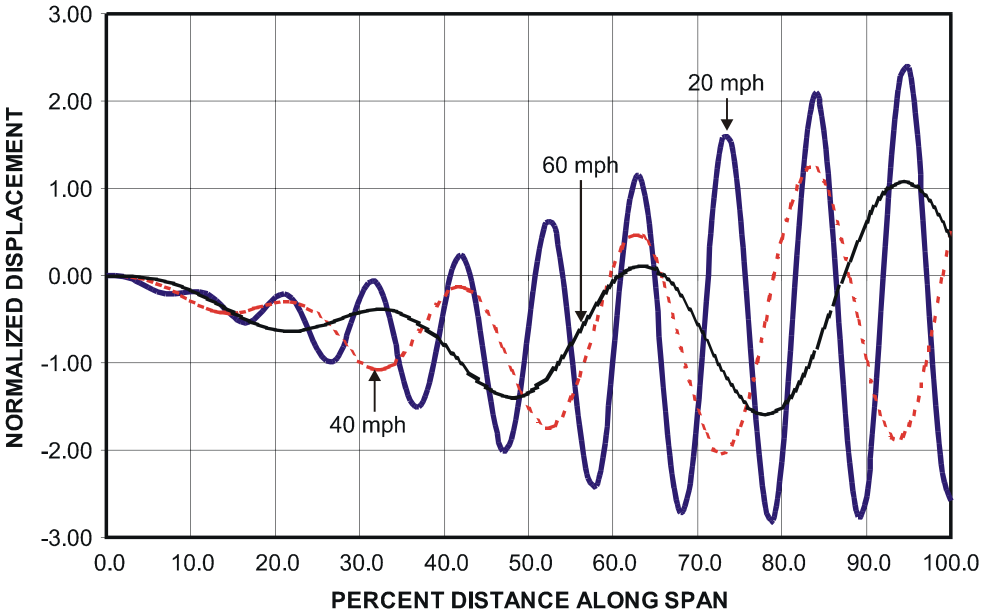

Figure 13. Midspan Response of Benchmark Girder-Slab Model to Harmonic Moving Load, Harmonic Increment = 20%, Load Velocities = 32.2, 64.4, 96.6 km/h (20, 40, 60 mph), Load Frequency = 2.83 Hz

SUMMARY

This investigation developed a capability for representing transient loads within a commercial finite element code and, employing this capability, evaluated the relative influence of a number of variables on the displacement response of a typical bridge model. Extensive analyses using both the benchmark beam finite element model and the benchmark girder-slab finite element model were conducted under a variety of loading conditions.

Results from these analyses can be conveniently discussed in terms of the validation of the procedures for implementing moving and harmonic loads in the commercial finite element code ANSYS and the relative effect of various design parameters on dynamic response.

Validation of the Finite Element Models and Loading Procedures Developed

The following findings were notable:

- The algorithm developed to represent transient loads in the finite element beam model solution provided results essentially identical with those determined from theory. This was true for both a constant moving load and a harmonic moving load.

- The response of a structure attributable to a moving load is characterized by a dynamic response about the pseudo-static displacement, with the particular features of the dynamic response dependent on the type and velocity of the load.
- The dynamic displacement of a beam attributable to a load moving at a constant velocity normalized with respect to the static displacement is independent of the moment of inertia and the cross-sectional area of the beam as long as the ratio of moment of inertia to area is constant.

Relative Influence of Various Design and Load Parameters

The relative influence of various design and load parameters was investigated using a finite element model of a section of an actual bridge. Midspan displacements of the bridge were calculated and normalized with respect to the static displacement. Changes in displacement attributable to modifications to the bridge characteristics and to loading parameters were determined. The following findings were notable:

- With respect to velocity, maximum normalized displacement increased slightly with increasing velocity; increases, within normal vehicle velocities, were in the range of 5% to 20%.

- With respect to stiffness and mass, as long as the same percentage changes were made in both, the fundamental frequency remained constant but the normalized displacements changed. For example, for a 40% increase in both stiffness and mass, the normalized displacement was reduced approximately 20%, whereas for a 40% reduction in both stiffness and mass, the maximum displacement was increased approximately 80%.

- If the mass was held constant and the stiffness modified, the changes in normalized displacement were similar to those when both stiffness and mass were changed proportionally. An increase in stiffness resulted in a decrease in normalized displacement, whereas a decrease in stiffness resulted in an increase in normalized displacement. However, if the stiffness remained unchanged while the mass was varied, the changes in midspan normalized displacement were relatively small.

A more realistic representation of a moving vehicle load is that of a constant force with a superimposed harmonic component moving at a constant velocity. For this loading configuration, in which the harmonic component of load was defined as 20% of the total load, the variables evaluated included vehicle velocity and frequency of the harmonic component of the load. The following findings were notable:

- For a nominal load velocity of 64.4 km/h (40 mph), the normalized midspan response of the structure was calculated for load frequencies from 2 Hz to 5 Hz. When the load frequency was 5 Hz, the normalized displacement was actually slightly less than the static displacement. However, at a load frequency of 3 Hz, the normalized displacement was almost twice the static displacement. This was not unexpected since the fundamental frequency of the bridge model was 2.83 Hz and no damping was included in the bridge model. Nevertheless, the fundamental frequencies of many bridges are in the range of the frequencies of many vehicles and, thus, this characteristic can be important in assessing dynamic response.

- For higher velocities of the moving load, and for the same frequency range of load just described, the maximum normalized displacement was reduced somewhat since the load was on the structure for a shorter time.

CONCLUSIONS

- The finite element models and the procedures developed to represent moving loads of any character in the existing commercial finite element code ANSYS are valid.

- The most important factors affecting dynamic response are the basic flexibility of the structure and, more specifically, the relationship between the natural frequency of the structure and the exciting frequency of the vehicle.

RECOMMENDATIONS

The results of this study provided a relatively simple numerical procedure for representing moving loads within the framework of a commercial finite element code and demonstrated the use of this procedure for evaluating the effects of particular loadings a bridge may undergo. A knowledge of the parameters that play a major role in bridge response and their relative importance should assist bridge designers in developing new design procedures that will minimize undesirable response characteristics.

Two issues in this regard warrant further consideration:

1. the development of a procedure for determining the response of a bridge to an actual moving vehicle that includes such vehicle characteristics as suspension stiffness and shock absorber damping

2. a procedure for including parameters not considered in this study, such as pavement roughness.

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