

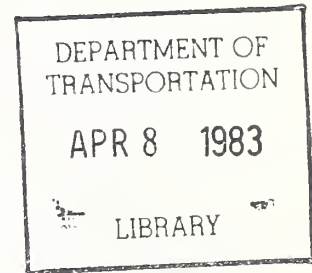
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MBTA PASSENGER DEMAND ANALYSES, 1977

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U.S. DEPARTMENT OF TRANSPORTATION
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FINAL REPORT

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16. Abstract A survey was made of the number of passengers using the Highland Branch of the MBTA Green Line. All the old PCC streetcars on that line were to be replaced by new light rail vehicles within the year. Therefore, this count was intended to represent the "before" segment of a before-and-after survey to estimate increase in demand due to the new vehicles. Analysis of the data confirmed a "market share" theory for the stations and suggested that fairly sparse sampling could yield estimates of total passenger movement acceptable at the 90 percent confidence level.					
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PREFACE

The present study is performed under Project Plan Agreement UM-37, sponsored by the U.S. Department of Transportation, Urban Mass Transportation Administration, Office of Planning Management and Demonstrations, Office of Transportation Management, UPM-40. It is undertaken for the calibration and the analyses of MBTA passenger data to be used as an input to the main operational performance simulation model being developed under the same PPA. Acknowledgement is given to Mary Roos and George H. Wang of TSC Code 20 for their direction and helpful advice in the study.

METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures				Approximate Conversions from Metric Measures			
Symbol	When You Know	Multiply by	To Find	Symbol	When You Know	Multiply by	To Find
LENGTH							
in	inches	2.5	centimeters	mm	millimeters	0.04	inches
ft	feet	30	centimeters	cm	centimeters	0.4	inches
yd	yards	0.9	meters	m	meters	2.3	feet
mi	miles	1.6	kilometers	km	kilometers	0.8	miles
AREA							
m ²	square inches	6.9	square centimeters	cm ²	square centimeters	0.16	square inches
ft ²	square feet	0.09	square meters	m ²	square meters	1.2	square yards
yd ²	square yards	0.9	square meters	ha ²	hectares	0.4	square miles
mi ²	square miles	2.6	square kilometers	ha	hectares (10,000 m ²)	2.6	square miles
ac	acres	0.4	hectares				acres
MASS (weight)							
oz	ounces	28	grams	g	grams	0.035	ounces
lb	pounds	0.45	kilograms	kg	kilograms	2.2	pounds
	short tons (2000 lb)	0.9	tonnes	t	tonnes (1000 kg)	1.1	short tons
VOLUME							
teaspoon	teaspoons	5	milliliters	ml	milliliters	0.03	fluid ounces
fluid ounce	fluid ounces	30	milliliters	ml	milliliters	2.1	pints
cup	cup	0.24	liters	l	liters	1.06	quarts
pint	pints	0.47	liters	m ³	cubic meters	0.26	gallons
quart	quarts	0.95	liters	m ³	cubic meters	36	cubic feet
gallon	gallons	3.8	liters	m ³	cubic meters	1.3	cubic yards
cubic foot	cubic feet	0.03	cubic meters				
cubic yard	cubic yards	0.76	cubic meters				
TEMPERATURE (exact)							
F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	C	Celsius temperature	9/5 (when add 32)	Fahrenheit temperature

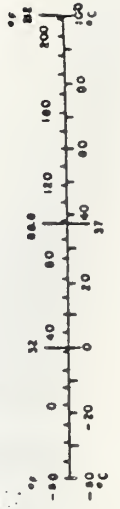
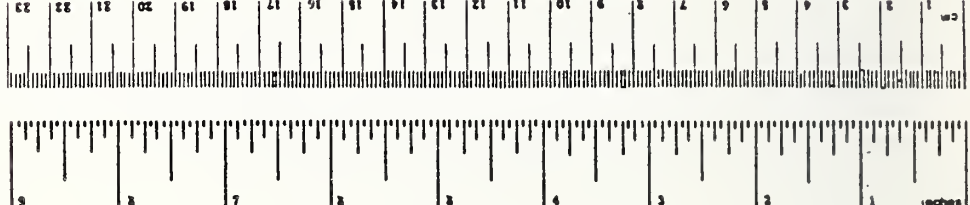


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I. INTRODUCTION

This report summarizes findings resulting from a special transit study which included 67 inbound trips (Riverside-Fenway) and 69 outbound trips (Fenway-Riverside) of patronage data collected between May 23 and June 1, 1977. This was about 15% of the trips for that period. The parameters measured during each trip for each of the 13 stations are: the time of arrival, the total boarding passengers, total alighting passengers, and the dwell time with comments on extraneous delay. Subsequently, statistics such as the total movement (= total on + total off), the load of vehicle upon immediate departure from the station, and the total trip time can also be calculated. The 15% sample was collected from over a wide spectrum of time periods and days of the week so that an average profile of ridership and transit operation can be delineated. Also, the reliability of the data thus collected is assessed and discussed with respect to the requirement of the MBTA Green Line operational simulation model being developed under Project Plan Agreement UM-37.

It is assumed throughout the course of analysis that the number of passengers boarding an inbound train (or the number of passengers alighting an outbound train) at a station truly reflects the passenger demand of the system at that station during the period, and is independent of the number of cars associated with that trip. It is of course conceivable that during rush hours, this assumption may be invalid because the volume of patrons boarding a train depends upon the load already carried by the train. However, data on the load factors show

that this measure rarely goes beyond "medium" even during rush hours. Hence train capacity (number of cars) is not considered.

A few trips are deleted from the inbound data because they reflected the effects of unusual conditions (scheduled baseball games) which resulted in a large influx of people into Fenway or Kenmore stations. These trips were on 5/31/77 at 4:58pm, 5:42pm, 6:32pm and 6:42pm.

The sample is distributed over time as outlined in Table 1.

TABLE 1. TRIP DISTRIBUTION SAMPLE

<u>Date</u>			<u>Trips Observed</u>	<u>Trips Generated by MBTA over the same period</u>	<u>Sampling Fractions</u>
5/23 Mon	6am-1pm	Inbound	9	57	.16
		Outbound	10	53	.19
5/24 Tues	1pm-8pm	Inbound	10	52	.19
		Outbound	9	53	.17
5/25 Wed	6am-1pm	Inbound	8	59	.14
		Outbound	8	53	.15
5/26 Thurs	1pm-8pm	Inbound	11	52	.21
		Outbound	10	53	.19
5/27 Fri	6am-1pm	Inbound	10	57	.18
		Outbound	10	53	.19
5/31 Tues	1pm-8pm	Inbound	11(-4)	52	.21
		Outbound	13	53	.25
6/1 Wed	6am-8pm	Inbound	10	107	.09
		Outbound	8	106	.08

II. ANALYSIS OF PASSENGER FLOW BY TEMPORAL DIVISION

A. Estimated daily volume of patronage (from the 13 stations)

It was discovered in the early stages of the study by ranking the trips according to their passenger volume, and testing the distribution of the ranks within each day, that the average daily passenger demand does not vary significantly from day to day. Hence all trip data thus collected are treated as if they have come from a single population rather than from five (Mōn-Fri) different ones. The matrices in Tables 2.a and 2.b show the results of stratifying the trips by the hours.

Estimated daily inbound volume is:

$$Y = \frac{N \sum N_h \bar{y}_h}{\sum N_h} = 107 \times 81.62 = 8734$$

where N is the total number of inbound trips in one day, (=107) and \bar{y}_h is the average number of trips per day in stratum (hour) h .

The variance of Y is:

$$V(Y) = \frac{N^2 \sum N_h^2 (1 - f_h) \sigma_{\bar{y}_h}^2}{(\sum N_h)^2} = 245,380.84$$

standard error = $\sqrt{V(Y)}$ = 495 or 5.7%.

Hence a 95% confidence interval for the actual total inbound volume is $(Y \pm 1.96 * \sqrt{V(Y)})$, which is (7793,9704).

Similarly, from the outbound matrix, estimated daily outbound volume, X , is :

$$X = \frac{N \sum N_h \bar{x}_h}{\sum N_h} = 106 \times 93.33 = 9893$$

where N is the total number of outbound trips from 6:00am to 8:00 pm. In this case, $N = 106$.

$$V(X) = \frac{N^2 \sum N_h^2 (1 - f_h) \sigma_{\bar{x}_h}^2}{(\sum N_h)^2}$$

Standard error = $\sqrt{V(X)}$ = 558 or 5.8%.

Hence a 95% confidence interval for the actual total outbound volume is $(X \pm 1.96 * \sqrt{V(X)})$, which is (8799,10987).

TABLE 2. AVERAGE PASSENGERS PER TRIP

a. Inbound Boarding

	6-7 am.	7-8	8-9	9-10	10-11	11-12	12-1pm	1-2	2-3	3-4	4-5	5-6	6-7	7-8
Approx. # of trips, N_h , generated during sampling period	32	36	32	28	36	28	32	28	36	32	36	24	24	24
# of trips in sample, n_h	5	3	5	5	5	4	4	5	5	5	3	3	4	8
sampling fraction, n_h/N_h	.16	.08	.16	.18	.14	.14	.13	.18	.14	.16	.08	.13	.17	.33
Avg. total loading passengers per trip, \bar{y}_h	44	109	170	84	72	49	74	83	88	107	73	63	61	40
sample variance, $\sigma_{y_h}^2$	13	64	55	19	19	22	23	51	48	46	12	24	21	9
standard error of \bar{y}_h , $\sigma_{\bar{y}_h}$	6	37	25	9	9	11	11	23	22	21	7	14	11	3

b. Outbound Deboarding

Time of Day	6-7 am	7-8	8-9	9-10	10-11	11-12	12-1pm	1-2	2-3	3-4	4-5	5-6	6-7	7-8
Approx. # of trips, N_h , generated during sampling period	24	32	36	28	32	28	32	28	32	32	36	32	28	24
# trips in sample, n_h	7	3	4	4	6	3	5	5	4	5	4	5	6	4
sampling fraction, n_h/N_h	.29	.09	.11	.14	.19	.11	.16	.13	.13	.16	.11	.16	.21	.17
Avg. total deboarding passengers per trip, \bar{x}_h	42	45	126	106	54	89	72	84	104	117	166	110	87	77
Standard error of \bar{x}_h , $\sigma_{\bar{x}_h}$	5	7	17	22	16	27	10	24	35	28	15	24	25	21

B. Time distribution of passenger demand by time of day

Comparing the inbound demand (dominated by boarding passengers) with the outbound demand (dominated by disembarking passengers) shows that one time series is almost the mirror image of the other, except that the latter is more erratic and the demand remains relatively high in the evening hours. This latter fact could explain the difference in the total passenger volume estimated earlier. The 95% confidence intervals around the total inbound and total outbound passenger volume estimates overlap, which indicates that the difference as supported by the data is not necessarily significant. Note also that the afternoon peak for outbound trains (4-5:00pm) occurs one and a half hours later than for inbound trains (2-3:00pm).

Another interesting observation from Tables 2.a and 2.b is the reliability of the estimates for the passenger demand for an average trip. Even though an average demand statistic is obtained for each time period, the variation, σ_{y_h} (or σ_{x_h}) of the individual trip demand around the mean is quite high. In fact, the average variability for any trip, regardless of which time period it falls into, is, for inbound trips,

$$\sigma_y = \sqrt{\frac{\sum(n_h - 1) \sigma_{y_h}^2}{(\sum n_h) - 13}} \approx 34$$

and for outbound trips,

$$\sigma_x = \sqrt{\frac{\sum(n_h - 1) \sigma_{x_h}^2}{(\sum n_h) - 13}} \approx 44$$

If an estimate is required of a flow rate at a particular time, then a sampling window of 60 minutes or more permits considerable shifting of the mean. The high variability alludes to not only the fluctuating nature of passenger demand, but also the effect which any departure from the train schedule may have on the load factor.

III. SPATIAL DISTRIBUTION OF PASSENGER DEMAND

As reported earlier, marked differences exist in the level of inbound passenger demand among the thirteen surface stations with Newton Center, Riverside, Brookline Village, Fenway, Woodland being the busier stations. This section attempts to quantify the spatial distribution of demand across these stations. The first question is whether such a spatial distribution is similar from hour to hour, so that an overall cross-sectional profile can be obtained for all time periods.

Define $n_j^{(k)}$ = the average number of loading passengers from station j when the trip originates during time period k .

$n^{(k)}$ = total loading passengers from all stations when the trip originates during time period k

$p_j^{(k)}$ = $n_j^{(k)} / n^{(k)}$, station j 's share of the trip demand for the time period k .

Our hypothesis is:

$$H_0: p_j^{(1)} = p_j^{(2)} = \dots = p_j^{(K)} = p_j \text{ for all } j$$

The χ^2 test of homogeneity is employed, for which the test statistic,

$$D = \sum \sum \frac{n^{(k)} [p_j^{(k)} - p_j]^2}{p_j},$$

is calculated for both the morning and afternoon shifts.

Under the hypothesis H_0 , D will be distributed as a χ^2 statistic with $(13-1)*(K-1)$ degrees of freedom, where K is the number of hourly periods. Since over 20% of the expected values in the subsequent contingency table (see Appendix 1A) is less than 5, a modified test is used. The details are shown in Appendixes 1, A-E, with the results of the test clearly indicating the acceptance of our hypothesis. Hence, for any inbound or outbound trip, the distribution of demand across the stations is depicted by Figure 1.

A note of interest is that while 77% of the total passengers on an inbound trip go beyond the Fenway Station and into the underground, only an estimated 69% of those on the outbound train originate from the underground stations. Although this difference seems significant, percentages can be misleading since 77% of the inbound passengers is approximately $8734 \times .77 = 6725$, and 69% of the outbound passengers is $9893 \times .69 = 6826$. Thus it is reasonable to presume that people using the line to get in town generally get back by the same means.

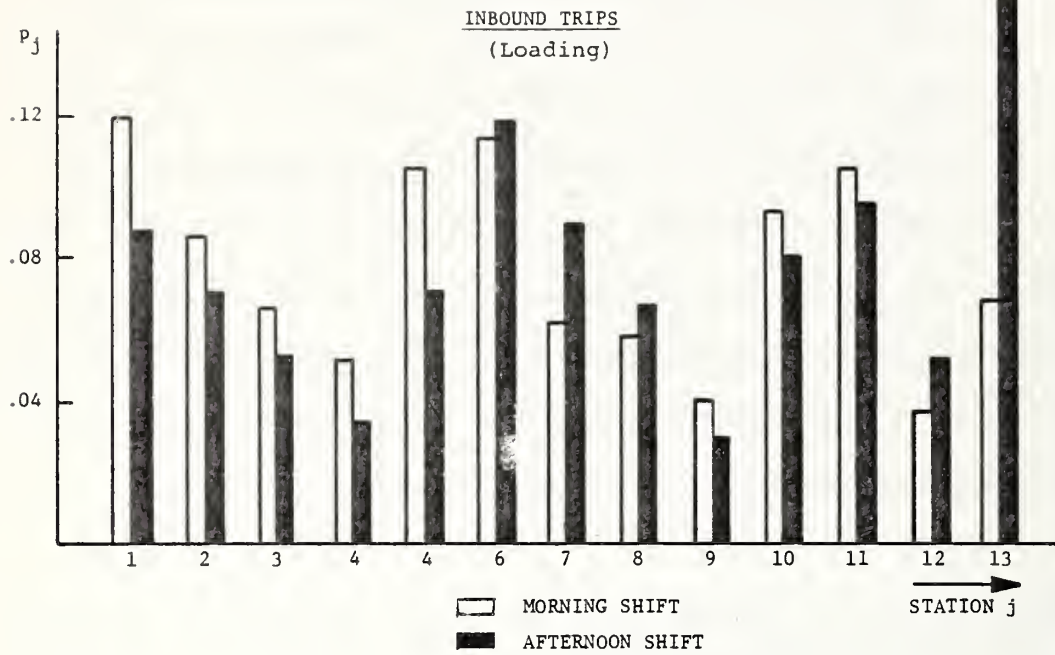


Fig.3.b.

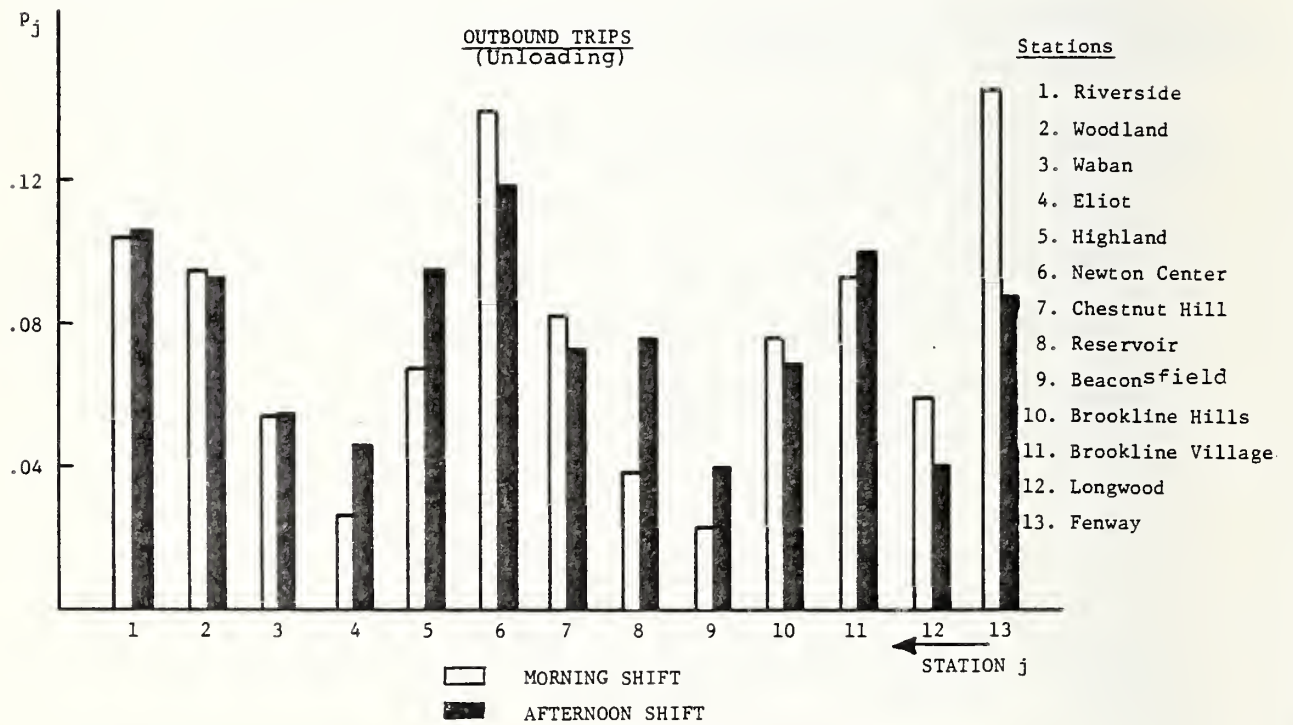


FIGURE 1. DEMAND DISTRIBUTION

IV. GENERATION OF PASSENGER DEMAND

To derive probability distribution functions, $f_{jt}(x)$, for the generation of the number of passengers getting on or off at station j and time t , previous conclusions on constant market shares for the thirteen stations prove to be useful. Suppose a train leaving the originating station at time t has its expected total trip passengers represented by X_t , then the expected number of people getting on this trip from station j is $p_j X_t$. If the probability distribution which generates the total trip demand is a Poisson distribution with parameter $\lambda_t h$, where h is the arbitrary headway, then $f_{jt}(x)$ is a Poisson distribution with parameter $p_j \lambda_t h$. The choice of the Poisson distribution follows from the hypothesis that the batch size of passengers arriving at a station within the time interval h has a probability expressed by the Poisson function.¹ $p_j \lambda_t$, then, becomes the rate of arrival at station j when the train leaves the originating station at time t .

It remains to determine the set of values λ_t 's. However, the estimation of such parameters requires repeated sampling at time t , which is not available at present. The next preferable solution is to regard our data series as one analogous to a discrete time series, u_t , $t=1,2,\dots$, (interpolating

¹Feller, William, "An Introduction to Probability Theory and Its Applications," p.156-164.

if necessary to estimate the missing u_t 's) and fitting a time trend to the series by a simple moving average of certain length, say $2L+1$. Then,

$$\hat{u}_t = 1/(2L+1) [u_{t-L} + \dots + u_{t-1} + u_t + u_{t+1} + \dots + u_{t+L}]$$

For example, for a length of 5,

$$\hat{u}_t = 1/5 [u_{t-2} + u_{t-1} + u_t + u_{t+1} + u_{t+2}]$$

Figures 2 through 5 show the inbound and outbound raw series and the extracted time trends using simple moving averages of length 5 and 11 and data from Tables 3.a and 3.b. It is worthy to note that these smoothed series are by no means a differentiable function of time, so that they cannot be modeled by any deterministic function such as a polynomial of a high order.

Having derived a smoothed series \hat{u}_t , $t=1,2,\dots$, the λ_t 's are obtained by simply dividing \hat{u}_t by Δt , where Δt is the time elapsed between trip $t-1$ and t . Table 4 shows the actual schedule of the inbound trips, together with the time of arrival at each station along the line. Note also that while Δt denotes the time in minutes, the subscript t represents the trip number, which in turn can be translated into time using Table 4. Figure 6 clarifies the application of λ_t in the Poisson probability density.

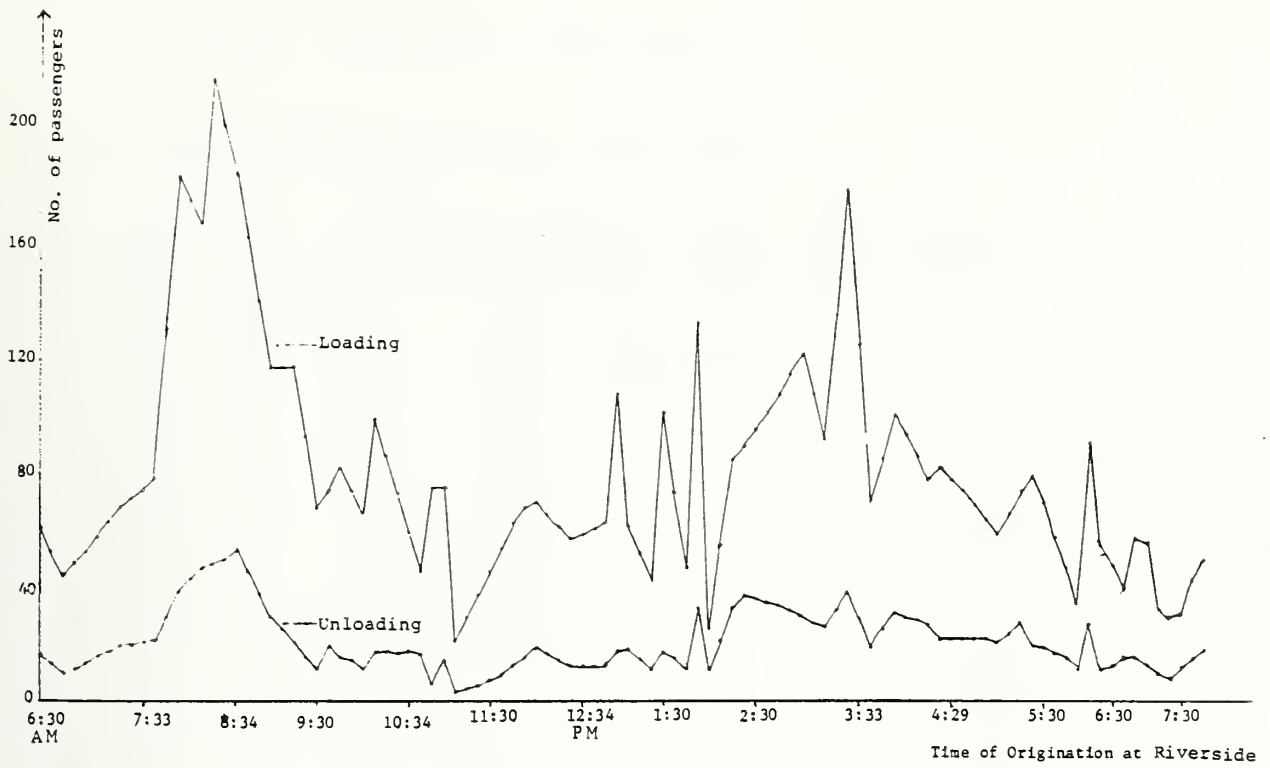


FIGURE 2. INBOUND RAW SERIES

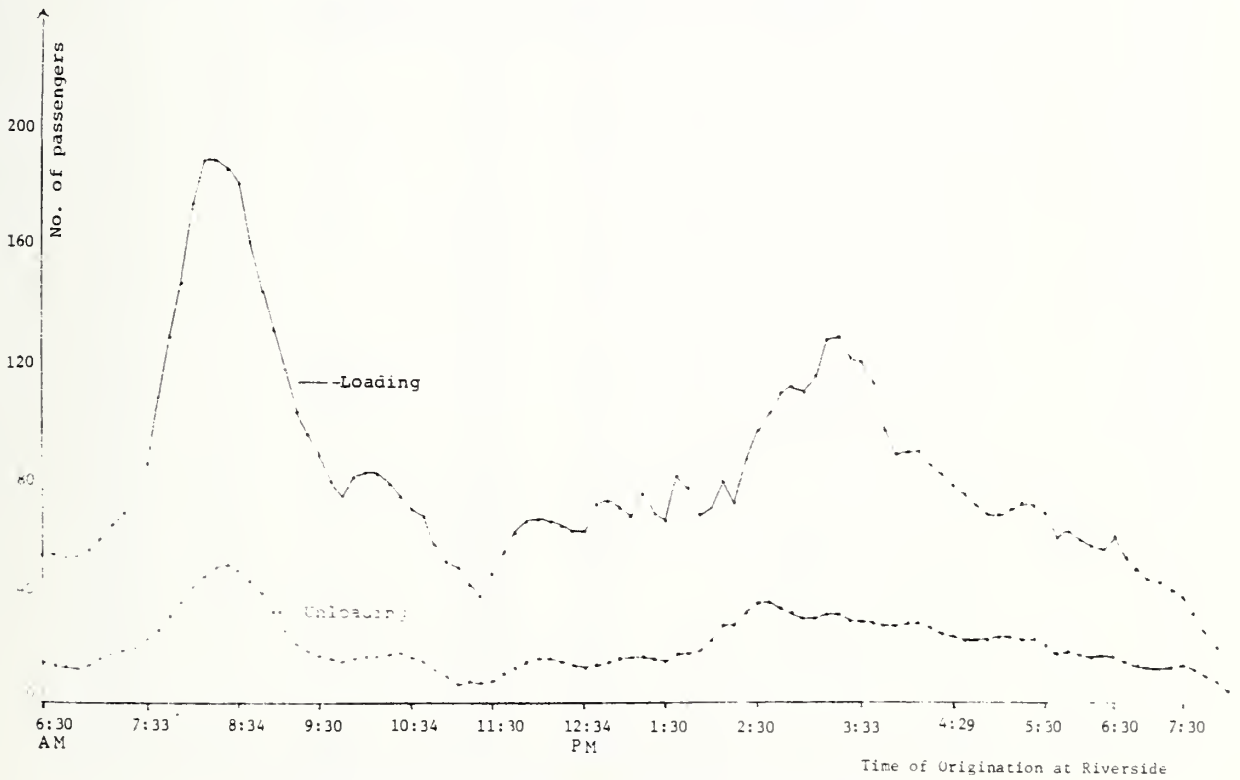


FIGURE 3. INBOUND SMOOTH SERIES
(Smoothed over a length of 5)

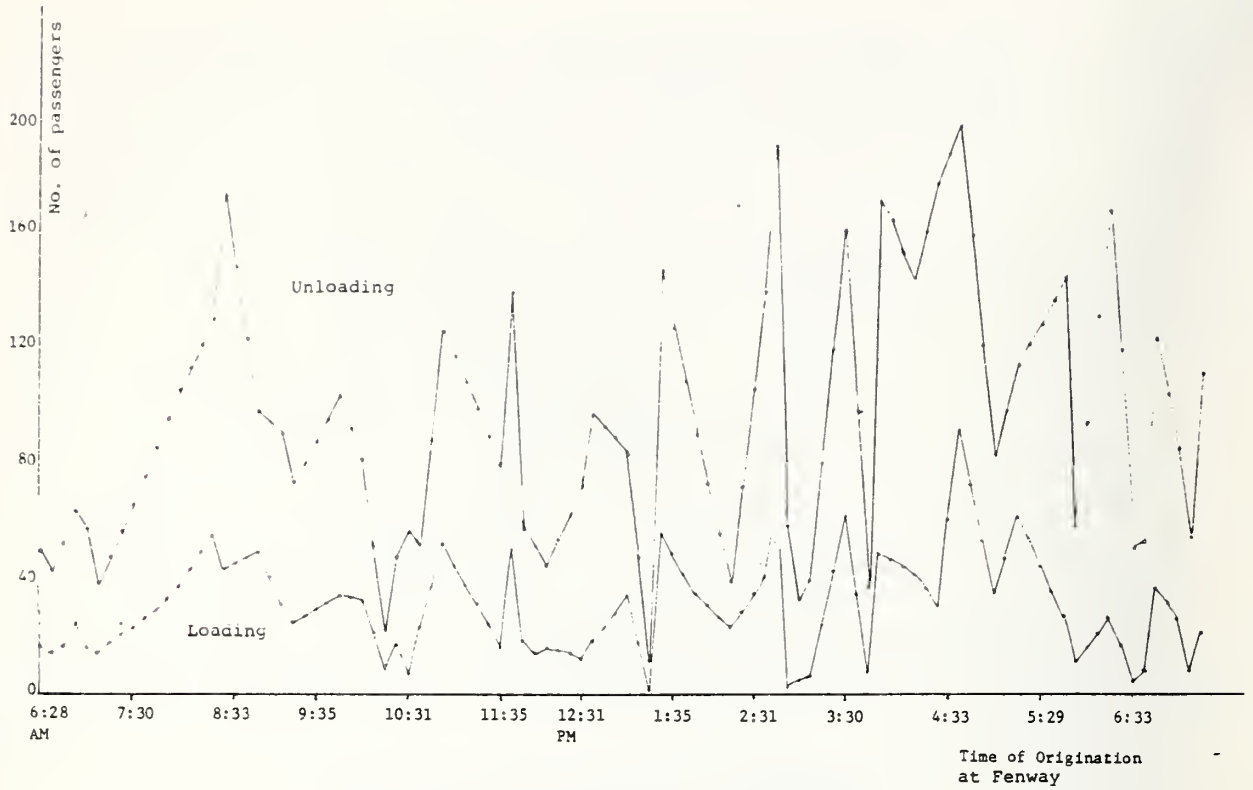


FIGURE 4. OUTBOUND RAW SERIES

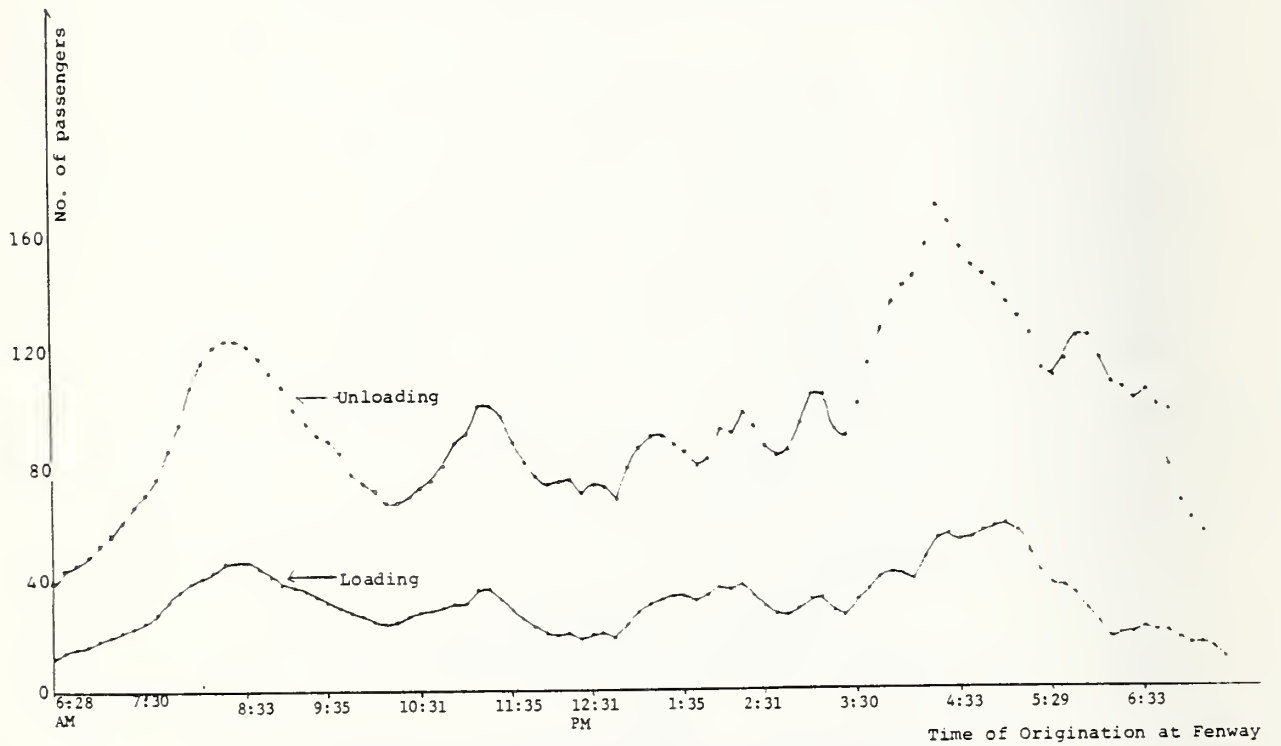


FIGURE 5. OUTBOUND SMOOTH SERIES
(Smoothed over a length of 11)

TABLE 3. SMOOTHED PASSENGER DATA FOR TOTAL TRIPS

a. Boarding/Deboarding Data Smoothed over 5 Trips (Inbound Trips)

TRIP #	SCHED. START at Riverside	MON	BOARDING COUNTS			DEBOARDING COUNTS				
			RAW	INTERP.	SMOOTHED	RESID.	RAW	INTERP.	SMOOTHED	RESID.
1	5:48	360		23	17	6		4	4	0
2	5:59	11	24	24	27	-3	5	5	7	-2
3	6:9	10		38	37	1		11	10	1
4	6:16	7	48	48	45	3	16	16	13	3
5	6:23	7		54	50	4		16	14	2
6	6:30	7	60	60	52	8	16	16	14	2
7	6:37	7		52	52	0		13	13	0
8	6:44	7	44	44	51	-7	10	10	13	-3
9	6:51	7		48	51	-3		11	12	-1
10	6:58	7		52	53	-1		13	13	0
11	7:5	7		57	57	0		15	15	0
12	7:12	7		62	62	0		17	17	0
13	7:19	7	67	67	66	1	19	19	18	1
14	7:26	7		70	70	0		19	19	0
15	7:33	7		73	83	-10		20	22	-2
16	7:40	7	77	77	106	-29	21	21	25	-4
17	7:47	7		129	127	2		29	30	-1
18	7:54	7	182	182	146	36	38	38	35	3
19	8:2	8		174	173	1		42	40	2
20	8:10	8	166	166	188	-22	46	46	44	2
21	8:18	8	216	216	188	28	47	47	47	0
22	8:26	8		200	185	15		49	48	1
23	8:34	8	183	183	180	3	52	52	46	6
24	8:42	8		161	160	1		45	42	3
25	8:50	8		139	143	-4		37	38	-1
26	8:58	8	116	116	130	-14	29	29	31	-2
27	9:6	8		116	116	0		25	25	0
28	9:14	8	116	116	101	15	20	20	20	0
29	9:22	8		92	93	-1		16	18	-2
30	9:30	8	67	67	86	-19	11	11	16	-5
31	9:38	8	73	73	77	-4	19	19	15	4
32	9:46	8	81	81	72	9	16	16	14	2
33	9:54	8		73	78	-5		14	15	-1
34	10:2	8	65	65	80	-15	11	11	15	-4
35	10:10	8	98	98	79	19	17	17	15	2
36	10:18	8		85	76	9		17	16	1
37	10:26	8		72	72	0		17	17	0
38	10:34	8		59	67	-8		17	15	2
39	10:42	8	45	45	65	-20	16	16	14	2
40	10:50	8	74	74	55	19	6	6	11	-5
41	10:58	8	74	74	49	25	14	14	9	5
42	11:6	8	21	21	47	-26	3	3	6	-3
43	11:14	8		29	41	-12		4	7	-3
44	11:22	8		37	37	0		5	6	-1
45	11:30	8	45	45	45	0	7	7	7	0
46	11:38	8		53	52	1		9	10	-1
47	11:46	8	62	62	59	3	12	12	12	0
48	11:54	8		65	63	2		15	14	1
49	12:2	8	69	69	64	5	18	18	15	3
50	12:10	8		65	63	2		16	15	1
51	12:18	8		61	62	-1		14	14	0
52	12:26	8	57	57	60	-3	12	12	13	-1
53	12:34	8		58	60	-2		12	12	0
54	12:42	8		60	69	-9		12	13	-1
55	12:50	8	62	62	70	-8	12	12	14	-2
56	12:58	8	107	107	68	39	17	17	15	2

TABLE 3.a (Cont.)

TRIP #	SCHED. START	HDWY	BOARDING COUNTS				DEBOARDING COUNTS			
			RAW	INTERP.	SMOOTHED	RESID.	RAW	INTERP.	SMOOTHED	RESID.
57	13:06	8	61	61	68	-4	18	18	15	3
58	13:14	8		52	73	-21		15	16	-1
59	13:22	8	43	43	66	-23	11	11	15	-4
60	13:30	8	101	101	63	38	17	17	14	3
61	13:38	8		73	79	-6		15	17	-2
62	13:46	8	45	45	75	-30	12	12	17	-5
63	13:54	8	132	132	66	66	32	32	18	14
64	14:02	8	26	26	68	-42	11	11	22	-11
65	14:09	7		55	77	-22		21	27	-6
66	14:16	7	84	84	70	14	32	32	27	5
67	14:23	7	89	89	85	4	37	37	32	5
68	14:30	7		95	95	0		36	35	1
69	14:37	7		101	101	0		35	35	0
70	14:44	7		107	108	-1		34	33	1
71	14:51	7		114	110	4		32	32	0
72	14:58	7	121	121	108	13	30	30	30	0
73	15:05	7		107	114	-7		28	30	-2
74	15:12	7	92	92	127	-35	26	26	31	-5
75	15:19	7		135	128	7		32	31	1
76	15:26	7	179	179	120	59	38	38	29	9
77	15:33	7		125	119	6		29	29	0
78	15:40	7	71	71	112	-41	19	19	28	-9
79	15:47	7		85	95	-10		25	27	-2
80	15:54	7	100	100	87	13	31	31	27	4
81	16:01	7		91	88	3		30	28	2
82	16:08	7		86	88	-2		29	28	1
83	16:15	7	78	78	83	-5	27	27	26	1
84	16:22	7	82	82	80	2	22	22	24	-2
85	16:29	7		78	76	2		22	23	-1
86	16:36	7		74	73	1		22	22	0
87	16:43	7		69	69	0		22	22	0
88	16:50	7		64	66	-2		22	22	0
89	16:57	7	59	59	66	-7	21	21	23	-2
90	17:05	8		66	68	-2		24	23	1
91	17:13	8	74	74	70	4	28	28	22	6
92	17:21	8	79	79	69	10	20	20	22	-2
93	17:29	8		70	66	4		19	20	-1
94	17:40	11		58	58	0		17	17	0
95	17:50	10		47	60	-13		15	18	-3
96	18:00	10	35	35	57	-22	12	12	17	-5
97	18:10	10	91	91	55	36	27	27	16	11
98	18:20	10	55	55	54	1	12	12	16	-4
99	18:30	10		48	58	-10		13	16	-3
100	18:40	10	40	40	51	-11	15	15	14	1
101	18:50	10	58	58	47	11	15	15	13	2
102	19:00	10	56	56	43	13	13	13	12	1
103	19:10	10	33	33	42	-9	10	10	12	-2
104	19:20	10	30	30	39	-9	8	8	12	-4
105	19:30	10	31	31	37	-6	12	12	13	-1
106	19:40	10	43	43	31	12	15	15	11	4
107	19:50	10	50	50	25	25	18	18	9	9
108	20:00	10		0	19	-19		0	7	-7
109	20:10	10		0	10	-10		0	4	-4
110	20:20	10		0	0	0		0	0	0
111	20:30	10		0	0	0		0	0	0
112	20:40	10		0	0	0		0	0	0
113	20:50	10		0	0	0		0	0	0
114	21:00	10		0	0	0		0	0	0
115	21:10	10		0	0	0		0	0	0
116	21:20	10		0	0	0		0	0	0
117	21:30	10		0	0	0		0	0	0

TABLE 3.a (Cont.)

TRIP #	SCHED, START	MOBY	BOARDING COUNTS			DEBOARDING COUNTS		
			RAW	INTERP, SMOOTHED	RESID,	RAW	INTERP, SMOOTHED	RESID,
118	21:40	10	0	0	0	0	0	0
119	21:50	10	0	0	0	0	0	0
120	22:00	10	0	0	0	0	0	0
121	22:10	10	0	0	0	0	0	0
122	22:20	10	0	0	0	0	0	0
123	22:30	10	0	0	0	0	0	0
124	22:40	10	0	0	0	0	0	0
125	22:50	10	0	0	0	0	0	0
126	23:00	10	0	0	0	0	0	0
127	23:10	10	0	0	0	0	0	0
128	23:20	10	0	0	0	0	0	0
129	23:30	10	0	0	0	0	0	0
130	23:40	10	0	0	0	0	0	0
131	23:50	10	0	0	0	0	0	0
132	24:00	10	0	0	0	0	0	0
TOTALS								
132	18:12		8396	8384	235	2170	2167	10

TABLE 3. SMOOTHED PASSENGER DATA FOR TOTAL TRIPS

b. Boarding/Deboarding Data Smoothed Over 11 Trips (Outbound Trips)

TRIP #	SCHED. START at Fenway	HDWY	BOARDING COUNTS			DEBOARDING COUNTS				
			RAW	INTERP. u _t	SMOOTHED G _t	RESID.	RAW	INTERP. u _t	SMOOTHED G _t	RESID.
1	5146	364		7	6	1		28	21	7
2	6111	15		7	8	-1		29	27	2
3	6115	14	8	8	10	-2	31	31	32	-1
4	6128	13	16	16	11	5	49	49	35	14
5	6138	10	14	14	13	1	42	42	40	2
6	6141	3	16	16	14	2	52	52	45	7
7	6152	11	24	24	16	8	63	63	48	15
8	712	12	16	16	18	-2	57	57	52	5
9	719	7	14	14	20	-6	38	38	37	-1
10	7116	7		17	21	-4		47	61	-14
11	7123	7		20	23	-3		56	67	-11
12	7130	7		23	26	-3		65	73	-8
13	7137	7		26	28	-2		73	78	-5
14	7144	7		29	31	-2		85	85	0
15	7151	7		33	34	-1		95	97	-2
16	7158	7	37	37	37	0	105	105	106	-1
17	815	7		43	39	4		113	112	1
18	8112	7		49	41	8		121	115	6
19	8119	7	55	55	43	12	130	130	117	13
20	8126	7	43	43	43	0	173	173	117	56
21	8133	7		45	42	3		148	115	33
22	8140	7		47	41	6		123	113	10
23	8147	7	49	49	40	9	98	98	111	-13
24	8155	8		40	38	2		94	108	-14
25	913	8	31	31	36	-5	90	90	106	-16
26	9111	8	25	25	36	-11	73	73	98	-25
27	9119	8		27	34	-7		80	92	-12
28	9127	8		29	32	-3		87	86	1
29	9135	8		31	28	3		95	79	16
30	9143	8	34	34	26	8	103	103	75	28
31	9151	8		33	24	9		92	71	21
32	9159	8	32	32	24	8	81	81	69	12
33	1017	8		21	25	-4		31	70	-39
34	10115	8	9	9	27	-18	21	21	74	-53
35	10123	8	17	17	28	-11	47	47	76	-29
36	10131	8	7	7	29	-22	56	56	76	-20
37	10139	8	23	23	28	-5	51	51	77	-26
38	10147	8		37	28	9		88	78	10
39	10155	8	52	52	27	25	126	126	80	46
40	1113	8		45	31	14		117	91	26
41	11111	8		38	31	7		128	92	36
42	11119	8		31	32	-1		99	91	8
43	11127	8		24	31	-7		89	91	-2
44	11135	8	16	16	29	-13	79	79	87	-8
45	11143	8	50	50	25	24	139	139	82	57
46	11151	8	18	18	23	-5	57	57	78	-21
47	11159	8	14	14	21	-7	51	51	77	-26
48	1217	8	16	16	20	-4	44	44	76	-32
49	12115	8		15	20	-5		33	78	-45
50	12123	8		14	22	-8		62	76	-14
51	12131	8	12	12	19	-7	72	72	68	4
52	12139	8	18	18	18	0	97	97	64	33
53	12147	8		23	21	2		93	73	20
54	12155	8	23	28	24	4	89	89	80	9
55	1313	8	34	34	27	7	83	83	85	-2
56	13111	8		18	29	-11		47	88	-41

TABLE 3.b (Cont.)

TRIP #	SCHED. START	MONY	BOARDING COUNTS				DEBOARDING COUNTS			
			RAW	INTERP.	SMOOTHED	RESID.	RAW	INTERP.	SMOOTHED	RESID.
57	13119	8	1	1	30	-29	11	11	88	-77
58	13127	8	56	56	31	29	147	147	84	63
59	13135	8		49	31	18		128	79	49
60	13143	8		42	31	11		109	78	31
61	13151	8	35	35	31	4	90	90	80	10
62	13159	8		31	34	-3		73	88	-15
63	1417	8		27	40	-13		56	105	-49
64	14115	8	23	23	35	-12	39	39	97	-58
65	14123	8		29	31	-2		72	88	-16
66	14131	8		35	28	7		106	82	24
67	14139	8	41	41	27	14	140	140	81	59
68	14147	8	70	70	28	42	191	191	85	106
69	14155	8	2	2	31	-29	59	59	95	-36
70	1512	7	5	5	32	-27	33	33	100	-67
71	1519	7	6	6	30	-24	40	40	97	-57
72	15116	7		24	31	-7		82	103	-23
73	15123	7		43	32	11		120	105	15
74	15132	7	62	62	30	32	161	161	102	59
75	15137	7		35	33	2		99	109	-10
76	15144	7	8	8	36	-28	37	37	121	-84
77	15151	7	49	49	38	11	171	171	134	37
78	15158	7		47	42	5		163	143	20
79	1615	7		45	46	-1		154	150	4
80	16112	7	42	42	47	-5	145	145	150	-5
81	16119	7		37	49	-12		161	152	9
82	16126	7	31	31	51	-20	178	178	157	21
83	16133	7		61	51	10		188	150	38
84	16140	7	92	92	53	39	198	198	146	52
85	16147	7		73	54	19		160	143	17
86	16154	7		54	54	0		122	141	-19
87	1711	7	35	35	54	-19	84	84	139	-55
88	1718	7		48	53	-5		99	136	-37
89	17115	7	62	62	49	13	115	115	125	-10
90	17122	7		54	42	12		122	115	7
91	17129	7		45	37	8		129	113	16
92	17136	7		36	35	1		137	117	20
93	17143	7	27	27	33	-6	145	145	120	25
94	17150	7	11	11	29	-18	59	59	116	-57
95	17158	8		16	24	-8		95	110	-15
96	1816	8		21	23	-2		132	110	22
97	18114	8	26	26	21	5	169	169	108	61
98	18122	8		17	20	-3		120	104	16
99	18133	11	4	4	19	-15	51	51	95	-44
100	18143	10	8	8	20	-12	53	53	100	-47
101	18153	10	37	37	18	19	124	124	92	32
102	1913	10		32	16	16		105	80	25
103	19113	10	26	26	14	12	86	86	64	22
104	19123	10	8	8	12	-4	55	55	53	2
105	19133	10	21	21	12	9	112	112	49	63
106	19143	10		0	11	-11		0	44	-44
107	19153	10		0	8	-8		0	33	-33
108	2013	10		0	5	-5		0	23	-23
109	20113	10		0	3	-3		0	15	-15
110	20123	10		0	2	-2		0	10	-10
111	20133	10		0	0	0		0	0	0
112	20143	10		0	0	0		0	0	0
113	20153	10		0	0	0		0	0	0
114	2113	10		0	0	0		0	0	0
115	21113	10		0	0	0		0	0	0
116	21123	10		0	0	0		0	0	0
117	21133	10		0	0	0		0	0	0

TABLE 3.b (Cont.)

TRIP #	SCHED. START	MOBY	BOARDING COUNTS			DEBOARDING COUNTS		
			RAW	INTERP.	SMOOTHED RESID.	RAW	INTERP.	SMOOTHED RESID.
118	21143	10	0	0	0	0	0	0
119	21153	10	0	0	0	0	0	0
120	2213	10	0	0	0	0	0	0
121	22113	10	0	0	0	0	0	0
122	22123	10	0	0	0	0	0	0
123	22133	10	0	0	0	0	0	0
124	22143	10	0	0	0	0	0	0
125	22153	10	0	0	0	0	0	0
126	2313	10	0	0	0	0	0	0
127	23113	10	0	0	0	0	0	0
128	23123	10	0	0	0	0	0	0
129	23133	10	0	0	0	0	0	0
130	23143	10	0	0	0	0	0	0
131	23153	10	0	0	0	0	0	0
132	2413	10	0	0	0	0	0	0
133	24113	10	0	0	0	0	0	0
134	24123	10	0	0	0	0	0	0
135	24133	10	0	0	0	0	0	0
136	24143	10	0	0	0	0	0	0
137	24153	10	0	0	0	0	0	0
TOTALS								
137	1917		3189	3176	131	9860	9818	859

TABLE 4. A CONSOLIDATED ONE-DAY INBOUND SCHEDULE

Arriving at:

Trip	Riverside	Woodland	Waban	Eliot	Highland	Newton Center	Chestnut Hill	Beacon Reservoir	Brookline Hills	Brookline Village	Longwood	Fenway
1	6:30	6132	6:34	6136	6:38	6140	6143	6:45	6:47	6149	6151	6153
2	6:37	6139	6:41	6143	6:45	6147	6152	6:52	6:54	6156	6158	7111
3	6:44	6146	6:48	6150	6:52	6154	6157	6:59	7:1	7113	7115	7117
4	6:51	6153	6:55	6157	6:59	7111	7114	7:6	7:8	7116	7118	7120
5	6:58	7112	7:2	7114	7:6	7117	7121	7:13	7:15	7117	7119	7121
6	7:5	7117	7:9	7111	7:13	7115	7118	7:20	7:22	7124	7126	7128
7	7:12	7114	7:16	7118	7:20	7122	7125	7:27	7:29	7131	7133	7135
8	7:19	7121	7:23	7125	7:27	7129	7132	7:34	7:36	7135	7137	7139
9	7:26	7128	7:30	7132	7:34	7136	7139	7:41	7:43	7145	7147	7149
10	7:33	7135	7:37	7139	7:41	7143	7146	7:48	7:50	7152	7154	7156
11	7:40	7142	7:44	7146	7:48	7150	7153	7:55	7:57	7159	8111	8113
12	7:47	7149	7:51	7153	7:55	7157	8110	8:2	8:4	8114	8116	8118
13	7:54	7156	7:58	810	8:2	814	817	8:9	8:11	8113	8115	8117
14	8:2	814	8:6	818	8:10	8122	8125	8:17	8:19	8121	8123	8125
15	8:10	8112	8:14	8116	8:18	8120	8123	8:25	8:27	8129	8131	8133
16	8:18	8120	8:22	8124	8:26	8126	8129	8:33	8:35	8137	8139	8141
17	8:26	8128	8:30	8132	8:34	8134	8137	8:41	8:43	8145	8147	8149
18	8:34	8136	8:38	8140	8:42	8142	8145	8:49	8:51	8153	8155	8157
19	8:42	8144	8:46	8148	8:50	8150	8153	8:57	8:59	9111	9113	9115
20	8:50	8152	8:54	8156	8:58	9110	9113	9:5	9:7	9115	9117	9119
21	8:58	9110	9:2	9114	9:6	9116	9119	9:13	9:15	9117	9119	9121
22	9:6	9118	9:10	9122	9:14	9124	9127	9:21	9:23	9125	9127	9129
23	9:14	9126	9:18	9130	9:22	9130	9133	9:29	9:31	9135	9137	9139
24	9:22	9134	9:26	9138	9:30	9140	9143	9:37	9:39	9141	9143	9145
25	9:30	9142	9:34	9146	9:38	9148	9151	9:45	9:47	9149	9151	9153
26	9:38	9150	9:42	9154	9:46	9156	9159	9:53	9:55	9157	9159	1011
27	9:46	1010	9:50	9162	9:54	1012	1015	10:1	10:3	1013	1015	1017
28	9:54	1018	9:58	1014	10:2	1016	1019	10:9	10:11	1013	1015	1017
29	10:2	1014	10:6	1018	10:10	1012	1015	10:17	10:19	1021	1023	1025
30	10:10	1012	10:14	1016	10:18	1020	1023	10:25	10:27	1029	1031	1033
31	10:18	1020	10:22	1024	10:26	1028	1031	10:33	10:35	1037	1039	1041
32	10:26	1028	10:30	1032	10:34	1036	1039	10:41	10:43	1045	1047	1049
33	10:34	1036	10:38	1040	10:42	1044	1047	10:49	10:51	1053	1055	1057
34	10:42	1044	10:46	1048	10:50	1052	1055	10:57	10:59	1111	1113	1115
35	10:50	1052	10:54	1056	10:58	1100	1103	11:5	11:7	1109	1111	1113
36	10:58	1100	11:2	1104	11:6	1106	1109	11:13	11:15	1117	1119	1121
37	11:6	1108	11:10	1112	11:14	1116	1119	11:21	11:23	1125	1127	1129
38	11:14	1116	11:18	1120	11:22	1124	1127	11:29	11:31	1133	1135	1137
39	11:22	1124	11:26	1128	11:30	1132	1135	11:37	11:39	1141	1143	1145
40	11:30	1132	11:34	1136	11:38	1140	1143	11:45	11:47	1149	1151	1153
41	11:38	1140	11:42	1144	11:46	1148	1151	11:53	11:55	1157	1159	1211
42	11:46	1148	11:50	1152	11:54	1156	1159	12:1	12:3	1211	1213	1215
43	11:54	1156	11:58	1200	12:2	1204	1207	12:9	12:11	1213	1215	1217
44	12:2	1204	12:6	1208	12:10	1212	1215	12:17	12:19	1221	1223	1225
45	12:10	1212	12:14	1216	12:18	1220	1223	12:25	12:27	1229	1231	1233
46	12:18	1220	12:22	1224	12:26	1228	1231	12:33	12:35	1237	1239	1241
47	12:26	1228	12:30	1232	12:34	1236	1239	12:41	12:43	1245	1247	1249
48	12:34	1236	12:38	1240	12:42	1244	1247	12:49	12:51	1253	1255	1257
49	12:42	1244	12:46	1248	12:50	1252	1255	12:57	12:59	1311	1313	1315

TABLE 4 (Cont.)

Arriving at:												
TRIP	Riverside	Woodland	Waban	Eliot	Highland	Newton Center	Chestnut Hill	Beacon Reservoir	Brookline Field Hills	Brook. Village	Longwood	Fenway
50	12:50	12:52	12:54	12:56	12:58	1:1	1:3	1:5	1:7	1:9	1:11	1:13
51	12:58	1:1	1:2	1:4	1:6	1:8	1:11	1:13	1:15	1:17	1:19	1:21
52	1:6	1:8	1:10	1:12	1:14	1:16	1:19	1:21	1:23	1:25	1:27	1:30
53	1:14	1:16	1:18	1:20	1:22	1:24	1:27	1:29	1:31	1:33	1:35	1:38
54	1:22	1:24	1:26	1:28	1:30	1:32	1:35	1:37	1:39	1:41	1:43	1:46
55	1:30	1:32	1:34	1:36	1:38	1:41	1:43	1:45	1:47	1:49	1:51	1:54
56	1:38	1:41	1:42	1:44	1:46	1:49	1:51	1:53	1:55	1:57	1:59	2:1
57	1:46	1:49	1:50	1:52	1:54	1:57	1:59	2:1	2:3	2:5	2:7	2:9
58	1:54	1:57	1:58	2:0	2:2	2:4	2:7	2:9	2:11	2:13	2:15	2:18
59	2:2	2:4	2:6	2:8	2:10	2:12	2:15	2:17	2:19	2:21	2:23	2:26
60	2:9	2:11	2:13	2:15	2:17	2:19	2:22	2:24	2:26	2:28	2:30	2:33
61	2:16	2:18	2:20	2:22	2:24	2:26	2:29	2:31	2:33	2:35	2:37	2:40
62	2:23	2:25	2:27	2:29	2:31	2:33	2:36	2:38	2:40	2:42	2:44	2:47
63	2:30	2:32	2:34	2:36	2:38	2:41	2:43	2:45	2:47	2:49	2:51	2:54
64	2:37	2:39	2:41	2:43	2:45	2:47	2:50	2:52	2:54	2:56	2:58	3:1
65	2:44	2:46	2:48	2:50	2:52	2:54	2:57	2:59	3:1	3:3	3:5	3:7
66	2:51	2:53	2:55	2:57	2:59	3:1	3:4	3:6	3:8	3:11	3:12	3:15
67	2:58	3:1	3:2	3:4	3:6	3:8	3:11	3:13	3:15	3:17	3:19	3:22
68	3:5	3:7	3:9	3:11	3:13	3:15	3:18	3:20	3:22	3:24	3:26	3:29
69	3:12	3:14	3:16	3:18	3:20	3:22	3:25	3:27	3:29	3:31	3:33	3:36
70	3:19	3:21	3:23	3:25	3:27	3:29	3:32	3:34	3:36	3:38	3:40	3:43
71	3:26	3:28	3:30	3:32	3:34	3:36	3:39	3:41	3:43	3:45	3:47	3:50
72	3:33	3:35	3:37	3:39	3:41	3:43	3:46	3:48	3:50	3:52	3:54	3:57
73	3:40	3:42	3:44	3:46	3:48	3:50	3:53	3:55	3:57	3:59	4:1	4:3
74	3:47	3:49	3:51	3:53	3:55	3:57	4:1	4:2	4:4	4:6	4:8	4:11
75	3:54	3:56	3:58	4:0	4:2	4:4	4:7	4:9	4:11	4:13	4:15	4:18
76	4:1	4:3	4:5	4:7	4:9	4:11	4:14	4:16	4:18	4:20	4:22	4:25
77	4:8	4:11	4:12	4:14	4:16	4:18	4:21	4:23	4:25	4:27	4:29	4:32
78	4:15	4:17	4:19	4:21	4:23	4:25	4:28	4:30	4:32	4:34	4:36	4:39
79	4:22	4:24	4:26	4:28	4:30	4:32	4:35	4:37	4:39	4:41	4:43	4:46
80	4:29	4:31	4:33	4:35	4:37	4:39	4:42	4:44	4:46	4:48	4:50	4:53
81	4:36	4:38	4:40	4:42	4:44	4:46	4:49	4:51	4:53	4:55	4:57	5:0
82	4:43	4:45	4:47	4:49	4:51	4:53	4:56	4:58	5:0	5:1	5:2	5:4
83	4:50	4:52	4:54	4:56	4:58	5:0	5:3	5:5	5:7	5:9	5:11	5:13
84	4:57	4:59	5:1	5:3	5:5	5:7	5:10	5:12	5:14	5:16	5:18	5:21
85	5:5	5:7	5:9	5:11	5:13	5:15	5:18	5:20	5:22	5:24	5:26	5:29
86	5:13	5:15	5:17	5:19	5:21	5:23	5:26	5:28	5:30	5:32	5:34	5:37
87	5:21	5:23	5:25	5:27	5:29	5:31	5:34	5:36	5:38	5:40	5:42	5:45
88	5:29	5:31	5:33	5:35	5:37	5:39	5:42	5:44	5:46	5:48	5:50	5:53
89	5:40	5:42	5:44	5:46	5:48	5:50	5:53	5:55	5:57	5:59	6:1	6:3
90	5:50	5:52	5:54	5:56	5:58	6:0	6:3	6:5	6:7	6:9	6:11	6:13
91	6:0	6:2	6:4	6:6	6:8	6:10	6:13	6:15	6:17	6:19	6:21	6:23
92	6:10	6:12	6:14	6:16	6:18	6:20	6:23	6:25	6:27	6:29	6:31	6:33
93	6:20	6:22	6:24	6:26	6:28	6:30	6:33	6:35	6:37	6:39	6:41	6:43
94	6:30	6:32	6:34	6:36	6:38	6:40	6:43	6:45	6:47	6:49	6:51	6:53
95	6:40	6:42	6:44	6:46	6:48	6:50	6:53	6:55	6:57	6:59	7:1	7:3
96	6:50	6:52	6:54	6:56	6:58	7:0	7:3	7:5	7:7	7:9	7:11	7:13
97	7:0	7:2	7:4	7:6	7:8	7:10	7:13	7:15	7:17	7:19	7:21	7:23
98	7:10	7:12	7:14	7:16	7:18	7:20	7:23	7:25	7:27	7:29	7:31	7:33
99	7:20	7:22	7:24	7:26	7:28	7:30	7:33	7:35	7:37	7:39	7:41	7:43
100	7:30	7:32	7:34	7:36	7:38	7:40	7:43	7:45	7:47	7:49	7:51	7:53
101	7:40	7:42	7:44	7:46	7:48	7:50	7:53	7:55	7:57	7:59	8:1	8:3
102	7:50	7:52	7:54	7:56	7:58	8:0	8:3	8:5	8:7	8:9	8:11	8:13
103	8:0	8:2	8:4	8:6	8:8	8:10	8:13	8:15	8:17	8:19	8:21	8:23
104	8:10	8:12	8:14	8:16	8:18	8:20	8:23	8:25	8:27	8:29	8:31	8:33
105	8:20	8:22	8:24	8:26	8:28	8:30	8:33	8:35	8:37	8:39	8:41	8:43
106	8:30	8:32	8:34	8:36	8:38	8:40	8:43	8:45	8:47	8:49	8:51	8:53

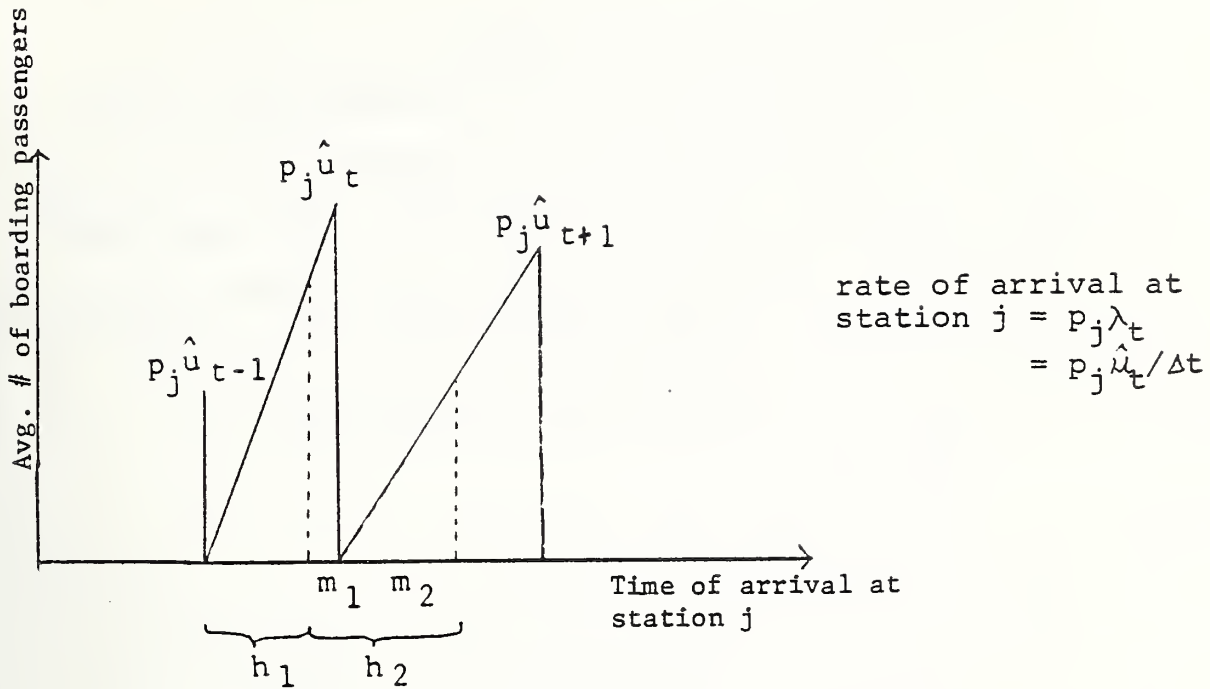


FIGURE 6. EXPECTED NUMBER OF PASSENGERS WAITING AT STATION J

1. Given a train has arrived at $t-1$, the second train, arriving h_1 minutes later, should expect the probability of having exactly x passengers accumulated at station j to be:

$$\Pr(x; p_j \lambda_t h_1) = \frac{e^{-p_j \lambda_t h_1} (p_j \lambda_t h_1)^x}{x!}$$

2. The third train arriving $h_2 (=m_1+m_2)$ minutes later, should expect the probability of having exactly y passengers to be:

$$\Pr(y; p_j \lambda_t m_1 + p_j \lambda_{t+1} m_2) = \frac{e^{-p_j (\lambda_t m_1 + \lambda_{t+1} m_2)} (p_j \lambda_t m_1 + p_j \lambda_{t+1} m_2)^y}{y!}$$

V. DISTRIBUTION OF THE DWELL TIME

The density function for the derivation of dwell time at each station is found to be dependent on the total "on-off" movement, M , taking place while the train remains stationary. This relationship is significant regardless of the hour of the day, the direction of the trip, or even the individual station configuration.

That is,

$$T_d = \alpha + \beta M + \epsilon$$

where α is the minimum dwell time, β the average rate of boarding and unloading, and ϵ , a random variable with zero mean and variance σ^2 .

This relationship was first shown for the PCC (President's Conference Committee) trains. The coefficients α , β , and σ , estimated by means of simple least squares regressions for a sample of stations, are listed in Table 5. Data where extra delay is indicated by the presence of equipment or fare problems, etc., are taken out of the data base before the regression analyses were performed. The figures in parentheses below the coefficients represent their respective standard errors. All the regressions are significant and the linear trends are readily observable from the scattergrams shown in Appendix 2. The low R^2 's, however, indicate the magnitude of the random fluctuation of dwell time even when a portion of it can be accounted by the delay incurred by boarding and unloading passengers.

TABLE 5. STATISTICAL ANALYSIS OF DWELL TIME (PCC TRAINS)

<u>Station j</u>	<u>n_j</u>	<u>Regression j</u>	<u>σ_j</u>	<u>residual sum of squares, SSE_j</u>
Highland	35	$T_d = 10.75 + .89M$ (1.58) (.18)	4.99	821.70
Newton Center	34	$T_d = 9.88 + .92M$ (1.88) (.15)	5.51	971.52
Brookline Village	37	$T_d = 8.19 + .99M$ (1.08) (.07)	3.60	453.60
Reservoir	34	$T_d = 8.45 + .96M$ (1.04) (.11)	3.65	426.32
Fenway	37	$T_d = 9.88 + .77M$ (1.19) (.09)	4.14	599.89
Chestnut Hill	34	$T_d = 9.29 + 1.03M$ (1.16) (.13)	3.74	447.60
Woodland	37	$T_d = 7.79 + 1.35M$ (1.03) (.13)	4.72	779.74
Combined	248	$T_d = 9.49 + .93M$ (.48) (.04)	4.47	4915.30

It is quite natural to suppose that a generalized dwell time vs. total movement relationship will be adequate for all stations and for both inbound and outbound trips. The scattergrams suggest that the seven regression lines could be pooled together to give a better precision on the estimation of the general level and slope. A formal test was accomplished to demonstrate whether they are in fact identical.

To test H_0 : all α_j 's are equal and
all β_j 's are equal,

against H_1 : either the α_j 's are not equal or the β_j 's are
not equal or both,

we need to examine the ratio of the "between station
variations" (which is the total variation minus the
within station variation) to the "within station variation".
Hence, the following statistic is defined.

$$F = \frac{SSE - \sum SSE_j}{(n-2) - \sum (n_j - 2)} \bigg/ \frac{\sum SSE_j}{\sum (n_j - 2)}$$

The decision to reject or accept H_0 is based on whether
 F is too large or too small. Compared to the 95th percentile
of an $F(12, 234)$ distribution, $F (=1.80)$ is small. Hence
the hypothesis of the adequacy of a general relationship
to represent all stations is accepted. This is also valid
for the outbound trips, the details of the comparisons are
shown in Appendix 2.

To generate or simulate dwell time at any station,
therefore, one may simply use:

$$T_d = 9.5 + .9 \times \text{total movement} + \varepsilon + \text{delay}$$

where ε is a random number generated from a $N(0, 4.5^2)$
distribution. The delay is an arbitrary nonnegative
number, incorporated into the equation for any delay due
to equipment problems, fare problems, or waiting for
passengers etc.

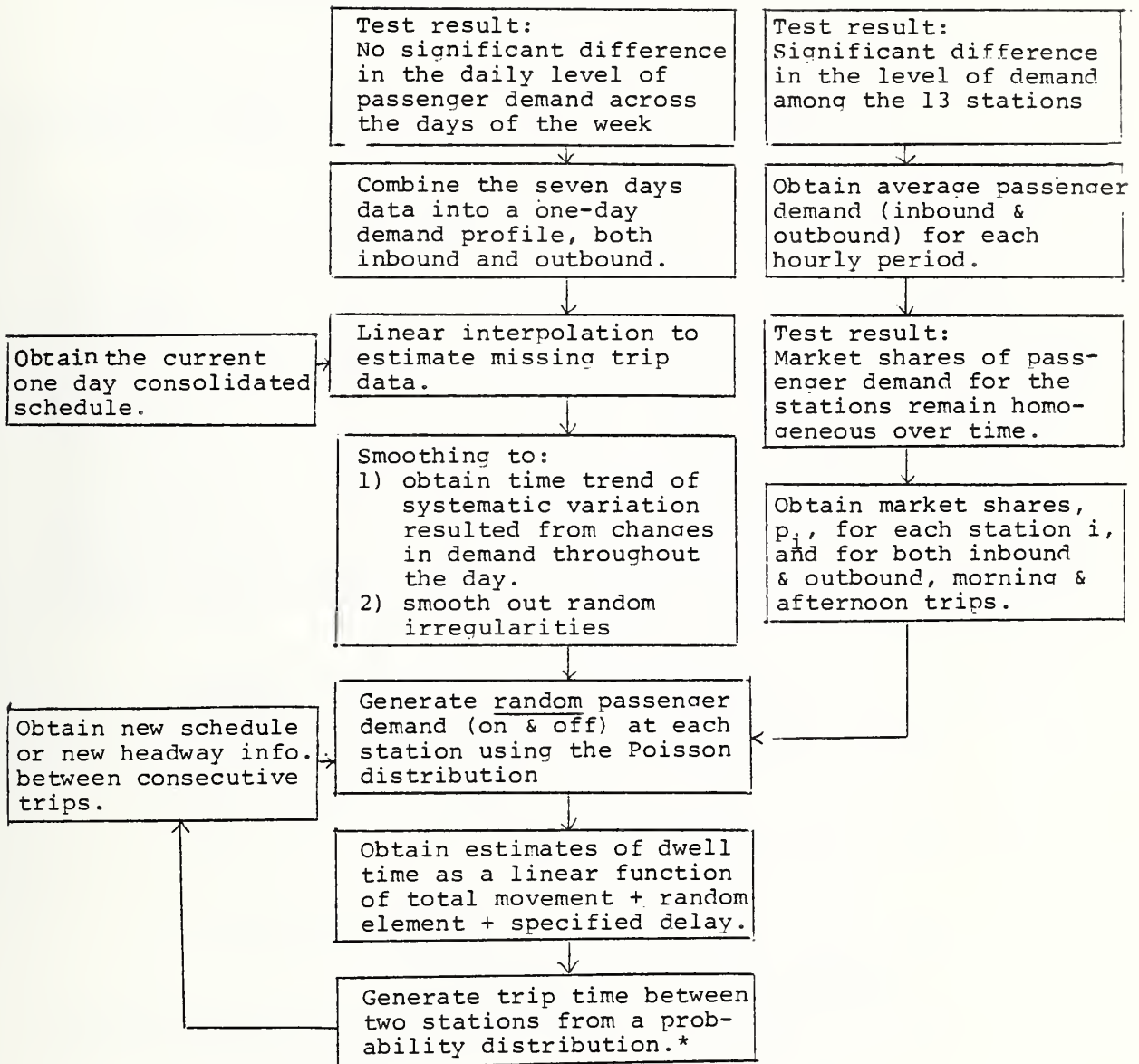
The dwell times for the Light Rail Vehicles display a different relationship, however; and are subjected to much variation. The linear regression is still significant, ($R^2 = .69$) even though it results in different coefficients. The generalized form for the LRV is:

$T_d = 10.75 + 1.46 \text{ total movement} + \psi + \text{delay}$, where ψ is again a $N(0, 7.16^2)$ random variable.

VI. CONCLUSION AND SUMMARY

Generation of the passenger demands and the dwell times at the stations based on the distributions and regression equations developed is important to the operational performance model which seeks the optimum train schedule to accommodate the undulating demand throughout the day. The high variability of the trip data does not allow for the estimation of passenger demand profile for each station, nor can it be used to test the assumption that the total trip demand at a certain time follows a Poisson distribution. This is because the time series thus presented represents only a single sample out of the many possible series from the sampling population. However, the choice of the discrete Poisson distribution is a most logical one because the arrival of passengers can be thought of as a series of random events in a time continuum. Hence the number of passengers per time period would be expected to form a Poisson distribution. Figure 7 is a flow chart illustrating the steps necessary for the simulation of passenger loading and unloading activities at each station.

The estimates of the total inbound and outbound passenger volumes with their respective standard errors set the lower and upperbounds for the general level of daily passenger activity. This can be helpful in the determination of the number of trains to be dispatched on any working day when no unusual circumstance affecting the passenger load is imminent. To conclude, this study has examined the input passenger data to the model, and developed estimation procedures to meet the model requirements.



*To be estimated.

FIGURE 7. DECISION FLOW CHART

APPENDIX 1: MARKET SHARE ANALYSIS

APPENDIX 1A: AVERAGE LOADING PASSENGERS
(Inbound morning trips)

Stations	Hourly	Row total						$P_i = R_i/N$	
	Period	6-7	7-8	8-9	9-10	10-11	11-12		R_i
Riverside		4	17	18	11	11	5	66	.119
Woodland		3	11	15	10	6	3	48	.086
Waban		2	6	16	6	3	4	37	.066
Eliot		4	6	11	3	3	2	29	.052
Highland		3	9	17	14	9	6	58	.104
Newton Center		5	10	19	12	10	8	64	.115
Chestnut Hill		2	7	12	4	5	4	34	.061
Reservoir		5	6	13	4	3	1	32	.058
Beacon Field		1	4	10	2	2	3	22	.040
Brookline Hill		3	9	16	7	6	10	51	.092
Brookline Village		7	13	19	8	5	6	58	.104
Longwood		2	3	7	4	1	3	20	.036
Fenway		3	7	10	4	5	8	37	.066
Column total, C_j		44	108	183	89	69	63	$N=556$	1.00

$$\chi^2 = \sum \frac{(O_{ij} - E_{ij})^2}{E_{ij}} = 38.2 \quad \text{where } E_{ij} = \frac{R_i C_j}{N}$$

$$E(\chi^2) = \frac{(r-1)(c-1)N}{N-1} = \frac{(13-1)(6-1)556}{556-1} = 60.11$$

$$V(\chi^2) = \frac{2N}{N-3} (n_1 - u_1)(n_2 - u_2) + \frac{N^2}{N-1} u_1 u_2$$

$$\text{where } n_1 = \frac{(r-1)(N-r)}{N-1}, \quad n_2 = \frac{(c-1)(N-c)}{N-1}$$

$$u_1 = \frac{N \sum R_i^{-1} - r^2}{N-2}, \quad u_2 = \frac{N \sum C_j^{-1} - c^2}{N-2}$$

$$\text{Hence } V(\chi^2) = 152.0$$

Under the null hypothesis that is stated on page 6, the statistic,

$$Z = \frac{\chi^2 - E(\chi^2)}{\sqrt{V(\chi^2)}} = -1.78$$

is distributed as $N(0,1)$ and its value is compared to the 95th percentile (=1.96 or -1.96) of a standard normal distribution. Since -1.78 is greater than -1.96, the hypothesis is accepted.

APPENDIX 1B: AVERAGE LOADING PASSENGERS
(Inbound afternoon trips)

Hourly Period	12-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	R _i	p _i =R _i /N
<u>Stations</u>										
Riverside	5	6	4	10	12	6	6	5	54	.088
Woodland	4	7	5	9	3	5	8	2	43	.070
Waban	2	4	6	6	5	3	3	3	32	.052
Eliot	4	1	2	4	4	1	4	1	21	.034
Highland	7	3	5	5	9	6	4	3	42	.069
Newton Center	11	8	13	12	6	9	8	5	72	.118
Chestnut Hill	6	4	8	11	3	14	5	3	54	.088
Reservoir	4	5	11	5	9	4	1	2	41	.067
Beacon Field	1	3	6	2	2	2	1	1	18	.029
Brookline Hill	6	6	19	7	3	2	2	4	49	.080
Brookline Village	9	5	11	12	4	7	5	5	58	.095
Longwood	7	2	6	7	2	3	3	2	32	.052
Fenway	8	8	13	23	25	9	5	6	97	.158
Column Total C _j	74	62	109	113	87	71	55	42	613	1.000

As in Appendix 1A, the Z statistic is derived.

$$\chi^2 = 112.04$$

$$E(\chi^2) = 84.14$$

$$V(\chi^2) = 163.2$$

$$Z = \frac{\chi^2 - E(\chi^2)}{V(\chi^2)} = 2.18$$

The 95th percentile of a N(0,1) distribution is 1.96. Since Z is very close to 1.96, for all practical purposes, the null hypothesis is again accepted.

APPENDIX 1C: AVERAGE NUMBER OF UNLOADING PASSENGERS
(Outbound morning trips)

Hourly Period	6-7	7-8	8-9	9-10	10-11	11-12am.	Pow	$p_i = R_i / N$
							Total R_i	
<u>Stations</u>								
Fenway	13	11	17	9	7	10	67	.145
Longwood	3	2	9	6	4	3	27	.059
Brookline Village	2	3	18	8	6	6	43	.093
Brookline Hill	1	1	13	9	5	6	35	.076
Beacon Field	0	0	3	1	2	3	9	.019
Reservoir	2	2	3	5	2	4	18	.039
Chestnut Hill	1	5	15	7	4	6	38	.082
Newton Center	1	5	19	17	8	14	64	.139
Highland	4	5	7	5	4	6	31	.067
Eliot	1	3	2	2	2	2	12	.026
Waban	2	2	7	7	3	4	25	.054
Woodland	4	4	9	10	6	11	44	.095
Riverside	6	10	13	4	6	9	48	.104
Column Total C_j	40	53	135	90	59	84	N=461	1.000

$$\chi^2 = 56.12$$

$$E(\chi^2) = 60.13$$

$$V(\chi^2) = 115.8$$

$$\text{Hence, } z = \frac{\chi^2 - E(\chi^2)}{V(\chi^2)} = -.37, \text{ which is greater than } -1.96,$$

therefore, the null hypothesis is accepted.

APPENDIX 1D: AVERAGE NUMBER OF UNLOADING PASSENGERS
(Outbound afternoon trips)

Hourly Period:	12-1	1-2	2-3	3-4	4-5	5-6	6-7	7-8	Row Total R_i	$p_i = R_i / N$
<u>Stations</u>										
Fenway	11	9	8	10	11	10	5	8	72	.088
Longwood	2	7	1	8	4	3	5	3	33	.040
Brookline Village	7	9	6	12	13	15	11	8	81	.099
Brookline Hill	3	4	7	8	9	12	6	8	57	.069
Beacon Field	3	3	4	4	4	7	5	3	33	.040
Reservoir	6	9	8	8	12	9	6	4	62	.076
Chestnut Hill	7	6	8	8	13	7	6	5	60	.073
Newton Center	5	12	5	11	19	23	10	13	98	.119
Highland	15	5	5	9	15	12	8	9	78	.095
Eliot	3	4	3	6	8	7	2	6	39	.047
Waban	2	4	2	5	11	11	5	5	45	.055
Woodland	7	12	7	12	18	11	4	5	76	.093
Riverside	7	9	10	12	21	12	6	10	87	.106
Column Total, C_j	78	93	74	113	158	139	79	87	821	1.000

$$\chi^2 = 63.24$$

$$E(\chi^2) = 84.10$$

$$V(\chi^2) = 164.13$$

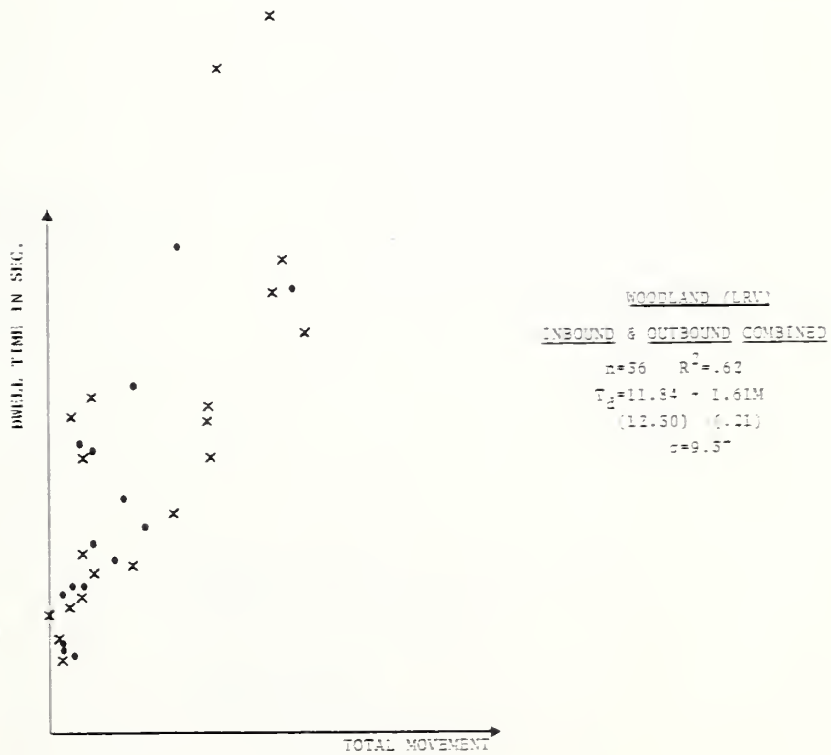
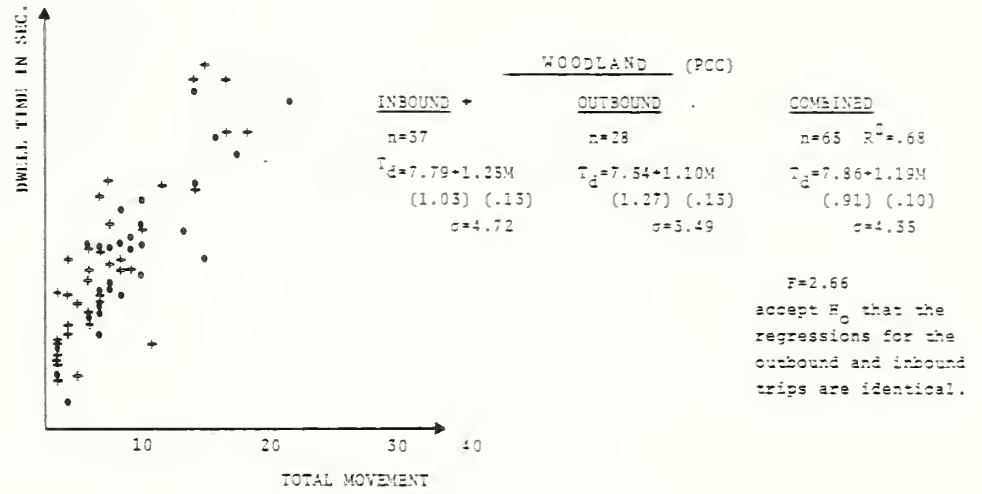
Hence,
$$Z = \frac{\chi^2 - E(\chi^2)}{\sqrt{V(\chi^2)}} = -1.63$$
 which is greater

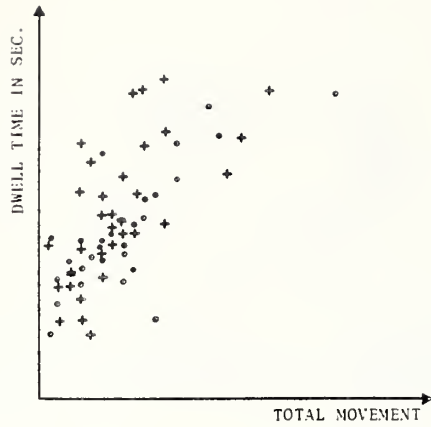
than -1.96, therefore, the null hypothesis is again accepted.

APPENDIX 1E: MARKET SHARES OF PASSENGER
ACTIVITIES FOR EACH STATION

	Unloading Passengers		Loading Passengers	
	<u>Inbound Morning Trips</u>	<u>Inbound Afternoon Trips</u>	<u>Outbound Morning Trips</u>	<u>Outbound Afternoon Trips</u>
Station i	P _i	P _i	P _i	P _i
Riverside	.00	.00	.00	.00
Woodland	.00	.00	.01	.00
Waban	.01	.01	.01	.00
Eliot	.00	.01	.01	.02
Highland	.01	.04	.05	.02
Newton Center	.05	.07	.06	.07
Chestnut Hill	.07	.04	.01	.07
Reservoir	.07	.18	.19	.09
Beacon Field	.01	.07	.05	.03
Brookline Hill	.08	.08	.08	.14
Brookline Village	.12	.25	.31	.14
Longwood	.43	.13	.10	.32
Fenway	.15	.12	.12	.10

APPENDIX 2: ANALYSIS OF DWELL TIME





HIGHLAND (PCC)

INBOUND +

n=55

$$T_d = 10.75 + .89M$$

(1.58) (.18)

$\sigma = 4.99$

OUTBOUND o

n=33

$$T_d = 9.05 + .83M$$

(1.19) (.13)

$\sigma = 3.90$

COMBINED

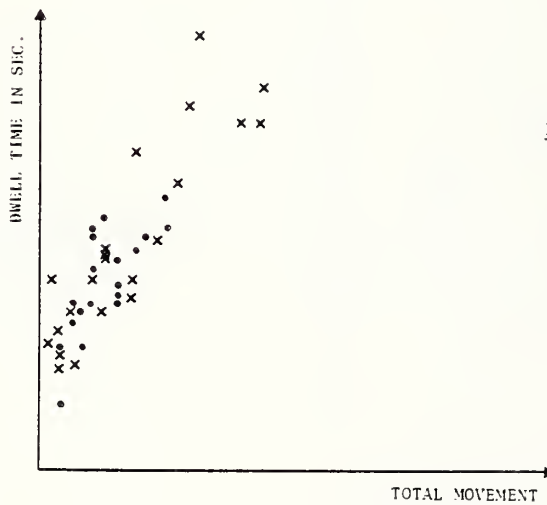
n=68 $R^2 = .48$

$$T_d = 9.99 + .85M$$

(1.0) (.11)

$\sigma = 4.56$

F=1.96
 accept H_0 that the
 the regressions for
 the outbound and inbound
 trips are identical.



HIGHLAND (LRV)

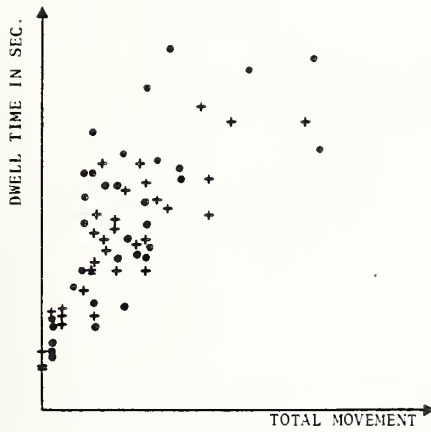
INBOUND AND OUTBOUND COMBINED

n=42

$$T_d = 10.17 + 1.31M$$

(1.06) (.12)

$\sigma = 3.97$



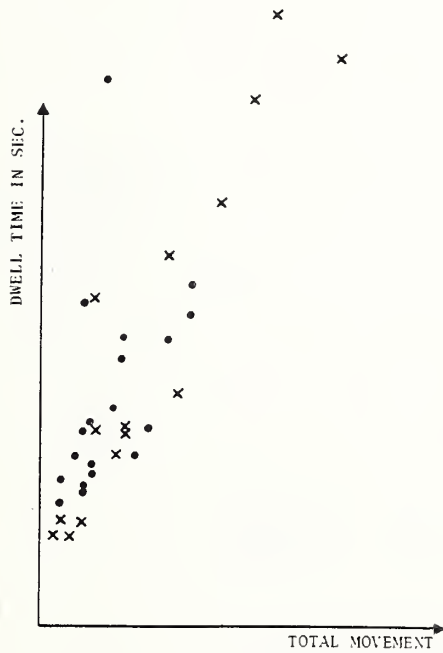
RESERVOIR (PCC)

INBOUND +
 n=34 $R^2=.70$
 $T_d=8.45+.96Mov$
 (1.04) (.11)
 $\sigma=3.65$

OUTBOUND •
 n=33 $R^2=.44$
 $T_d=11.28+.85Mov$
 (1.73) (.17)
 $\sigma=6.21$

COMBINED
 n=67 $R^2=.53$
 $T_d=9.84+.90Mov$
 (1.01) (.10)
 $\sigma=5.11$

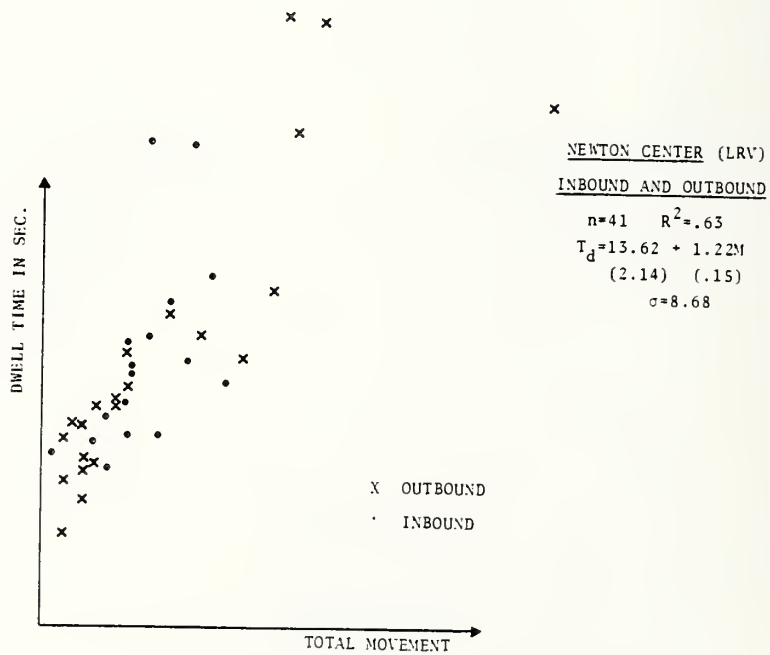
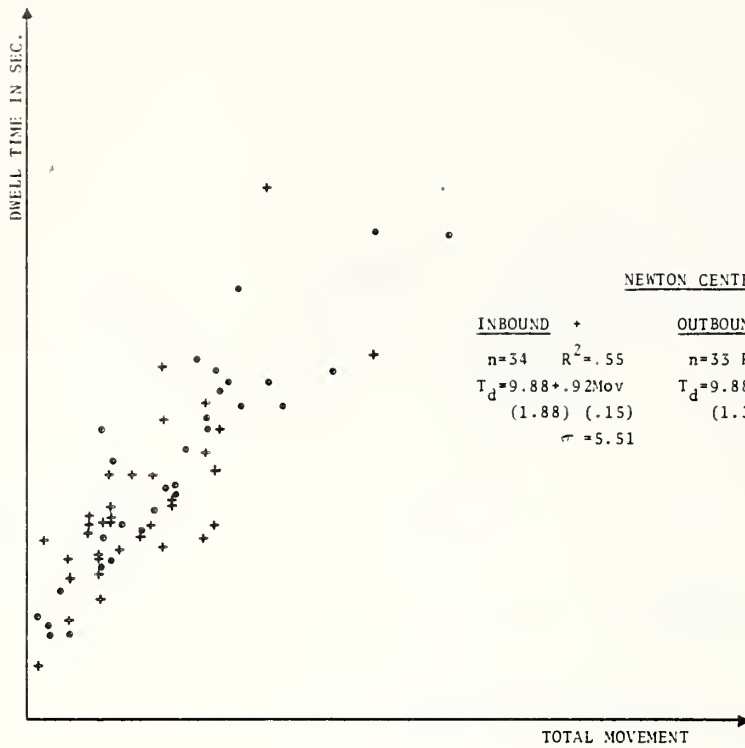
F=1.47
 accept H_0 that the regressions for the outbound and inbound trips are identical.

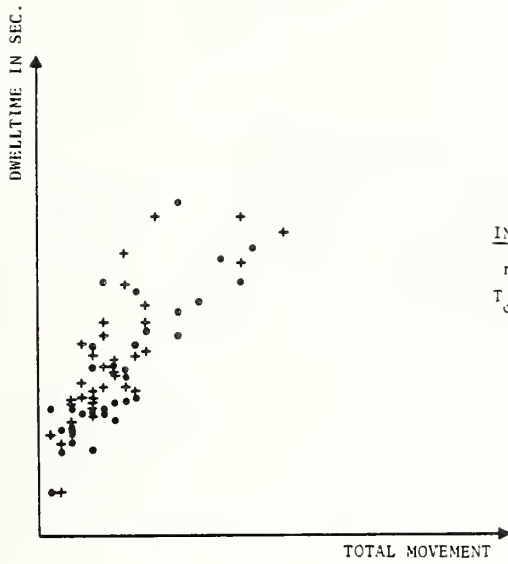


RESERVOIR (LRV)

INBOUND AND OUTBOUND COMBINED

n=34 $R^2=.80$
 $T_d=7.86 + 1.81M$
 (1.66) (.16)
 $\sigma=5.71$



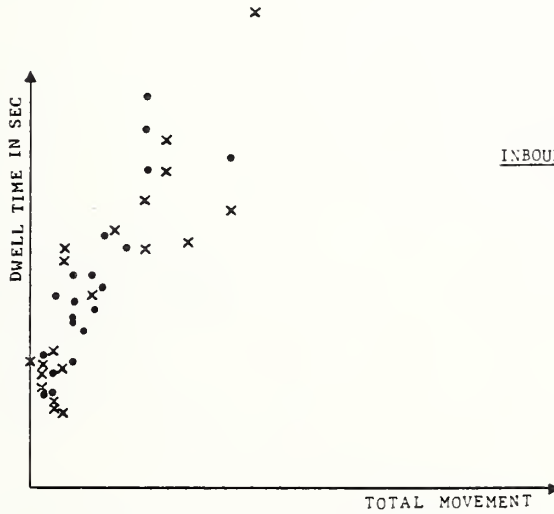


CHESTNUT HILL (PCC)

<u>INBOUND</u> +	<u>OUTBOUND</u> -	<u>COMBINED</u>			
n=34	R ² =.65	n=35	R ² =.69	n=69	R ² =.66
T _d =9.29+1.03M	T _d =7.61+1.05M	T _d =8.43+1.04M			
(1.16) (.13)	(1.10) (.12)	(.80) (.09)			
σ=3.74	σ=3.58	σ=3.69			

F=1.56

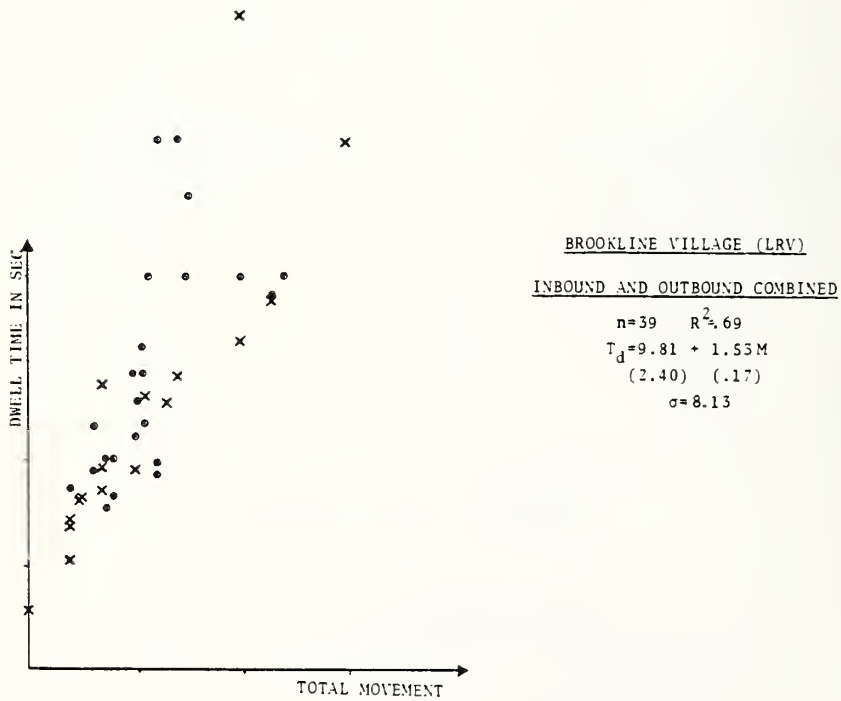
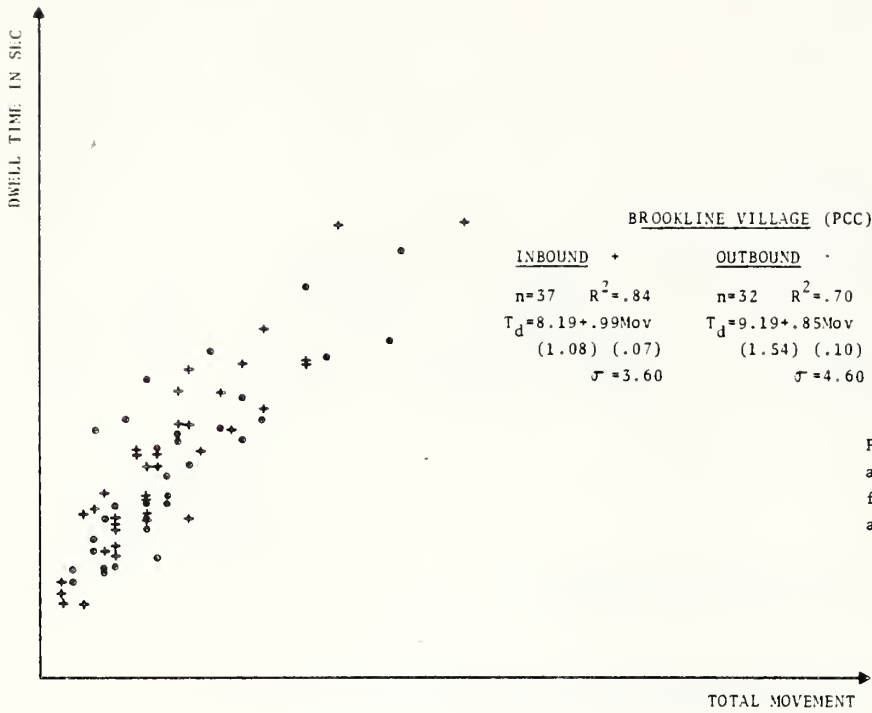
accept H₀ that the regressions for the outbound and inbound trips are identical.

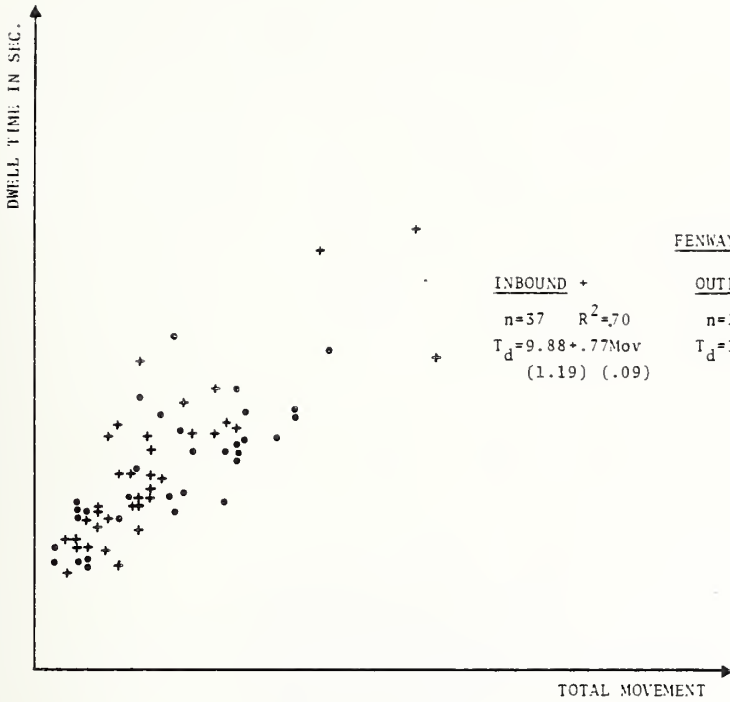


CHESTNUT HILL (LRV)

INBOUND AND OUTBOUND COMBINED

n=39	R ² =.74
T _d =10.1 + 1.38M	
(1.12)	(.13)
σ=4.48	





FENWAY (PCC)

INBOUND +

n=37 R²=.70
 $T_d = 9.88 + .77\text{Mov}$
 (1.19) (.09)

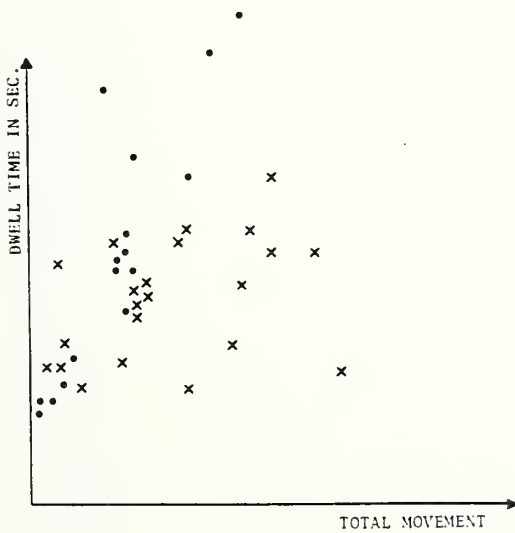
OUTBOUND o

n=37 R²=.56
 $T_d = 11.25 + .59\text{Mov}$
 (1.26) (.09)

COMBINED

n=74 R²=.63
 $T_d = 10.45 + .68\text{Mov}$
 (.87) (.06)

F=1.55
 accept H₀ that
 the regressions for
 outbound and inbound
 trips are identical.



FENWAY (LRV)

(INBOUND AND OUTBOUND COMBINED)

*The data vary too much for
 any estimation of linear
 relationship between the dwell
 time and the total movement.

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UMTA- 80-45
Kwok, Betty S

MBTA passenger
analyses, 19

Form DOT F 17
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