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PHYSICAL FOUNDATIONS FOR SOCIO-ECONOMIC MODELING FOR TRANSPORTATION PLANNING

Part I: Interaction Between Urban Centers as a Potential Process

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16. Abstract

The objective of this research is to make use of a physically based social system model to study the determinants of city sizes and their interactions in a nation. In particular, it was required that attention be paid to how new transportation systems affect city sizes.

In this first part of a final report, the character of the distribution function for settlements of man is investigated. The distribution for weakly interacting settlements (early man as hunter-gatherer) is developed and experimentally tested against historical data. The distribution function for interacting settlements (since agricultural settlements) - Zipf's law - is then treated, first as a pure information theoretic, namely as a communicational "living" language, and then as a communicational language for communities of man loosely bound to the earth. To keep the ensemble alive, the need for good cheap transportation among a significant mobile fraction of the population is discussed.

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PREFACE

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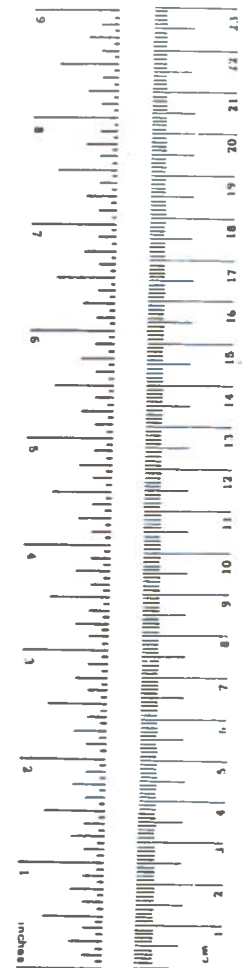
METRIC CONVERSION FACTORS

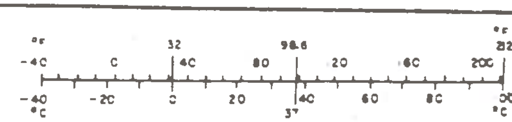
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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1. INTRODUCTION

Under a previous phase contract, a physical basis was developed for modeling a social system (1).* That phase was heavily indebted to earlier reports developed in Army contracts (2). In the current contract, the objective is to use these developments to study the determinants of city size, and how transportation systems are related to city sizes. More broadly, the purpose of this phase is to study the relation between socio-economic factors, including demographic factors, governing cities and transportation planning (such as the introduction of new transportation systems).

Since there are some existing notions of a potential-like nature (Zipf's law, demographic potential theory) extant in urban planning (see (3), (4), (5), (6), (7), (8); also the information theoretic literature (9), (10), (11), (12)), it was requested that the effort be made to clarify their foundations and relate them to the new theory of a social physics. This is the problem to be addressed in this report. The intent of this study has been to develop a solid physical distribution function basis for some such demographic potential methodology to be used in national transportation planning.

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*It is useful to remember that the model related to the concept of the following 5 physical compartments, and 5 potentials:

1. Materials balance
2. Energy balance
3. Human activity balance
4. Population balance
5. Economic balance.

1. Environmental temperature
2. Environmental sources of free energy and materials
3. Genetic value systems (whereby human reproduction is insured)
4. Epigenetic (learned) value system
5. Technological potential in the human mind.

2. PRELIMINARY DISCUSSION OF ZIPF'S LAW

A first task that was proposed was: given a nation and its field potentials, provide a constructive derivation for Zipf's law as it related to the size distribution of cities. Zipf's law states that the distribution in city sizes in an interacting organization is distributed inversely approximately as the first power of the rank ordering of size:

$$pr = p_o$$

$$P_1 = \Sigma p = p_o + \frac{p_o}{1} + \frac{p_o}{2} + \ldots + \frac{p_o}{k}$$

where:

- p = population of particular cities
- r = rank order of cities in size
- Σp = cumulative urban population (P_1)
- p_o = a characteristic population size
- k = the limiting rank order of city size.

According to our notions of social physics, we are restricting our task to national systems possessing urban centers which form autonomous thermodynamic systems.* These systems possess all of the required potentials, and operate freely in an ergodic sense within the essential thermodynamic compartments. That is, they can freely exchange materials, energies, persons and current technology. Further, we believe that there tends to be a ragged cut-off for k.

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*This is opposed, say, to slave societies, in which some outside agency runs the society by fairly strict rules. Note how some such restricted performance is placed by man on his own imprisoned species: rats, chickens, wheat, cows, bacteria, or even "wild" species in a national park when he rasies or cultivates them.

That is, below some critical size p_o/k, any small population concentration is no longer part of an urban center. In some sense it is largely part of an agricultural community. The particular size p_o/k, as well as p_o, are characteristic, in an a priori sense, of a nation, its ecology, a technological era, and the historical status of its civilization. With these restrictions (we are not trying to derive a notion of Zipf's law as an absolutely general statistical property of any or all ensembles), we can move toward a constructive discussion of Zipf's law. In order to achieve this goal we have to assemble a considerable number of disparate pieces. A final resolution, we believe, will bring them all into line.

At this point we would simply like to offer the following approximate summary of Zipf bound social ensembles:

$$p_1 = p_o \left[\ln \left(\frac{2\ p_o}{'250'} + 1 \right) - 0.1 \right]$$

where:

p_1 = total nonfarm population (all associated with urban-rural settlements or transiently so associated. We regard these settlements as essentially greater than pre-Neolithic settlements which range from perhaps 40-200. Its cut off is assumed a priori to be at a '250' level)

p_o = the population of the largest city (the characteristic population in Zipf rank-ordering).

As a rough derivation, we have:

$$pr = p_o \qquad \text{Zipf's law}$$

$$p_1 = \sum_1^k p = p_o \sum_1^k \frac{1}{r} :: p_o + p_o \int_{1.5}^{k+.5} \frac{1}{r}\, dr$$

$$= p_o \left[\ln\ (2k+1) - 0.10 \right]$$

But $k = \frac{P_o}{'250'}$, if '250' is taken as limiting the settlement size that can be Zipf rank-ordered.

We hope to convince the careful reader that a second segment of a social physics is in progress. If we regard the earlier segment (Army Research Institute reports (2), 4 DOT reports (1)) as a preliminary philosophic and descriptive outlining of a system's thermodynamics for living systems, then we are beginning now to outline some hard physical principles. We are not yet up to the detailing of long range equations of change; we can describe the physical consequences of thermodynamic closure for systems that persist in spite of life and death processes. In particular, we propose to convince the reader that we are dealing with the "communications" aspects of such thermodynamic systems.

3. ON A PRIMITIVE BASIS FOR DISTRIBUTION FUNCTIONS

The problem is to provide the most parsimonious scientific construct for physical distribution functions. The following seems to be a minimal path for very minimally constrained ensembles:

a. Given an ensemble of material atomisms with well-locked up internal form.

b. Its motion is supported by a radiation field.* What is the nature of the most primitive distribution function that develops, and why?

c. As a minimal result of low loss collisions, in which the radiation interactions will make up for the low losses, there will be certain conservations that will be maintained over the collision cycles. Namely, an extremum entropy state will be developed,** characterized by a mean kinetic energy, a mean monentum, and mean mass. Also, a fluctuating entropic cycle will develop, characterized by a mean fluctuating energy, momentum, and mass.

d. Now these dynamically constrained interactions form discrete time sequence sets in a measurement sense. The resulting reaction of a collision provides a measure of the reaction "force" according to Newtonian physics. Thus we can apply statistical reasoning to these "observational sets"; in fact, they become directly observable in the Einstein modeling of Brownian motion.

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*Its motion may be supported by a deus ex machina according to any prescription that the outside agency wishes to maintain, but we are concerned with autonomous support. That basically means by fixed potential fields - "sources" - which the material atomisms can draw from. Thus we must visualize the source to be a radiation field originating at boundaries, or traceable to that source. In the living system, the sun is the radiation source, and the living system motion is traceable (via its genetic code and biochemistry) to that source - directly in the case of photosynthetic species, or indirectly through the ecology of flora and fauna to that source.

**That is, the system will be incapable of doing external work.

e. We select the philosophy that Lees and Janes have offered to clarify a more unified LaPlace-Bayes information theoretic for classical statistics, but we are not interested in the general argument: we are satisfied with a narrow view - given a random process with limited moments, then the distribution is fixed (e.g., fixing two moments under this condition gives rise to a Gaussian distribution).

f. We will start with only three assumptions:

1. What we believe to be the Boltzmann assumption - that as a result of the collisions among particles, the distribution of mechanical fluctuations can be described by "independent" statistical events whose associative probability is the product of independent probabilities (i.e., a "usual" statement about mutually independent causal systems).

2. What we believe to be the essential foundation of Newtonian mechanics - that as a result of collisions, given m (an intrinsic characteristic of material systems, a parameter that cannot be transformed away), then there are three existing conservations, which in fact are integral:

$$mnu^0,\ mnu^1,\ mnu^2$$

where

m	=	mass
n	=	number
u	=	velocity
Σmn	=	number, which is conserved
Σmnu	=	momentum, which is conserved
Σmnu^2	=	vis viva (or more modern 1/2 = kinetic energy), which is conserved.

For an ensemble, we are thus given that these "three" moments - a zeroth momentum of number; a first moment of momentun; a second moment of kinetic energy - are conserved throughout the collision cycle among the ensemble.

3. What we believe to be a composite statistical-physical assumption, consistent with both fields, including irreversible thermodynamics, and the specific arguments of Boltzmann, Maxwell, Einstein, Nyquist, Shannon, Lees, Jayne, and now us, that the logarithm of the distribution function f has to be a linear association of these integral moments:

$$\text{Ln } f = Amn + B_1 mnu + B_2 mnv + B_3 mnw + C_1 mnu^2 + C_2 mnv^2 + C_3 mnw^2$$

Since Newtonian mechanics is a vector law there are three velocity and thus momentum components (f is the associated probability).

Then the usual arguments condense the results. If the ensemble is isolated, then mn is a constant; u^2 can be rearranged to $(u-u_o)^2$; lack of preference makes $B_1 = B_3$, $C_1 = C_2 = C_3$.

$$\text{Ln } f = \text{Ln } A + C\left[(u-u_o)^2 + (v-v_o)^2 + (w-w_o)^2\right]$$

and if the first moment is fixed by choice of axes and isolation to $u_o = v_o = w_o = 0$,

$$f = A\,e^{C(u^2 + v^2 + w^2)}$$

then the standard procedure - C being an intensive distributed parameter, contact with an ideal gas thermometer - provides the metric

$$f = A\,e^{-\frac{(u^2 + v^2 + w^2)}{kT}}$$

and we have the Maxwell-Boltzmann distribution.

How was it gotten? From statistical independence of the moments, independent of how many members were in the colliding ensemble, and the fact that the laws of physics gave values to a limited and integral number of moments, for the act of simple collisions.

This derivation,

$$ff' - \overline{f}\overline{f'}$$

strikes us as more primitive than all of the arguments wherein two particles in a collision with probability distributions f and f' for each, prior to collision and $\bar{f}, \bar{f}'$ for each, after a collision (see Maxwell, Joos, Tolman, etc.).

Basically, such proofs try elaborately to show that a collisional cycle can be treated by mechanical reversibility. We simply assert that if particles don't coalesce, they will go through cycles of performance; these are thermodynamically lossy, so that they must sink down to the lowest sustainable limit cycle. That provides the equilibrium of a uniform extremum entropy.

Thus the nature of the velocity distribution function has been described. What is the nature of the displacement function? We shall show a similar moment argument due originally to Einstein (Langevin derivation). What is the nature of the displacement distribution function and why?

g. The comparable argument is based on the impulse fluctuations that rain on a particle by virtue of collisions. We make only three assumptions:

1. The impulse forces f are randomly distributed.

2. The average motion imparted to a particle is opposed by a linear drag force, the effect of the other members of the ensemble in opposing a free motion of the particle under consideration. This makes the process consistent with thermodynamics.

3. Motion is governed by Newtonian mechanics.

h. Examining motion on one independent axis, Newton's law becomes:

$$f - a\dot{x} = m\ddot{x}$$

where:

f = random impulsive forces

$a\dot{x}$ = effective lossy drag.

Take the first moment:

$$\overline{f - a\dot{x}} = (m\dot{x})_o = m V_o$$

If the ensemble is at rest relative to the observer, the arbitrary nature of the choice of particle makes the result, summed over the ensemble, vanish. In other words, the mean momentum of the ensemble per member is zero. This result has been obtained from the first moments.

For the second moment, multiply by x and rearrange:

$$\frac{m}{2}\,(\ddot{x^2}) - m(\dot{x})^2 = fx - \frac{a}{2}\,(\dot{x^2})$$

Now time-average over a collision cycle (the impulses are not really random, but periodic fluctuations whose collisional coordination possess random phasing):

$$< \frac{m}{2}\,(\ddot{x^2}) > + < \frac{a}{2}\,(\dot{x^2}) > - < m\,(\dot{x})^2 > = 0$$

since $< fx > = 0$, "left"- and "right"-handed collisions being equiprobable. If any appreciable interval of time - if the particles remain in the bounded space - there can be no pileup of impulses in one direction.

Also, from the Maxwell distribution, we can show:

$$< m\,(\dot{x})^2 > = kT$$

The average kinetic energy is proportional to temperature. Thus in continuum (averaged) macroscopic variables:

$$m\,(\ddot{x^2}) + a\,(\dot{x^2}) - 2kT = 0$$

$$x^2 = \frac{2kTt}{a}\left[1 - \frac{bm}{2kTt}\left(1 - e^{-at/m}\right)\right]$$

where:

b = a constant of integration.

We can imagine the x^2 motion to be a small saw tooth around, at most, a constant velocity. This as $t \to 0$, the motion may be visualized as starting from rest. Thus b can be evaluated to make the bracket vanish as $t \to 0$:

$$x^2 = \frac{2kTt}{a}\left[1 - \frac{m}{at}\left(1 - e^{-at/m}\right)\right]$$

At $t = 0$, the velocity is:

$$x^2 = \frac{kT}{m} t^2$$

But very "quickly," beyond the very short time constant m/a, e.g., $t > 1.5$ m/a, the bracket approaches unity and:

$$\bar{x}^2 = \frac{2kT}{a} t$$

This is Einstein diffusion. It does not hold for $t < m/a$, but for $t > (2-3)$ m/a.

Thus we have achieved a description of the dynamic character of the summational invariants (conservations) of a simple autonomous ensemble. All we have needed are the laws of physics, the note that the laws of physics provide integral constraint moments for the ensemble, and the randomness of phase. We thus found for a stationary (and thus also a quasi-stationary) ensemble that the most complex form of organization compatible with the element species will develop in accord with the available underlying energetics. This will develop quantized structures of that complex form. If the energetics of its quantum levels, e.g., its lower "ground" state, and a subsequent high "excited" state are sufficiently far apart that only one complex form (e.g., simple molecularity) can exist, then we find in fact very simple distribution properties in equilibrium with available energy sources.

The mass will be conserved. The density will be uniform. The mean displacement of a molecularity will be zero in time. The variance of the displacement (mean square) will vary linearly with time; namely, the field will exhibit diffusive properties in any conservation introduced (e.g., mass, momentum, energy). The velocity distribution will be Maxwellian-Gaussian.

Distortions from these simple findings can now take place because of any of a number of factors: the molecularities have joint interparticle forces other than simple collisions; there is more than one competing quantized state; the molecularities have evolutionary properties; the confinement is open to both mass and energy and the external potentials have their own historical evolution.

4. A PRIMITIVE APPLICATION TO HUMAN SOCIETY - THE BOLTZMANNIAN

In a freely interacting spatial domain (e.g., political, if the system is already bound by value-in-trade, or a land surface upon which there is "free" collisional interaction of population groups), in which social "atomisms" of differing size exist, which "collide elastically" generation by generation, the equation is: what distribution will be found among the populations of the ensemble of such atomisms? Statistical mechanics would suggest the following. The additivity of atomistic summational invariants should determine the logarithm of the distribution function: the probability of finding atomisms in the particular states. So first, what are the summational invariants?

a. The population (p) of each atomistic active surviving group.

b. The total mass (Mp) of the group, where M is the genetically coded mass of the species member, approximately 150 lbs

c. The total daily energy consumption (Hp) of the group, where H is the daily energy requirement (power) of the species member, approximately 2000 Kcal/day

d. Plus a constant.

Momentum, or action, is also genetically coded (as are the internal organs of the system - these develop from internal-external action modalities. Since action is energy time, it is essentially the switch characterizations, mode by mode, which are internally programmed). It is not necessary to take this into account in the summational invariant metrics. Namely, there is approximate closure each day when:

$$H = \frac{\sum_{1}^{n} H_i \tau_i}{\sum \tau_i}$$

where i are the 1-n modes, τ_i their times, H_i their empowering energetics and H the daily power. The genetic code tends to insure the switching among the 1 to n (approximately 20) modes. Thus:

$$\ln N = Ap + BMp + CHp + \ln N_o$$

$$= \left[A + BM + CH\right] p + \ln N_o$$

$$N = N_o e^{\left[A + BM + CH\right] p} = N_o e^{-ap}$$

where:

N = no. of ensemble members

p = population of isolate ensemble members.

Since it is thermodynamically impossible that a monotonic increasing probability be associated with infinite population, a negative exponent is indicated.

Let us derive the result another way that later can be generalized further later on. We imagine a ground territory that is marked by N nucleating centers of population. These "compartments" may have populations, e.g., 1,2,...6k. Thus let there be n_1, n_2, ... n_k centers. We have two constraints:

$$N = \sum_1^k n_i$$

is the number of population centers.

$$P = \sum_1^k i\ n_i$$

is the total population.

Now the number of complexions W, with which the centers can be filled is:

$$W = \frac{N!}{\prod_{n}^{k} n_i!}$$

The statistical mechanical construct requires that the log W (entropy S = k ln W according to the Boltzmannian construct), be an extremum for the most probable complexion:

$$\ln W = N! - \sum_{i}^{k} \ln n_i!$$

$$\ln x! = (x + .5) \ln \left(\frac{x + .5}{e}\right) + 1.5 \ln \frac{e}{1.5}$$

Maximizing ln W, while conserving N and P, is achieved by Lagrangian multipliers (variations are with respect to n_i, i.e., δn_i):

$$\delta \ln W + A\delta N + B\delta P = 0$$

$$\sum_i \left[- \ln (n_i + .5) + A + B_i\right] \delta n_i = 0$$

$$n_i + .5 = a e^{-bp_i}$$

Outside of the term 1/2 (included for some minor accuracy) we get the Boltzmannian. Note again, that no interactions are provided for, only elastic collisions.

Let us look at some experimental data. First, from the Ethnographic Atlas ((14), see abstracted data in (2)), we look at data regarding the size of independent isolated cultures. The criteria given in the Atlas for "isolated" are separation by 200 miles or 1000 years. The "statistically valid" sample* for small cultures (two distributions are shown - the larger distribution is likely for interacting populations) is about as shown in Figure 1.

*Murdock (1) provides justification why that Atlas data can be validly queried.

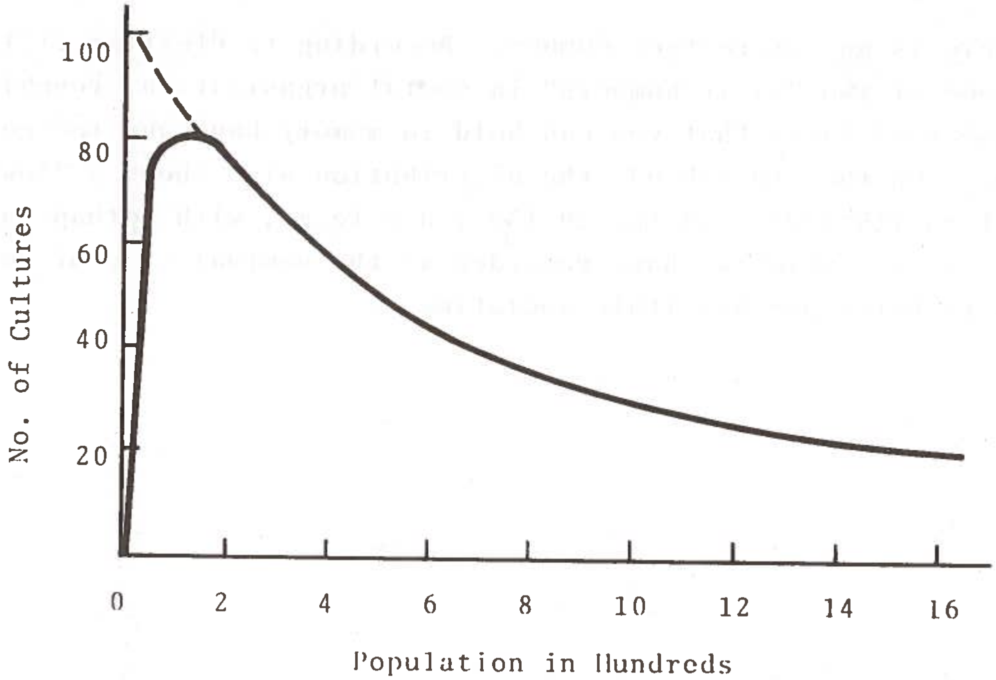


FIGURE 1. NUMBER OF CULTURES AS A FUNCTION OF POPULATION

As we discussed in an earlier report, Murdock provided a near-exhaustive catalogue of all well-documented cultures. Here we have concerned ourselves with the ones that are isolated. They seem to range in population size from about 15 to 2000; they seem to be flat in the range 25 to 200.

We find that isolate cultures seem to have a finite size cut-off at perhaps 10-15 people, and there are some flattening issues in the range 10-150 people, but beyond that population we find the exponential tail that our very elementary Boltzmannian type of distribution suggests. The small size cut-off issues tend to be problems that can be associated with quantization issues related to non Maxwellian distributions, particularly those related to other kinds of statistics.

Thus, outside of some small size quantization issues, a roughly exponential distribution in populations is found. The "average" size (a common first use of a distribution function) is about 1000.*

This is an interesting number. According to Pfeiffer (15), it is one of two "magic numbers" in social organization, roughly the number of faces that you can hold in memory bank and recognize by name. On the other hand, the distribution also shows a "low" population flattening in the 20-150 range (e.g., with perhaps a 50-70 center), which we have regarded as the nominal size of mobile hunter-gatherer pre-Neolithic societies.

*An approximation for the data is:

$$N = 95.25\, e^{-.1155\, p/100}$$

for the range N = 80 (a cut-off at p = 150/100) to N = 1 (assumed). Average value of population is thus:

$$\frac{\bar{p}}{100} = \frac{\int_1^{80} \frac{p}{100}\, dN}{\int_1^{80} dN} = 8.8$$

namely, $\bar{p} \approx 1000$.

5. DISCUSSION OF THE BOLTZMANNIAN POPULATION DISTRIBUTION

Note what we have shown in that the populations of isolated living societies - "molecularities" - have a number distribution like a Boltzmannian exponential with a quantizing cut-off at small populations, and a limiting size at a large population. In log population units, the number distribution resembles a somewhat distorted normal distribution. What can we say about this result?

We wish to bring in the issues that a metasystem's physics faces. They are similar to those faced by Maxwell in beginning a kinetic theory of gases.

As we discussed in an earlier paper* on population dynamics, we pointed out that the Malthusian law,

$$\frac{dP}{dt} = \left[b-d\right] P$$

was only a kinematic relation, compatible with a homogeneous population, but - by itself - incompatible with thermodynamics. In the long run, in order to be compatible $b \approx d$, namely, in some significant way societies of living systems self-regulated their population. We further pointed out that the homogeneity breaking by which b (the birth rate) and d (the death rate) would be kept in step was in fact the problem of the complete (i.e., irreversible) thermodynamics of living societies.

We now realize that we were pointing out meta-issues (i.e., atomistic kinetics) to social thermodynamics in the same sense that Maxwell and Boltzmann were pointing out meta-issues when they began, in kinetic theory, with the notion that molecules are in sustained motion. Thus, we too are starting out with a meta-issue. Living systems persist, as a species, by division or reproduction.

---

* A thermodynamic note on population, presented at the 1975 annual American Association for the Advancement of Science (AAAS) conference.

Individually, they live and die, but the species goes on. While kinematic record is kept by

$$\frac{dp}{dt} = (b-d)\ P$$

it does not reveal the kinetic strategy (except to say b ≈ d somehow).

Note, we have asked that the isolated molecularities in a region (e.g., bacteria in isolated pool regions; grass in stands; mice colonies, each with its living range; nomadic hunter-gatherers) fulfill thermodynamic near-equilibria. The living system (species) does not fulfill its thermodynamic system characteristics at a particular point. Instead it can breed and it can broadcast; that is sufficient for them to be distributed throughout their available space with an exponential Boltzmannian character. But, kinetically, how can they survive?

What does that mean? Each generation, the isolate society communicationally "collides" (communicates with) with its next generation.* The groups are small, the fluctuations large. The question is, how can the species survive? The subtle answer is that it must do so both by an inbreeding and a broadcast "outbreeding." It must have a strategy for scheduling off toward the labile small population size.**

Namely, every species evolves a mixed genetic and epigenetic technique whereupon it broadcasts its seed. Some young are blown away, some young are killed, some degree of outbreeding mating takes place too. The important thing is that enough young are broadcast so that they have some chance of start-up.

---

*"Generation" loosely denotes the population equilibrium scale, characterized by the choice function of having children now and a period of time "ago" (a suitable integrated average over the past), such that the smoothed rate of change of population now is the difference in choice function now and "ago," divided by the time interval.

**We received a clue to this notion from Darlington (16).

We have no atomistic theory for start-up, but sufficient individuals are tossed out to start up new population molecularities. Each species may use its own techniques, or one or more techniques, for broadcast and outbreeding. The important thing is that a robust broadcast technqiue is required that fits the growth style of the species.*

The basic point is that the Boltzmannian distribution function among isolated (and thereby "elastically" colliding) stands** is supported by a growth of the entire species' population. At any time, there are individuals (or individual pairs) who have been broadcast into the milieu. They may or may not persist beyond one generation. They may attach themselves to other groups, they may coalesce into a group of their own, they may die out. They probably represent a significant percentage of the total population.† They are the labile "front" of the distribution from which new groups form.

Correspondingly, we are not certain about the rare large-size stand. That tail of the Boltzmannian (which, if rank-ordered in size, would represent low-number rank-orders) is nourished by the growing species population, but there is no guarantee as to its long-term stability. It may decline; it may split up.

Whatever stability there is, lies in the "middle" of the Boltzmannian curve. That "stability" is precarious. It is maintained by a sufficient overall growth in population.

$$\frac{dP}{dt} - (b-d)\ P$$

---

*Actually, the deeper issues are how in fact life itself may likely have started by similar processes, but that is beyond our current scope.

**Rather than settlements. These standards each have a roaming range.

†How significant, we cannot say in general. One has a suspicion that it may vary with species and ecology (i.e., as it changes in time). Thus, it may range from 5% to 50% as an a priori estimate.

b-d > 0, so that there is a diffusion toward the small, toward a "broadcast" diffusion of individuals.

This represents a rudimentary metasystems thermodynamic theory of small stands of population which are dilute in the ecology and which are not yet particularly interacting. As we judge from Murdock (and our own estimates), when these groups are hunter-gatherer humans, they may be separated at spatial ranges of the order of 70 miles.*

---

*A 200-mile separation would mean complete isolation, and a 20-mile separation would mean quite heavy interaction.

6. INTERACTION WITH GROWTH - THE ZIPFIAN DOMAIN - MANDELBROT "LANGUAGE" USAGE DERIVATION

Now that we have some notion of how a population of energetically entrained entities in a living system becomes viable - namely, is born, persists, dies - and is tied to its supporting "earth" substrate, at least in an ideal gas collisional sense, we wonder how one goes toward a more interactional ensemble existence, e.g., the Zipfian distribution.

The Boltzmann distribution will not account for the Zipfian distribution. The former involves essentially isolate ("elastic") collision, generation to generation with its own group. It is nourished by a small rate-governed "pressure" to broadcast diffusion of small seed groups. The latter involves a new kind of interaction.

In the first place, in the Zipfian case, one finds the beginning formation of a van der Waal interaction: there is more of a binding and sensing of neighboring groups. In the second place, there is more of a locking-in of each group in a state space. Thus, a spaital mean free path (in a state space sense), a notion of a roaming range in which the particular group "permanently" preempts the space, instead of it being any group's "for the moment" occupied space, develops.

We would like to get to our derivation of Zipf's law, but first we would like to present a synopsis of the Mandelbrot derivation of the Mandelbrot-Zipf law. This is adapted from Brillouin (12). We do not subscribe in depth to the derivation. Nevertheless, its flavor is quite important. While the Mandelbrot form suggests that it is related to language usage among the most common words, we adopt the spirit of Zipf and regard it as an ensemble selection from a broader state space.

The Mandelbrot extension of Zipf's law is:

$$p = \frac{A}{(r + r_o)^n}$$

where:

p = a metric of probability or size of an element in an ensemble

r = rank-ordering of that metric

r_o = a rank-ordering constant that displaces rank from its normal 1,2,3... ordering

n = a constant, near unity.

For Zipf's law, $r_o = 0$, $n = 1$.

We can plot the distribution relationships we have thus far alluded to as shown in figures 2 and 3.

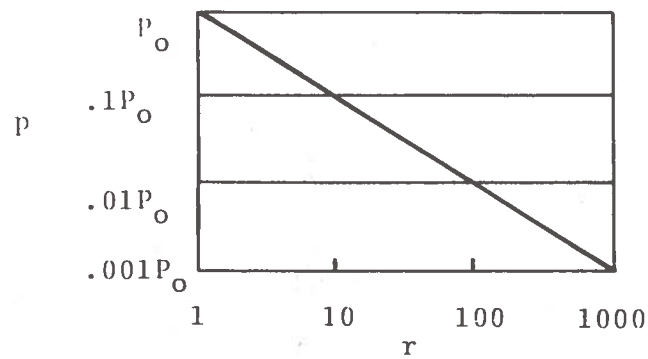


FIGURE 2. ZIPF'S LAW

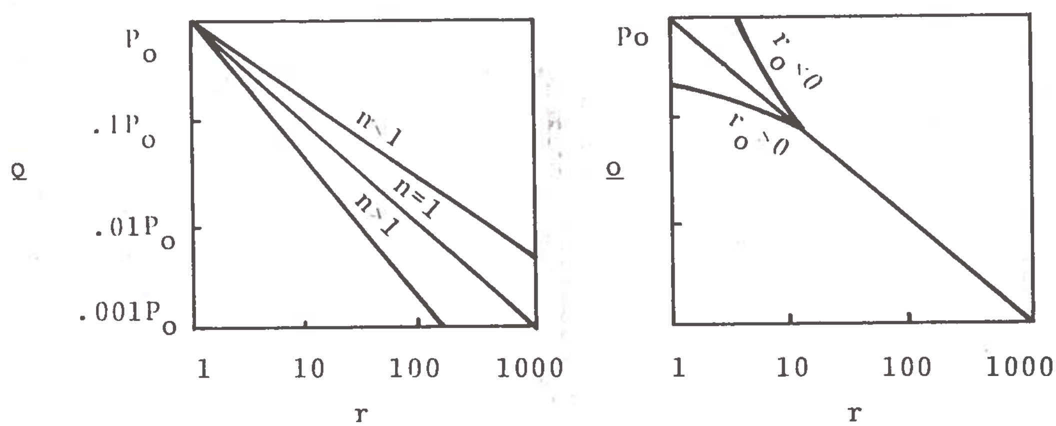


FIGURE 3. MANDELBROT DEVIATIONS

For the Boltzmannian, we have:

$$N = N_o e^{-ap} = e^{a(p_o - p)}$$

with a cut-off at some small population p_1 and at some large population p_o. We need to relate the number of members in the ensemble to the rank-order r.

N	r
1	1
2	2
2	3
3	4
3	5
3	6
4	7
4	8
4	9
4	10
...	

The smoothest selection are the N, r pairs (1,1), (3,5), (5,13), etc. This is represented by:

$$r = \frac{N^2 + 1}{2}$$

Thus:

$$2r - 1 = e^{2a\,(p_o - p)}$$

This resembles Figure 4.

The mismatch, except for a very limited range, is obvious.

We will now provide the Mandelbrot derivation of the Zipf-Mandelbrot law, as it is developed via an information theoretic. The derivation will involve two notions: one, a notion of efficient coding in time, and the second, a notion of the rank-ordering

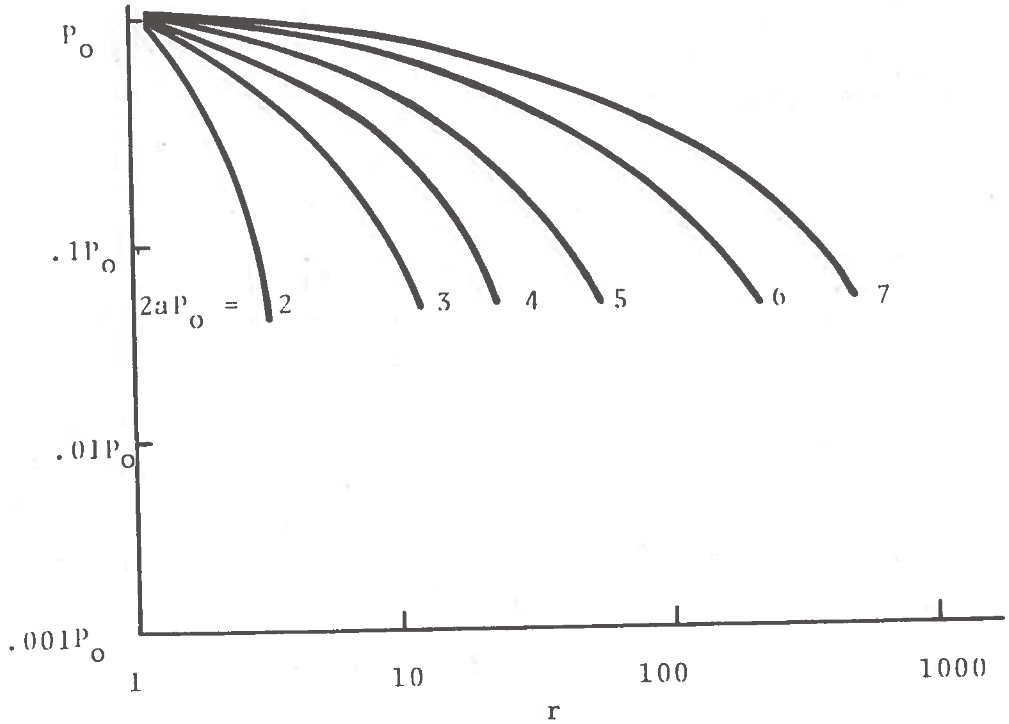


FIGURE 4. POPULATION AS A FUNCTION OF RANK ORDER

of messages in time. Elimination of the parameter time gives the Zipf-Mandelbrot law in the rank-order - probability space.

We turn now to Brillouin (12) and a viable "language" selected from an ensemble. (These ultimately have to relate to populations of cities): Suppose $n_1 \ldots n_m$ symbols of type 1...m, each of "costly" duration $t_1 \ldots t_m$, and each with an a priori probability p_i.

[Examples of ensemble "languages."

(a) $n_1 = 1$, type $1 \equiv 0$, duration 1 unit, $p_1 = 1/2$

$n_2 = 2$, type $2 \equiv 1$, duration 1 unit, $p_1 = 1/2$

(b) $n_1 = 1$, type $1 \equiv$ space, duration 699 units, $p_1 = 0.20$
(-log p = .699)

$n_2 = 1$, type $2 \equiv$ E, duration 979 units, $p_2 = .105$
(-log p = .979)

$n_3 = 1$, type 3 = T, duration 1143 units, $p_3 = .072$
(-log p = 1.143)

(c) $n_1 = 1$, type 1 = the $p_1 = 0.08$

$n_2 = 1$, type 2 = of $p_2 = .06$

$n_3 = 1$, type 3 = and $p_3 = .04$

$n_{10} = 1$, type 10 = I $p_{10} = 0.015$

$n_{4000} = 1$, type 4000 - nerves $\sum p_1 = 1$]

"Cost" is undefined. It is identified here with "sending cost," involving as a metric "time" t. We wish to send "messages" in that ensemble "language." "Information," which is undefined, is maximum per symbol at a given signal rate if p_i is related to t_i, in the utilization of the symbol, by

$$p_i = e^{-\beta_1 t_i}$$

The following proof is offered for this theorem. We imagine a large string of symbols G_o, which may have many possible arrangements. It will consist of:

$$G_o = N_1 + \ldots N_m = \sum_1^m N_i$$

Namely, N_1 of type 1 symbols are used, ... N_m of type m symbols are used. The total sending time T will be given by:

$$T = N_1 t_1 + \ldots N_m t_m = \sum_1^m N_i t_i$$

We are given that the probability of symbols will be in accord with their a priori probability (it is a long string, and the current "message" comes from the same a priori "language" as previous messages from which the a priori probabilities were derived):

$$p_i = \frac{N_i}{G_o} \quad \sum_1^m p_i = 1 \quad T = - G_o \sum_1^m p_i t_i$$

The number of total arrangements N are:

$$M = \frac{G_o\,!}{\prod_1^m N_i\,!}$$

as the standard statistical selection process. Using the Stirling approximation for large factorial numbers:*

$$\ln x\,! = x\,(\ln x - 1)$$

Thus:

$$\ln N \approx G_o \left(\ln G_o - 1\right) - \sum_1^m N_i \left(\ln N_i - 1\right)$$

$$= - G_o \sum_1^m p_i \ln p_i$$

If, for a given amount of "information" contained in the string G_o, ($G_o \Sigma p_i \ln p_i$, which is thus proportional to ln N) we wish to minimize the time T,

$$\frac{\ln N}{T} = - \frac{G_o \sum p_i \ln p_i}{G_o \sum p_i t_i} = - \frac{\sum p_i \ln p_i}{\sum p_i t_i}$$

---

*Adding up unit-wide strips from 1/2 to 1-1/2, and from 1-1/2 to x + 1/2, we find an even more accurate integration result: (x + 1/2) ln (x + 1/2) - (x + 1/2) + 0.89.

should be a maximum, then

$$\ln p_i = -\beta_1 t_i + \ln C_1{}^*$$

This is roughly the standard Shannon-Hartley proof of the efficient coding theorem.

A few comments are in order. We are guided by earlier remarks in (10). "Messages" are really not the total string G_o but, per Hartley, a choice s times from each of m symbols. As far as the message generator system is concerned, his selections are random. But the message generator is accustomed to operate in the language. So the G_o string, the number of such messages that will be delivered are G_o/s, out of a universe of $m^s\ G_o/s$ messages.

But this selection hasn't met the language expectations; we have to weigh the selections by language probabilities. Reconsider s choices of m symbols. While each selection, in a short message, appears independent, the selections are being made as if from a shorter language m'. Thus the effective number of messages is:

$$(m')^s = \left(\frac{m'}{m}\right)^s m^s$$

The total universe of messages is thus:

$$\left(\frac{m'}{m}\right)^s \frac{m^s\ G_o}{s}$$

In any case, the number of messages grows with G_o, and so does ln N. Thus, selecting symbols "naturally," in the light of their a priori language frequency, should make the sending rate a minimum independent of string length G_o and message length just dependent on assigning a specific cost to symbols. Then a simple Lagrangian multiplier argument on

*A more accurate result is $\ln\left(p_i + \frac{1}{2G_o}\right) = -\beta_1 t_i + \ln C_1$. Small probability symbols are thereby treated more correctly.

$$\sum p_i \qquad \sum p_i \ln p_i \qquad \sum p_i t_i$$

leads to:

$$\ln p_i = -\beta_1 t_i + \ln C_1$$

$$p_i = C_1 e^{-\beta_1 t_i}$$

Also the relation,

$$\frac{1}{G_o} \ln N = -\Sigma P_i \ln P_i$$

does not seem to be particularly dependent on a requirement for a long string as long as messages are properly coded for. Actually the more accurate result is:

$$\ln \left(p_i + \frac{1}{2 G_o} \right) = -\beta_1 t_i + \ln C_1$$

which affects small probability symbols, but we will consider them outside of the Zipfian range.

For the second part of the argument, we consider strings of various total times T, but we will rearrange our symbols by units of duration. We will find a fine-grained unit of time, so that all our symbols are expressed in minimum integral units, that is, there will be:

a_1 symbols of duration 1 unit

a_2 symbols of duration 2 units

a_m symbols of duration $m = \sigma$ units

$$a_i \geq 0. \qquad \sum_1^m a_i = n$$

The probabilities associated with these symbols are given by:

$$p_i = C_1 e^{-\beta_1 t_i}$$

The magnitude m may be much larger than before because there may be many zero values of a_i. However C_1, β_1 permitted a stretching to accommodate. There will be N (T) distinct arrangements in any total duration T. We will define

$$N(o) = 1$$

$$N\begin{pmatrix}\text{negative}\\ \text{integer}\end{pmatrix} = 0$$

Let us construct a general difference function. For one minimum time unit,

$$N(1) = a_1 N(1-1) + a_2 N(1-2) + \dots + a_m N(1-m)$$

$$= a_1$$

Namely, for one unit slots a_1 symbols can be used:

$$N(2) = a_1 N(2-1) + a_2 N(2-2) + \dots + a_m N(2-m)$$

$$= a_1^2 + a_2$$

There are a_1 choices by a_1 choices for each of the two time slots, or a_2 choices for a two time slot unit:

$$N(3) = a_1 N(3-1) + a_2 N(3-2) + \dots + a_m N(3-m)$$

$$= a_1^3 + a_1 a_2 + a_2 a_1 + a_3$$

The time slots can be taken 1 at a time, 2 at a time plus 1 at a time, 1 at a time plus 2 at a time, or 3 at a time. Continuing the construction,

$$N(T) = a_1 N(T-1) + a_2 N(T-2) + \dots + a_m N(T-t_m)$$

Let

$$N(t) = A_1 \rho_1^t + A_2 \rho_2^t + \dots + A_m \rho_m^t$$

where Λ_i's are constants and ρ_i's are m independent roots of

$$\rho_i^{t_m} - a_1 \rho^{t_{m-1}} + \ldots + a_m = 0$$

This can be derived by substituting the previous form for N (t) into the recursion formula and letting each value of Λ_i but one, vanish one at a time. This gives:

$$\Lambda_i \rho_i^{t} - a_1 \Lambda_i \rho_i^{t-1} + \ldots - \Lambda_i a_m \rho_i^{t-t_m} = 0$$

Multiplying by $\rho^{t_{m-t}}$ gives the result: we find m independent solutions for ρ for a given set $a_1 \ldots a_m$. The values of $\Lambda_1 \ldots \Lambda_m$ are given by the set,

$$N\,(o) = 1 = \Lambda_1 + \Lambda_2 + \ldots + \Lambda_m$$

$$N\,(-1) = 0 = \frac{\Lambda_1}{\rho_1} + \frac{\Lambda_2}{\rho_2} + \ldots + \frac{\Lambda_m}{\rho_m}$$

$$N\,(-m+1) = 0 = \frac{\Lambda_1}{\rho_1^{m-1}} + \frac{\Lambda_2}{\rho_2^{m-1}} + \ldots + \frac{\Lambda_m}{\rho_m^{m-1}}$$

m roots exist with only one positive root, because of one variation in sign. The other roots are negative or complex.

If $\rho = 1$, $1 - \Lambda_1 - \ldots - \Lambda_m = 1-n$, which is negative (since $n \geq 2$). There is only one real positive root which is greater than one. Isolate that positive root and call it

$$\rho_1 = e^{\beta_2} \; (\beta_2 > 0).$$

$$N(t) = A_1 e^{\beta_2 t} + \sum_{i=2}^{m} A_i \rho_i^t$$

$$= A_1 e^{\beta_2 t} + \sum \text{decaying terms}$$

$$+ \sum \text{oscillatory terms}$$

Absolute or relative stability exists if the ρ_i's are less than unity in absolute value, or less than e^{β_2}.

We shall be concerned with stable systems, and in particular where the first term is dominant $N \approx N'$

$$N'(t) = A_1 e^{\beta_2 t}$$

Now we shall be concerned with the rank-order r(t), which will comprise the number of arrangements of string length smaller than or equal to t:

$$r(t) = N(1) + N(2) + \ldots + N(t) = \sum_{s=1}^{t} N(s)$$

$$= \sum_{s=1}^{t} \sum_{i=1}^{m} A_i \rho_i^s = \sum_{i=1}^{m} A_i \left(\sum_{s=1}^{t} \rho_1^s \right)$$

But

$$\sum_{s=1}^{t} \rho_i^s = \rho_i \left(1 + \rho_i + \rho_i^2 + \ldots + \rho_i^{t-1} \right) = \rho_i \frac{1-\rho_i^t}{1-\rho_i}$$

$$r(t) = \sum_{i=1}^{m} \frac{A_i \rho_i}{\rho_i - 1} \left(\rho_i^t - 1 \right)$$

$$r(t) = \frac{A_1}{1-1/\rho_1} e^{\beta_2 t} + \sum_{i=2}^{m} \frac{A_i \rho_i^t}{1-1/\rho_i} - \sum_{i=1}^{m} \frac{A_i \rho_i}{\rho_i - 1}$$

$$= \frac{A_1}{1-1/\rho_1} e^{\beta_2 t} - B_o + \sum_{i=2}^{m} \frac{A_i \rho_i^t}{\rho_i - 1}$$

Again, for stable systems, the dominant term is

$$r' = B_1 e^{\beta_2 t} - B_o$$

$$B_1 = \frac{A_1}{1-1/\rho_1}$$

$$B_o = \sum_{i=1}^{m} \frac{A_i \rho_i}{\rho_i^{-1}}$$

If now we combine these two results,

$$\beta_2 t = -\ln B_1 + \ln (r' + B_o)$$

$$\beta_1 t = \ln C_1 - \ln p$$

so as to eliminate "cost" t, we find:

$$\ln p\,(r' + B_o)^{\beta_1/\beta_o} = \ln C_1 + \frac{\beta_1}{\beta_2} \ln B_1$$

Mandelbrot's law.

It is not our intention to dwell on this derivation. We simply wanted to present one facet by which the Zipf-Mandelbrot relation has been viewed as an information theoretic. At the present, while we believe that the same relation holds for cities and other human institutions for "communications" reasons, we do not believe that the formalism is the same as in this compact coding story. The connection, we believe, is more related to the Boltzmann relation.

7. INTERACTION WITH GROWTH - THE ZIPFIAN DOMAIN (A PHYSICAL FOUNDATION)

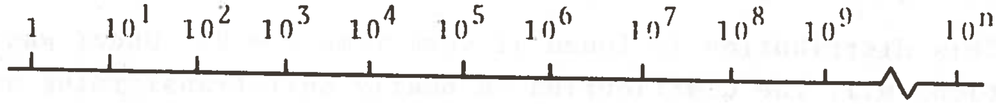
We are concerned with a moderately interacting ensemble of active atomisms - social modecularities - which form viable stands, which reproduce, but which interact somewhat. These atomisms are self-serving thermodynamic, namely, they satisfy summational invariants of mass, mass species, momentum modalities, energies. But since they are living, they are a reproductive species. They show

$$\frac{dP}{dt} = (b-d)\ P$$

where: $b - d \geq 0$

at least slightly. The materials used for reproduction, maintenance, and growth, are at hand, and the processes are sufficiently rich to include communications capabilities. Communications capabilities involve the capability to switch modal states of the ensemble, but this notion will mean little at this time.

We want a concept structure rich enough to include language and population and cities, so we start with the notion of an extended logarithmic scale related to some ensemble produced by the living system:



Note there is a limiting size 1 and say 10^n, where n is some large integer. Actually there is some minimum size 10^a, $a > 0$, for which the repertoire is stable, and some thermodynamic bound governs the largest size n that can exist in the field. For post-Neolithic humans bound to the earth by an agriculturally based energy mode, we surmise that 10^a is of the order of 250.

Now we wish to approach the "demographic" question. Given a population distribution (of symbols, stands, colonies, populations) that is sufficiently dense that the elements "interact."

"Interact," in the first place, will mean that the population changes in time due to internal system processes - inbreeding - and to some process external to the local generator; it will also mean that there is some degree of emigration and immigration as well as outbreeding and inbreeding. It would appear that living systems cannot survive without such breeding and broadcast. Thus, populations not only "collide" with their own group, generation by generation, but they inbreed and outbreed, immigrate and emigrate. They act as if there were attractive and repulsive forces, but not derived from spatial potentials.

The question we will ask is: what are the conditions under which an ensemble of such interacting centers will remain stable or nearly stable? "Stability" will mean that they preserve their form against disturbances, so we can imagine a distribution of settlement of molecularities, large compared to the isolate Boltzmannian distribution, placed on the logarithmic scale:

$$q = \ln_b p.$$

(We will be indifferent about b. For computation, b = e; for arithmetic, b = 10; for information theoretic, b = 2).

This distribution is found at some time t = 0. Under what conditions will the distribution be nearly self-transforming and stationary in time, namely, at time $t = t_1$, $2t_1$, $3t_1$, etc., will the distribution function be nearly fixed? The argument to discern the fixed distribution is loosely as follows.

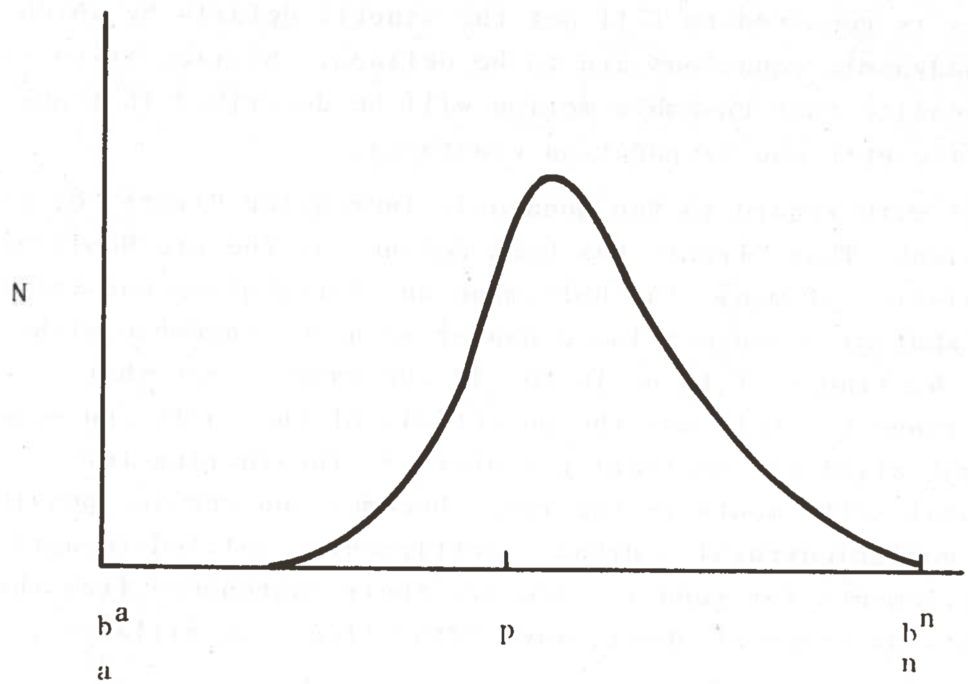


FIGURE 5. NUMBER OF SETTLEMENTS AS A FUNCTION OF POPULATION

If we consider an "impulsive" slice of N settlements whose size lies between q and q + Δq at time t, in the next interval of time there will be a diffusive dispersion. That tends to flatten out the distribution.

One can sense, from that result, that any symmetric distribution (e.g., Gaussian, or near a peak) has that kind of spreading result. Thus peaked distributions likely cannot be preserved in form. The only possible distributions that might preserve their character are monotonic increasing or monotonic decreasing functions.

But whence are they driven, since there is a relaxation process toward a more uniform distribution? One must surmise that an "ingathering" from some pool of creation takes place. Whether hydrogen galactic dust, or transient living entities, the same kind of force-binding structure acts as an attractor to start up ensembles. Beyond that point, a much more detailed examination of the forces is required to fill out the kinetic details by which the thermodynamic equations are to be defined. Suffice it to say that we realize that ensemble motion will be described in transport coefficients and propagation constants.

First with regard to the monotonic increasing "front" of the distribution. That "front" has been defined by the pre Neolithic characteristics of man. The Boltzmannian distribution has indicated in what size range a loose hunter-gatherer ensemble might survive. We find that to be in the 40-200 range. But that ensemble range has felt out the potentials of the earth and worked out the potential for nucleating centers.* Ingathering for agricultural settlements in the range becomes and remains possible. Then the nonagricultural - urban - settlements, related to agricultural settlements for support, receive their sustenance from those settlements at sizes of about, say, "250" (i.e., as villages).

This does not resolve the details of this quantization process, but it indicates that the quantum formation of settlements goes back to Neolithic trade interactive start-up issues. What is important is that a considerable transient population that may be "equally" attracted for forming agricultural settlement or urban settlement exists, and it is the galactic dust which can sustain the pressure for growth of urban settlements.

On the monotonic decreasing side, where the urban centers are nourished from this transient population, it is the combination of inbreeding and broadcast (whether ingathering, immigration, or emigration), with a modest growth, that supports the monotonic decreasing distribution.

But there is a high lability at the two ends - of small settlements and the very large settlements. Small settlements will show large fluctuations. They can come and go. The largest

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*For example, we would surmise that man in the Americas marked these centers out prior to 1500, and that the European settlers (1500-1700) reconfirmed and scored more densely those centers for agricultural settlement purposes.

settlements will also fluctuate and not maintain their position forever. The large end will begin to partake seriously in the 300-500-year rhythm of civilization.

Thus, in summary, what is the Zipfian distribution? It is a labile distribution of highly used elements by which a complex "living" thermodynamic system communicates with its milieu. The distribution - in summation - represents the species. These molecularity elements represent the results of mass-energy-action-space-time interaction (in cities, also value-in-trade interaction). As long as the species lives, there is a growth process by which the Zipfian character is maintained as a push outward. When the species begins to die, the distribution tends to flatten out, and its components escape. This notion was presented by Sorokin.

8. IMPLICATIONS FOR TRANSPORTATION - FIRST ROUND

We were asked the question, in the contract, what the effect of the introduction of a new high-speed transportation would be on city size distributions in the United States. At this point we are prepared, surprisingly, to suggest a preliminary answer.

It would seem, if our notion is true that it requires an in-borne slight growth pressure to maintain the Zipfian distribution as an essential characteristic of a viable system, that it is necessary to provide good internal transport and perhaps slightly impeded external transport.

We have the suspicion, from examining experimental data (13), that prior to 1900, while perhaps 40 percent of the population was fixed in agricultural settlements (and devoting their efforts to farming), that fraction has since shrunk because of extensive automation, the rest of the population (60 percent) could be accounted for in Zipfian distribution down to size "250." But this includes a sizeable number of "transients" (e.g., 20 million people in 1950 out of 150 million, who were neither urban (95 million) nor farm (25 million) nor in rural settlements (10 million)). It seems to us that it is the "transient" and mobile population that maintains the living Zipfian character.

Thus, while class transportation will always take care of itself - the elite and middle class will insist on service for their needs - the essential matter to keep a nation viable is to provide good, fast, cheap transport for the more transient portion of its population. That is, the lowest quarter of the population must find it easy to move about. If that is true, the open market characteristics can be maintained. People can be attracted and move where the opportunity is.

Reflection on the internal character of mobility in American cities suggests that movement at the range of perhaps 5-10 years is the typical pattern, and under a generation or so between cities. Very seldom do people stay put for 3 generations or more of thermodynamic equilibrium. Good transportation has supported that possibility, and good cheap transportation is what is needed to keep our future from a civilizational death.

Our system might become moribund for other economic reasons (e.g., escape of multinationals, the joint competition of dictatorships), but there is no reason to encourage an easy death by not understanding this issue of transports. That the jet-set can travel or escape is not the issue; can the person of marginal subsistence - the "Oakie" - get on a bus, a train, a subway, etc. to seek out his living? That maintains the system. This Zipfian character and the spatial potential-like character of the ecological potentials then make planning for a future possible.*

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*As a final note on Zipf's law as a "communicational" law, or a rule of transactional possibilities in a viable language, we recognize somewhat that this relation is related to perceptual cost. Thus, if

$$f = \frac{A}{(d+B)^n}$$

were to be a spatial transactional form of Zipf's law, known also as a gravity or potential model, we ought to recognize that d can be any metric associated with a perceptual space (see, for example, (11) for a discussion of Fairthorne's view of this as a non-physical, even legal view; in our terms, a perceptual view). You may "walk a mile for a Camel," or men may divert their business trips half way around the world for a liaison with a mistress. Thus, any number of scalings for n equal to, less than, or greater than one, are possible. Yet the rank-ordering of an "objective" function still seems to arise. Our basic criticism - related to Fairthorne's - is that the space is not a totally stationary space in general, but undergoes historical change, as the system bothers to learn or attend. Pierce (17, p. 241) shows wide experimental fluctuation as in most psychological experiments which draw from a dynamically stored perceptual "field dictionary" of "words." When sufficient time has elapsed for near equilibrium, the pressure of the distribution function - and whatever quantizes the cutoff at the epoch - gives the operating region. For example, Fairthorne shows Richardson's data for regional populations on earth distributed as extremely as:

$$P = \frac{A}{(r+B)^3}$$

Pierce also discusses Mandelbrot's notion of n. "The wealth of the vocabulary is measured chiefly by [n]; if [n] is much greater than 1, a few words are used over and over again; if [n] is nearer to 1, a greater variety of words are used." For a child, $n \approx 1.6$, which drops toward 1 when finally "the child happens to be James Joyce." Cost, in transportation, has similar properties.

APPENDIX

REPORT OF INVENTIONS

The work performed under this contract, while leading to no invention, has led to several innovative concepts on the use of near-equilibrium thermodynamics for living social systems. Compartmental balance of food materials, energetics, manpower, productive function, economic balance, and technology were introduced as concepts for social systems modeling, as well as the idea of summational invariants for social systems.

REFERENCES

1. A. Iberall, S. Cardon, "Systems Models for Transportation Problems," Parts 1-4, reports to Transportation Systems Center, U.S. Dept. of Transportation, Cambridge, MA, March 1976; on file at the DOT/TSC Library.

2. A. Iberall, S. Cardon, "Toward a General Science of Man Systems," reports to U.S. Army Research Institute, Arlington, VA, May 1973, September 1973, December 1973, April 1974, April 1975.

3. G. Zipf, Human Behavior and the Principle of Least Effort, Addison Wesley, Reading MA, 1949.

4. J. Stewart, "Empirical Mathematical Rules Concerning the Distribution and Equilibrium of Population," Geographical Review, 37, 461, 1947.

5. W. Warntz, Toward a Geography of Price, University of Pennsylvania, Philadelphia, 1959.

6. H. Simon, "On a Class of Skew Distribution Functions," in Models of Man, Wiley, New York, 1957.

7. A Wilson, "Advances and Problems in Distribution Modeling," Transportation Research, 4, 1, 1970.

8. G. Olsson, "Distance and Human Interaction," Regional Scientific Research Institute, Philadelphia, 1965.

9. G. Cherry, On Human Communication, Wiley, New York, 1957.

10. A. Iberall, "Information Science," report to Army Research Office, AD #635-809, June 1966.

11. R. Fairthorne, "Progress in Documentation," Journal of Documentation, 25, 319, 1969.

12. L. Brillouin, Science and Information Theory, Academic Press, New York, 1956.

13. M. Clawson, R. Held, C. Stoddard, Land for Future, Johns Hopkins Press, Baltimore, MD, 1960

14. G. Murdock, Ethnographic Atlas, University of Pittsburgh, 1967.

15. J. Pfeiffer, The Emergence of Man, Harper and Row, New York, 1969.

16. C. Darlington, The Evolution of Man and Society, Simon & Schuster, New York, 1969.

17. J. Pierce, Symbols, Signals, and Noise, Harper and Row, New York, 1961.

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