

EVOLUTION OF DYNAMIC ANALYSIS IN GEOTECHNICAL EARTHQUAKE ENGINEERING

by

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INTRODUCTION

Probably one of the most significant events which contributed to the rapid development of geotechnical earthquake engineering in the 1960's was the application of finite element methods to the analysis of embankment dams for the first time by Clough and Chopra (1966). This was followed by the seismic analysis of slopes by Finn (1966a, b) and the analysis of central and sloping core dams by Finn and Khanna (1966). The latter study demonstrated the effects of the stress transfer between core and shell. All these analyses were conducted using a visco-elastic constitutive model of the soil and therefore were not capable of modelling the porewater pressure development or permanent deformations. To overcome this problem, Finn (1967) outlined a procedure for interpreting the effects of the dynamic stresses computed by the visco-elastic analysis with the help of data on porewater pressures and strains from laboratory cyclic loading tests.

A major improvement in analysis occurred in 1972 when Seed and his colleagues at the University of California in Berkeley developed the equivalent linear method of analysis for approximating non-linear behaviour. This method was incorporated in the program SHAKE (Schnabel et al., 1972). The technique was extended to two dimensional finite element analysis by Idriss et al. (1974) and Lysmer et al. (1975) in the programs QUAD-4 and FLUSH respectively. These programs led to more realistic analyses of embankment dams under earthquake loading, especially under strong shaking and updated versions are the backbone of engineering practice to the present day.

While this program development was going on, the capability of testing soils under cyclic loading was also being developed. The cyclic triaxial test was developed by Seed and Lee (1966) and made possible the study of liquefaction potential and seismically induced deformations. At around the same time the resonant column test was developed for measuring dynamic shear modulus and damping at low strains (Drnevich, 1967). In the early 1970s the use of the cyclic simple shear test was pioneered by Seed and Peacock (1970) and Finn et al. (1971). Therefore by 1975 geotechnical engineers had many of the analytical and laboratory capabilities necessary for realistic assessments of the seismic response of soil structures.

These methods were put to the test when Seed et al. (1973, 1975a, 1975b), undertook a comprehensive study of the liquefaction induced failure of the lower San Fernando Dam which occurred as a result of the San Fernando, California earthquake of 1971. The analyses predicted

that the dam would fail by undergoing very large deformations upstream during the earthquake. In fact the dam did not deform significantly until after the earthquake. This post earthquake failure was attributed by Seed (1979) to porewater pressure redistribution.

The equivalent linear method of analysis used in the study of the San Fernando Dam is a total stress analysis and therefore does not take into account the effect of porewater pressures on soil properties and dynamic response during the earthquake. Therefore the analyses tend to predict a stronger response than actually occurs. As a result, the San Fernando case history provided the stimulus for the development of effective stress methods of dynamic analysis which could take the effects of porewater pressures into account directly. The Martin-Finn-Seed (MFS) model for generating porewater pressures during earthquake loading based on the strain response of the soil was developed by Martin et al. (1975) and paved the way for dynamic effective stress analysis.

The first non-linear dynamic effective stress analysis based on the MFS porewater pressure model was developed by Finn et al. (1975; 1976; 1977) and was incorporated in the 1-D program DESRA-2 (Lee and Finn, 1978). A rudimentary 2-D version of this program was developed by Siddharthan and Finn (1982). An updated comprehensive program TARA-3 was developed by Finn et al. (1986). This program was verified over a three year period by centrifuge tests conducted at Cambridge University in the UK on behalf of the United States Nuclear Regulatory Commission (Finn, 1988). TARA-3 has the capability to conduct both static and dynamic analysis under total stress or effective stress conditions and can compute permanent deformations directly. The program uses properties that are normally measured in connection with important engineering projects. Indeed, the program can be run entirely on the basis of data from in-situ tests using correlations between in-situ parameters such as normalized Standard Penetration Resistance $(N_1)_{60}$ and Cone Bearing Pressure q_{c1} , and soil moduli, strength parameters and liquefaction resistance.

Since the mid 1980's, a spate of non-linear effective stress programs have been developed, for the most part based on some version of plasticity theory. Detailed presentations of some of these programs may be found in Pande and Zienkiewicz (1982) and comprehensive critical reviews in Finn (1988) and Finn (1993). These programs are mathematically and analytically quite powerful but use some properties which are not routinely measured in the laboratory or the field.

The estimation of post-liquefaction deformations is a very important part of assessing the consequences of liquefaction in embankment dams. Finn and Yogendrakumar (1989) developed the program TARA-3FL to track the post-liquefaction deformations. TARA-3 and TARA-3FL have been used by the US Army Corps of Engineers at Waterways Experiment Station on many embankment dams since 1989.

EQUIVALENT LINEAR ANALYSIS

The dynamic response of earth structures is often computed in engineering practice using an equivalent linear (EQL) method of two-dimensional (2-D) analysis, such as that incorporated in the computer programs QUAD-4 (Idriss et al., 1973) or FLUSH (Lysmer et al., 1975).

The EQL analyses are conducted in terms of total stresses and so the effects of seismically induced porewater pressures are not reflected in the computed stresses and accelerations. Also since the analyses are elastic, they cannot predict the permanent deformations directly. Therefore equivalent linear methods are used to get the distribution of accelerations and shear stresses in the dam. The semi-empirical methods shown in Fig. 1 are often used to estimate the permanent deformations using either the acceleration or stress data from the equivalent linear analyses.

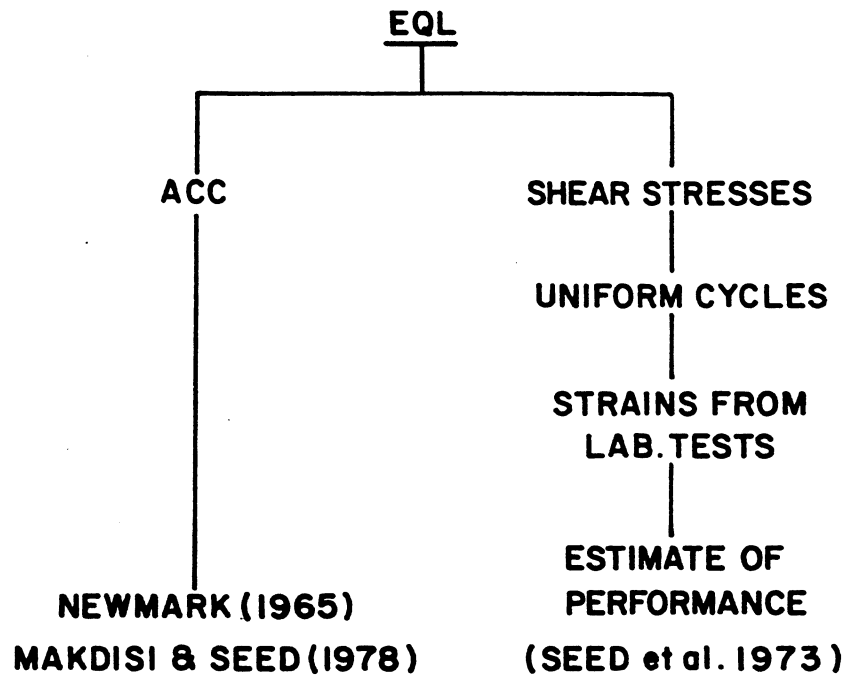


Fig. 1. Procedures for predicting porewater pressures and permanent deformations

Deformations from Acceleration Data

Various potential sliding surfaces in the embankment are analyzed statically to find the inertia force $F_I = (W/g)a_y$ required to cause failure (Fig. 2). The average yield acceleration a_y is then deduced from this force. The average acceleration time-history of the sliding block is obtained usually from a QUAD-4 analysis. The yield acceleration is deduced from the average acceleration time-history and the net acceleration (the shaded area in Fig. 2), is available to generate permanent displacements. The analysis is conducted on the equivalent model of a horizontal sliding block on a plane with only one way motions allowed (Fig. 2).

This method of analysis was pioneered by Newmark (1965). The version described above was developed by Makdisi and Seed (1978) and differs from the Newmark approach in generating relative displacements by the net accelerations above the sliding surface, whereas Newmark used the net accelerations below the sliding surface. The difference results from the fact that the QUAD-4 accelerations are determined in the sliding block without taking the yield acceleration into account. Thus, in many cases, estimates of displacement will be conservative.

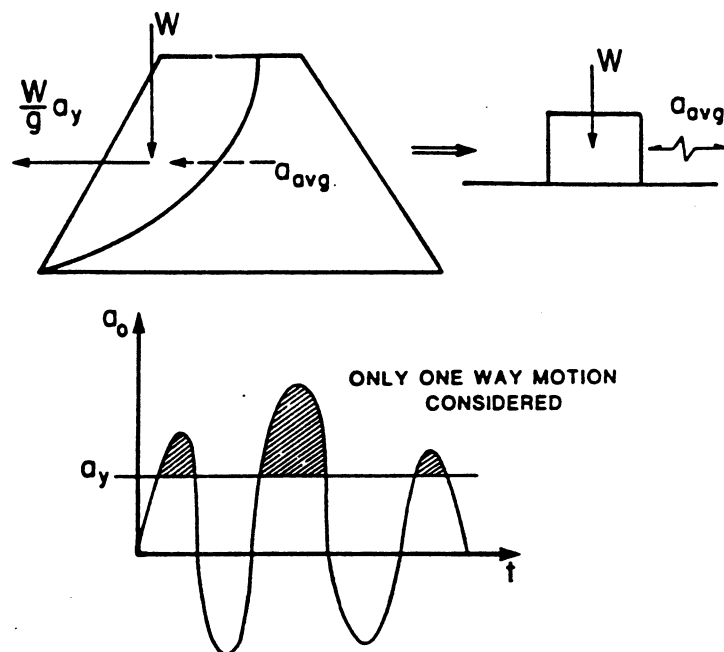


Fig. 2. Elements of Newmark's deformation analysis

The Newmark approach was introduced at a time when there were no direct methods of computing permanent deformations. It is still widely used despite all the evidence that embankment dams do not deform as assumed in the analysis and despite advances in the state of the art that make its use unnecessary. Whatever benefits the method has as an index of behaviour can be achieved by using Makdisi and Seed's (1978) simplified approach. The deformations can be calculated directly using current technology, giving a global picture of dam behaviour.

Deformations from Stress Data

A more detailed picture of potential strains and deformations is obtained using Seed's semi-empirical method (Seed, 1979). The computed dynamic stresses in soil elements in the dam are converted to equivalent uniform stress cycles and are applied to laboratory specimens in consolidated states similar to corresponding elements in the dam. The resulting strains in the laboratory specimens are assigned to the corresponding elements in the dam. This procedure gives an incompatible set of strains which, however, are an indication of the potential for straining at selected locations within the dam. A further static analysis can be conducted to smooth out the strains throughout the dam.

These procedures were used to investigate the failure of the San Fernando Dam during the 1971 earthquake. Large upstream displacements were predicted to occur during the earthquake. In fact the failure occurred under static loading conditions shortly after the earthquake shaking had ceased. This illustrates that total stress analyses may be very conservative.

Therefore a major motivation for the development of more general constitutive relations has been the need to model nonlinear behaviour in terms of effective stresses and to provide reliable estimates of porewater pressures and permanent deformations under seismic loading.

Nonlinear Methods of Analysis

A hierarchy of constitutive models is available for the nonlinear dynamic response analysis of embankments to earthquake loading. The models range from the relatively simple hysteretic nonlinear models to complex elastic-kinematic hardening plasticity models. Detailed critical assessments of these models may be found in Finn (1988) and Finn (1993). This review presents the main true nonlinear procedures used in current practice and outlines their advantages and limitations.

Elastic-Plastic Methods

Plasticity theory has been a very fertile field for the development of constitutive models of soil response to cyclic loading. Twenty-six of the thirty-two constitutive models listed in the preprint volume of the Cleveland Workshop on Constitutive Modelling (Saada and Bianchini, 1987) are based on elastic-plastic theory and these, by no means, exhaust the number of available models.

It is generally recognized that elastic-plastic models of soil behaviour under cyclic loading should be based on a kinematic hardening theory of plasticity based on multi-yield surfaces or a boundary surface theory with a hardening law giving the evolution of the plastic modulus. These constitutive models are complex and incorporate some parameters not usually measured in field or laboratory testing. Soil is treated as a two phase material using coupled equations for the soil and water phases. The coupled equations and the more complex constitutive models make heavy demands on computing time (Finn, 1988, 1993).

Models based on the classical isotropic theory of plasticity such as the critical state model (Roscoe, Schofield and Wroth, 1958; Roscoe and Burland, 1968) cannot simulate the porewater pressures and permanent deformations generated by cyclic loading (Carter et al., 1982).

Research over the last ten years has been devoted to the development of more complex elastic plastic models with the potential for simulating cyclic loading effects while retaining some of the convenient features of classical plasticity theory. Detailed descriptions of these developments can be found in Pande and Zienkiewicz (1982), in a report on constitutive laws prepared for the XI International Conference on Soil Mechanics and Foundation Engineering (Murayama, 1985) and in the proceedings of the following conferences; International Conference on Numerical Methods in Geomechanics (Kawamoto and Ichikawa, 1985; Swoboda, 1988), International Symposium on Numerical Models in Geomechanics (Pande and Van Impe, 1986) and the 2nd International Conference on Constitutive Laws for Engineering Materials (Desai et al., 1987).

Typical elastic-plastic methods used in practice to evaluate the seismic response of embankment dams are DNAFLOW (Prevost, 1981), DIANA (Kawai, 1985), DSAGE (Roth, 1985) and DYNARD (Moriwaki et. al., 1988), FLAC (Itasca, 1996).

Validation studies of the elastic-plastic models suggest that, despite their theoretical generality, the quality of response predictions is strongly path dependent (Saada and Bianchini, 1987; Finn, 1988). When loading paths are similar to the stress paths used in calibrating the models, the predictions are good. As the loading path deviates from the calibration path, the prediction becomes less reliable. In particular, the usual method of calibrating these models using data from static compression and extension tests, does not seem adequate to ensure reliable estimates of dynamic response for the shear loading paths that are important in many kinds of seismic response studies. It is recommended that calibration studies of elastic-plastic models for dynamic response analysis should include appropriate cyclic loading tests, such as triaxial, torsional shear, or simple shear tests. The accuracy of pore pressure prediction in the coupled models is highly dependent on the accurate characterization of the soil properties. It is difficult to characterize the volume change characteristics which control porewater pressure development in loose sands, because it is difficult to obtain undisturbed samples representative of the field conditions. A good check on the calibration of the models is achieved by comparing the computed liquefaction resistance curves with the field liquefaction resistance curves.

DIRECT NONLINEAR ANALYSIS

The direct nonlinear approach is presented as incorporated in the program TARA-3 (Finn et al., 1986) because there is extensive experience in using this method in practice. In addition, the program has been validated in an extensive series of centrifuge tests conducted on behalf of the U.S. Nuclear Regulatory Commission (Finn, 1988).

In this model the behaviour of soil in shear is assumed to be nonlinear and hysteretic. The response of the soil to uniform all round pressure is assumed to be nonlinearly elastic and dependent on the mean normal effective stress.

The objective during analysis is to follow the stress-strain curve of the soil in shear during both loading and unloading (Fig. 3). Checks are built into the program to determine whether or not a calculated stress-strain point is on the stress-strain curve and correction forces are applied to

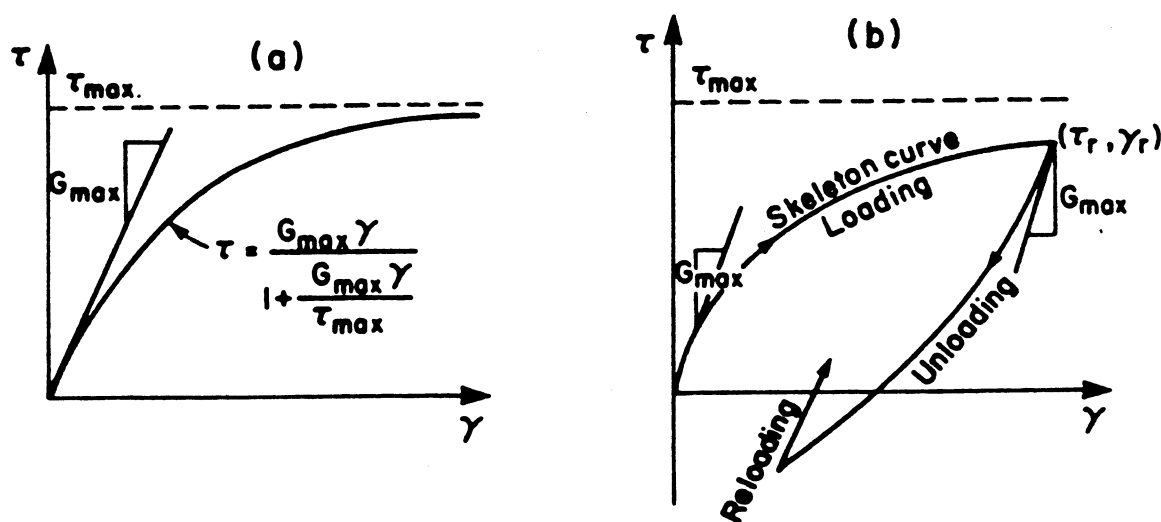


Fig. 3. Nonlinear hysteretic loading paths.

bring the point back on the curve if necessary. To simplify the computations, the stress-strain curve is assumed to be hyperbolic. This curve is defined by two parameters which are fundamental soil properties, the strength τ_{\max} and the in situ small strain shear modulus, G_{\max} (Fig. 3).

The equation for initial loading is given by (Fig. 3a),

$$\tau = f(\gamma) = \frac{G_{\max} \gamma}{1 + (G_{\max} / \tau_{\max}) |\gamma|} \quad (1)$$

and the equation for the unloading-reloading segments by eqn. (2).

$$\frac{\tau - \tau_r}{2} = \frac{G_{\max} (\gamma - \gamma_r) / 2}{1 + (G_{\max} / 2\tau_{\max}) |\gamma - \gamma_r|} \quad (2)$$

The parameters in eqn. (2) are defined in Fig. 3b.

The response of the soil to an increment in load, either static or dynamic, is controlled by the tangent shear and tangent bulk moduli appropriate to the current state of the soil. The moduli are functions of the level of effective stress and therefore excess porewater pressures must be continually updated during analysis, and their effects on the moduli taken progressively into account.

During seismic shaking two kinds of porewater pressures are generated in saturated soils; transient and residual. The residual porewater pressures are due to plastic deformations in the sand skeleton. These persist until dissipated by drainage or diffusion and therefore they exert a major influence on the strength and stiffness of the soil skeleton. These pressures are modelled in TARA-3 using either the Martin-Finn-Seed porewater pressure model (Martin et al., 1975), or the simpler version proposed by Byrne (1991).

TARA-3 conducts both static and dynamic analysis. A static analysis is first carried out to determine the stresses and strains in the dam at the end of construction. The program can simulate the gradual construction- of the dam.

Dynamic analysis of the dam starts from the static stress-strain condition in each element. Methods of dynamic analysis commonly used in practice ignore the static strains in the dam and start from the origin of the stress-strain curve in all elements, even in those which carry high shear stresses (Fig. 4). TARA-3 also allows the analysis to start from the zero stress-strain condition, if it is desired to follow current practice. The program takes into account the effects of the porewater pressures on moduli and strength during dynamic analysis and estimates continuously the additional deformations due to gravity acting on the softening soil and due to consolidation.

An example will now be given to illustrate the global picture of dam response that the general nonlinear methods of analysis can provide an engineer. The example shows clearly that these

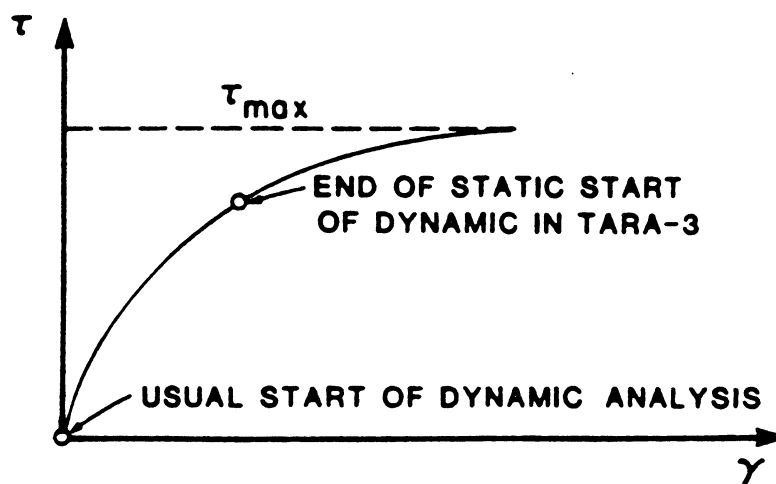


Fig. 4. Starting points for dynamic analyses.

analyses can provide very clear pictures of how the different zones in the dam behave individually and how they interact on each other. This kind of information is very useful in assessing the potential for failure or large deformations and for the planning of remedial measures. It also provides the design engineer with a good framework for exercising professional judgement.

LUKWI TAILINGS DAM, NEW GUINEA

The analysis of Lukwi Tailings Dam by TARA-3 is a good illustration of the kind of global information on dam behaviour provided by a dynamic nonlinear deformation analysis. The finite element representation of the proposed dam is shown in Fig. 5. The sloping line in the foundation is a plane between two foundation materials. Upstream to the left is a limestone with shear modulus $G = 6.4 \times 10^6$ kPa and a shear strength defined by $c' = 700$ kPa and $\phi' = 45^\circ$. The material to the right is a siltstone with a very low shearing resistance given by the parameters $c' = 0$ and $\phi' = 12^\circ$. The shear modulus is approximately $G = 2.7 \times 10^6$ kPa.

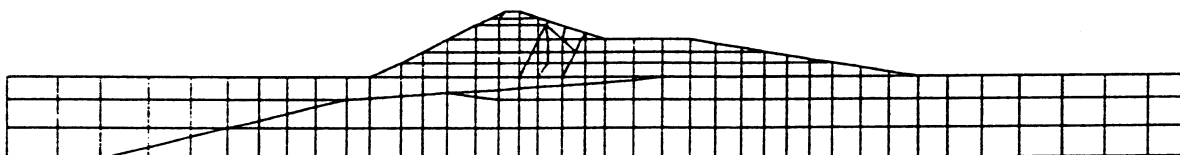


Fig. 5. Section of Lukwi Tailings Dam showing finite element mesh

The difference in strength between the foundation soils is reflected in the dam construction. The upstream slope on the limestone is steeper whereas the downstream slope on the weaker foundation is much flatter and has a large berm to ensure stability during earthquake shaking. The designers were interested in whether the berm could absorb the motions of the siltstone without large displacements or failure of the dam.

The dam was subjected to strong shaking with a peak acceleration of 0.33 g (Finn, 1988; Finn, 1991). The shear stress-shear strain response of the limestone foundation is almost elastic (Fig. 6). The response of the siltstone foundation is strongly nonlinear. The deformations increase progressively in the direction of the initial static shear stresses as shown in Fig. 7. Since the analysis starts from the initial post-construction stress-strain condition, subsequent

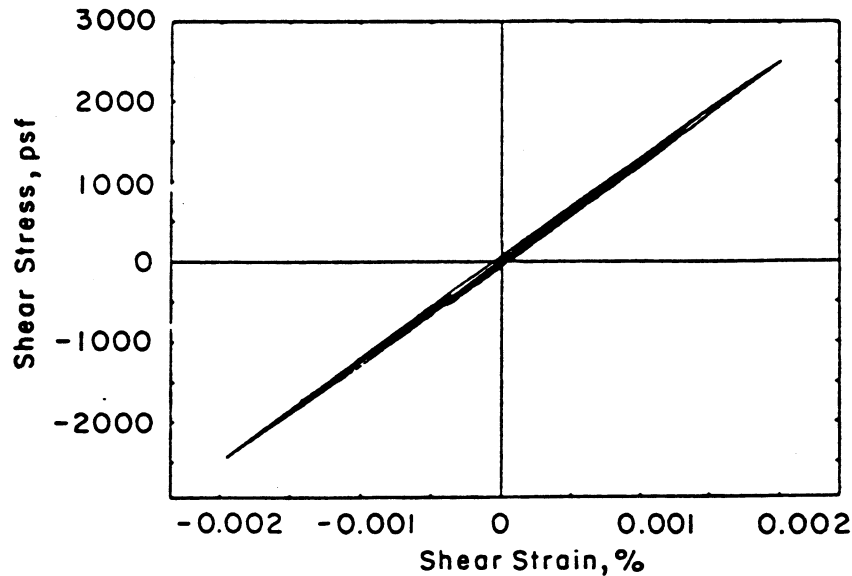


Fig. 6. Elastic response of the foundation limestone

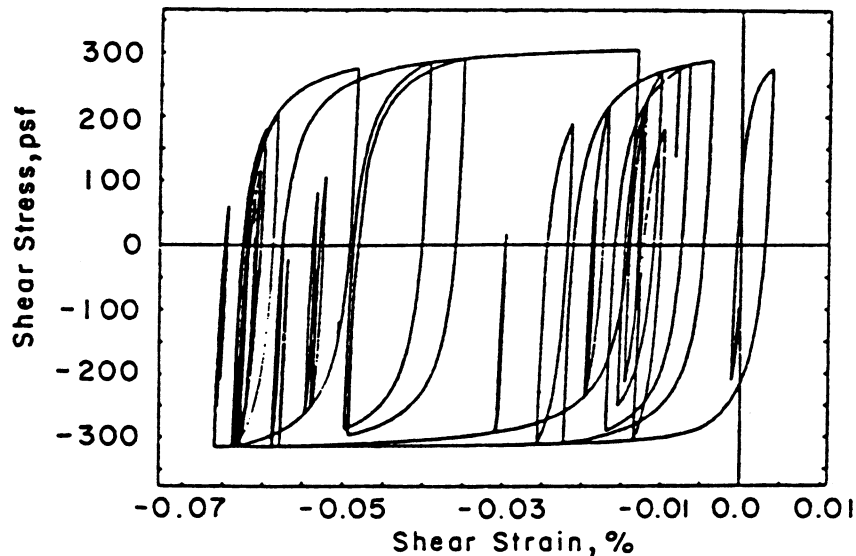


Fig. 7. Strongly nonlinear behaviour of foundation silt

large dynamic stress impulses move the response close to the highly nonlinear part of the stress-strain curve. It may be noted that the hysteretic stress-strain loops all reach the very flat part of the stress-strain curve. An element in the berm also shows strong nonlinear response with considerable hysteretic damping resulting from the large strains in the siltstone (Fig. 8).

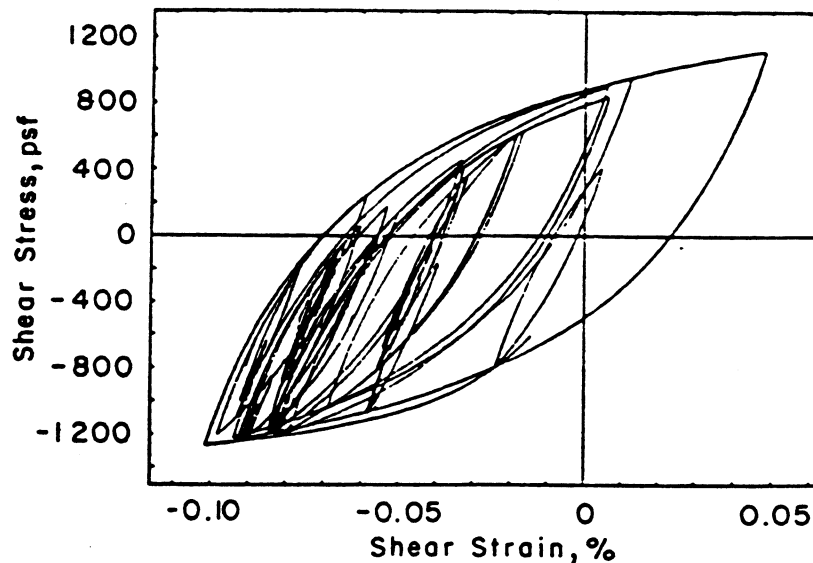


Fig. 8. Strongly hysteretic response of the downstream berm

Despite the highly nonlinear response of the foundation, the estimated maximum displacements of the dam were limited to about 250 mm because of the effect of the berm in reducing initial static stresses. The deformed shape of the central portion of the dam is shown in Fig. 9.

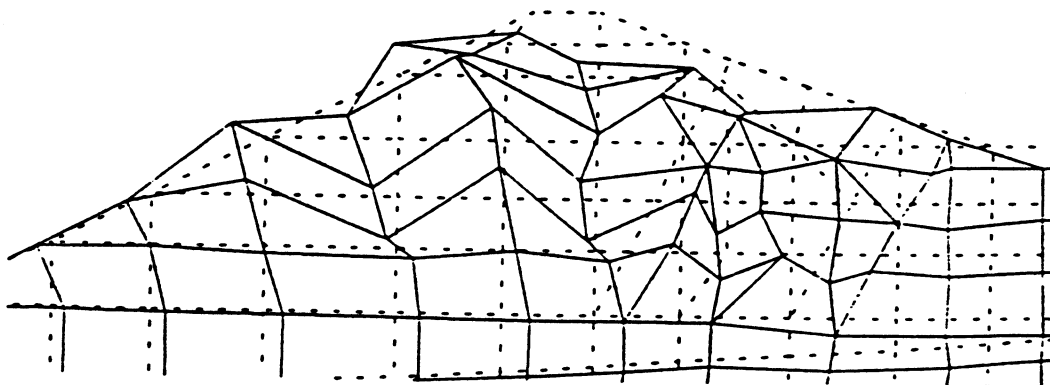


Fig. 9. Deformed post-liquefaction shape of Lukwi Tailings Dam

VALIDATION OF CONSTITUTIVE MODELS

General

Constitutive models are normally validated by using them to predict response in single element tests such as the static or cyclic triaxial test. However, single element tests do not provide an adequate validation of the predictive capability of a model because the stresses or the strains are known a priori and there is no need to solve the boundary value problem, using the constitutive model to predict the response. However, all practical applications involve the solution of the equilibrium equations and the continuity equations under a prescribed set of boundary conditions and a prescribed input. In other words, adequate model validation requires an inhomogeneous stress field.

The centrifuge tests offers the best opportunity for validating models by the solution of boundary value problems. The centrifuge models can be extensively instrumented, prepared under very controlled conditions and shaken by prescribed input. A constitutive model can be clearly tested by seeing how well it can predict the performance of the centrifuge model. Very little of this validation testing has been conducted on the models used in geotechnical earthquake engineering practice. An exception is the TARA-3 model which has been subjected to validation studies on the centrifuge over a three year period. The tests were conducted at Cambridge University in the UK on behalf of the Nuclear Regulatory Commission of the United States. Results from the validation studies have been reported by Finn (1988) and Finn (1991).

A similar validation program based on centrifuge tests was conducted in the United States under the auspices of the National Science Foundation called the VELACS program. The acronym arose from the title Verification of Liquefaction Aalysis by Centrifuge Studies. A conference on predictions made under this program was held at the University of California at Davis in October 1993 (Arulanandan and Scott, 1993).

Opportunities for quantitative validation by case histories in the field are quite limited, primarily because structures are not generally adequately instrumented. The 1987 Edgecumbe Earthquake in New Zealand $M=6.7$ provided an opportunity to see whether the TARA-3 program could model the acceleration response and the permanent deformations adequately in the Matahina Dam (Finn, 1992, 1993).

Validation of Plasticity Models

Case Western Reserve University and the Institute de Mecanique of the University of Grenoble have established a data centre for cohesionless soils that could be used to test constitutive models. The data bank contains data from over 200 tests on 3 sands, obtained from a wide variety of loading paths. One of the sands is the well-known Reid Bedford Sand from near Vicksburg, Mississippi. Half the tests were conducted in a hollow cylinder device at Case; the other half in a cubical testing device at the University of Grenoble.

A workshop was held at Case in July 1987 to publicize the data base to the geotechnical engineering profession and to test the predictive capabilities of various constitutive models (Saada and Bianchini, 1987).

Data from 24 tests were given to each of the predictors to calibrate his model. Tests were divided into four groups. Each group consisted of 3 compression test, 2 extension tests and an isotropic consolidation test with unloading/reloading. Each group of tests was conducted on each type of sand and in each of the testing devices. Having calibrated his model on the 4 test groups, the predictor was asked to predict the results of the other tests with more complicated stress paths.

Of particular interest for this review are the predictions of response to cyclic torsional shear stresses in the hollow cylinder test. This test was conducted in three stages. The specimen was loaded first in compression to a specified stress level and then 5 cycles of sinusoidally varying torsional stress with fixed amplitude were applied to the specimen. Finally, failure was induced by increasing the torsional shear stress monotonically. Axial force and deformation, volume change, torque and rotational displacement were measured during the test.

Typical results for cyclic loading tests are shown in Figs. 10-15. Dotted lines indicate the predictions and solid lines the measured responses. Results for a multi-yield surface model are shown in Fig. 10 and for three bounding surface models in Figs. 11-13. Although these models are considered among the best of their type, the predictions of the stress-strain paths are crude and the volumetric strains are over-estimated by amounts up to a factor of 2.

Two multi-mechanism models did not fare any better as exemplified by the predictions of the Matsuoka et al. (1986) model in Fig. 14 and those of the Miura-Finn (1987) model in Fig. 15.

Predictions of the stress paths and of volumetric strains in the case of the circular loading paths in the cubical device were similarly disappointing.

Poor predictions of volumetric strains lead to poor estimates of isotropic hardening and hence of the evolution of bounding surface. Also if there are difficulties in predicting reliably the response to 5 cycles of relatively slow loading in a drained test, it is difficult to have confidence in predictions of porewater pressures in undrained tests at high frequency seismic loading.

It is hard to escape the conclusion that despite their theoretical generality, the quality of the predictions of the elastic plastic methods are strongly path dependent. The predictions are good for loading paths close to those used to calibrate the models. But for paths far removed from these, such as the cyclic torsional shear and circular loading paths, the predictions were disappointing. This was also the writer's experience with the Miura-Finn multi-mechanism model. Despite obtaining a satisfactory fit with the calibration data, the predictions for cyclic loading were poor.

Path dependent calibration of model parameters is inconsistent with the theoretical generality of the elastic-plastic models. If calibration along stress paths similar to those expected in the field

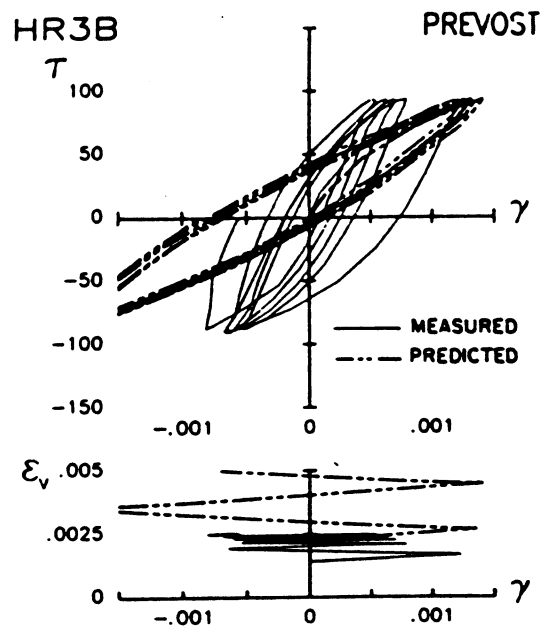


Fig. 10. Prediction of stress-strain loops and volumetric strains by Prevost.

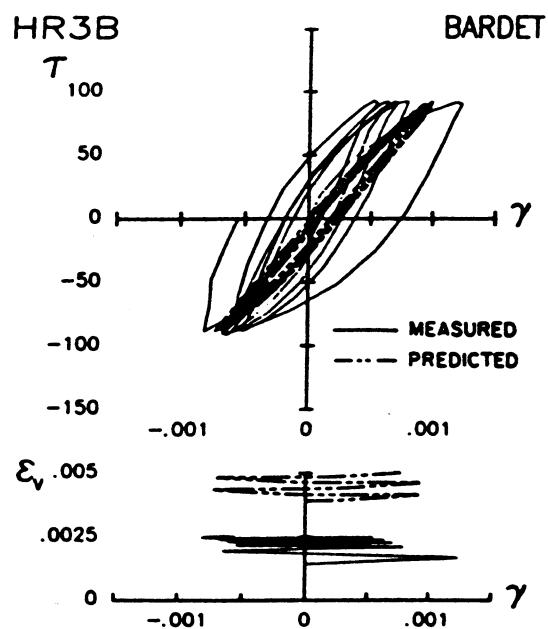


Fig. 11. Prediction of stress-strain loops and volumetric strains by Bardet.

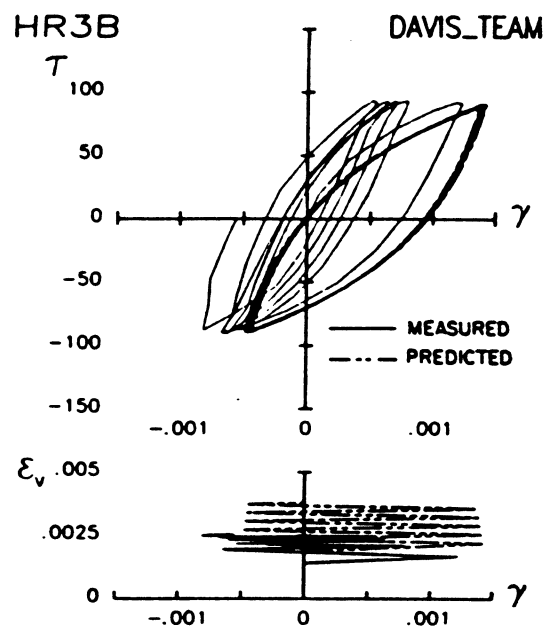


Fig. 12. Prediction of stress-strain loops and volumetric strains by Davis-Team.

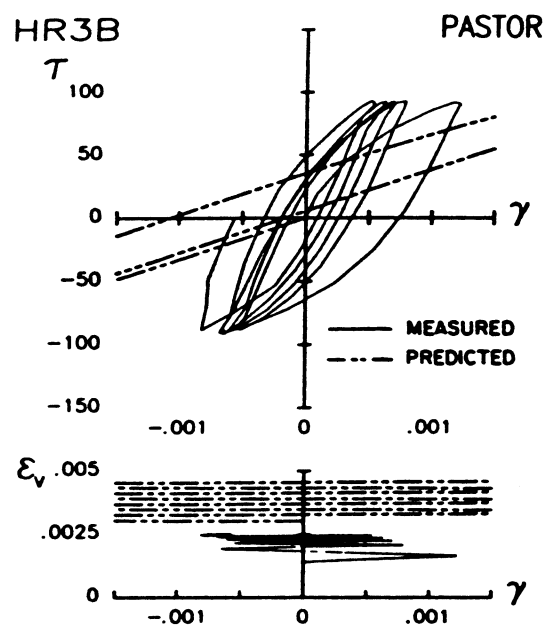


Fig. 13. Prediction of stress-strain loops and volumetric strains by Pastor.

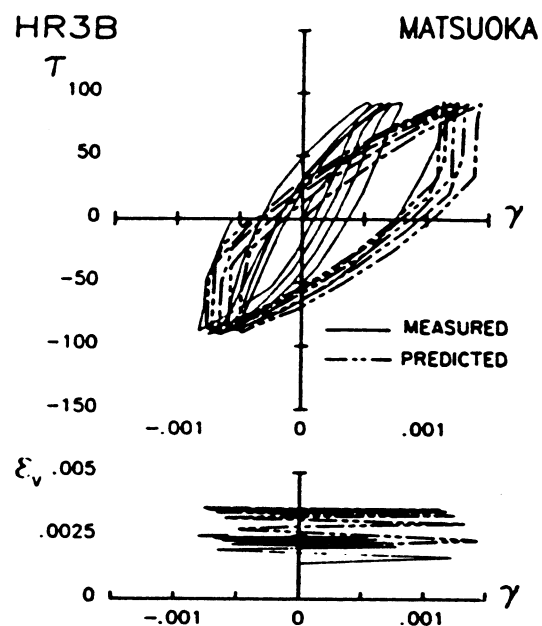


Fig. 14. Prediction of stress-strain loops and volumetric strains by Matsuoka.

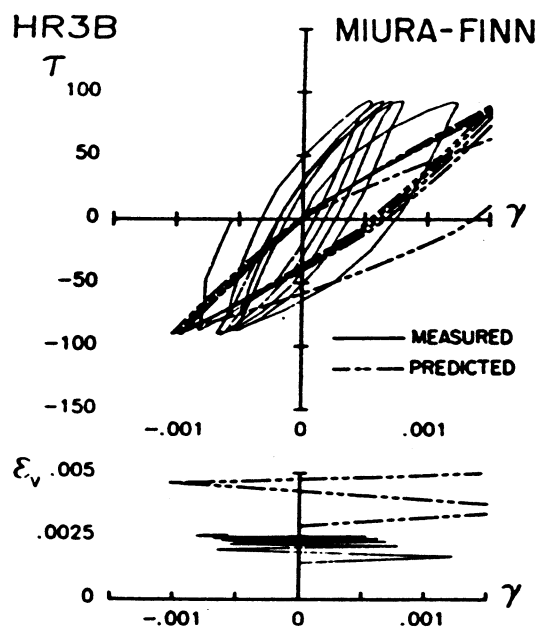


Fig. 15. Prediction of stress-strain loops and volumetric strains by Miura-Finn.

should be found necessary for satisfactory predictions of response, then much simpler models might prove adequate at much less cost.

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