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GEOMETRIC ANALYSIS OF WING SECTIONS

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SUMMARY

This paper describes a new geometric analysis procedure for wing sections. This procedure is based on the normal mode analysis for continuous functions. A set of special shape functions is introduced to represent the geometry of the wing section. The generators of the NACA 4-digit airfoils were included in this set of shape functions. It is found that the supercritical wing section, Korn airfoil, could be well represented by a set of ten shape functions. Preliminary results showed that the number of parameters to define a wing section could be greatly reduced to about ten. Hence, the present research clearly advances the airfoil design technology by reducing the number of design variables.

STATEMENT OF PROBLEM

In the optimization procedure, wing sections are perturbed by linear combinations of shape functions to render an optimal geometry which satisfies certain mission requirements. The final optimal geometry and the efficiency of the design tool depends upon the choice and the number of shape functions used. The selection of shape functions to suitably represent wing sections is one of the fundamental problems to airfoil designers.

Mathematically, any continuous function defined on a closed interval can be represented by an infinite series of normal modes which form a complete set of bases. The set of Fourier sine functions is an example of such a complete set. However, practical application does not call for an infinite series but a finite partial sum to approximate the geometry of the wing section within a prescribed tolerance in some norm.

There are several well known shape functions for wing section modification introduced in recent papers. Hicks functions, Wagner functions, and aerofunctions are some of them (refs. 1–3). Although these shape functions are useful, there are no systematic guidelines to effectively select these shape functions. The number of shape functions used in current design optimizations is easily more than 20—too many to be efficient.

The present research pondered this problem and proposed a solution to it via normal mode analysis (ref. 4).

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NORMAL MODE ANALYSIS

In the following, we briefly describe the normal mode analysis of the continuous function.

An inner product of two integrable functions, g and h, on the closed unit interval is denoted as (g, h) and is defined by the relation

$$(g, h) = \int_0^1 g(x)\, h(x)\, dx$$

A norm of f is defined as the square root of (f, f). Two functions, g and h, are orthogonal to each other if $(g, h) = 0$. A set of functions $\{f_i(x)\}$ is orthogonal if $(f_i, f_j) = 0$ for $i \neq j$ and $(f_i, f_i) \neq 0$ for all i. A set of functions $\{f_i(x)\}$ is orthonormal if it is orthogonal and the length of each member is unity, $(f_i, f_i) = 1$. A set of functions is linearly independent if the norm of any linear combination of its members is zero, provided that the coefficient of each term of the combination is zero.

Let $\{g_i(x)\}$ be a set of functions which is linearly independent. The Gram-Schmidt orthonormalization process for the set $\{g_i(x)\}$ can be described as follows.

First, an orthogonal set $\{h_i(x)\}$ is formed from the following relations:

$$h_1(x) = g_1(x)$$

$$h_2(x) = g_2(x) - a_{21} h_1(x)$$

$$\vdots$$

$$h_n(x) = g_n(x) - \sum_{i=1}^{n-1} a_{ni} h_i(x)$$

$$\vdots$$

where $a_{ni} = (g_n, h_i)/(h_i, h_i)$ is the projection of g_n in the direction of h_i. Finally, the orthonormal set $\{f_i(x)\}$ is found by normalizing $h_i(x)$ as follows:

$$f_i(x) = h_i(x)/(h_i, h_i)$$

The least-squares problem is to seek a vector $\hat{f}(x)$ in the subspace spanned by $\{g_i(x)\}$ which lies the shortest distance from the given vector $f(x)$. Namely, the norm of error function, $f - \hat{f}$, is as small as possible. This least-squares problem has a unique solution:

$$\hat{f}(x) = \sum_{i=1}^{n} b_i f_i(x)$$

where the orthonormal coefficient is $b_i = (f, f_i)$. The function $\hat{f}$ is called the reconstruction of function f via the normal mode analysis.

PRACTICAL APPLICATION

An airfoil geometry is usually defined by a discrete set of points. There are three steps involved in the process to normalize the airfoil geometry into the canonical form which requires that the leading edge be located at the origin of the coordinate system and the trailing edge at unit length of the horizontal axis. First, for an airfoil with finite thickness at the trailing edge, some work has to be done to close the trailing edge of the airfoil. Second, the leading edge of the airfoil has to be determined. The leading edge of the airfoil is determined as the farthest point from the trailing edge which lies on the circle passing through the three leading points of the discrete set which are farthest away from the trailing edge. Finally, a scaling and a rigid body transformation (rotation and translation) may be needed to map the airfoil geometry onto the canonical form.

A complete geometry of wing section entails two parts, upper and lower surfaces. They can be combined to define the camber and thickness distributions by the following relations. Namely, let the net $\{x_i\}$ be a discrete set of abscissas on the closed unit interval and $\{u_i\}$ and $\{v_i\}$ the upper and lower surface functions defined on the net. Then, the camber distribution $\{c_i\}$ is given by the relation

$$c_i = 0.5(u_i + v_i)$$

and the thickness distribution $\{t_i\}$ by the relation

$$t_i = u_i - v_i$$

Conventional Airfoil

There are several books which include airfoil definitions in the literature (refs. 5–7). The NACA 0012 airfoil is a typical example for conventional airfoils. It falls in the category of NACA 4-digit airfoils (ref. 5) which are defined by the following equation:

$$y = \frac{t}{0.2}(0.2969\sqrt{x} - 0.126x - 0.3516x^2 + 0.2843x^3 - 0.1015x^4)$$

where t is the maximum thickness expressed as a fraction of the chord which is of unit length. It is clear that function y can be rewritten in terms of the following four shape functions:

$$g_1(x) = \sqrt{x} - x \tag{1}$$

$$g_2(x) = x(1 - x) \tag{2}$$

$$g_3(x) = x^2(1 - x) \tag{3}$$

$$g_4(x) = x^3(1 - x) \tag{4}$$

where x is on the closed unit interval [0,1]. These four functions are referred to as the generators of the NACA 4-digit airfoils.

The geometry of the original NACA 0012 airfoil defined above is not closed at the trailing edge. An effort was made to close the trailing edge of the airfoil. Figure 1 shows the contour of the normalized

NACA 0012 airfoil. The set of shape functions, $\{g_i(x), i = 1, 4\}$, is plotted in figure 2(a). The set of orthonormalized duals, $\{f_i(x), i = 1, 4\}$, of the shape functions is displayed in figure 2(b). The orthonormal coefficients of the upper surface of the NACA 0012 with respect to the orthonormal duals, $\{f_i(x), i = 1, 4\}$, are shown in figure 3. The guppy plot of the reconstructed airfoil, generated by the orthonormal coefficients and the orthonormal duals, is displayed along with the input airfoil for comparison in figure 4. The maximum norm of the error function of the NACA 0012 airfoil was about 0.18E-4. This showed that the NACA 0012 airfoil was practically recovered through the normal mode analysis.

Supercritical Airfoil

The extension of the NACA shape functions to represent a supercritical wing section is now presented. The following six shape functions which are in the homotopic neighborhood of the NACA shape functions were added to the NACA listing. They are as follows:

$$g_5(x) = x^4(1-x) \tag{5}$$

$$g_6(x) = x^5(1-x) \tag{6}$$

$$g_7(x) = \sqrt[3]{x} - \sqrt{x} \tag{7}$$

$$g_8(x) = \sqrt[4]{x} - \sqrt[3]{x} \tag{8}$$

$$g_9(x) = \sqrt[5]{x} - \sqrt[4]{x} \tag{9}$$

$$g_{10}(x) = \sqrt[6]{x} - \sqrt[5]{x} \tag{10}$$

Now there are ten shape functions in this new NACA collection.

The supercritical wing section, Korn airfoil, was chosen to be studied by these ten shape functions. The Korn airfoil was not defined by an algebraic equation but was devised by an inverse design method on the hodograph plane. The geometry definition of the Korn airfoil can be found in reference 6. Its trailing edge is closed. The leading edge of the airfoil was not given in the original discrete set for the airfoil geometry. The leading edge is determined to be $(-0.000053, 0.002228)$ and the pitching angle between the leading edge and trailing edge was calculated as 0.127663 degree (nose up). Figure 5 shows the geometry of the normalized Korn airfoil. The set of shape functions, $\{g_i(x), i = 5, 10\}$, is plotted in figure 6(a). The set of orthonormal modes, $\{f_i(x), i = 5, 10\}$, of the shape functions is shown in figure 6(b). The orthonormal coefficients of the upper surface and lower surface of the Korn airfoil corresponding to the orthonormal modes, $\{f_i(x), i = 1, 10\}$, are shown in figure 7. The guppy plots of the reconstructed Korn airfoil along with the input Korn airfoil are displayed in figure 8. The maximum norm of the error function of the Korn airfoil was about 5.70E-4. It shows that the Korn airfoil can practically be represented by these ten shape functions.

CONCLUDING REMARKS

A geometric analysis was described for practical wing sections. The approach of the analysis was based on the approximation theory. The least-squares approximant to the geometry of the wing section was determined via the normal mode analysis. A suitable set of shape functions was newly gathered to generate a set of orthonormal modes through the Gram-Schmidt orthonormalization process. The generators of the NACA 4-digit airfoils were included in this set of NACA bumps. The geometry contour of the NACA 0012 airfoil was represented by a combination of only four normal modes within a very tight tolerance, whereas the Korn airfoil, a supercritical wing section, was well represented by the set of only ten orthonormal modes. Preliminary results showed that the number of parameters to define a practical wing section can be reduced to about ten of these newly identified shape functions. The significance of the present research is to provide a promising method for advancing the wing section design in reducing the number of design variables.

Additional work remains to be done. One area of particular interest is to apply the present geometric analysis to other advanced airfoils to further confirm the usefulness of the NACA bumps. Another is to make direct comparison of airfoils with their reconstructed duals in terms of aerodynamic characteristics.

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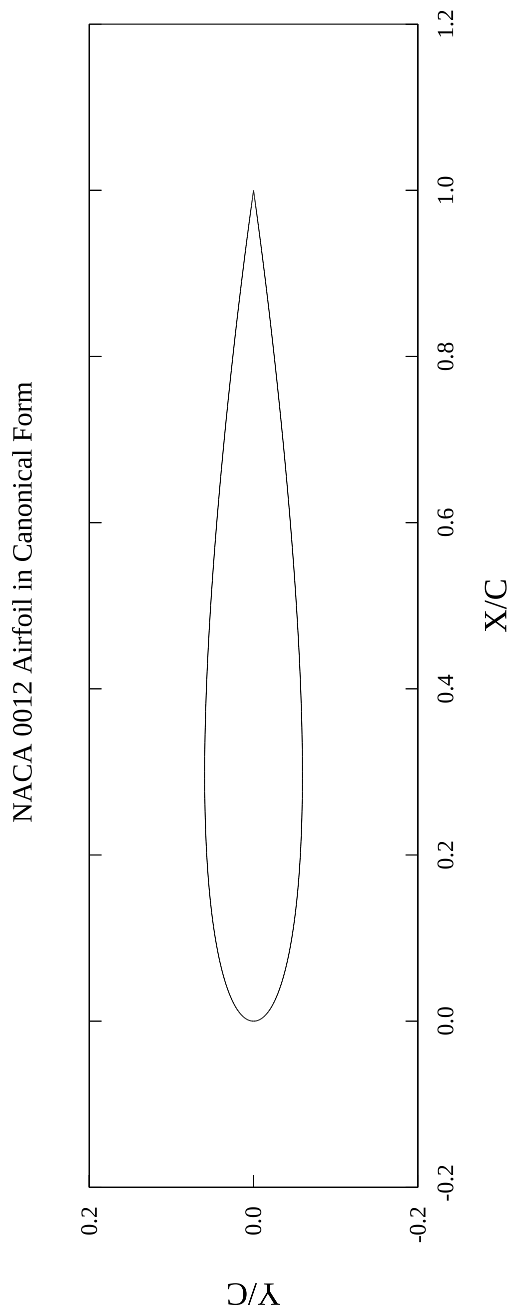
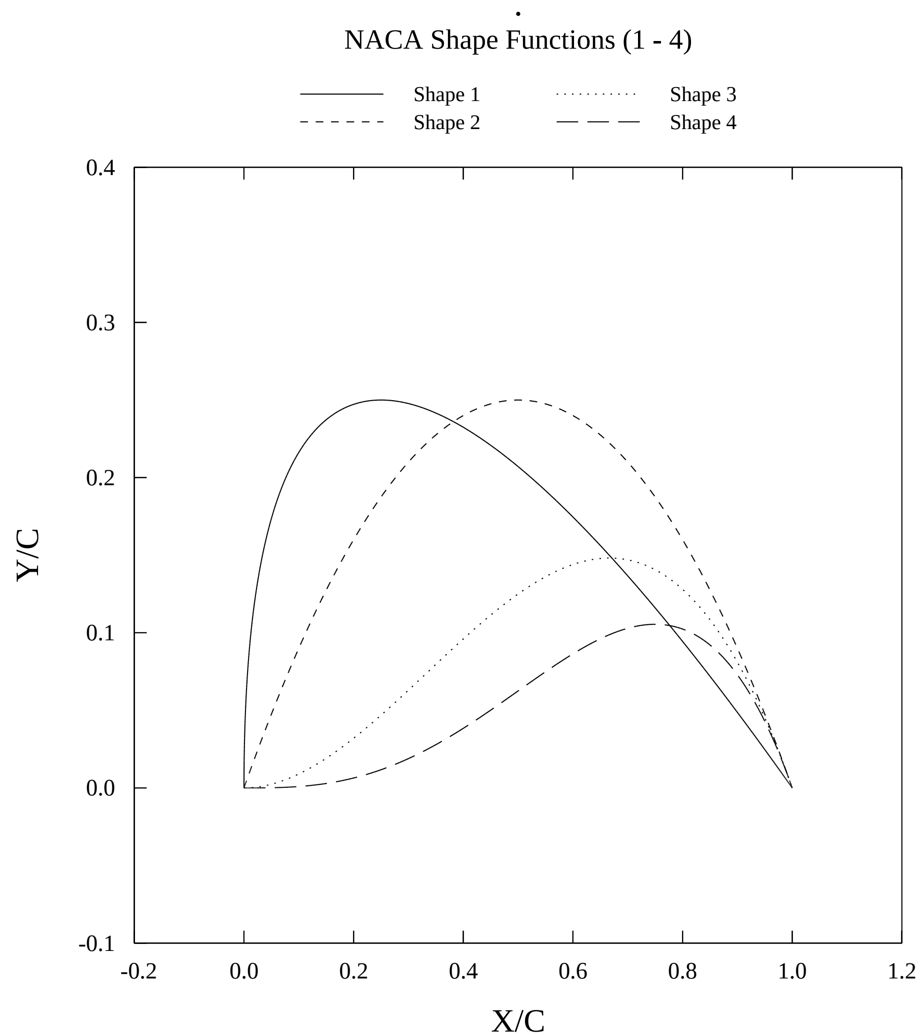
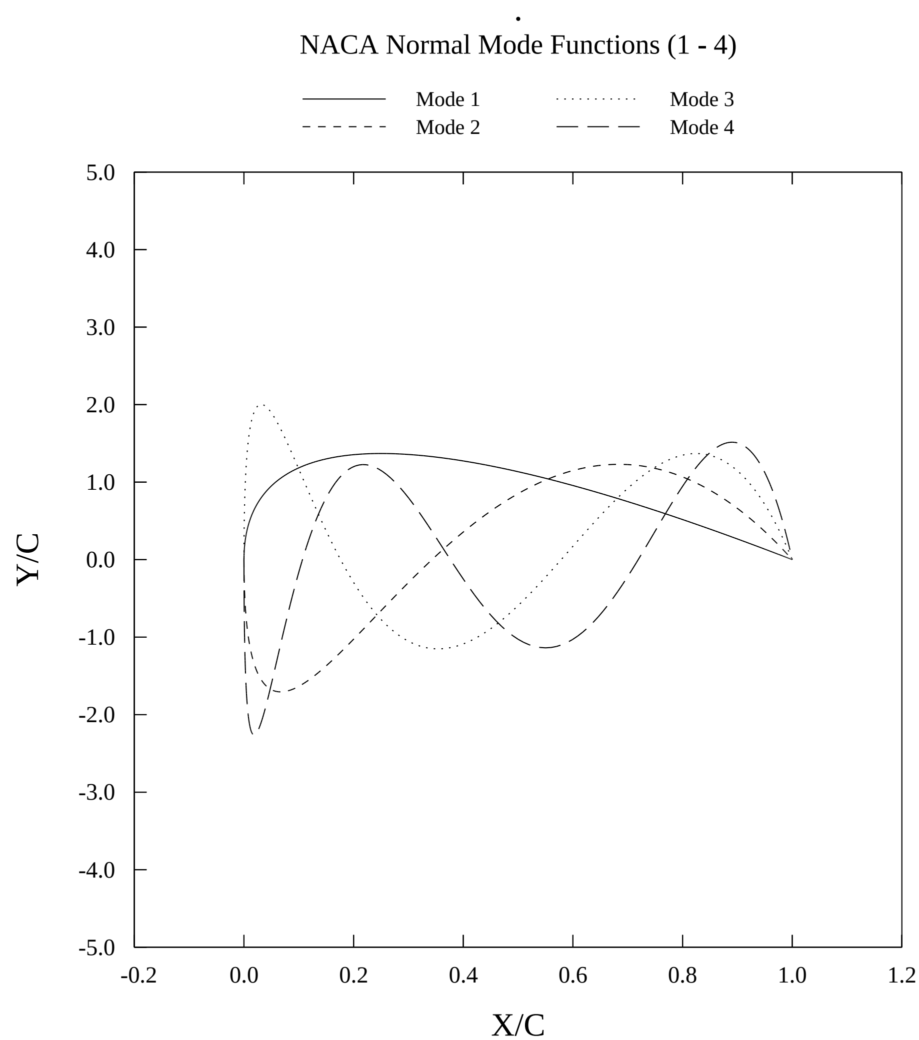


Figure 1. The normalized NACA 0012 airfoil with the trailing edge closed.



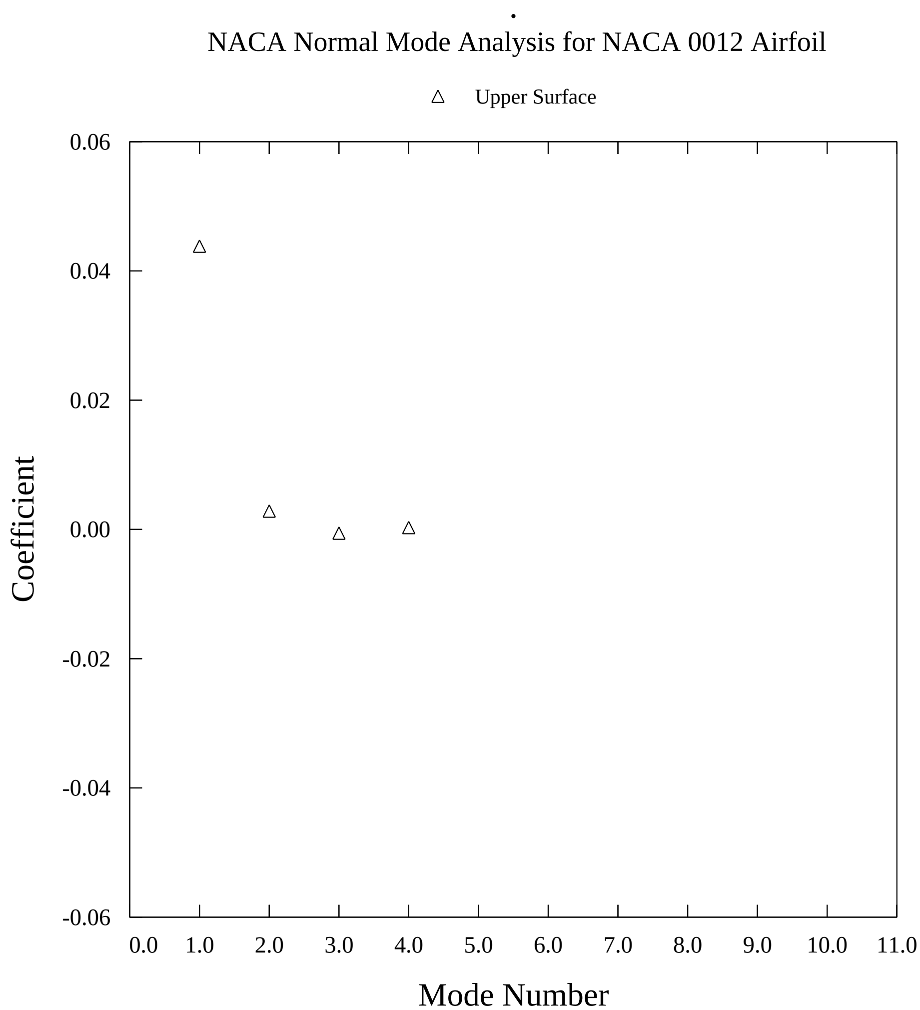
$\{g_i(x)$, i=1,4$\}$.

Figure 2(a). The first set of four NACA shape functions, $\{g_i(x)$, i = 1,4$\}$.



$\{f_i(x), i=1,4\}$.

Figure 2(b). The first set of four NACA normal modes, $\{f_i(x)$, i = 1,4$\}$.



$\{f_i(x), i=1,4\}$.

Figure 3. The normal mode coefficients of the upper surface of the NACA 0012 airfoil corresponding to the normal modes, $\{f_i(x)$, i = 1,4$\}$.

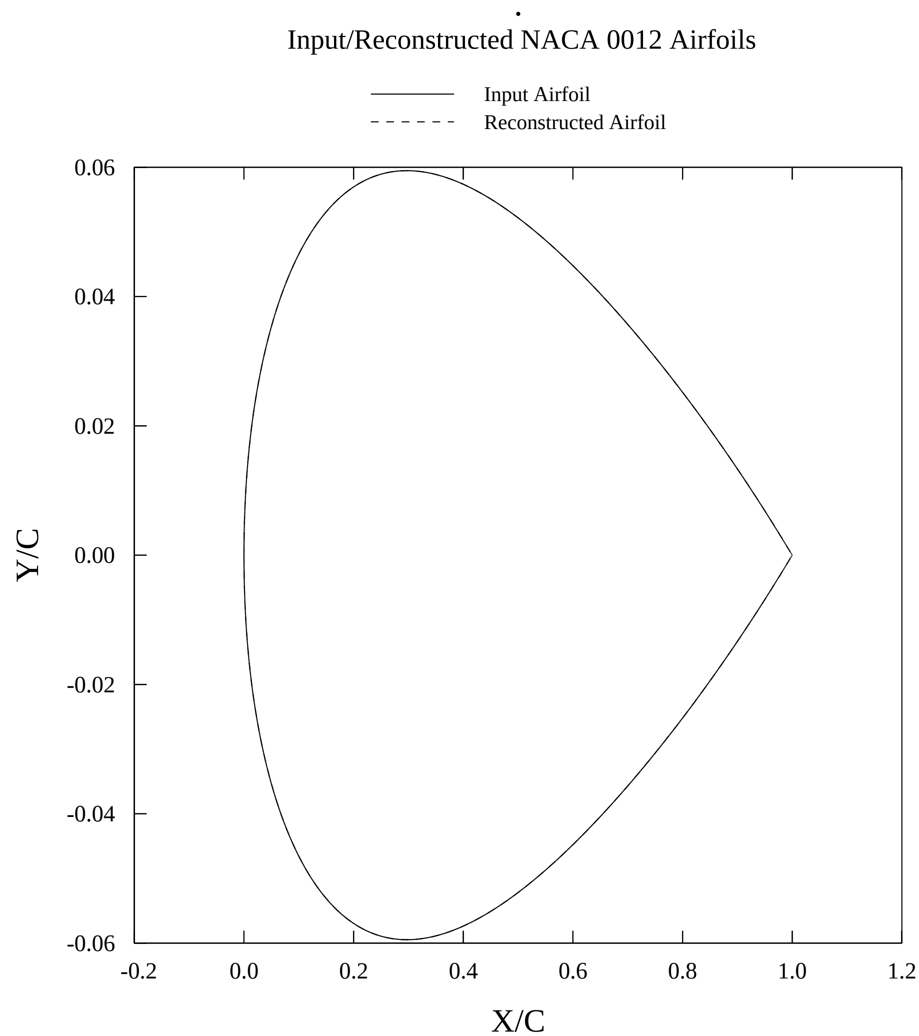


Figure 4. The guppy plots of the input NACA 0012 airfoil and its reconstructed airfoil via the normal mode analysis.

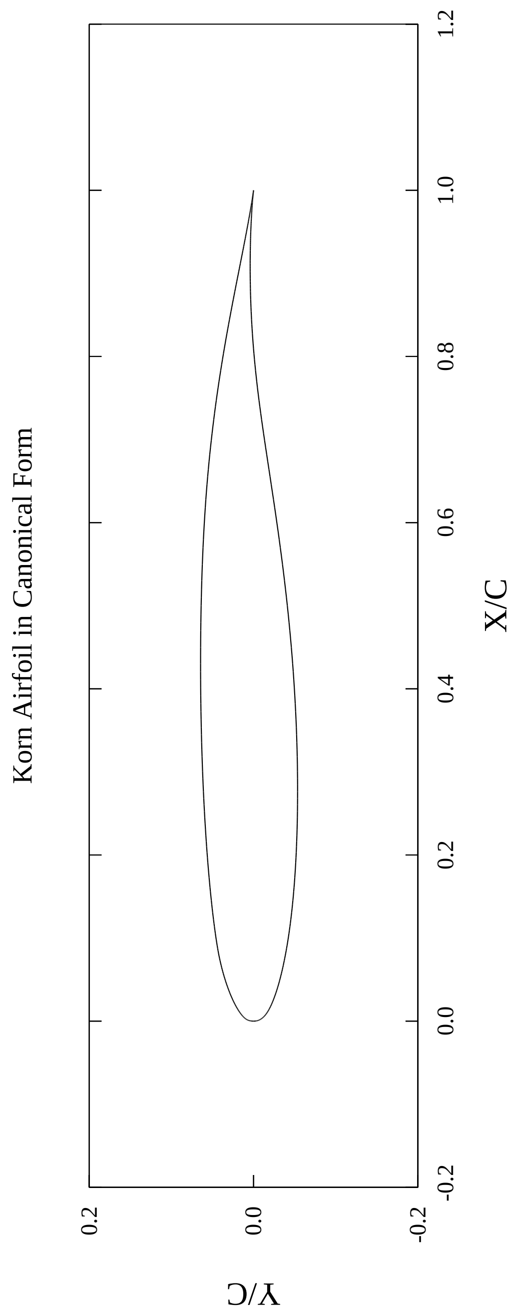
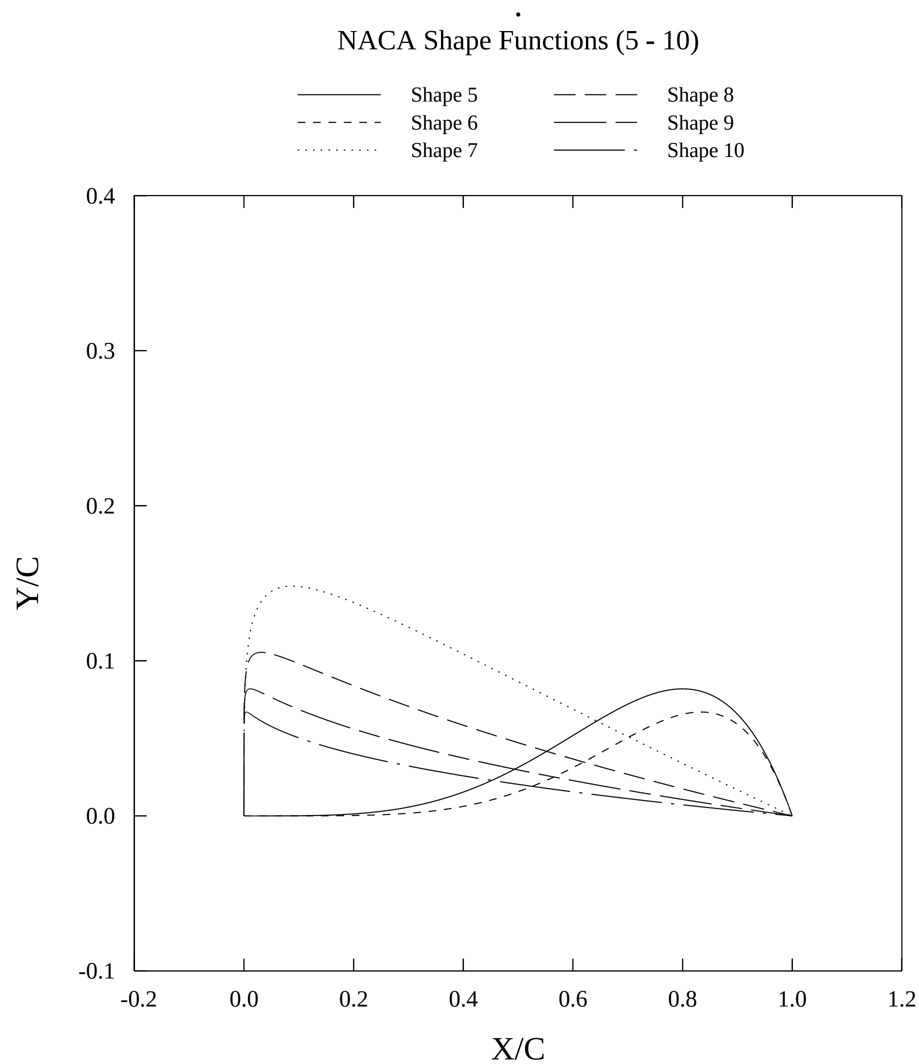
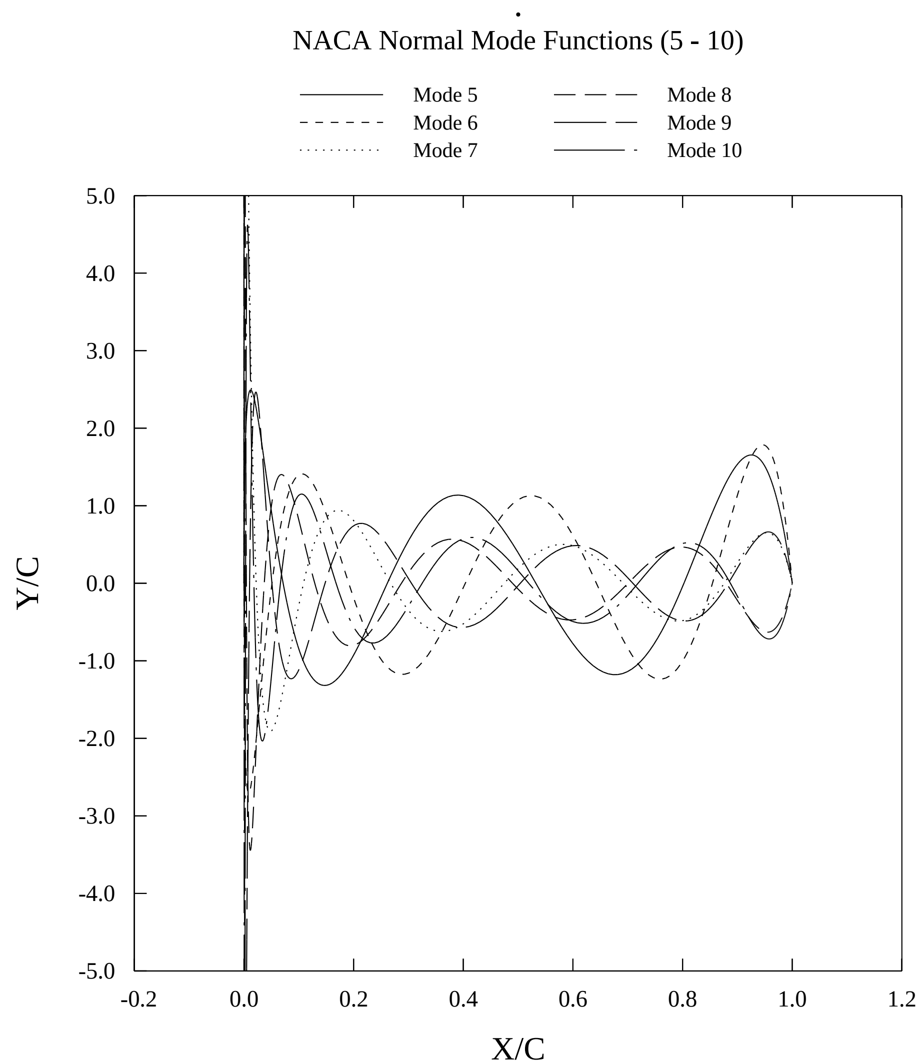


Figure 5. The normalized Korn airfoil with the fitted-in leading edge.



$\{g_i(x), \text{i}=5,10\}$.

Figure 6(a). The next set of six NACA shape functions, $\{g_i(x)$, i = 5,10$\}$.



$\{f_i(x), i=5,10\}$.

Figure 6(b). The next set of six NACA normal modes, $\{f_i(x)$, i = 5,10$\}$.

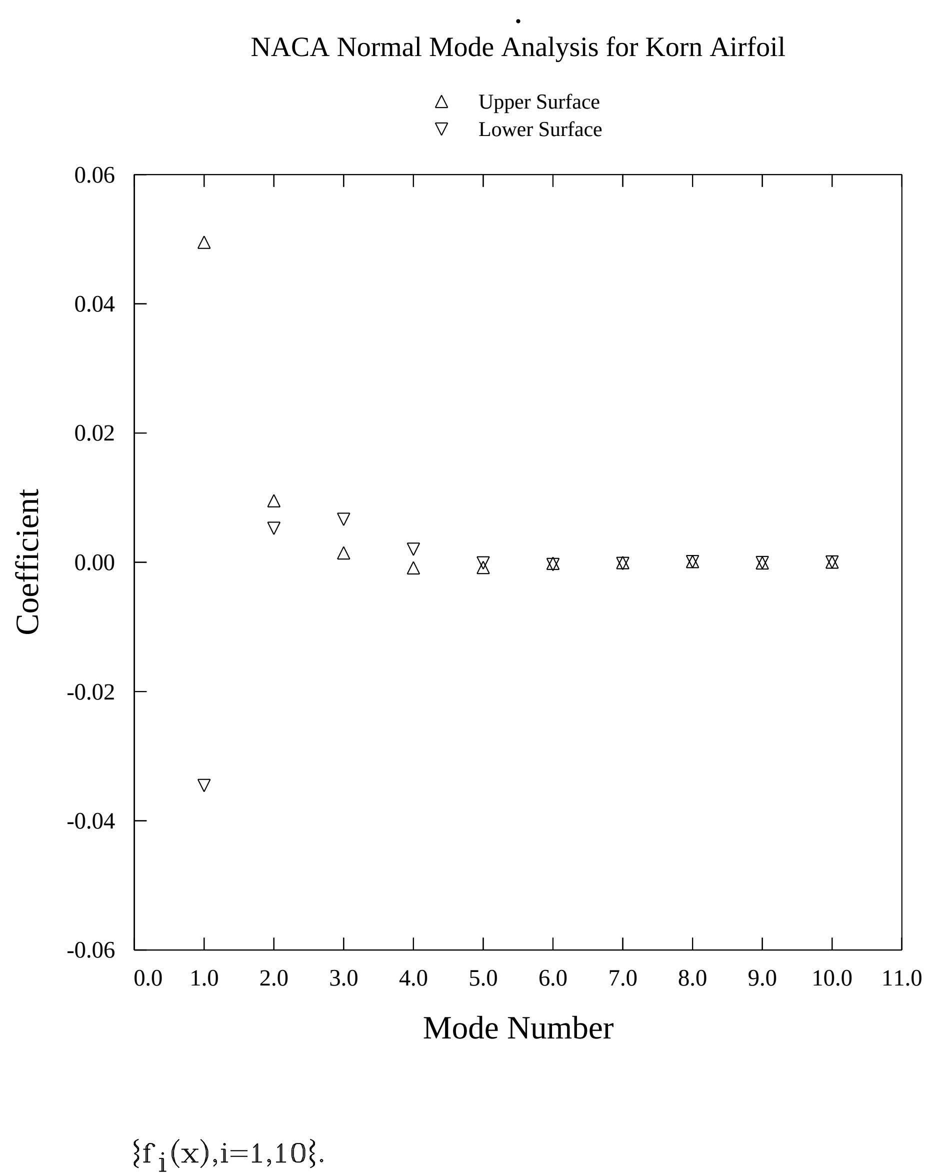


Figure 7. The normal mode coefficients of the upper and the lower surfaces of the Korn airfoil corresponding to the normal modes, $\{f_i(x)$, i = 1,10$\}$.

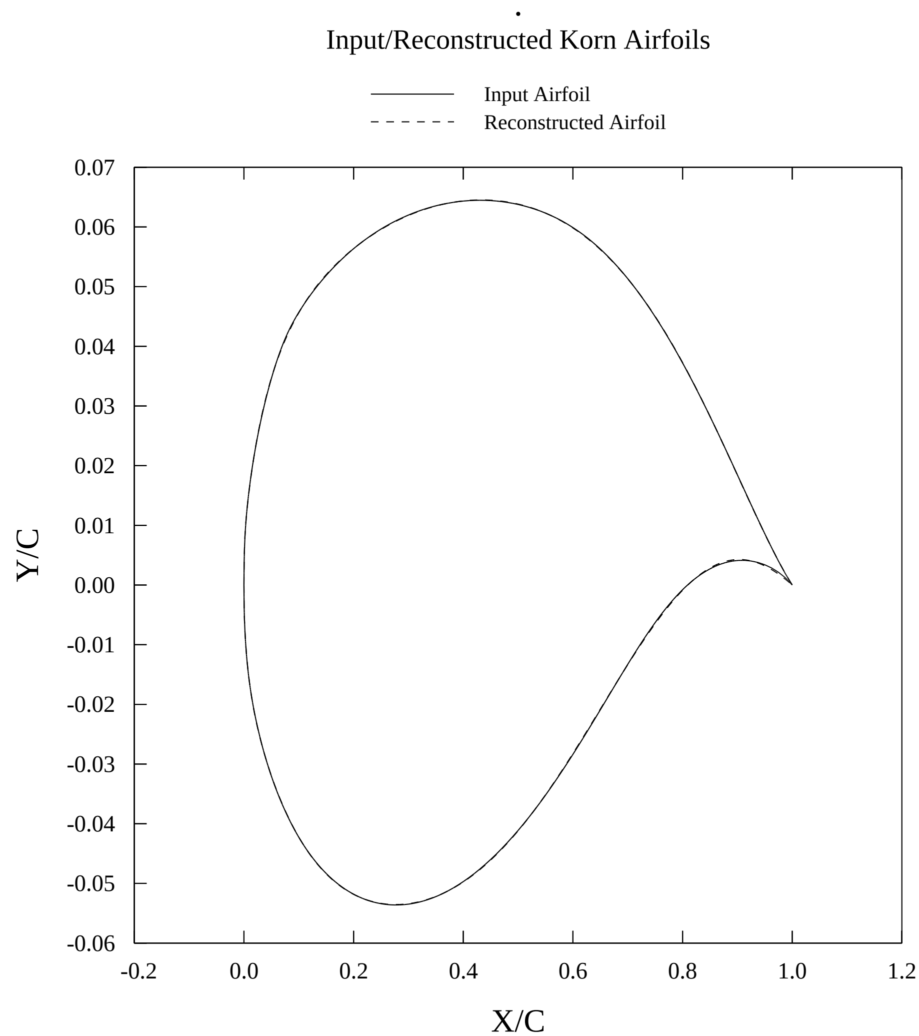


Figure 8. The guppy plots of the input Korn airfoil and its reconstructed airfoil via the normal mode analysis.

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