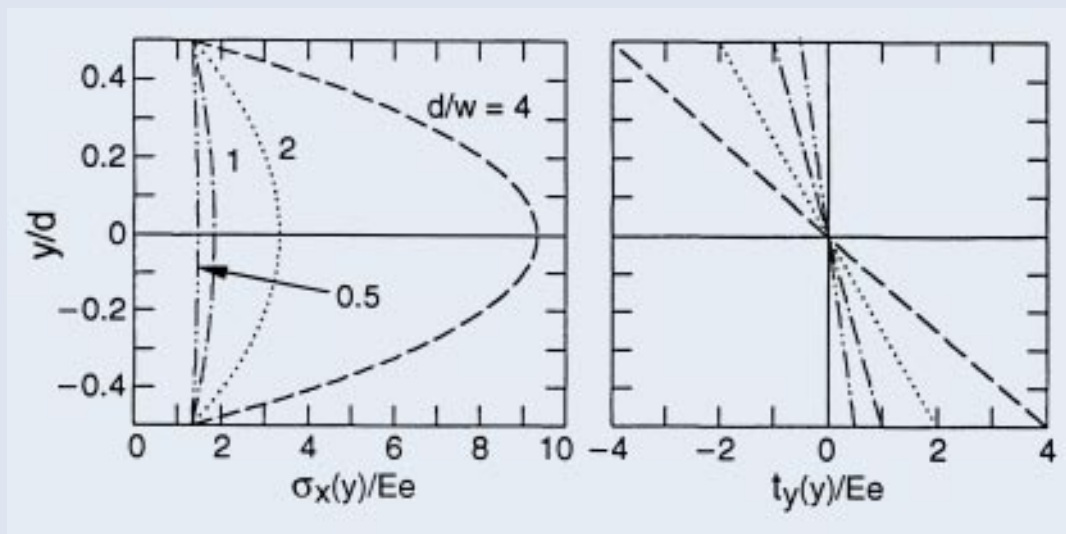# Structural Mechanics Solutions for Butt Joint Seals in Cold Climates

Stephen A. Ketcham

August 1996

Abstract: An effective, formed-in-place joint seal will respond with elastic or viscoelastic behavior over a reasonable design life to any large movement of the joint without adhesive or cohesive failure. For a given joint movement, seals with lower stiffness are most able to deform without cohesive or adhesive failure of the seal or of the structure to which it is bonded. It is in recognition of this desirable response feature that lower-modulus, rubber-based elastomeric materials have been formulated and promoted as joint sealants. For a seal formed from an elastomeric sealant, it should generally be expected that the modulus of elasticity will depend upon temperature and loading rate, such that the modulus increases (sometimes dramatically) with a reduction in temperature and an increase in loading rate, and it should be expected that the seal stiffness will depend upon the material modulus and the shape of the seal. Measurements from testing techniques that are routinely used to evaluate the temperature and rate-dependent mechanical properties of rubber-like materials, together with simple structural mechanics solutions for the load vs. deflection behavior of rubber in the configuration of rectangular-shaped joint seals, allow these dependencies to be modeled, and form the basis of a practical analysis technique that could be used by civil and mechanical engineers for sealant selection and seal design.

Cover: Nondimensionalized normal and tangential stress distributions, $\bar{\sigma}_x(y)/E$ and $t_y(y)/E$, divided by the nominal strain e, for joint seals with depth-to-width ratios 0.5, 1, 2, and 4.

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**US Army Corps
of Engineers**
Cold Regions Research &
Engineering Laboratory

Structural Mechanics Solutions for Butt Joint Seals in Cold Climates

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August 1996

PREFACE

CONTENTS

ILLUSTRATIONS

Structural Mechanics Solutions
for Butt Joint Seals in Cold Climates

STEPHEN A. KETCHAM

INTRODUCTION

An effective joint seal* that is formed in a build-ing or pavement joint by the curing of a sealant will respond with elastic or viscoelastic behavior over a reasonable design life to any movement of the joint without adhesive or cohesive failure. Such a seal is not meant to transfer significant forces across the joint. On the contrary, for a given joint movement, seals with lower stiffness are most able to deform without cohesive or adhe-sive failure of the seal or of the structure to which it is bonded. It is in recognition of this desirable response feature that lower modulus, rubber-based, elastomeric materials have been formu-lated and promoted as joint sealants. For a seal formed from an elastomeric sealant, it should gen-erally be expected that the modulus of elasticity will depend upon temperature and loading rate, such that the modulus increases with a reduction in temperature and an increase in loading rate, and it should be expected that the seal stiffness will depend upon the material modulus and the shape of the seal.

In the field of rubber technology, conventional engineering design of rubber structures incorpo-rates engineering mechanics-based structural analysis techniques and corresponding material

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* In this paper the standard terminology for "seal" and "sealant," given by ASTM C717-88c (ASTM 1991a) for buildings, is adopted. Specifically, "seal" describes a barrier against the passage of liquids and solids, and "sealant" describes a material that has the adhesive and cohesive capabilities to form a seal. These defini-tions are used in an engineering mechanics sense to allow distinction between the material properties of the sealant and the load-deflection behavior of the seal. The discussion here is limited to formed-in-place seals.

properties. When temperature and loading rate variations are expected, these properties are mea-sured as a function of temperature and loading rate so that the effect on structural response can be evaluated. This is in contrast to the current practice for the design of building and pavement seals, which, being based on the "movement ca-pability" of a model seal structure (e.g., ACI 1993, Panek and Cook 1991), does not utilize structural analysis and does not incorporate measurements of the stress–strain mechanical properties of seal-ants. As such, the design practice is not compat-ible with conventional thermal analysis measures for rubber materials, such as the modulus of elas-ticity vs. temperature and the coefficient of ther-mal expansion vs. temperature.

As indicated by the shear modulus vs. temper-ature data in Figure 1, measurements of the modu-lus of elasticity as a function of temperature can be very revealing to the designer of a rubber struc-ture. The data in Figure 1 were published by Nashif and Lewis (1991) as an example of a large database of the properties of rubbers and other materials. The curves shown are of a natural rub-ber and a polysulfide sealant tested at a 50-Hz harmonic loading frequency and at several tem-peratures, and were obtained using measurement techniques that are included in standard test meth-ods (ASTM 1991b). The shear modulus variations of the two materials illustrate the dramatic in-crease in material stiffness that can occur over a narrow, low temperature range in rubber materi-als, as well as a more subtle increase that is pos-sible. By examining the data of the natural rub-ber, for example, a designer might suggest that –20°C should be the lowest temperature at which this rubber is used for loading applications at the 50-Hz frequency. Although the data shown are from high-frequency loading tests, a designer
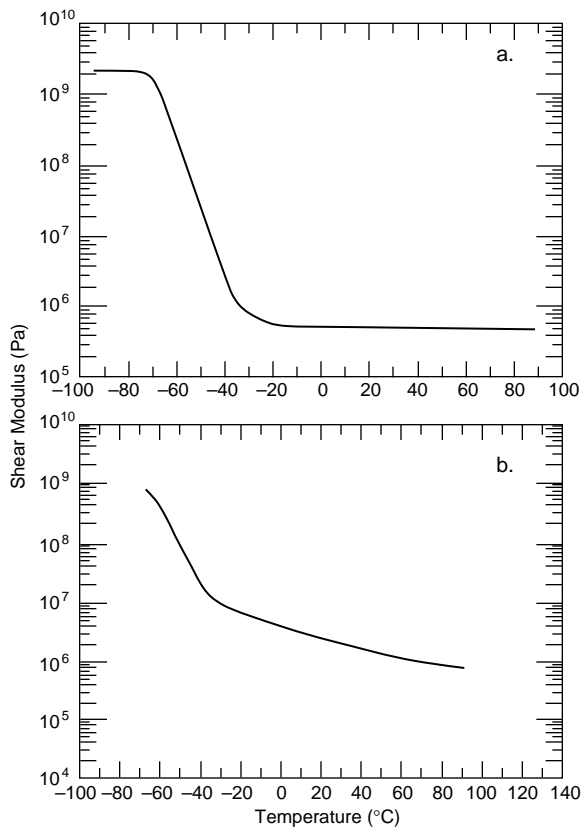
Figure 1. Real part of shear modulus G *as a function of temperature* T *for (a) a natural rubber and (b) a polysulfide sealant, from 50-Hz harmonic loading tests (Nashif and Lewis 1991).*

of measurements of the modulus of elasticity as a function of temperature for sealant materials, and in recognition of the incompatibility of such data with movement capability-based design calculations, this paper presents a review of simple engineering mechanics-based analysis techniques for the structural design of rubber materials subjected to tension, compression, and shear loading in long rectangular joint seal configurations.

Solutions and techniques described by Rivlin and Saunders (1949), Payne (1956), Gent and Lindley (1958, 1959), Gent (1974), and Gent et al. (1974), for rubber materials that can be considered incompressible under hydrostatic loading, are summarized here and in some cases extended to the plane strain configuration of a joint seal. Previous reviews of these solutions, and of the corresponding engineering practice in general, have been presented by Payne and Scott (1960), Lindley (1967), Gent and Meinecke (1970), Gent (1978a, b), and Stanton and Roeder (1982). Recent examinations and extensions of these solutions have been presented by Chaloub and Kelly (1991). A design example illustrating use of solutions from Payne (1956) and Gent and Lindley (1959) is shown in Appendix A.

EXTENSION AND COMPRESSION LOADING

Gent and Lindley (1959), using small deformation, linear elasticity theory and realistic assumptions regarding structural deformations, generated solutions for the stress distribution in bonded rubber blocks during compression of the blocks. From these solutions they obtained expressions for the nominal stress–strain relations of the blocks. Payne (1956) and Gent and Lindley (1959) also suggested approximate relations for the corresponding large deformation problem. These and other solutions described here for a block in a plane strain configuration are directly applicable to the extension and compression of long, formed-in-place, rectangular butt joint seals, and can be presented as such with only slight modification of terminology.

Shape factor, modulus of elasticity, and apparent modulus

The apparent modulus of a butt joint seal in extension or compression was given by Gent and Lindley as

could make similar suggestions about the rubber subjected to slower loads from modulus vs. temperature results of quasi-static loading tests, which are also conventional thermal analysis tests. Lewandowski et al. (1992) have demonstrated the measurement and usefulness of such data for pavement sealants.

It is easy to envision the potential practicality of such information for the selection of cold climate joint sealants. For example, a designer could use modulus vs. temperature data of the candidate sealants together with climatic temperature data for the region of interest, and make a selection using a rationale that incorporates a severe winter design temperature. This is in contrast to current practice, which typically utilizes standard bond tests of model seal structures at a given low temperature (e.g., ASTM 1991c, d), but which does not, in general, reveal explicitly the temperature range at which the model seal or the sealant material stiffens. In recognition of the potential uses

$$E_a = \left[\frac{4}{3} + \frac{1}{3} \left(\frac{d}{w} \right)^2 \right] E \qquad (1)$$

where E is the Young's modulus of the sealant, and d and w are the depth and width of the seal within the joint, respectively, as illustrated in Figure 2a. In the literature of building and pavement seals, d/w is often called the shape factor of a seal. The term "apparent" was used by Gent and Lindley to distinguish the bonded extension or compression deformation as an inhomogeneous structural deformation and to refer to the nominal stress and strain of the structural response in explicit contrast to the homogeneous stress and strain of a material property test. The two terms

a

b

c
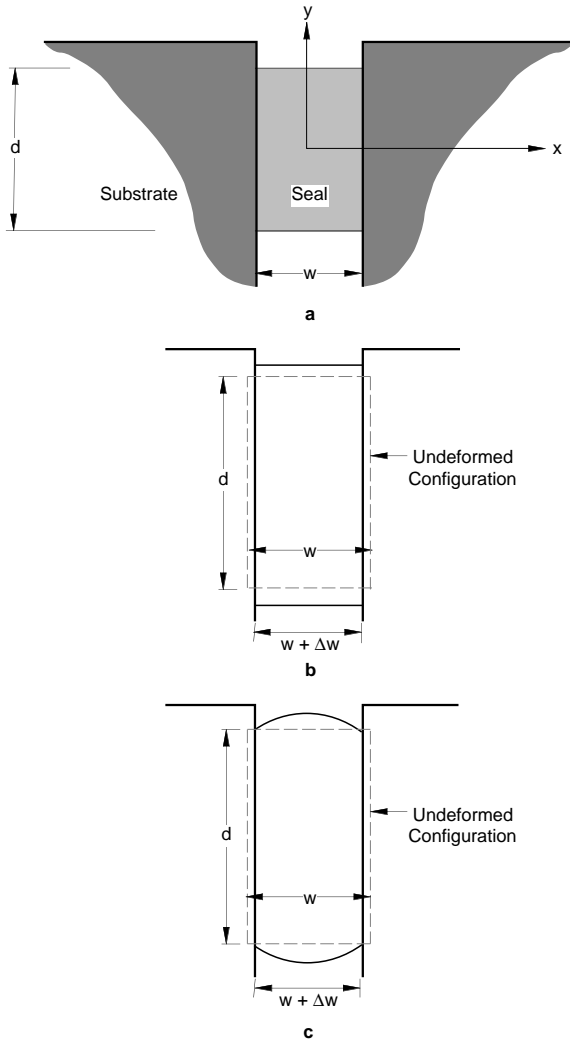
Figure 2. (a) Original butt joint seal configuration, (b) homogeneous deformation, and (c) final, inhomogeneous deformation, corresponding to solution of Gent and Lindley (1959) for the compression of bonded rubber blocks.

in eq 1 originate, respectively, from solutions to (a) a plane strain deformation in which the material is free to slip on the bonded interface and to deform homogeneously in the section, while remaining constrained in the long direction, and (b) a subsequent inhomogeneous shear deformation that restores the material of the bonded interface to the bonded position. These deformations are depicted schematically in Figure 2, in parts b and c, respectively. The material is assumed to be incompressible, i.e., it is assumed that there is no volume change during deformation. For the inhomogeneous shear deformation, it is assumed that planes parallel to the bonded surface remain plane, and that planes normal to these distort to form parabolas in the cross section. The solution to the homogeneous deformation problem (a) is the uniform normal stress, σ_{x1}, i.e.,

$$\sigma_{x1} = \frac{4}{3} Ee \qquad (2)$$

where $e = \Delta w/w$ is the joint extension or compression, i.e., the nominal strain of the seal, Δw is the total joint movement in the x direction, and x refers to the coordinate axis of Figure 2. The solution to the shear deformation problem (b) is a hydrostatic pressure distribution, $p(y)$, that varies with the joint extension or compression, and along the interface of the seal in the y direction, according to

$$p(y) = \left[\frac{1}{2} \left(\frac{d}{w} \right)^2 - 2 \left(\frac{y}{w} \right)^2 \right] Ee \quad . \qquad (3)$$

(See Chaloub and Kelly [1991] for an illustrative derivation of the governing equation that $p(y)$ satisfies.) A normal stress distribution, $\sigma_{x2}(y)$, is in equilibrium with this pressure. The total normal stress distribution is found by the superposition of the stresses σ_{x1} and $\sigma_{x2}(y)$, i.e.,

$$\sigma_x(y) = \left[\frac{4}{3} + \frac{1}{2} \left(\frac{d}{w} \right)^2 - 2 \left(\frac{y}{w} \right)^2 \right] Ee \qquad (4)$$

where σ_x is the total normal stress. The average of this stress, i.e., the nominal stress $\overline{\sigma}_x$, is found by integrating $\sigma_x(y)$ over the area $d \times 1$ of the interface and dividing by this area. The nominal stress–strain relation that follows is

$$\overline{\sigma}_x = \left[\frac{4}{3} + \frac{1}{3} \left(\frac{d}{w} \right)^2 \right] Ee$$
$$= E_a e \quad . \qquad (5)$$

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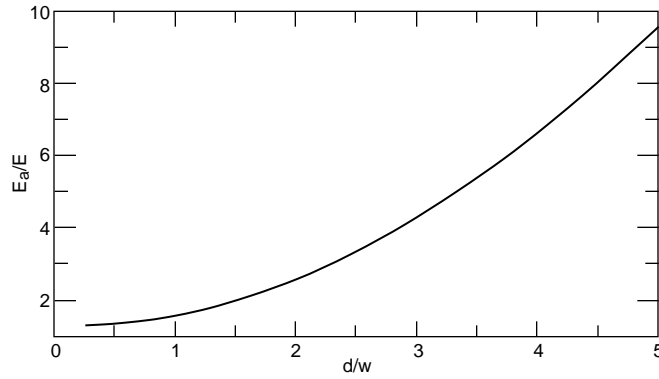
Figure 3. Ratio of the apparent modulus to the Young's modulus, E_a/E, as a function of the depth-to-width ratio d/w.

The apparent modulus is thus greater than the Young's modulus of the associated homogeneous plane strain deformation by the value of the second term in eq 1, which models the additional structural stiffness that arises from the constraint of the bond. Gent and Lindley showed that eq 1 and 5 correctly represented results of load and deflection experiments of rubber structures with length-to-width ratios of three. The joint compressions in these experiments were less than 5% and the material modulus of elasticity was approximately 1800 kPa. Lindley (1967) suggested that eq 1 is applicable up to nominal strains of about 10%.

The plane strain structural stiffness, $F/\Delta w$, which corresponds to the apparent modulus E_a, is given by

$$\frac{F}{\Delta w} = \frac{d}{w}\,E_a \quad . \tag{6}$$

In this equation F is the resultant compressive or extensive force at the interface per unit length along the seal.

The relationships of eq 1 and 6 describe the structural stiffness of a butt joint seal in extension or compression and its dependence on shape factor and elastic modulus. The relationship of eq 1 is depicted in Figure 3 in the form of the ratio of the apparent modulus to the Young's modulus, for depth-to-width ratios from 0.25 to 5. As indicated in the figure, the modulus ratio increases from 4/3 to nearly 10 as the depth-to-width ratio increases from 0.25 to 5, revealing that the average normal stress in the seal can increase by a factor of 7 over this small range of d/w. It is readily observed from Figure 3 that in order to keep the seal stresses at reasonably low levels,

e.g., below failure stress levels, both the depth-to-width ratio of the seal and the modulus of elasticity of the sealant should be kept optimally small. It is in this context that measurements of the shear modulus or the Young's modulus as a function of temperature appear practical for seal design.

Recognizing the limitations of the above relations beyond small deformations, Payne (1956) and Gent and Lindley (1959) suggested the following expression for a large deformation, nominal stress–strain relationship of a rubber block.

$$\overline{\sigma}'_x = \frac{1}{3}\left(\lambda - \frac{1}{\lambda^2}\right)E_a \tag{7}$$

where $\lambda = 1 + e$ is the ratio of the deformed seal width to the original width. The expression is a modification of the large, homogeneous deformation relationship for the simple extension or uniaxial compression of an elastic material (Treloar 1975). The Payne/Gent and Lindley expression accounts for the inhomogeneous deformation in an approximate manner by the use of the apparent modulus of the structure at small strains E_a rather than the Young's modulus E. Gent and Lindley presented experimental results which suggest that, for accuracy consistent with that required for design of building and pavement seals, the relationship of eq 7 is valid for nominal compression strains up to 30%.

The expression of eq 7 is illustrated by non-dimensional relationships in Figures 4 and 5. Figure 4 shows the ratio of the nominal stress to the apparent modulus as a function of nominal compression and extension strain, and Figure 5 shows similar relationships for the nominal stress di-
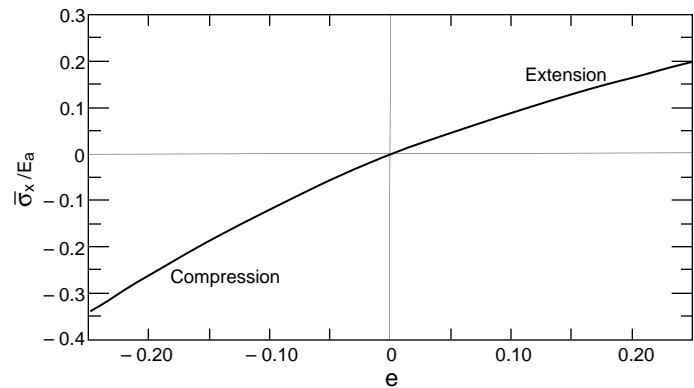

Figure 4. Ratio of the nominal stress to the apparent modulus, $\overline{\sigma}_x/E_a$, as a function of nominal compression and extension strain, e, for large deformations.
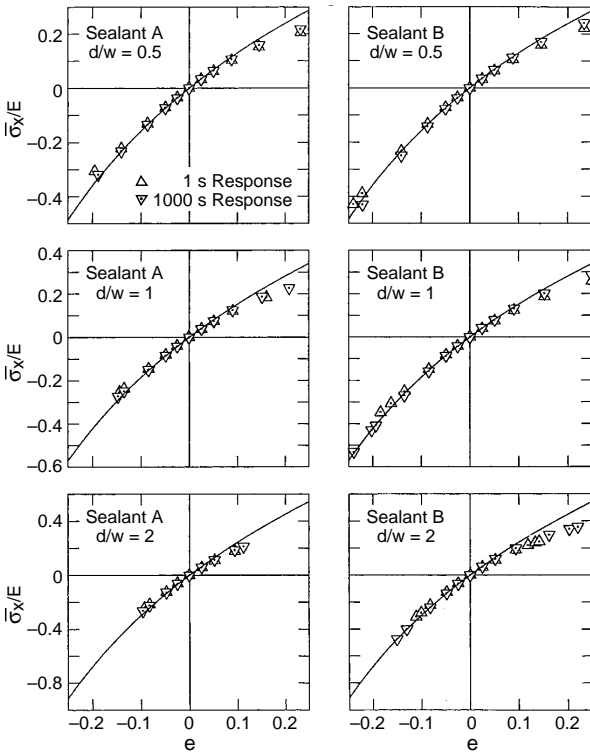
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Figure 5. Ratio of the nominal stress to the Young's modulus, $\overline{\sigma}_x/E$, for joint seals with depth-to-width ratios 0.5, 1, and 2, as a function of nominal compression and extension strain e *(continuous curves). Finite element data for silicone sealants A and B at 0°C and relaxation times 1 s and 1000 s (discrete points).*

vided by the elastic modulus E, which is a function of the depth-to-width ratio. Predictions for compressions and extensions up to 0.25 are depicted. In the latter figure, predictions are shown for three depth-to-width ratios: 2, 1, and 0.5. The curves in both figures demonstrate the nonlinear stress–strain response predicted by the Payne/Gent and Lindley expression, and the curves in Figure 5 further demonstrate the effect of increasing d/w values on the average bond stress acting on a seal.

Figure 5 also includes results from large deformation numerical analyses of butt joint seals. These analyses were finite element analyses, using the commercially available software ABAQUS (Hibbitt, Karlsson, and Sorenson, Inc. 1993), that incorporated strain energy constitutive models of two silicone sealants at 0°C for 1 s and 1000 s relaxation times (Ketcham et al. 1996). The sealants are designated "sealant A" and "sealant B" in Figure 5. The measured Young's moduli of the sealants at 0°C were $E = 540$ kPa and $E = 350$ kPa

for the respective $t = 1$ s and $t = 1000$ s responses of sealant A, and $E = 330$ kPa and $E = 170$ kPa for the 1 s and 1000 s responses of sealant B. The analyses were conducted to allow comparison of predictions of eq 7 with the more realistic numerical results. The general nonlinearity of the finite element predictions and the effect that the depth-to-width ratio has on the behavior are indeed captured by the eq 7 approximation. For compressive strains the differences between the predictions in Figure 5 are typically less than 10%. For extensive strains, however, the eq 7 relation predicts considerably stiffer responses than the finite element analyses. As indicated in Figure 5, the eq 7 predictions are closest to the numerical results for the $d/w = 0.5$ and $d/w = 1$ seals. In this d/w range, and for extensions nearing 25%, the comparisons indicate that the Payne/Gent and Lindley equation would provide conservative estimates of the average bond stress that are roughly 20–30% high.

The small and large deformation relations of eq 5 and 7 indicate that butt joint seals with large depth-to-width ratios should be avoided. It should be noted that, for asphalt pavement crack seals, these relations provide a structural analysis-based argument for preparing a joint at a crack rather than simply filling the crack. A hypothetical example for an asphalt pavement crack seal design, illustrating the use of eq 5 and 7, is presented in Appendix A.

**Stress distributions
at bonded interface**

Equation 4 gives the normal stress distribution at the interfaces between the seal and the joint for small deformations. The corresponding tangential stress distribution $t_y(y)$ is found, as suggested by Gent et al. (1974), from the pressure distribution $p(y)$ according to

$$t_y(y) = \frac{w}{2} \frac{\partial p}{\partial y}$$

$$= -2\frac{y}{w} Ee \ . \tag{8}$$

Expressions like eq 4 and 8 for bonded rubber cylinders have been derived and validated by experiment by Gent et al. (1974).

The normal and tangential stress distributions are illustrated in Figure 6 for the d/w values 4, 2, 1, and 0.5. The stresses, per unit nominal strain, are shown in a nondimensional form divided by the elastic modulus E as a function of the position y/d

5

along the interface. As indicated, the peak of the normal stress σ_x (y) occurs at the mid-depth of the seal, and the maximums of the tangential stress t_y (y) occur at the upper and lower edges of the seal. For a given material and joint extension, the effect of an increasing depth-to-width ratio on the peak normal or tangential stress is dramatic. Also, for higher *d/w* values, the contribution of the shear deformation to the total, normal stress is much greater than the contribution of the homogeneous deformation, which is constant at the σ_x (y)/*Ee* ratio of 4/3.
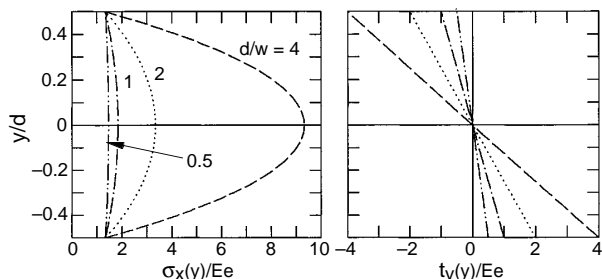
Figure 6. Nondimensionalized normal and tangential stress distributions, $\overline{\sigma}_x$(y)/E and t_y(y)/E, divided by the nominal strain e, *for joint seals with depth-to-width ratios 0.5, 1, 2, and 4.*

Elastic instability

Gent and Lindley (1958) observed in experiments of rubber cylinders that an internal rupture was possible at a comparatively small tensile load when the diameter-to-thickness ratio of the cylinder was high. The rupture was described as consisting of the sudden appearance of internal cracks at a repeatable, small tensile load. The experiments and analysis described by Gent and Lindley showed the internal rupture to be governed by an elastic instability and the failure stress to depend upon the elastic modulus. The cracking stress was found to be independent of the extensibility and strength of the rubber material.

The elastic instability was shown by Gent and Lindley to occur when a small cavity or imperfection within the rubber is subjected to a tensile and primarily hydrostatic stress, such as the maximum normal stress of the curve for *d/w* = 4 in Figure 6. At a critical pressure the cavity expands, forming a crack. Using a large deforma-

tion elasticity analysis, this pressure was found to be approximately (5/6)*E*. Gent and Lindley validated this solution with experimental results, and used the solution with normal stress predictions from small deformation relations like eq 5 to predict the nominal stress and extension values of a rubber structure at failure.

Applying Gent and Lindley's analysis technique to the plane strain structure of a butt joint seal, the critical average stress $\overline{\sigma}_x'$ at which an internal rupture occurs can be found as a function of the material elastic modulus *E* and the depth-to-width ratio of the seal. This relation is

$$\overline{\sigma}_x' = \frac{5}{3}\left[\frac{4}{3}\left(\frac{w}{d}\right)^2 + \frac{1}{3}\right]E \quad . \tag{9}$$

The critical extension, $e' = \overline{\sigma}_x'/E_a$, is

$$e' = \frac{5}{3}\left(\frac{w}{d}\right)^2 \quad . \tag{10}$$

These relations are illustrated in Figure 7 for depth-to-width ratios of a seal from 4 to 8. Below *d/w* = 4, the relations should not be applied since the resulting critical extensions are large and vio-
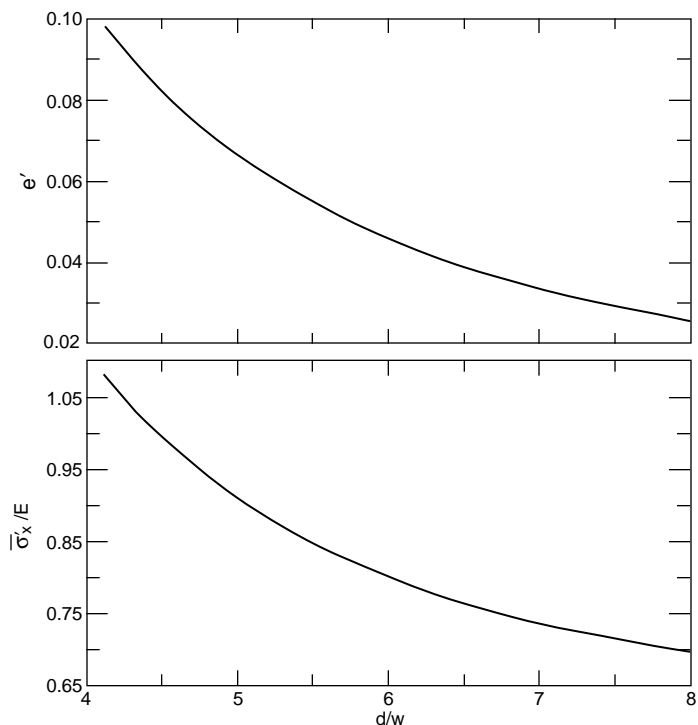
Figure 7. Critical rupture extension, e′, *and ratio of the critical rupture stress to the Young's modulus,* $\overline{\sigma}_x'$/E, *as functions of the depth-to-width ratio* d/w.

late the small deformation assumption of the derivation. At $d/w = 4$, the critical extension is about 10%, which occurs at a critical stress approximately equal to the modulus E. At higher d/w values, the critical values become lower. Using this information, it is possible to attribute some seal failures, e.g., some of the "cohesive" failures observed in butt joint seals with high d/w values, to the elastic instability mechanism.

Shrinkage or expansion stresses

Using a superposition of solutions, Gent (1974) demonstrated the use of the nominal stress–strain relations of bonded rubber structures to evaluate interface stresses generated by the shrinkage of the rubber. Gent suggested that prediction of the interface stresses caused by thermally induced volume changes would be an application of the technique. For example, Gent suggested that interface stresses generated after a rubber structure is formed and bonded at an elevated temperature could be predicted. Using the small deformation, nominal stress–strain relation for a rubber block in a plane strain configuration, the prediction of stresses generated during the cooling of a hot-applied seal would be a specific example of this suggested application.

Following Gent's analysis procedure, when there is no joint movement, and when the seal, if unconstrained, would otherwise extend or compress homogeneously by $e_1 = \Delta w/w$ due to shrinkage or expansion, the following expression for the average, normal interface stress applies.

$$\overline{\sigma}_x = -\frac{2}{3}\left[2+\left(\frac{d}{w}\right)^2\right]Ee_1 \ . \tag{11}$$

e_1 must be known in order to evaluate the stress. For volume changes due to temperature changes, e_1 can be related to the coefficient of linear thermal expansion of the material, α, by $e_1 = \alpha\Delta T$, where ΔT is the temperature change. When α should be treated as a function of temperature, an integration must be performed to calculate e_1.

The normal stress of eq 11, divided by Ee_1, is illustrated in Figure 8 as a function of the depth-to-width ratio of the seal. The figure indicates that, as expected, the average normal stress increases dramatically with d/w for a given material and constrained strain.

Expressions for the stress distributions $\sigma_x (y)$ and $t_y (y)$ for a constrained strain e_1 could also be
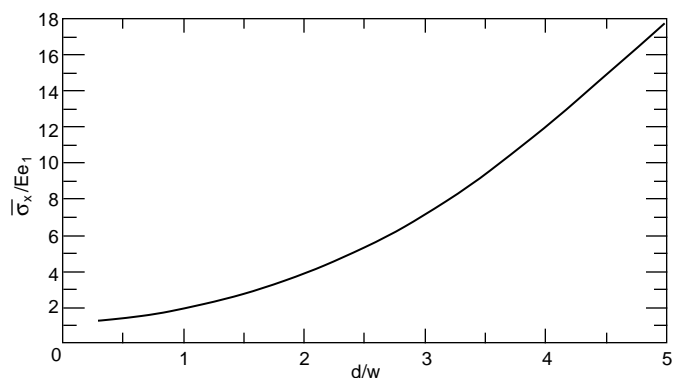
Figure 8. The nondimensionalized average normal stress $\overline{\sigma}_x/$ E divided by the constrained strain e_1, as a function of the depth-to-width ratio d/w.

found by following Gent's procedure. Gent suggested that these stresses could be superimposed with stresses generated by the joint movement in order to evaluate their significance.

SHEAR LOADING

Rivlin and Saunders (1949) studied the effect of shape on the shear behavior of rubber cylinders bonded at their ends. In particular they analyzed the problem in which one end of a cylinder is displaced parallel to the other. They demonstrated by experiment and theory that, when the diameter-to-height ratio of the cylinder is relatively small, the actual deformation can be considered to be made up of a component due to bending in addition to a component due to shear. For cylinders where the bending deformation is significant, Rivlin and Saunders suggested the use of an apparent shear modulus G_a to describe the combined deformation behavior. Their analysis can be applied to the problem of the shear loading of a long, rectangular joint seal, as indicated by the analysis of long, bonded rubber blocks by Lindley (1967) and Gent and Meinecke (1970). Neglecting inertial effects, the problem corresponds to the transverse shear displacement of a pavement joint seal by traffic loading. The geometry of the seal, and the shear and bending deformations, are illustrated schematically in Figure 9.

For the joint seal geometry, the apparent shear modulus derived by Rivlin and Saunders has the form

$$G_a = \frac{G}{1+\frac{1}{3}\left(\frac{w}{d}\right)^2} \tag{12}$$

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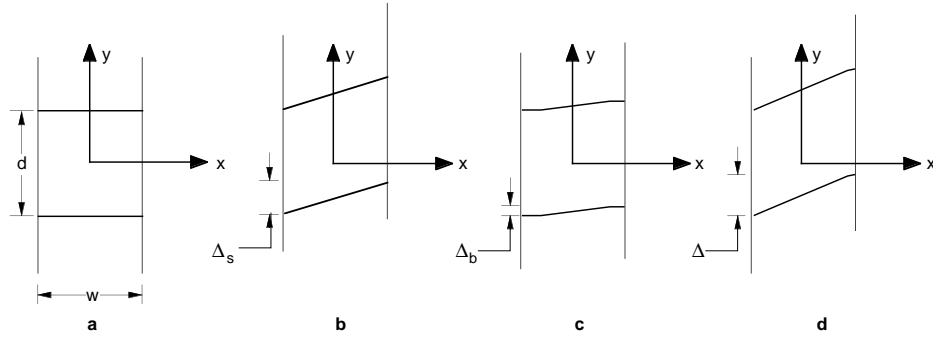
Figure 9. The geometry and deformation of a joint seal in shear loading. (a) original configuration, (b) shear component of total deformation, (c) bending component of total deformation, and (d) total deformation, corresponding to solution of Rivlin and Saunders (1949) for the shear behavior of rubber cylinders bonded at their ends.

where G is the shear modulus of elasticity. G_a can be used in the nominal stress–strain relation

$$\bar{t}_y = G_a \gamma_a$$
$$= G_a \frac{\Delta}{w} \qquad (13)$$

where $\bar{t}_y$ denotes the nominal or average interface shear stress that is generated by the transverse joint displacement Δ, and $\gamma_a = \Delta/w$ is the corresponding apparent shear strain. The expression for the apparent shear modulus can be derived by assuming small shear and bending displacements, Δ_s and Δ_b, respectively, as the superimposed components of the total displacement. For the shear component, $\Delta_s = \gamma w$, where γ is the shear strain, and thus

$$\Delta_s = \frac{\bar{t}_y}{G} w \ . \qquad (14)$$

For the bending component, beam theory yields

$$\Delta_b = \frac{1}{3} \frac{\bar{t}_y}{G} \left(\frac{w}{d} \right)^2 w \qquad (15)$$

for a volume-incompressible material. G_a can be found by substituting the displacement $\Delta = \Delta_s + \Delta_b$ into eq 13. The plane strain shear stiffness F/Δ that corresponds to G_a is

$$\frac{F}{\Delta} = \frac{d}{w} G_a \qquad (16)$$

where, for shear loading, F is the resultant shear force at the interface per unit length along the seal.

A significant bending contribution has the effect of reducing the apparent shear modulus of the seal relative to the actual shear modulus of the sealant. This is illustrated in Figure 10, which depicts the relation of eq 12 in a nondimensional form. For example, at $d/w = 1$, the apparent shear modulus is 75% of the shear modulus. Like the expressions of eq 1 and 5 for extension and compression loading, the shear loading relations of eq 12 and 13 allow the use of the modulus of elasticity, measured as a function of temperature, in design calculations.

CONCLUSION

The relations reviewed here constitute the basis of a practical analysis technique for evaluating load vs. deflection responses of rectangular joint seals subjected to tension, compression, and shear. The nominal stress–strain relations presented have
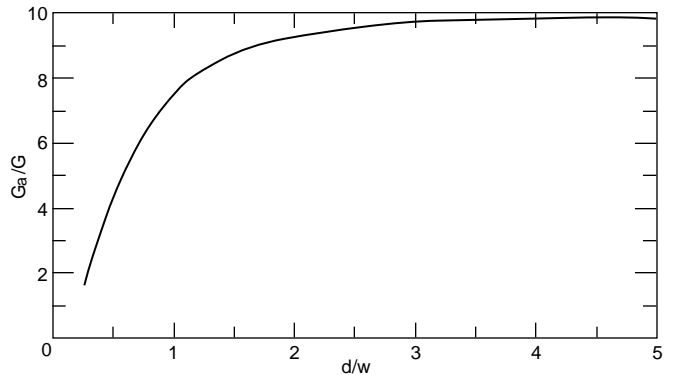
Figure 10. Ratio of the apparent shear modulus to the shear modulus of elasticity, G_a/G, as a function of the depth-to-width ratio d/w.

8

been shown by their developers to be valid for experimental structures formed with rubber materials. The relations provide a simple and rational mechanics-based approach to the selection of seal shape factor, and allow use of the modulus of elasticity of the sealant as a design variable. In this way the effect of temperature on the modulus of elasticity, which is routinely measured for elastomeric materials that are used in cold climates, can be directly incorporated into the sealant selection and seal design process, as can the effect of loading rate or time. Future work in this area should focus on (1) incorporating temperature and rate-dependent mechanical properties in the sealant selection and seal design process, and (2) establishing by field evaluation how results from standard tests of model seals, in which displacements and loads are measured, can be used in conjunction with the relations described here for a practical mechanics-based seal design.

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APPENDIX A: HYPOTHETICAL DESIGN EXAMPLE, PAVEMENT BUTT JOINT SEAL

A maintenance engineer at a northern airfield has received two contract bids for sealing cracks in asphalt concrete pavement. Both contracts specify good installation techniques, e.g., both contractors have specified that cracks will be routed, cleaned, and sealed with a "low-modulus" sealant over a rectangular backer material, without bonding of the sealant to the backer material. Contractor 1 has specified that low-modulus sealant 1 will be used to form seals of 20-mm width and a depth-to-width ratio of 1. Contractor 2 has specified that low-modulus sealant 2 will be used to form seals of 15-mm width, but with a depth-to-width ratio of 3 in order to have "better adhesion."

The engineer has material property data given in Figure A-1 for the two sealants. (Data of this form could be generated using the techniques of Lewandowski et al. [1992]). He/she also has results of standard bond tests conducted at –29°C. From these tests, average normal stresses across the bond interface at failure are calculated. (These data could be generated by extending the standard bond tests to include load measurements.) Failure stress for the model seal formed with sealant 1 for an asphalt concrete substrate was 2×10^6 Pa, and for the seal formed with sealant 2 was 8×10^5 Pa.

The engineer has noticed in the past that seals made with sealant 1 have debonded during cold winter periods, and knows that every five years or so it gets down to –40°C. Although the engineer expects routed joint openings as large as 5 mm resulting from the thermal contraction of the pavement, the exact opening is not known. To evaluate the combined effects of temperature and depth-to-width ratio on the adhesive bond stresses, the engineer does some simple calculations using eq 5 and 7.
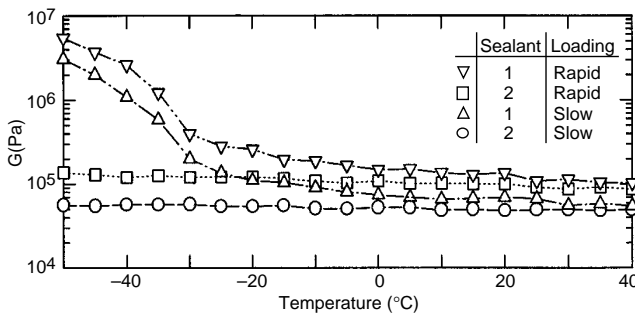
From eq 5 the engineer plots the ratio of the apparent seal modulus to the sealant Young's modulus for a given sealant temperature, i.e., Figure 3. The plot and eq 5 show the engineer that the average normal stress across the bond interface for a $d/w = 3$ seal made of sealant 2 should be about 2.5 times that of a $d/w = 1$ seal also made of sealant 2. In order to keep the bond stresses as far below failure stresses as reasonable, he/she decides that the depth-to-width ratio of 3 suggested by contractor 2 should be abandoned and that a depth-to-width ratio of 1 should be considered for both sealants. Thus the engineer's calculations will compare the performance of seals formed with sealant 1 and sealant 2 at 20-mm width and depth-to-width ratio of 1 for a 5-mm joint opening.

The engineer calculates the average bond stress of a 20-mm^2 seal cross section for a 5-mm (25%) joint opening when the temperature of the sealant is –40°C, which is the design condition he/she feels is appropriate. He/she conservatively selects the rapid-loading shear modulus curve for the material property because it is suspected that the wintertime movement of the joint is a rapid stick–slip movement. The engineer assumes the material to be volume-incompressible and uses the corresponding relation between Young's modulus of elasticity E and the shear modulus of elasticity G in the calculations, i.e., $E = 3G$.

Sealant 1: $G \approx 3 \times 10^6$ Pa, and so $E \approx 9 \times 10^6$ Pa; $d/w = 1$; $e = 25\%$. Thus the linear result from eq 5 is $\bar{\sigma}_x = 3.75 \times 10^6$ Pa and the nonlinear result from eq 7 is $\bar{\sigma}_x = 3.05 \times 10^6$ Pa.

Sealant 2: $G \approx 1.5 \times 10^5$ Pa, and so $E \approx 4.5 \times 10^5$ Pa; $d/w = 1$; $e = 25\%$. Thus the linear result from eq 5 is $\bar{\sigma}_x = 1.875 \times 10^5$ Pa and the nonlinear result from eq 7 is $\bar{\sigma}_x = 1.525 \times 10^5$ Pa.

Based on these calculations, the engineer realizes that sealant 2 results in much less bond stress at large extensions and cold temperatures than does sealant 1, which suggests that sealant 2 should be the selected material. However, the engineer must also compare these stresses with the bond failure stresses. He/she notes that the calculated average normal stresses in the seal formed with sealant 2 at –40°C and 25% extension is less than the failure stress of the standard bond test (8×10^5 Pa),

Figure A-1. Hypothetical sealant shear modulus vs. temperature data for design example.

whereas the average normal stress in the sealant 1 seal under these conditions is greater than the bond failure stress of the sealant 1 standard test (2×10^6 Pa). As a result, the engineer no longer considers sealant 1.

The engineer performs further calculations to evaluate the additional factor of safety provided by using a wider joint, and what effect a joint opening larger than 5 mm would have. The engineer recognizes that the constructed seals will not be exactly square or rectangular in shape, that the sealant has a more complicated viscous behavior than modeled in the analysis, that other effects such as material aging have not been accounted for, and that his/her calculations are theoretical approximations of the actual response of the seal. However, he/she is satisfied that the 20-mm-wide joint, as designed above to include the effects of both temperature and shape on the bond stress, is adequate.

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13. ABSTRACT (*Maximum 200 words*)

An effective, formed-in-place joint seal will respond with elastic or viscoelastic behavior over a reasonable design life to any large movement of the joint without adhesive or cohesive failure. For a given joint movement, seals with lower stiffness are most able to deform without cohesive or adhesive failure of the seal or of the structure to which it is bonded. It is in recognition of this desirable response feature that lower-modulus, rubber-based elastomeric materials have been formulated and promoted as joint sealants. For a seal formed from an elastomeric sealant, it should generally be expected that the modulus of elasticity will depend upon temperature and loading rate, such that the modulus increases (sometimes dramatically) with a reduction in temperature and an increase in loading rate, and it should be expected that the seal stiffness will depend upon the material modulus and the shape of the seal. Measurements from testing techniques that are routinely used to evaluate the temperature and rate-dependent mechanical properties of rubber-like materials, together with simple structural mechanics solutions for the load vs. deflection behavior of rubber in the configuration of rectangular-shaped joint seals, allow these dependencies to be modeled, and form the basis of a practical analysis technique that could be used by civil and mechanical engineers for sealant selection and seal design.

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