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DYNAMIC URBAN GROWTH MODELS

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16. Abstract

In this report the concept of "order by fluctuation," that has appeared recently in physico-chemical and biological systems, is applied to the description of urban growth. It is shown that fluctuations play a vital role in the evolutionary process of urban growth. The evolution of a complex system cannot be known simply by studying deterministic equations describing the system. It is necessary, in addition, to study the effects of fluctuations, or historical accident, which can drive the system to new modes of behavior. Taking account of both the deterministic elements of urban growth and the appearance of innovations at chance locations in an economic region, a transportation-sensitive dynamic model of the evolution of the organization of urban centers and the evolution of the spatial distribution of urban populations was developed.

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PREFACE

This is an Interim Report whose objective is to introduce a methodology capable of an integrated view of transportation planning, permitting the analysis of the impacts of changing transport on the socio-economic and demographic structure of the system.

This report introduces two preliminary models and simulation results on the process of urbanization on a national or regional scale and the evolutionary growth of a single urban area.

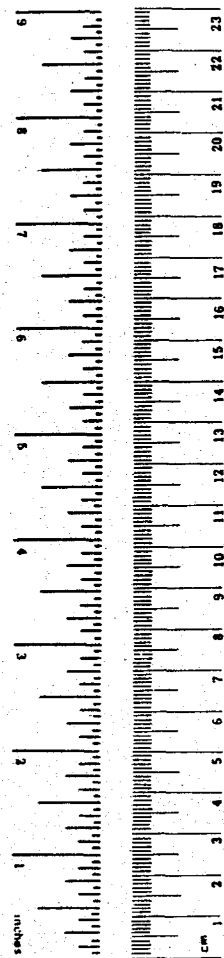
We note that this Interim Report has been published after the Final Report entitled "The Dynamics of Urban Evolution, Volumes I and II," Report Numbers DOT-TSC-RSPA-78-20,I,II, October 1978. The reason for this was to not delay publication of the Final Report. The present Interim Report contains research results not published in the Final Report (This is especially true of Sections 4 and 5.) and hence should be read as a companion piece to the Final Report.

The Technical Monitor of this research effort, D. Kahn, would like to acknowledge the help he received from L. Levine for his copy editing and production editing which has led to this final version of the Interim Report.

METRIC CONVERSION FACTORS

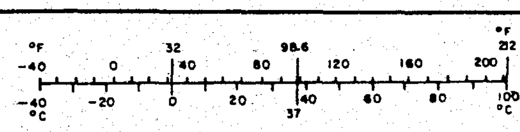
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	AREA			
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
	MASS (weight)			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
	VOLUME			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
	TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
	AREA			
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
	MASS (weight)			
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
	VOLUME			
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
	TEMPERATURE (exact)			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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1. INTRODUCTION

In this Interim Report we derive a mathematical model of the process of urbanization based on the "autocatalytic" effects of population and economic activity. The guiding principle behind such a model is the expression of the effects of an economic law relating the spatial distribution of supply, demand, and transportation, allowing for the fact that the actual location of a particular economic function can be the result of random, unpredictable events. Thus, we look at the interplay of a deterministic economic law with a stochastic implantation of economic functions, and we study particularly the resulting spatial structures of population density.

In the first part of this report we will describe the model and give the results of numerous computer simulations concerning the number, size and territory appertaining to the urban centers of a region. From there, we can study the effects of:

1) an inhomogeneous initial population distribution,
2) improvements in transportation,
3) changes of scale of the units of produciton.

We outline the manner in which the model can be generalized to describe real situations where industrial location of basic industries is subject to the inhomogeneous distribution of raw materials.

In this first part, however, we do not distinguish between place of residence and place of work, and therefore describe the process of urbanization on a regional or national scale. In the second section of this report we turn to the evolution within an urban center of such a region. We allow for the spatial relationships existing between places of residence and places of work and develop a model which describes the evolution of the urban population distribution resulting from these various processes. In the preliminary simulations that have been performed, we find most encouraging results which seem to be in agreement with experiment. The future development of such a model seems to offer the possibility of studying the effects of modifications in the urban transport system, of employment capacities and character, and of the evolution of segregation (racial, class, religious, etc.....) within the city.

2. THE EVOLUTION OF THE URBAN HIERARCHY

The principle underlying our dynamic model of urban growth is that the population which can be supported at a given locality depends upon the number of economic functions existing at that point, but the possibility and survival of economic functions there, in turn, depend upon the population existing at and around that point.

Let us first write down the basic terms that will be considered in our model and then analyze, term by term, the exact form required. The equation governing population growth is given by:

$$\frac{dx_i}{dt} = kx_i(P + \sum_k R^K S_i^{\,K} - X_i) - dX_i \quad ,$$

where P is the basic, self-sufficient carrying capacity, and RS_i^k the "extra" carrying capacity at the point i due to the presence of a unit of production of economic function k. The equation governing the development of this carrying capacity S_i^k at the point i can be written briefly as:

$$\frac{dS_i^K}{dt} = \alpha_i S_i^K \quad \left[(\text{Demand})_i^K - (\text{competition}) - \gamma S_i^k \right] .$$

Let us discuss each term in detail.

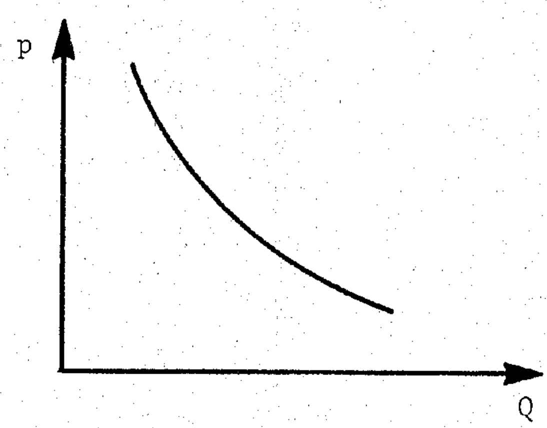
The term α_i

This is a parameter which characterizes the "dynamism" of the entrepreneurs at point i. It will depend on several factors such as the size of interest rates, the availability of capital, and the "psychology" of the locality. An interesting feature, which will be brought out in later studies, is that we may imagine that for extractive industries, or for the work associated with a port, this "dynamism" is a function of position. That is to say α_i^K, for these functions, is only nonzero where the mineral resources exist, or in the case of a port on the coast or a river, where there exists the possibility of setting up such a function. This will allow us, later, to study the effects of such location factors on these basic industries. The manufacturing, service, and retail sectors can therefore be treated within the spatial framework given by the richness of the soil of a region, the existence of mineral deposits, or the possibility of establishing a port.

The Demand

This demand for the function K situated at point i must be divided into two parts. The first part is the demand for private consumption by individuals; the second is the demand on one function made by another economic function.

The private demand will be based, we suppose, on the well-established experimental relation existing between the quantity demanded, Q and the price, P at which a good or service is offered.



We will suppose some law for the quantity of K demanded per individual:

$$Q = \frac{\varepsilon^K}{(P^K)^e}$$

and hence, allowing for the cost of transportation between the point of "production" i and the point of consumption j, we have:

$$\text{Private Demand} = \sum_j \frac{\chi_j \; \varepsilon^K}{\left[P_i^K + \phi^K(r_j - r_i)\right]^e},$$

where ϕ will depend on the efficiency of the means of distribution and on the importance for the function K of the distribution costs in the total price.

Up to this point we have considered that the region of study is homogeneous and that the costs of production of K at any point i, p_i^K, are uniform. However, an interesting new dimension is introduced into our model if we allow for costs of transportation inputs. This, in manufacturing industries for example, is a major factor influencing price. Thus, we may further analyze our spatial interactions by adding:

$$\text{Private Demand} = \sum_j \frac{\chi_j \varepsilon^K}{\left[P_i^K(0) + \sum_I \phi^I(\zeta_I - \zeta_i) + \phi^K(r_j - r_i)\right]^e},$$

where I is the sum over the costs of transporting the various inputs from their origin ζ_I to ζ_i.

This brings us naturally to the second demand factor, that existing between economic functions. This we may write as:

$$\text{Industrial Demand:} \quad \sum_{K'} \sum_j \omega^{KK'} \; \chi_j^{K'} \quad .$$

Thus each worker $\chi_j^{k'}$ at point j in function K' demands a quantity $\omega^{KK'}$ of the good or service K situated at i. We see from this that the total flow of goods, from the extractive industry through manufacturing to the private consumer, will be all the greater if the distances involved in input transportation and distribution are small. The service and retailing occupations are, of course, less dependent on "input costs" than on distribution and will therefore follow more closely the population distribution.

As we shall see, when distribution costs are a main factor, the model shows us (Figure 3-13) that any initial differences in population densities tend to be amplified by the urbanization process. Thus, it could well be that the locational factors that influenced the initial growth of manufacturing industries, where "input" and "output" transport costs played a role, may be the predominant factors in the location of urban centers. The relative sizes and number of urban centers will then depend on the autocatalytic/competitive processes described by the model. The locational factors for an industry with several input materials and a costly distribution will not lead to a single, easily calculable point of optimum location. What will occur is that entrepreneurs at different points in the system will attempt to launch an industry. Their success will depend on the price of the product, in turn a reflection of the "input" and "output" transport costs. The population distribution, reacting to these industries and others, will undergo a differentiated nonlinear growth, and only certain enterprises will find sufficient demand to survive. While certain areas near to important "input factor" locations may be favored, the final result will certainly bear witness to the historical order in which enterprises appeared. As we shall see, competition is never perfect because of the effect of space. Thus, a small enterprise may well survive in a relatively isolated region although it may be far from an "optimum" location for the function.

Summarizing the "Demand," we find that the two parts, "Private" and "Industrial," are characteristic of different types of occupation. We have:

a) extractive industries, where industrial demand is the most important;
b) manufacturing industries and ports, where both private and industrial demand may be equally important;
c) service industries, where private demand is much more important.

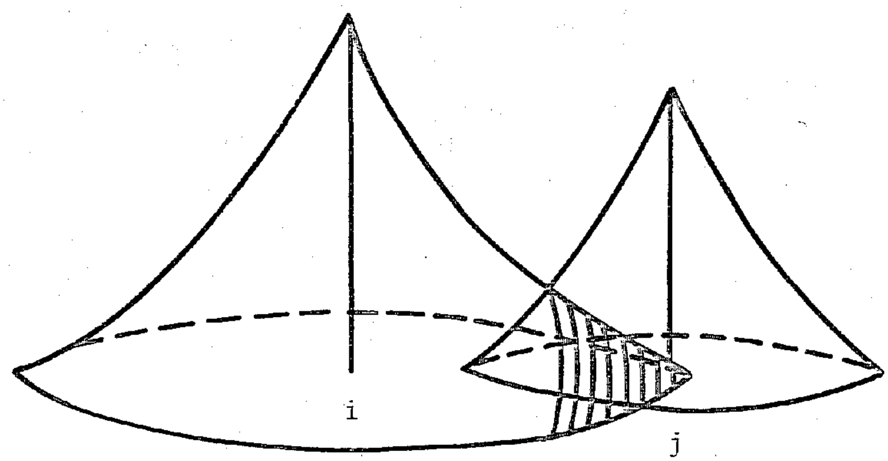
The "Competition" Term

With this term we attempt to put into the equation the demand for the function K at i which will be diverted to rival units of production. There may be either identical scale units of K situated at different points j or different units of K situated either at the same point i or another point j. Thus we write:

$$- \beta^{KK'} \sum_{k'} \sum_{j} \psi_{ij} \gamma^{K'} S_j^{K'} \quad ,$$

where β=0 if there is no competition between K and K' and K ≠ K', and β=1 if K = K' or if the two functions are substitutional.

ψ_{ij} is a numerical factor related to the amount which the two cones of demand of K and I, and K' and j overlap.



Thus, we calculate the volume in common divided by the total volume of the demand cone. Thus, ψ_{ij} varies between zero and unity.

The Market Threshold $\gamma^K S^K$

This term expresses the fact that for one unit of production of $S_i^K = 1$ to exist there is a certain minimum demand requirement which must be met if the function S_i^K is not to disappear. We suppose then that γ^K is a measure of the scale of production of the unit, and that it has two components - labor costs and the capital costs of maintaining the plant. Thus, we pose $\gamma^K = R^K + C^K$, and R^K appears as the increase in population carrying capacity associated with each unit of S^K.

We can allow for economics of scale in our problem by supposing that the "price" appearing in the demand term is in fact a function of γ^K. Thus:

$$P_i^K (o) = \sigma^K + \frac{\Delta^K}{\gamma^K} .$$

Thus, we may consider the effects of competition between units of production of different scales by supposing that:

$$\beta^{KK'} = 1, \text{ that } \gamma^{K'} > \gamma^K, \text{ for example,}$$

but that $\sigma^K = \sigma^{K'}$; $\Delta^K = \Delta^{K'}$. By choosing different values of σ and Δ we may look at cases where there exist either large or small economies of scale, and by launching these competitive functions in our system we may test under which conditions of population density, transportation characteristics, and economic development different scales of production can impose themselves.

By launching successively a series of economic functions, the manner in which the economic relations and exchanges structure the population distribution can be studied, and we can discover which features of the resulting urban hierarchy are fortuitous and which are "imposed" by economic law. We can also test the veracity of certain economic beliefs, similar to ideas shared by certain Darwinsts in biology: that only "optimum structures" will subsist after a long evolution, that the system can only evolve toward greater "efficiency" if left to itself, and that human intervention will always prove "wasteful" in the long run.

3. SIMULATION RESULTS

Let us now turn to the description of the computer simulations which have been performed so far. In these we have used a slightly simplified version of our general equations in that interindustrial demand is not considered. Thus, the equation actually simulated in the results which follow is:

$$\frac{dS_i^K}{dt} = \alpha\, S_i^K \left[\sum_j \frac{X_j \varepsilon^K}{(\sigma^K + \frac{\Delta^K}{\gamma^K} + \phi^K(r_j - r_i)} e - \beta^{KK'} \sum_{K'} \psi_{ij} \gamma^{K'} S^{K'} - \gamma^K S_i^K \right],$$

where we suppose that

$$X_j = P + \sum_K R^K S_i{}^K$$

and that $$R^K = \frac{\gamma}{2}\ .$$

Using this equation, we find a time evolution such as the one shown in Figure 3-1 for the values of parameters:

P=60

$\gamma_1 = 10.$	$\gamma_2 = 10.$
$\varepsilon_1 = .7$	$\varepsilon_2 = .8$
$\phi_1 = 10.$	$\phi_2 = 2.$
$\sigma_1 = 1$	$\sigma_2 = .1$
$\Delta_1 = 9.$	$\Delta_2 = 9.$

Thus, we are looking at the spatial distribution of the population as two economic functions appear in the system. First, at T=0, we have at each point the first function, which has for each unit an increased carrying capacity of $\gamma/2 = 5$ and is characterized by very high transport costs $\phi_1 = 10$, implying that the frequency with

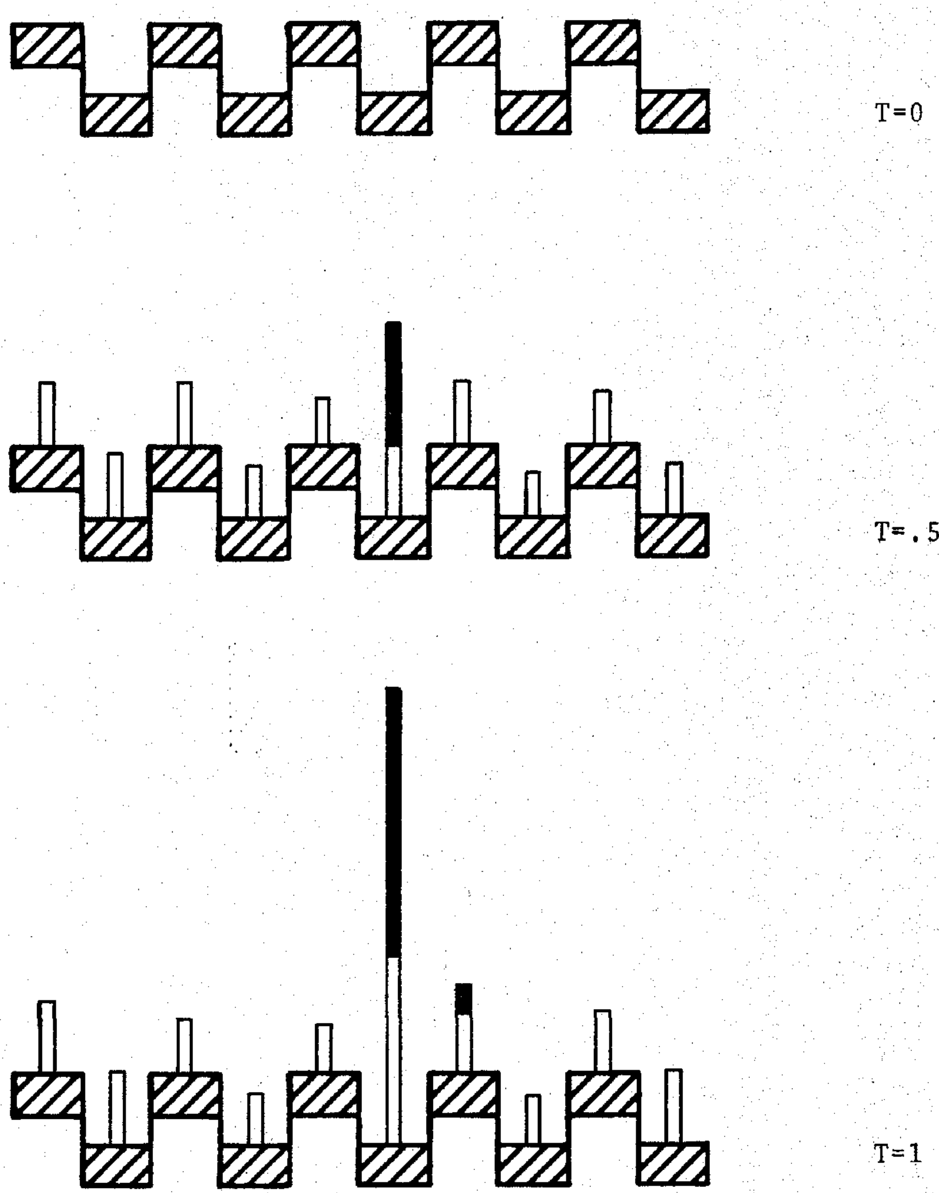


FIGURE 3-1. TIME EVOLUTION OF SPATIAL DISTRIBUTION OF POPULATION AS NEW ECONOMIC FUNTIONS ARE INTRODUCED

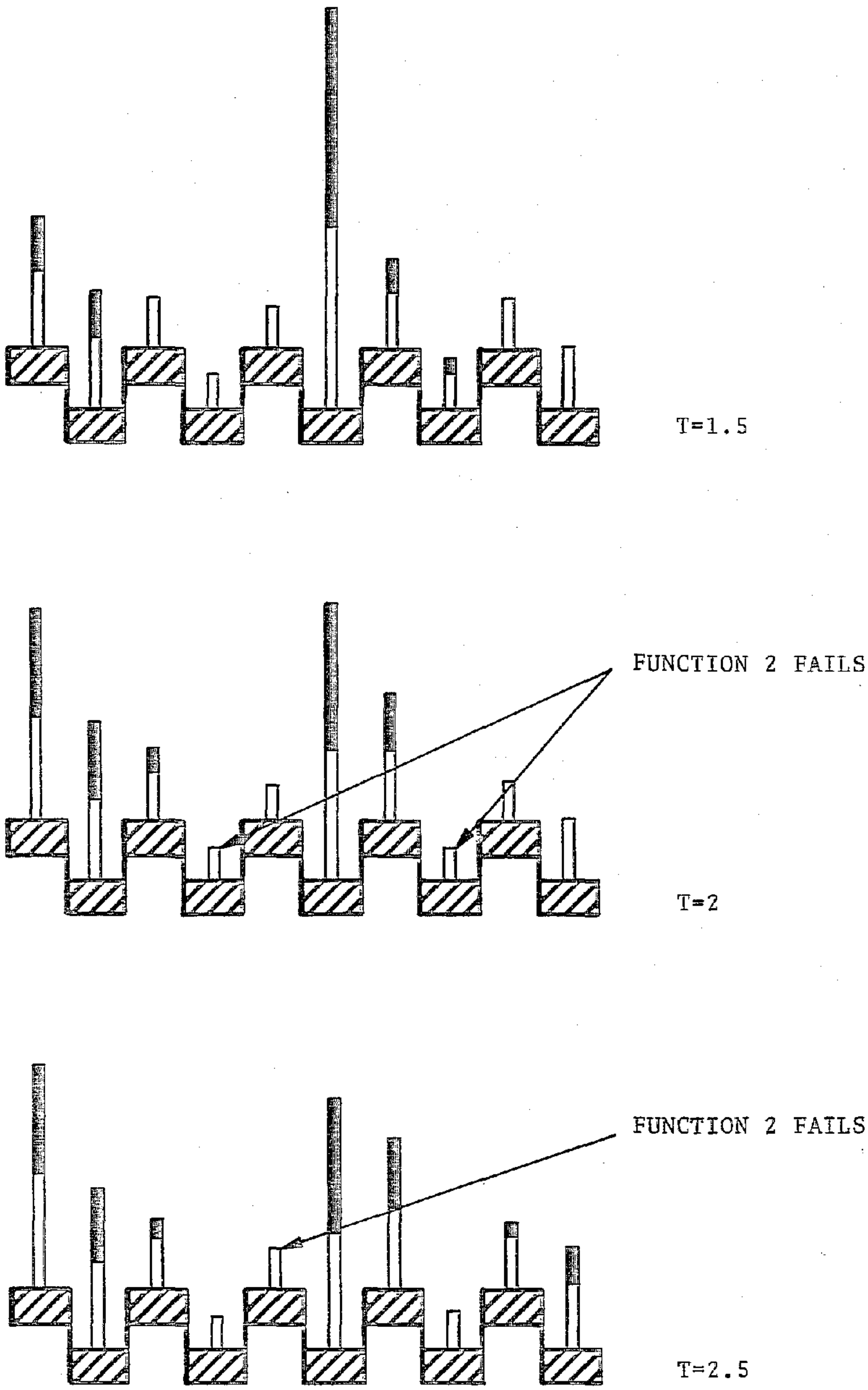


FIGURE 3-1. (CONTINUED)

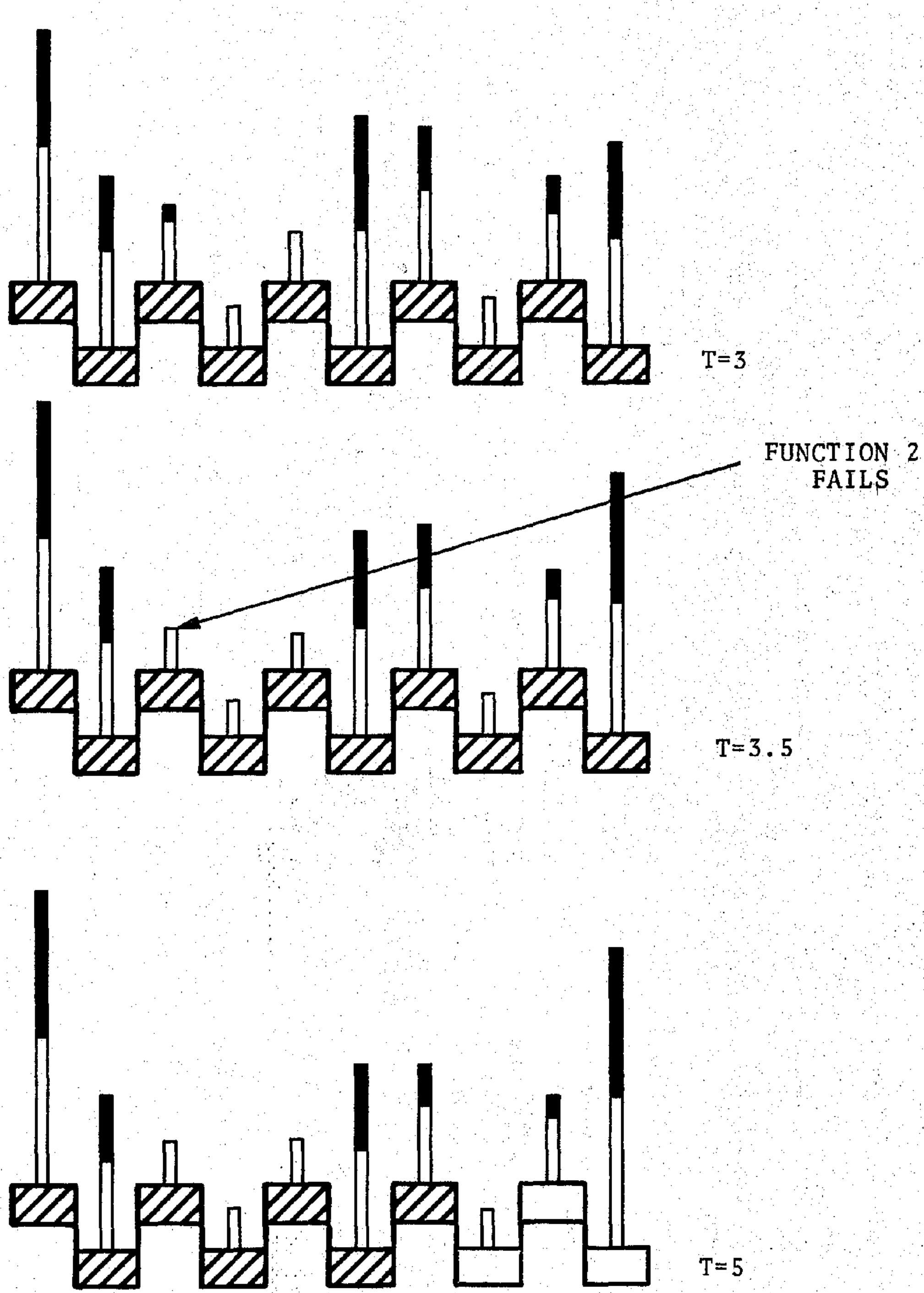


FIGURE 3-1. (CONTINUED)

which this service or good must be obtained is very high. We may think of a general store, butchers, bakers, or a school for example. We see that several units of this type of function can be supported by each point for P = 60 and $\varepsilon_1 = .7$.

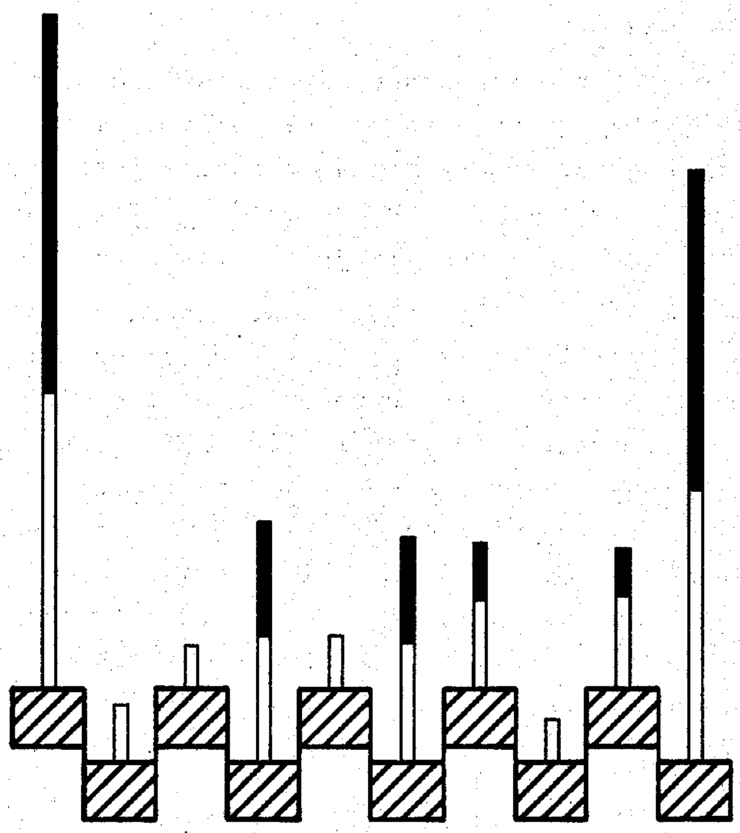
After this, we look at the appearance in the system of the second function, which, having appeared at one point at a random moment, is imitated throughout the system at random points and moments by other entrepreneurs trying the same idea. This function we have characterized by a lower transport cost $\phi_2 = 2$.

We see that the presence of the second function at a point automatically increases employment in the local functions (Figure 3-1). As the simulation proceeds, the enterprise of type 2 is imitated on more and more points, failing to find sufficient demand on certain locations to sustain one unit of production. As the simulation proceeds, the population distribution structures such that details depend on the exact historical sequence of events. Certain broad features, however, are predictable and depend on the parameters characterizing the functions, the initial population distribution, and the efficiency of the transportation system.

In the following series of figures we can see the urbanization resulting from the impact of 2, 3, 4 or 5 economic functions launched successively. Figures 3-2, 3, and 4 show the effect of increasingly effective transportation within the system. When $\phi_2 = 3$, we have 6 centers with functions 1 and 2, while with $\phi_2 = 2$ and $\phi_2 = 1$, we have 4 and 2 respectively. The results, which are typical, tell us that the effect of improving transportation decreases the number of centers having the function 2 but increases the importance of those that do.

Figures 3-5a and 5b show the time evolution of our system for 3 functions where 2 and 3 are equally demanded ($\varepsilon_2 = \varepsilon_3$) but have transport costs of 3 and 1 respectively. It seems here that function 3 imposes its pattern of two centers on the total final structure.

Figures 3-6 to 3-12 show us the structure occurring when five functions are launched successively. In these we have supposed



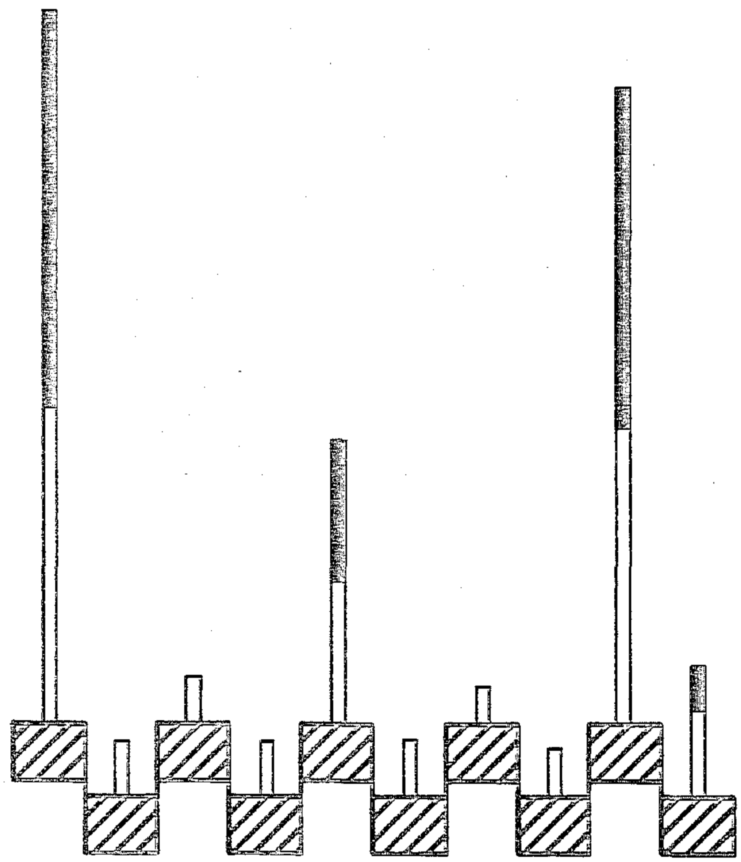
P=80

$\gamma_1 = 10$ $\gamma_2 = 20$

$\varepsilon_1 = .7$ $\varepsilon_2 = .9$

$\phi_1 = 10$ $\phi_2 = 3$

FIGURE 3-2. URBAN STRUCTURE AND TRANSPORT IMPROVEMENT: POOR TRANSPORT



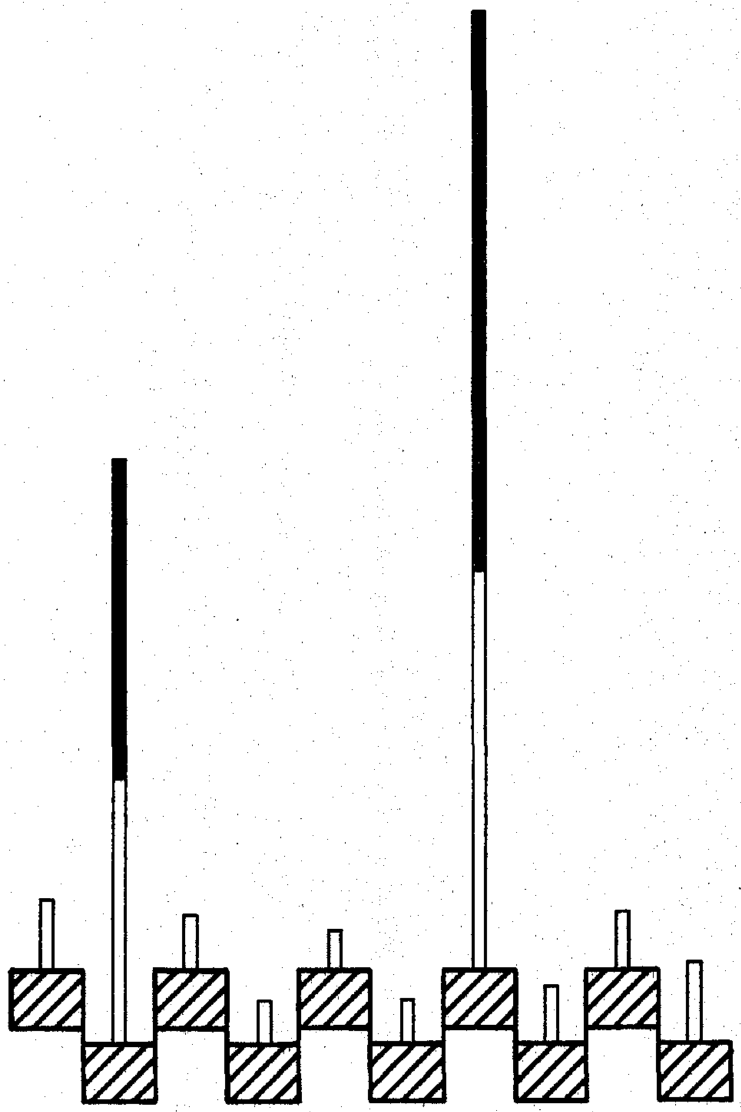
P=80

$\gamma_1 = 10$ $\gamma_2 = 20$

$\varepsilon_1 = .7$ $\varepsilon_2 = .9$

$\phi_1 = 10$ $\phi_2 = 2$

FIGURE 3-3. URBAN STRUCTURE AND TRANSPORT IMPROVEMENT: BETTER TRANSPORT



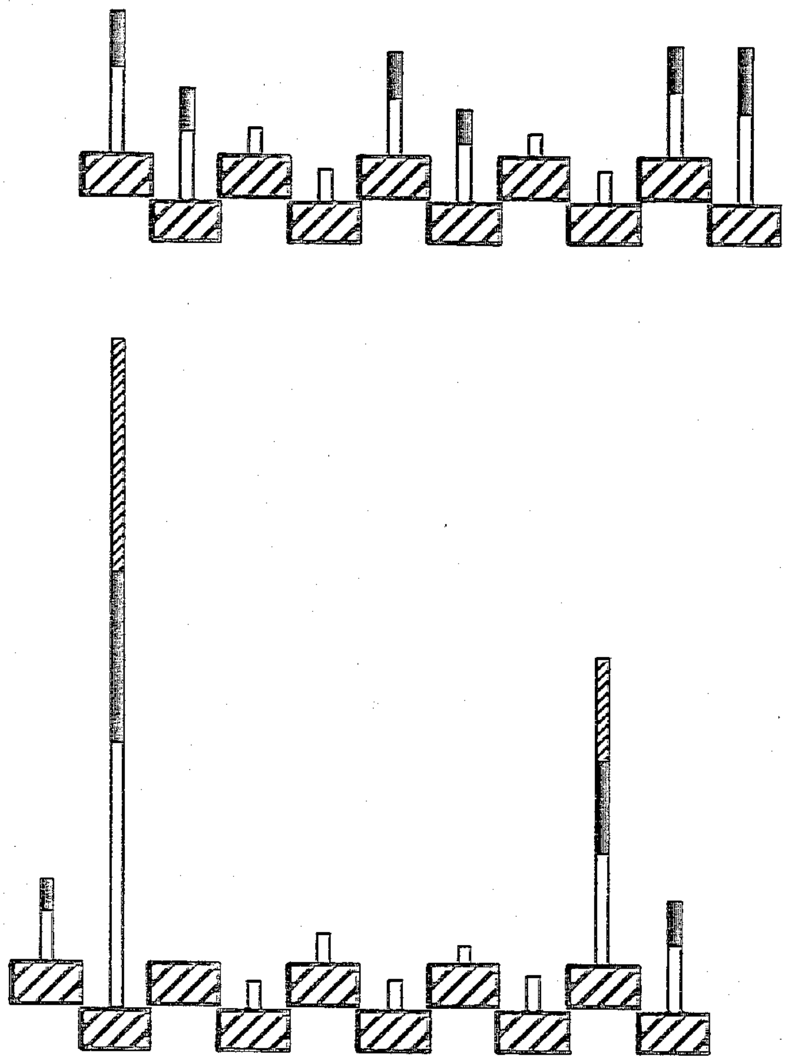
P=80

$\gamma_1 = 10$ $\gamma_2 = 20$

$\varepsilon_1 = .7$ $\varepsilon_2 = .9$

$\phi_1 = 10$ $\phi_2 = 1$

FIGURE 3-4. URBAN STRUCTURE AND TRANSPORT IMPROVEMENT: VERY GOOD TRANSPORT



T=2.5

T=3.5

β(2,3)=0

$\gamma_1 = 10$ $\gamma_2 = 20$ $\gamma_3 = 30$

$\delta_1 = 9$ $\delta_2 = 18$ $\delta_3 = 27$

$\phi_1 = 5$ $\phi_2 = 3$ $\phi_3 = 1$

$\varepsilon_1 = .7$ $\varepsilon_2 = .5$ $\varepsilon_3 = .5$

FIGURE 3-5a. TIME EVOLUTION OF URBANIZATION: TRANSPORT COSTS FOR THIRD FUNCTION THREE TIMES GREATER THAN FOR SECOND

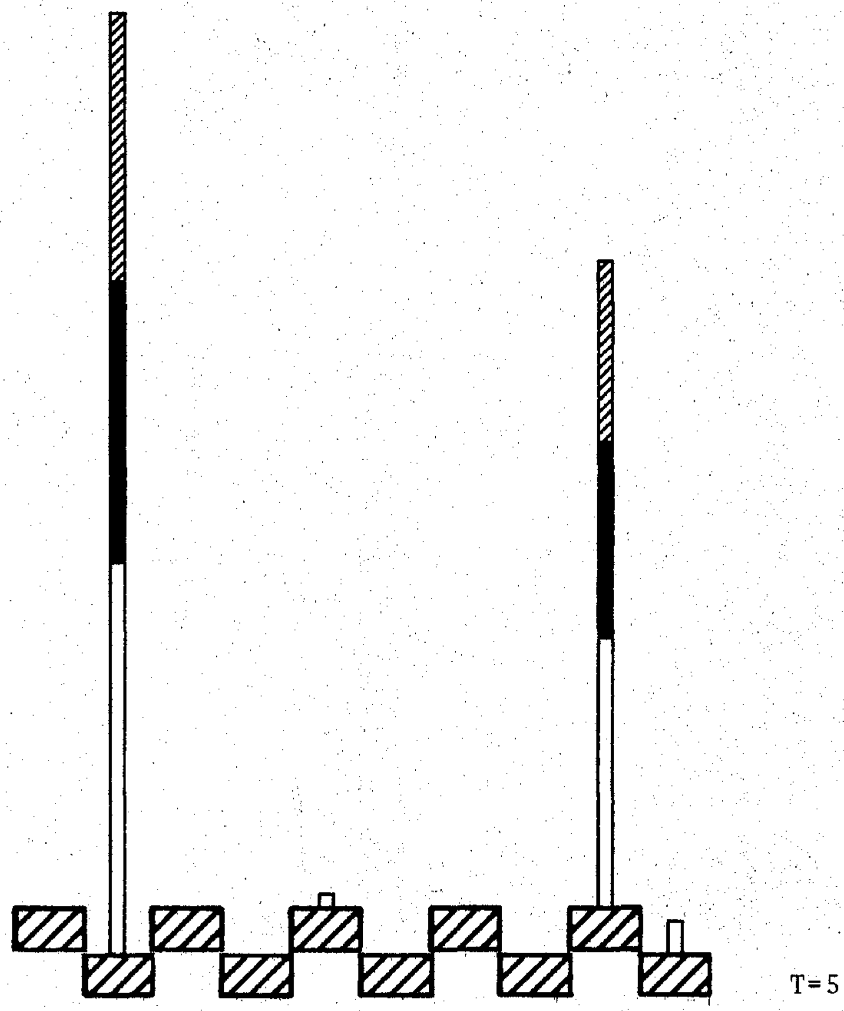
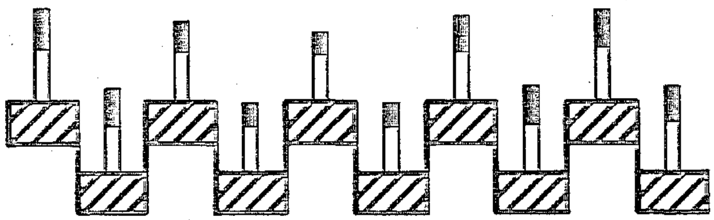


FIGURE 3-5b. TIME EVOLUTION OF URBANIZATION: TRANSPORT COSTS FOR THIRD FUNCTION THREE TIMES GREATER THAN FOR SECOND

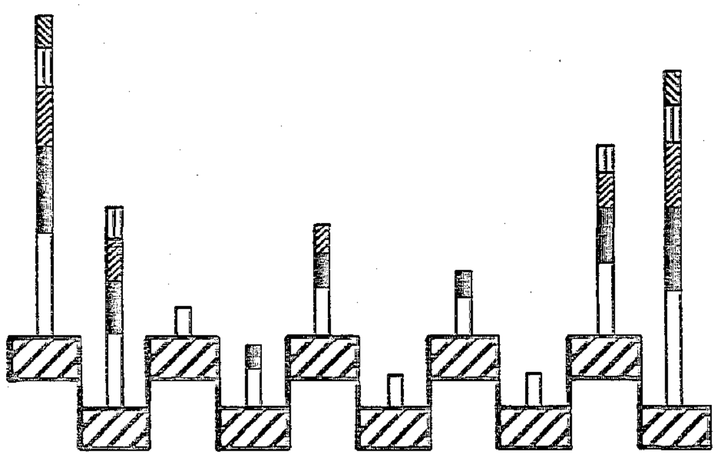
5 FUNCTIONS



P=60

After the launching of functions 1 and 2

$\gamma_1 = 10$	$\gamma_2 = 10$	$\gamma_3 = 10$	$\gamma_4 = 10$	$\gamma_5 = 10$
$\varepsilon_1 = .5$	$\varepsilon_2 = .4$	$\varepsilon_3 = .3$	$\varepsilon_4 = .2$	$\varepsilon_5 = .1$
$\phi_1 = 10$	$\phi_2 = 4$	$\phi_3 = 3$	$\phi_4 = 2$	$\phi_5 = 1$



P=60

After the launching of all five functions

FIGURE 3-6. URBAN STRUCTURE FOR GIVEN VALUES OF PARAMETERS

$\gamma_1 = 10$	$\gamma_2 = 10$	$\gamma_3 = 10$	$\gamma_4 = 10$	$\gamma_5 = 10$
$\epsilon_1 = .5$	$\epsilon_2 = .3$	$\epsilon_3 = .3$	$\epsilon_4 = .2$	$\epsilon_5 = .1$
$\phi_1 = 10$	$\phi_2 = 4$	$\phi_3 = 3$	$\phi_4 = 2$	$\phi_5 = 1$

$P = 60$

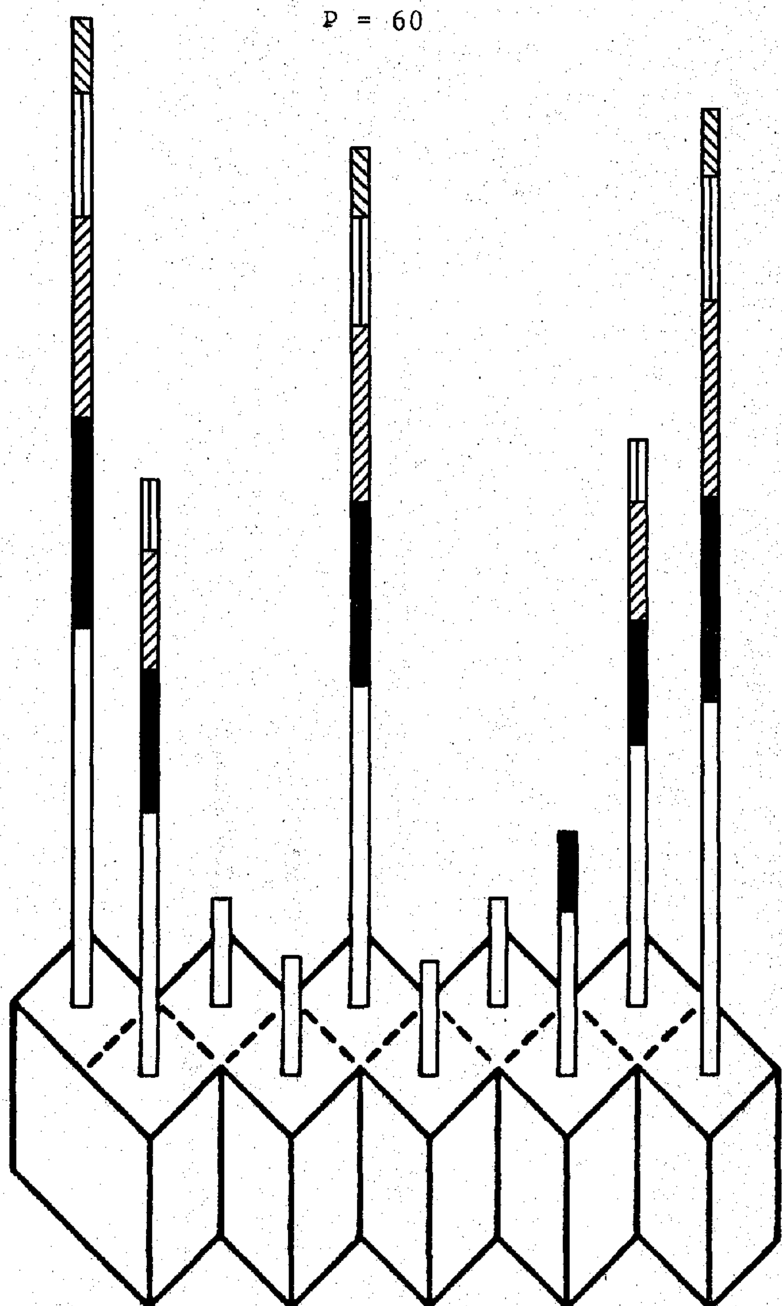


FIGURE 3-7. URBAN STRUCTURE FOR GIVEN VALUES OF PARAMETERS

$\gamma_1 = 10$ $\gamma_2 = 10$ $\gamma_3 = 10$ $\gamma_4 = 10$ $\gamma_5 = 10$

$\varepsilon_1 = .5$ $\varepsilon_2 = .3$ $\varepsilon_3 = .3$ $\varepsilon_4 = .2$ $\varepsilon_5 = .1$

$\phi_1 = 10$ $\phi_2 = 4$ $\phi_3 = 3$ $\phi_4 = 2$ $\phi_5 = 1$

$P = 60$

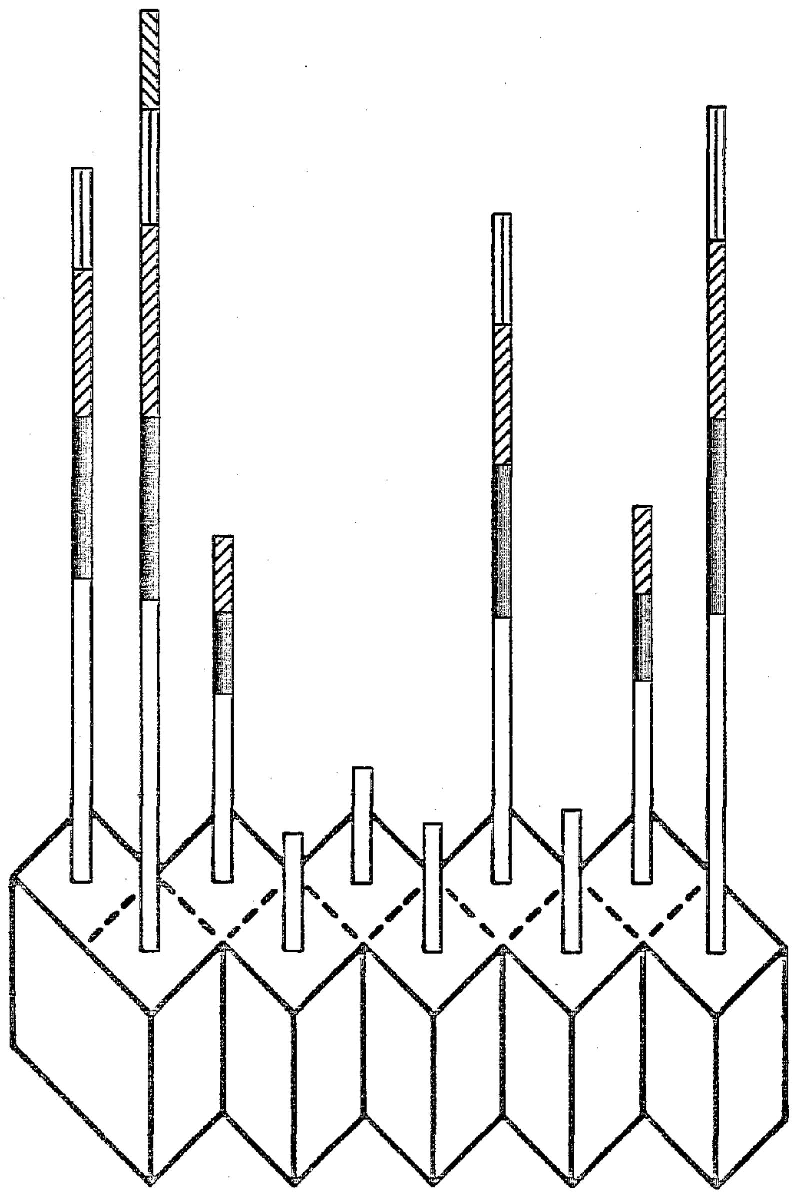


FIGURE 3-8. URBAN STRUCTURE FOR GIVEN VALUES OF PARAMETERS

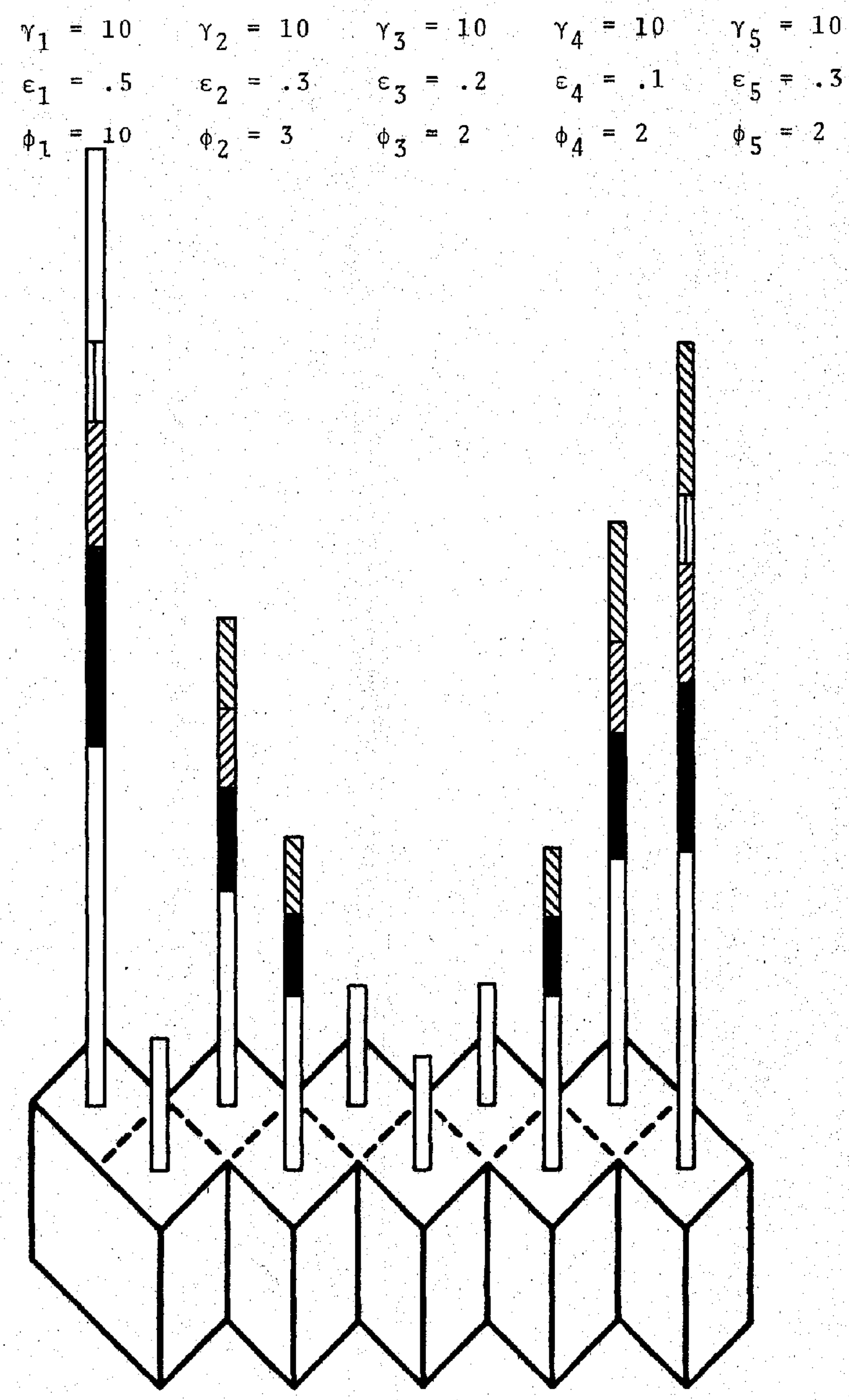


FIGURE 3-9. URBAN STRUCTURE FOR GIVEN VALUES OF PARAMETERS

$\gamma_1 = 10$	$\gamma_2 = 10$	$\gamma_3 = 10$	$\gamma_4 = 10$	$\gamma_5 = 10$
$\varepsilon_1 = .5$	$\varepsilon_2 = .4$	$\varepsilon_3 = .3$	$\varepsilon_4 = .2$	$\varepsilon_5 = .1$
$\phi_1 = 10$	$\phi_2 = 2$	$\phi_3 = 2$	$\phi_4 = 2$	$\phi_5 = 2$

$\varepsilon_{tot}=1.5$

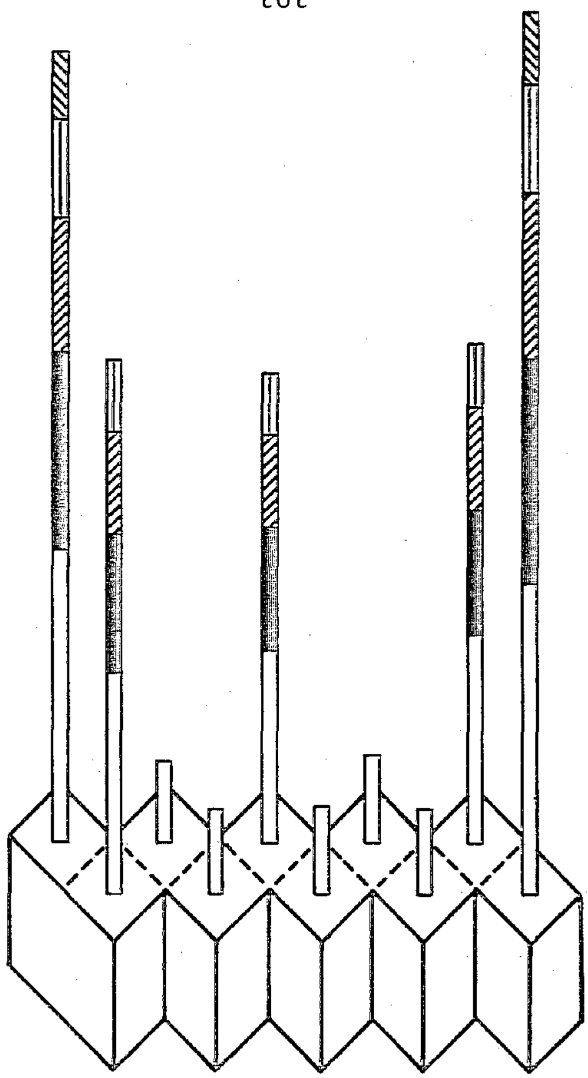


FIGURE 3-10. URBAN STRUCTURE FOR GIVEN VALUES OF PARAMETERS

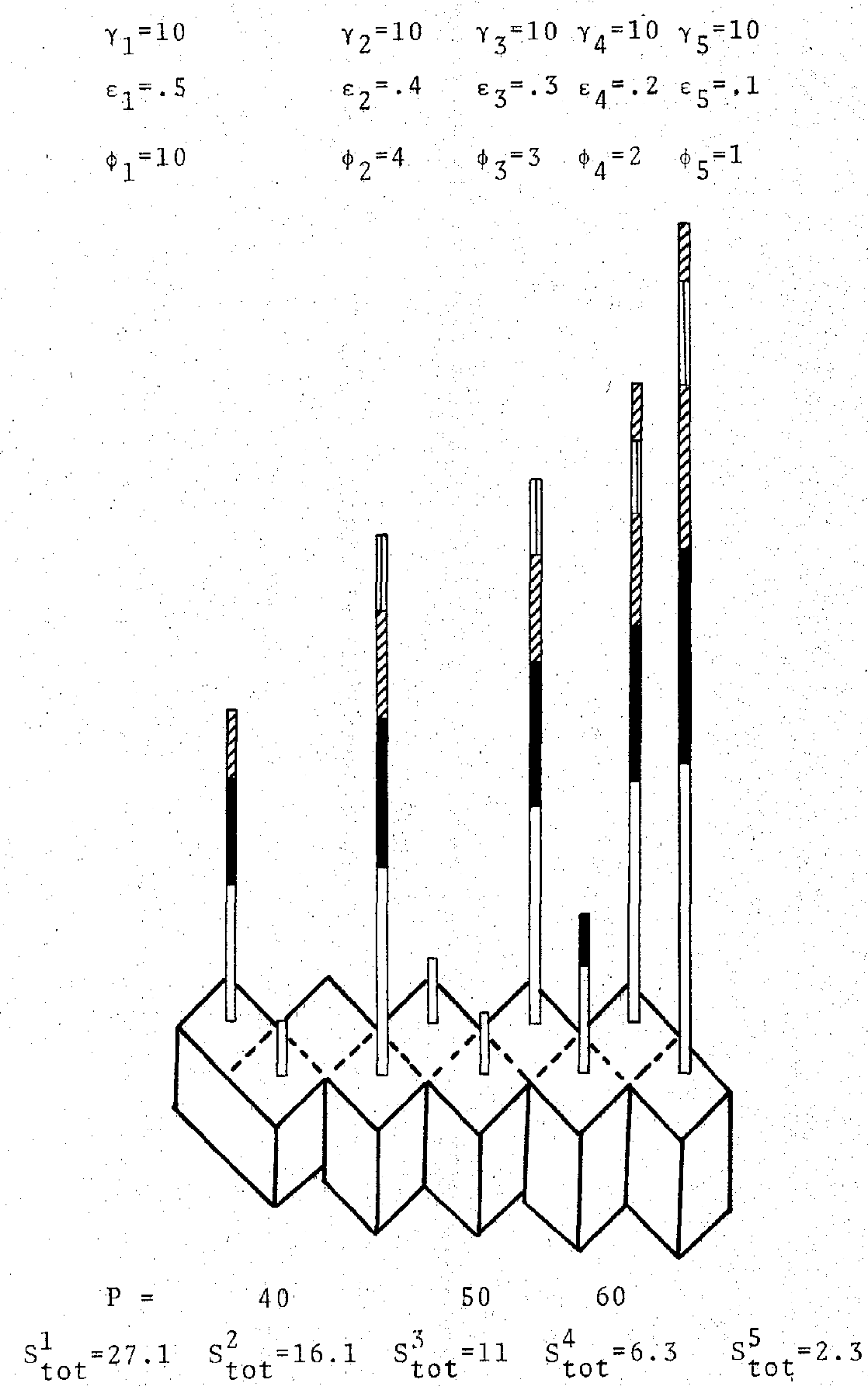


FIGURE 3-11. URBAN STRUCTURE FOR GIVEN VALUES OF PARAMETERS

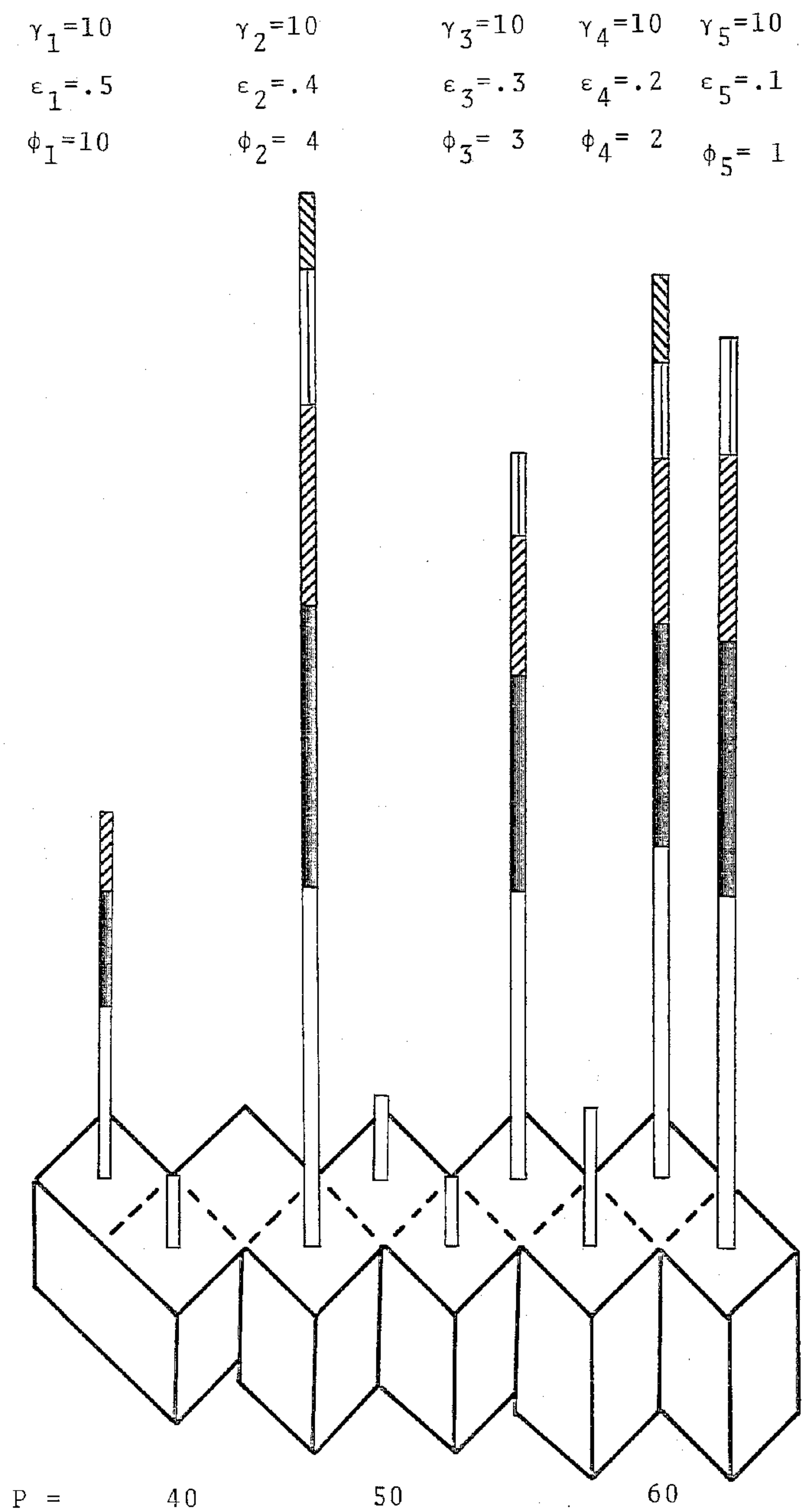


FIGURE 3-12. URBAN STRUCTURE FOR GIVEN VALUES OF PARAMETERS

that the functions 2, 3, 4, and 5 are increasingly specialized: that is, the individual demand per unit time for function 5 is less than function 4, and for 4 is less than 3 $\varepsilon_2 > \varepsilon_3 > \varepsilon_4 > \varepsilon_5$. We see that in fact functions 4 and 5 can only appear after the establishment of 2 and 3 because otherwise the necessary concentration of population has not occurred. We find a considerable stochastic variation in the final results, reflecting the importance of the historical sequence that has occurred, but giving rise nevertheless to a spatially organized hierarchy. The variability of the patterns would seem to reflect the small values involved in the higher order functions, which cause only a minor structuration, thus allowing more "choice" for future events.

In Figures 3-13 and 3-14 we show the time evolution of a system with an initially inhomogeneous distribution of population, corresponding to initially richer farming land. We see that this initial inhomogeneity acts powerfully to localize the area within which growth will occur.

Figures 3-15, 16, and 17 concern the change of scale of the units of production satisfying a given demand. Thus, $\beta^{23} = 1$, and the scale of function 3 is greater than function 2. $\gamma_3 = 30$, $\gamma_2 = 20$; and the economies of scale result in a unit price which falls from

$$p^{(2)} = .5 + \frac{10}{20} = 1 \text{ to}$$

$$p^{(3)} = .5 + \frac{10}{30} = .833.$$

In Figure 3-17 we show the result of competition between different scale industries for different transportation efficiencies. Thus, the large scale industry (3) has a higher market threshold ($\gamma^3 = 30$) but a lower unit cost ($P_3 = .833$). The question is whether it can reach sufficient individuals to make up for its higher market threshold. As Figure 3-17 shows us, the more efficient the transportation system, the more the "large" scale of production wins. Consequently, we will find an increasing centralization--that is, the suppression of many centers as the transportation techniques improve.

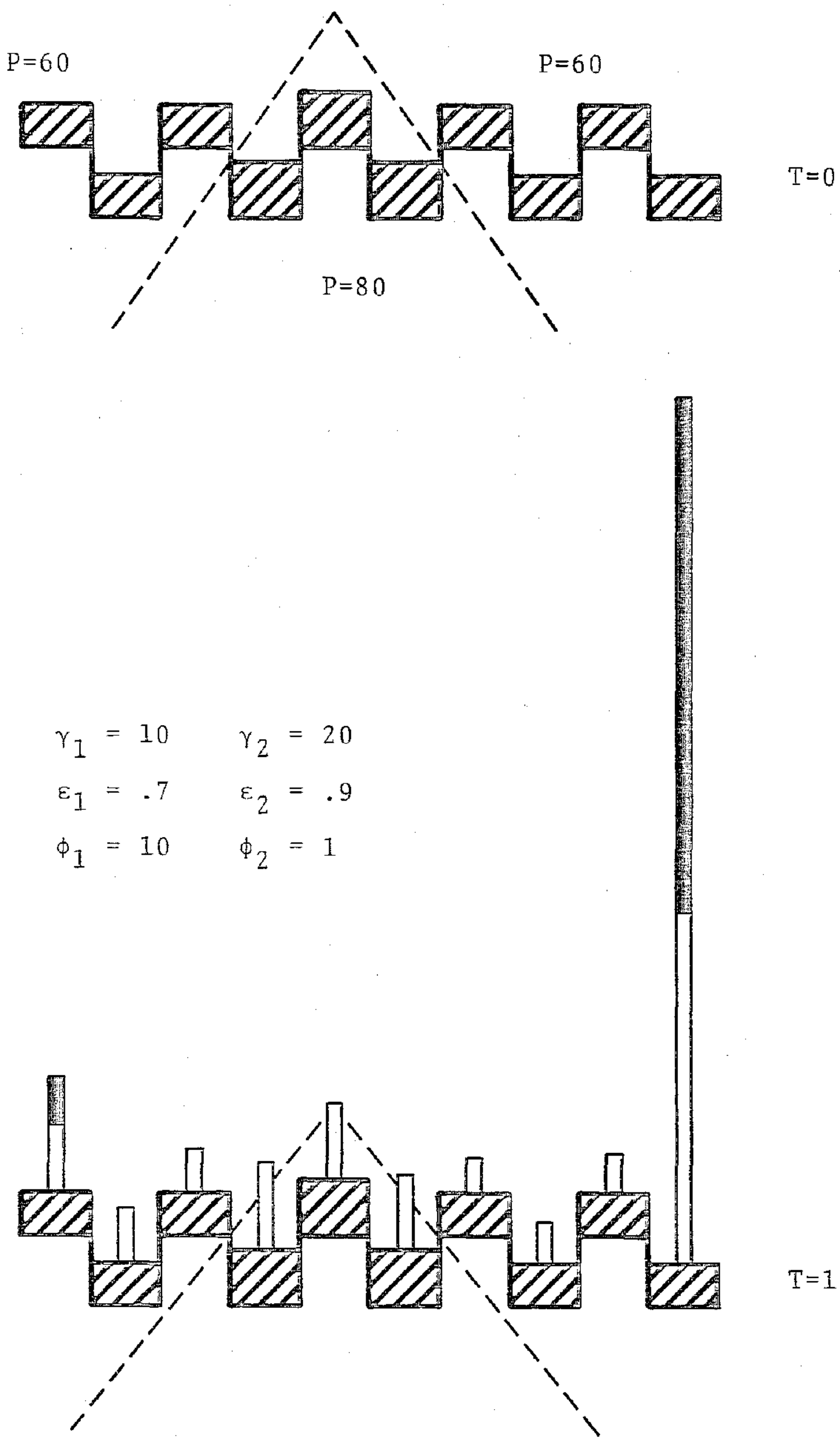


FIGURE 3-13. TIME EVOLUTION OF URBANIZATION: INITIALLY INHOMOGENEOUS POPULATION DISTRIBUTION

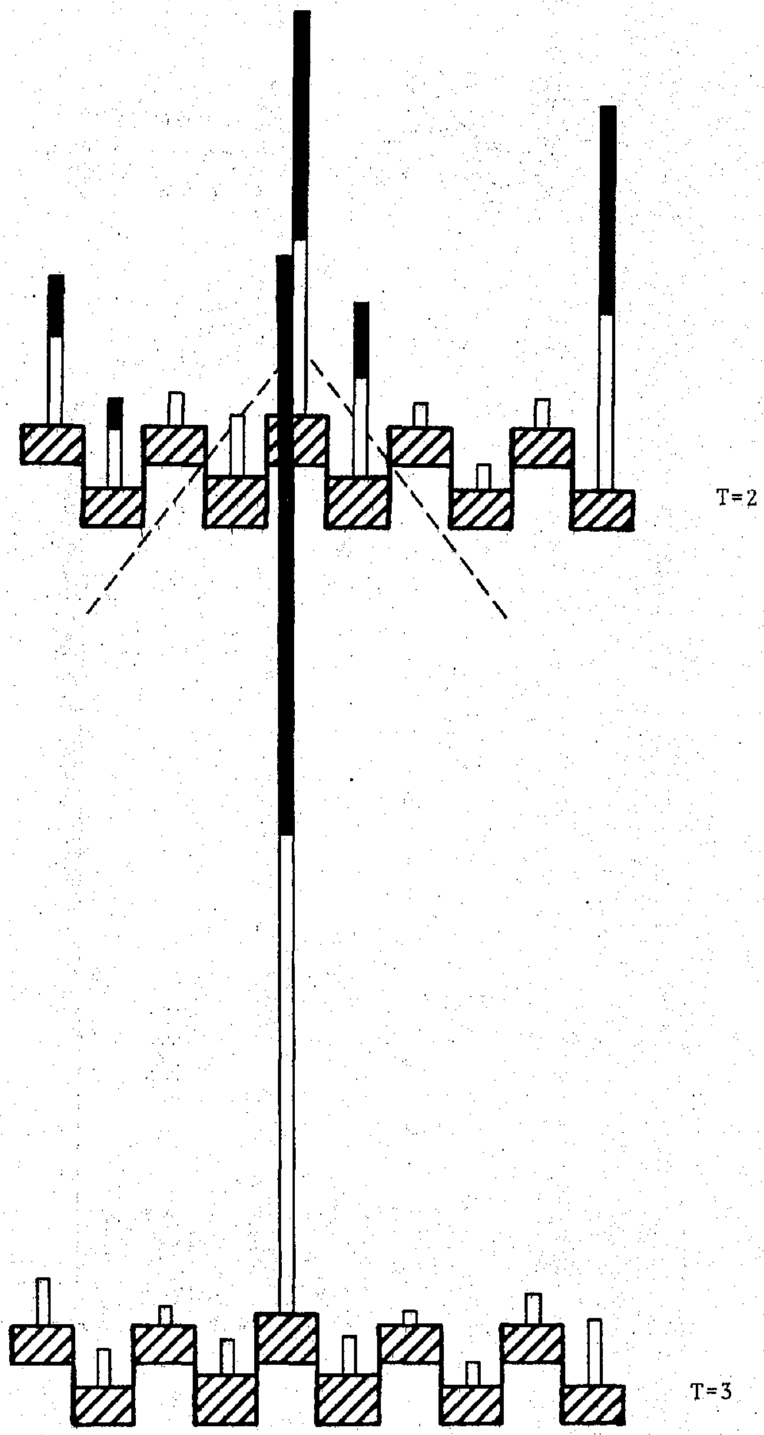


FIGURE 3-13. (CONTINUED)

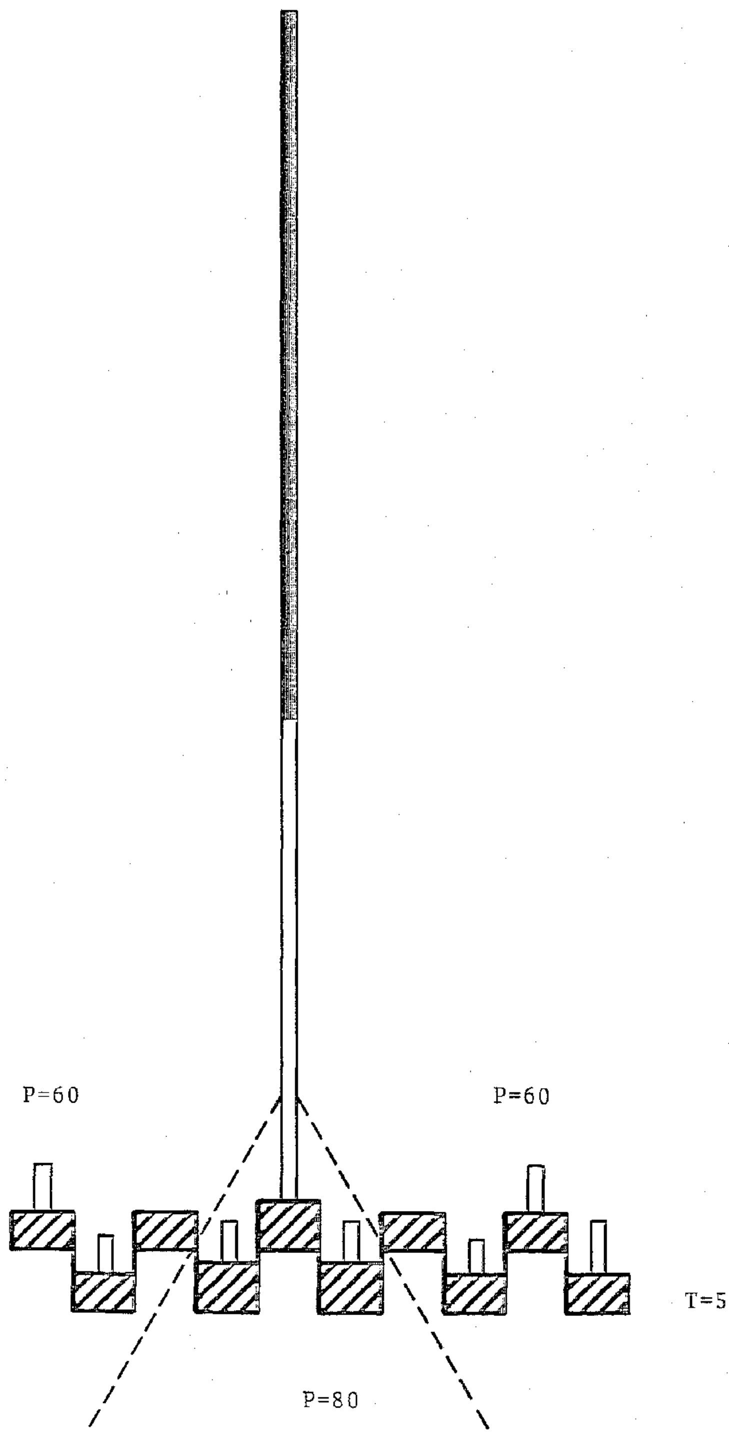


FIGURE 3-13. (CONTINUED)

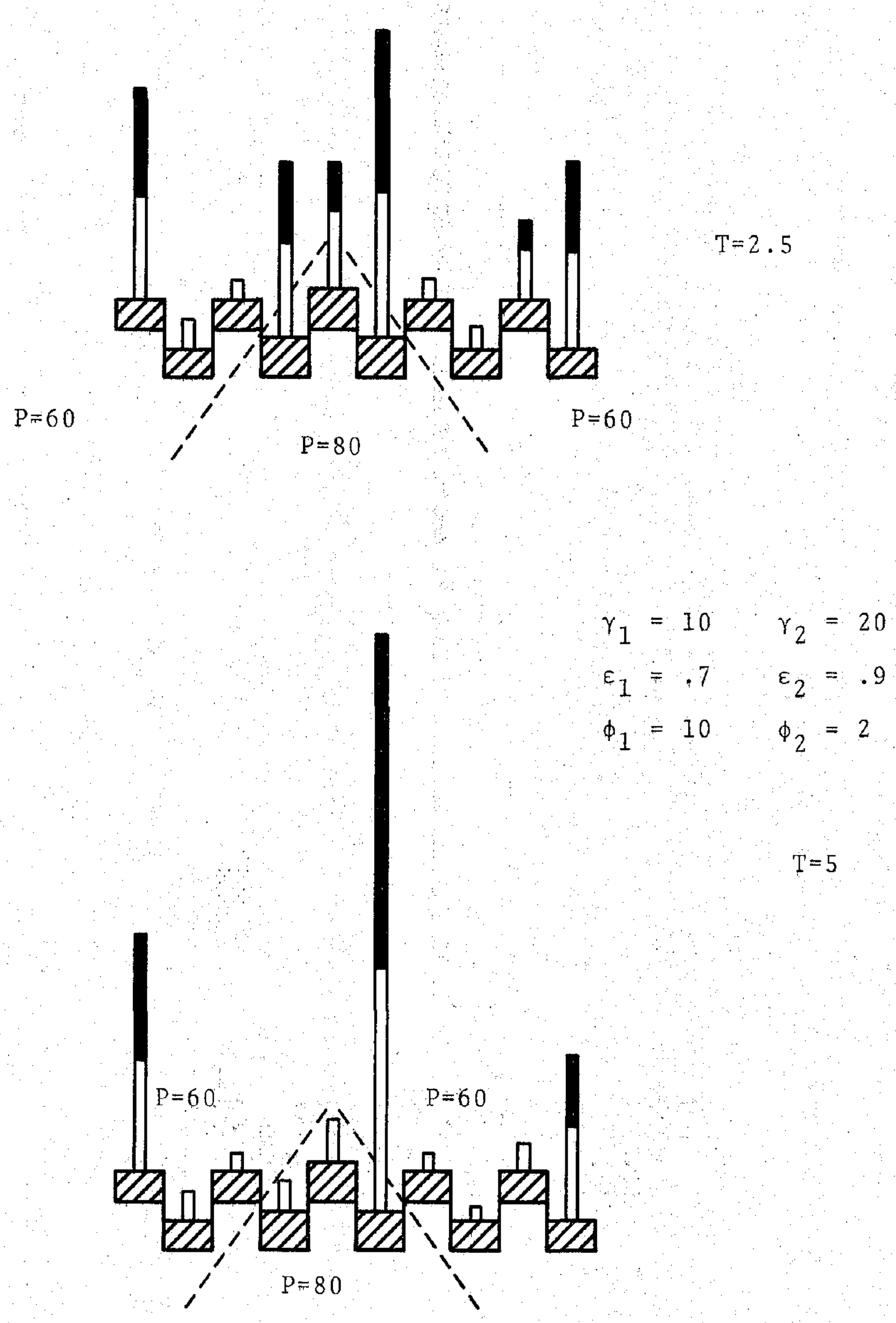


FIGURE 3-14. TIME EVOLUTION OF URBANIZATION:
INITIALLY INHOMOGENEOUS POPULATION DISTRIBUTION

$\gamma_1 = 10 \qquad \gamma_2 = 20 \qquad \gamma_3 = 30$

$\varepsilon_1 = .7 \qquad \varepsilon_2 = .4 \qquad \varepsilon_3 = .4 \qquad \beta\ (2,3) = 1$

$\phi_1 = 5 \qquad \phi_2 = 1 \qquad \phi_3 = 1$

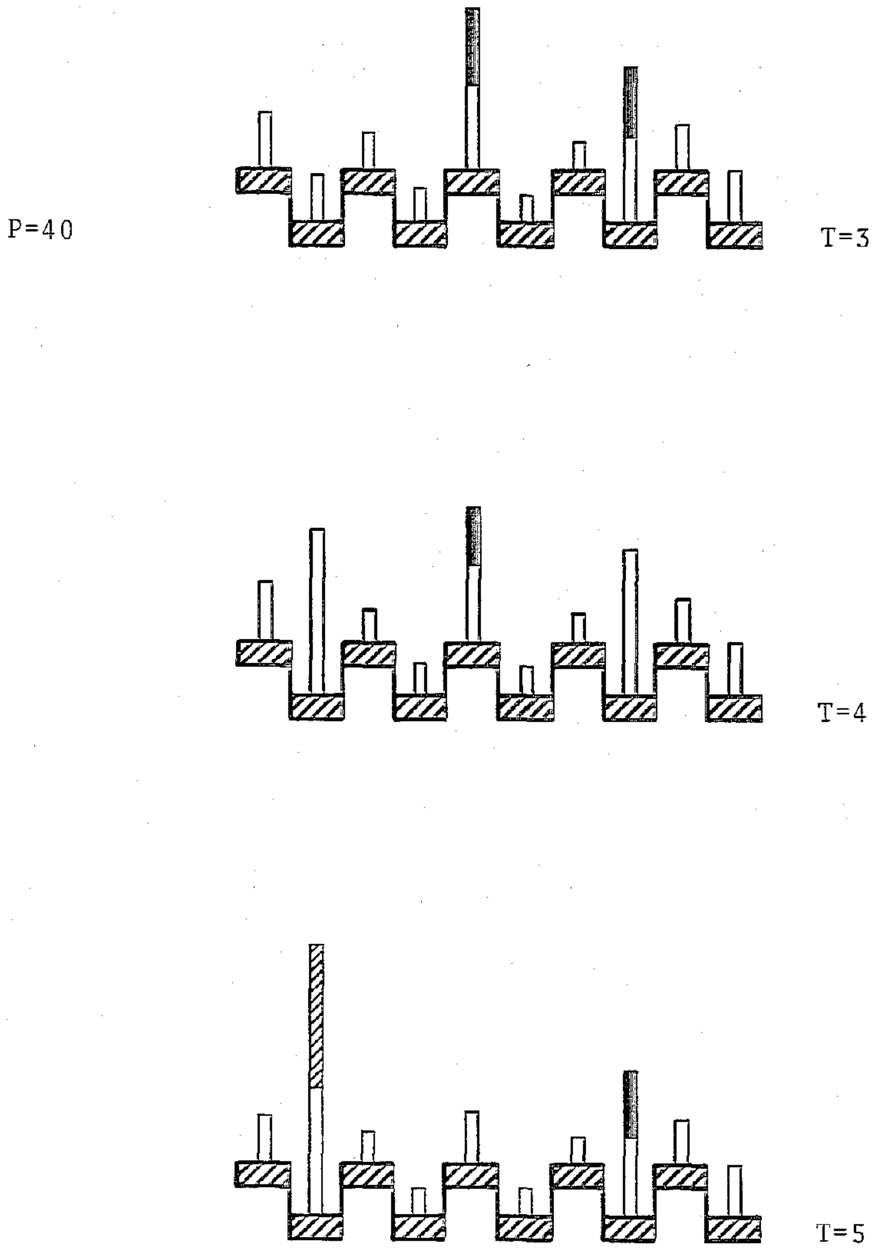


FIGURE 3-15. TIME EVOLUTION OF URBANIZATION: DIFFERENT MARKET THRESHOLDS

$\gamma_1 = 10$ $\gamma_2 = 20$ $\gamma_3 = 30$

$\epsilon_1 = .7$ $\epsilon_2 = .4$ $\epsilon_2 = .4$ $\beta\ (2,3)=1$

$\phi_1 = 5$ $\phi_2 = 2$ $\phi_3 = 2$

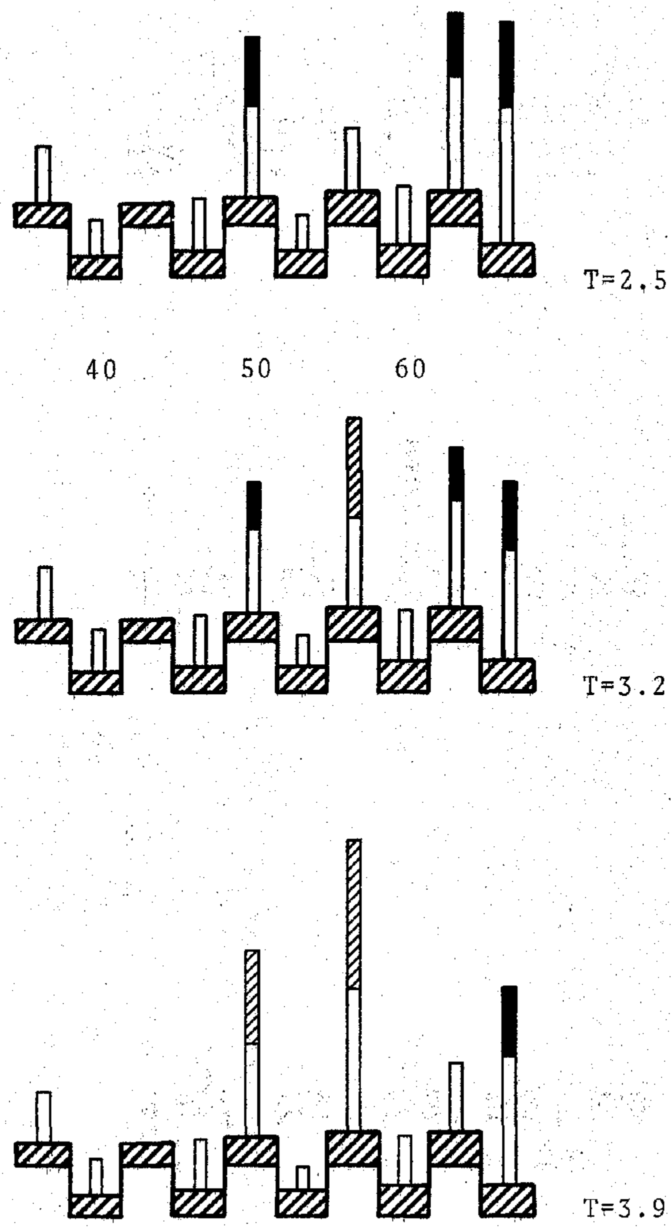


FIGURE 3-16. TIME EVOLUTION OF URBANIZATION; DIFFERENT MARKET THRESHOLDS

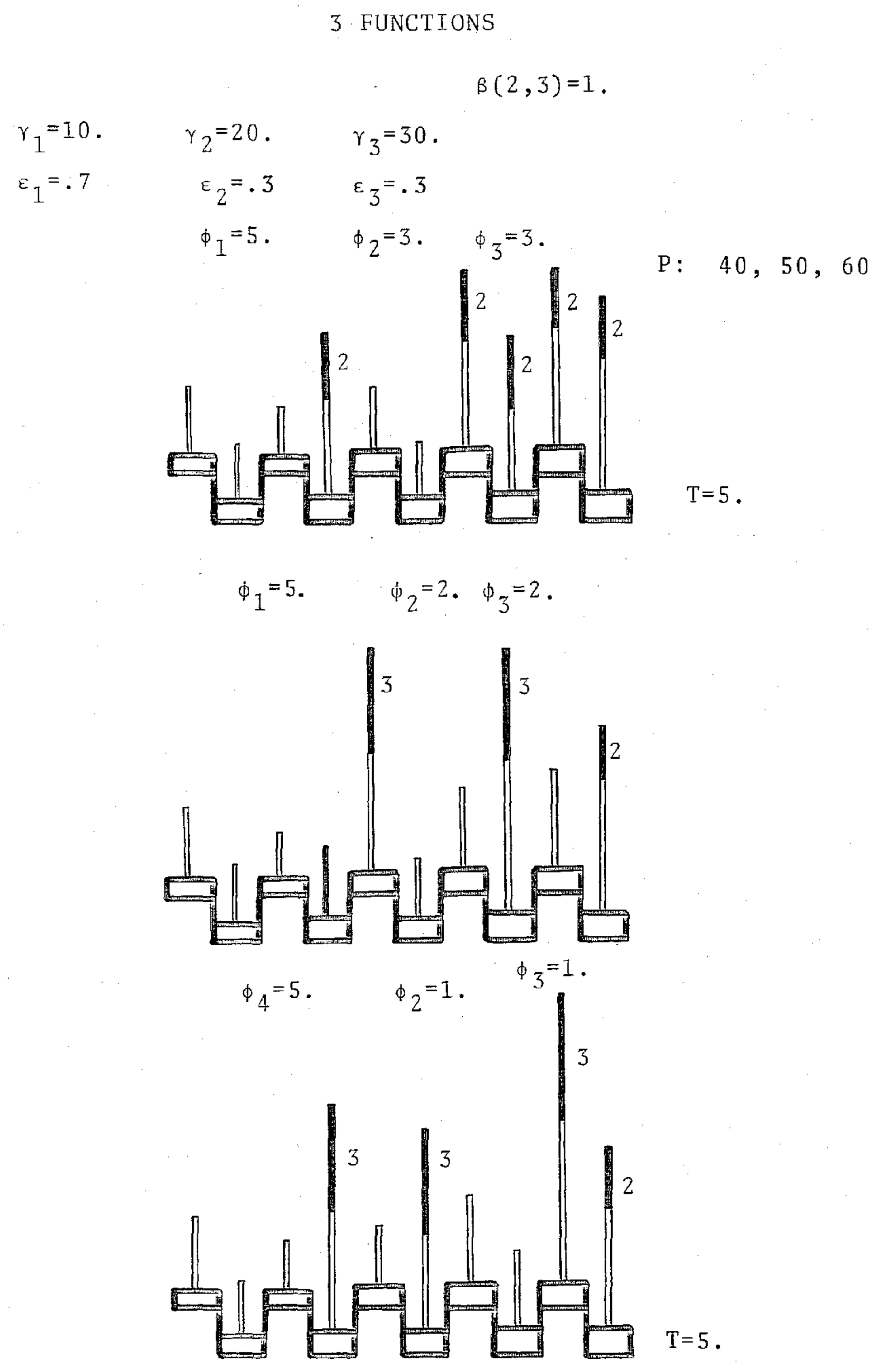


FIGURE 3-17. TIME EVOLUTION OF URBANIZATION: DIFFERENT MARKET THRESHOLDS AND THE EFFECT OF DIFFERENT TRANSPORT EFFICIENCIES

4. DYNAMIC SPATIAL ORGANIZATION OF THE CITY

In this section we consider the spatial organization which will take place <u>within</u> a growing urban center when we distinguish between place of work and place of residence. Clearly, the structure of a city results from the nonlinear interaction of many different elements, and very probably the organization that occurs is due to the complex interplay of both random and deterministic factors. In this preliminary model, however, we have chosen to simplify the problem by considering three basic elements:

a) the places of residence of the population,

b) the places of work of the population,

c) the transportation system which allows the movement between the two.

In order to write down differential equations describing the various interactions between these factors, we must consider the basic mechanisms operating within the system.

Let us consider first the forces of attraction between the elements. For example, an industry will attract other industries with common interests; and business offices, banks, and administrative centers will be subject to a mutual attraction. Equally, there is a tendency for residences to be grouped because of the necessity for common services, such as water, electricity, and transport. There is also an attraction between places of work and residence because, <u>excluding other factors</u>, the average employee would live as close as possible to his place of work. We must also take account of the fact that jobs connected with local services will also be attracted to clusters of residences. All these correspond to centripetal "forces."

Let us consider next the "centrifugal" mechanisms operating in the city. First, there is a spatial impossibility of placing everything at a point, and because of the economic constraints imposed by the "centripetal" forces (high land value at center), a selective repulsion acts on places of residence and work. Also, psychological factors manifest themselves in the unwillingness (if given the choice) of the population to live directly in the shadow of a large factory and in the desire to inhabit a semirural environment away from an industrial center.

These various mechanisms can be included in a scheme of interdependent differential equations, which we will now describe. The variables whose time evolution the equations describe are:

- H_i - the number of residences at the point i.
- J_i - the number of jobs in the city's export sector at point i (offices, industries, etc.).
- S_i - the number of jobs in the local service sector (schools, shops, medical services).

This represents, of course, a rather simplified view of the components in interaction but leads to results which are relatively satisfying.

The means of transport within the city are considered to be distributed homogeneously, and therefore only the trip distance will play a role in the parameters characterizing the interactions in this preliminary model. In neglecting the difference between the radial and circumferential transport facility, we may, of course, eliminate the possibility of angle-dependent structures forming in our model, but this should not influence the radial distribution pattern which is of interest for the comparison with the observation of Buissière and Berry, for example. We propose to describe modifications in the transportation network by discrete variation of the parameters depending on it, and later we could introduce the possibility of a spatially dependent distribution of transportation.

The first equation describing the evolution of H is:

$$\frac{dH_i}{dt} = \alpha \left(\sum_{j=1}^{m} D_{ji} - H_i\right) - \theta_i H_i J_i^{n1} - \theta_2 H_i^2$$

$$+ \frac{1}{m-1}\left(\theta_1 \sum_{\substack{j=1 \\ j \neq i}}^{m} H_j J_j^n + \theta_2 \sum_{\substack{j=1 \\ j \neq i}}^{m} H_j^2\right).$$

The term D_{ji} corresponds to the demand for housing at the point i resulting from employment at point j, and H_i subtracts off the amount already satisfied. This will be discussed in more detail below. The terms $\theta_1 H_i J_i^{n1}$ represent the repulsive mechanism operating between the places of residence and employment; and the power "n" can be changed to give different strengths to this effect, while the term $\theta_2 H_i^2$ represents the repulsion between residences due to crowding. These repulsions cause the "diversion" of residences from one point to another, and therefore we write the final terms which take account of the arrival at i of residences due to the repulsive mechanisms at other points. This supposes that the choice of a "new site" is uniform over the city area, but clearly if the choice is unfortunate (high HJ or H^2), then displacement will occur again rapidly.

The equation for J_i is:

$$\frac{dJ_i}{dt} = \left(\sum_{j=1}^{m} \beta_{ij} J_j + \sum_{j=i}^{m} \gamma_{ij} J_j^2\right)\left(N - \sum_{i=1}^{m} J_i\right),$$

where the growth of the number of jobs at i depends on, first of all, N, the exterior demand, minus the sum for the whole city of the jobs satisfying it, $\sum_{i=1}^{m} J_i$. The term $\beta_{ij} J_j + \gamma_{ij} J_j^2$ are the first terms of a polynomial expressing how the spatial distribution, within the city, affects the growth of J_i; that is to say, how the cooperativities between different export sectors depend on their proximity. Thus, β_{ij} γ_{ij} and the demand in the previous equation will depend on some inverse power law of distance.

The equation for the local services is:

$$\frac{dS_i}{dt} = \mu\left(\sum_{j=i}^{m} \bar{D}_{ji} - \sum S_i\right),$$

where $\bar{D}_{ji}$ is the demand for local services at the point i, resulting from residences at the point j. Again we will invoke an inverse power law of distance for this function. Clearly, S_i substracts the demand already satisfied.

Now let us consider explicitly the role of "space" in these various equations. In the equation for H_i, we have the term:

$$D_{ij} = J_j \left\{ \frac{\dfrac{\rho + qH_i}{1 + \theta d_{ij}^n}}{\sum_{K=1}^{m} \dfrac{\rho + qH_k}{(1 + \theta d_{kj}^n)}} \right\},$$

where θ and n_1 characterize the efficiency of the transportation system taking employees to and from their place of work. When q=0, there is no cooperative effect of a "cluster" of residences.

Similarly, the terms β_{ii} and γ_{ii} are taken to be of the form:

$$\beta_{ij} = \frac{\nu}{(1 + \tilde{\theta}\, d_{ij}^n)^{m_1}} \qquad \gamma_{ij} = \frac{\nu'}{(1 + \tilde{\theta}\, d_{ij}^n)^{m_1}} \quad .$$

The presence of the power m_1 takes into account the fact that the distance of effective cooperation between industries, offices, etc., is much shorter than the range over which an industry may draw employees.

The final term is $\bar{D}_{ij}$, the demand for services at i from residences at j. This has the form:

$$\overline{D_{ij}} = H_j \left\{ \frac{\dfrac{1}{1 + \bar{\theta}\, d_{ij}^n}}{\sum_{K=1}^{m} \dfrac{1}{1 + \bar{\theta}\, d_{kj}^n}} \right\} \quad .$$

5. SIMULATION RESULTS FOR SECTION 4

Several simulations have been performed on a regular rectangular lattice of 49 points, and in Figure 5-1 we show the resulting structure for different values of the parameters for the growth of a city which starts initially at the center of the lattice. For the values of the parameters corresponding to Figures 5-1, 3, 4, and 5, the density of residences falls off from the center in approximately exponential fashion, e^{-br}, r being the distance from the center. The value of b in these arbitrary distance units is, for Figure 5-1, $b \approx .65$, and Figure 5-5, $b \approx .8$.

For other values of the parameters, as in Figures 5-2 and 5-6, 7, and 8, the density of residences at the central point falls below that of its neighbors as the evolution proceeds. The exponential "law" breaks down with the formation of a "central business district." The comparison between the growth of our simulated city (Figure 5-9) and the observed growth pattern of Chicago (Figure 5-10) shows a close correspondence.

Summarizing the results of these preliminary simulations, we have found that the differential equations proposed by us, which describe the effects of individual interactions without reference to a preferred direction or center, lead to a fairly realistic distribution of residences, jobs, and services.

In further studies it will be possible, by changing the parameters concerned, to look at how modifications in the urban transportation sector affect the city structure. Comparisons could also be made very easily between cities which grow on a plain and cities which grow on the coast (ports), telling us how the geometrical asymmetry affects the city structure.

Clearly, such methods offer the possibility of understanding the full effects of any innovation within a single sector as nonlinear mechanisms modify the whole urban structure. Global social and economic costs of decision could perhaps be more accurately estimated.

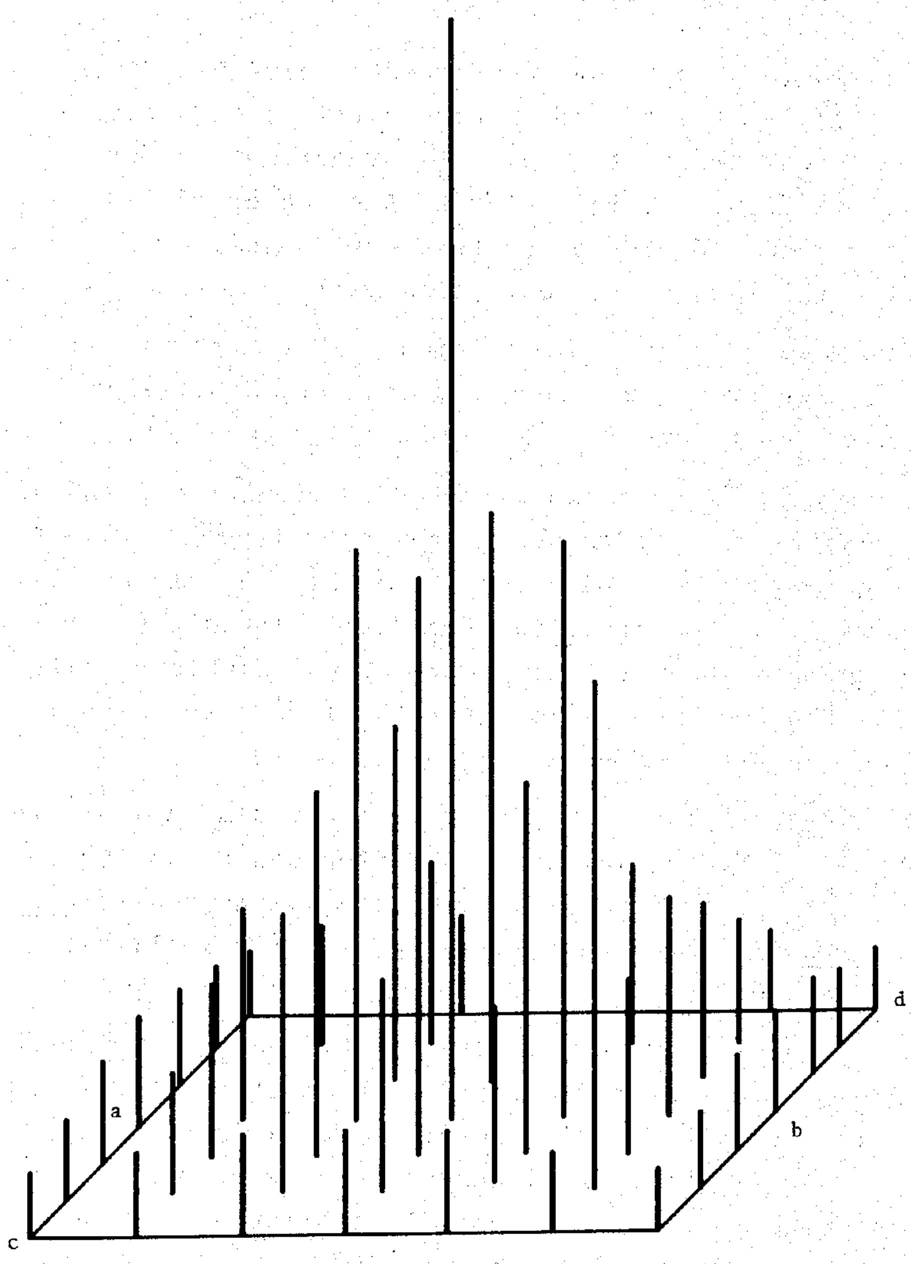


FIGURE 5-1a. THE DISTRIBUTION OF RESIDENCES

$N = 1.7$	$\nu = 2$	$\nu' = .5$	$\tilde{\theta} = 2$	$n = 2$
$m_1 = 5$	$\alpha = 30$	$\theta_1 = 5$	$n_1 = 2$	$\mu = 10$
$\varepsilon = 10$	$\theta = 2$	$\bar{\theta} = 30$	$\theta_2 = 0$	$\rho = 1$

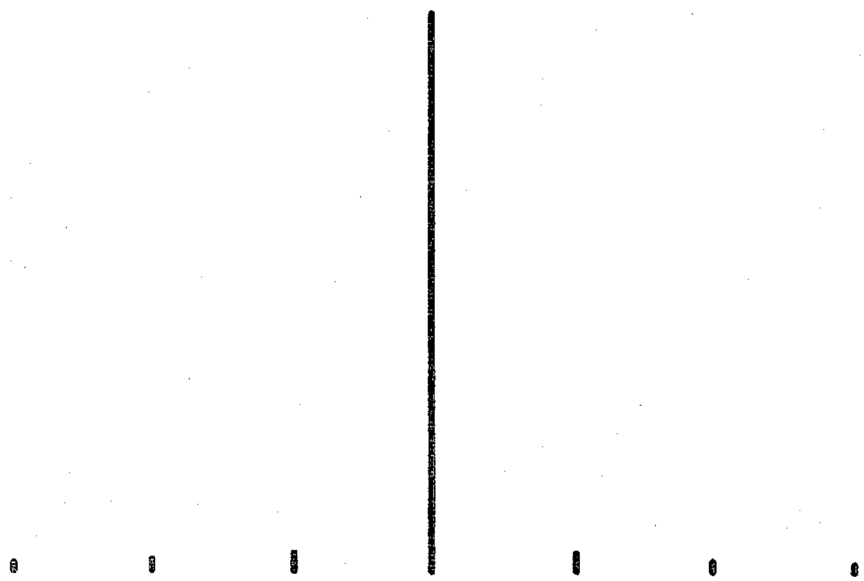


FIGURE 5-1b. THE DISTRIBUTION OF JOBS ACROSS THE CUT FROM a TO b (PARAMETERS SAME AS FIGURE 5-1a)

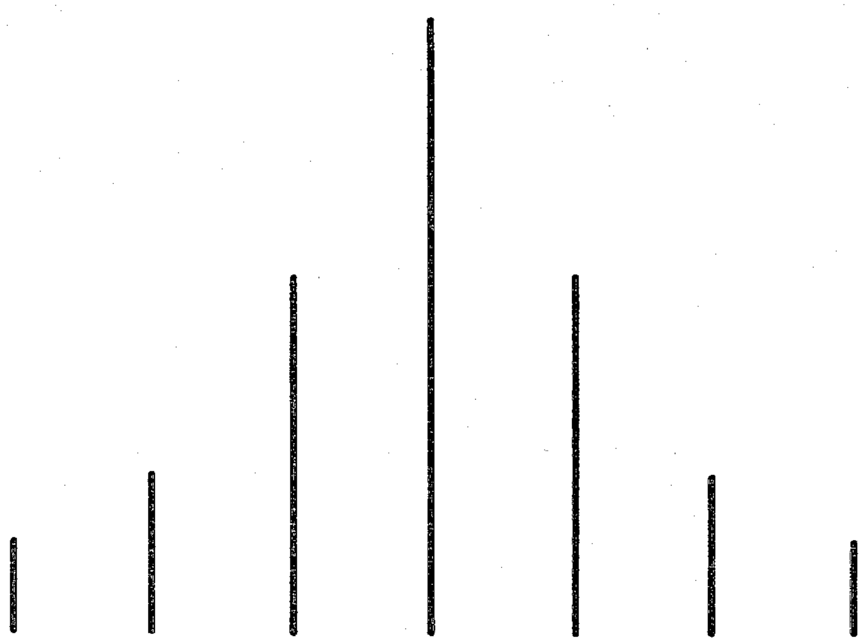


FIGURE 5-1c. DISTRIBUTION OF SERVICES ACROSS THE CUT FROM a TO b (PARAMETERS SAME AS FIGURE 5-1a)

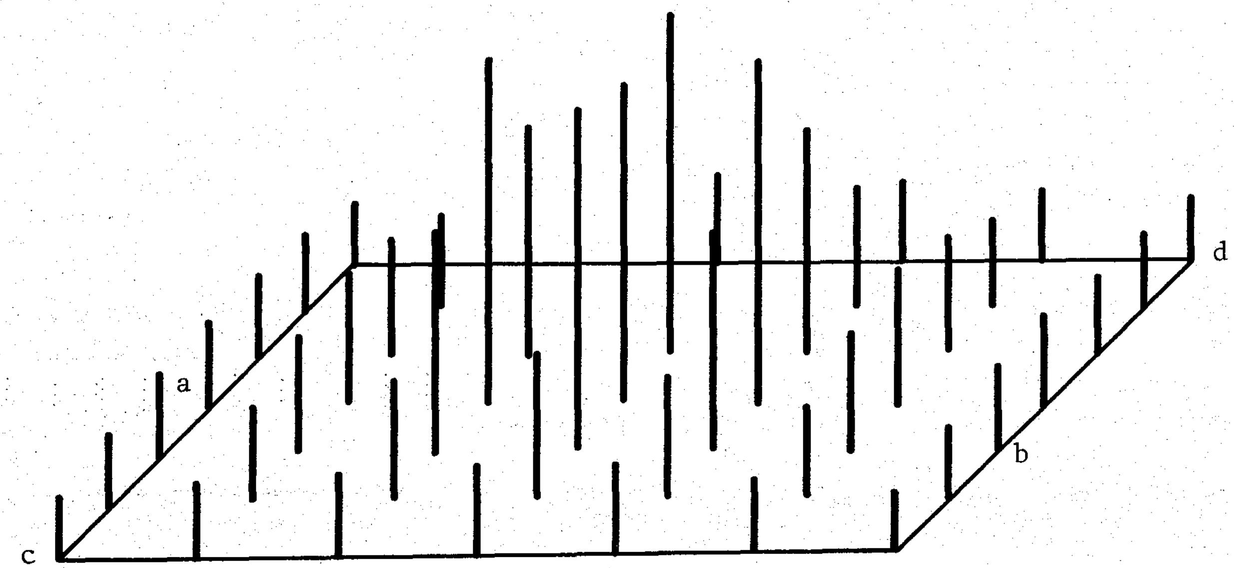


FIGURE 5-2. DISTRIBUTION OF RESIDENCES

Increased "repulsion" between jobs and residences diminishes the population at the center.

$N = 1.7$	$\nu = 2$	$\nu' = .5$	$\tilde{\theta} = 5$	$n = 2$
$m_1 = 5$	$\alpha = 30$	$\theta_1 = 50$	$n_1 = 2$	$\mu = 10$
$\varepsilon = 10$	$\theta = 2$	$\overline{\theta} = 10$	$\theta_2 = 0$	$\rho = 1$

Increased "repulsion" between jobs and residences diminishes the population at the center.

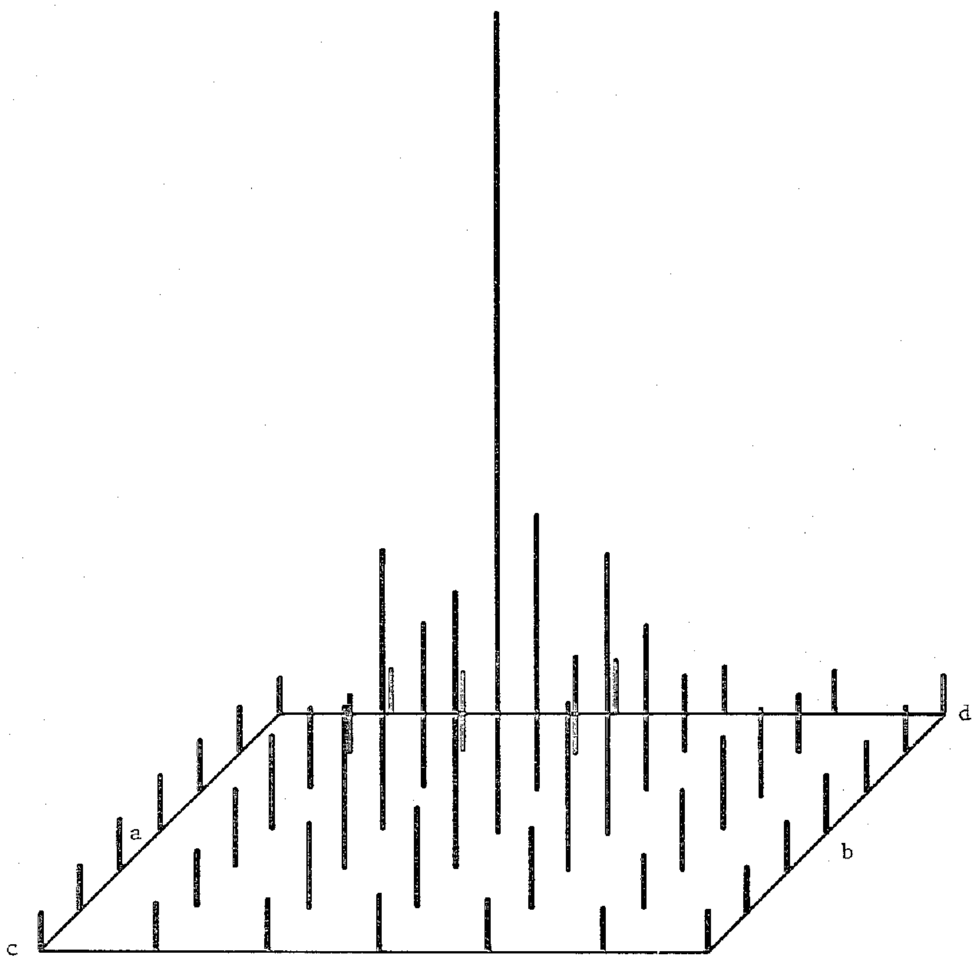


FIGURE 5-3. DISTRIBUTION OF RESIDENCES
A centralized structure results.

$N = 1.7$	$\nu = 2$	$\nu' = .5$	$\tilde{\theta} = 5$	$n = 2$
$m_1 = 5$	$\alpha = 30$	$\theta_1 = 10$	$n_1 = 10$	$\mu = 10$
$\varepsilon = 10$	$\theta = 5$	$\bar{\theta} = 10$	$\theta_2 = 0$	$\rho = 1$

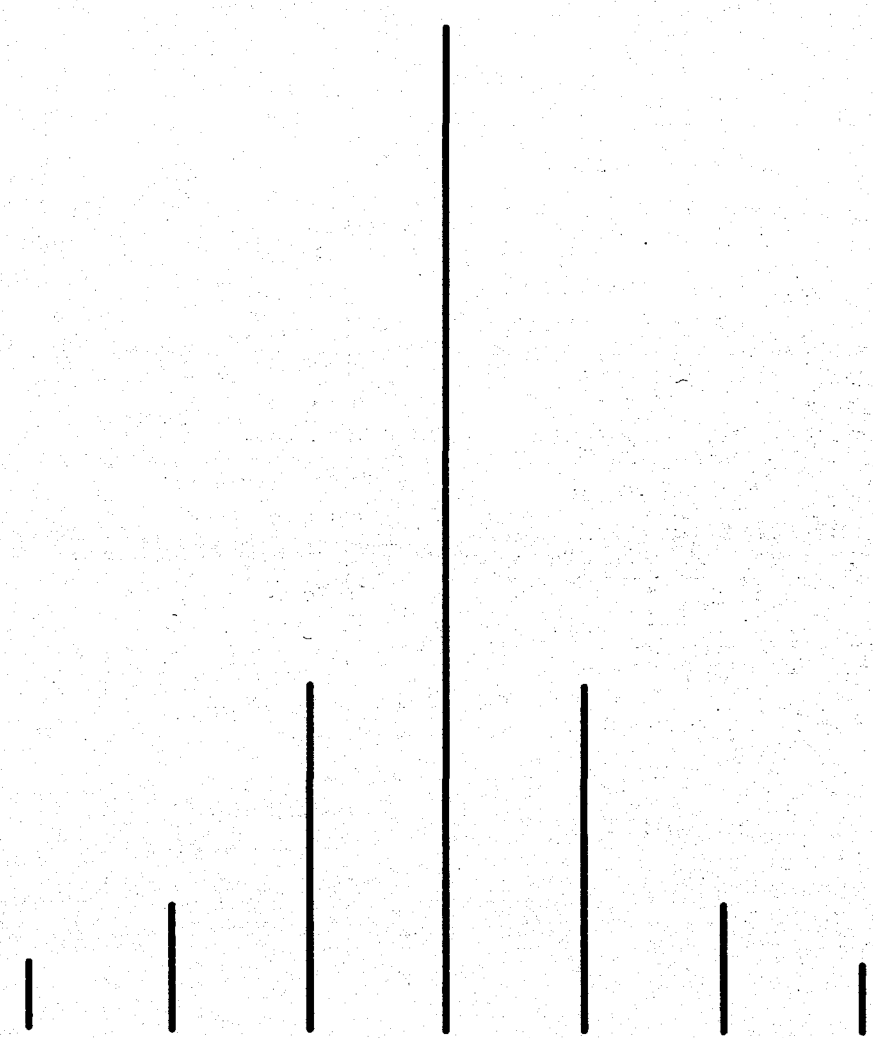


FIGURE 5-4. DISTRIBUTION OF RESIDENCES OF FIGURE 5-3 ACROSS THE CUT a TO b

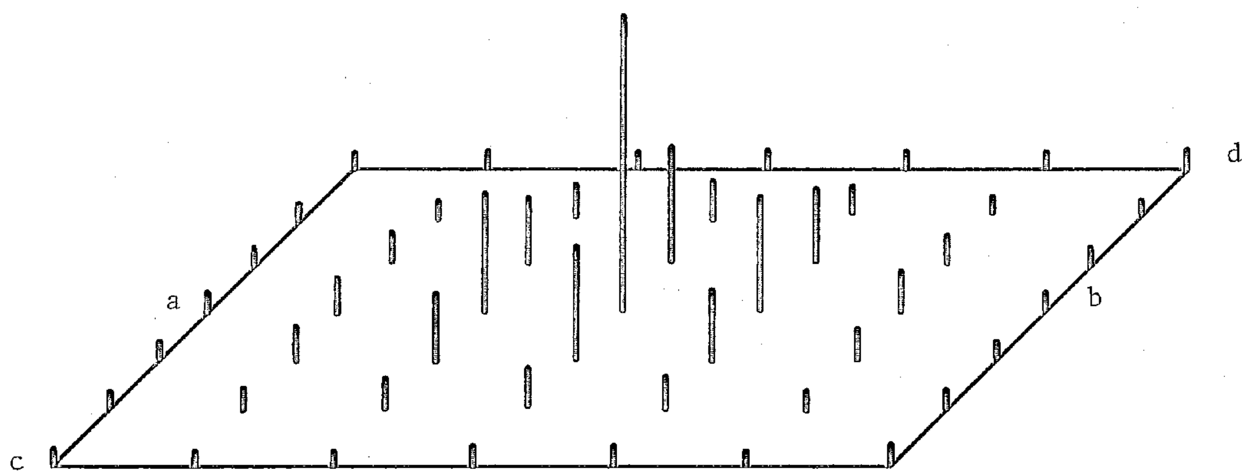


FIGURE 5-5. DISTRIBUTION OF RESIDENCES AT TIME t = 1.01

$N = 1.7$
$m_1 = 5$
$\varepsilon = 10$

$\nu = 2$
$\alpha = 30$
$\theta = 2$

$\nu' = .5$
$\theta_1 = 70$
$\frac{1}{\theta} = 10$

$\tilde{\theta} = 2$
$n_1 = 2$
$\theta_2 = 0$

$n = 2$
$\mu = 10$
$\rho = 1$

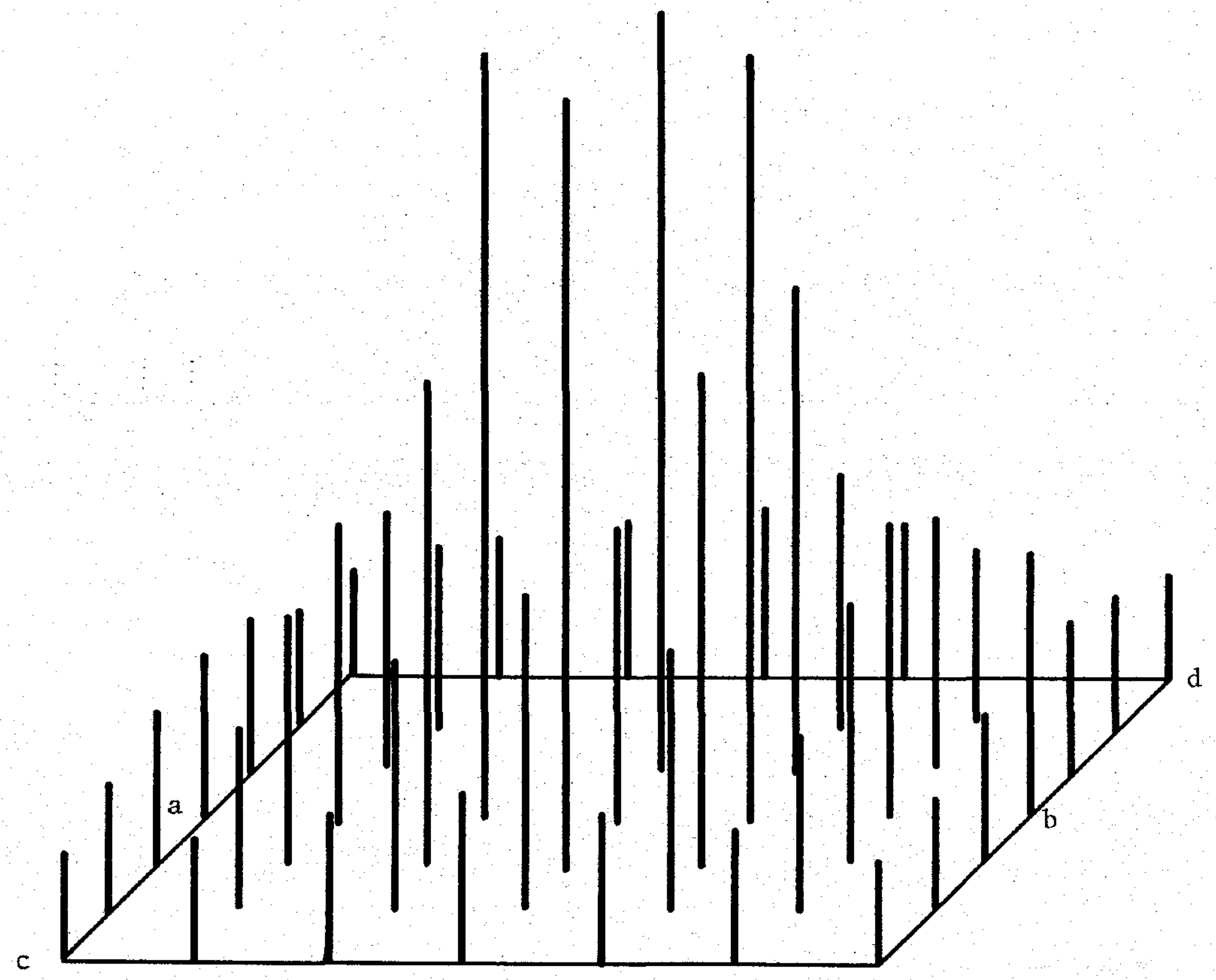


FIGURE 5-6. THE DISTRIBUTION OF THE RESIDENCES AT t = 3

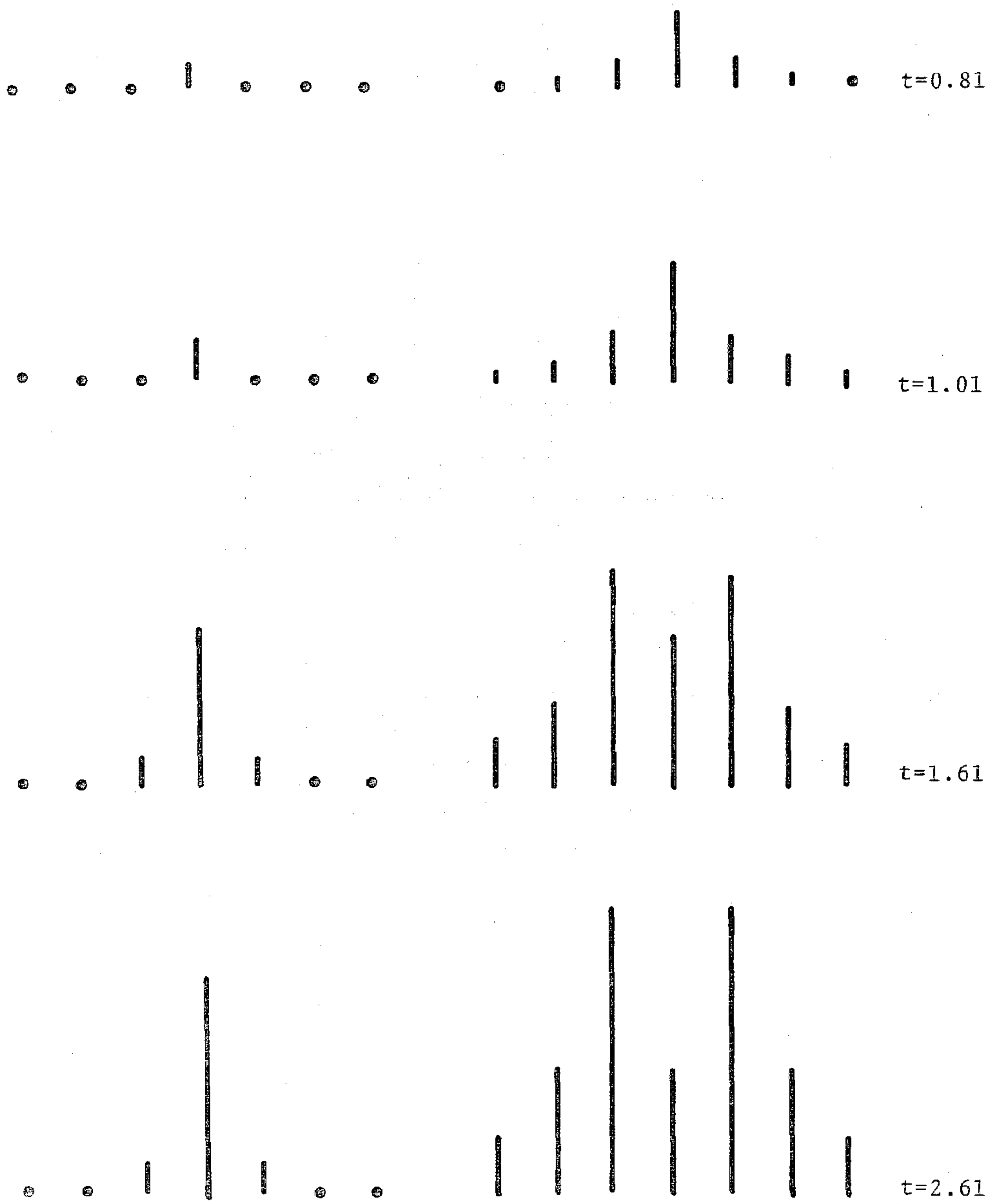


FIGURE 5-7a. THE TIME EVOLUTION OF JOBS (LEFT-HAND COLUMN) AND RESIDENCES (RIGHT-HAND COLUMN) ACROSS THE CUT a TO b

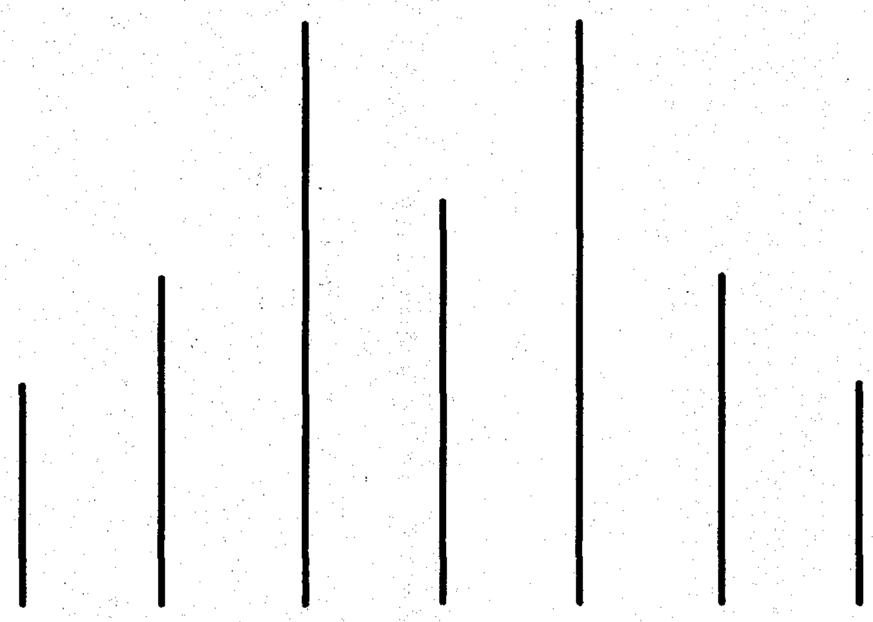


FIGURE 5-7b. THE FINAL DISTRIBUTION OF SERVICES ACROSS a TO b

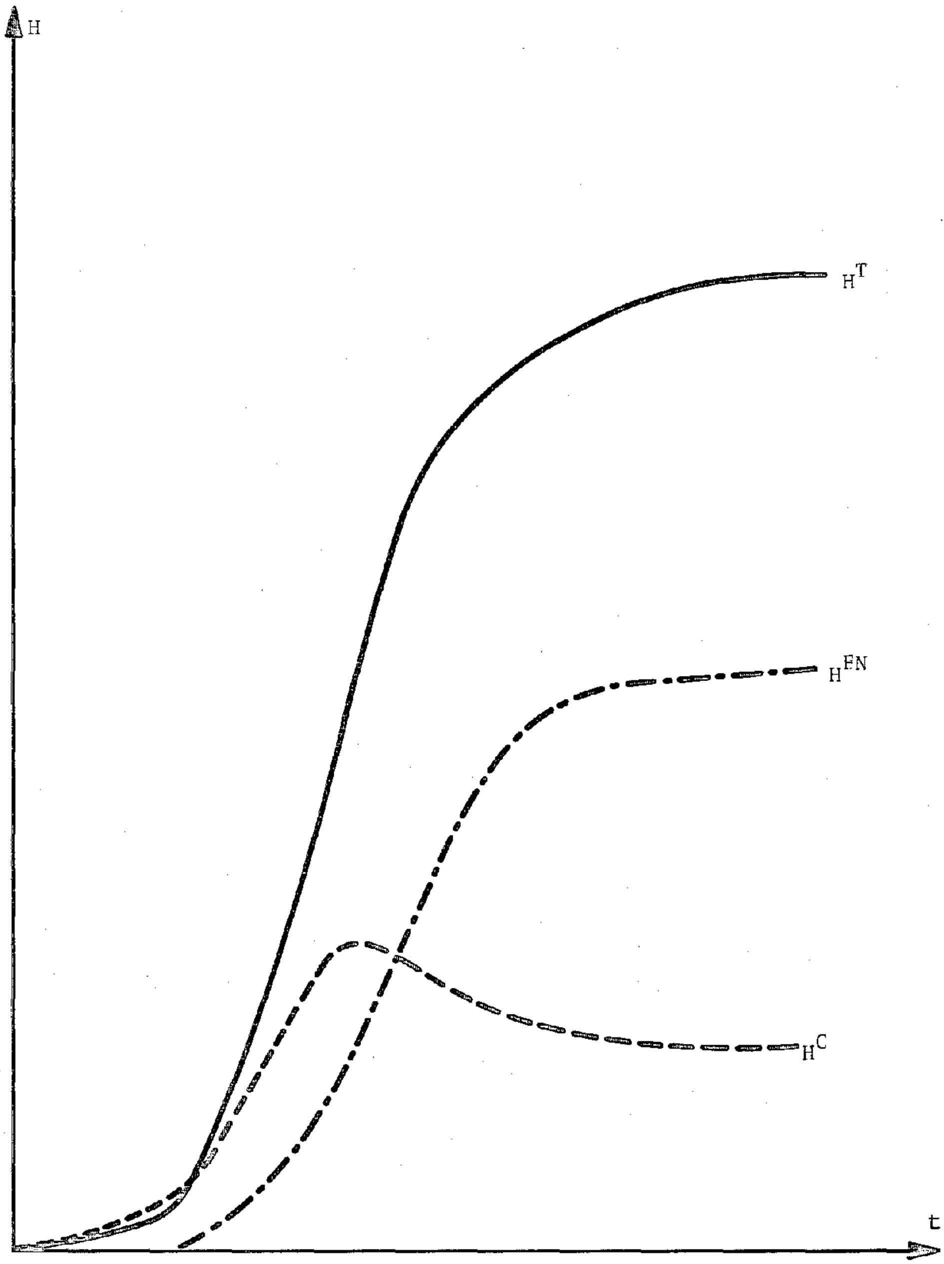


FIGURE 5-8a. THE GROWTH OF RESIDENCES AT THE CENTRAL POINT (H^C) AT THE FIRST NEIGHBOR POINTS (H^{FN}) AND, ON A DIFFERENT SCALE, THE TOTAL NUMBER H^T FOR THE LATTICE, AS FUNCTIONS OF TIME

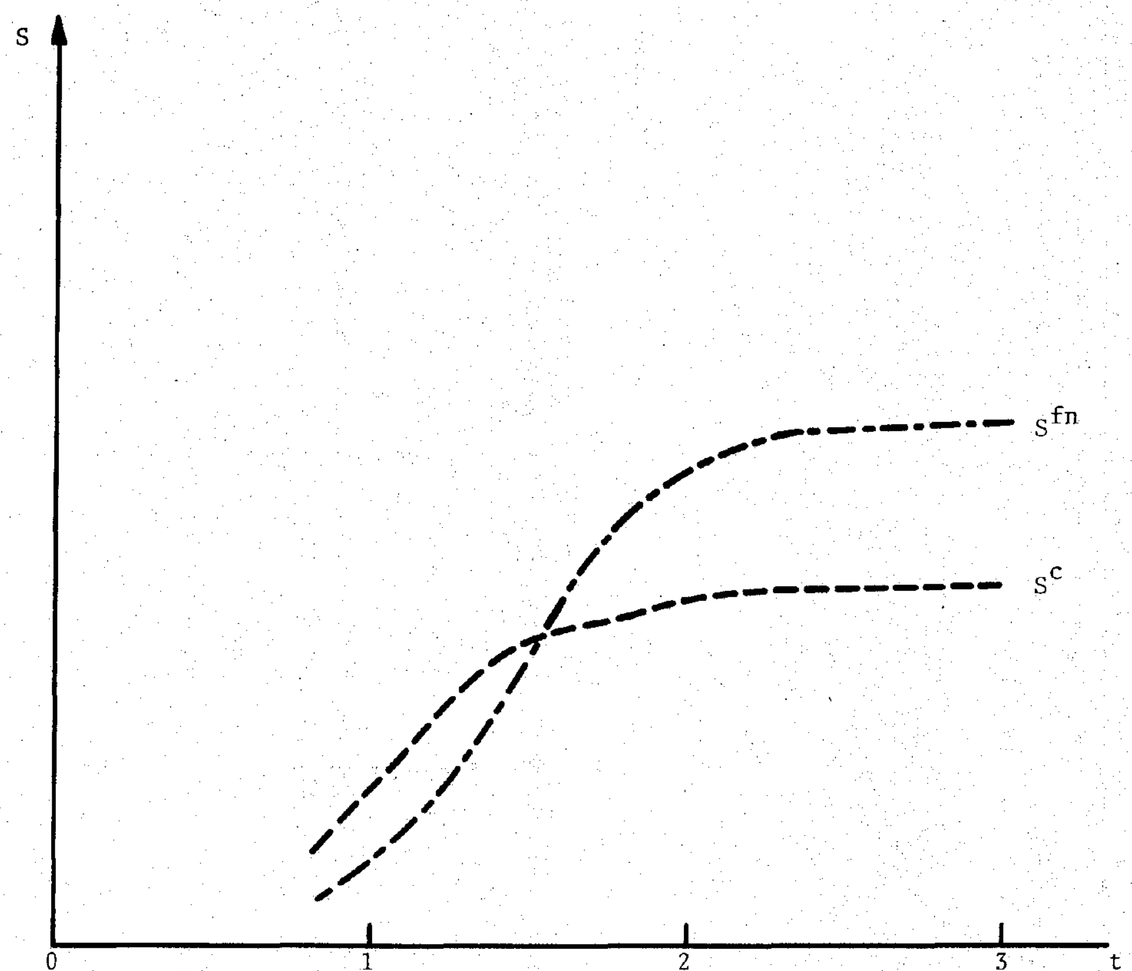


FIGURE 5-8b. THE GROWTH OF SERVICES AT THE CENTER S^{C} AND ON THE FIRST NEIGHBORS S^{FN}

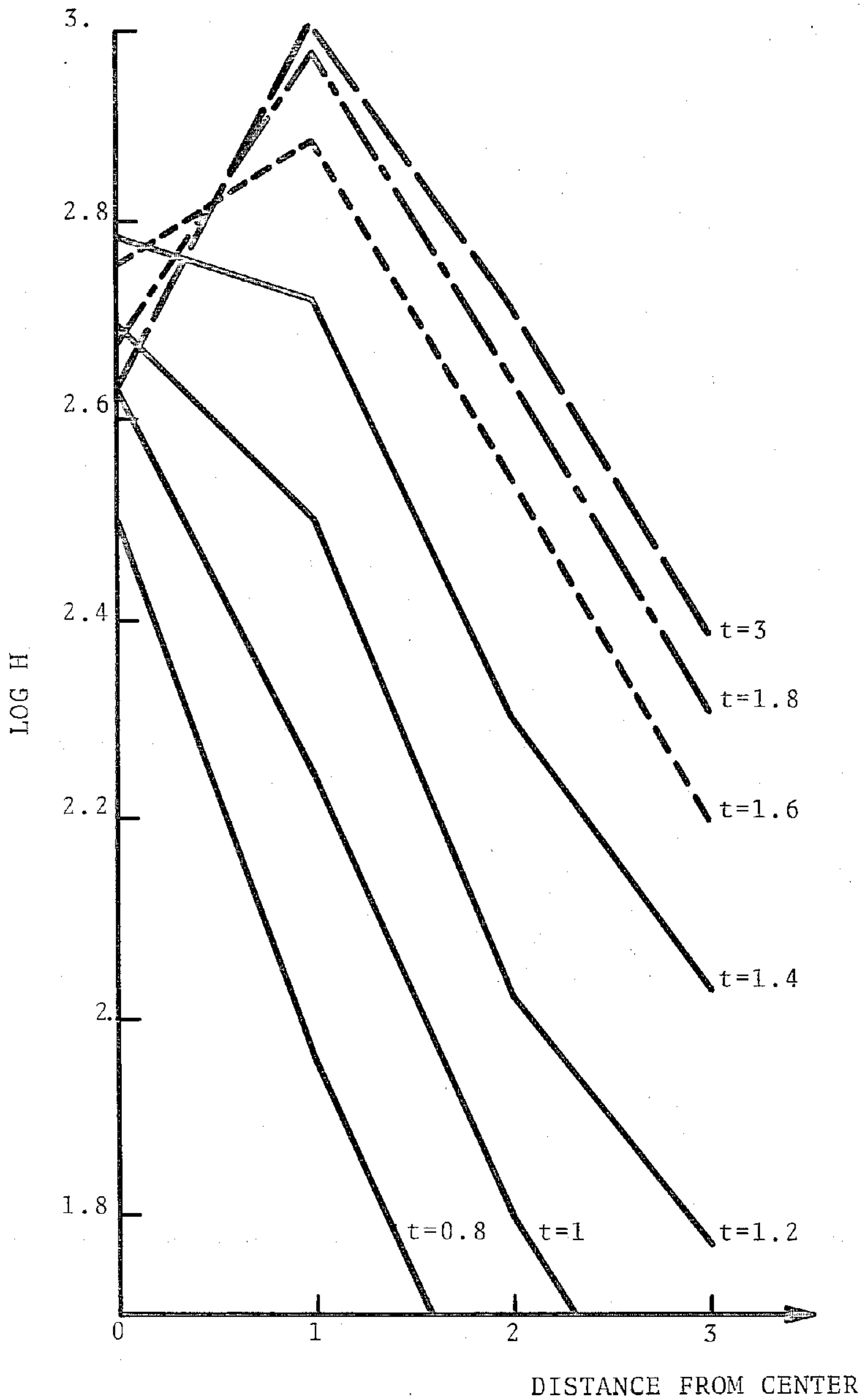


FIGURE 5-9. THE PLOT OF LOG H AGAINST DISTANCE FROM THE CENTER, AT DIFFERENT TIMES OF THE SIMULATION OF FIGURE 5-5

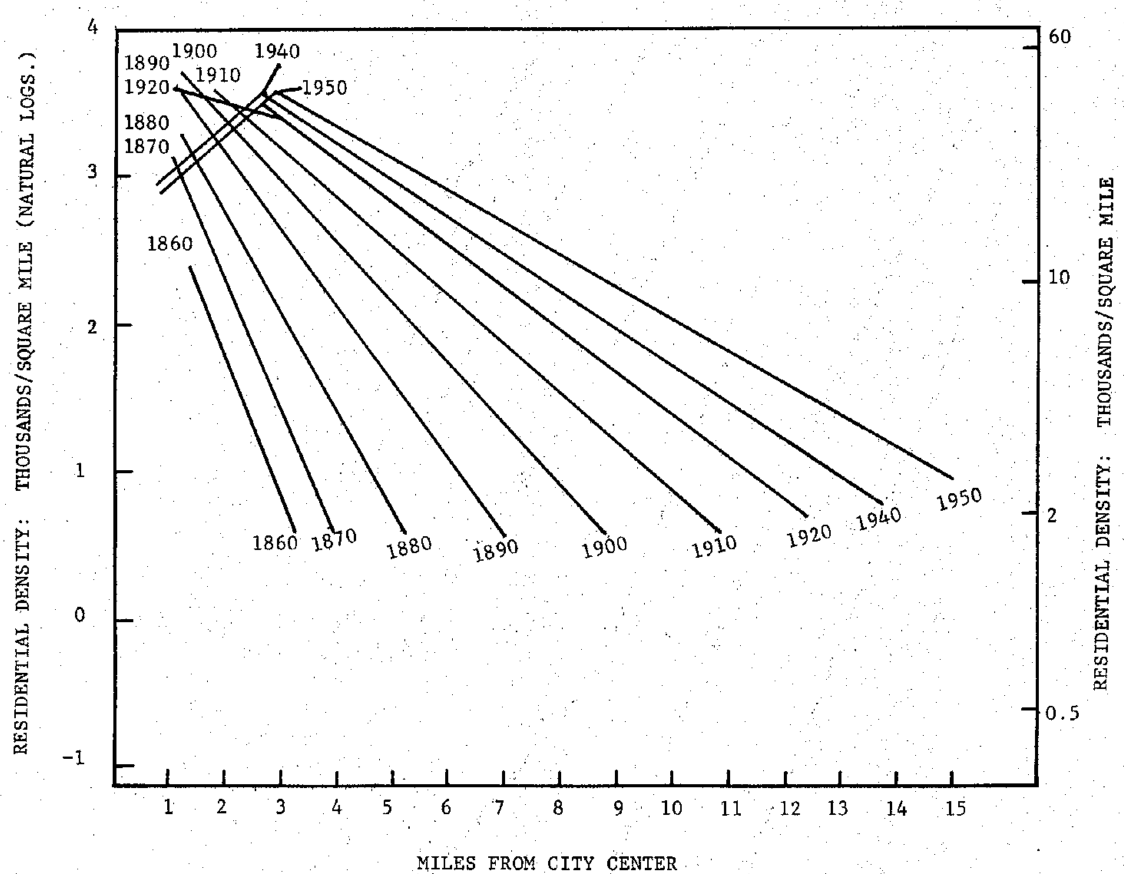


FIGURE 5-10. PLOT OF LOG H AGAINST DISTANCE FROM THE CENTER AT DIFFERENT MOMENTS IN THE HISTORY OF CHICAGO (BERRY)

APPENDIX: REPORT OF NEW TECHNOLOGY

The work performed under this contract, while leading to no invention, had produced preliminary dynamic models of the evolution of the spatial organization of urban centers and urban populations based on the concept of "order by fluctuation," which are improvements over previous urban models. It was shown that fluctuations play a vital role in the evolutionary process of urban growth (Section 1). The evolution of a complex system cannot be known simply by studying the deterministic equations describing its internal dynamics. It is necessary, in addition, to study the effects of fluctuations or historical accident which can drive the system to new modes of behavior. Taking account of both the deterministic elements of urban growth and the appearance of innovations at chance locations in an economic region, a dynamic model of the evolution of the spatial organization of urban centers was developed in Section 2 and simulations produced in Section 3. The dynamic model of the evolution of the spatial distribution of urban populations was developed in Section 4 and simulation results presented in Section 5.

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