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Wheel Climb Derailment Criteria for Evaluation of Rail Vehicle Safety

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ABSTRACT

Criteria for evaluating safety of rail vehicles with respect to wheel climb derailment are reviewed. The relationship between flanging wheel lateral to vertical force ratio at impending derailment and angle of attack, lateral velocity and longitudinal creep is discussed. This ratio has no explicit relationship with time duration. Empirical relations in current use result from specific vehicle and track operating conditions tested. It is observed that the lateral to vertical force ratios on the non-flanging wheel at impending derailment are controlled by the same kinematic parameters of axle motion as on the flanging wheel. The information available from measurements on the non-flanging wheel are sufficient to permit definition of new wheel climb safety criteria that are consistent with current wheel-rail interaction force theory which include situations where lateral to vertical forces in excess of Nadal's limit may be achieved.

INTRODUCTION

In conducting tests to evaluate the derailment safety of rail vehicles under prescribed track and operating conditions it is necessary to employ criteria which provide a measure of the margin of safety associated with the measured dynamic performance. These criteria are necessary to establish the probable safety of proceeding to test at the next higher speed in the planned test series or to proceed to a more severe track or operating condition. The criteria are also necessary to provide a ranking of the relative safety of different rail vehicle designs and configurations. The modes of derailment of concern in these tests are generally:

1-Excessive car body movement resulting in the car separating from the truck and rolling over on the track

2-Excessive wheel-rail forces causing either rail failure or gage widening permitting the wheels to fall to the ties and ballast.

3-Wheel climb where the rolling motion of the wheels combined with lateral force causes the wheel to climb over the rail without a preceeding structural failure being necessary to the derailment.

The relative safety of the first two modes is readily established by comparing measured car body motions and wheel rail forces with prescribed motion limits and rail restraint capability limits. The tendency towards wheel climb is more difficult to establish since the difference between the axle motions associated with safe and unsafe operation is small compared to typical axle motions. The ratio of lateral to vertical force on the flanging wheel $(L/V)_F$ is typically used as a surrogate measure of wheel climb tendency. Between 1896 and 1908, Nadal (1),(2), based upon a highly simplified wheel-rail interaction force model, developed a derailment quotient that defines the minimum $(L/V)_F$ required for wheel climb to ocurr. For a typical flange angle of 65-degrees and a coefficient of friction of 0.5 the Nadal limit is about 0.8. Gilchrist and Brickle in 1976 (2) using current theory of wheel-rail interaction forces combined with measurements from laboratory experiments studied the relationship between the $(L/V)_F$ required for derailment and angle of attack and longitudinal creep. These results showed that Nadal's results were correct for high angles of attack and low longitudinal creep. However for lower angles of attack and large longitudinal creep the Nadal limit was found to be quite conservative.

In the early 1960s the conservative nature of the Nadal limit was demonstrated by Japanese investigators in both laboratory and full scale derailment experiments. (3),(4),(5) In tests conducted on locomotives in the United States (6) it was observed that flanging wheel L/V ratios as large as 2 were achieved without derailment. In these data it was observed that when large $(L/V)_F$ ratios were observed the time duration associated with the force pulse was small. Reference (6) also notes that the increase in the $(L/V)_F$ ratio was typically a result of a decrease in the vertical force rather than an increase in lateral force. Based upon the observed data and a heuristic momentum transfer formulation (4) the Japanese formulated a relationship between the $(L/V)_F$ ratio at incipient derailment and the time duration of the force pulse. This criteria was adopted by the Japanese National Railways and has become popularly known as the JNR Criteria for dynamic wheel climb derailment. Since the momentum transfer formulation considers an effective unsprung axle mass in the calculations, the relation between critical $(L/V)_F$ and time duration would be expected to be a function of the wheel size and axle mass. Accordingly, investigators at Electromotive Division of General Motors Corporation (6) have been using a scaled up version of the JNR Criteria, known as the EMD Criteria, permitting higher $(L/V)_F$ ratios for a given pulse duration based upon the test data they have obtained.

Since the analytic formulations associated with the JNR Criteria contained a number of assumptions which are difficult to fully justify in terms of current understanding of wheel-rail interaction forces, the U.S. Department of Transportation contracted with Princeton University to perform a series of scale model experiments to investigate the wheel-rail forces developed under dynamic derailment conditions. The results of these measurements as reported by Sweet (7),(8) showed excellent agreement between the predictions of current wheel-rail force theory (as described in References (9) and (10)) and measurement of wheel-rail force. In the derailment experiments under dynamic conditions, however, the measurements showed a number of cases where a derailment occurred and the JNR Criteria predicted no derailment. Furthermore, there were a number of cases where the JNR Criteria predicted derailment and no derailment actually occurred. In reviewing the results of his experiments and dynamic simultaion studies Sweet concludes that a general relationship between the L/V ratio at incipient derailment and time duration does not exist. These results caused investigators to believe that the JNR and EMD criteria were at best valid for specific vehicle configurations and operating conditions. Pragmatically, however, investigators continued to use the JNR Criteria as a guideline for conducting performance tests.

In conducting such tests using instrumented wheelsets, investigators at Transportation Systems Center began to note that critical information could be extracted from the forces measured on the non-flanging wheel to provide additional confidence in the safety of test operations. For example, the $(L/V)_T$ on the non-flanging wheel could be used to estimate the friction coefficient between wheel and rail. The direction of the lateral force on the tread provides an indication of the sign of the angle of attack. These observations combined with the results of current theoretical and experimental findings on the nature of wheel-rail forces at incipient derailment have formed the basis for formulation of a new set of dynamic derailment criteria which are the subject of this paper.

FLANGING WHEEL DERAILMENT FORCES

For purposes of this paper, an incipient derailment is the condition where the forces acting on an axle have caused a wheel flange to climb the rail to the point where the maximum possible angle between the plane of contact of the flange and rail and the track surface has been reached, as shown in Figure 1. For typical wheel and rail profiles this angle ranges from 60 to 70-degrees. Although the calculations presented here are for a flange contact angle of 65-degrees, similar results would be obtained for typical flange angles.

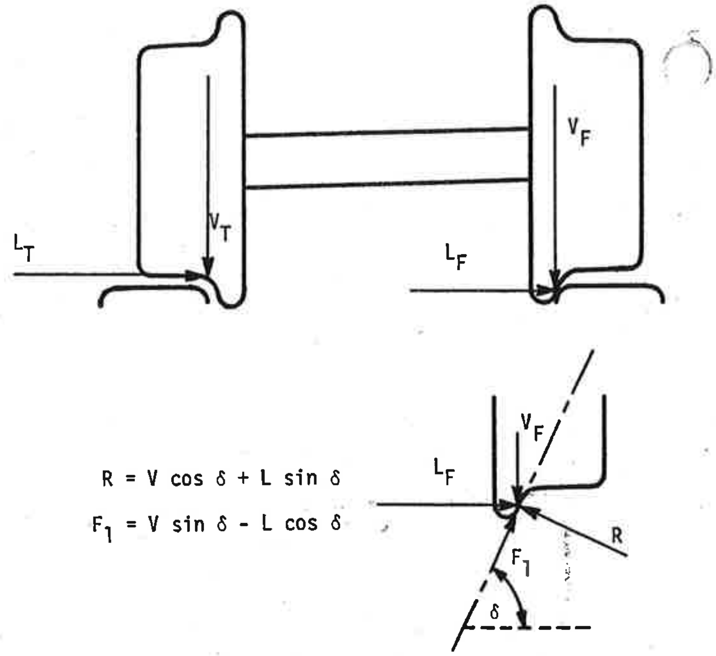


FIGURE 1. WHEEL-RAIL FORCES ON AXLE AT INCIPIENT DERAILMENT

Figure 1 shows the equilibrium between the lateral and vertical forces exerted by the vehicle on the wheel-rail contact point with the normal and tangential forces exerted by the rail with respect to the wheel-rail contact plane at an incipient derailment condition. Since the point of contact is massless, equilibrium requires:

$$R = V\cos\delta + L\sin\delta \tag{1}$$

$$F_1 = V\sin\delta - L\cos\delta \tag{2}$$

If we define μ_e as the ratio between the tangential force F_1 and the normal to the contact plane we may substitute:

$$F_1 = \mu_e R$$

to obtain:

$$\frac{L}{V} = \frac{\sin\delta - \mu_e \cos\delta}{\cos\delta + \mu_e \sin\delta} = \frac{\tan\delta - \mu_e}{1 + \mu_e \tan\delta} \tag{3}$$

If we substitute the coefficient of friction μ we obtain the Nadal limit for the minimum L/V ratio required to produce derailment.

If we assume that the measurement of the lateral and vertical forces is located at a distance from the point of contact and that the resultant of the vehicle and rail contact forces acts on an effective mass M_e, we obtain:

$$L(\cos\delta + \mu_e \sin\delta) - V(\sin\delta - \mu_e \cos\delta) = M_e \ddot{s} \tag{4}$$

If these forces act for a period DT, a velocity $\dot{s}$ is produced. Defining a critical velocity for derailment to occurr $\dot{s}^*$, we obtain the L/V required for derailment as:

$$\frac{L}{V} = \frac{\sin\delta - \mu_e \cos\delta}{\cos\delta + \mu_e \sin\delta} + \frac{\dot{s}^* M_e}{V(\cos\delta + \mu_e \sin\delta)(DT)} \tag{5}$$

Equations (4) and (5) represent a simplified derivation of the momentum formulation used to develop the JNR and EMD time duration criteria. However the quantities $\dot{s}^*$ and M_e are to a high degree vehicle and measurement system dependent and at best extremely difficult to estimate analytically. Moreover, the measurements reported recently with instrumented wheels have been demonstrated to have no associated effective mass and are in good agreement with way side measurements. The large L/V ratios of 2 reported in Reference (6) could not be accounted for by mass effects.

A more satisfying explanation for the large L/V ratios observed consistently in dynamic tests is that the coefficient μ_e is less than the coefficient of friction and under the observed test conditions was probably a negative number as defined here. As depicted in Figure 2 the force F_1 is not the only component of the friction force. The resultant friction force in the plane of contact also has a component in the longitudinal direction produced by the longitudinal creep of the wheel. This longitudinal creep rate is a function of the difference in rolling radius of the flanging wheel and non-flanging wheel contact points and the yaw rate of the axle. The derailing friction force F_1 itself is made up of two components. One is due to the spin creep produced by the component of the axle angular velocity in the plane of contact and the other is produced by the angle of attack. The true angle of attack is defined here as the angle between the direction of the velocity of the axle center and the normal to the axle center of rotation, as shown in Figure 3.

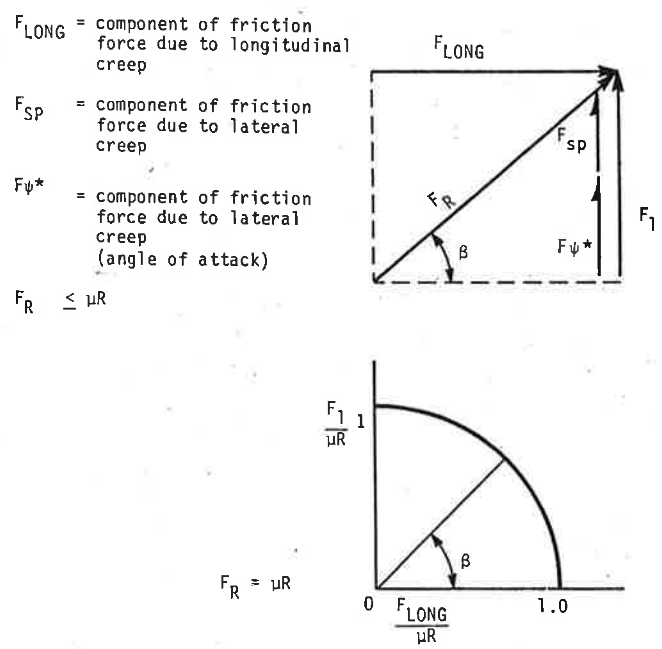
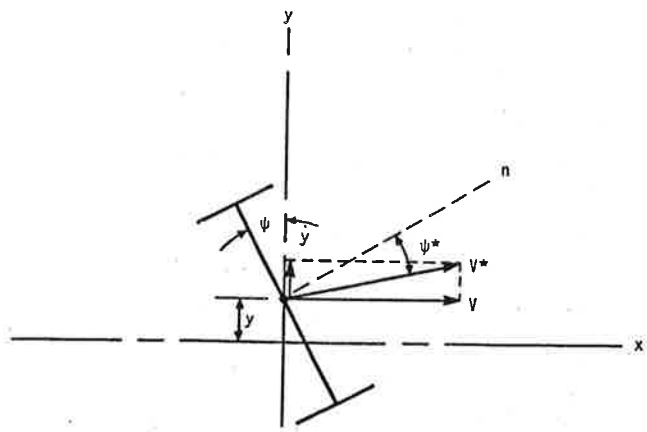


FIGURE 2. COMPONENTS OF WHEEL-RAIL FRICTION FORCE IN CONTACT PLANE

It is conventional to define an axle coordinate system at the nominal centerline of the track, with the angle ψ representing the angle between the normal to axle center of rotation and the track centerline and to define an axle lateral displacement y and lateral velocity $\dot{y}$ as shown in Figure 3. In this notation the true angle of attack is $\psi - \dot{y}/V$.



y = lateral displacement from track nominal centerline
V = axle center velocity in direction of nominal track centerline
x = nominal direction of axle movement along track
$\dot{y}$ = lateral velocity of axle centerline
ψ = axle yaw angle
n = normal to axle centerline
$\psi^* = (\psi - \frac{\dot{y}}{V})$ = "true angle of attack"
V* = instantaneous axle translational velocity

FIGURE 3. DEFINITION OF CONVENTIONAL AXLE COORDINATE SYSTEMS AND "TRUE ANGLE OF ATTACK"

The magnitude and sign of the lateral creep is a variable with the angle of attack, so that the force F_1 may be either positive or negative. With negative angles of attack the friction forces act to inhibit derailment and much larger L/V ratios can be sustained.

Current theory of rolling contact as developed by Kalker (11) has been applied to the development of accurate algorithms for the prediction of wheel-rail forces as a function of wheel and rail geometry and kinematic conditions by Elkins and Gostling (9) and Elkins and Eickhoff (10). These algorithms have been validated by both laboratory and field test data.

Figure 4 shows the L/V ratio for the flanging wheel of a typical freight car axle with no yaw rate and equal loads on the flanging and non-flanging wheels, as calculated by the algorithms developed in References (9) and (10).

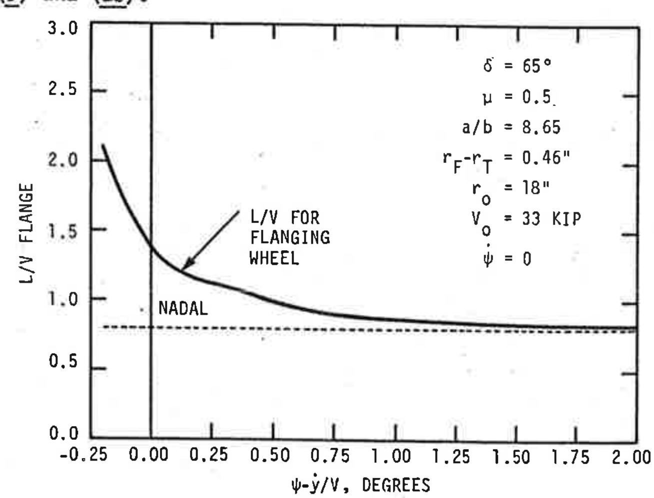


FIGURE 4. L/V RATIO OF FLANGING WHEEL AT INCIPIENT DERAILMENT FOR AXLE WITH ZERO YAW RATE, VS. EFFECTIVE ANGLE OF ATTACK

As seen in Figure 4 the L/V ratio required to climb the rail is very much a function of the angle of attack. At large angles of attack, the creep forces reach saturation and the longitudinal creep due to the difference in rolling radii is small compared to the angle of attack. This results in the force F_1 approaching the product of the coefficient of friction μ and the normal force to the plane of contact R. This is the assumption of the Nadal limit derivation so that at high angles of attack the L/V ratio approaches the Nadal limit. As the angle of attack is decreased, the lateral creep forces are reduced and the ratio of longitudinal creep to lateral creep increases changing the direction of the resultant friction force shown in Figure 2. This has the effect of reducing the derailing force F_1 and the coefficient μ_e in Equation (3), producing a higher L/V ratio required for derailment. At zero angle of attack the force F_1 is produced totally by the spin creep and is limited in magnitude by the longitudinal creep. At negative angles of attack, the lateral creep force is in the opposite direction to the spin creep force and the sign of the derailing force F_1 changes so that it acts to inhibit derailment. At a given angle of attack as seen in Figure 2, an increase in longitudinal creep results in an increase the L/V ratio required for derailment at positive angles of attack.

For a given load, and contact geometry, the only parameters that influence the L/V calculations are the true angle of attack and the longitudinal creep. This suggests that if the longitudinal creep and the angle of attack could be measured in field tests it would be possible to use these calculations to evaluate the margin of safety from derailment. Until recently longitudinal force measurements were not included in field test programs. Although there have been successful measurements of the difference in angle of attack of successive axle passings, measurement of the true angle of attack has proved to be elusive because of the small angles involved and the difficulty in defining an appropriate absolute frame of reference.

NON-FLANGING WHEEL FORCES AT INCIPIENT DERAILMENT

As noted above, the forces on the non-flanging wheel are also functions of the angle of attack and longitudinal creep. The L/V ratio for the non-flanging wheel of the axle discussed above with zero yaw rate and equal vertical loads is shown in Figure 5. Spin creep on the non-flanging wheel is small and the lateral force is controlled by the lateral and longitudinal creep. For the condition of equal wheel loads and zero yaw rate the longitudinal creep produced by the rolling radius difference between the contact points of the flanging and non-flanging wheel is large enough to saturate the friction force. For zero angle of attack, the non-flanging wheel is totally sliding longitudinally with a longitudinal friction force equal to μV. Since the axle is not accelerating rotationally and the rolling radii of the flanging and non-flanging wheel are equal to within 5%, the longitudinal creep force on the flanging wheel is approximately equal in magnitude and opposite in direction to the longitudinal force on the non-flanging wheel. As the angle of attack is increased the friction force on the non-flanging wheel changes direction to that of the resultant of the lateral and longitudinal creep. At large angles of attack the lateral creep is much larger than the longitudinal creep and the L/V ratio approaches the sum of the friction coefficient and the component of the normal reaction force to the vertical load.

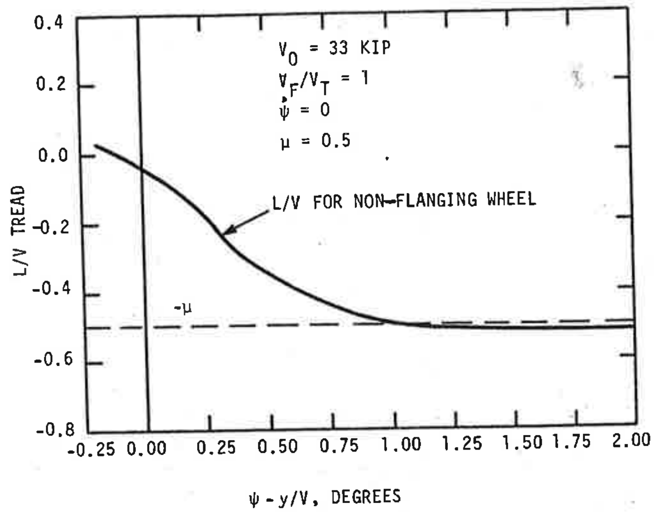


FIGURE 5. L/V RATIO OF NON-FLANGING WHEEL AT CONDITION OF INCIPIENT DERAILMENT OF FLANGING WHEEL FOR AXLE WITH ZERO YAW RATE

DERAILMENT CRITERIA HYPOTHESIS

Figure 6 plots the L/V ratios of both the flanging and non-flanging wheels at incipient derailment on the same scale for the condition of equal vertical loads and zero yaw rate. Inspection of these two curves suggests that the difference between the two L/V ratios is independent of angle of attack for positive angles of attack and increases as the angle of attack becomes more negative for negative angles of attack.

This suggested the hypothesis that derailment could not occur if:

$$(L/V)_F - (L/V)_T < \mu + \text{NADAL LIMIT} = \text{DRCRT} \quad (6)$$

This hypothesis would be true, if for all axle kinematics and loading situations the ratio:

$$Q = \frac{(L/V)_F - (L/V)_T}{\text{DRCRT}} > 1 \quad (7)$$

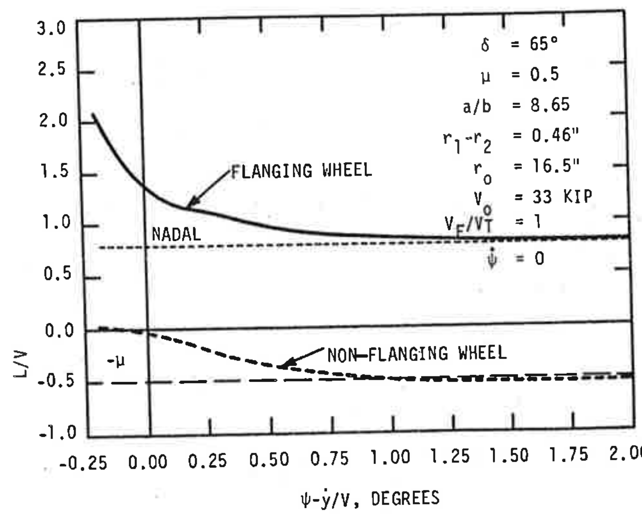


FIGURE 6. COMPARISON OF L/V RATIOS OF FLANGING AND NON-FLANGING WHEELS AT INCIPIENT DERAILMENT

Figure 7 plots DRCRT and the Nadal limit against the friction coefficient for a typical wheel profile with a 65-degree flange angle. It is seen that for the range of friction coefficients normally encountered, DRCRT is relatively constant while Nadal's limit changes rapidly. This feature of the proposed criteria is particularly useful when the friction coefficient is not known or varies during the course of testing.

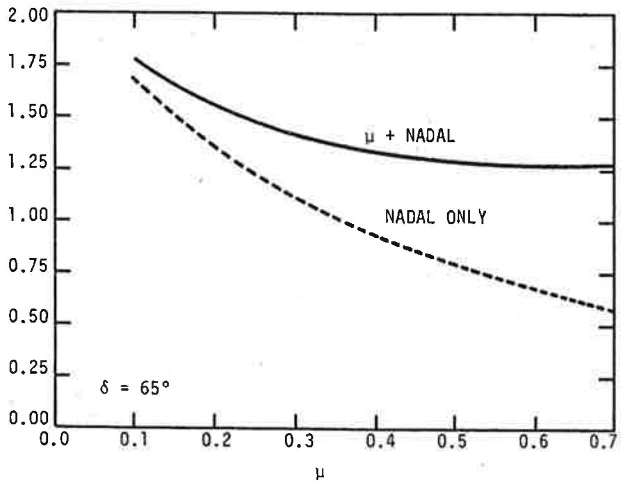


FIGURE 7. VARIATION OF "DERAILMENT QUOTIENTS" WITH FRICTION COEFFICIENT

As discussed below, parametric studies were conducted to test the criteria for a range of kinematic and loading conditions.

INFLUENCE OF VERTICAL WHEEL LOAD

Figure 8 shows the behavior of the ratio of the difference in L/V ratios to DRCRT as a function of angle of attack for an axle load of 66 kips and varying distributions of vertical load between the flanging and non-flanging wheel with zero yaw rate. It is seen from Figure 6 that the criteria is valid when the vertical load on the flanging wheel is less than the vertical load on the non-flanging wheel for positive angles of attack and is always valid for negative angles of attack. As noted in Reference (6) the most common observed situation where L/V ratios well above Nadal's limit occur are wheel unloading situations where the vertical force on the flanging wheel is less than that on the non-flanging wheel.

Failure of the criteria to hold when the non-flanging wheel is unloading is due to the behavior of the longitudinal creep force on the flanging wheel. As noted above the longitudinal force on the non-flanging wheel is limited to the product of μ and the vertical load and the longitudinal force on the flanging wheel is approximately equal to that on the non-flanging wheel. As vertical load is reduced on the non-flanging wheel the longitudinal creep force on the flanging wheel decreases limiting its ability to restrict the derailing friction force F_1 produced by the spin creep and angle of attack.

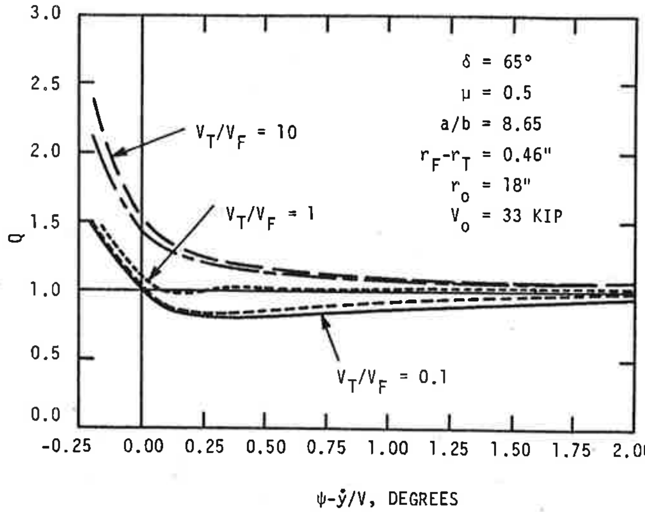


FIGURE 8. INFLUENCE OF VERTICAL LOAD DISTRIBUTION ON PROPOSED DERAILMENT CRITERIA FOR 66 KIP ANGLE LOAD

Figure 9 shows the sum of the magnitudes of the two L/V ratios as a function of angle of attack without the normalization to DRCRT. It is seen that even for situations where the vertical force on the non-flanging wheel is 10% of that on the flanging wheel, the sum of the magnitudes of the two L/V ratios is never less than 1.0.

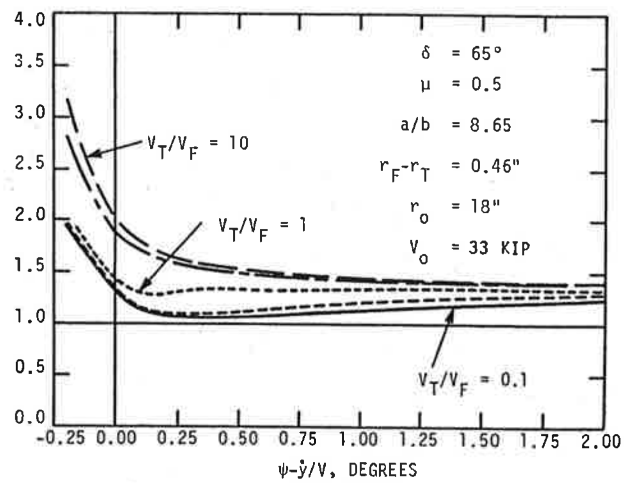


FIGURE 9. INFLUENCE OF VERTICAL LOAD VARIATION ON THE SUM OF THE MAGNITUDES OF L/V RATIOS OF FLANGING AND NON-FLANGING WHEELS AT INCIPIENT DERAILMENT VS. ANGLE OF ATTACK FOR 66 KIP AXLE LOAD

Figures 10 and 11 show the same results for the case of an 18 kip axle load. The results appear relatively independent of axle load for this range (axle loads from 18 to 66 kips) which encompasses most U.S. rolling stock.

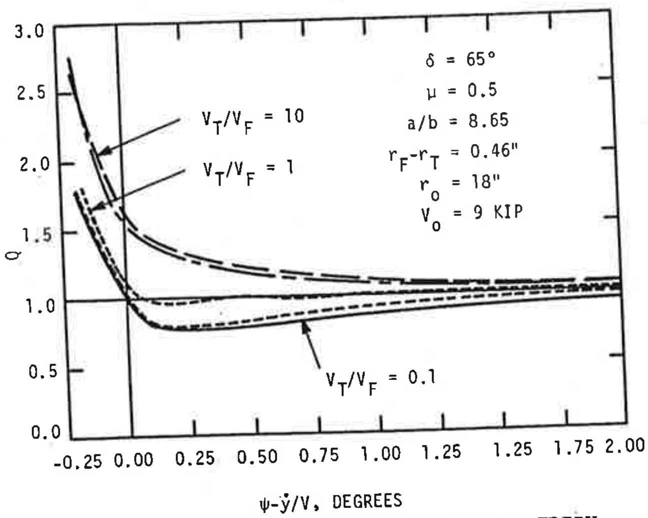


FIGURE 10. INFLUENCE OF VERTICAL LOAD DISTRIBUTION ON PROCESSED DERAILMENT CRITERIA FOR 18 KIP AXLE LOAD

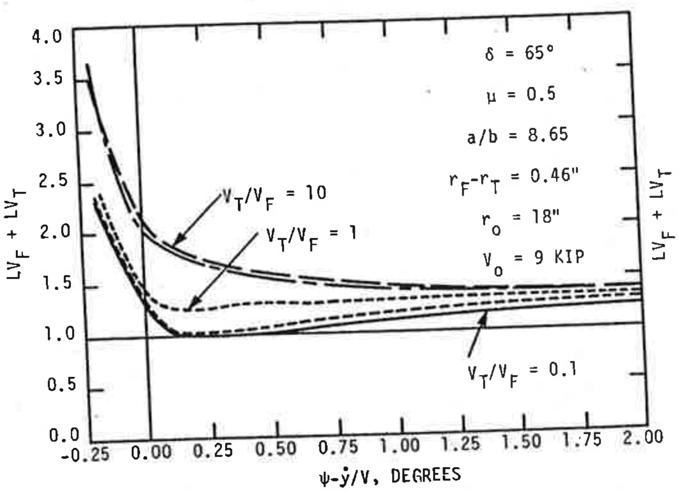


FIGURE 11. INFLUENCE OF VERTICAL LOAD VARIATION ON THE SUM OF THE MAGNITUDE OF THE L/V RATIOS OF FLANGING AND NON-FLANGING WHEELS AT INCIPIENT DERAILMENT VS. ANGLE OF ATTACK FOR 18-KIP AXLE LOAD

INFLUENCE OF YAW RATE

For a given wheel-rail geometry and vertical load ratio between the flanging and non-flanging wheels, the only factors that influence the L/V ratios of the two wheels of an axle are the true angle of attack and the axle yaw rate. The effect of yaw rate is to increase or decrease the longitudinal creep at each wheel produced by the rolling radius difference between the contact points on the flanging and non-flanging wheel. As discussed above, an increase in longitudinal creep has the effect of increasing the L/V ratio required for the flanging wheel to climb over the rail and of reducing the L/V ratio of the forces generated on the non-flanging wheel. This results in a partial cancellataion of the effect of yaw rate on the sum of the two L/V ratios.

Figure 12 shows the behavior of the sum of the two L/V ratios normalized to DRCRT for yaw rates of 3.25, 1Ø.3 and 32.5-degrees of axle rotation for 1ØØ-feet of forward movement. At a speed of 6Ø mph these yaw rates would correspond to axle yaw angular velocities of 2.9, 9.1 and 29-degrees per second. As seen in Figure 12 for the range of yaw rates and track curvatures expected in test applications the yaw rate has only a minor influence on the proposed derailment criteria.

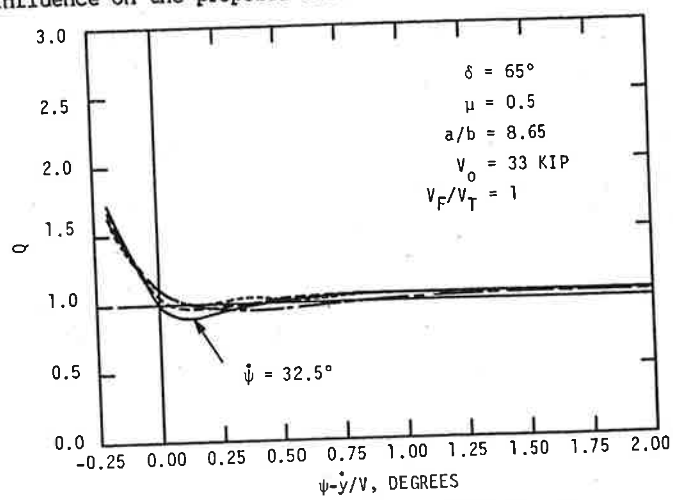


FIGURE 12. INFLUENCE OF YAW RATE (CURVATURE) ON PROPOSED DERAILMENT QUOTIENT

CONCLUSIONS

Experience obtained in both laboratory and field tests combined with the parametric studies of the behavior of wheel rail forces described above indicates that judicious use of the L/V ratios on both the flanging and non-flanging wheel permits development of criteria for assuring safety of dynamic vehicle track interaction situations where the flanging wheel L/V ratio can safely exceed the Nadal limit.

Wheel climb derailment will not occur if any of the following conditions are met:

A-The lateral to vertical force ratio on all wheels is less than Nadal's limit.

B-The sum of the magnitudes of the lateral to vertical force ratios on both the flanging and non-flanging wheels of an axle are less than 1.Ø.

C-The vertical force on the flanging wheel is less than the vertical force on the non-flanging wheel and the sum of the magnitudes of the L/V ratios on both wheels of an axle is less than the sum of Nadal's limit and the coefficient of friction.

D-If the lateral forces on both wheels of the axle are in the same direction (the angle of attack is negative) and the sum of the two L/V ratios on the axle is less than the sum of the Nadal limit and the coefficient of friction.

When both the flanging and non-flanging wheel lateral forces are in the same direction the true angle of attack is negative and even higher lateral to vertical force ratios can be achieved. For special situations, detailed knowledge of field conditions combined with judicous use of all of the force measurement information available can be combined to provide less conservative criteria than those given above. Since the above criteria are consistent with current theoretical formulations of wheel-rail interaction behavior and with existing experimental data, it is recommended that the above criteria be used in future evaluations of rail vehicle track interaction safety.

ACKNOWLEDGEMENT

The work described here was performed as part of the Track Safety Research Program sponsored by the Federal Railroad Administration of the U.S. Department of Transportation. The author would like to thank Mr. John Elkins, Dr. Fred Blader and Professor David Wormley for helpful discussions on the behavior of wheel-rail interaction forces and for encouraging the author to formalize the criteria that had developed in the course of test programs conducted by Transportation Systems Center. The calculations of wheel-rail forces used in this paper were performed by Mr. G. Mealey of The Analytic Sciences Corporation under Contract to Transportation Systems Center.

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