

Discussion Paper No. 10

LEFT-TURN BAYS

prepared for the

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Salem, Oregon**

by the

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Discussion Paper No. 10

LEFT-TURN BAYS

DISCLAIMER

This discussion paper represents the viewpoints of the authors. Although prepared for the Oregon Department of Transportation (ODOT), they do not represent ODOT policies, practices nor procedures.

GENERAL OBJECTIVE

This and other discussion papers were prepared for the purpose of stimulating discussion among interested individuals representing a variety of agencies having an interest in Oregon's highways.

SPECIFIC OBJECTIVES

The specific objectives of this discussion paper are:

1. Provide information for discussion leading to the adoption of warrants for left-turn bays (lanes) on Oregon highways, and
2. Provide information for discussion leading to standards for queue storage and the design of left-turn bays.

ACKNOWLEDGMENTS AND CREDITS

Mr. Del Huntington is project manager for ODOT. Dr. Robert Layton, Professor of Civil Engineering at OSU is project director for the TRI. This discussion paper was prepared by Dr. Vergil G. Stover, consultant to the TRI. The content of this discussion paper is an elaboration on information which Dr. Stover has published elsewhere.

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LEFT-TURN BAYS

OVERVIEW

Introduction

The topic of left-turn bays (left-turn lanes) involves the following three issues:

1. Warrants
2. Bay Length
3. Design Details

This discussion paper deals with warrants and bay length -- including queue storage at signalized and unsignalized left-turns.

Principal Discussion Topics

1. The elements involved in a left-turn.
 2. Where left-turn bays (lanes) should be provided.
 3. The factors affecting left-turn bay length (volumes, signal/cycle length, unsignalized, number of approaches lanes, trucks/large vehicles, progression efficiency, risk of queue exceeding storage).
 4. Basic design features.
-

Major questions to be addressed and for which some conclusion needs to be reached include the following:

Major Questions to be Answered

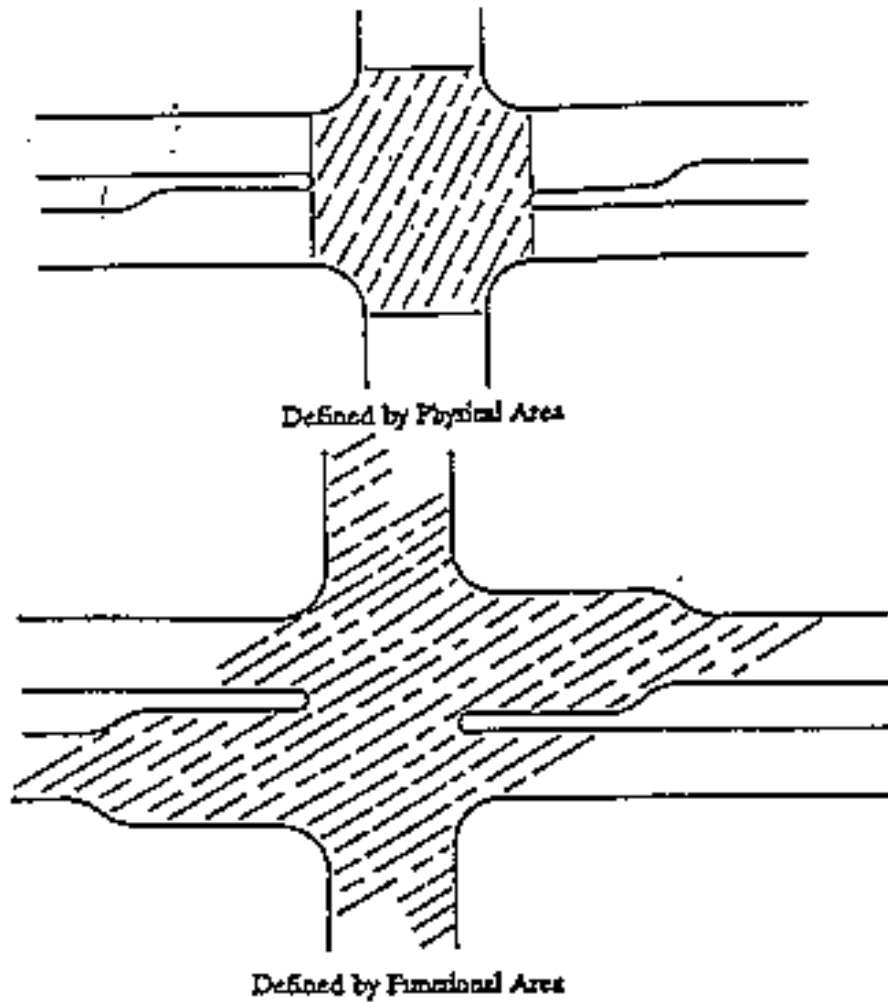
1. What warranty should be adopted for the provision of left-turn bays on Oregon highways? Should the same warrants apply to all highways?
2. What speed differential is acceptable between left-turning and through traffic? Is it reasonable to accept a higher speed differential on roadways of lower functional classification than on high functional class?
3. What criteria should be used for queue storage length? Should the criteria vary by functional classification?
4. What procedure(s) should be used to determine queue storage requirements at signalized intersections? At unsignalized intersections.
5. What minimum queue storage should be required on urban streets? On rural streets?
6. What allowance should be made for trucks?

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ELEMENTS OF THE LEFT-TURN

Functional Intersection Area

Although AASHTO (1, p. 841) suggests that the functional area of an intersection is larger than the physical area (See Figure 1) it presents no information as to the functional length.



Source: Reference (4), Figure 4-16, p. 100

Figure 1 - Boundary of Intersection

LEFT-TURN BAYS

ELEMENTS OF THE LEFT-TURN (Continued)

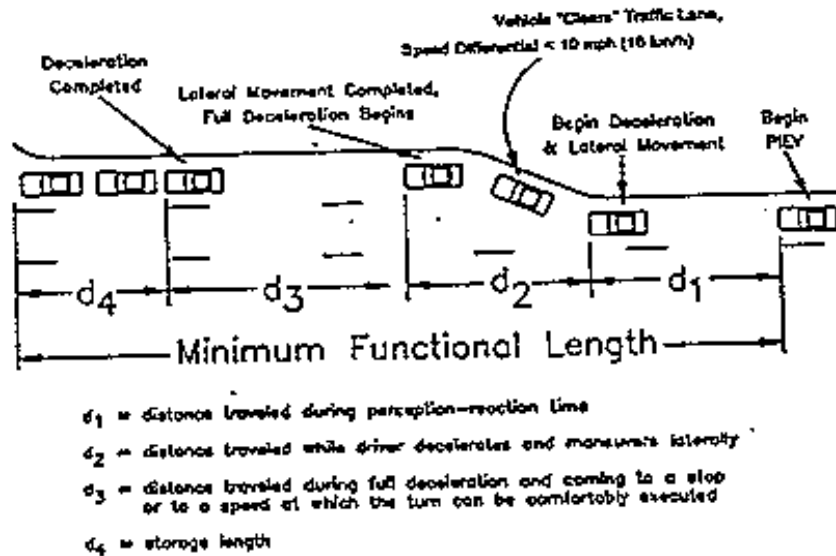
Functional Intersection Area (Continued)

Logic suggests that the functional area should be comprised of the four elements:

- d_1 = distance traveled during perception-reaction time
- d_2 = distance traveled while driver decelerates and maneuvers laterally
- d_3 = distance traveled during full deceleration and coming to a stop or to a speed at which the turn can be comfortably executed
- d_4 = storage length

The same elements apply to left-turn bays and right-turn bays in Figure 2.

As illustrated in Figure 2, the taper is included in the deceleration distance. The distance traveled during the driver's perception-reaction time adds an additional length to the total intersection maneuver distance. The turn bay should be designed so that a turning vehicle will develop a speed differential of 15 km/h (10 mph) or less at the point it clears the through traffic lane. The length of the bay should allow the vehicle to come to a comfortable stop prior to reaching the end of the expected queue in the turn bay.



Source: Reference (4)

Figure 2 - Elements of the Functional Area of an Intersection

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LEFT-TURN BAYS

ELEMENTS OF THE LEFT-TURN (Continued)

**Other Factors
Influencing
Left-Turn
Bay Length**

In addition to the maneuver plus storage lengths (i.e., distances d_1 plus d_3) the minimum length of left-turn bay may be determined by the following:

1. Length of maximum expected queue in the through traffic lanes. This is necessary in order for the left-turn to operate efficiently, especially if a "leading green arrow" is used. This control will commonly apply when there is poor progression due to closely irregularly spaced signals and/or traffic demand approaches or exceeds capacity.
2. The intersection is beyond the crest of a vertical curve and the bay taper and an initial section of the full bay width are not visible to drivers prior to reaching the crest of the vertical curve.

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LEFT-TURN BAYS

SAFETY BENEFITS OF LEFT-TURN BAYS

Introduction

Left-turn bays reduce the "shock wave" effect caused by a speed differential. Shock waves occur when left-turning vehicles are forced to decelerate in the through lanes, thereby causing through traffic to decelerate. The flow of traffic through intersections will be improved by ensuring that left-turn bays are designed with lengths sufficient to meet storage and deceleration requirements.

Safety Comparisons

Agent (33), illustrated the desirability of medians in order to provide left-turn lanes at intersections. He compared crash rates (left-turn crashes per million left-turning vehicles) at signalized and unsignalized intersections in Lexington, Kentucky. As shown in Table 1, the crash rate at unsignalized intersections the average crash rate with a left-turn bay was only 23% of that at those not having a left-turn bay. Signalized intersections with a left-turn bay experienced an average crash rate only 46% of that where a left-turn bay was not provided. The average crash rate was only 0.8 at signalized intersections with turn lanes and a separate left-turn phase. These data clearly suggest the value of a median on left-turn lanes on major roadways.

Left-turn maneuvers have been found to be involved in a disproportionately high percentage of crashes. For streets without medians or sufficient left-turn storage provisions, left-turns delay through traffic and reduce street capacity. In a 1967 report based on 21 months of crash data for 388 miles of divided urban and rural highways in North Carolina, Cribbins, et al (35), found that left-turn, rear-end crashes can be greatly reduced by construction of median area storage lanes. The authors concluded that median openings without left-turn bays are not necessarily hazardous under conditions of low-volume, wide median, and light roadside development. However, as volume and development increase, the frequency of median openings has a significant effect on increasing the potential for vehicular crashes.

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SAFETY BENEFITS OF LEFT-TURN BAYS (Continued)

Safety
Comparisons
(Continued)

**Table 1 - Comparison of Average Crash Rates⁽¹⁾ at Intersections
With and Without a Left-Turn Bay**

<u>Left-Turn Bay</u>	<u>Signalized Intersections⁽²⁾</u>	<u>Unsignalized Intersections</u>
With	3.6	1.3
Without	7.9	5.7
Comparison (With : Without)	0.46%	0.23

⁽¹⁾Crashes per million vehicle miles

⁽²⁾No Separate left-turn phase.

Source: Adopted from Reference 33)

In 1967 Wilson (37), also found a significant reduction in crashes where channelized left-turn lanes were added at unsignalized medial access points (intersections and high-volume driveways). Before-and-after studies were made at locations where the left-turn lanes were delineated using raised bars, curbs, and paint. As shown in Table 1, all three methods produced a significant reduction in crashes. Painted channelization produce a 32% reduction whereas curbed and raised bars (rumble strip) resulted in 59% and 67% reduction in crash frequency and 64% and 69% reductions in crash rates.

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SAFETY BENEFITS OF LEFT-TURN BAYS (Continued)

Safety Comparisons (Continued)

Table 2 - Before-and-After Crashes by Left-Turn Channelization at Unsignalized Access Points

Type of Channelization	Number of Projects	Condition	Million Vehicle-Miles	Total Crashes	Severity			Condition	
					Property Damages	Injury	Fatal	Day	Night
Painted	27	before	134.5	157	84	71	2	98	51
		after	134.1	106*	64	50*	2	58*	48
		% change		-32	-24	-30	0	-41	-6
Curbed	7	before	68.8	61	61	15	2	38	23
		after	77.7	25*	25*	3*	0	18*	7*
		% change		-50	-50	-80		-53	-70
Raised	6	before	64.4	95	54	40	1	67	28
		after	69.6	31*	18*	12*	1	18*	13*
		% change		-67	-67	-70	0	-73	-54

* Reduction in number of crashes is significant of 0.10 significance level using Chi Square Test
 Source: Adapted from Reference (67)

Arterial streets in Vancouver, British Columbia are spaced at approximately one kilometre intervals (38). The initial street system was constructed without left-turn bays. The city's engineering department developed a benefit/cost measure to evaluate and rank various turn bay projects. Each year the city spends about \$2.5 million to construct 6 to 10 left-turn bays. These improvements are reported to have resulted in a 20% increase in through capacity and a 25% to 50% reduction in accident rates.

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LEFT-TURN BAYS

WARRANTS FOR LEFT-TURN BAYS

Introduction

Various guidelines, standards or warrants have been developed for left-turn bays. Most notable are those proposed by Harmelink (15) and modified by ITE Committee 4A-2 (27) and the standards used by the Colorado DOT (28). Harmelink's work, and ITE, consider the turning and opposing volume. Recent research by TTI (29) considers the left-turn volume and the opposing volume as well as the advancing volume from which the left-turns are made. The guidelines given in Figure 3 are based on the following two criteria: 1) Minimizing total vehicular delay; and 2) A 0.01 (1%) probability that a left-turning vehicle will interfere with a following vehicle. The horizontal lines at 325, 350 and 375 vph directional result from the conflict between a left-turning vehicle and a following through vehicle at a maximum probability of 0.01.

TTI

Guidelines

The research by TTI considered various directional splits over a range of directional volumes. This analysis indicated that the results are not sensitive to directional splits between 50/50 and 70/30. Therefore, it is suggested that the average of the opposing and advancing volumes be used as the "directional" volume in Figure 3. This also simplifies the comparison with Harmelink and the Colorado DOT curves which are for the advancing volume only.

The TTI curves consider whether a turning left from a through lane will affect a following advancing vehicle as well as the opposing volume. The TTI curves also account for the fact that, under low advancing volumes, through vehicles can change lanes prior to slowing because of a left-turning vehicle on multilane roadways. The TTI curves show that a left-turn lane should be provided at directional volumes of 325, 350 and 375 vph or more, depending upon speed. Again, this is due to limiting the probability that a left-turning vehicle will interfere with a following advancing vehicle to 0.01 or less.

**Comparison
with
Colorado
Warrants**

When compared to the Colorado DOT warrants (Figure 4), the TTI curves are more liberal at low directional volumes (i.e., higher left-turn volumes are required). This is due to a combination of two factors. One, when the turn volume is high compared to the advancing volume, the change of a conflict with a following vehicle is small. And two, at low advancing volumes, a driver of a following vehicle has ample opportunity to change lanes to avoid a vehicle turning left from a through lane.

**Provision
of Left-Turn
Bays**

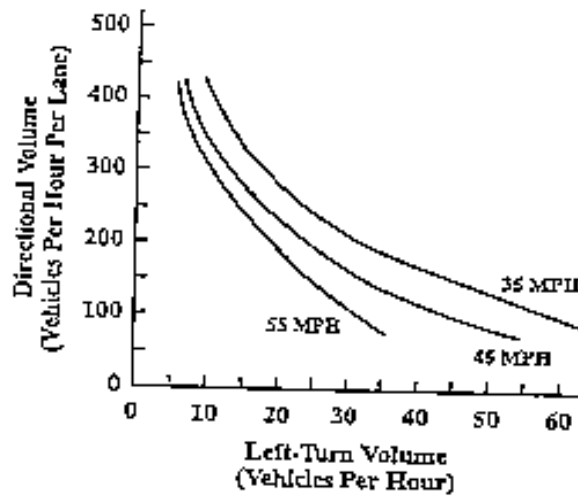
The curves given in Figure 3, or similar guidelines (Harmelink) or standards (Colorado DOT) indicate when a left-turn bay is to be

LEFT-TURN BAYS

WARRANTS FOR LEFT-TURN BAYS (Continued)

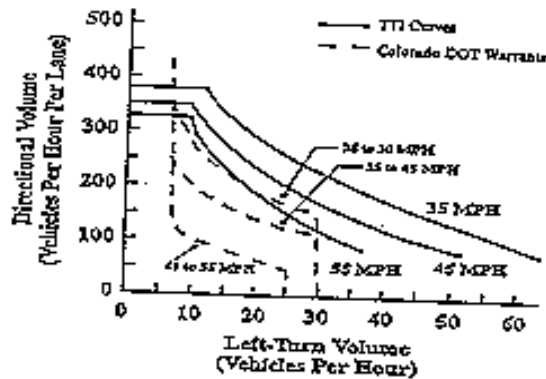
**Provision
of Left-Turn
Bays
(Continued)**

provided. Such curves are most applicable to rural areas and suburban locations where headways between vehicles are distributed in a random manner (i.e., vehicular flow is not platooned). In urban areas, left-turn bays need to be provided at all median openings, signalized and unsignalized. This is because the design hour volumes per lane in urban areas will greatly exceed the volume and even a small number of left-turning vehicles will produce high delays and a high probability of conflicts with following through vehicles. Even off-peak volumes on major urban streets commonly exceed the 325 to 375 vehicles per hour per lane "cut-off-volumes."



Source: Reference (29)

Figure 3 - TTI Guidelines for Left-turn Lanes



Source: Reference (4)

Figure 4 - Comparison of the TTI Curves and Colorado DOT Warrants

LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE

Turn Bay Length

Once it has been determined that a left-turn bay is warranted, or should be provided, the question becomes: "How long should it be?" The required physical length is the sum of the distance required for the driver to move laterally into the left-turn bay and decelerate to a stop plus the required queue storage. The distance required for the lateral movement and deceleration to a stop is addressed in Discussion Paper No. 1, "Functional Intersection Area". This section of discussion paper deals specifically with the queue storage issue.

Queue Storage Criteria

The storage length should be sufficient to have a high probability of storing the longest expected queue. As the functional class of the intersection increases, the probability of storing all arriving vehicles should increase. The storage length for a 98% probability is only about one vehicle longer than for a 95% probability. As shown in Figure 5, the expected queue length increased rapidly once the v/c ratio (coefficient of utilization) exceeds 0.75 to 0.80.

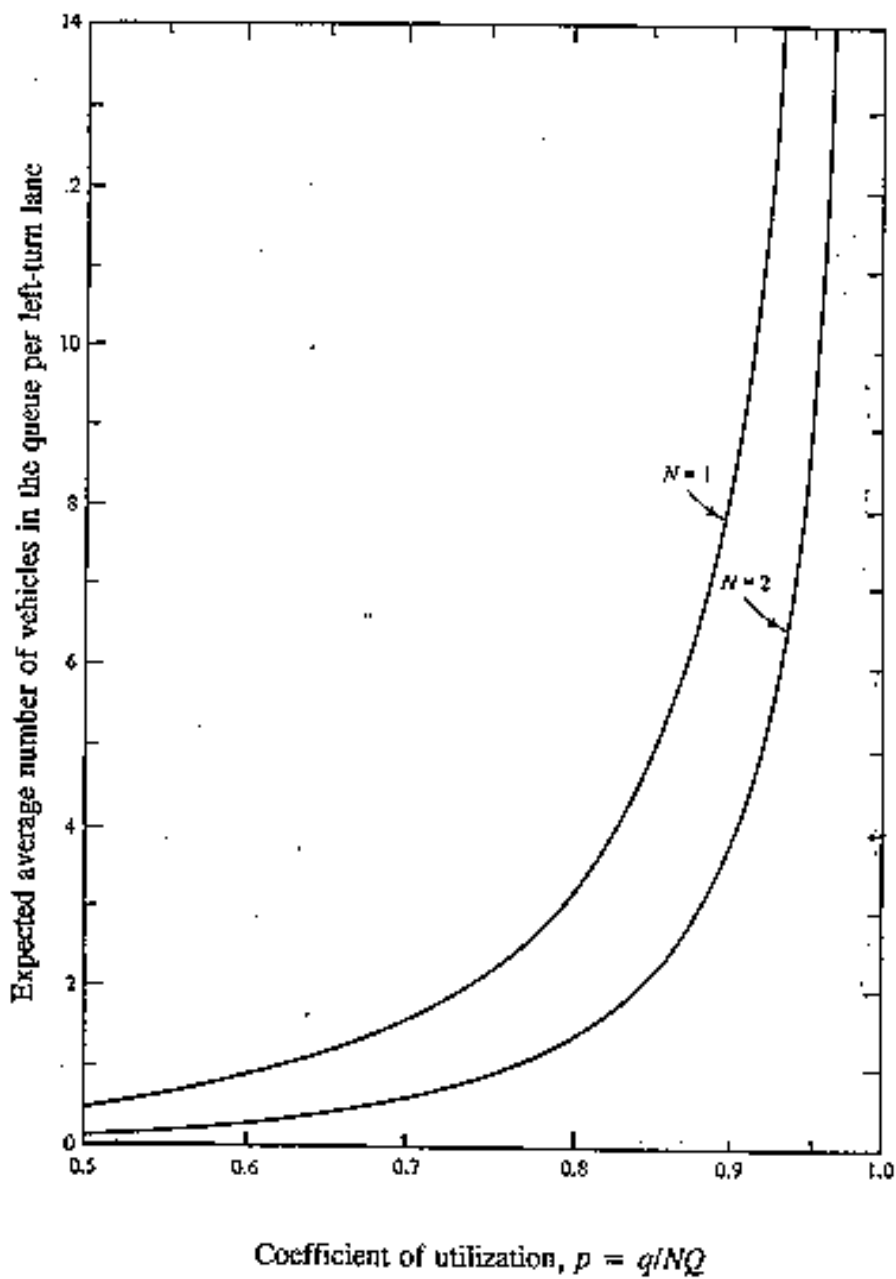
The following probabilities for storing all vehicles are offered for purposes of facilitating discussion.

<u>Intersection</u>	<u>Probability of Storing All Vehicles</u>
Major Arterial - Major Arterial	98%, all approaches
Major Arterial - Minor Arterial	98%, major arterial approaches 90%, minor arterial approaches
Major Arterial - Major Collector	98%, major arterial approaches 90%, major collection approaches
Minor Arterials - Major Collector	90%, minor arterial approaches 85%, major collector approaches
Minor Arterial - Minor Collector	90%, minor arterial approaches 80%, minor collector approaches

LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

Queue Storage
Criteria
(Continued)



Coefficient of utilization, $p = q/NQ$

Figure 5 - Average Queue Length Per Left-Turn Lane

LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

**Variations in
Left-Turn
Demand**

Left-turn volumes can not be foremost with percussion. Consequently, variations in traffic volumes and/or patterns frequently requires lengthening of the left-turn storage at a major intersection. This may necessitate the elimination of left-turns at a nearby intersection of a public street or private access.

**Determination
of Storage
Length**

The required storage for any selected probability of storing all vehicles can be determined using the queuing analysis. However, nomographs and "rules of thumb" have been developed that provide simpler solutions.

A variety of authors have presented guidelines for queue storage at signalized and unsignalized intersections. These include the following:

Signalized Intersections

- J. E. Leish, (14), a nomograph for cycle length, percent trucks and two probabilities of storage.
- Rules of Thumb, based on turn volume or on turn volume and cycle length.
- Stover, et al (5), a table for different red phases (this table needs to be expanded to reflect the use of longer red phases resulting with 120 to 180 second cycles and a greater range of cycle splits).
- J. C. and J. E. Oppenlander (34), a set of tables of required queue storage for 50%, 85% and 95% probability of storing all vehicles for a range of cycle lengths (60, 75, 90, 105, 120, 150 and 180 seconds), various volumes (up to 800 vehicles per hour per lane at 50 vph intervals) and various "effective queue times" (10 sec. intervals up to 40 sec. at 60 sec. cycle and to 120 sec. at 180 sec. cycle).
- Institutional Transportation Engineers District 7 Canada, theoretical analysis composed to observed conditions used to develop curves for queue storage versus probability of queue exceeding the average queue.

LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

**Determination
of Storage
Length
(Continued)**

Unsignalized Intersections

- M. D. Hamerlink (15), a nomograph for 4-way stops and a family of nomographs for two-way stops.
- Stover, et al (5), a table for queue storage as a function of the approach service rate (capacity). The table can be applied to four-way as well as two-way stop intersections. However, the approach service rate must first be estimated using traffic flow theory.

Additionally, queue storage can be calculated using the queuing equations given in Appendix B or using the simplified equation and Table B-1.

**Nomograph for
Storage at
Signalized
Intersections**

The storage for a single-lane left-turn lane at a signalized intersection can be estimated by queuing analysis or by the nomograph shown in Figure 6. This nomograph is based upon queuing analysis which assumes, 1) random (Poisson) arrivals in the left-turn bay, 2) negative exponential service times which are a function of the cycle length, 3) a weighted average "length of vehicle for different percent trucks and 4) two selected probabilities (95% and 90%) that the longest queue can be stored (i.e., the storage will be inadequate 5% and 10% of the time).

Based on Figure 6, with a left-turn volume of 240 vehicles per hour (vph), a 90-second cycle, and 0% trucks, a storage lengths of about 65 metres (220 feet) is required for desirable conditions and about 50 metres (160 feet) for a minimum. These storage lengths would accommodate 9 passenger cars for the desirable conditions and about 6 for the minimum.

LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

Nomograph for
Storage at
Signalized
Intersections
(Continued)

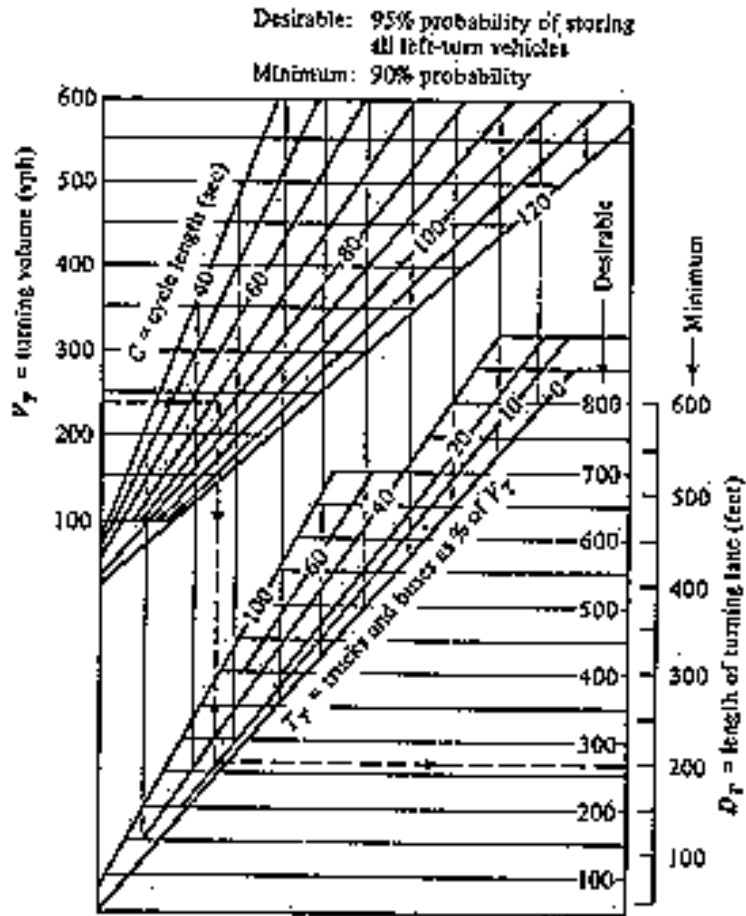


Figure 6 - Storage at Signalized Intersections (14)

Rule of Thumb
Methods for
Signalized
Intersections

The following "rules of thumb" have also been used for left-turn storage at signalized intersections.

Rules of Thumb

- #1 Storage Length = 1 foot for each vehicle per hour (vph) turning left during peak hour.
- #2 Storage Length = (vph/number of cycles per hr) x (t) x (25 ft).

where t is a variable, the value of which is selected based on the minimum acceptable likelihood that the storage length will be adequate to store the longest expected queue. Suggested values are:

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LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

Rule of Thumb Methods for Signalized Intersections (Continued)	<u>Minimum t</u>	<u>Approximate Probability of Storing all Vehicles</u>
	2.0	>0.98
	1.85	0.98
	1.75	0.95

**Adjustment
for Trucks**

The length of 25 feet (7.6 metres) is an average distance, front bumper-to-bumper of a queue. If the queue is comprised mostly of passenger cars, this distance provides for an average distance between vehicles of about one-half car length. If more than 1% trucks are expected, the average length, including gap, per vehicle must be increased as follows:

<u>Percent Trucks</u>	<u>Average Queue Storage Length</u>
<2%	7.6 m (25 ft)
5%	2.7 m (27 ft)
10%	9.0 m (29 ft)

**Examples of
Signalized
Queue Storage
Required**

Example A

300 vph left-turns
60 sec cycle (60 cycles per hour)
0% trucks

Rule of Thumb #1
(300 vph) (1 ft/veh) = 300 ft.

Rule of Thumb #2
(300 vph/60 cycles per hr) (2) (25) = 250 ft.

Nomograph

- desirable = 250 ft
- minimum = 180 ft

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LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

**Examples of
Signalized
Queue Storage
Required
(Continued)**

Example B

300 vph left-turns
120 sec cycle (30 cycles per hour)
0% trucks

Rule of Thumb #1

$$(300 \text{ vph}) (1 \text{ ft/veh}) = 300 \text{ ft}$$

Rule of Thumb #2

$$(300 \text{ vph}/30 \text{ cycles per hr}) (2) (25) = 500 \text{ ft}$$

Nomograph

- desirable = 500 ft
- minimum = 375 ft

Example C

300 vph left-turns
60 sec cycle (60 cycles per hour)
10% trucks

Rule of Thumb #1

300 ft, as before

Rule of Thumb #2

$$(300 \text{ vph}/60 \text{ cycles per hr}) (2) (30 \text{ ft}) = 300 \text{ ft}$$

Nomograph

- desirable = 270 ft
- minimum = 200 ft

Example D

300 vph left-turn
120 sec cycle (30 cycles per hr)
10% trucks

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LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

Examples of Signalized Queue Storage Required (Continued)

Rule of Thumb #1
300 ft, as before

Rule of Thumb #2
(300 vph/30 cycles per hr) (2) (30) = 580 ft

Nomograph

- desirable = 540 ft
 - minimum = 400 ft
-

Comparison

Comparison of the above example calculators reveals that:

1. Rule of Thumb #2 and the nomograph, desirable value, produce very close to the same results at both short and long cycle lengths
2. Rule of Thumb #1 over estimates queue storage for a 60 second cycle and very seriously under estimates the required storage for a 120 second cycle.

Application of Rules of Thumb

It is suggested that Rule of Thumb #2 offers a simple process for routine estimation of queue storage requirements at signalized intersections over a range of cycle lengths. It is easy to apply and there is no need to refer to tables, figures, or complex equations.

Canadian Capacity Manual

The Canadian Highway Capacity Manual, Canadian HCM, (39, pp. 67-69) contains a procedure for estimating the maximum queue length. It was developed to apply to the lanes where queues may impede the operation of other lanes, such as left-turn bays or four conditions where queue spillback may block an up-stream intersection, or access drive. It is also presumably applicable to queuing in left-turn and/or right-turn bays as well. Queue lengths are in terms of passenger car units (pcu's). This procedure considers the probability that a given queue length (number of vehicles) will be exceeded. the process is akin to Rule of Thumb #2 using a variable in place of the constant value of 2.0.

The graph presented in Figure 7 was developed to facilitate design. Use of the figure is illustrated by the following:

- 300 left-turn vehicles per hour

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ESTIMATING REQUIRED STORAGE (Continued)

Canadian
Capacity
Manual
(Continued)

- 120 sec. cycle

$$\text{This: } Q = q (c/3600) = 300 (120/3600) = 9.0$$

Where: Q = average vehicles arriving per cycle
 q = the average arrival rate (pcu/h)
 c = cycle length
 $c/3600$ = number of cycles per hour

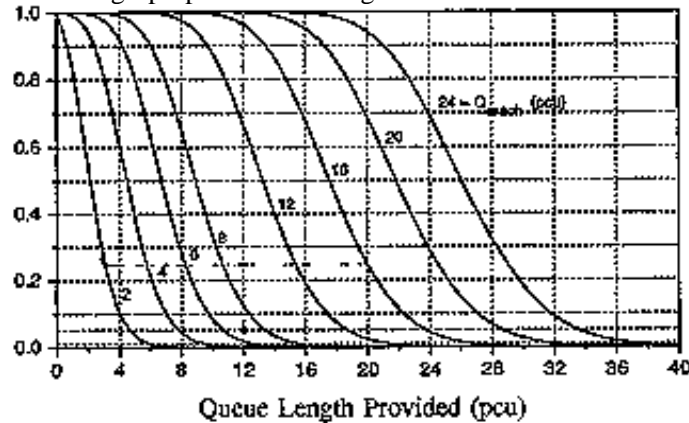
For a probability that the longest queue will be exceeded less than 5% of the time:
 $P(Q > Q) = 0.05$. Eq. 1

Where $P(Q_i > Q) =$ the probability of any given queue length (Q) will be exceeded by a longer queue (Q).

Interpolating using Figure 7, a design queue length of 15 pcu's should be provided.

If all vehicles to be stored are autos and 25 ft. per vehicle (front bumper-to-front bumper is assumed, the storage length is $15 \times 25 = 375$ ft. (excluding deceleration distance).

It will be observed that this amounts to a queuing factor of 1.67 (15/9) as opposed to the value of 2.0 used in Rule of Thumb #2. It is also to be noted that it gives a shorter storage length than the generally accepted application of queuing theory such as the nomograph presented in Figure 7.



Source: Adopted from Reference (39, p. 69)

Figure 7 - Maximum Probable Queue Length (Reach)

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LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

Undersaturated Conditions

The average queue at the end of the red phase (assuming no vehicles are in the queue at the end of the green plus yellow) is given by:

$$Q_{red} = q(c-g)/3600 \quad \text{Eq. 2}$$

Where: Q_{red} = average queue length of the end at the red phase (pcu)

q = arrival flow rate (pcu/h)

c = cycle length in seconds

g = effective green phase in seconds

However, average queue length is more critical than the end of red queue because it is an indication of how far upstream a queue may extend. This average is given by:

$$Q_{avg} = [q(c-g)] / [(3600)(q/s)] \quad \text{Eq. 3}$$

Where: Q_{avg} = average queue length

s = saturation flow rate (pcu/h)

And the other variables are the same as defined as above.

Comments

It should be recognized that the above procedures (the nomograph, rules of thumb and the Canadian HCM procedure) assume both of the following:

1. All vehicles arriving during a cycle join the left-turn queue, and
2. All vehicles in a queue clear on the following green phase (i.e., there is no queue carryover from cycle cycle).

Thus, when saturated conditions are encountered (queue carryover occurs), the storage indicated by these methods will be inadequate. Also, under unsaturated conditions, shorter queues will occur. Excellent traffic signal progression and protected/permissive left-turn or fully permissive left-turn signal operation may also result in shorter maximum queue lengths. As traffic flow rates approach the saturation flow rate, the unsaturated model approaches the saturated model (i.e., g/s approaches 1.0).

LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

Congested Conditions

During periods of traffic congestion (arriving traffic flow rate exceeds saturation flow rate), queue carryover from cycle-to-cycle will occur. Under these conditions the maximum queue length may be estimated by:

$$Q_{sat} = [t (q-c)] + Q_{avg} \quad \text{Eq. 4}$$

Where: Q_{sat} = the maximum queue length during the congested period (pcu)
 t = the length of the congested (oversaturated) period in minutes
 q = arrival flow rate (pcu/h)
 c = capacity (pcu/h)
 Q_{avg} = average queue length (pcu), Equation 3

Figure 8 illustrates the concept of queue carryover under oversaturated conditions.

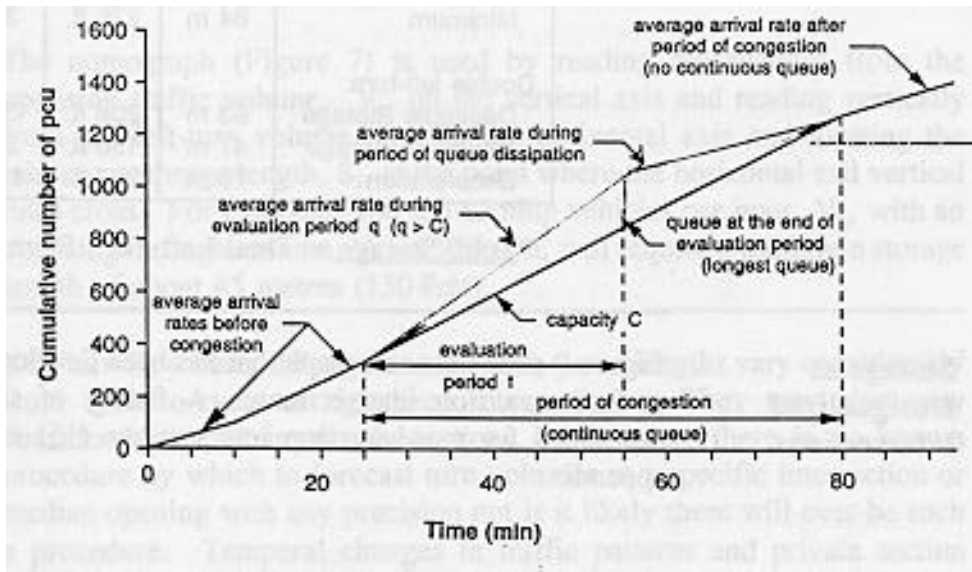


Figure 8 - Schematic Illustration of Queuing for Oversaturated Conditions

LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

Storage Length for Dual-Left Turns

The storage for a dual left-turn lane at a signalized intersection can be estimated by queuing analysis, or by the nomograph in Figure 2. The storage length is estimated for a dual left-turn bay by dividing this storage length by 1.8. This practice is suggested even though recent research (10) has shown that the saturation flow rate for a dual left-turn bay is about the same as for two through traffic lanes. The use of the value 1.8 recognized that the left-turn traffic is not equally distributed between the two turn lanes. In usual cases, the imbalance between dual turn lanes may be much greater. Example calculations are given in Table 3.

Table 3 - Example Calculation for Dual Left-Turn Bay

Condition	Peak		Off Peak	
	<u>SI</u>	<u>U.S.</u>	<u>SI</u>	<u>U.S.</u>
Left-turn volume, vph	200	200	100	100
Cycle length, sec	120	120	60	60
Speed, %	56	35	72	45
Trucks %	<1	<1	5	5
Total Storage				
Desirable	114 m	375 ft.	53 m	175 ft.
Minimum	84 m	275 ft.	38 m	125 ft.
Double left-turn:				
Desirable Storage	63 m	208 ft.	30 m	97 ft.
Minimum Storage	47 m	153 ft.	21 m	69 ft.
Deceleration	76 m	250 ft.	130 m	425 ft.

$$\frac{\text{*Total Storage}}{1.8} = \text{Dual Left-Turn Storage Length}$$

Storage at Unsignalized Intersections

Figure 9 shows a nomograph that has been developed for left-turn storage at four-way stop intersections. A family of similar nomograph was developed for two-way stop intersections (15) and are included in the appendix.

LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

Storage at Unsignalized Intersections (Continued)

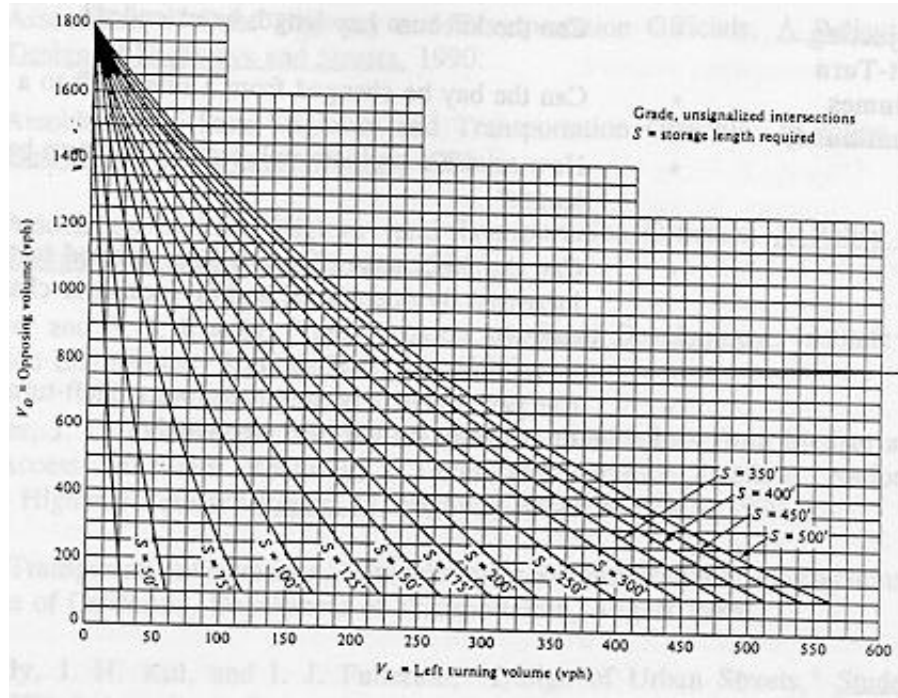


Figure 9 - Storage for Unsignalized Four-Way Stop Intersections (15)

The nomograph (Figure 7) is used by reading horizontally from the opposing traffic volume, V_o , on the vertical axis and reading vertically from the left-turn volume, V_L , on the horizontal axis and locating the minimum storage length, S_i , at the point where the horizontal and vertical lines cross. For example, 100 left-turning vehicles per hour, V_L , with an opposing through volume, V_o , of 950 vph, will require a minimum storage length of about 45 metres (150 feet).

Projecting Left-Turn Volumes

Left-turn flow rates and, in turn, left-turn queue lengths vary considerably from cycle-to-cycle at signalized intersections. They may also vary considerably at unsignalized locations. Moreover, there is no known procedure by which to forecast turn volumes at a specific intersection or median opening with any precision not is it likely there will ever be such a procedure. Temporal changes in traffic patterns and private section development decisions will continue to result in changing volumes at individual left-turn locations. Therefore, flexibility to adjust to unknown future conditions needs to be considered when designing each left-turn bay. Consideration include the following:

LEFT-TURN BAYS

ESTIMATING REQUIRED STORAGE (Continued)

**Projecting
Left-Turn
Volumes
(Continued)**

- Can the left-turn bay length be extended?
- Can the bay be changed from a single left to a dual left?
- How severe a problem will result if the turn bay is of inadequate length?
- Can the percentage of green time devoted to the major street be increased by operational and/or geometric changes on the cross-street?
- Can permissive or permissive/protected left-turns be allowed in lieu of left-turns on left-turn arrow only?

Discussion Paper No. 10

LEFT-TURN BAYS

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LEFT-TURN BAYS

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APPENDIX A

Curves for Left-Turn Storage at Unsignalized Intersections

Source: Harmelink, Reference (15)

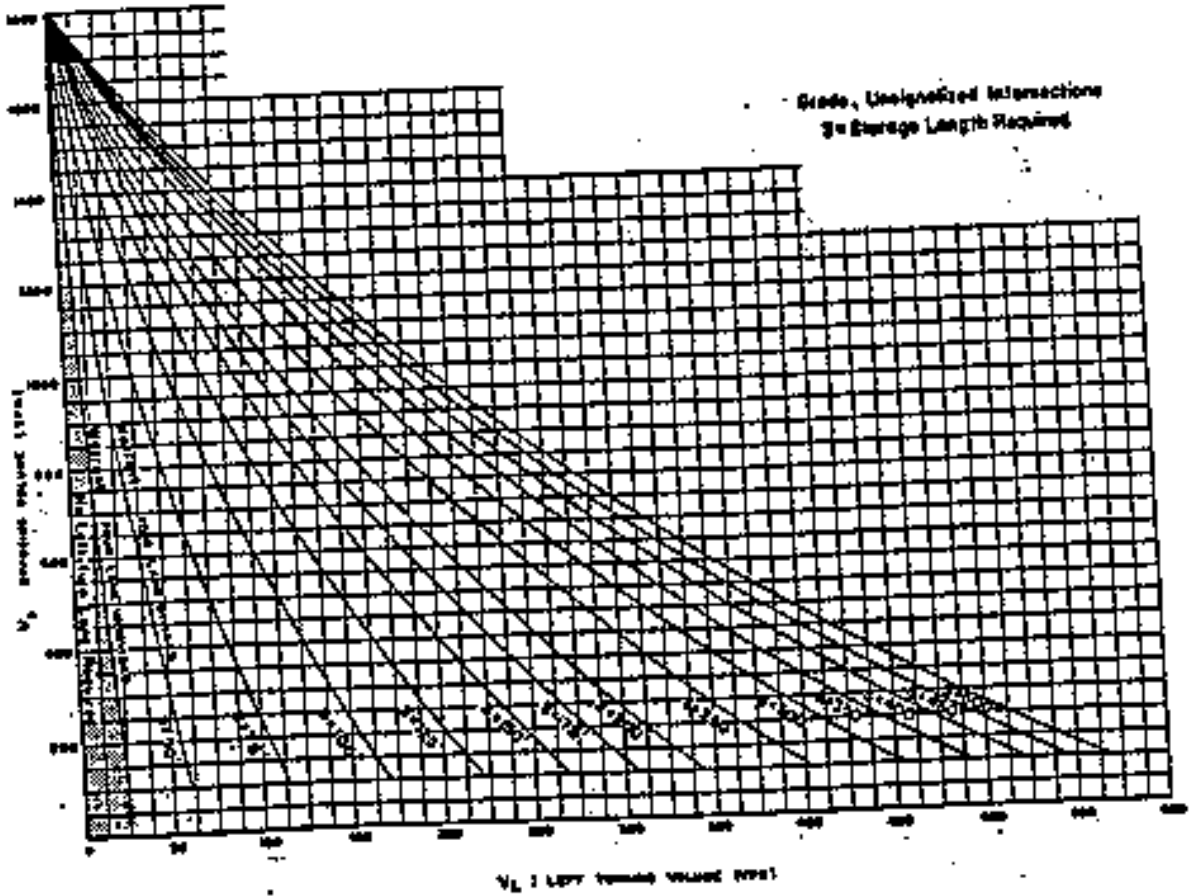


Figure 1. Warrant for left-turn storage lanes on four-lane highways.

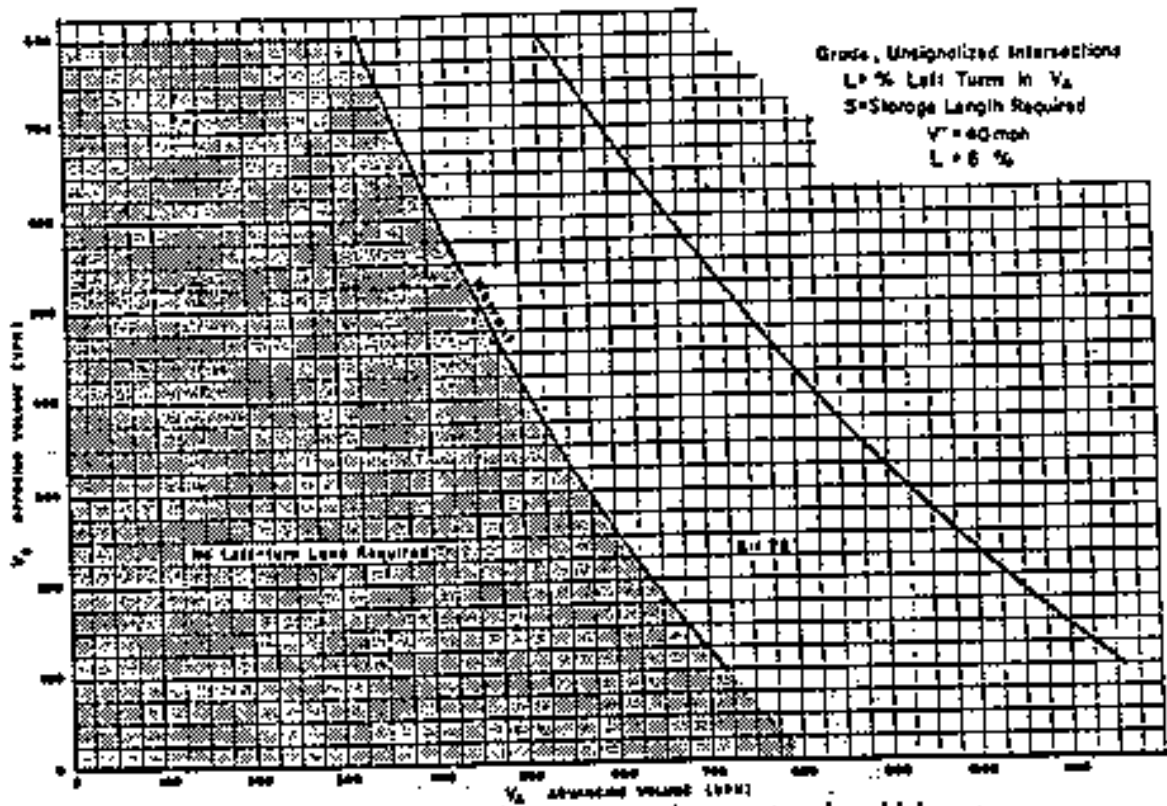


Figure 2. Warrant for left-turn storage lanes on two-lane highways.

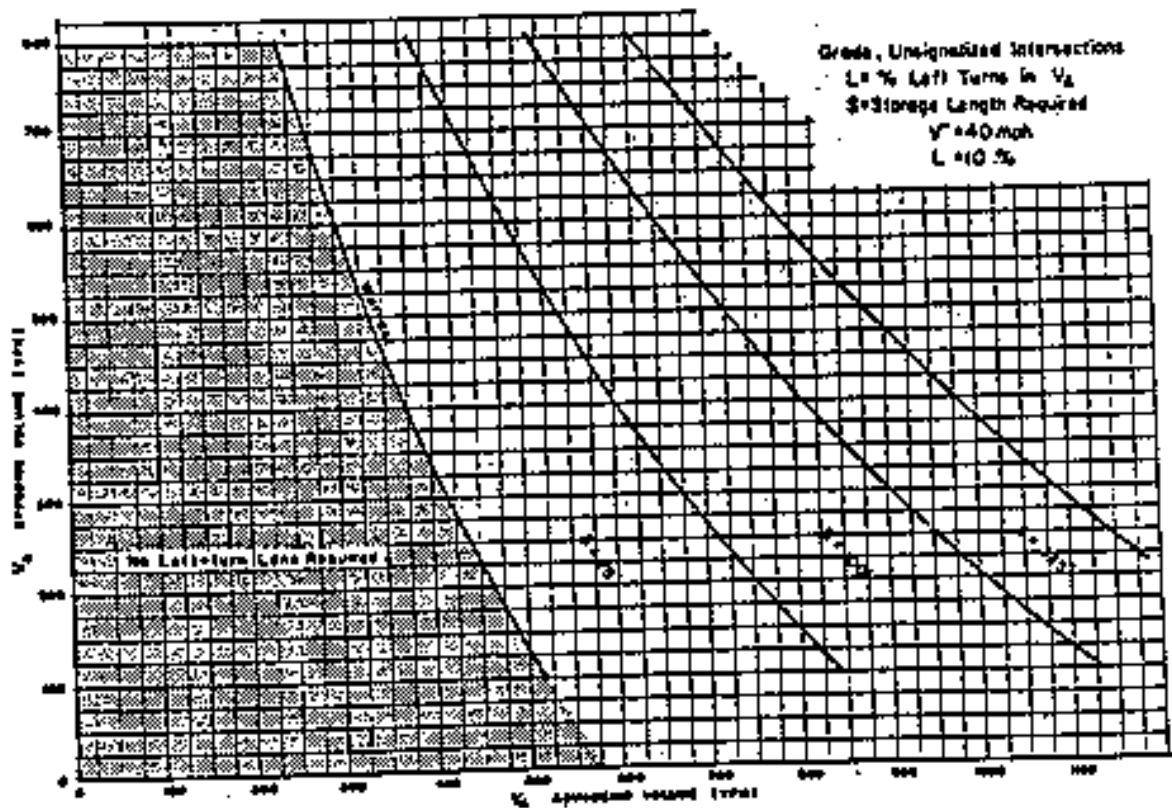


Figure 3. Warrant for left-turn storage lanes on two-lane highways.

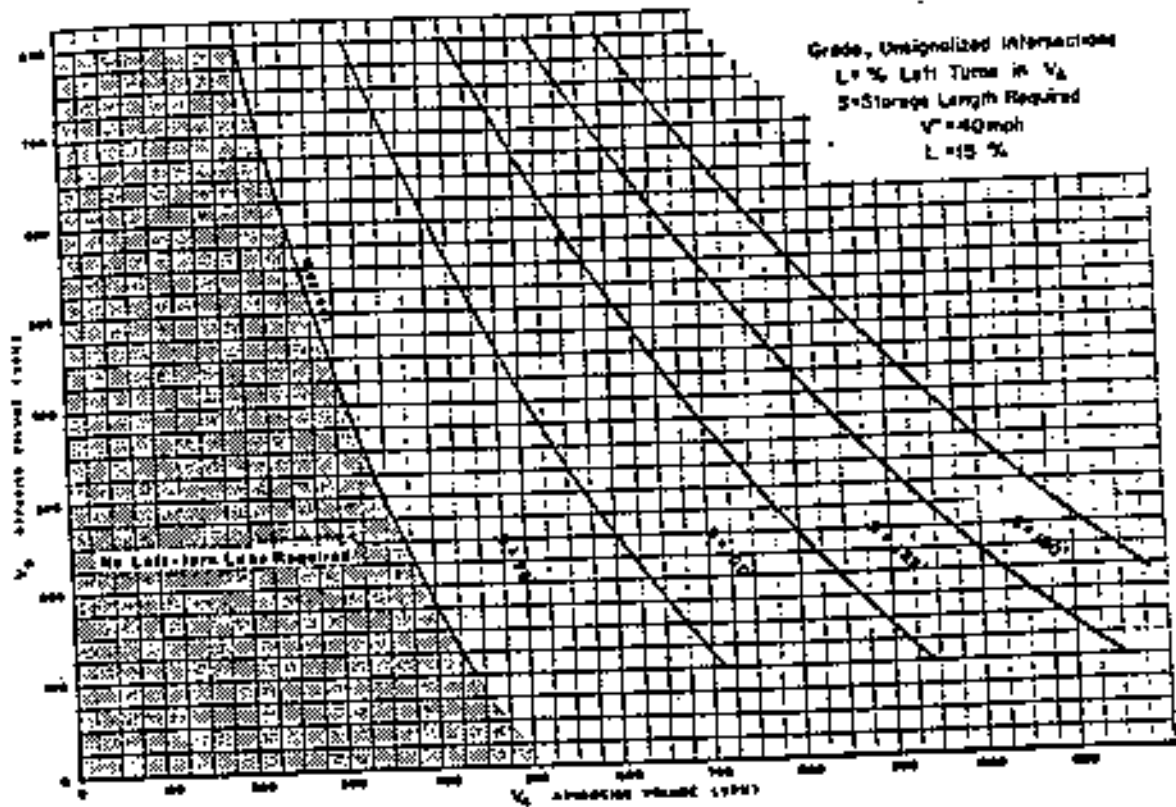


Figure 4. Warrant for left-turn storage lanes on two-lane highways.

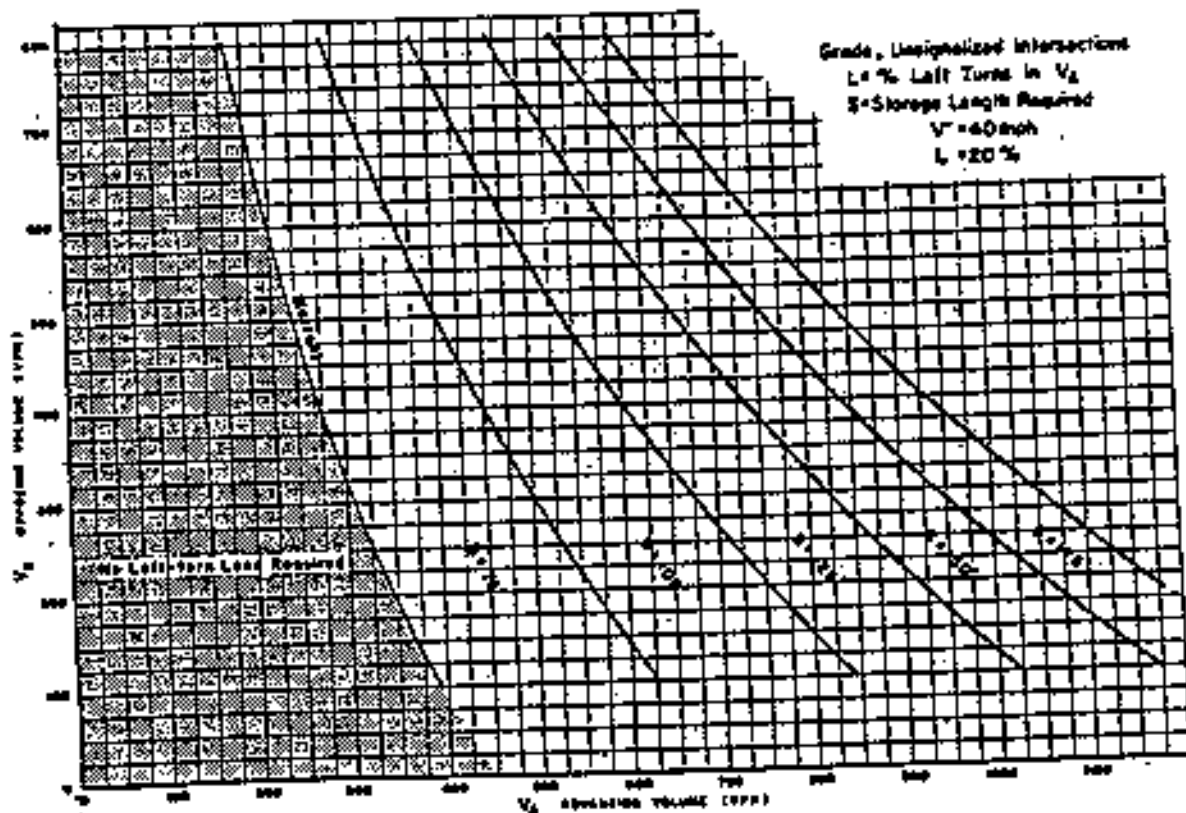


Figure 5. Warrant for left-turn storage lanes on two-lane highways.

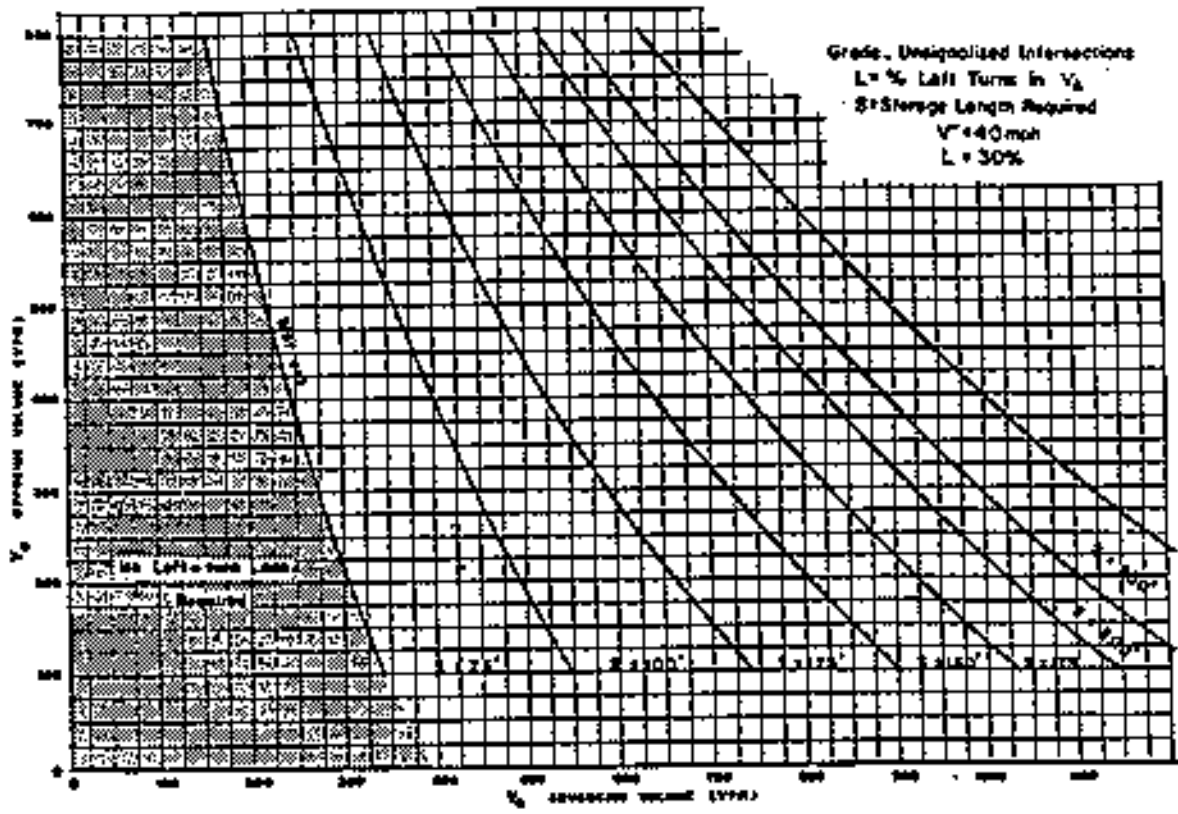


Figure 6. Warrant for left-turn storage lanes on two-lane highways.

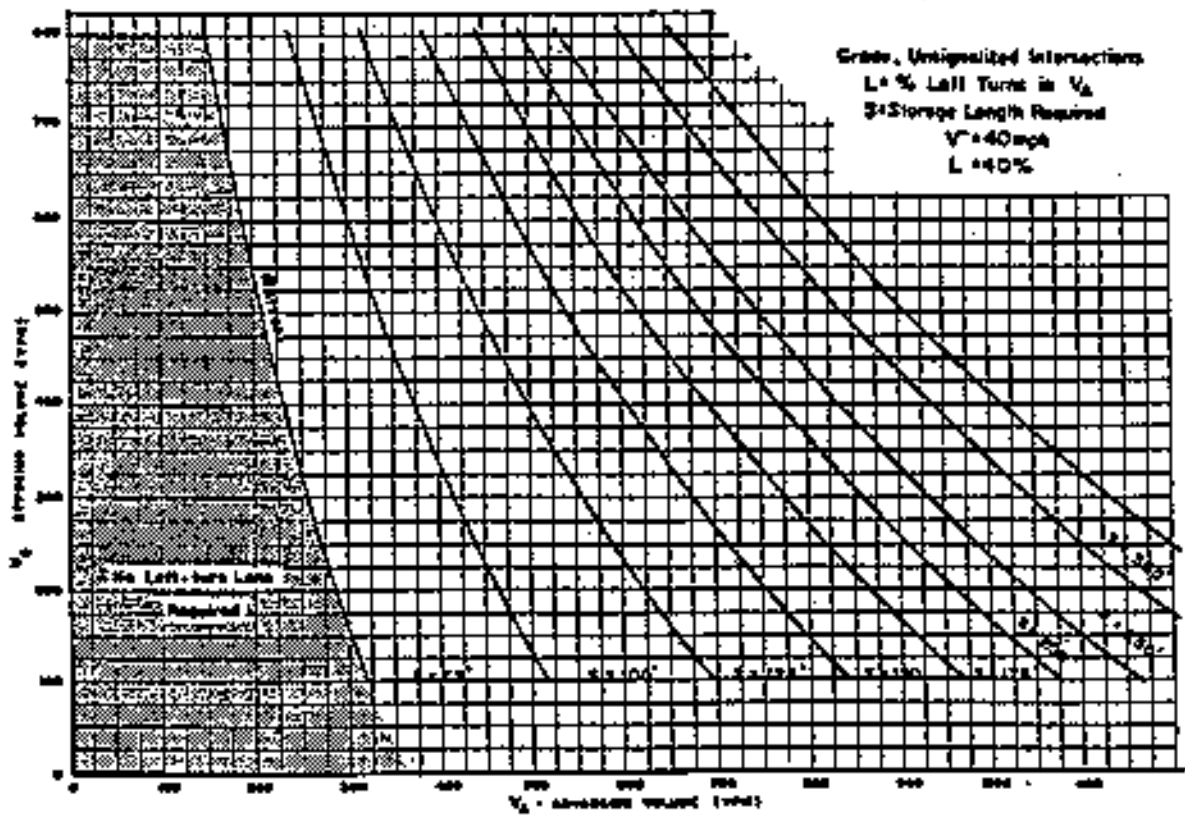


Figure 7. Warrant for left-turn storage lanes on two-lane highways.

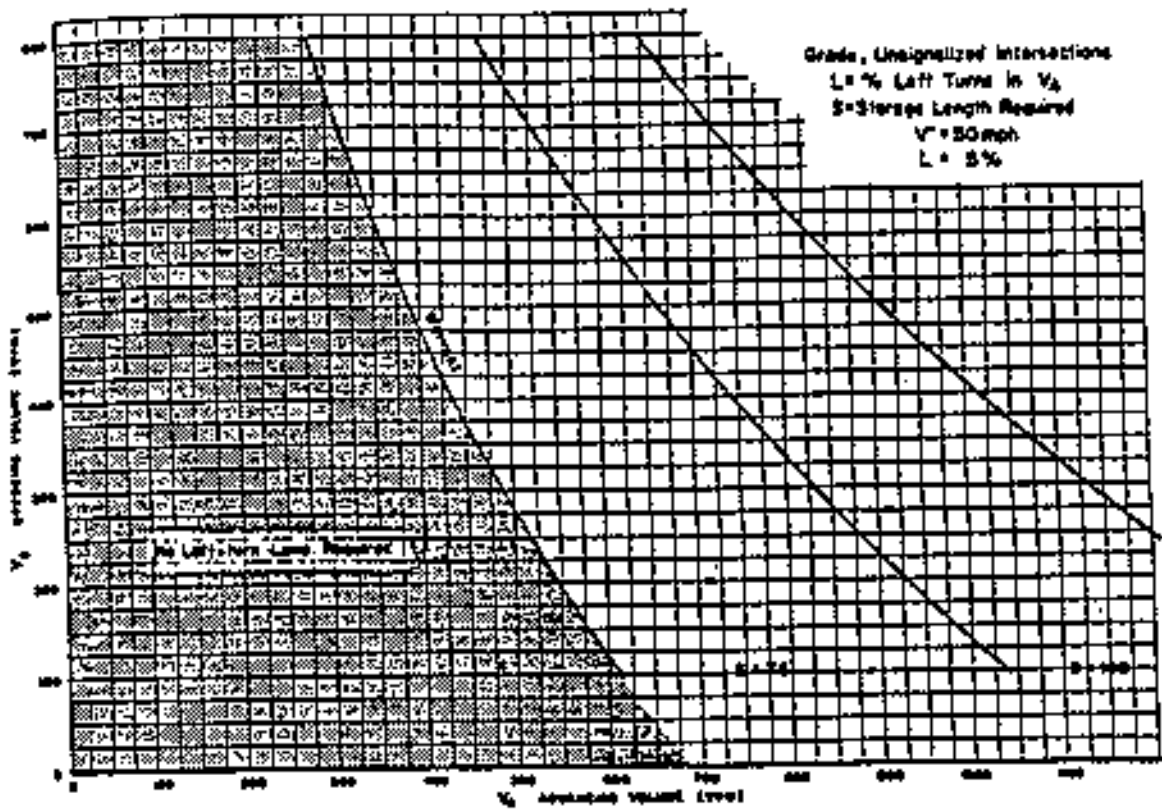


Figure 8. Warrant for left-turn storage lanes on two-lane highways.

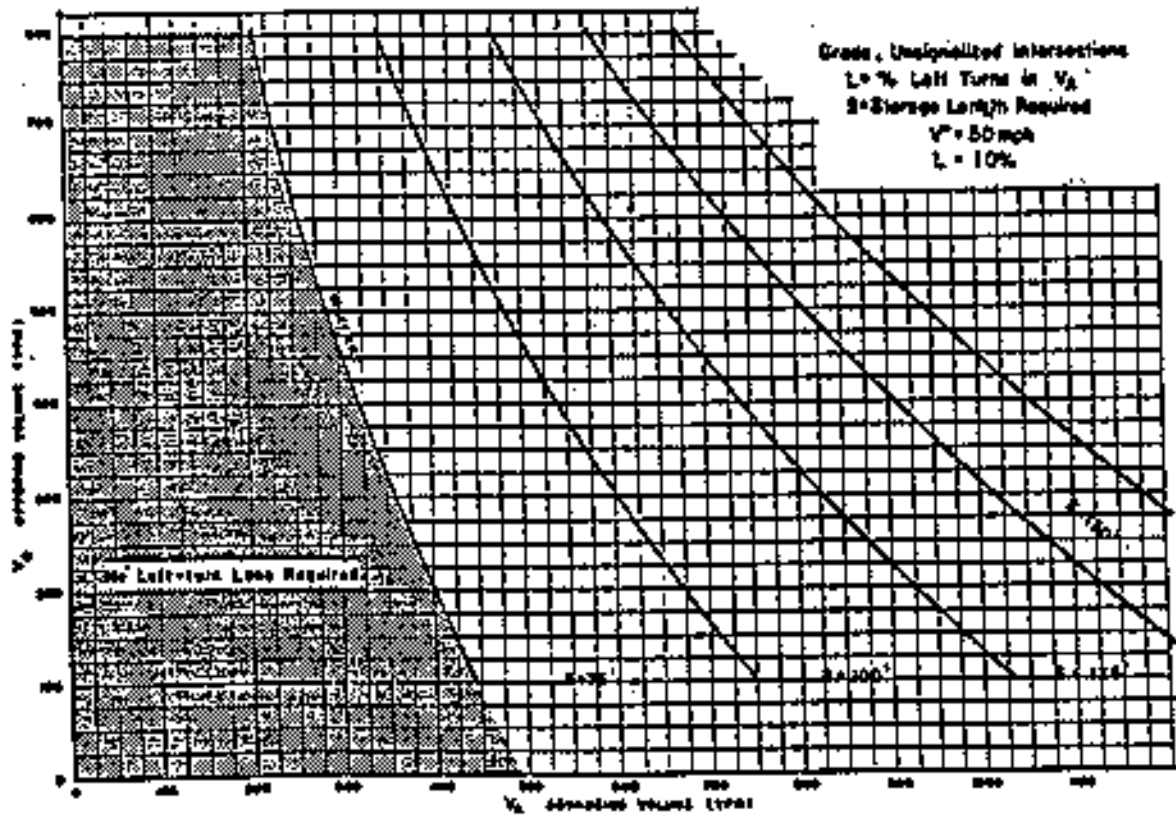


Figure 9. Warrant for left-turn storage lanes on two-lane highways.

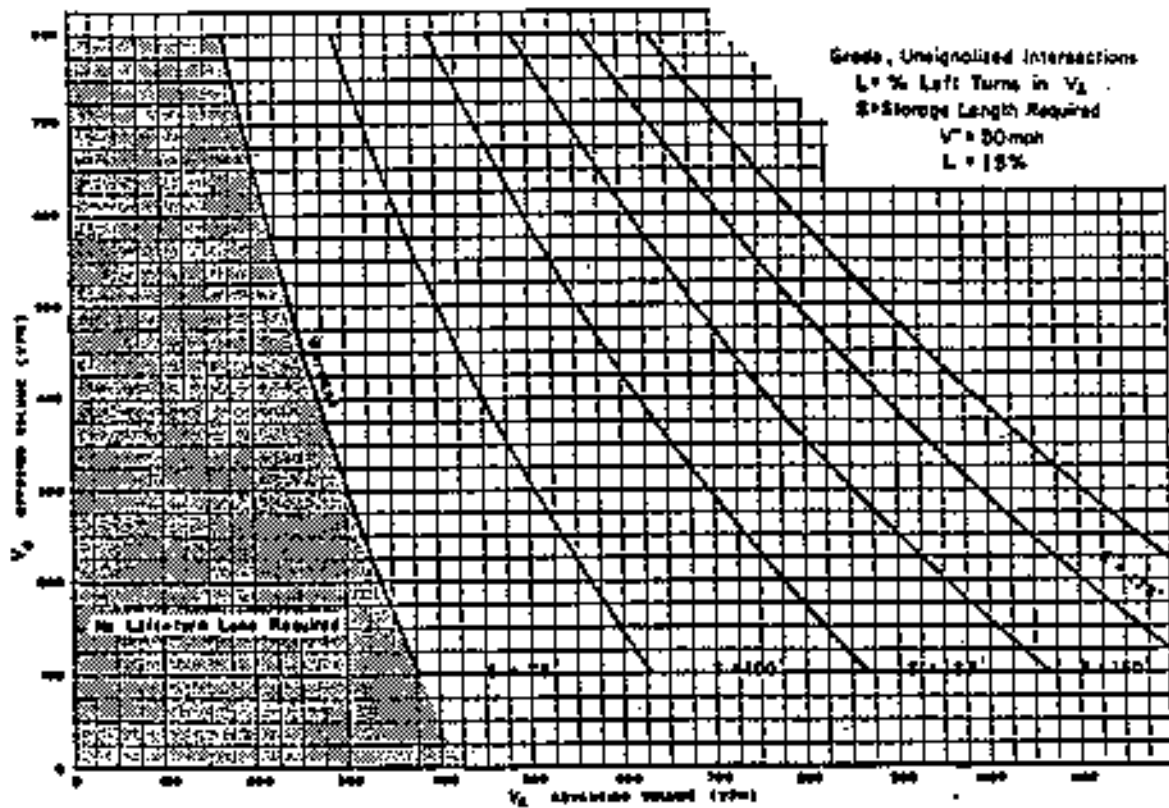


Figure 10. Warrant for left-turn storage lanes on two-lane highways.

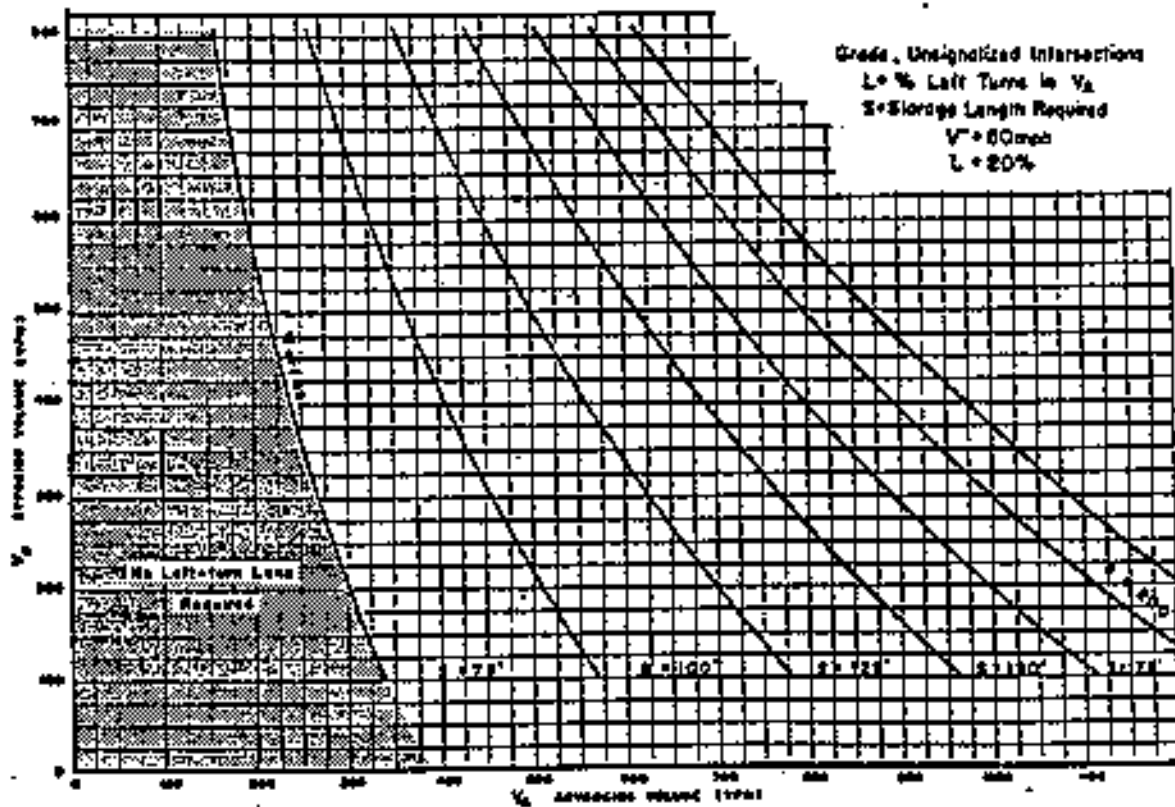


Figure 11. Warrant for left-turn storage lanes on two-lane highways.

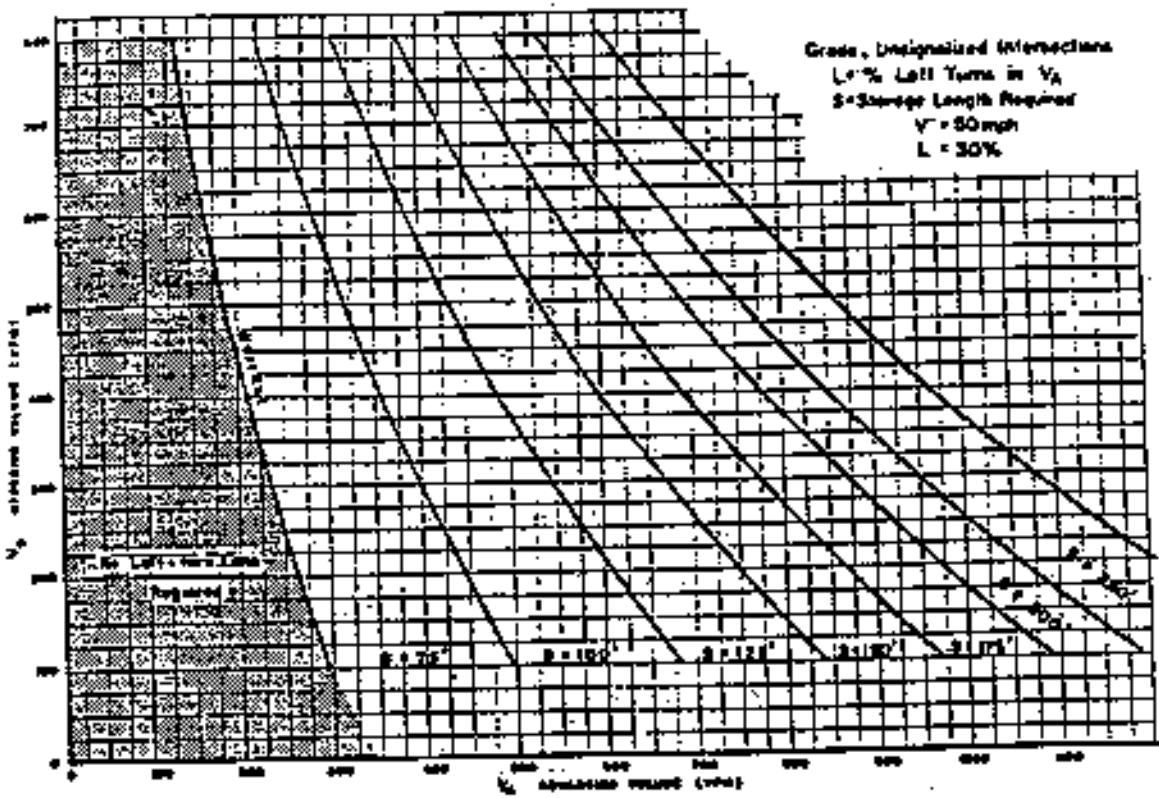


Figure 12. Warrant for left-turn storage lanes on two-lane highways.

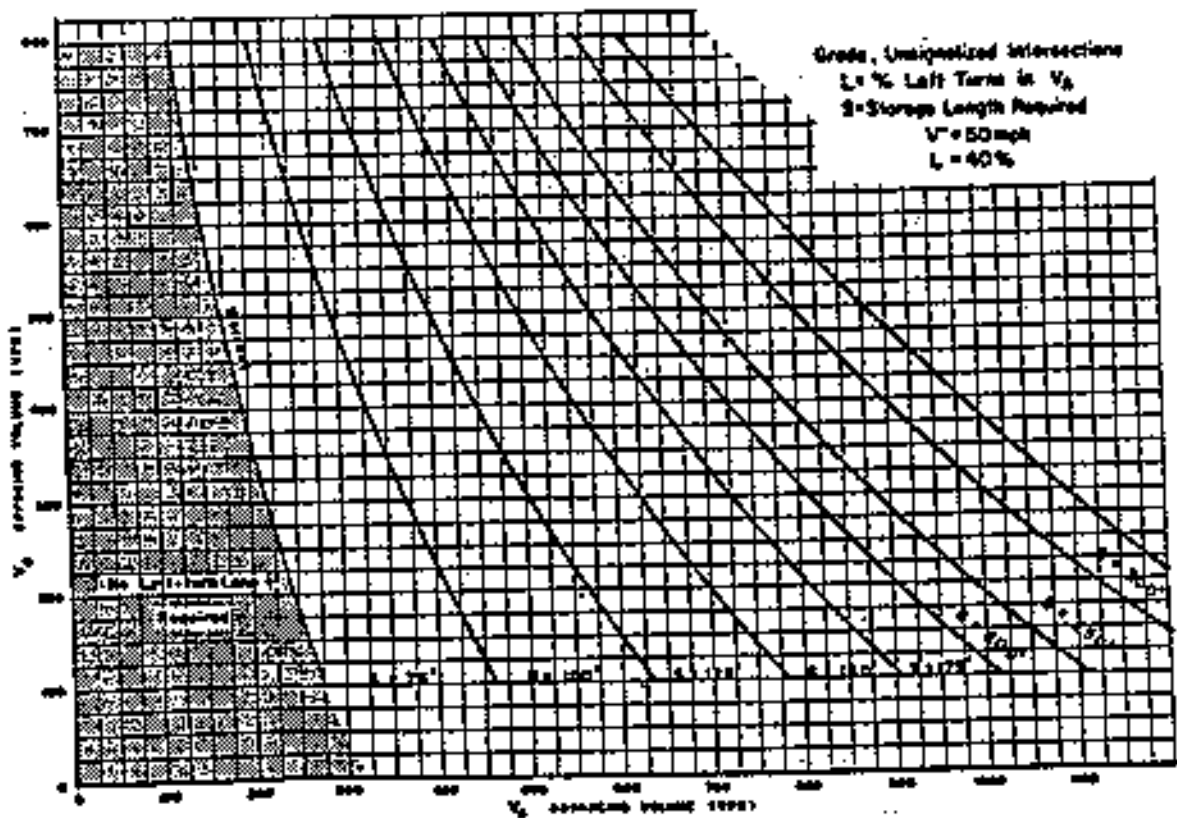


Figure 13. Warrant for left-turn storage lanes on two-lane highways.

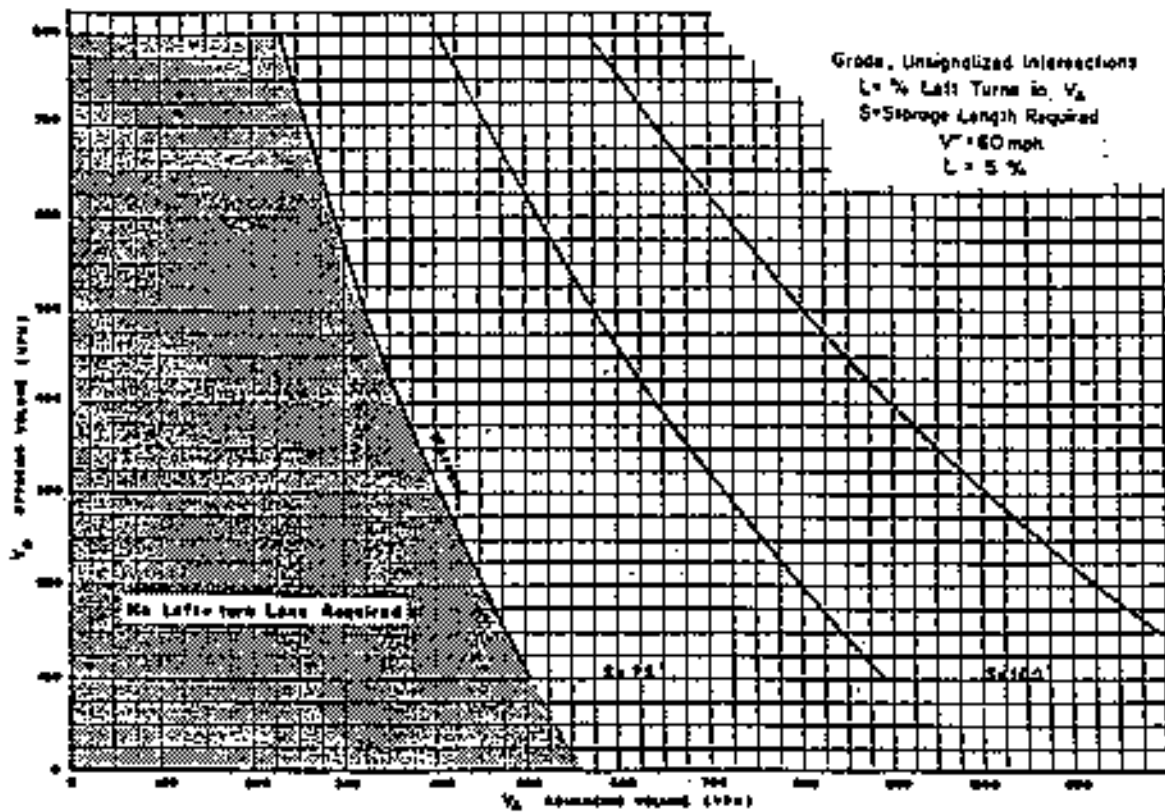


Figure 14. Warrant for left-turn storage lanes on two-lane highways.

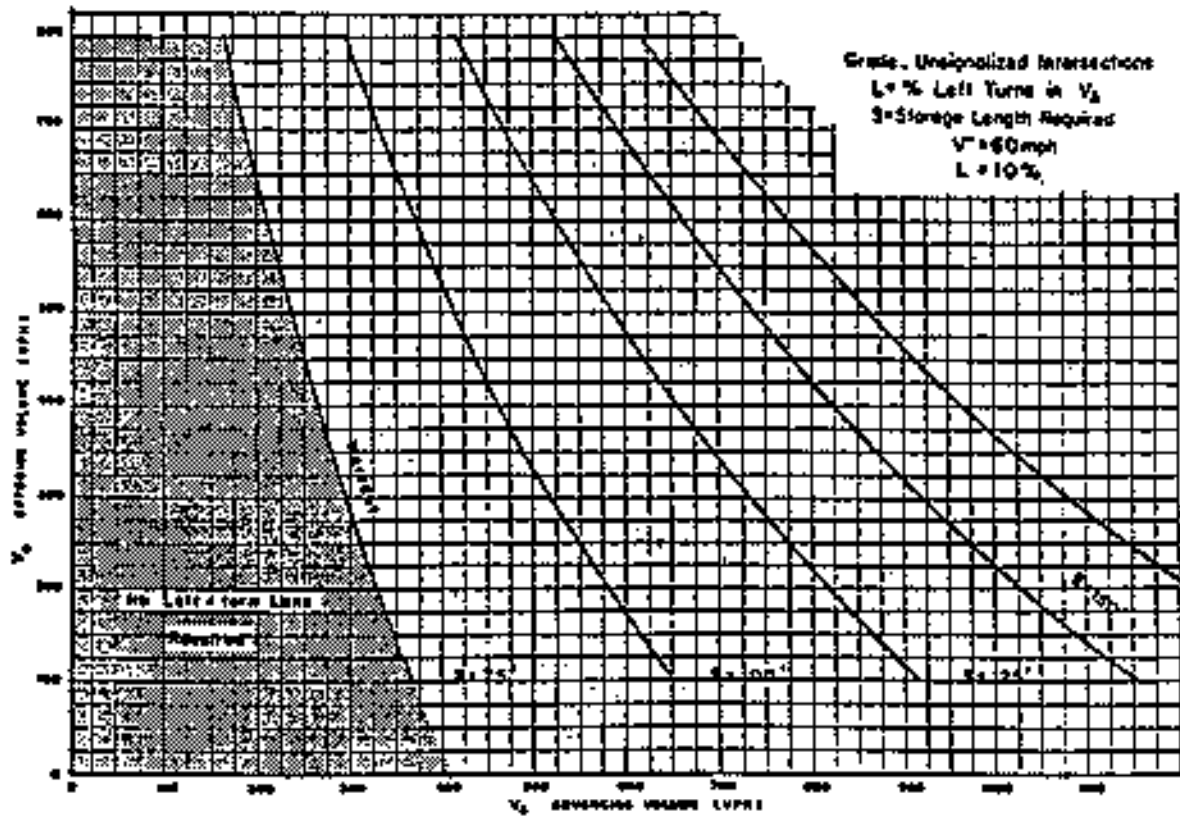


Figure 15. Warrant for left-turn storage lanes on two-lane highways.

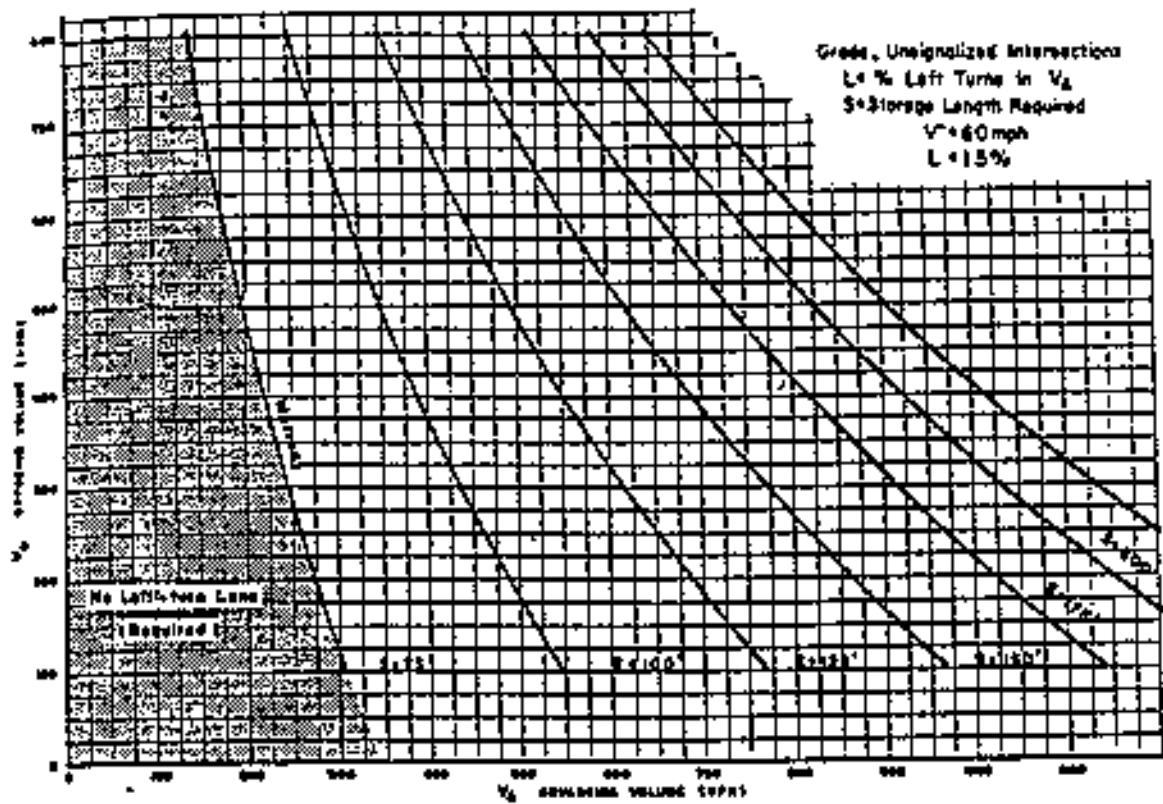


Figure 16. Warrant for left-turn storage lanes on two-lane highways.

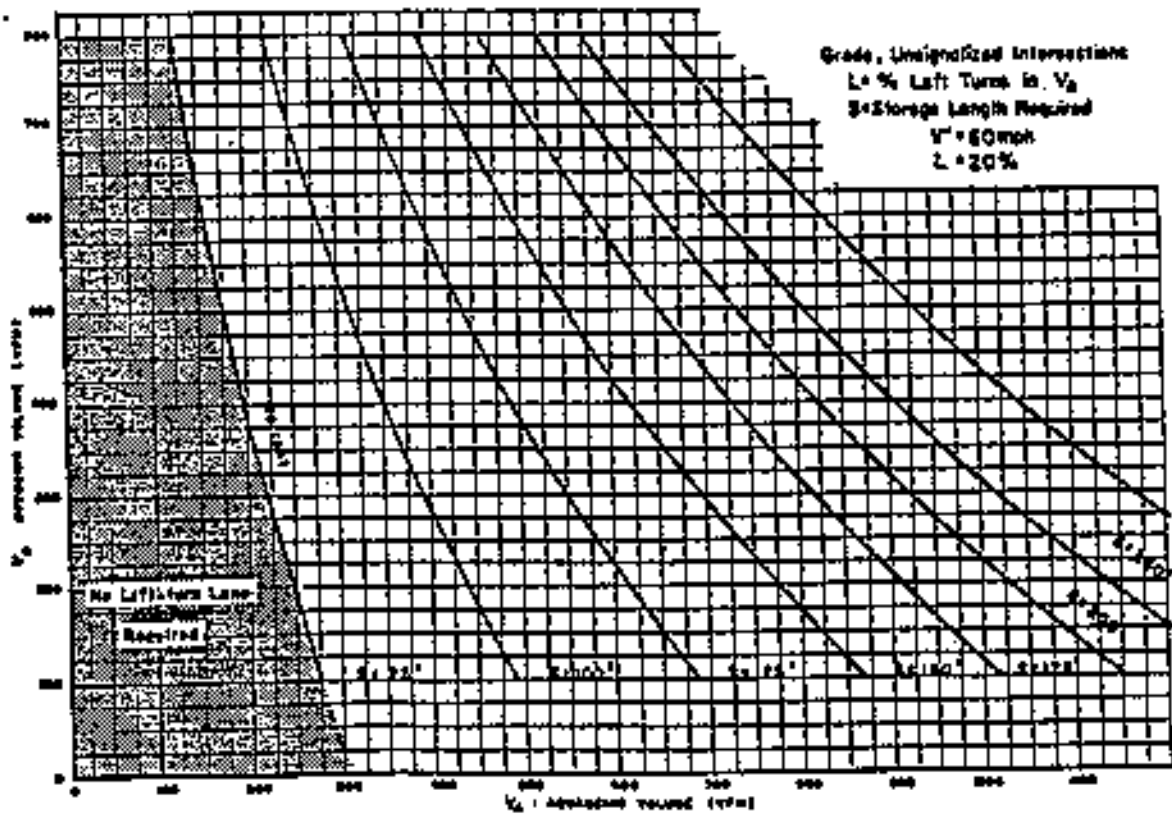


Figure 17. Warrant for left-turn storage lanes on two-lane highways.

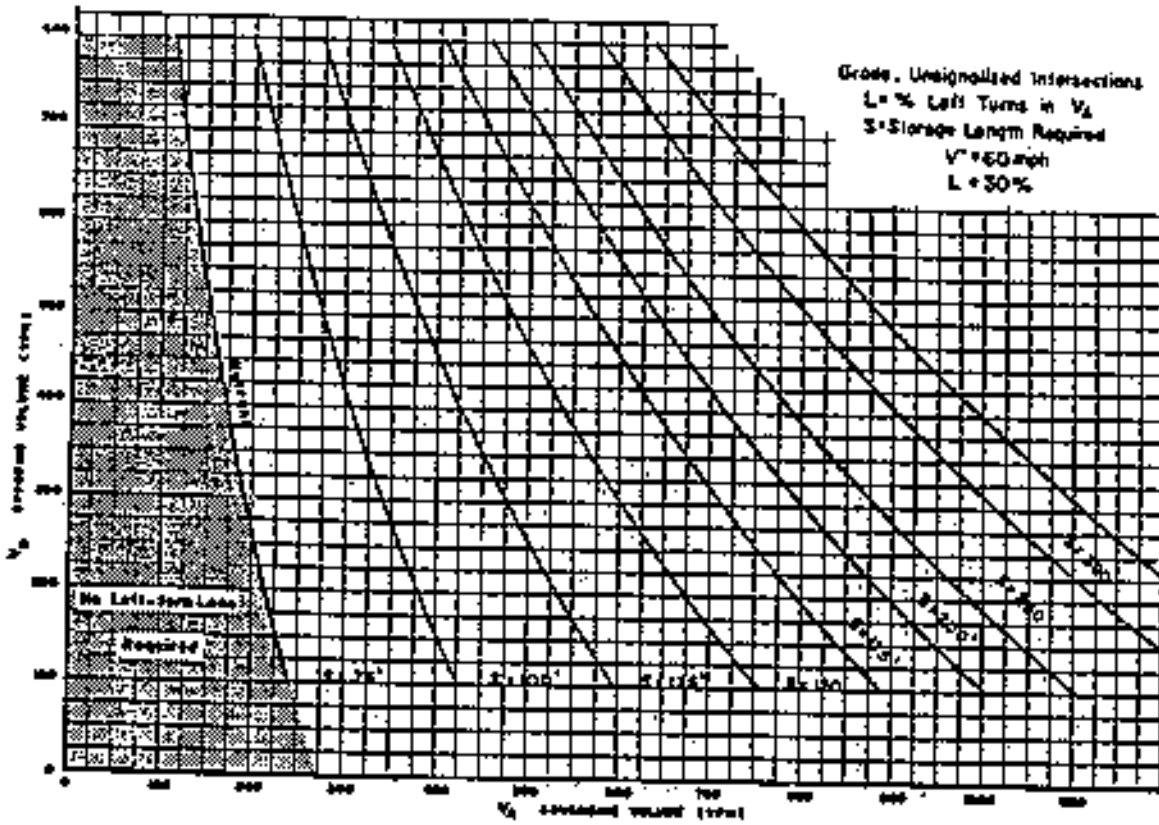


Figure 18. Warrant for left-turn storage lanes on two-lane highways.

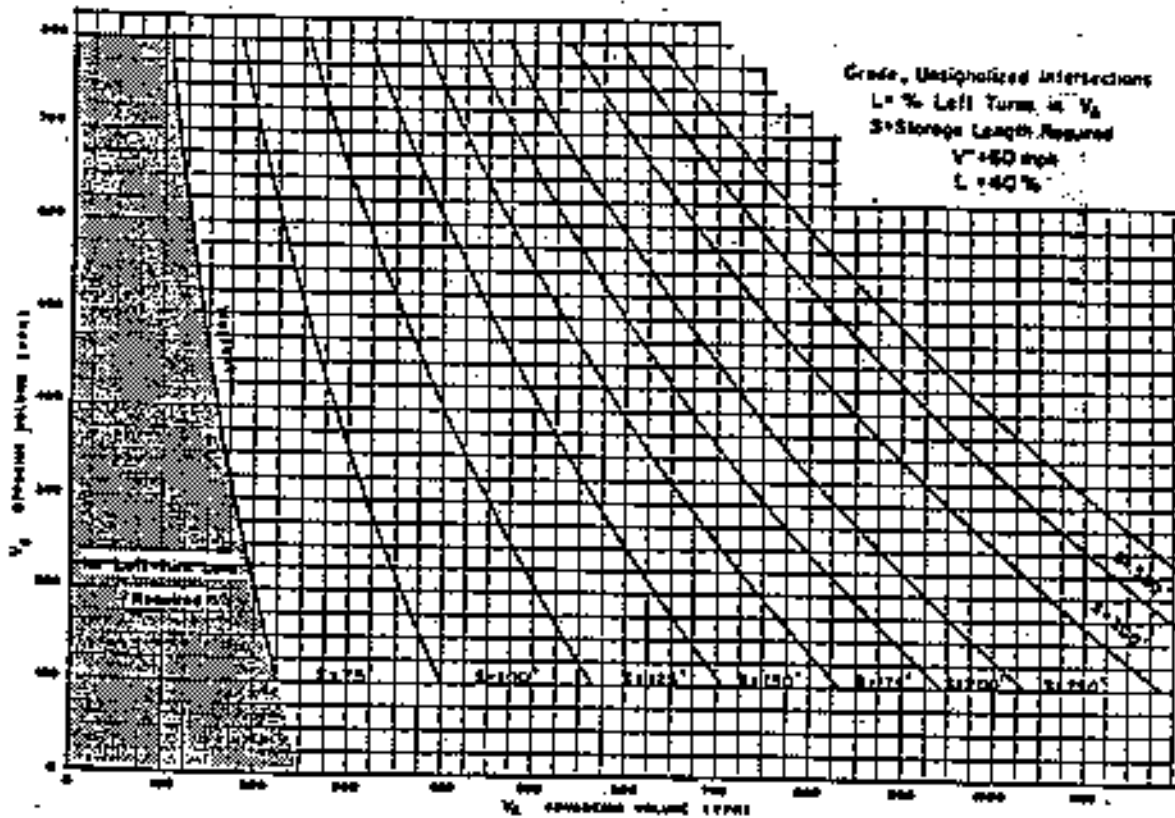


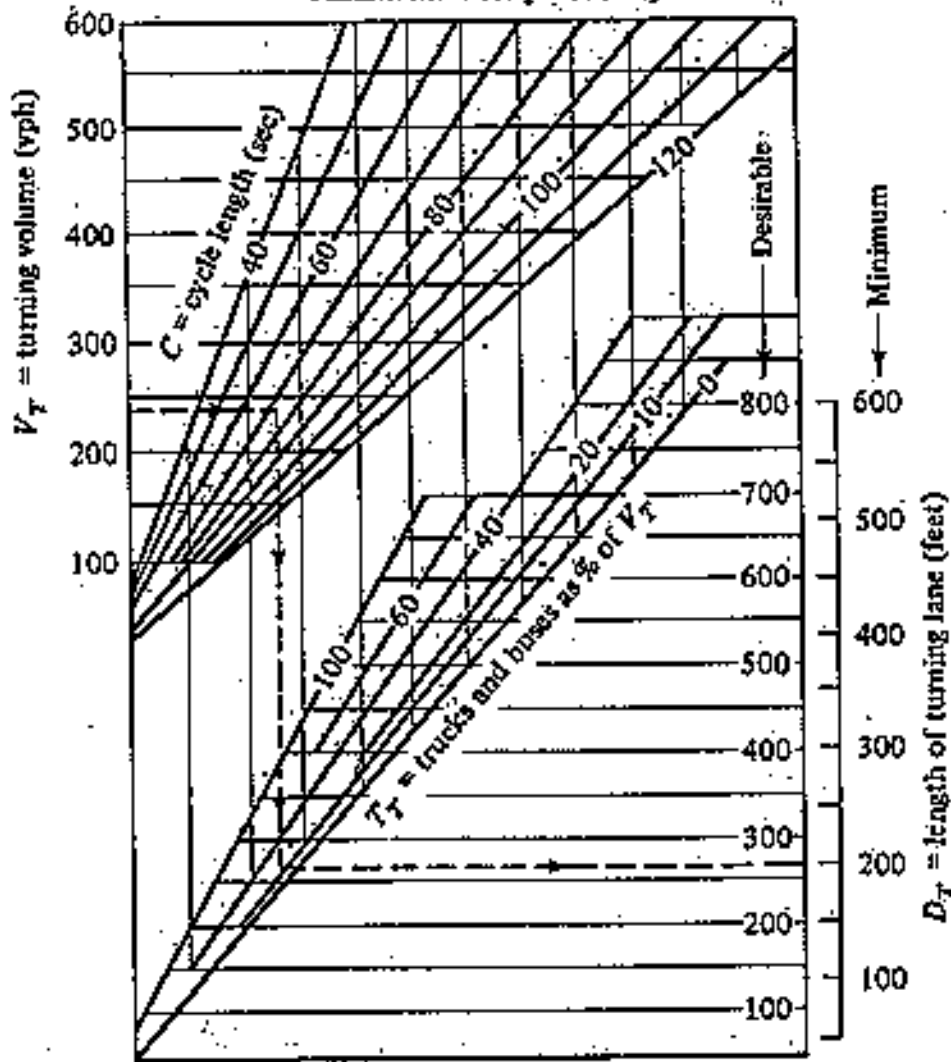
Figure 19. Warrant for left-turn storage lanes on two-lane highways.

APPENDIX B

Nomograph for Left-Turn Storage

Source: Reference (14)

Desirable: 95% probability of storing
all left-turn vehicles
Minimum: 90% probability



Nomograph for a single-lane left-turn storage at signalized intersections. As illustrated, with a left-turn volume of 240 vehicles per hour (vph), a 70-second cycle, and 10% trucks, a storage length of about 260 feet for a minimum. These storage lengths would accommodate 10 or 11 vehicles for the desirable conditions and about 8 for the minimum. The figure can be used to estimate the storage length (excluding taper length) of a double left-turn bay by dividing by 1.8. Thus for the desirable conditions, a double left-turn bay of about 145 feet (excluding taper) would be required.

APPENDIX C

SELECTED TABLES FOR STORAGE REQUIREMENTS FOR SIGNALIED INTERSECTION APPROACHES

The complete set of tables is available in Reference [34](#))

Table 1
50th-, 85th-, and 90th-percentile Storage Lengths
(vehicle units)

Seperate Phase		Cycle length = 60 sec						
Lane Volume	Percentile Value	Effective Green Time - sec						
		10	15	20	25	30	35	40
50	50th	1	0	0	0	0	0	0
	85th	2	1	1	1	1	1	1
	90th	2	2	2	2	2	2	1
100	50th	1	1	1	1	1	1	0
	85th	3	2	2	2	2	2	1
	90th	4	3	3	3	3	2	2
150	50th	2	2	2	1	1	1	1
	85th	4	3	3	3	2	2	2
	90th	6	4	4	4	3	3	3
200	50th	4	2	2	2	2	1	1
	85th	9	4	4	4	3	3	2
	90th	13	5	5	4	4	3	3
250	50th	∞	3	3	2	2	2	1
	85th	∞	6	5	4	4	3	3
	90th	∞	8	6	5	5	4	4
300	50th		5	3	3	2	2	2
	85th		10	6	5	4	4	3
	90th		14	7	6	5	5	4
350	50th		32	4	3	3	2	2
	85th		∞	7	5	5	4	3
	90th		∞	9	7	6	5	5
400	50th		∞	5	4	3	3	2
	85th		∞	9	6	5	5	4
	90th		∞	12	8	7	6	5

Table 1
(continued)

50th-, 85th-, and 90th-percentile Storage Lengths
(vehicle units)

Seperate Phase		Cycle length = 60 sec						
Lane Volume	Percentile Value	Effective Green Time - sec						
		10	15	20	25	30	35	40
450	50th			11	5	4	3	2
	85th			21	7	6	5	4
	90th			27	10	6	6	5
500	50th			∞	6	4	3	3
	85th			∞	10	7	6	5
	90th			∞	13	9	7	6
550	50th				9	5	4	3
	85th				16	8	6	5
	90th				23	10	8	6
600	50th				∞	6	4	3
	85th				∞	10	7	6
	90th				∞	12	9	7
650	50th					8	5	4
	85th					15	8	6
	90th					19	10	7
700	50th					19	6	4
	85th					43	9	6
	90th					55	12	8
750	50th					∞	7	4
	85th					∞	13	7
	90th					∞	19	10
800	50th						12	5
	85th						25	9
	90th						33	12

Table 3

50th-, 85th-, and 90th-percentile Storage Lengths
(vehicle units)

Seperate Phase		Cycle length = 90 sec										
Lane Volume	Percentile Value	Effective Green Time - sec										
		10	15	20	25	30	35	40	45	50	55	60
50	50th	1	1	1	1	1	1	1	0	0	0	0
	85th	2	2	2	2	2	2	2	1	1	1	1
	90th	3	3	3	3	3	2	2	2	2	2	2
100	50th	15	2	2	2	2	1	1	1	1	1	1
	85th	28	4	3	3	3	3	3	2	2	2	2
	90th	37	5	5	4	4	4	3	3	3	3	3
150	50th	∞	3	2	2	2	2	2	2	2	1	1
	85th	∞	5	5	4	4	4	4	3	3	3	2
	90th	∞	7	6	6	5	5	5	4	4	4	3
200	50th		6	4	3	3	3	3	2	2	2	2
	85th		10	6	6	5	5	5	4	4	3	3
	90th		13	8	7	7	6	6	5	5	4	4
250	50th		∞	5	4	4	4	3	3	3	2	2
	85th		∞	9	7	6	6	5	5	4	4	3
	90th		∞	11	9	8	7	7	6	6	5	5
300	50th			11	6	5	5	4	4	3	3	2
	85th			22	9	8	7	6	6	5	5	4
	90th			29	11	9	8	8	7	7	6	5
350	50th			∞	8	6	5	5	4	4	3	3
	85th			∞	13	9	8	7	7	6	5	5
	90th			∞	18	11	10	9	8	7	7	6
400	50th				4	8	3	6	5	4	4	2
	85th				6	12	9	8	8	7	6	3
	90th				8	15	11	10	9	8	8	7

**Table 3
(Continued)**

50th-, 85th-, and 90th-percentile Storage Lengths
(vehicle units)

Seperate Phase		Cycle length = 90 sec										
Lane Volume	Percentile Value	Effective Green Time - sec										
		10	15	20	25	30	35	40	45	50	55	60
450	50th					12	7	6	6	5	4	4
	85th					24	11	9	8	7	7	6
	90th					32	14	11	10	9	8	7
500	50th					∞	10	7	6	5	5	4
	85th					∞	16	11	9	8	7	6
	90th					∞	21	13	11	10	9	8
550	50th						30	9	7	6	5	5
	85th						∞	14	11	9	8	7
	90th						∞	17	12	11	10	9
600	50th						∞	13	8	7	6	5
	85th						∞	24	12	10	8	7
	90th						∞	33	16	12	10	9
650	50th							∞	10	8	6	5
	85th							∞	16	11	9	8
	90th							∞	22	14	11	10
700	50th								19	9	3	6
	85th								34	14	10	8
	90th								57	18	13	11
750	50th								∞	12	8	6
	85th								∞	22	12	10
	90th								∞	28	16	12
800	50th									∞	10	7
	85th									∞	16	11
	90th									∞	22	13

**Table 5
(Continued)**

50th-, 85th-, and 90th-percentile Storage Lengths
(vehicle units)

Seperate Phase		Cycle length = 120 sec														
Lane Volume	Percentile Value	Effective Green Time - sec														
		10	15	20	25	30	35	40	45	50	55	60	65	70	75	80
450	50th						∞	14	10	9	8	7	7	6	6	5
	85th						∞	25	14	12	11	10	10	9	8	7
	90th						∞	33	17	15	13	12	11	11	10	9
500	50th							∞	13	10	9	8	8	7	6	5
	85th							∞	23	15	13	12	11	10	9	8
	90th							∞	29	18	15	14	13	12	11	10
550	50th								∞	13	11	10	8	8	7	6
	85th								∞	21	15	13	12	11	10	9
	90th								∞	26	18	15	14	13	12	11
600	50th									∞	13	11	9	8	8	7
	85th									∞	1	15	13	12	11	10
	90th									∞	24	18	15	14	12	11

**Table 5
(Continued)**

50th-, 85th-, and 90th-percentile Storage Lengths
(vehicle units)

Seperate Phase		Cycle length = 120 sec															
Lane Volume	Percentile Value	Effective Green Time - sec															
		10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	
650	50th											26	12	10	9	8	7
	85th											51	19	15	12	11	10
	90th											∞	24	17	15	13	12
700	50th											∞	21	12	10	9	8
	85th											∞	38	18	14	12	11
	90th											∞	55	22	17	14	13
750	50th											∞	18	12	10	8	
	85th											∞	31	17	14	12	
	90th											∞	41	22	17	14	
800	50th												∞	15	11	9	
	85th												∞	26	17	13	
	90th												∞	35	21	15	

APPENDIX D

TABLE D-1: QUEUING EQUATIONS

Equation Number	Variable	Equation
1	Coefficient of utilization	$p = \frac{q}{NQ}$
2	Probability of no customers in the system	$P(0) = \left[\sum_{n=0}^{N-1} \frac{\left(\frac{q}{Q}\right)^n}{n!} + \frac{\left(\frac{q}{Q}\right)^N}{N!(1-p)} \right]^{-1}$
3	Mean number in the queue	$E(m) = \left[\frac{p\left(\frac{q}{Q}\right)^N}{N!(1-p)^2} \right] P(0)$
4	Mean number in the system	$E(n) = E(m) + \frac{q}{Q}$
5	Mean wait time in the queue (hours)	$E(w) = \frac{E(m)}{q}$
6	Mean time in the system (hours)	$E(t) = E(w) = \frac{1}{Q}$
7	Proportion of customers who wait	$P[E(w) > 0] = \left[\frac{\left(\frac{q}{Q}\right)^N}{N!(1-p)} \right] P(0)$
8	Probability of a queue exceeding a length M	$P(x > M) = (p^{N+1}) P[E(w) > 0]$
9a	Queue storage required	$M = \left[\frac{\ln P(x > M) - \ln E(w) > 0}{\ln p} \right] - 1$
9b*	Queue storage required	$M = \left[\frac{\ln P(x > M) - \ln Q_m}{\ln p} \right] - 1$

* Q_M is a statistic which is a function of the utilization rate and the number of service channels (service positions); see Table.

The table of Q_M values and use of Equation 9b greatly simplifies the calculations.

TABLE D-2: TABLES OF Q_M VALUES

C	Number of Left Turn Lanes	
	1	2
0.00	0.0000	0.0000
0.05	0.5000	0.0091
0.10	0.1000	0.0182
0.15	0.1500	0.0424
0.20	0.2000	0.0666
0.25	0.2500	0.10253
0.30	0.3000	0.1385
0.35	0.3500	0.1386
0.40	0.4000	0.2286
0.45	0.4500	0.2810
0.50	0.5000	0.3333
0.55	0.5500	0.3917
0.60	0.6000	0.4501
0.65	0.6500	0.5134
0.70	0.7000	0.5766
0.75	0.7500	0.6438
0.80	0.8000	0.7111
0.85	0.8500	0.7818
0.90	0.9000	0.8526
0.95	0.9500	0.9263
1.00	1.0000	1.0000