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THE DYNAMICS OF URBAN EVOLUTION
Volume II: Intra-Urban Evolution

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16. Abstract

In this report the concept of "order by fluctuation," that has appeared recently in physico-chemical and biological systems, is applied to the description of urban growth. It is shown that fluctuations play a vital role in the evolutionary process of urban growth. The evolution of a complex system cannot be known simply by studying the deterministic equations describing the system. It is necessary, in addition, to study the effects of fluctuations or historical accident which can drive the system to new modes of behavior. Taking account of both the deterministic elements of urban growth and the occurrence of fluctuations in population in an urban region, a transportation-sensitive, dynamic model of the evolution of the spatial distribution of urban populations was developed.

This is the second of two volumes. Volume I presented a model of the evolution of the organization of urban centers.

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PREFACE

This is Volume 2 of a final report. The objective of this research was to develop a methodology capable of a multi-disciplinary treatment of transportation planning. This methodology should allow the prediction of the consequences of modification in the transport system on the social, demographic, economic, and ecological structure of the total system.

This volume contains a first model and simulation results of the dynamic evolution of an urban center. The model yields the time evolution of the urban spatial structure.

The authors wish to thank Professor I. Prigogine, whose ideas have inspired this work, for his constant interest and for many helpful discussions. Among the group at Brussels, we particularly wish to thank Professor G. Nicolis, S. Pahaut and I. Stengers for their interest and help. Also, from the C.R.U in Paris, we would like to thank R. Buissière for his suggestions concerning possible simulations, as well as for considerable data. We wish also to thank F. Hassler and D. Kahn at the Transportation Systems Center, Cambridge, for the most constructive and helpful oral briefings and for much data and useful references. Finally, we would like to thank P. Kinet for drawing the many figures that appear in our text as well as for technical assistance in the production of this report.

This Final Report was organized into its present form from the authors' draft final report by D. Kahn, the Technical Monitor of this research effort. However, no substantive technical changes have been made, and the only stylistic changes made have been for purposes of clarification. The style remains essentially that of the authors.

The Technical Monitor would like to acknowledge the invaluable help he received from L. Levine for his substantial copy editing and production editing which has led to this final version of the Final Report.

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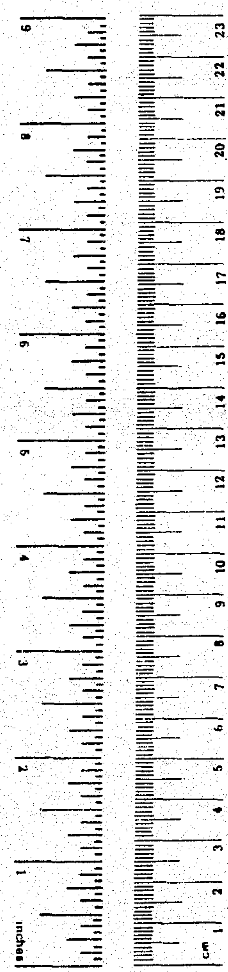
METRIC CONVERSION FACTORS

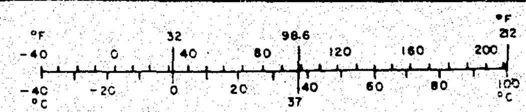
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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1. INTRODUCTION

In the preceding volume, because of the choice made in the scale of description, the urban center has been viewed as a simple point in space without consideration of its internal structure. We will now adopt this more microscopic point of view and, in a study that is entirely complementary to the foregoing one, concentrate precisely on the internal structure of the urban center.

First, let us outline a number of observed characteristics of towns and cities and discuss briefly the various factors influencing the locational decisions made by residents and enterprises within the urban space.

There are basically three different types of urban centers which are related to three different criteria of location characteristic of different economic activities.[1] They are:

1) specialized centers where activities are gathered which depend directly on some characteristic attribute of the particular locality (This may be the case for extractive industries and dependent heavy industry, for centers of tourism or pilgrimage, as well as administrative or political centers.);[2]

2) centers situated at key points of the communications network where transport dependent, often intermediate, industries are clustered;

3) market centers whose principle function is the distribution of a series of goods and services to their own population and that of their hinterland.

Whereas the first two types result from regional and national potentialities, the third type is a function of the demand coming from the population living within a limited radius. Thus, even though a large town is always a market center, it is only sometimes a specialized center or a center of communications in addition.

Let us consider the development and growth of an urban center in a liberal economic system.[3] By "development" we imply the creation or adoption of innovations which can entrain structural changes within the center. Suppose that we introduce a new factory into our town, a factory that manufactures goods which are very predominantly sold to clients living in other towns and cities. That is to say, our new factory is in the "export" sector of the town's economic activity. This event will trigger off a chain reaction. The creation of these new jobs will result in a certain number of families moving to the town from elsewhere, and (as explained in the previous volume) the "urban multiplier" will operate. The increase in population will swell the local demand for goods and services, resulting in the growth of the tertiary sector of the town's economy. Turnovers will increase and the employment offered by shops and local services will grow, resulting in a further influx of population which in its turn causes the whole cycle to repeat itself until, finally, the total increase in population and economic activity is much greater than that caused directly by implantation of the factory. Furthermore, new tertiary activities may appear as new market thresholds are attained, and increasingly "rare" activities can be sustained within the center as it grows. This mechanism has already been discussed in the preceding volume, but now we will concentrate on its effects on the internal structure of the town or city.

At the same time, another chain reaction is triggered off by the implantation of the factory. This is due to the appearance, with the factory, of a demand for "linked" industries — either those which could supply the factory or those which would buy its products. Thus, because of the external economies that would be possible, a series of new industries may be installed in the town following the initial implantation. In this way the industrial development of the urban center continues, with, of course, the accompanying rise in demand for local goods and services and the operation of the "urban multiplier."

This increase in employment contributes to the attraction of new residents to the town, especially as its activities diversify. The two linked chain reactions, the "urban multiplier" and the multiplication of economic activities themselves, consitutute the development cycle of the urban center, as local population and economic activity spiral upwards.

In addition to this, the intensification of contacts between individuals, resulting from a higher population density, as well as the increasing standard of living resulting from the diversification of the economy, will stimulate the appearance of still more new activities and behaviors. Finally, the management of this increasingly complex collectivity requires the creation of administrative employment which becomes more and more costly as the diseconomies of scale of the city develop.

Let us now turn to the observed spatial distribution of the various components making up the urban center (residences, jobs, etc.); that is, its internal structure. First, our definition of an urban center is taken as the quasi-continuous region of housing characterized by the presence of industrial and tertiary activity.

Each agent of an economic or social function is influenced by his relations with agents of the same category, as well as those of other urban functions. This results in spatial interactions. For example, the interest that company headquarters have in being close to one another and to the banking and financial centers illustrates the action of a cooperative mechanism; whereas the pollution engendered by some industries which causes residents to flee is an example of a repulsive mechanism. It is a superposition of a large number of these "microscopic" responses that results in the "macroscopic" structure of the town or city.

Several different types of urban structures have been observed, and the three classic models of Burgess (concentric zones), of Hoyt (sectorial zones), and of Harris and Ullman[4] (polynuclear structure) have been the object of considerable experimental study.

From these studies it has been possible to discern several basic facts concerning the spatial distribution of the various constituents of the urban center.

1) The activities of production are localized in a rather unsystematic fashion within the urban tissue according to their particular needs. For example, the requirement of good transport facilities will localize an activity along an axis of communication, or the need of a particular externality may localize an activity in the central business district (CBD), a need for an extensive site may lead to a peripheral location, etc.[5]

2) The centers of commerce and services are organized hierarchically, forming a fairly regular spatial lattice of which the most important point is the CBD.[6]

3) The resident population tends to distribute itself according to three essential factors: their socio-economic status (sectorial zones); their age (concentric zones), single people and childless couples tending to live centrally, households with children preferring the periphery in order to profit from the greater availability of space; their ethnic group - minority groups tend to reside in scattered aggregations.[7]

The global structure of the town or city will be the result of all these different basic reactions, a combination which will be conditioned by the proportion of specialized or intermediate activities existing in the economy of the center, which are, as we have seen, a function of external influences.

Figures 1-1A, B, and C represent respectively the concentric, the sectorial, and the polynuclear models. Figure 1-1D is a schematic representation of Vienna and illustrates the case of a combination of the basic models.[10] The total population is distributed according to an exponential envelope decreasing[1,8,9] from the center to the periphery. In certain cases a central crater develops in this distribution,[6] and Figure 1-2 shows the evolution

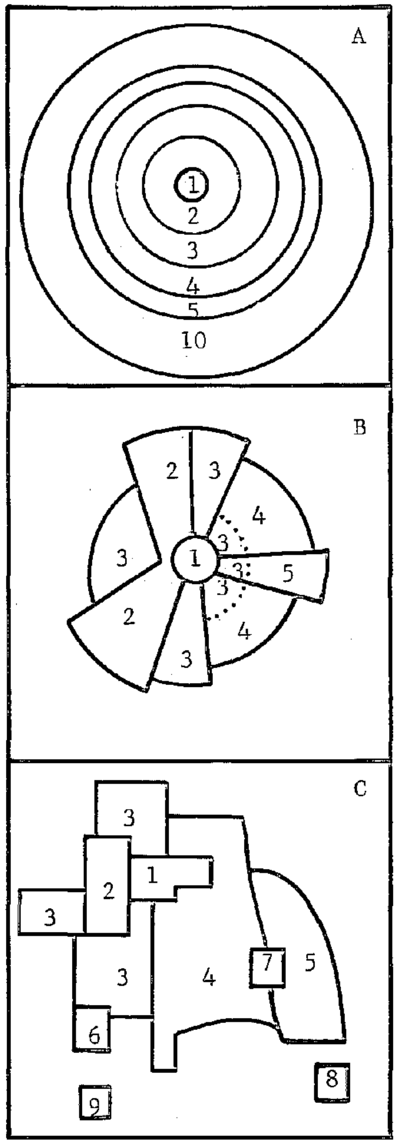


FIGURE 1-1A,B,C. URBAN STRUCTURES (FROM HARRIS AND ULLMAN)[4]

1. Central Business District
2. Wholesale Light Manufacturing
3. Low-class Residential
4. Medium-class Residential
5. High-class Residential
6. Heavy Manufacturing
7. Outlying Business District
8. Residential Suburb
9. Industrial Suburb
10. Commuters' Zone

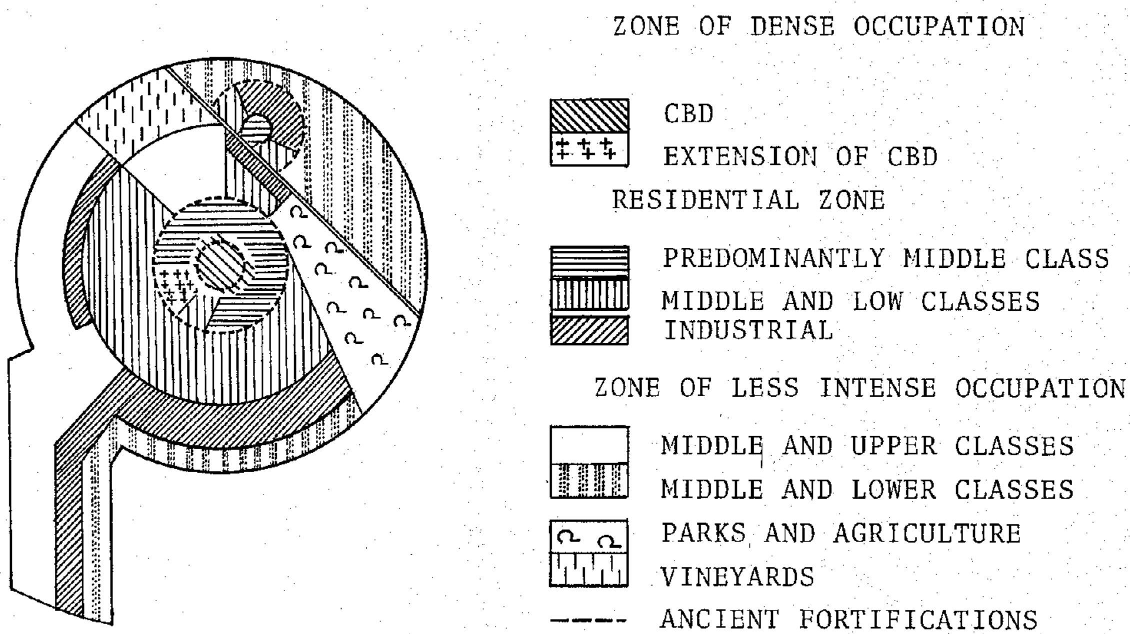


FIGURE 1-1D. SOCIO-ECONOMIC STRUCTURE OF A EUROPEAN CONTINENTAL CITY: EXAMPLE OF VIENNA (FROM LICHTENBERGER)[10]

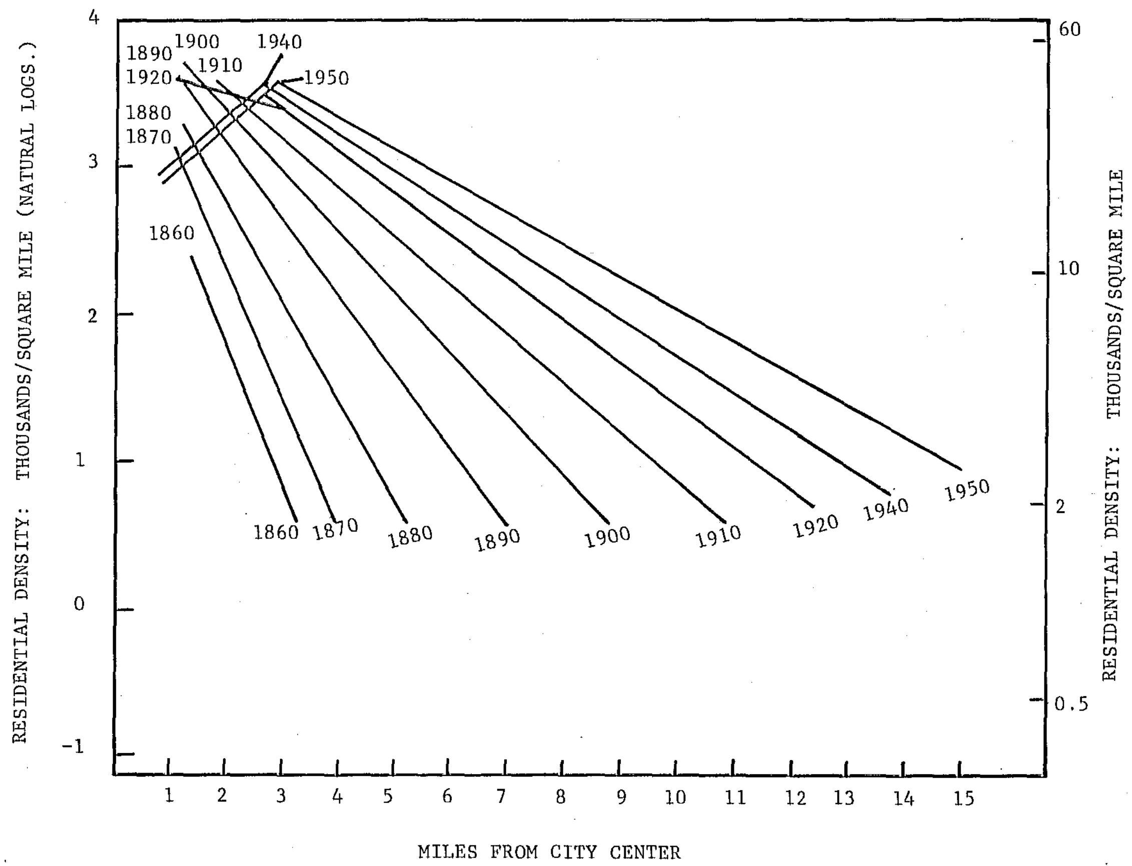


FIGURE 1-2. CHANGE OF POPULATION DENSITIES IN CHICAGO REGION (FROM BERRY)[6]

of the population distribution in time. Initially, the distribution is exponential and growing, but later the central region hollows out, giving rise to the crater.

2. THE GLOBAL EVOLUTION OF THE URBAN SYSTEM

In this section we shall develop equations describing the global evolution in time of the different elements of the town or city, without worrying, for the moment, about their spatial distribution within the urban space.

We consider the active individuals in their two fundamental economic roles: as producers, and as consumers. We do not yet take into account, explicitly, the presence of an inactive fraction of the population, although this is nevertheless included implicitly in the values which are given to the parameters characterizing the active population. Thus, the amount of space required for an active member of the population, for example, will be greater if he has children. The total and active populations are related by the fraction of activity.

It has been supposed, furthermore, that everyone working in the town or city considered resides there. This suggests that the active population must be equal to the number of jobs. Inversely, everyone who lives within the urban center works there, implying that there is no unemployment or that unemployed individuals emigrate immediately. If X is the number of jobs present in the center, and P the active population, then we have at all moments:

$$X = P. \qquad (2\text{-}1)$$

As we have seen from the preceding volume on regional growth, a demand can appear for a good or service exported from a particular locality and become the germ of an urban center. The initial export activity will develop gradually by word of mouth or advertisement, resulting in a growing clientele. This results in an increase in production and in the need to employ more workers. These new employees and their families consume various goods and services and occupy a residence within the urban area. They are in this way the basis of the appearance of a weak internal demand for goods and services which may not be sufficient in itself to exceed the market threshold of these functions. Businesses, which nevertheless will be installed in the town, will rely on an external demand coming from the town's hinterland. These are the "mixed" activities of our model, being themselves "job creating" and multipliers of internal demand.

As the process continues, the growth in employment in the town will increase the internal demand to a point sufficient in itself to render profitable the activities which before relied in part on an external demand. It is also possible that the passage from "mixed" to "internal" results from the increased demand for a higher standard of living. Therefore, "mixed" activities will tend to become "internal" activities.

We have been considering three types of economic activity in our town which differ in degree of dependence on the outside world. Their classification can be made by considering the ratio of the town's population to the market threshold of economic activity:

1) "Export" sector depends essentially on the demand coming from outside the urban center considered. The ratio of population to market threshold tends to zero.

2) "Mixed" sector relies on an economic demand coming both from within and without the town. The ratio, for example, is between .05 and 1. This corresponds to exporting industries having local clients or rare goods and services which are rendered profitable by the joint demand of the town and its hinterland.

3) "Domestic" or "internal" sector depends exclusively on an economic demand coming from within the urban center.

In Figure 2-1 we have illustrated these various relationships.

Mathematical Formulation of the Model

We first derive the equation governing the evolution of employment. For this purpose if D is the global demand for a given economic activity, D^s the demand that has been and is satisfied, and D^{ns} the demand which is still unsatisfied, then:

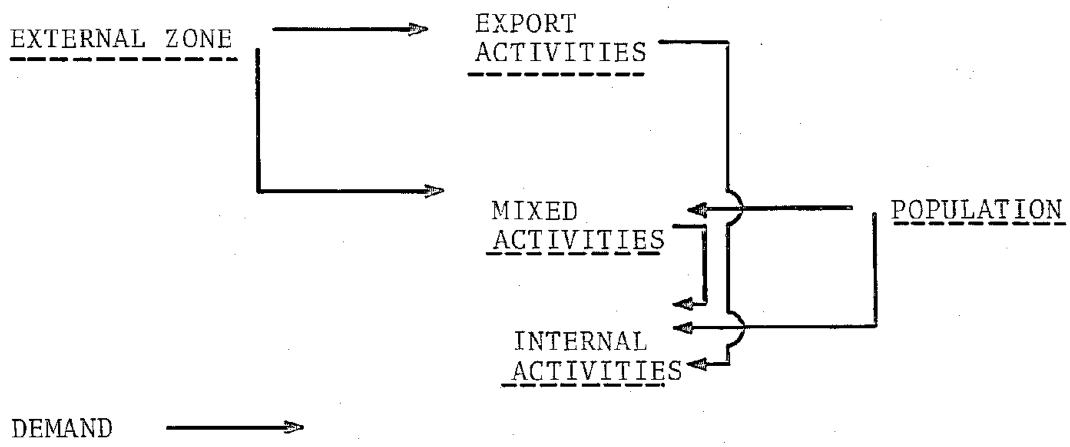


FIGURE 2-1. DEMAND-ECONOMIC ACTIVITY RELATIONSHIPS

$$D = D^{ns} + D^{s}. \tag{2-2}$$

At time $t = t_o$, when the activity is introduced, the total demand is unsatisfied. After t_o, however, the economic activity begins to satisfy this demand, and we may write a kinetic equation describing this conversion of D^{ns} into D^{s}. We suppose that the rate of transformation is proportional to the unsatisfied demand remaining:

$$\frac{dD^s}{dt} = \theta\ D^{ns} \tag{2-3}$$

where θ is the average rate of transformation of D^{ns} into D^{s}. Combining this equation with (2-2), we find:

$$\frac{dD^s}{dt} = \theta\ (D - D^{s}). \tag{2-4}$$

Let us now suppose that the satisfied demand D^{s} is proportional to the number of jobs required to sustain it. We may then write:

$$D^S = X \tag{2-5}$$

where D^S corresponds to the quantity of goods or services effectively available to buyers, while X represents the number of jobs necessary for their production.

By replacing D^S by X in Equation (2-4), we obtain an equation governing the evolution of employment for this particular sector:

$$\frac{dX}{dt} = \theta (D - X) \tag{2-6}$$

where θ is the average rate of job creation and is characteristic of the dynamism of each economic activity. It combines several mechanisms which may operate either simultaneously or successively. The two most important are:

1) for the "supply," the perception of the demand by the entrepreneur and his investment strategy;

2) for the "demand," the propagation of an innovation by the modification of the pattern of individual consumption.

Let us now formulate the demand coming from the various sectors of our urban economy which will, by their combined effect, govern the evolution of the working population as a whole.

a. The export sector.

In this case the demand D corresponds to the fraction of regional or national demand for a particular activity i which the urban center considered attracts. The evolution of this demand, which depends of course on the regional or national context, is treated in the inter-urban model discussed in the preceding volume. Here we shall restrict ourselves to the internal structure of an urban center and simply make different hypotheses concerning this external demand:

1) The external demand is constant.

2) It increases steadily over time.

3) Its growth will be dependent on the internal structure which has evolved in the urban center.

If we consider the case of a town or city having n exporting activities, then if D^1, D^2, ...D^n are the respective demands for these coming from the exterior, the equation for the evolution of the employment offered by each activity will be:

$$\frac{dX_i}{dt} = \theta^i (D^i - X^i) \quad i = 1, \ldots m \tag{2-7}$$

where θ^i is characteristic of activity i.

b. The domestic sector.

Let us consider first the development of a service to the population. For a particular service or several similar services it may be assumed that the demand will be a linear function of the population of the center. Let D^i be the demand for the particular service i and S^i the corresponding number of jobs. We may write for the evolution of S^i:

$$\frac{dS^i}{dt} = \theta^i (D^i - S^i) \quad i = 1, \ldots p \tag{2-8}$$

where $D^i = \sum_{j=1}^{n} a^{ij} p^j$. For n types of population, a^{ij} characterizes the importance of consumption of service i by the population of type j.

In the slightly different case of an industrial or perhaps tertiary activity, where the origin of the economic demand is an enterprise located within the urban center, we may derive a similar type of equation. If V^i is the employment resulting from this activity, then we have:

$$\frac{dV^i}{dt} = \theta^i (D^i - V^i) \quad i=1, \quad \ldots,q \tag{2-9}$$

where $D^i = \sum_{j=1}^{m} b^{ij} X^j$ and b^{ij} characterizes the rate of consumption of the product of the V^i, by the X^j (m types of activities).

c. The evolution of the populations.

Let us recall that we have made the supposition that all those who work in the town, reside there, and inversely.

Hence we have,

$$\sum_{i=1}^{m} X_i + \sum_{k=1}^{p} S_k + \sum_{\ell=1}^{q} V^{\ell} = P \tag{2-10}$$

where P is the total active population.

We must also take into account that population associated with a particular activity, P^i say, is not necessarily homogeneous because:

1) There is a salary scale within each activity for skilled, unskilled, office and executive staff, etc.

2) The demographic characteristics of the region will determine a second type of differentiation based on age, family structure, nationality, etc.

Let C^{ki} be the fraction of employment of activity i which is filled by an individual of socio-demographic type k. Clearly, from the above, C^{ki} is characteristic of the composition of the population in the region as well as of the type of enterprise. If P^{ki} is the population of socio-demographic type k employed in activity i, then:

$$p^{ki} = C^{ki}\, p^i \qquad k = 1, \ldots, n \tag{2-11}$$

where n is the number of distinct socio-demographic classes.

Clearly, $\sum_{k=1}^{n} C^{ki}\, p^i = p^i$. Thus, the equation governing the evolution of p^{ki} will have the form,

$$\frac{dP^{ki}}{dt} = \theta^i \left(C^{ki} D^i - p^{ki}\right) \qquad k = 1, 2\ldots\ldots n. \tag{2-12}$$

3. THE LOCATION OF RESIDENTS AND ECONOMIC ACTIVITIES WITHIN AN URBAN CENTER

We shall now consider the location of jobs and residences at each point in space, and our initial problem is to decide where, and for what reason, the different economic and social agents choose a particular locality within the urban space.

First, let us determine the form of the equations describing the evolution of an activity belonging to the export sector of the urban center's economy for each point of the lattice representing our urban space. Let X be the total employment for this activity within the urban zone and X_i the number of these X jobs that exist at the point i (i=1,...r). The urban space is represented by n points and clearly the X_i are subject to the constraint,

$$\sum_{i=1}^{r} X_i = X \qquad (3\text{-}1)$$

and

$$\sum_{i=1}^{r} \frac{dX_i}{dt} = \frac{dX}{dt}. \qquad (3\text{-}2)$$

We will further suppose that the equation governing the intra-urban localization of X_i at i is similar in form to that which we have used for the global equation for X. Hence, for X_i we write,

$$\frac{dX_i}{dt} = \theta\ (D_i - X_i) \quad i = 1, \ldots, r \qquad (3\text{-}3)$$

where D_i is that part of the total demand for X coming from outside the urban center which falls on the point i. Clearly, the relation between D and D_i is,

$$D = \sum_{i=1}^{r} D_i \qquad (3\text{-}4)$$

and, supposing a form $D_i = C_i D$, then we must have,

$$\sum_{i=1}^{r} C_i = 1 \qquad (3\text{-}5)$$

where C_i is the fraction of the global demand falling on i. We may then write,

$$C_i = F_i/A \tag{3-6}$$

where F_i is an "absolute" fraction attracted and A is a normalization factor. Using (3-5) and (3-6) we have,

$$\sum_{k=1}^{r} F_k/A = 1$$

and therefore

$$A = \sum_{k=1}^{r} F_k. \tag{3-7}$$

We see that C_i is simply the relative attractiveness of the point i in competition with all the other points of the urban lattice.

$$C_i = F_i/\sum_{k=1}^{r} F_k \tag{3-8}$$

$$\frac{dX_i}{dt} = \theta\left(\frac{DF_i}{\sum_{k=1}^{r} F_k} - X_i\right) \qquad i = 1,\ldots.r. \tag{3-9}$$

The value of F_i will be specific to the type of activity considered and to the "state of affairs" existing instantaneously at i. By "state of affairs" we mean the values of its populations and of its industrial and commercial activities. We shall return to discuss the explicit dependence of F_i on the "state" of i.

Let us now consider the equation governing the location of an activity in the service sector of the urban economy. We shall call S the sum of the services for the whole urban area and S_i those located at the point i. As we have already discussed in Section 2, the equation for the evolution of S is

$$\frac{dS}{dt} = \theta\ (D - S) \tag{3-10}$$

where D is some function of the total population. We shall suppose a similar form for S_i:

$$\frac{dS_i}{dt} = \theta \, (D_i - S_i). \tag{3-11}$$

Then clearly, $\sum_{i=1}^{r} D_i = D.$ (3-12)

The services situated at i "feed on" the population which is situated not only at i itself, but also at neighboring points, up to a distance which depends on the precise nature of the service. We can decompose the demand at i, D_i, into its components:

$$D_i = \sum_{k=1}^{r} D_{ik} \tag{3-13}$$

where D_{ik} is the demand originating from the point k which is attracted to i. The total demand coming from the point k radiating out to the various services situated in the surrounding area is considered to be proportional to the population resident at k, P_k. Thus, the demand of the P_k is some value $b_{ik}\ P_k$ where b_{ik} reflects the standard of living, or disposable income, of the P_k, as well as of the nature of the services and the pattern of consumption. The demand D_{ik} is therefore given by,

$$D_{ik} = b_{ik}\ P_k \tag{3-14}$$

where b_{ik} is a function of the state of the points i and k and of the relations existing between them. If we sum the D_{ik} over i, then clearly,

$$\sum_{i=1}^{r} D_{ik} = \sum_{i=1}^{r} b_{ik}\ P_k = P_k \tag{3-15}$$

which implies that

$$\sum_{i=1}^{r} b_{ik} = 1. \tag{3-16}$$

If we suppose the form,

$$b_{ik} = F_{ik}/A \tag{3-17}$$

where F_{ik} is the "absolute" attraction between i and k and A is a factor of normalization, then we find on combining (3-16) and (3-17), that

$$A = \sum_{i=1}^{r} F_{ik} \tag{3-18}$$

and hence,

$$D_{ik} = \frac{F_{ik}}{\sum_{i=1}^{r} F_{ik}} P_k. \tag{3-19}$$

Substituting these expressions into (3-13) and (3-11), we obtain:

$$\frac{dS_i}{dt} = \theta \left(\sum_{k=1}^{r} \frac{F_{ik}}{\sum_{i'=1}^{r} F_{i'k}} P_k - S_i \right). \tag{3-20}$$

This equation describes the evolution of the services to the residential population, and a similar analysis can be performed for services whose clients are either exporting industries or other services. For example, if V_i is the employment in the services for exporting industries which is located at i, then we will find the equation:

$$\frac{dV_i}{dt} = \theta \left(\sum_{K=1}^{r} \frac{F^{+}_{ik} X_k}{\sum_{i'=1}^{r} F^{+}_{i'k}} - V_i \right) \tag{3-21}$$

where we see, on comparing this with (3-20), that the residential population P_k has been replaced by the importance of the exporting industries at k, X_k, where F^{+}_{ik} is the corresponding function of attraction between V_i and X_k.

A final situation, which cannot be disregarded, is that of an activity for which the demand is both external and internal to the town or city. If Z_i is the employment offered by such activity, situated at the point i, then we may write, following the above discussion:

$$\frac{dZ_i}{dt} = \theta \left(\sum_{k=1}^{r} \frac{F_{ik}}{\sum_{i'=1}^{r} F_{i'k}} P_k + \frac{DF_i}{\sum_{i'=1}^{r} F_{i'}} - Z_i \right) \tag{3-22}$$

which is a combination of (3-20) and (3-9).

Finally, we must examine the equation describing the localization of the residents of our urban center. We suppose that P_{ij} is the population resident at i with employment at j. If, for a moment, we consider that there is only a single economic activity, situated at j, and that X is the corresponding number of jobs available, then the equation governing the total employment in the urban center will be the same as that for the point j, and the total employment demand is equal to that at j.

$$D = D_j \tag{3-23}$$

$$\frac{dX_j}{dt} = \theta (D_j - X_j) \tag{3-24}$$

The equation for the evolution of the population of the whole town or city can be written (see Equation (2-7)):

$$\frac{dP}{dt} = \theta (D - P). \tag{3-25}$$

If P_j is the population which works at the point j, then in this simple case, $P = P_j$, and

$$\frac{dP}{dt} = \frac{dP_j}{dt} = \theta (D - P) = \theta (D_j - P_j). \tag{3-26}$$

Now we study the repartition of this population among the points i and write an equation for P_{ij}, the population resident at i having employment at j.

$$\frac{dP_{ij}}{dt} = \theta (D_{ij} - P_{ij}) \tag{3-27}$$

where D_{ij} is the demand which permits the "creation" of a job at j for the resident at i. Clearly, the sum of D_{ij} over all points i must give D_j, since we assume that people working within the town or city also reside in it, which corresponds to the supposition of a coefficient of employment equal to unity for this particular center.

We now write D_{ij} as:

$$D_{ij} = e_{ij} D_j \tag{3-28}$$

$$\sum_{i=1}^{r} D_{ij} = D_j, \tag{3-29}$$

where e_{ij} is a function of the state of the points i and j, and the relations existing between i and j. Substituting into (3-29), we find:

$$\sum_{i=1}^{r} e_{ij} = 1. \tag{3-30}$$

Proceeding as before in the cases of the services and exporting industries, we have:

$$e_{ij} = \frac{F_{ij}^{-}}{A^{-}}, \tag{3-31}$$

where the normalization constant A^{-} is given by:

$$A^{-} = \sum_{i=1}^{r} F_{ij}^{-}. \tag{3-32}$$

We see that the site i is in competition with the other possible sites in attracting residents, which is another way of saying that individuals having a job at j must choose where they are going to reside. The point i is only one of the possible choices, and it will be chosen with a frequency which will depend on its relative merits and demerits.

The equation for the evolution of P_{ij} is therefore:

$$\frac{dP_{ij}}{dt} = \theta\left(\frac{F^{-}_{ij}}{\sum_{i=1}^{r} F^{-}_{ij}} D_j - P_{ij}\right). \tag{3-33}$$

If we consider next the more general problem where there are various centers of employment scattered through our town or city, then the evolution of the population resident at a given point i, P_i, is obtained simply by summing Equation (3-33) over the various points j which are seats of employment, be it in the export, domestic, or mixed sectors of the urban economy.

$$\frac{dP_i}{dt} = \theta\left(\sum_{j=1}^{r} \frac{F^{-}_{ij}}{\sum_{i=1}^{r} F_{ij}} D_j - P_i\right) \tag{3-34}$$

In the Equations (3-9), (3-20), and (3-34) we find terms which have the form of models of intervening opportunities, for example, in Equation (3-34) the term,

$$\frac{F^{-}_{ij}}{A^{-}_{j}} D_j. \tag{3-35}$$

This term reflects the fact that each site is in competition with the others in attracting residents. Quite generally, in the kinetic equations developed here, the rate of growth of a variable turns out to be proportional to the difference between a term of "intervening opportunities" and the number of such agents already present.[11,12]

4. THE LOCATION FUNCTION

4.1 SOCIAL CHARACTER OF LOCATION

The first point that must be made is that in this preliminary study we have considered a spatial lattice for which all points are initially equivalent, not only in terms of their physical environment, but also in their accessibility. The particular state of a given point or locality on the lattice will be of differing interest to each type of economic agent. This must be taken into account by their location functions. Quite generally, it may be supposed that each agent will try to make the most of the possibilities of communication in the socio-cultural and economic exchanges which are possible within the urban complex. At the same time he will try to avoid or minimize its unsatisfactory aspects. The location function therefore represents the effect of the simultaneous "attractions" and "repulsions" exercised by the different sites on an agent. It will be a function of various factors such as the number of and distance from the other agents, the density of occupation of the terrain, etc. The location function will therefore be written in terms of cooperative or competitive effects representing phenomena of social, ethnic, cultural, religious, and (of course) economic origin.

Consider for a moment the question of ethnic segregation where individuals of a particular ethnic origin are "attracted" to residences in neighborhoods already having a concentration of individuals of similar origin, while at the same time being "repulsed" by individuals of different origins. In the socio-cultural context, for example, the presence of families of high social status favors the further installation of similar households. This may be to profit from contacts made within such a group, to be near the particular services that have developed around it, or to be near people with similar tastes and incomes.

Quite generally, the settling of a particular locality will bring about the development of an infrastructure (sewers, water, gas, electricity, schools, hospitals, shops, etc.) which will favor the further attraction of new residents. This corresponds to a "cooperative" effect of population density on its own growth up to some level of crowding after which "repulsive" or "competitive" effects dominate.

Similar ideas of "cooperativity" and "competitivity" apply equally to industry. Consider, for example, an enterprise concerned with the transformation of some raw materials into a finished product. Its location function will favor sites near to industries with which exchanges must be made. This also depends, of course, on the relative costs of transporting both raw materials and finished products. In addition the unattractiveness of crowding at favorable locations must be considered. These various pressures result from the need to reduce production costs, maximize sales, etc....

As an example, let us consider X_j, V_j and Z_j to be the employment in the three basic economic sectors situated at the point j; and P_j is the population at j.

The location function of an economic activity at the point i will therefore have the form:

$$F_i = F_i\,(X_j,\ V_j,\ Z_j,\ P_j,\ d_{ij})\quad j = 1,2... \tag{4-1}$$

where d_{ij} is the distance between the points i and j. Similarly, the location function for the population at i having employment at k is:

$$F_{ik} = F_{ik}\,(X_j,\ V_j,\ Z_j,\ P_j,\ d_{ij},\ d_{ik})\quad j = 1,2.... \tag{4-2}$$

In order to illustrate this, let us consider a zone where heavy industry is concentrated together with all its accompanying unpleasantness. The persons employed there will try to find a place of residence which is reasonably close to the zone but which avoids (as much as possible) the various types of problems associated with living close to the industrial complex. The location function will therefore reflect these antagonistic tendencies. A function of the "unpleasantness" of the point r may be written:

$$U(r) = U(I(r'), r', r) \qquad (4\text{-}3)$$

where $I(r')$ is the volume of heavy industry at the point r' and U depends on $(r'-r)$, corresponding to the degree which the "unpleasantness" is apparent at distance $(r'-r)$.

We may write the function of attraction towards r' due to the effect of the cost of regular transportation between r and r'. Suppose:

$$T(r) = k\ (1 + c(r'-r)\ ). \qquad (4\text{-}4)$$

In the simulations which we shall discuss below, the continuous system will be replaced by a discrete lattice, having m sites where d_{ij} is the distance between the sites i and j. Relation (4-3) and (4-4) become:

$$U(i) = U(I(i'), d_{i'i}) \qquad (4\text{-}5)$$

$$T(i) = k\ (1 + cd_{i'i}). \qquad (4\text{-}6)$$

Several different combinations of these two factors are possible:

$$(a) \quad 1/U(i) + 1/T(i) \qquad (4\text{-}7)$$

$$(b) \quad 1/(U(i) + T(i)) \qquad (4\text{-}8)$$

$$(c) \quad 1/(U(i) \times T(i)) \qquad (4\text{-}9)$$

where (a) and (b) correspond to the addition of the inconveniences of the point i, while (c) corresponds to a mutual amplification. If, therefore, we consider only these two factors, the location function will have the form (a), (b), or (c) for example:

$$F_i = 1/U(i) + 1/T(i). \qquad (4\text{-}10)$$

4.2 ECONOMIC CHARACTER OF LOCATION

In this section we briefly outline how the approach which is more familiar in economics can be integrated into our model. We shall limit ourselves, however, simply to a discussion of residential location as an example.

The theories most often cited in this connection are[18] those of Alonso,[13] Becker,[14] Beckmann,[15] Kain,[16] Muth,[17] and Wingo.[20] These are, on the whole, static and deal with the individual behavior of households deciding on their location.

If we use the ideas of Beckmann, which describe a town or city with centralized functions where the space is homogeneous, then two elements suffice to characterize a given plot of land: its surface area and distance from the center.

Let us suppose that the satisfaction of an individual is measured by an expression U, called the utility function. In our case this will be a function of r, the distance from the center, and s, its surface area. These two factors r and s can be substituted one for another and the utility derived for any particular choice is clearly a characteristic of the particular population considered. For example, during a lifetime, the relative importance attached by an individual to the area of land at his disposition varies. One may suppose that the budget allotted to his residence is some fixed proportion of an individual's revenue. The cost of a given location must take into account the cost of transportation as well as the cost of a residential site,

$$Y = C_i A_i + T_i$$

where C_i is the cost per unit area of land at i, A_i the area and, T_i the cost of transport from i to the center. It is then supposed that the individual tries to maximize his level of satisfaction, given by U, taking into account the constraint exercised by Y.

Returning to the equations of residential location introduced in Section 2 :

$$\frac{d P_i}{dt} = \left(\frac{D F_i}{\sum_{1'=1}^{n} F_{i'}} - P_i \right) \quad i'=1, \ldots, n \tag{4-11}$$

where F_i is a function of U.

In the case of a homogenous population, consider two situations: (1) perfect and total information and (2) incomplete or misleading information. For the first situation each individual will choose a site where (taking the constraint Y into account) U is a maximum at that moment such that F_i will have the form:

$$F_i = 1 \quad i = i_o \qquad (4\text{-}12)$$

$$F_i = o \quad i \neq i_o$$

where i_o is the point at which U is maximum.

In the second situation, one may suppose that the probability of location at i is greater if the function of utility at i is large. A linear relation between U and F is the simplest representation of such a situation.

We therefore can write:

$$\frac{d\,P_i}{dt} = (D \frac{U_i}{\sum_{i=1}^{n} U_i} - P_i) \quad i = 1, \ldots, n \qquad (4\text{-}13)$$

where the surface area of a given plot has been eliminated by making use of the equation for Y. The complete solution of the system (4-13) requires, naturally, knowledge of the explicit form of the utility function U as well as a satisfactory model for calculating the price of land as a function of the other variables of the problem.

5. ANALYSIS AND SIMULATION

As we have seen in the preceding chapters, the simulation of a "real" urban center requires the simultaneous integration of a great number of differential equations which correspond to the behavior of the various types of economic agents and population. Here, we shall discuss the simulations which we have carried out involving, in a first model, two population variables distributed around an economic center.

As we have mentioned in the preceding volume, in any such system of coupled non-linear differential equations, a vital role in the evolution of the system is played by the fluctuations of the variables around their average values. In our simulations, therefore, we perform the integrations while subjecting the variables to an additional random "noise." These fluctuations can, as we have explained, sometimes cause a dramatic change in the structure of the system, as one type of solution becomes unstable and is replaced by another which may be qualitatively different.

5.1 MODEL OF RESIDENTIAL LOCATION

5.1.1 Model's Description

Here we study the manner in which a population distributes itself around a center where, we suppose, all the economic activities are situated. This basic model, proposed by Beckmann, is used widely by economists.[15,18] Our simulation, which is, of course, dynamic, describes the evolution of the spatial distribution of two populations of different socio-economic status around the point where all economic activities are concentrated. In this case, the total urban employment, X, is simply equal to X_j - the total employment at the center j.

The equation for the evolution of X_j is:

$$\frac{dX_j}{dt} = \theta\ (D - X_j). \tag{5-1}$$

D is kept constant, in which case multiplication effects are not taken into account. P^1 and P^2 are the total populations of socio-economic status 1 and 2 within the system. The equations governing their evolution are:

$$\frac{dP^k}{dt} = \theta\ (c^k D - P^k) \qquad k = 1,2 \tag{5-2}$$

where c^k is the fraction of employment for the population k. This set of equations is also subject to the relation:

$$X = P^1 + P^2. \tag{5-3}$$

Now let us consider the spatial distribution of these populations. The equations describing the evolution of the population of socio-economic type k, at the point i, P_i^k, are:

$$\frac{dP_i^k}{dt} = \theta \left(\frac{c^k\ F_i^k}{\sum_{i'=1}^{r} F_{i'}^k} - P_i^k \right) \qquad k = 1,2 \tag{5-4}$$

where F_i^k is a location function which takes into account the relative merits of each spot for the residence of each population type. It has been given the explicit form:

$$F_i^k = \frac{G_i^k \times (U_i^k/(C + U_i^k\ )}{(B1^k\ + AZ^k\ (\ P_i^k + P_i^{k'})^2 + CL^k\ d_{ij}^2)} \tag{5-5}$$

where k'=2 when k = 1 and vice versa. G_i^k is the term which describes how the state of affairs at other points influences the population k at i.

$$G_i^k = \sum_{h=h1}^{h2} (A12\ P_h^k + A13\ P_h^{k'}) \tag{5-6}$$

where h is the index of the neighboring sites. In the simulations presented here, which have been performed using a square lattice of points, we have only taken into account the influence of the eight points immediately surrounding i (see Figure 5-1). The term $U_i^k/(C+ U_i^k)$ allows for the cooperative effects occurring at the point i itself.* U_i^k has the form:

$$U_i^K = \exp(AV1(P_i^k - P_i^{k'})(P_i^k + P_i^{k'})). \tag{5-7}$$

Let us now explain this apparently complex choice for the form of the localization function.

The denominator contains the terms which account for the repulsion, or lack of attraction, of the point i for an individual. We have two types of mechanisms for this. First, we have the effect of crowding, which is unattractive once it attains a certain level. The constant AZ^k gives a measure of sensitivity of the population k to this phenomenon. We have chosen a parabolic form for the dependence on total population density in order to account for the fact that this relation is not linear. Low densities cause scarcely any inconvenience, but high densities cause a great deal. The second effect is that of the distance from the center of the point i. Here it is the parameter CL^k that takes this into account. The greater the value assigned to CL^k, the greater the sensitivity of the population k to the distance separating it from the center.

Let us now examine the form of the numerator. The term U_i^k, internal to each point, is supposed to cover two effects. First, there is the "attraction" felt by an individual for a group of the same type. They have roughly the same income and outlook. Inversely, there is a certain aversion to the choice of a residential location where individuals of a different socio-economic group are concentrated. The parameter AV1 measures the intensity of this effect of repulsion/attraction. If $P_i^{k'}$ is large with

*C in the simulations presented here is always 0.5.

respect to P_i^k, then U_i^k will be small. This effect will be even more pronounced if the total population at i is large. Inversely U_i^k will be large if P_i^k is large compared to $P_i^{k'}$. This type of function is in accord with observations concerning segregation which show that this latter is all the more marked as the total population density is great. The choice of the form:

$$U/(C + U)$$

is explained by the fact that it is necessary to impose a limit to these effects.

The form of the term G_i^k can be justified on similar grounds. There is, first and foremost, the positive effect of the presence of the population at neighboring points on the growth at i. Secondly, we have the saturation of this effect. In this particular simulation involving two variables this "cooperativity" between neighboring points is really the manifestation of the propagation of the urban infrastructure. We have supposed that this infrastructure is proportional to the population density, but becomes a constant above some threshold. The combination of the various factors of "cooperativity" both inter and intranodal imply simply that it is necessary for the urban infrastructure to have spread out to a particular locality for growth to occur there.

This choice of residential location function can be used to describe three different aspects of the locational behavior of individuals:

1) ethnic segregation in which the function U_i^k plays a dominant role,

2) residential location as a function of age (Couples with children seek low residential densities corresponding to a large value of AZ^k. Childless couples and single people prefer to be near the urban center where CL is large and AZ is small.),

3) according to income bracket, where, for example, car ownership endows a greater mobility to individuals, which may be reflected in a large value of AZ and a small CL.

5.1.2 Simulation

We have simulated the growth of an urban zone on a surface of area (L^2), divided into m^2 elements taken to be homogeneous and of area ($(L/m)^2$):

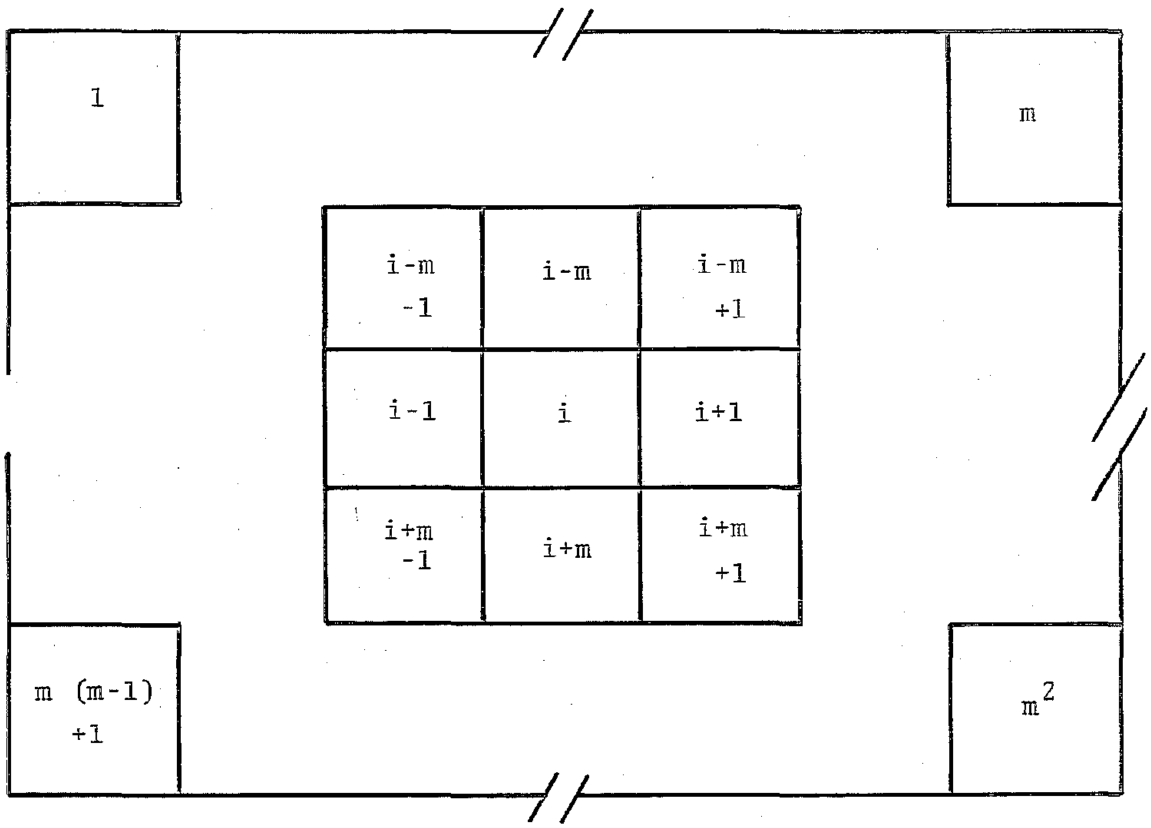


FIGURE 5-1. SIMULATION LATTICE FOR THE MODEL OF TWO VARIABLES m=7

In order to make a systematic analysis of the various distribution patterns obtained from our simulations and to facilitate their comparison with observed urban structures, we have calculated a series of positions and coefficients with which to characterize a given result.

a) Average position.

$$XM = \frac{\sum_{i=1}^{i=n} x_i P_i^j}{\sum_{i=1}^{i=n} P_i^j} \qquad j = 1,2 \qquad (5\text{-}8a)$$

$$YM = \frac{\sum_{i=1}^{i=n} y_i P_i^j}{\sum_{i=1}^{i=n} P_i^j} \qquad j = 1,2 \qquad (5\text{-}8b)$$

which give the average "x" and "y" coordinates and can be calculated for each group in turn.

b) Center of minimum distance.

These centers are calculated for each group. If d_{ik} is the distance between i and k, then first we calculate the value of A_k at the point k:

$$A_k = \frac{\sum_{i=1}^{i=n} d_{ik} P_i^j}{\sum_{i=1}^{i=n} P_i^j} \qquad j = 1,2 \qquad (5\text{-}9)$$

which gives us the average distance of the population P^j from the point k. The center of minimum distance is the point k' for which $A_{k'}$ attains its minimum value. This is rather like a center of gravity. An alternative definition is that of the harmonic center of a population, given by the value of k for which the following is a minimum:[21]

$$A'_k = \frac{\sum_{i=1}^{i=n} P_i^j}{\sum_{i=1}^{i=n} P_i^j / d_{ik}} . \tag{5-10}$$

c) Mean distances.

DD is the mean distance of the population j from the center of gravity. DN is the mean distance from the harmonic center which implies:

$$DD = \frac{\sum_{i=1}^{i=n} d_{ik'} P_i^j}{\sum_{j=1}^{j=n} P^j} \quad \text{where } k' \text{ is the center of gravity, and} \tag{5-11}$$

$$DN = \frac{\sum_{i=1}^{i=n} d_{ik} P_i^j}{\sum_{j=1}^{j=n} P^j} \quad \text{where } k \text{ is the harmonic center.} \tag{5-12}$$

d) Mean variation.

This is defined according to the relation:

$$VR_{jj'} = \frac{\sum_{i,i'=1}^{n} d_{ii'} P_i^j P_{i'}^{j'}}{\sum_{i,i'}^{n} P_i^j P_{i'}^{j'}} . \tag{5-13}$$

If j = j', then we have a measure of the "spread" of a particular group.

e) Coefficients of segregation.

Here we define a variable which will characterize the the degree of segregation existing between two populations:

$$CS = \frac{\sum_{i=1}^{i=n} (P_i^1 + P_i^2)}{P^1 + P^2} \left(\frac{P_i^j}{P_i^1 + P_i^2} - \frac{P^j}{P^1 + P^2} \right) \tag{5-14}$$

where $j = 1,2$.

With this definition, when there is no segregation, CS is equal to zero; but when we have maximum segregation, CS is given by:

$$\sum_{j=1}^{n} \frac{P^j}{P^1 + P^2} \left(1 - \frac{P^j}{P^1 + P^2} \right). \tag{5-15}$$

We have simulated urban growth on an area of surface L^2 which is divided into m^2 elements, which are supposedly homogeneous and of area L^2/m^2. Each of these elements corresponds to what we have hitherto called a "point" i (see Figure 5-1).

Each type of economic agent is described by m differential equations which describe the evolution of the type considered at each point or site. The urban system is thus supposed to be described completely by these n x m differential equations. The numerical solution of this system, in the presence of random events will lead us to the structure of the urban center.

The simulations which we present here have been performed with random events superimposed which modify only the spatial distribution of the agents. The global equation of evolution for the whole system remains purely deterministic.

If X is the total number of agents in the urban center, and X_i the number that are situated at i, then the equations of evolution are:

$$\frac{dX}{dt} = \theta (D - X) \tag{5-16}$$

and

$$\frac{dX_i}{dt} = \theta (D_i - X_i) \tag{5-17}$$

which are purely deterministic equations giving the evolution of the X and X_i.

The random, or stochastic, element is introduced in the following manner. At intervals given by:

$$t_{introduction} = k \Delta t \qquad k \text{ interger,} \tag{5-18}$$

the values which the X_i have attained are redistributed according to the following rule:

$$X_i^N = X_i^A (1 - E) + E X_i^A G_i \tag{5-19}$$

where G_i is a random number between 0 and 1 such that:

$$\sum_{i=1}^{n} G_i = 1. \tag{5-20}$$

X_i^N is the new value of X_i and X_i^A the old. This redistribution of the X_i simply takes into account the statistical nature of any "law" describing the behavior of people. It allows for the "unpredictable" deviations from the average result that will in fact be experienced. A stochastic modification of this sort is applied to each type of agent independently. Thus in the course of a simulation the deterministic system is subjected to a succession of random events which may be amplified if the deterministic conditions are favorable.

Among the results presented are ones for identical systems starting from the initial condition whose final structures are nevertheless different. This is possible because they did not undergo the same succession of random events, since the redistributions that occur at each interval are unique, being decided by the random number generator of the computer. These examples illustrate perfectly the concept of "order by fluctuation": a unique succession of random events, a unique "history" can, when the conditions of stability allow, lead to the evolution of a particular structure.

5.2 ANALYSIS OF THE RESULTS OF THE TWO-VARIABLE MODEL

A series of simulations was performed using the two-variable model not only to find the kind of spatial structures of residence that would result, but also to follow their time evolution and measure the influence of some of the parameters of the model on these structures. Let us first follow one of the simulations during its entire course.

5.2.1 Detailed Analysis of a Simulation Example

Figure 5-1a shows that the total population density decays from the center to the periphery; this decay is exponential (correlation coefficient 0.9995) in agreement with the experimental data (see Appendix B, Figure B-5). The growth in time of this density envelope is also realized as it is observed in actual situations, since the center reaches its maximum first while the other points reach theirs after some delay (Figure 5-1b and Appendix B). As a result the average distance of residence from the center increases in time (Figure 5-1c) and also with the size of the city (Figure 5-1d and Appendix B, Figure B-9).

At the final state of the simulation an already complex residential structure is included under this density enevelope (Figure 5-1e):

1) population 1 exhibits a clear concentric structure: it fills up the center and the second ring;

2) population 2 exhibits a concentric structure modified by sectorial tendencies: it occupies the first ring and extends radially towards the periphery.

This observation reveals, on the average, an already high segregation, stronger in the high density areas, weaker in some peripheral nuclei. Note, however, that the largest part of the urban area is well mixed. Residential segregation then increases with density as the interactions between people do. In time the two populations gradually separate from each other as is shown by the evolution of the segregation coefficient (Figure 5-1f) and

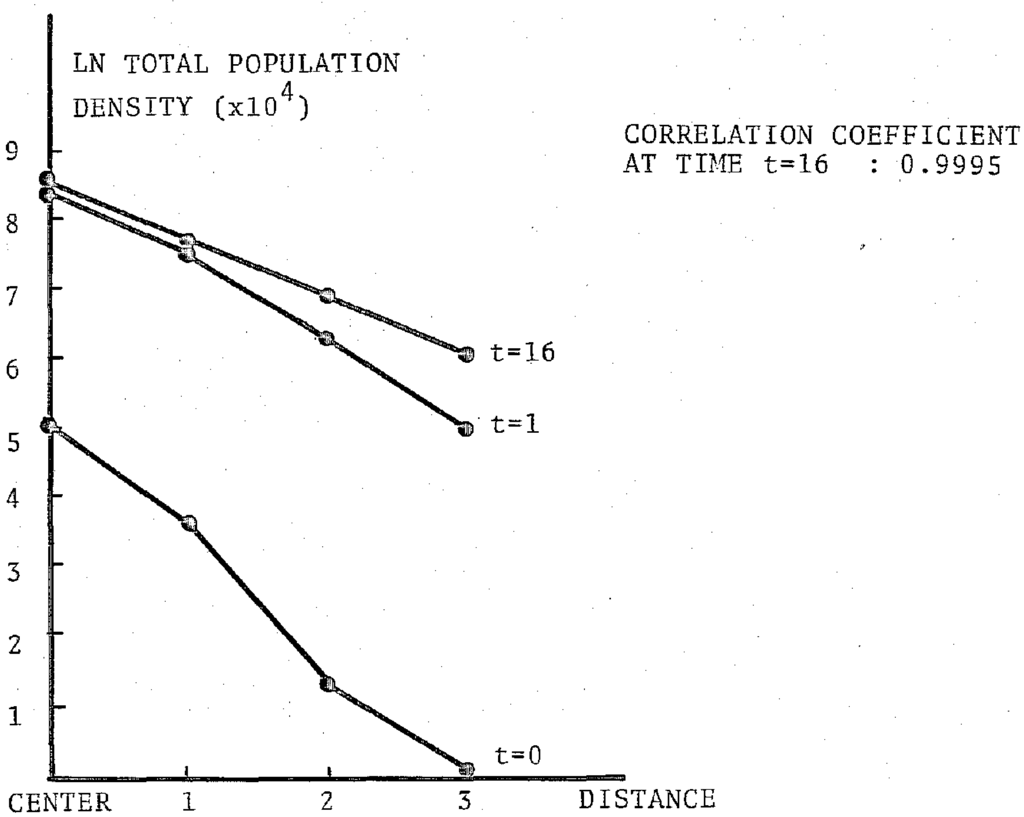


FIGURE 5-1a. LOG OF THE POPULATION DENSITY AS A FUNCTION OF DISTANCE FROM THE CENTER FOR DIFFERENT TIMES: EXPONENTIAL DISTRIBUTION

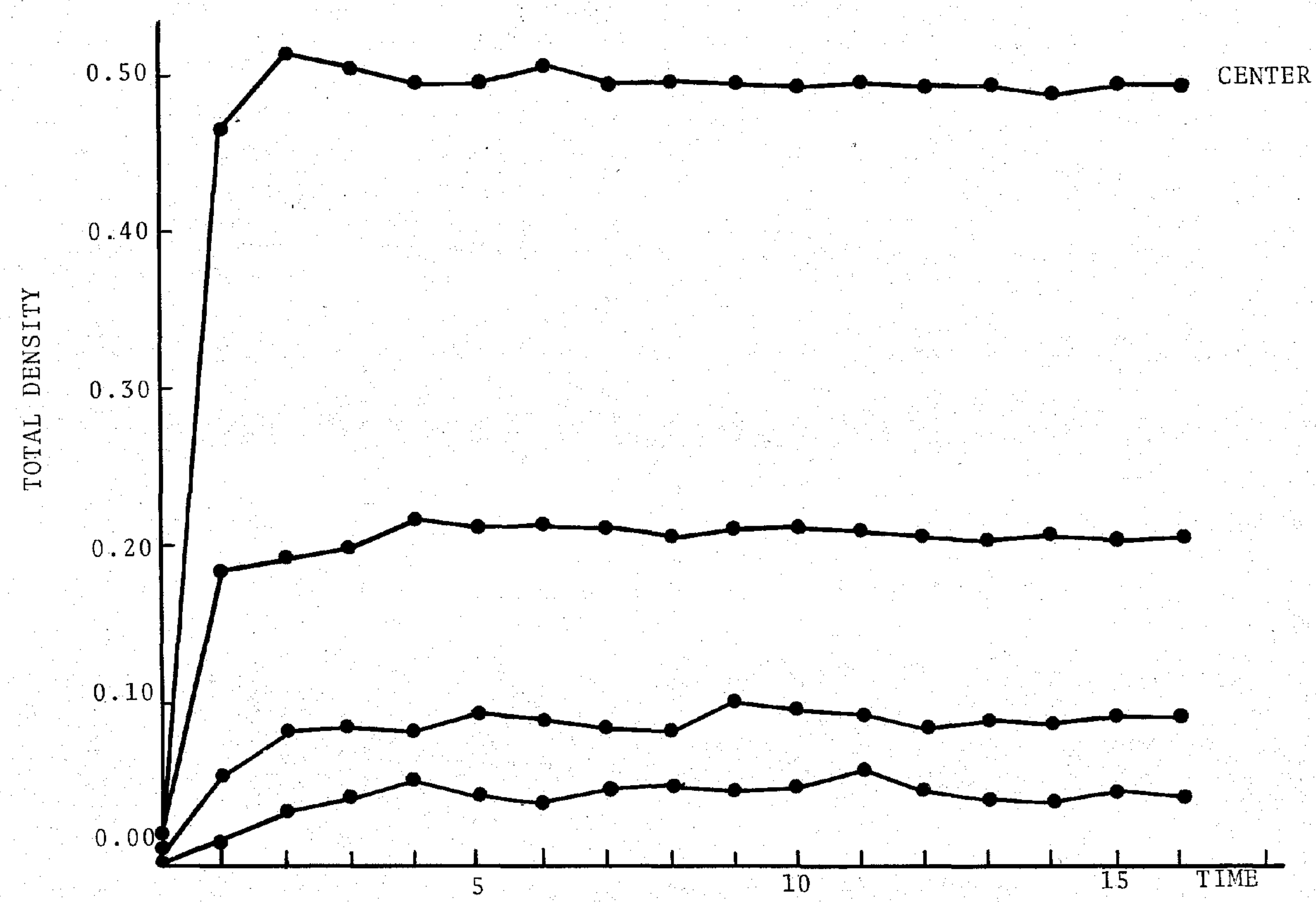


FIGURE 5-1b. GROWTH IN TIME OF POPULATION DENSITY

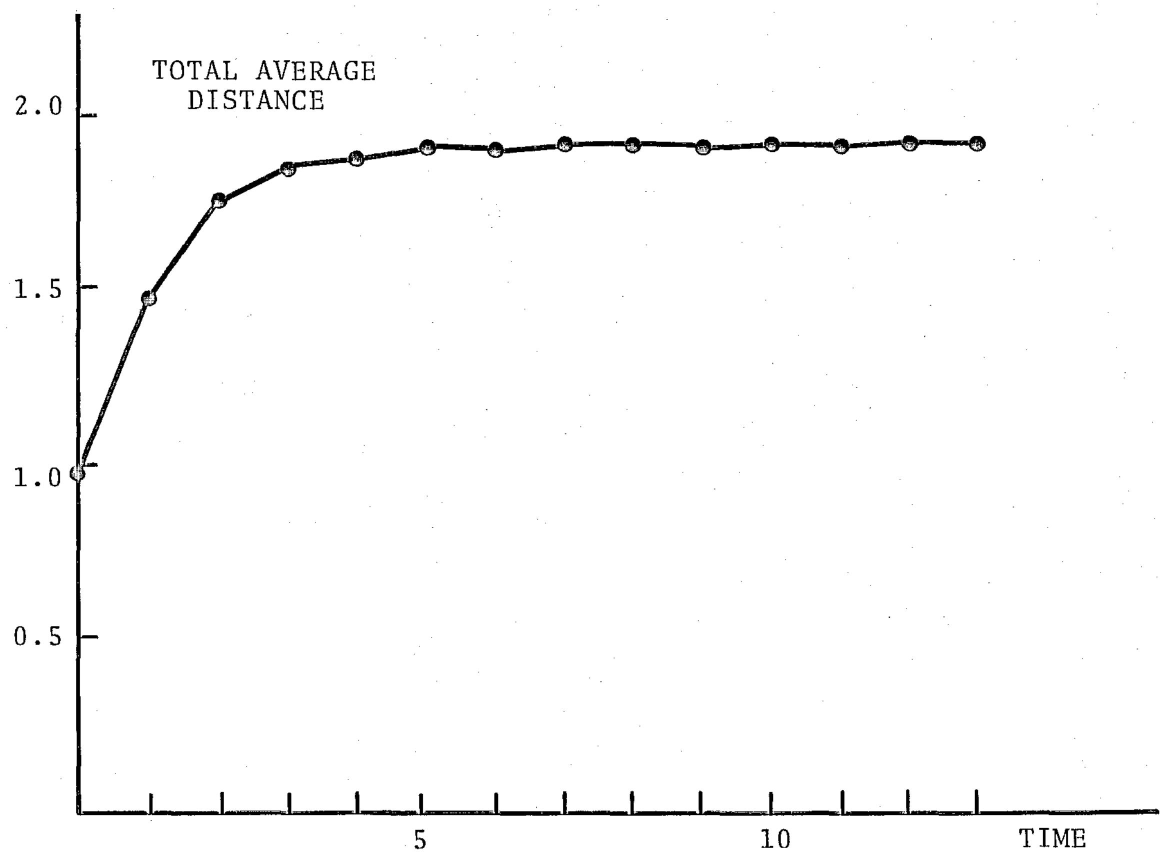


FIGURE 5-1c. AVERAGE DISTANCE OF RESIDENCE FROM THE CENTER AS A FUNCTION OF TIME

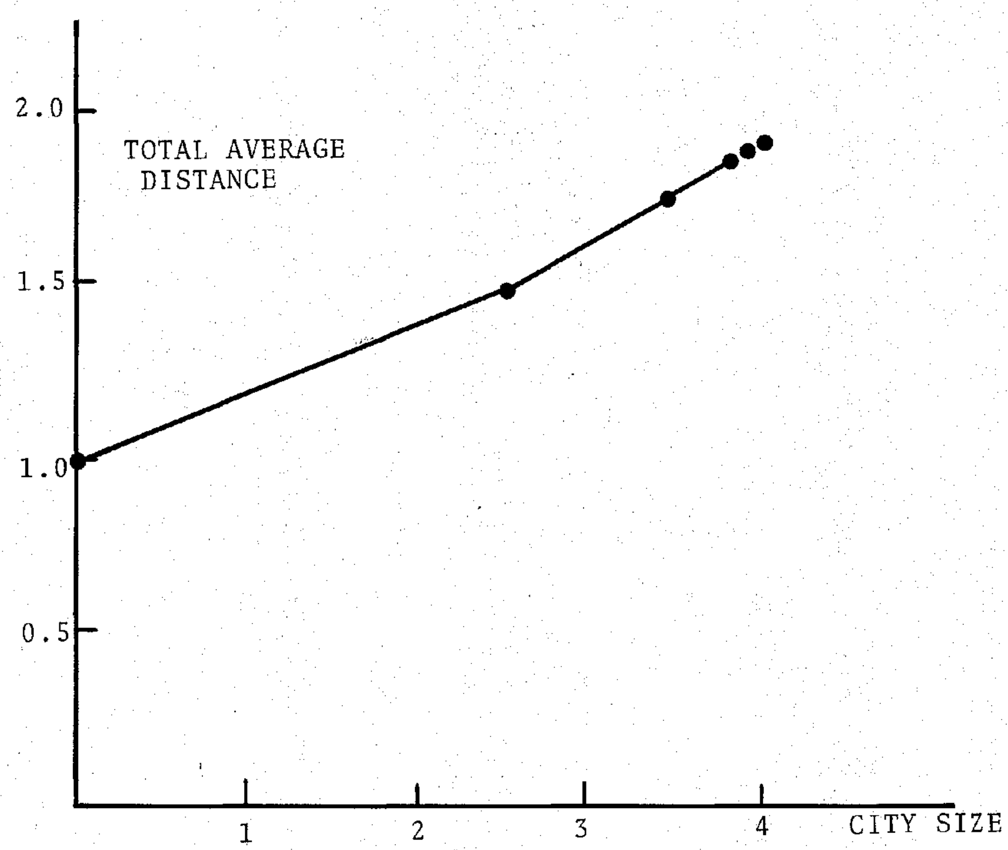
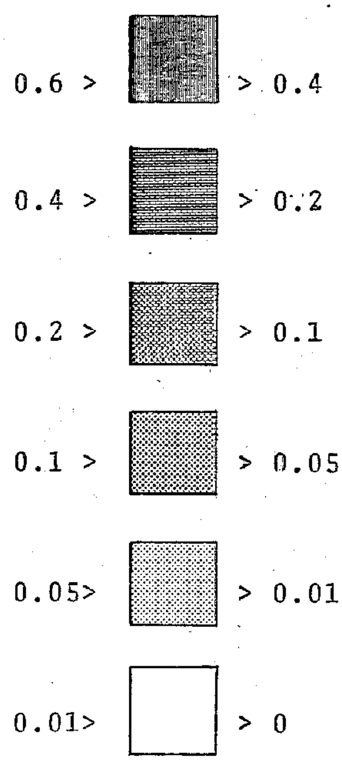


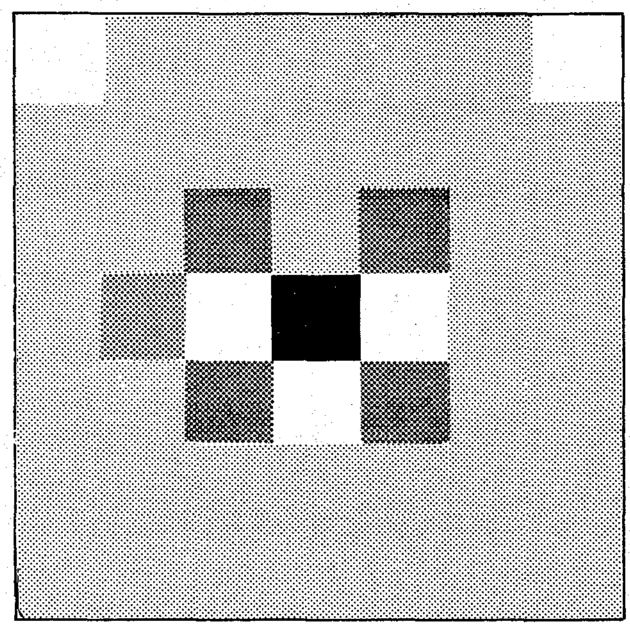
FIGURE 5-1d. AVERAGE DISTANCE OF RESIDENCE FROM THE CENTER AS A FUNCTION OF THE CITY POPULATION SIZE

Figure Captions: The densities are represented by the following symbols:



$A_{12} = 0.1$
$A_{13} = 0.01$
$AZ_1 = AZ_2 = 30$
$CL_1 = 30$
$CL_2 = 5$
$AV1 = 300$

VARIABLE N1



VARIABLE N2

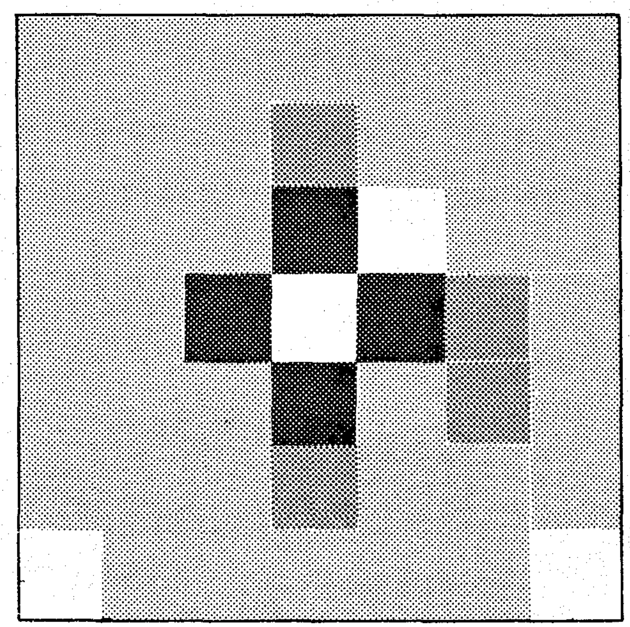


FIGURE 5-1e. SPATIAL DENSITY DISTRIBUTION

Population 1 is concentric; population 2 shows mixed concentric-sectorial distributions.

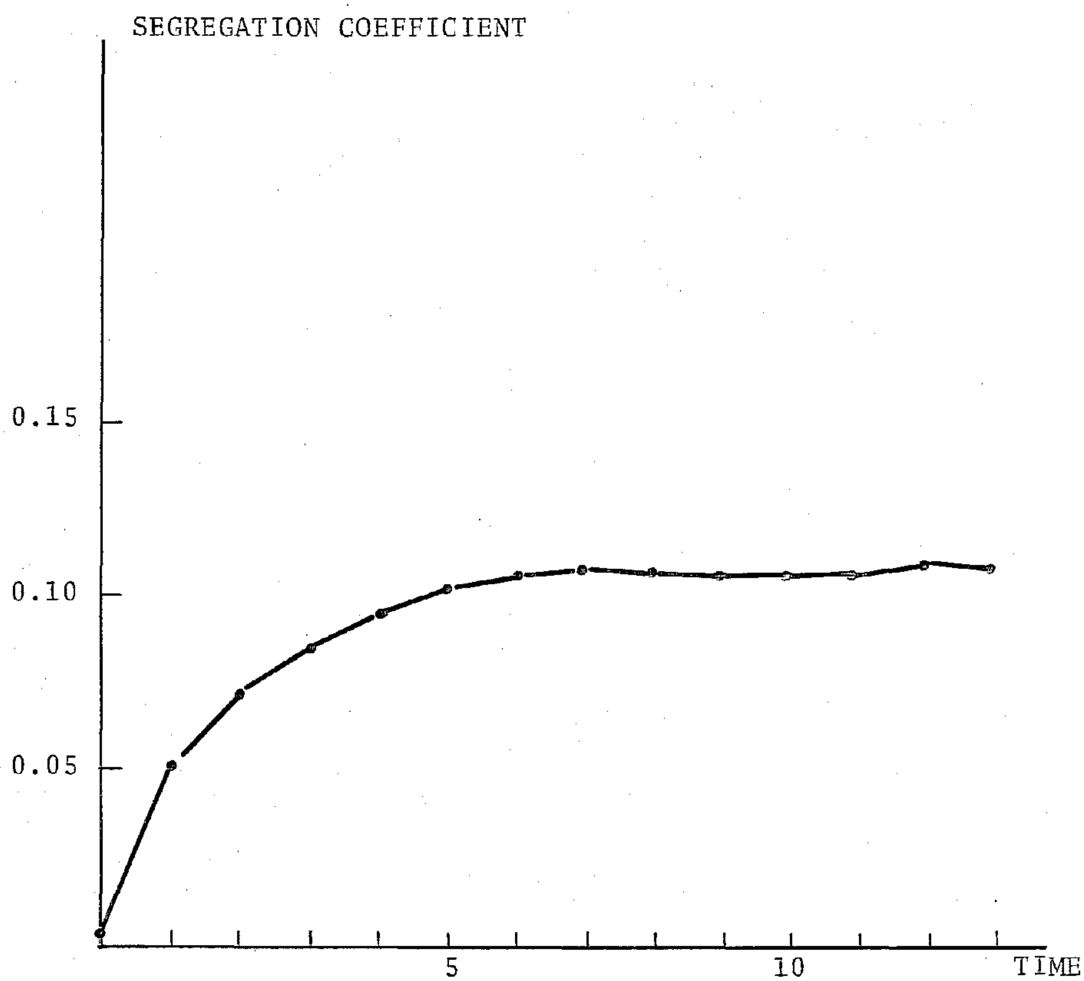


FIGURE 5-1f. EVOLUTION OF SEGREGATION

of the intergroup variance (Figure 5-1g). But it is from the beginning that they differentiate by their individual average distances (Figure 5-1h) and their respective internal variances (Figure 5-1i).

5.2.2 The Different Types of Residential Structures and Their Origin

The different residential structures obtained in the simulations can be analzyed in terms of the influence of some parameters.

The rate of space consumption (AZ)

The space consumption rate reflects the socio-economic status of the inhabitant (on the average his rate increases with his income) as well as the stage of his life cycle (on the average his rate increases with the presence of children in the family). One should then expect structures which result from the combination of sectorial tendencies (typical of socio-economic status) and of concentric tendencies (typical of the life cycle) within the density envelope.

The simulations lead to structures which are not sharply differentiated for the values of the parameter considered. This indicates the weak influence of this parameter under the condition of zero repulsion-attraction between people (AVI = 0). This observation is confirmed by the very low value of the segregation coefficient. In addition, the structures are clearly concentric. The introduction of some repulsion-attraction between people (AVI = 300), enhancing the influence of the parameter, increases even more this concentric city (Figure 5-2a and 5-2b). The absence of sectorial structure seems logical in this case since space is perfectly homogeneous and cooperativity weak. Actually, the sectorial tendency, as it occurs in real cities, seems to be related to the appearance of an industrial axis along an important transportation way. It is possible to partially reconstitute this type of situation in an homogeneous space by differentiating cooperativities between first neighbors: a higher cooperativity between people of the same kind than between people of different

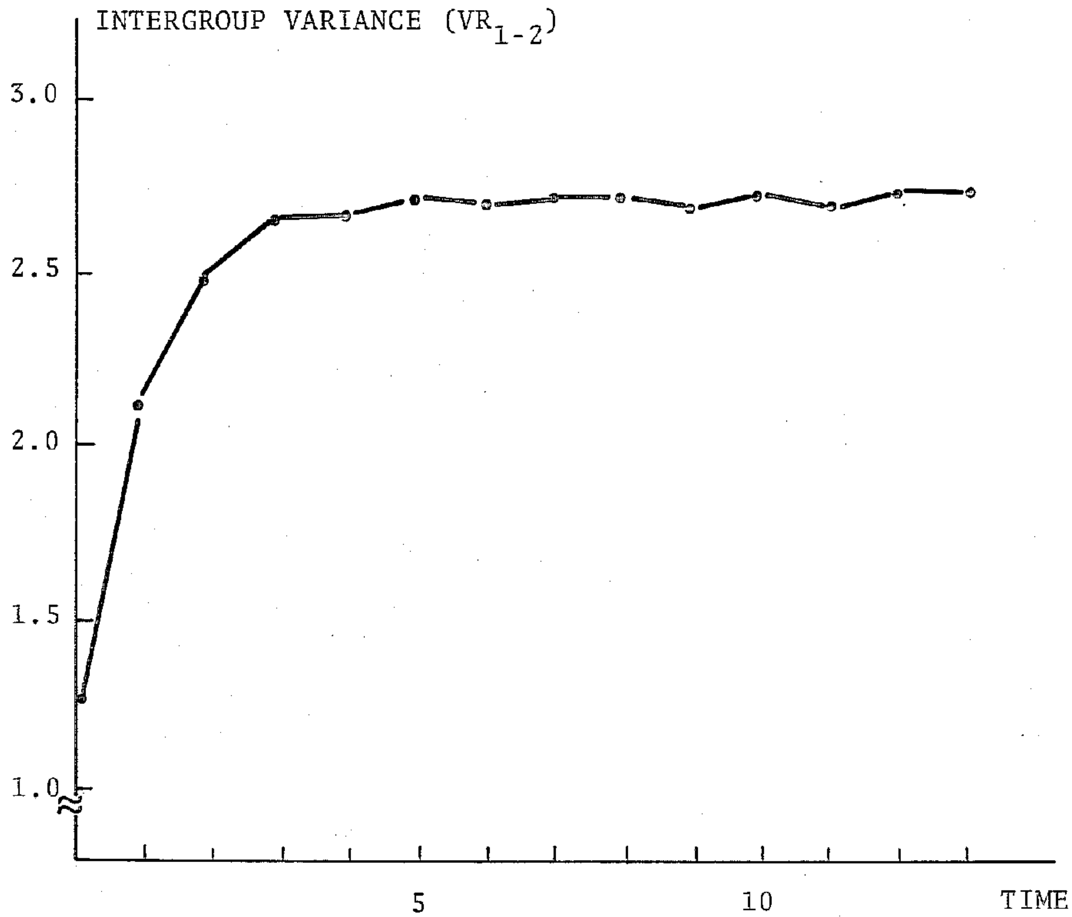


FIGURE 5-1g. EVOLUTION OF THE "SPREADING" OF THE TWO POPULATIONS

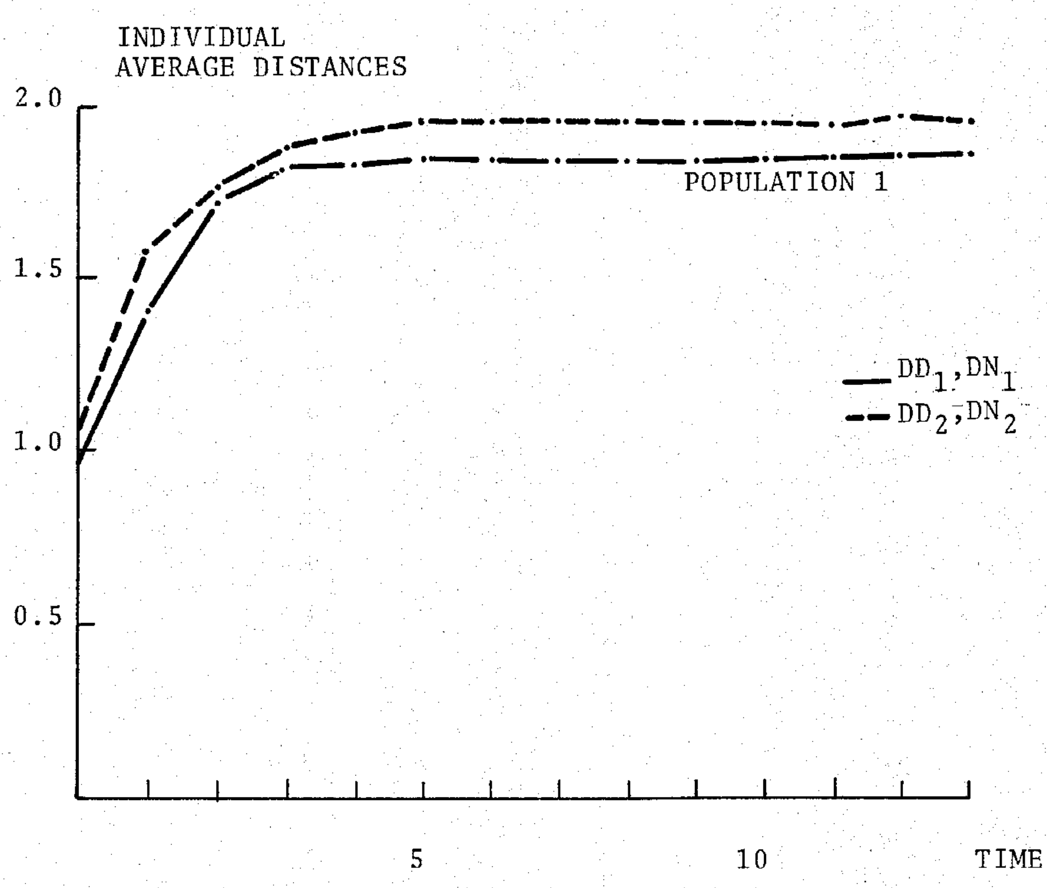


FIGURE 5-1h. EVOLUTION OF AVERAGE DISTANCES OF RESIDENCE FROM THE CENTER FOR EACH OF THE POPULATIONS

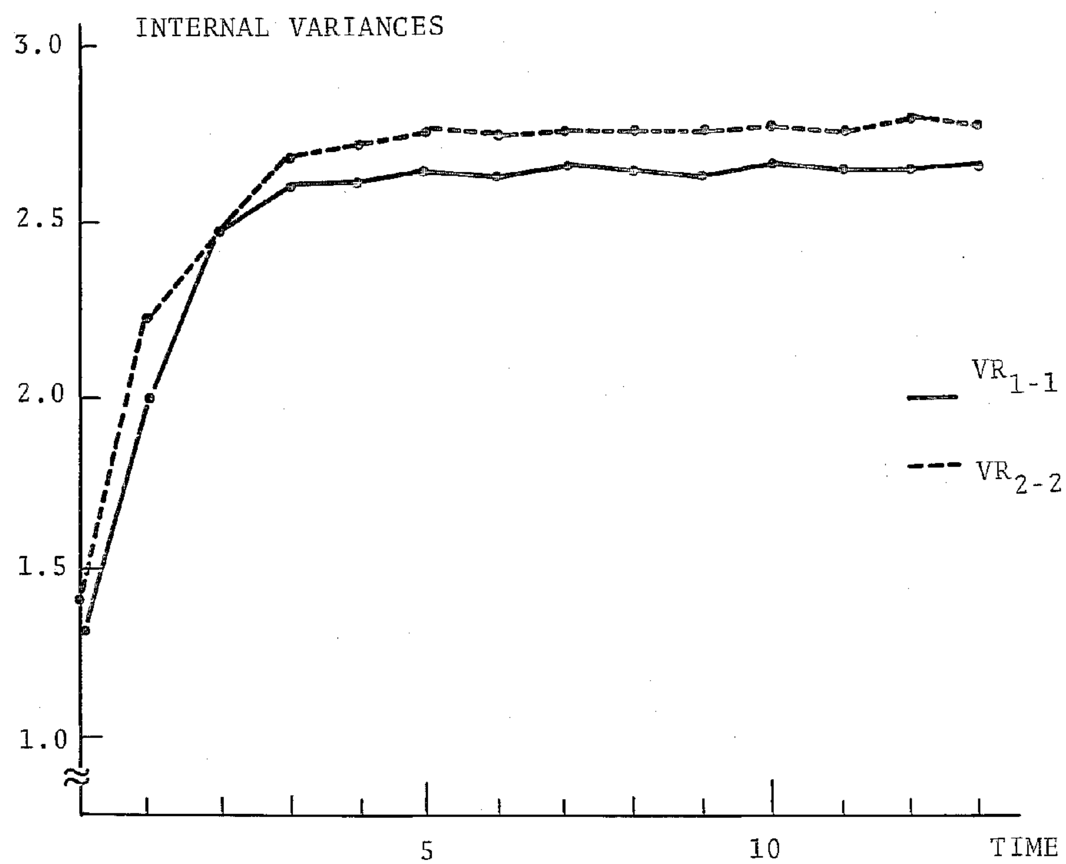


FIGURE 5-1i. EVOLUTION OF WITHIN POPULATION GROUP SPATIAL VARIANCES

$A_{12} = A_{13} = 0.1$

$AZ_1 = AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 0$

VARIABLE N1 VARIABLE N2

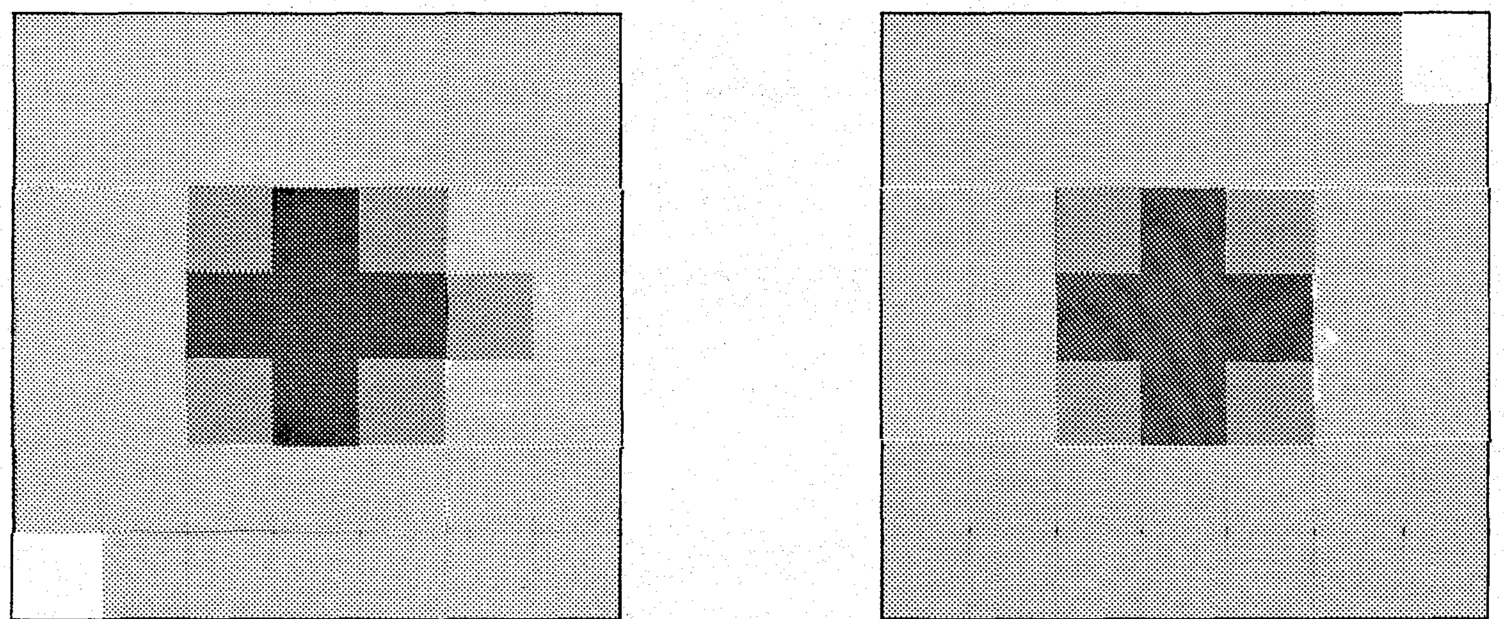


FIGURE 5-2a. SPATIAL DENSITY DISTRIBUTION, NO REPULSION OR ATTRACTION BETWEEN POPULATION GROUPS

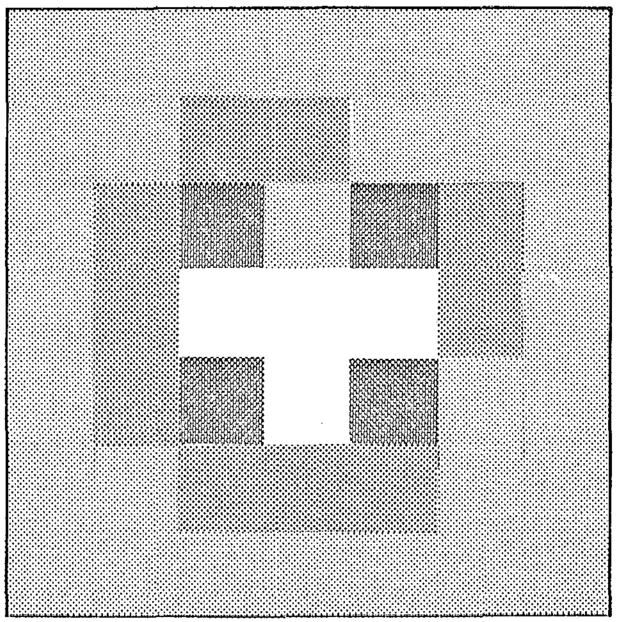
$A_{12} = A_{13} = 0.1$

$AZ_1 = AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 300$

VARIABLE N1



VARIABLE N2

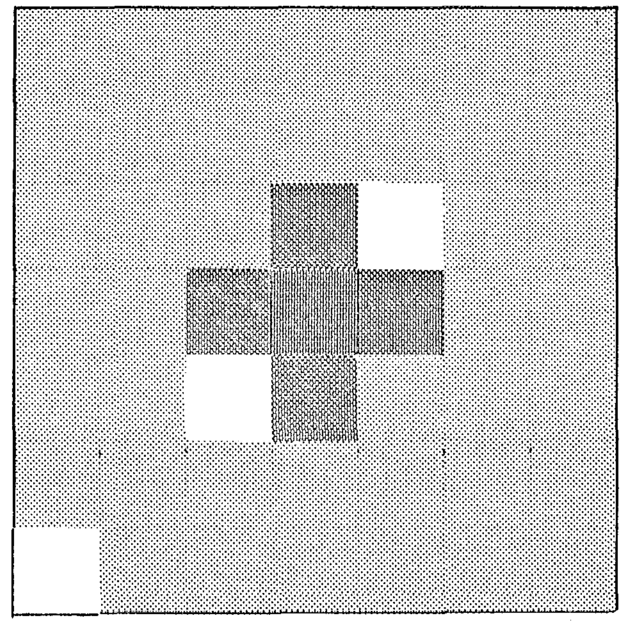


FIGURE 5-2b. SPATIAL DENSITY DISTRIBUTION WITH INTERACTION BETWEEN POPULATIONS-CONCENTRIC SPATIAL DISTRIBUTION

kind combined with some repulsion-attraction between groups (A_{12} = 0.1; A_{13} = 0.01; AVI = 300) creates a sectorial structure (Figure 5-2c).

If we systematically vary the value of the parameter for population 1 (AZ = 20, 10, 5, 1) while population 2 keeps the same characteristics (AZ = 30), we introduce a scale of differentiation between the two populations (Figure 5-3a, b, c, d). With an increasing contrast between them, population 1, whose space consumption goes down, locates closer to the center (its average distance decreases - Figure 5-4b and Table 5-1) and literally shrinks (its internal variance decreases greatly - Figure 5-4d). In doing so, it pushes away population 2 to more distant locations and contributes to its spreading out (P_2 internal variance and average distance slightly decrease). It is thus logical to observe an increase of the segregation coefficient (Figure 5-4c) and of the intergroup variance (Figure 5-4d). This result is confirmed by the facts: the urban lower classes are generally concentrated around the Central Business District (Appendix B, Figures B-13a,b, and B-14a,b) as are single people and couples without children; on the other hand, higher classes and families tend to go to the periphery and spread.

On the whole, the average distance of the two populations, taken together, decreases. This suggests that the poorer and the older the population of a city, the higher its density for cities of comparable size.

The rate of friction to distance (CL)

The rate of friction to distance is an economic as well as cultural parameter. On the one hand, it represents transport cost expressed in monetary or time units; on the other hand, it may reflect the need to escape from the city daily life.

In an homogeneous space undifferentiated from the transportation view point, one can expect a concentric structure for the residences. This is indeed what one observes in the simulations performed under the conditions of zero repulsion-attraction between

$A_{12} = 0.1$

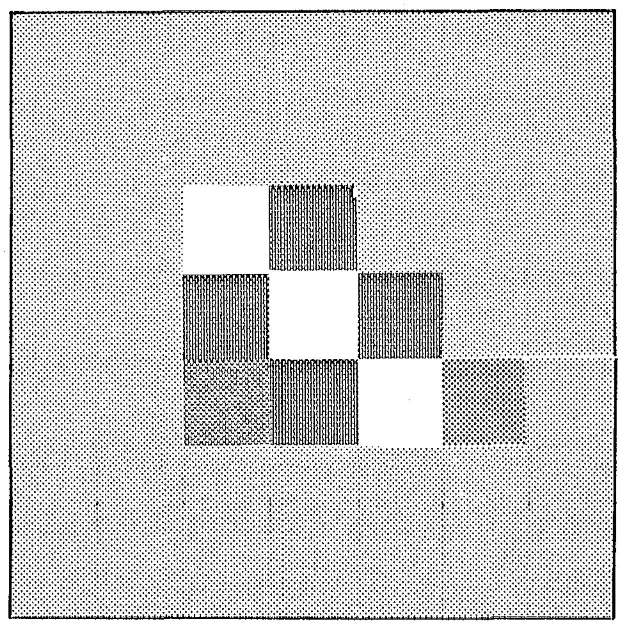
$A_{13} = 0.01$

$AZ_1 = AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 300$

VARIABLE N1



VARIABLE N2

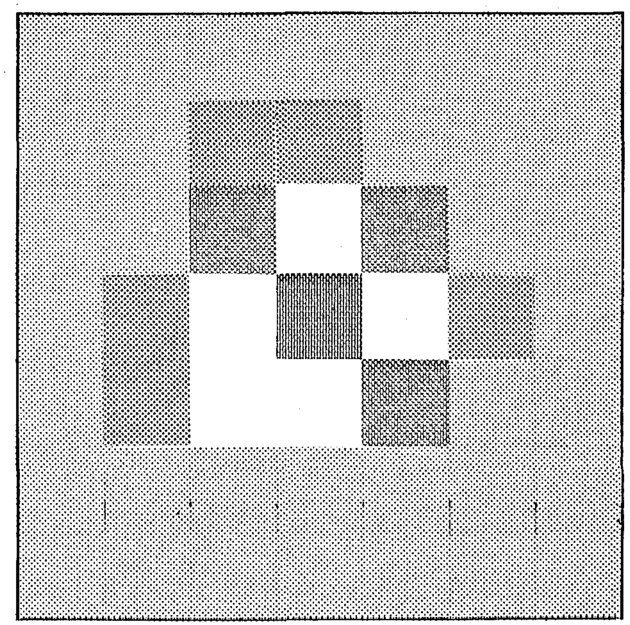


FIGURE 5-2c. SPATIAL DENSITY DISTRIBUTION WITH INTERACTION BETWEEN POPULATIONS AND HIGHER COOPERATIVITY BETWEEN PEOPLE OF THE SAME GROUP-SECTORIAL SPATIAL DISTRIBUTION

$A_{12} = A_{13} = 0.1$

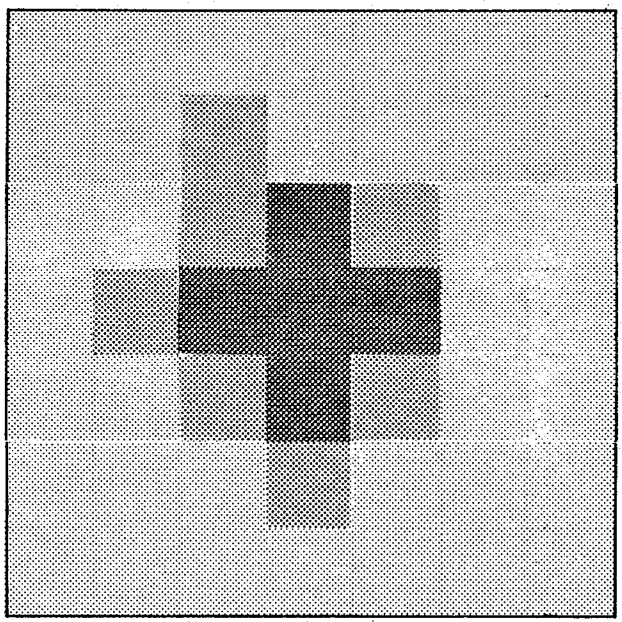
$AZ_1 = 20$

$AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 0$

VARIABLE N1



VARIABLE N2

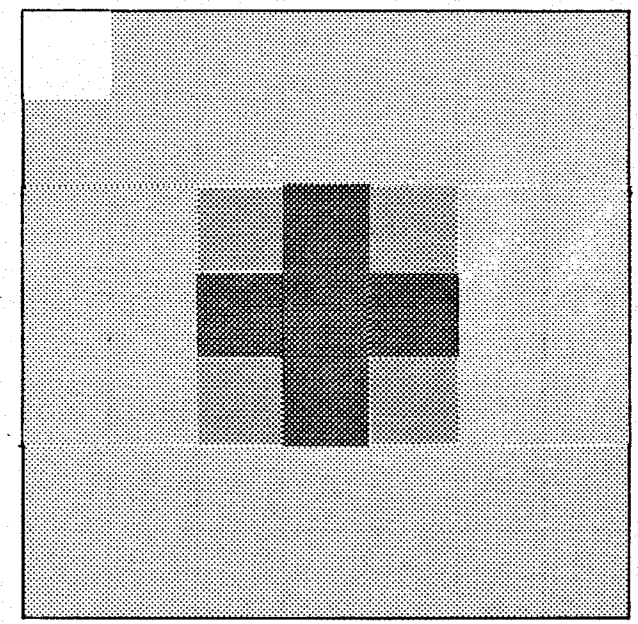


FIGURE 5-3a. SPATIAL DENSITY DISTRIBUTION

Sensitivity of population 2 to crowding is held constant in Figures 5-3a to 5-3d. Sensitivity of population 1 to crowding decreases in the series of figures.

$A_{12} = A_{13} = 0.1$

$AZ_1 = 10$

$AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 0$

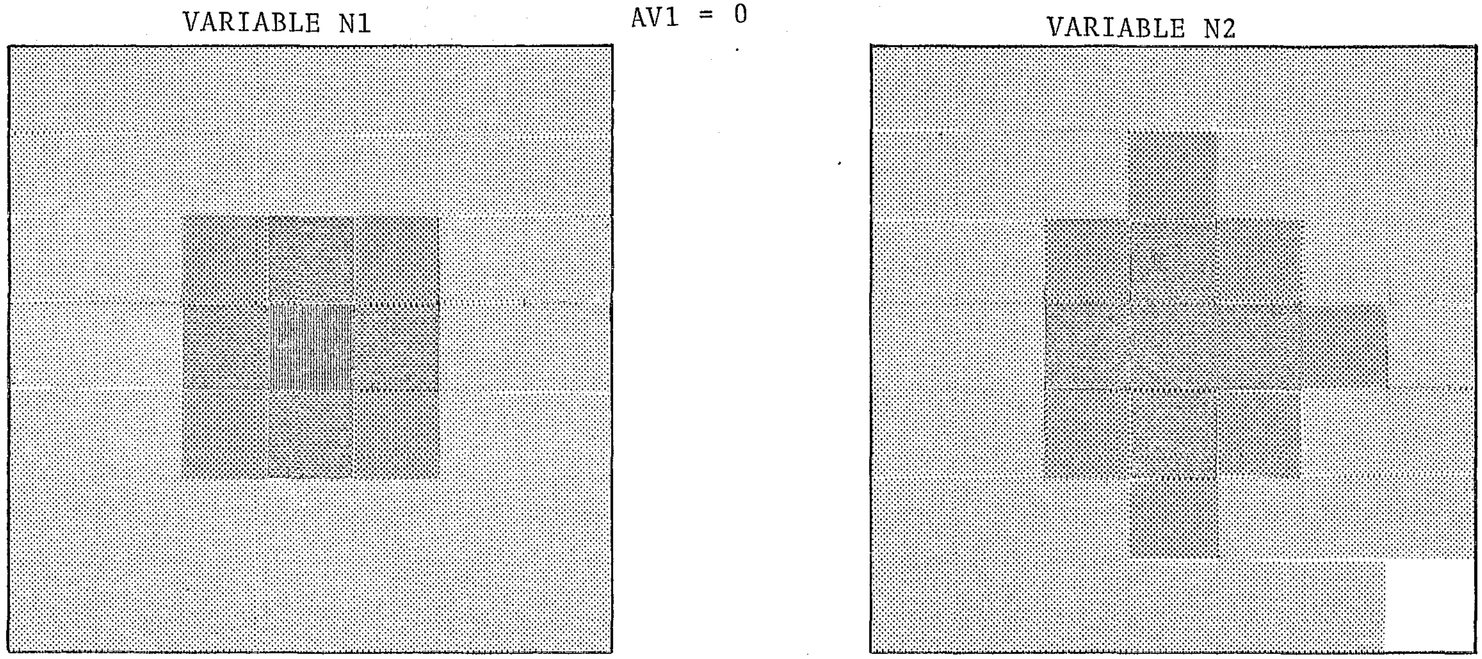


FIGURE 5-3b. SPATIAL DENSITY DISTRIBUTION

Sensitivity of population 1 to crowding is less here than in Figure 5-3a.

$A_{12} = A_{13} = 0.1$

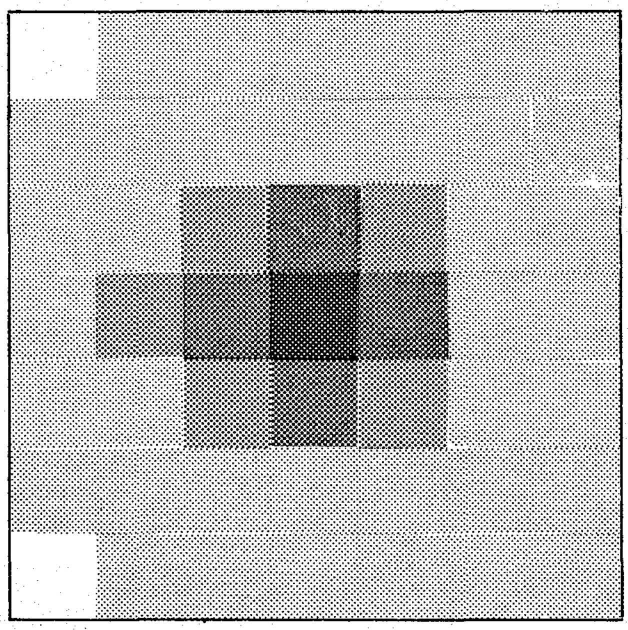
$AZ_1 = 5$

$AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 0$

VARIABLE N1



VARIABLE N2

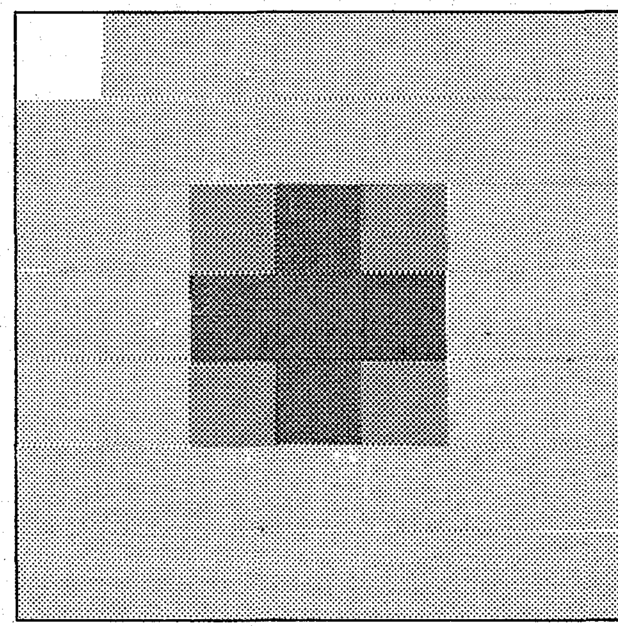


FIGURE 5-3c. SPATIAL DENSITY DISTRIBUTION

Sensitivity of population 1 to crowding is less than in Figure 5-3b.

$A_{12} = A_{13} = 0.1$

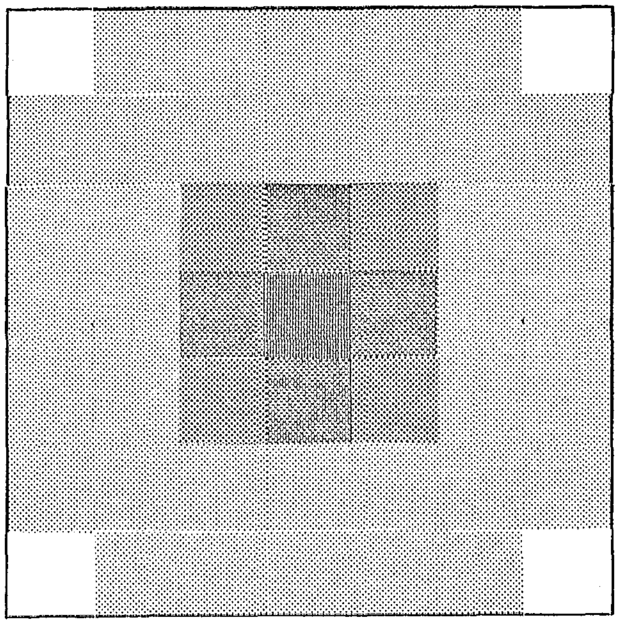
$AZ_1 = 1$

$AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 0$

VARIABLE N1



VARIABLE N2

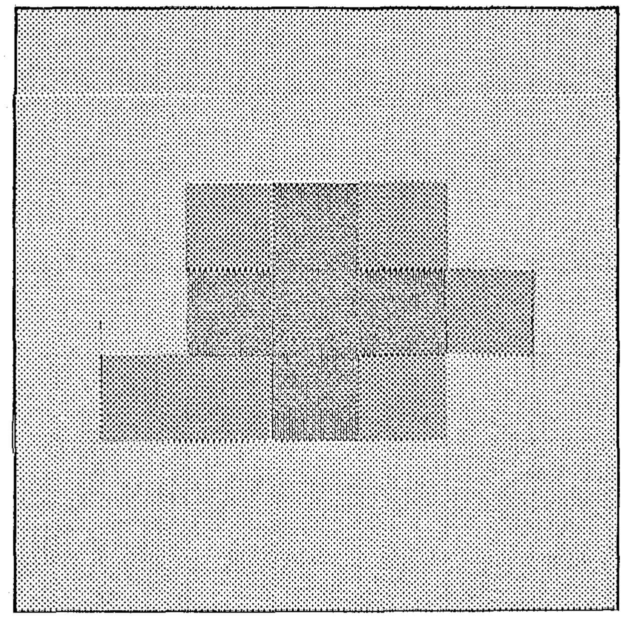


FIGURE 5-3d. SPATIAL DENSITY DISTRIBUTION

Sensitivity of population 1 to crowding is least in this last figure of the series. Population 1 locates in the city center contributing to the spreading out of population 2.

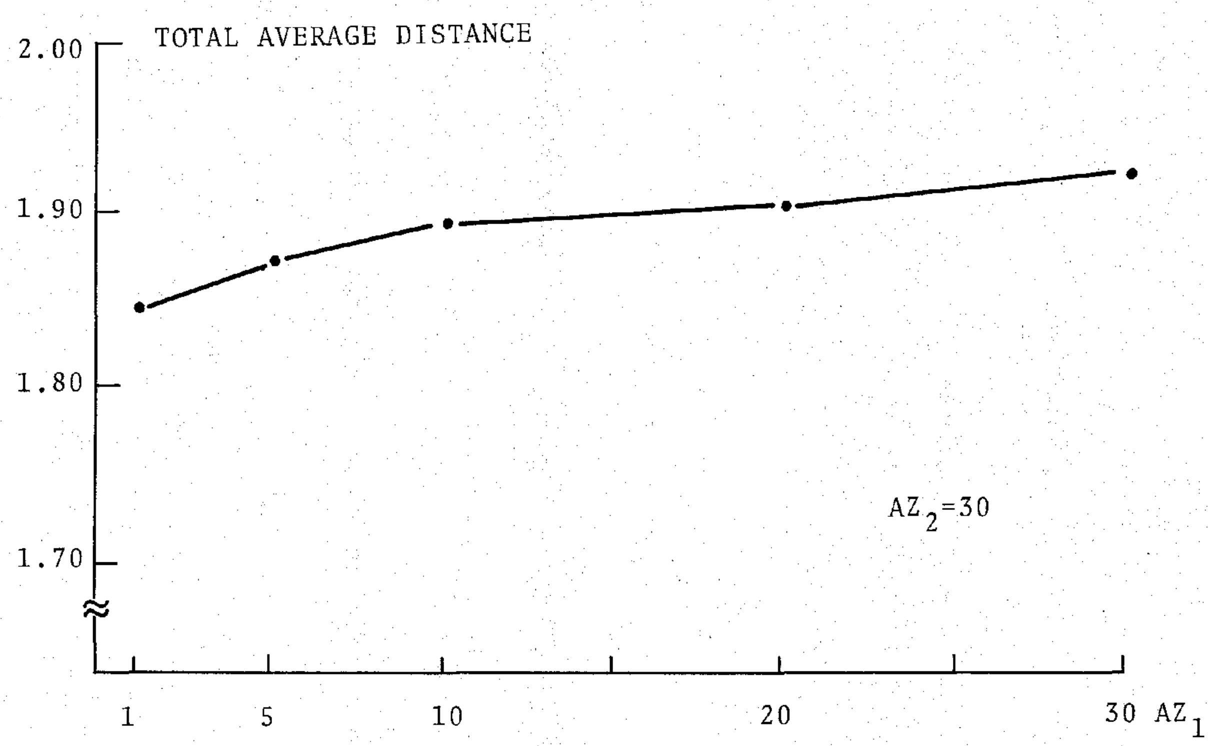


FIGURE 5-4a. TOTAL AVERAGE DISTANCE FROM THE CENTER AS A FUNCTION OF THE CROWDING COEFFICIENT AZ

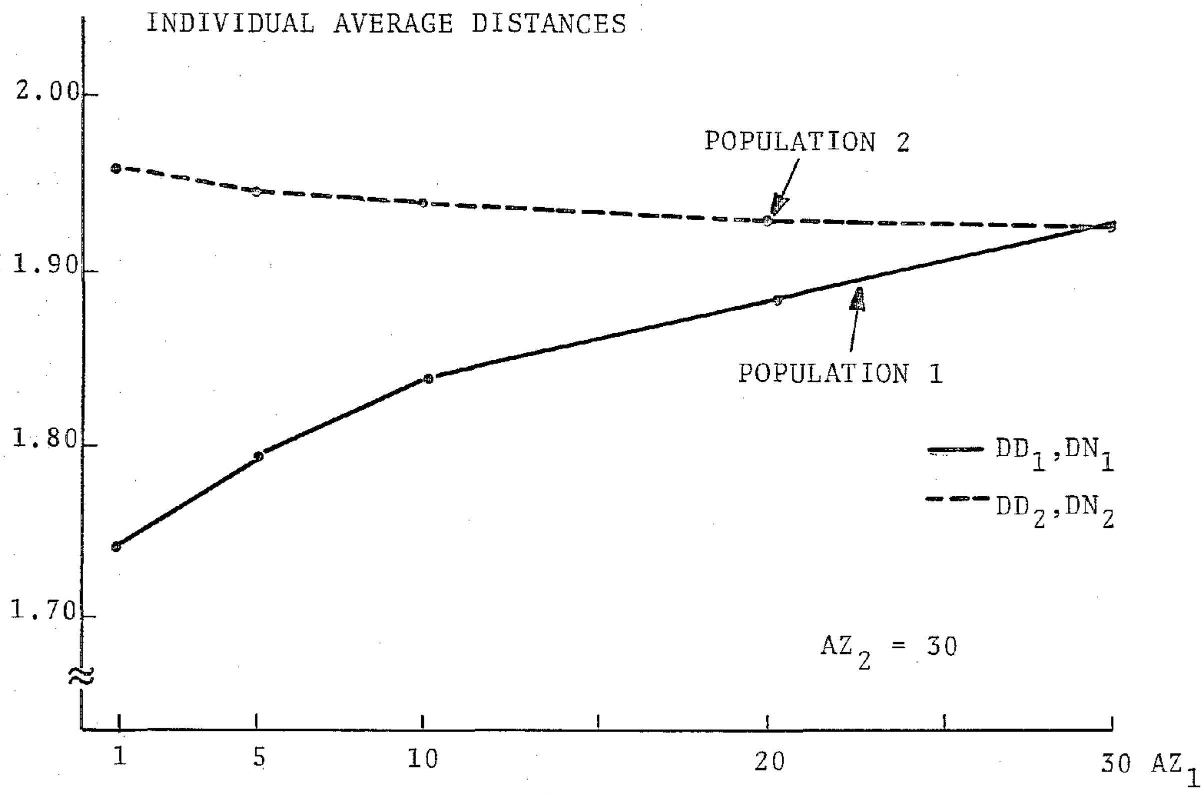


FIGURE 5-4b. INDIVIDUAL AVERAGE DISTANCES AS A FUNCTION OF THE CROWDING PARAMETER AZ (LARGER VALUES OF AZ INDICATE GREATER SENSITIVITY OF THE POPULATIN TO CROWDING.)

TABLE 5-1. URBAN STRUCTURE(*)

Variable parameters			BY POPULATION: Average distances DD_1	DD_2	DN_1	DN_2	Variances VR_{1-1}	VR_{2-2}	VR_{1-2}	TOTAL: Average dist.	Segregat. Coeffic.	Parameters kept constant
$\boxed{AZ_1}$	$\boxed{AZ_2}$	(**)										
1	30	(3)	1.7432	1.9622	1.7432	1.9622	1.4970	2.7609	2.6366	1.8527	.0099	$A_{12} = A_{13} = 0.1$
5	30	(3)	1.7942	1.9508	1.7942	1.9508	2.5598	2.7485	2.6583	1.8725	.0058	$CL_1 = CL_2 = 5$
10	30	(3)	1.8427	1.9439	1.8427	1.8439	2.6221	2.7424	2.6846	1.8933	.0035	AVI = 0
20	30	(3)	1.8857	1.9302	1.8957	1.9302	2.6712	2.7254	2.6993	1.9080	.0022	
30	30	(3)	1.9269	1.9282	1.9269	1.9282	2.7222	2.7265	2.7252	1.9275	.0020	
$\boxed{CL_1}$	$\boxed{CL_2}$											
1	5	(2)	2.1635	1.8893	8.1635	1.8893	3.0069	2.6813	2.8543	2.0264	.0091	$A_{12} = A_{13} = 0.1$
3	5	(1)	1.9929	1.9126	1.9929	1.9126	2.8042	2.7067	2.7567	1.9275	.0030	AVI = 0
5	5	(3)	1.9269	1.9282	1.9269	1.9282	2.7222	2.7265	2.7252	1.9274	.0020	$AZ_1 = AZ_2 = 30$
10	5	(2)	1.8583	1.9408	1.8583	1.9408	2.6388	2.7412	2.6920	1.8995	.0030	
15	5	(2)	1.8166	1.9460	1.8166	1.9460	2.5885	2.7440	2.6690	1.8813	.0044	
20	5	(2)	1.7991	1.9498	1.7991	1.9498	2.5680	2.7476	2.6614	1.8744	.0053	
30	5	(2)	1.7704	1.9561	1.7704	1.9561	2.5324	2.7546	2.6488	1.8632	.0072	
$\boxed{AVI}$			DD_P(***)	DD_C	DN_P	DN_C	VR_{P-P}	VR_{C-C}	VR_{P-C}			
10		(3)	1.9056	1.9127	1.9056	1.9127	2.6916	2.7007	2.6971	1.9091	.0020	$A_{12} = 0.1$
75		(1)	1.9245	2.0091	1.9245	2.1556	2.7301	2.8058	2.7774	1.9668	.0414	$A_{13} = 0.01$
100		(2)	1.9602	2.0045	1.9602	2.1220	2.7714	2.7932	2.8001	1.9823	.0595	$AZ_1 = AZ_2 = 30$
300		(3)	2.0719	1.9399	2.0719	8.0911	2.9031	2.7232	2.8327	2.0058	.1036	$CL_1 = CL_2 = 5$
650		(2)	2.0239	1.9664	2.0239	2.0390	2.8408	2.7713	2.8185	1.9952	.1257	
1000		(3)	1.9905	1.9746	1.9905	2.1162	2.7875	2.7905	2.8088	1.9825	.1524	

(*) The values given in this table are mean values calculated on the basis of identical parameters with a different noise factor.
(**) Number of simulations performed.
(***) P : population having a central "peak" in its density distribution.
C : population having a central "crater" in its density distribution.

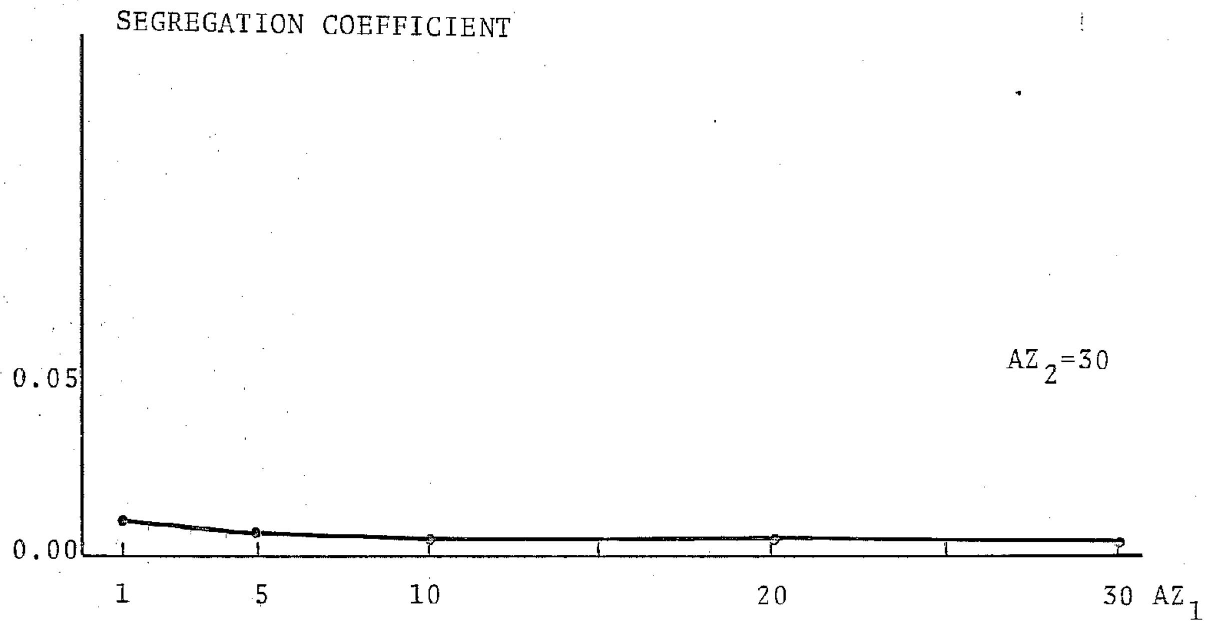


FIGURE 5-4c. SEGREGATION COEFFICIENT AS A FUNCTION OF THE CROWDING COEFFICIENT AZ

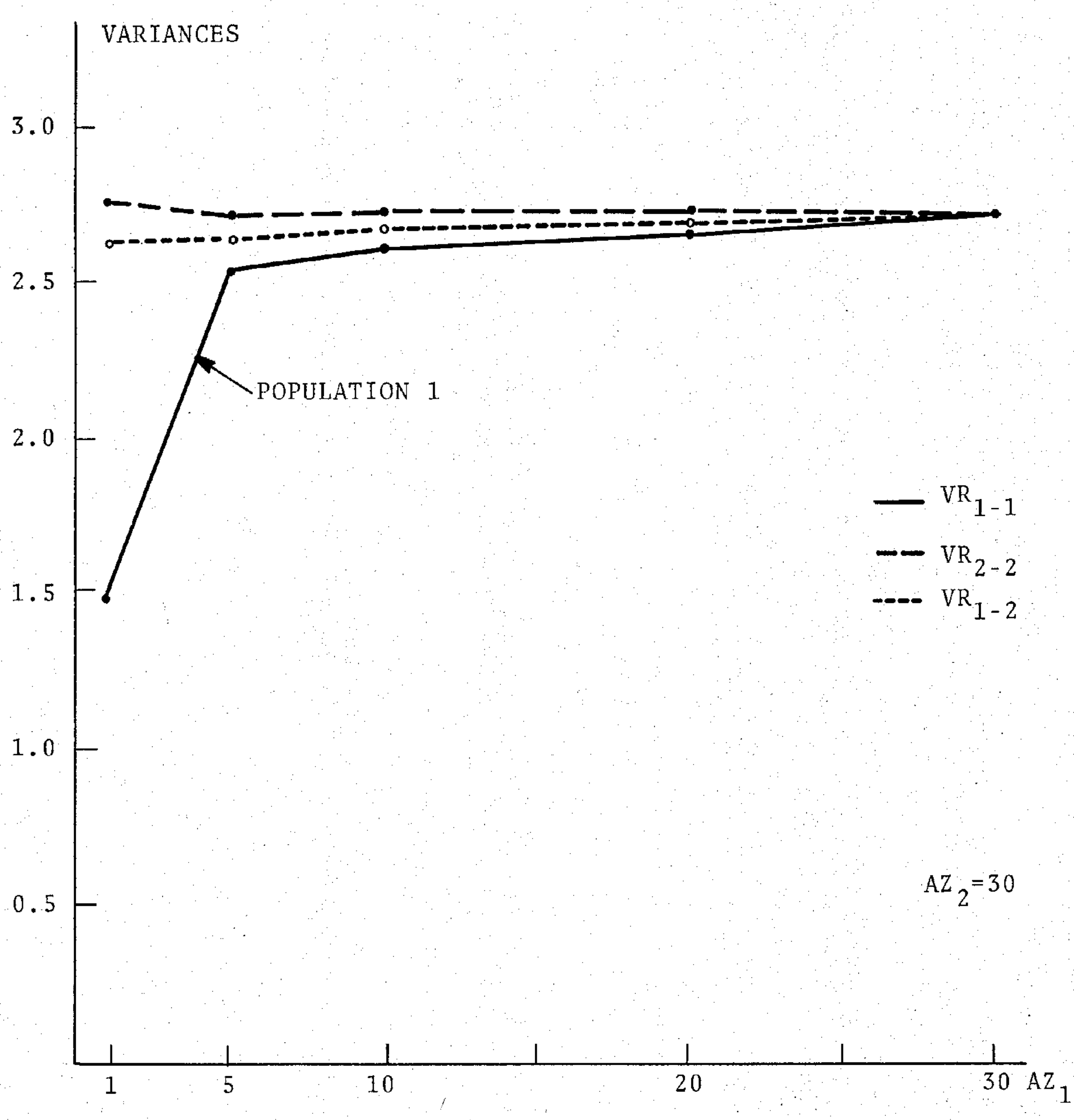


FIGURE 5-4d. VARIANCES, MEASURING THE "SPREAD" BETWEEN AND WITHIN GROUPS

groups (AVI = 0) and zero or equal cooperativity between first neighbors ($A_{12} = A_{13} = 0.1$).

When the parameter for population 1 varies (CL = 1, 3, 5, 10, 15, 20, 30), whereas that of population 2 is kept constant (CL = 5), a differentiated spatial behavior appears (Figures 5-5a, b, c, d, e, f, g). With an increasing value of its parameter, population 1 locates closer and closer to the center (its average distance decreases greatly - Figure 5-6b, Table 5-1) but its internal dispersion only slightly. In doing so, population 1 forces population 2 to relocate.

(These two parameters - rate of space consumption and rate of friction to distance - have similar effects as to the type of spatial structures and act in the same direction. The third parameter analyzed leads to very different patterns.)

<u>The rate of repulsion attraction between groups</u> (AVI)

The rate of repulsion-attraction between groups reflects a classical situation of segregation: mutual rejection of two populations which co-exist with a more or less high need to come closer to one's fellow-men.

In our simulations, the case leading to the most complex patterns is the one that combines intergroup segregation with a cooperativity between first neighbors favoring the meeting together of similar people ($A_{12} = 0.1$; $A_{13} = 0.01$). This is the residential ghetto, often described in the literature as leading to nucleation.

It is under these conditions that we have analyzed the influence of the final pattern, of an increasing value of repulsion-attraction parameter (Figure 5-7a, b, c, d, e, f). When the value of the parameter is small, there is practically no difference between the two distributions of population and the structure is clearly concentric. But quickly (AVI $\geq$ 75) one of the populations occupies the center and pushes away the other one. It can be one population or the other, according to the noise factor used in the simulation. So it is clearly the historical

$A_{12} = A_{13} = 0.1$

$AZ_1 = AZ_2 = 30$

$CL_1 = 1$

$CL_2 = 5$

$AV1 = 0$

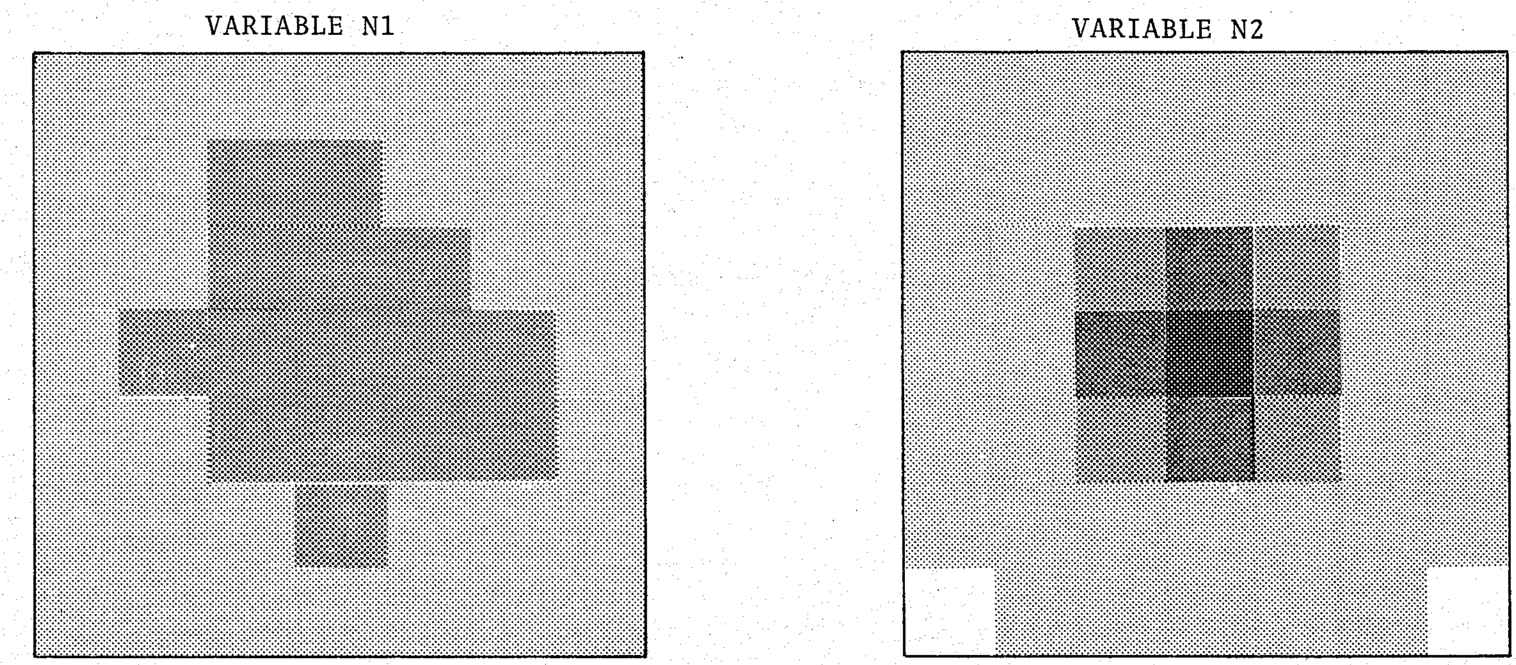


FIGURE 5-5a. SPATIAL DENSITY DISTRIBUTION

Figures 5-5a to 5-5g show the effect of increasing the sensitivity of population 1 to transport costs on the resulting urban structure. This figure shows urban structure when the sensitivity to transport costs for population 1 is minimum.

$A_{12} = A_{13} = 0.1$

$AZ_1 = AZ_2 = 30$

$CL_1 = 3$

$CL_2 = 5$

$AV1 = 0$

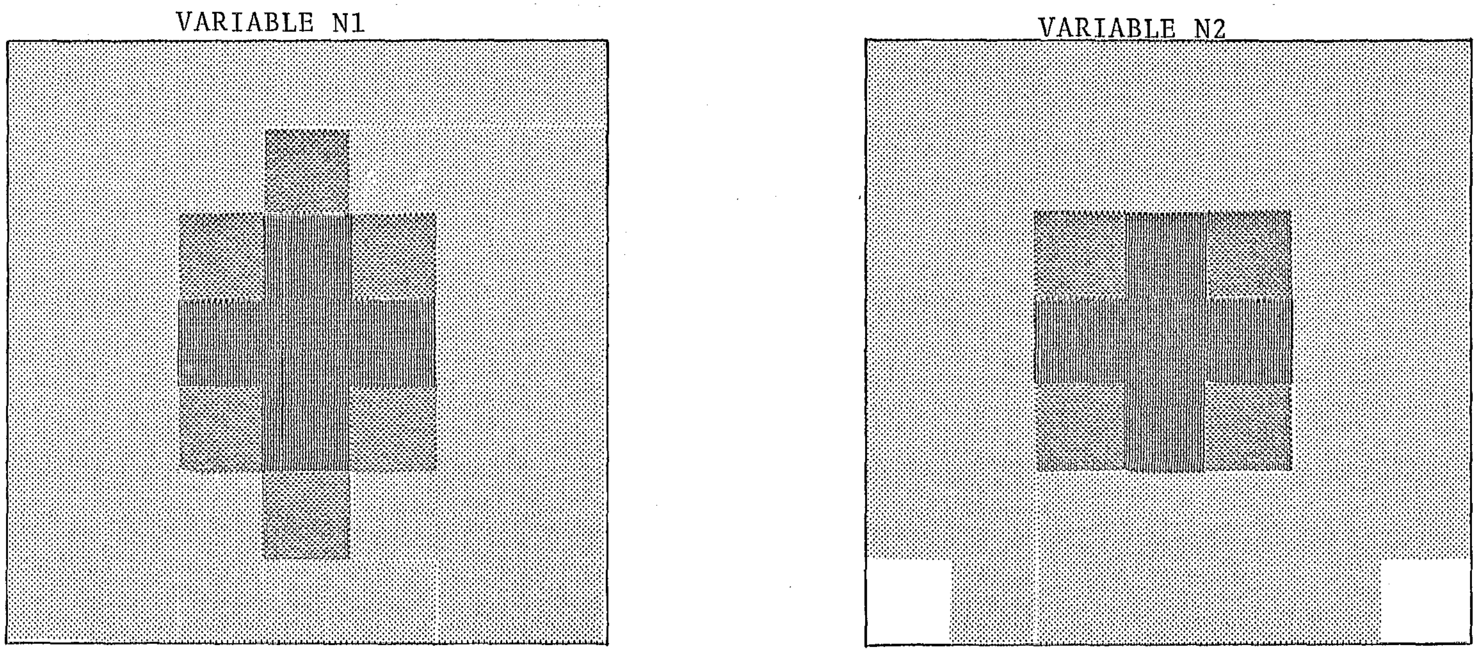


FIGURE 5-5b. SPATIAL DENSITY DISTRIBUTION

Sensitivity of population 1 to transport costs is higher than in the previous figure.

$A_{12} = A_{13} = 0.1$

$AZ_1 = AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 0$

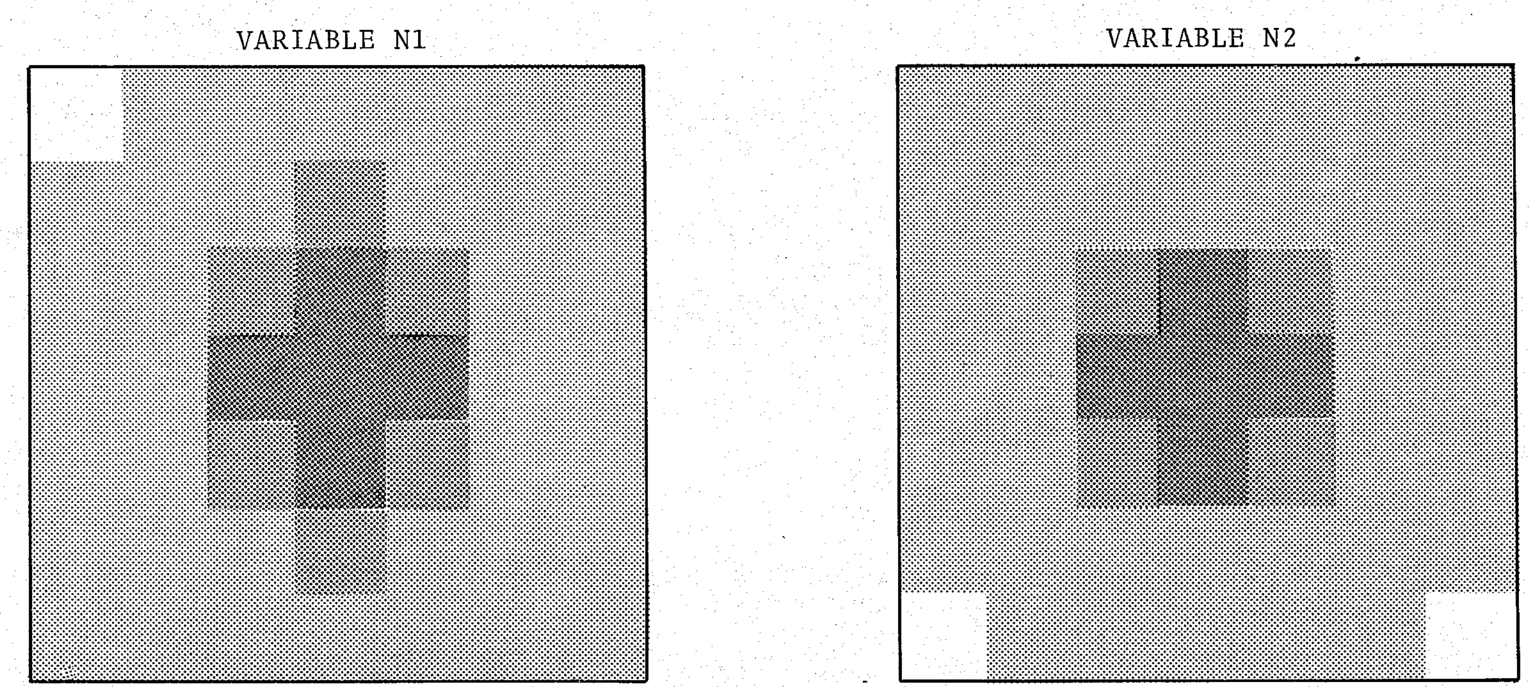


FIGURE 5-5c. SPATIAL DENSITY DISTRIBUTION

Sensitivity of population 1 to transport cost is higher than in the previous figure.

$A_{12} = A_{13} = 0.1$

$AZ_1 = AZ_2 = 30$

$CL_1 = 10$

$CL_2 = 5$

$AV1 = 0$

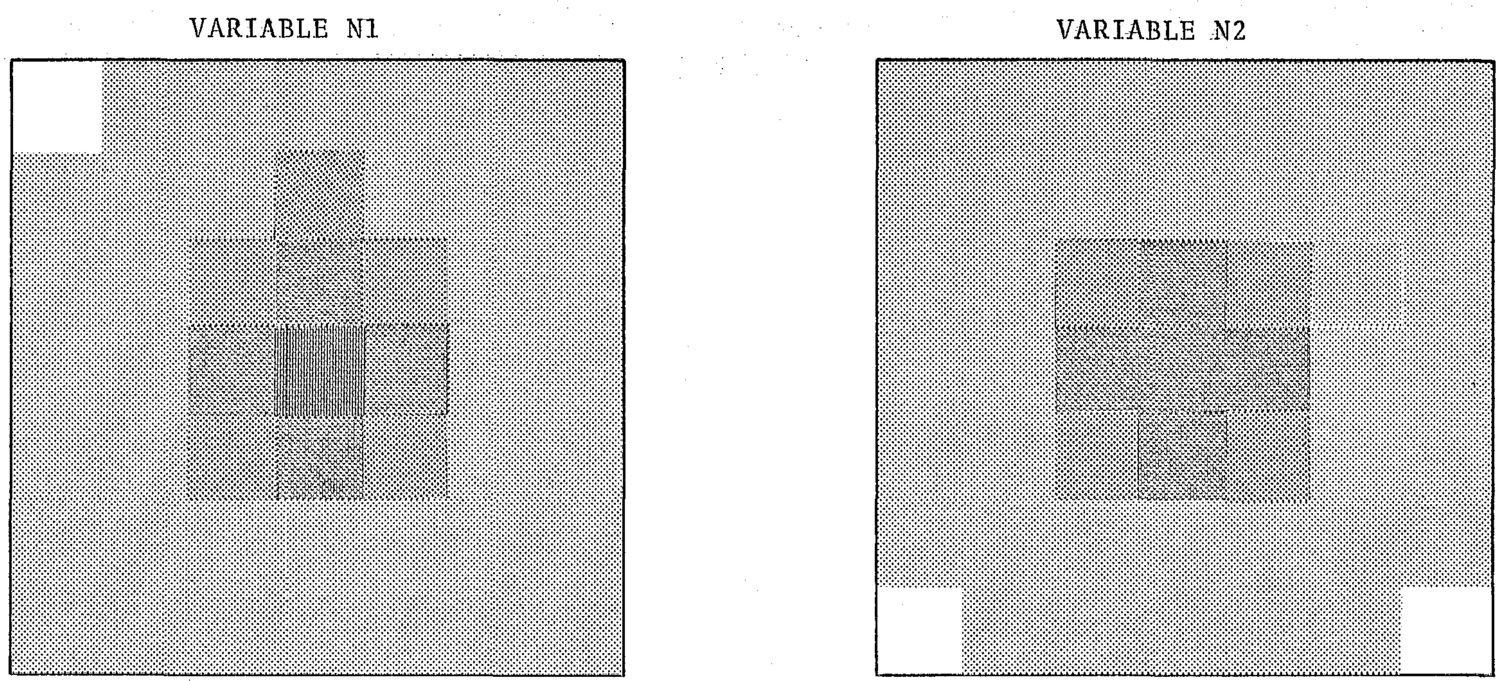


FIGURE 5-5d. SPATIAL DENSITY DISTRIBUTION

Sensitivity of population 1 to transport costs is higher than in the previous figure.

$A_{12} = A_{13} = 0.1$

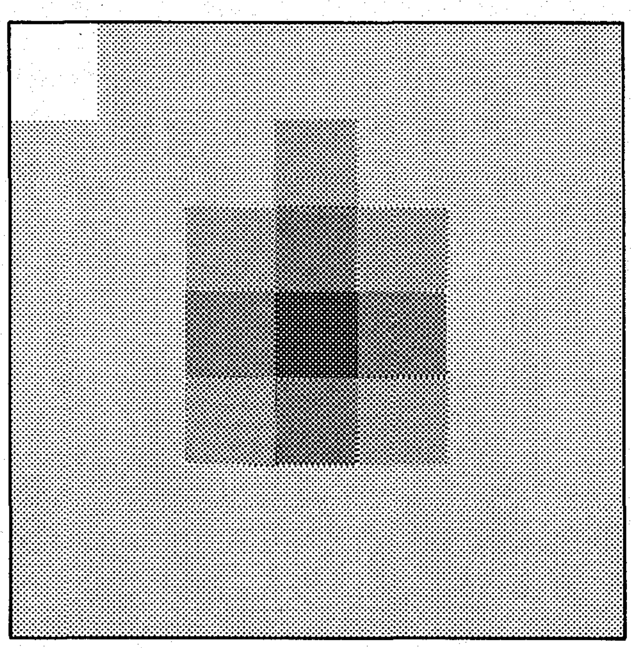
$AZ_1 = AZ_2 = 30$

$CL_1 = 15$

$CL_2 = 5$

$AV1 = 0$

VARIABLE N1



VARIABLE N2

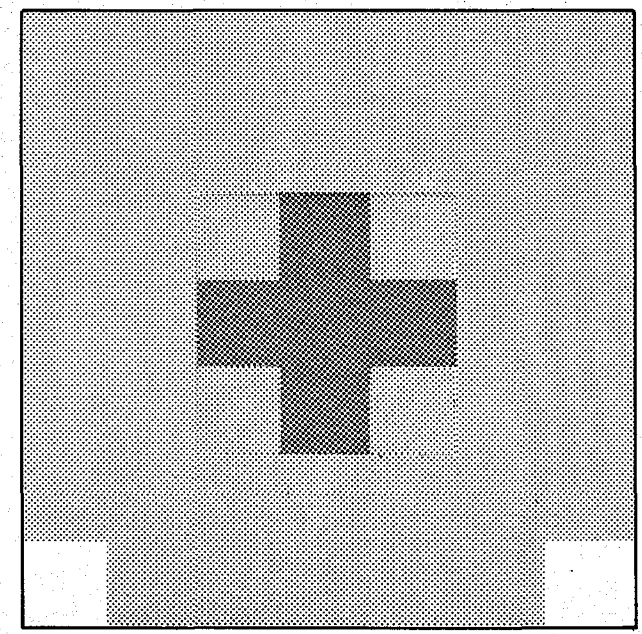


FIGURE 5-5e. SPATIAL DENSITY DISTRIBUTION

Sensitivity of population 1 to transport costs is higher than in the previous figure.

$A_{12} = A_{13} = 0.1$

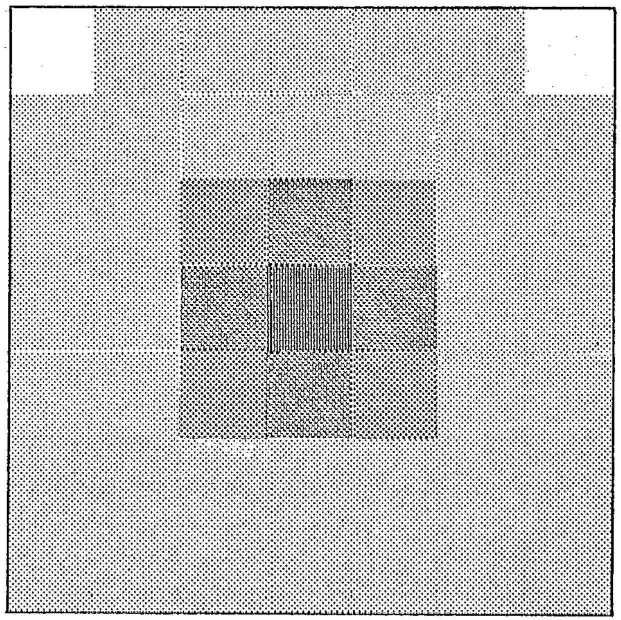
$AZ_1 = AZ_2 = 30$

$CL_1 = 20$

$CL_2 = 5$

$AV1 = 0$

VARIABLE N1



VARIABLE N2

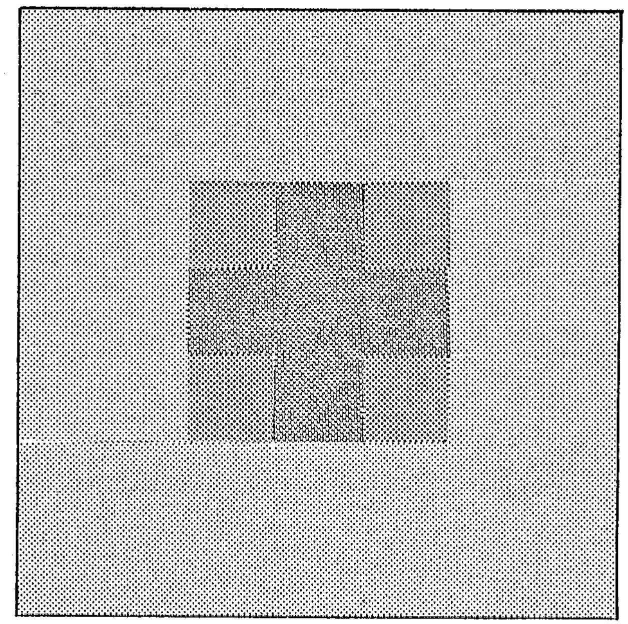


FIGURE 5-5f. SPATIAL DENSITY DISTRIBUTION

Sensitivity of population 1 to transport costs is higher than in the previous figure.

$A_{12} = A_{13} = 0.1$

$AZ_1 = AZ_2 = 30$

$CL_1 = 30$

$CL_2 = 5$

$AV1 = 0$

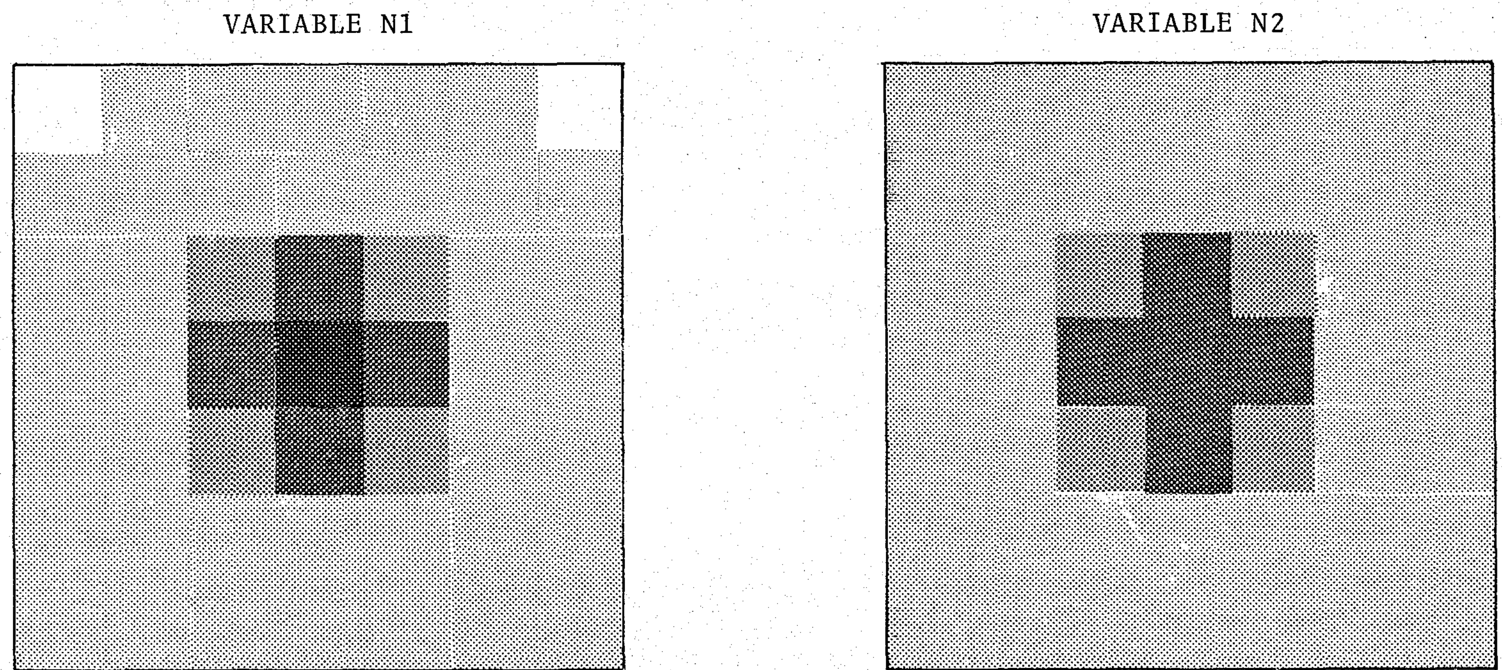


FIGURE 5-5g. SPATIAL DENSITY DISTRIBUTION

Sensitivity of population 1 to transport costs is higher than in the previous figure showing the location of population 1 closer and closer to the center.

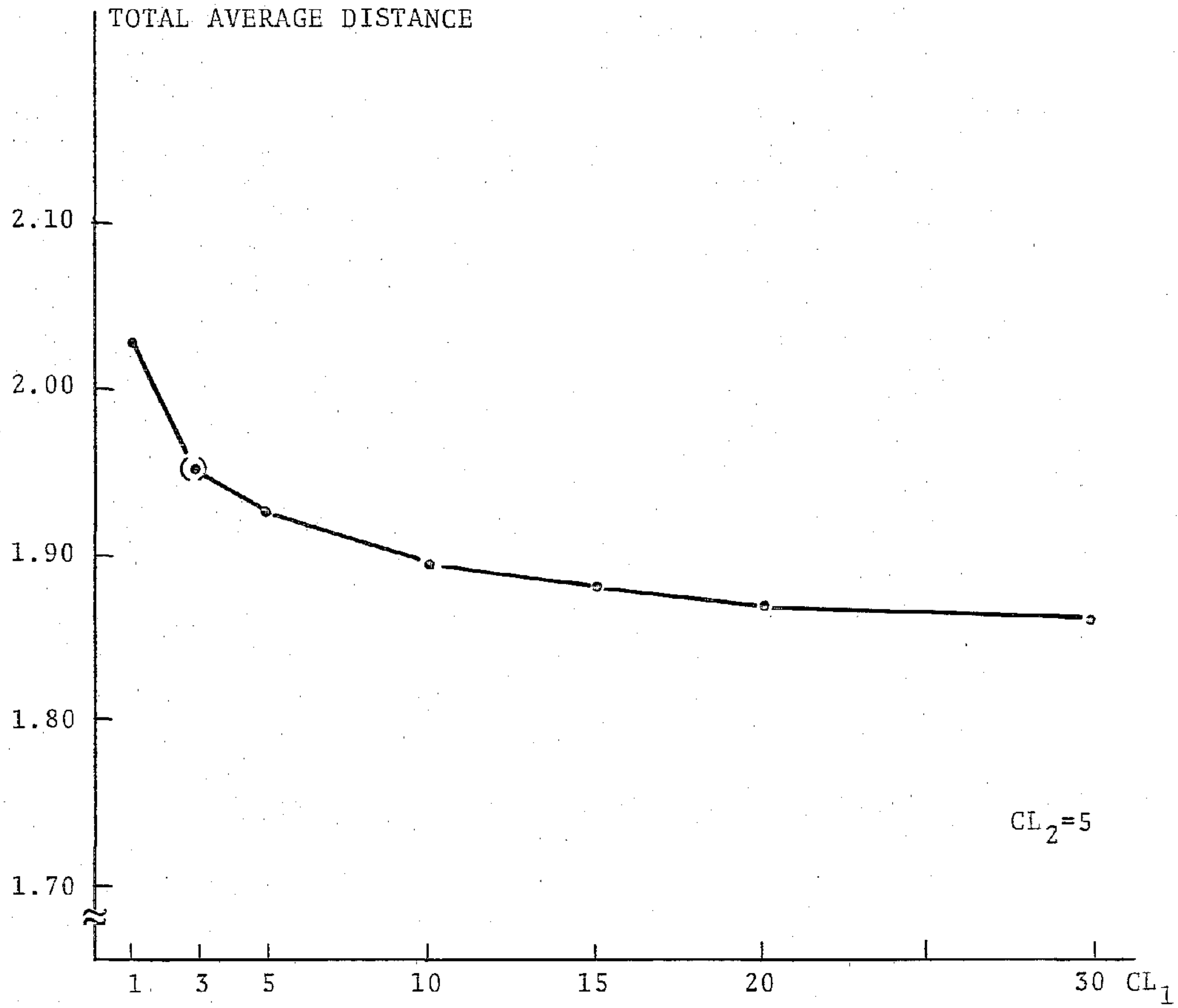


FIGURE 5-6a. TOTAL AVERAGE DISTANCE OF POPULATION FROM THE CENTER AS A FUNCTION OF THE "TRANSPORT" COEFFICIENT CL

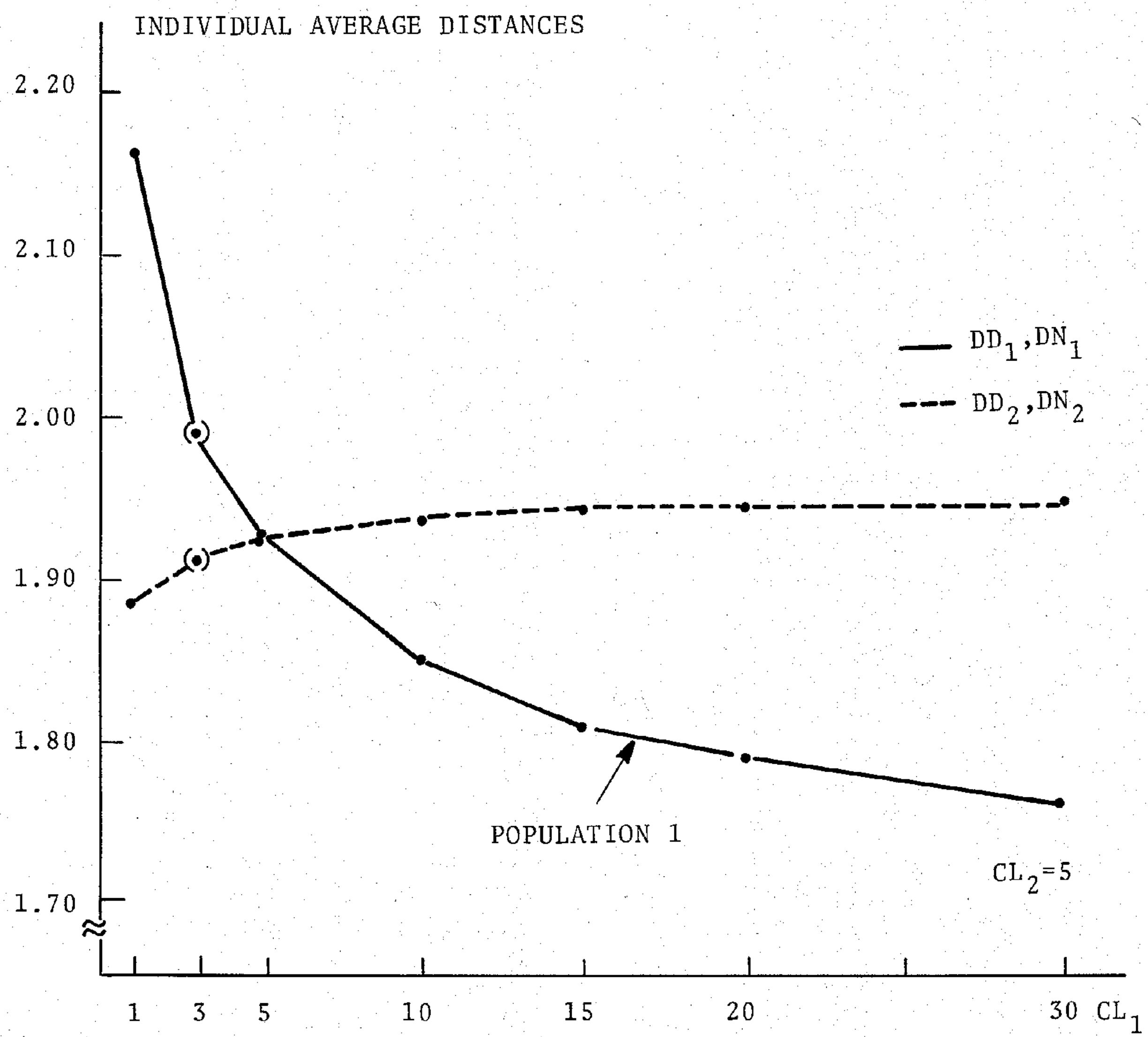


FIGURE 5-6b. AVERAGE DISTANCE OF POPULATION GROUPS FROM THE CENTER AS A FUNCTION OF INCREASING SENSITIVITY OF POPULATION 1 TO TRANSPORT COSTS

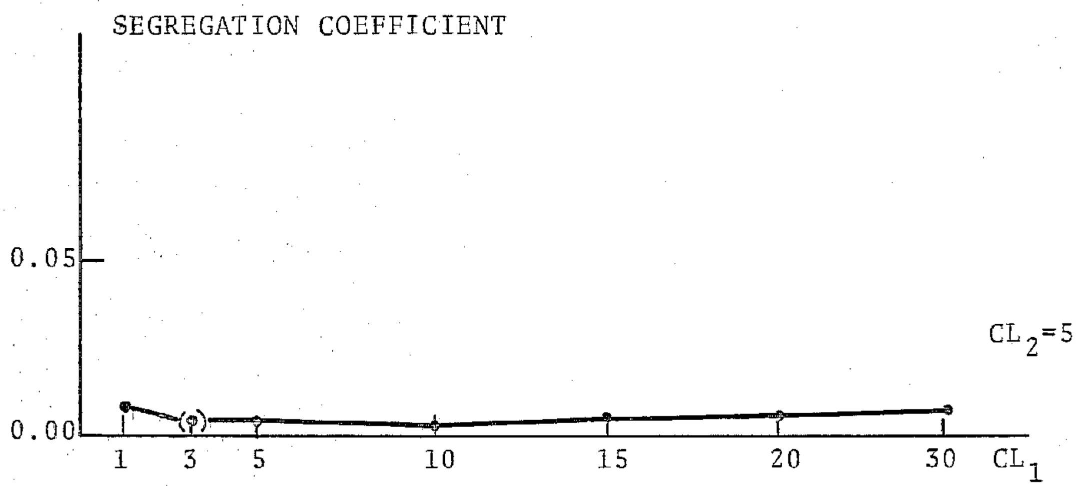


FIGURE 5-6c. SEGREGATION COEFFICIENT AS A FUNCTION OF SENSITIVITY OF POPULATION 1 TO TRANSPORT COSTS

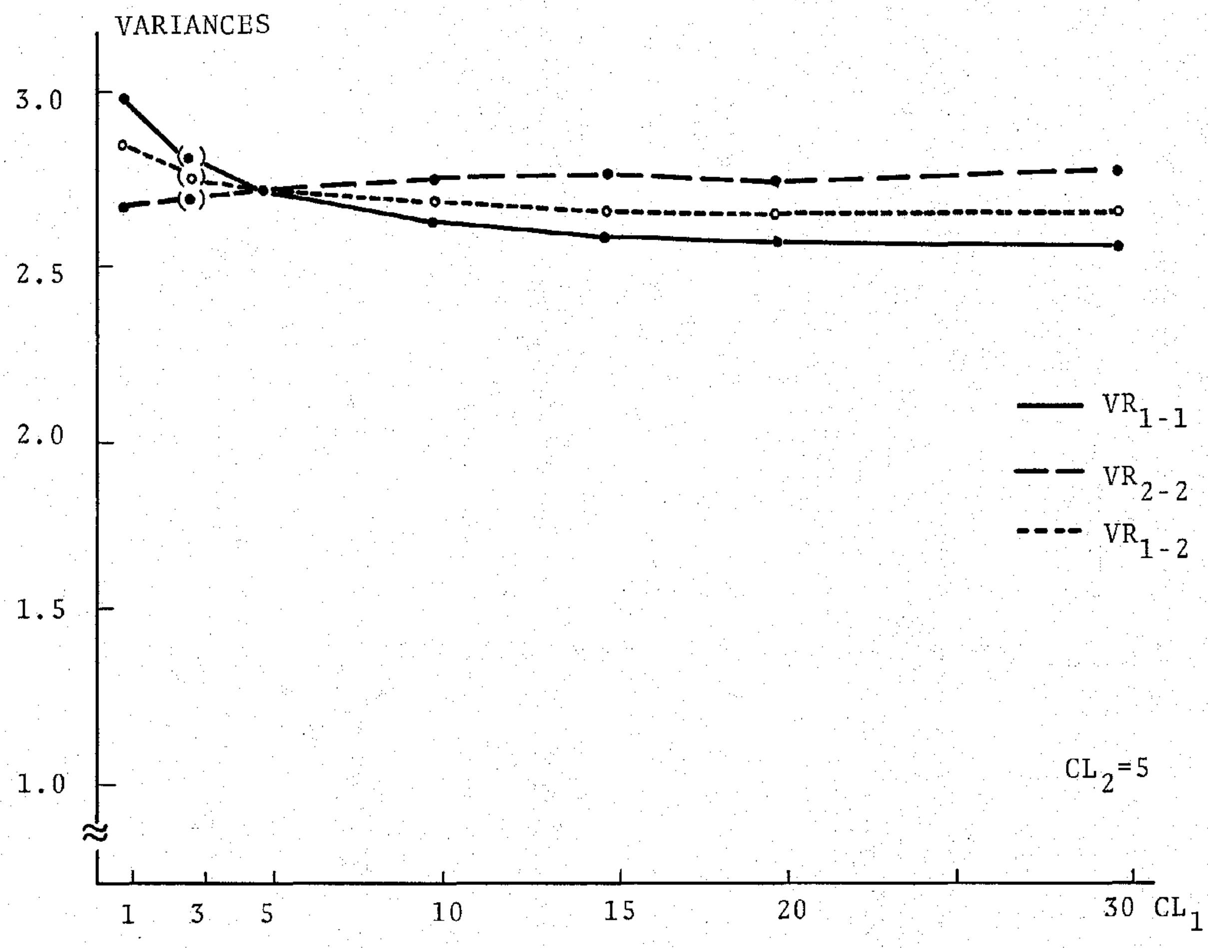


FIGURE 5-6d. INTER- AND WITHIN-GROUP "SPREADING" AS A FUNCTION OF THE SENSITIVITY OF POPULATION 1 TO TRANSPORT COSTS

$A_{12} = 0.1$

$A_{13} = 0.01$

$AZ_1 = AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 10$

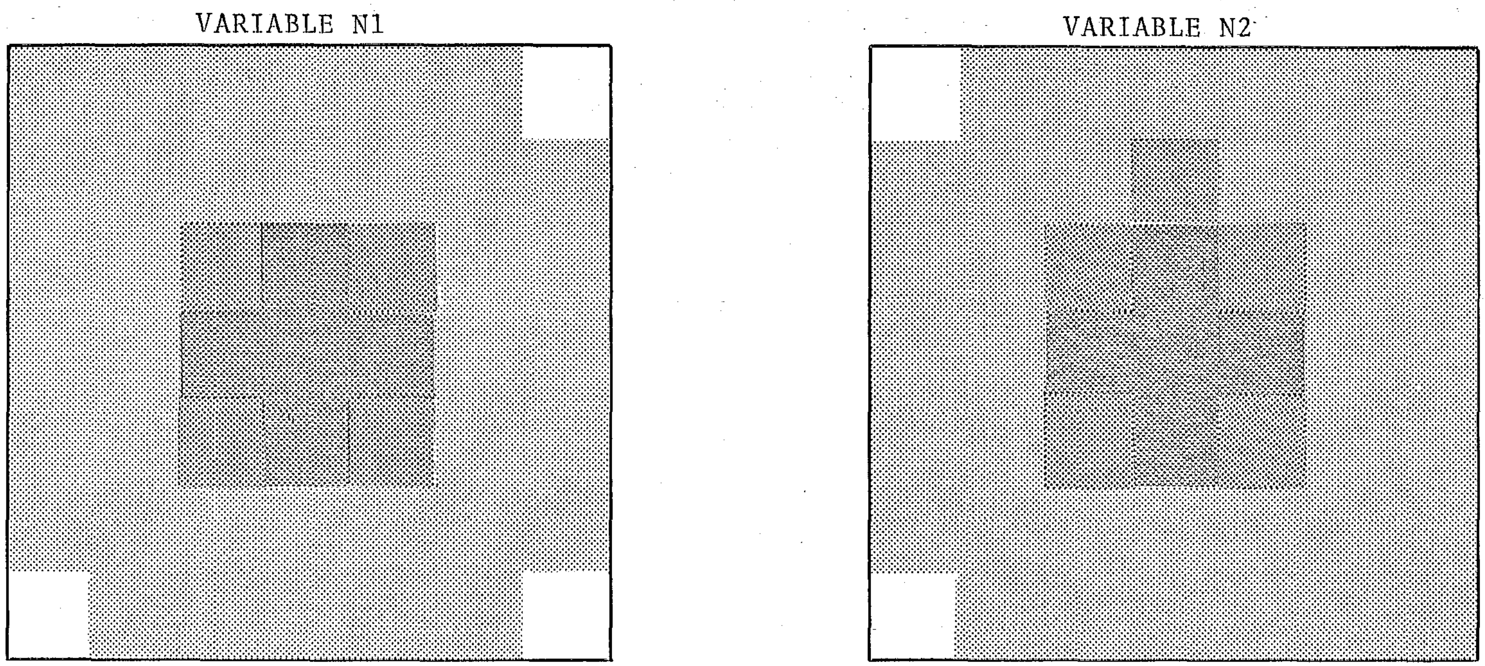
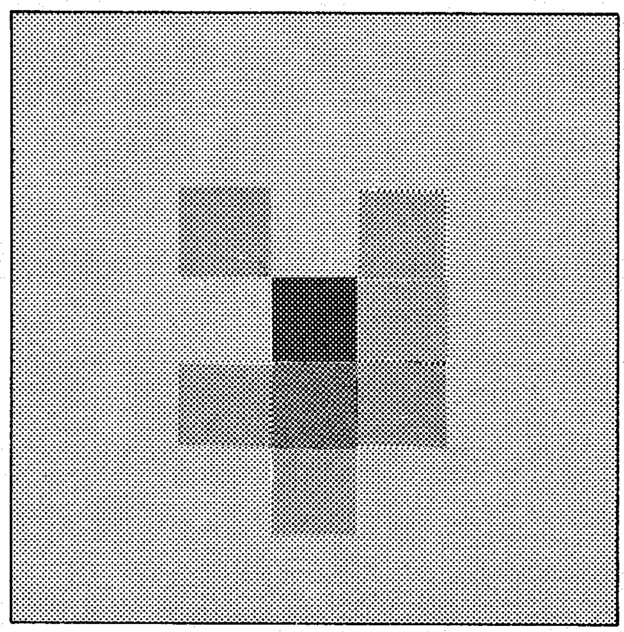


FIGURE 5-7a. SPATIAL DENSITY DISTRIBUTION AS A FUNCTION OF THE REPULSION-ATTRACTION PARAMETER, AV1

AV1 is minimum in this figure.

$A_{12} = 0.1$
$A_{13} = 0.01$
$AZ_1 = AZ_2 = 30$
$CL_1 = CL_2 = 5$
$AV1 = 75$

VARIABLE N1



VARIABLE N2

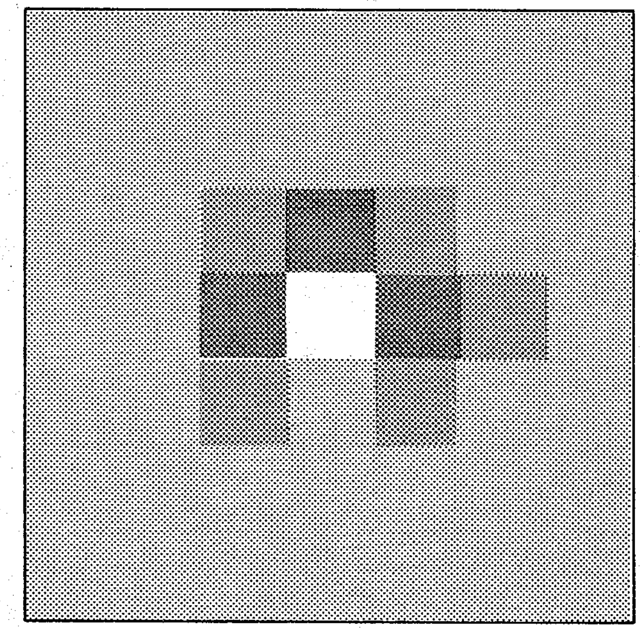


FIGURE 5-7b. SPATIAL DENSITY DISTRIBUTION AS A FUNCTION OF THE REPULSION-ATTRACTION PARAMETER, AV1

AV1 is larger than in the preceding figure.

$A_{12} = 0.1$

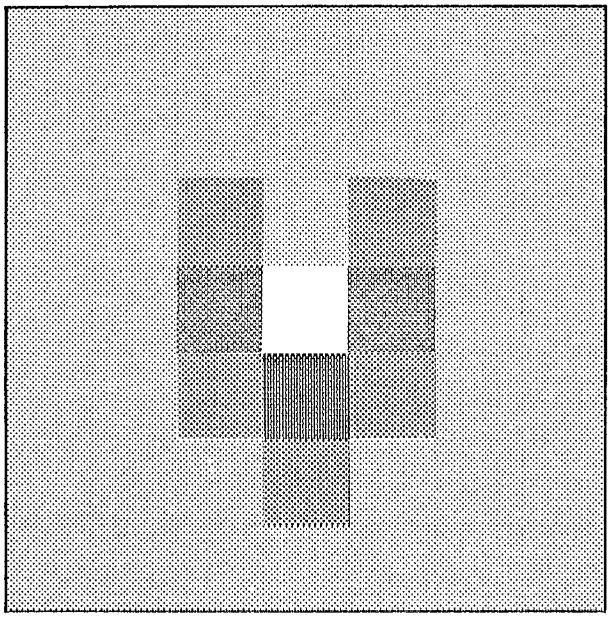
$A_{13} = 0.01$

$AZ_1 = AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 100$

VARIABLE N1



VARIABLE N2

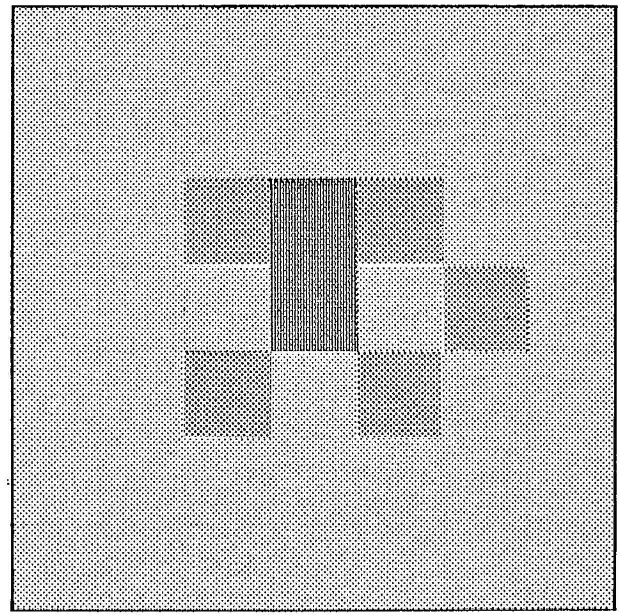


FIGURE 5-7c. SPATIAL DENSITY DISTRIBUTION AS A FUNCTION OF THE REPULSION-ATTRACTION PARAMETER, AV1

AV1 is larger than in the preceding figure.

$A_{12} = 0.1$

$A_{13} = 0.01$

$AZ_1 = AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 300$

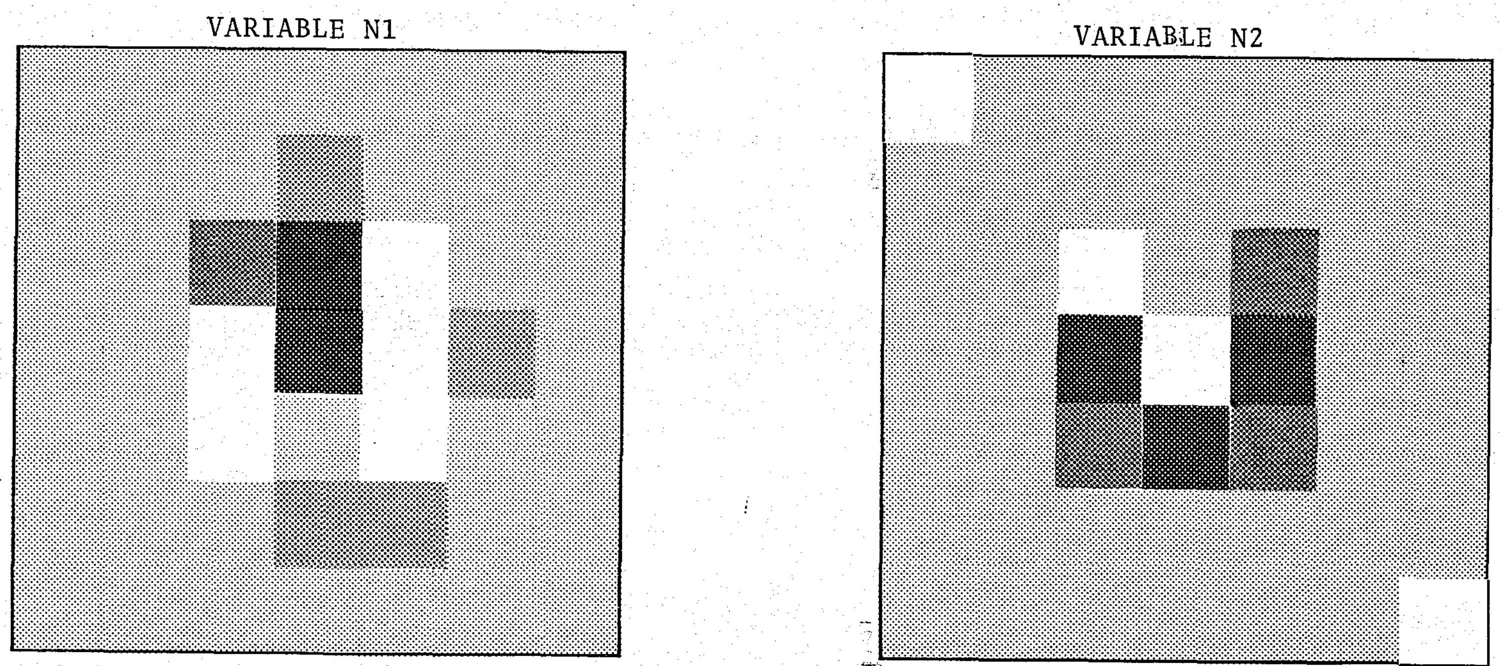


FIGURE 5-7d. SPATIAL DENSITY DISTRIBUTION AS A FUNCTION OF THE REPULSION-ATTRACTION PARAMETER, AV1

AV1 is larger than in the preceding figure.

$A_{12} = 0.1$

$A_{13} = 0.01$

$AZ_1 = AZ_2 = 30$

$CL_1 = CL_2 = 5$

$AV1 = 650$

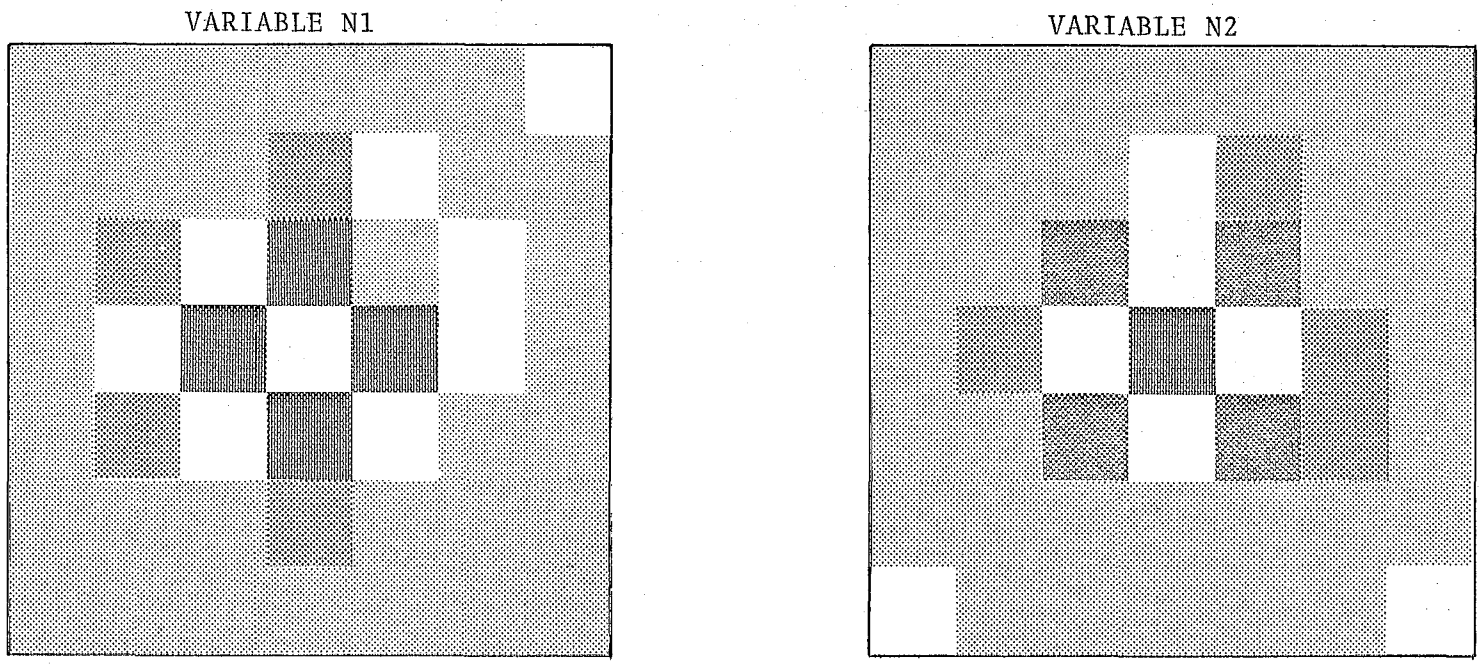


FIGURE 5-7e. SPATIAL DENSITY DISTRIBUTION AS A FUNCTION OF THE REPULSION-ATTRACTION PARAMETER, AV1

AV1 is larger than in the preceding figure.

$A_{12} = 0.1$

$A_{13} = 0.01$

$AZ_1 = AZ_2 = 30$

$CL_1 = CL_2 = 5$

AV1 = 1000

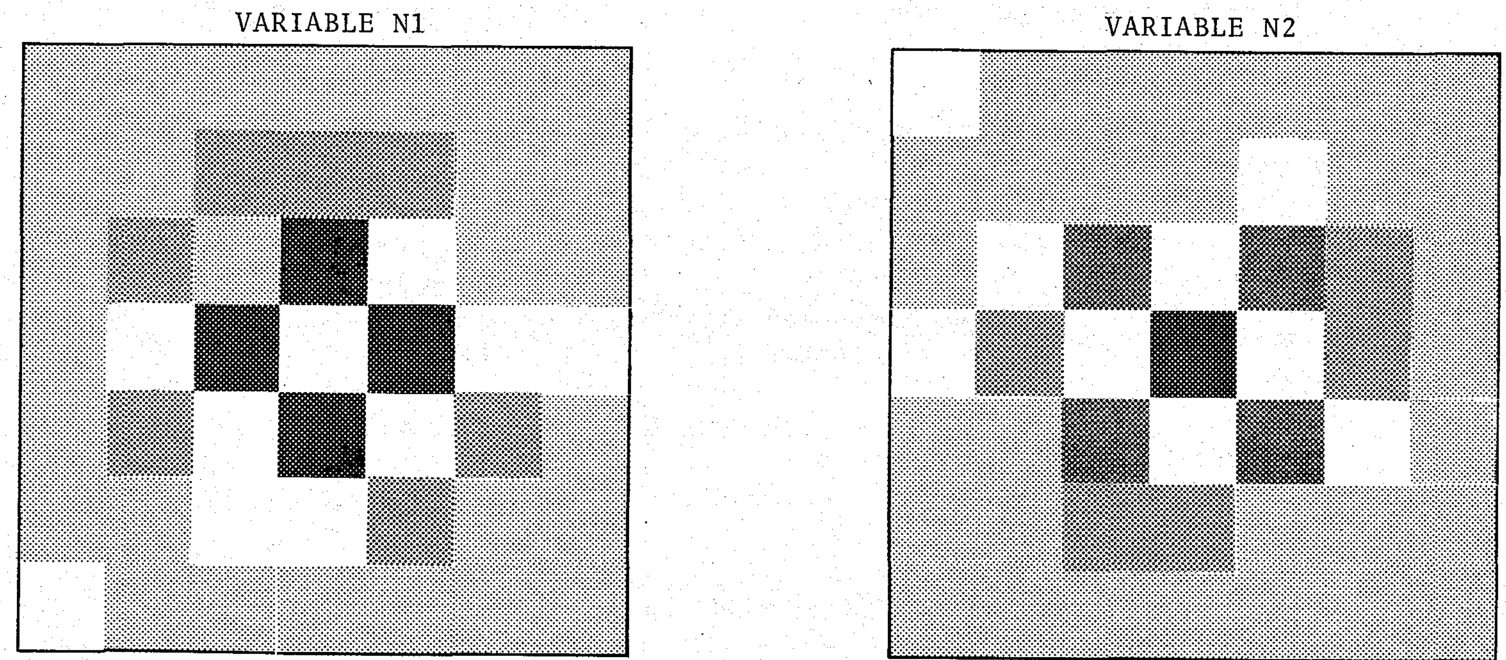


FIGURE 5-7f. SPATIAL DENSITY DISTRIBUTION AS A FUNCTION OF THE REPULSION-ATTRACTION PARAMETER, AV1

AV1 is larger than in the preceding figure.

sequence of events which is responsible for this spatial behavior. With the increase of the repulsion-attraction parameter, a sectorial tendency appears in the pattern and imposes itself, and nucleation takes place. It seems that the three classical tendencies of structuration - concentric, sectorial, and nuclear - combine in various proportions by chance. Whereas the pattern varies in a rather chaotic way with the gradual increase of the value of the parameter, the overall measurements of the spatial structure are relatively indifferent to the pattern (Figure 5-8a, b, c, d). The segregation coefficient regularly and strongly increases; the total average distance increases and stabilizes rather quickly; internal and intergroup variances show very little variation; and alone the individual average distances exhibit larger variations. It seems that in the course of an increasing segregation process the two populations do not see much change in their own spatial distributions but rather move with respect to the center.

Finally we have combined the case of ethnic type segregation (A_{12} = 0.1; A_{13} = 0.01; AVI = 300) with the case of an economic type segregation by differentiating the rate of friction to distance of the two populations (CL_1 = 30; CL_2 = 5). The occurrence of chance leads to two very different patterns which seem to be significant (Figure 5-9):

1. The first pattern (analyzed in detail as a typical example of the simulation) exhibits a share of the high density areas between the two populations: population 1, characterized by a lower income than the other one, occupies the center and the second ring; population 2 is located in the first ring. Each of them has a few isolated nuclei in the periphery. On the whole, the larger part of the urban area has a well mixed population.

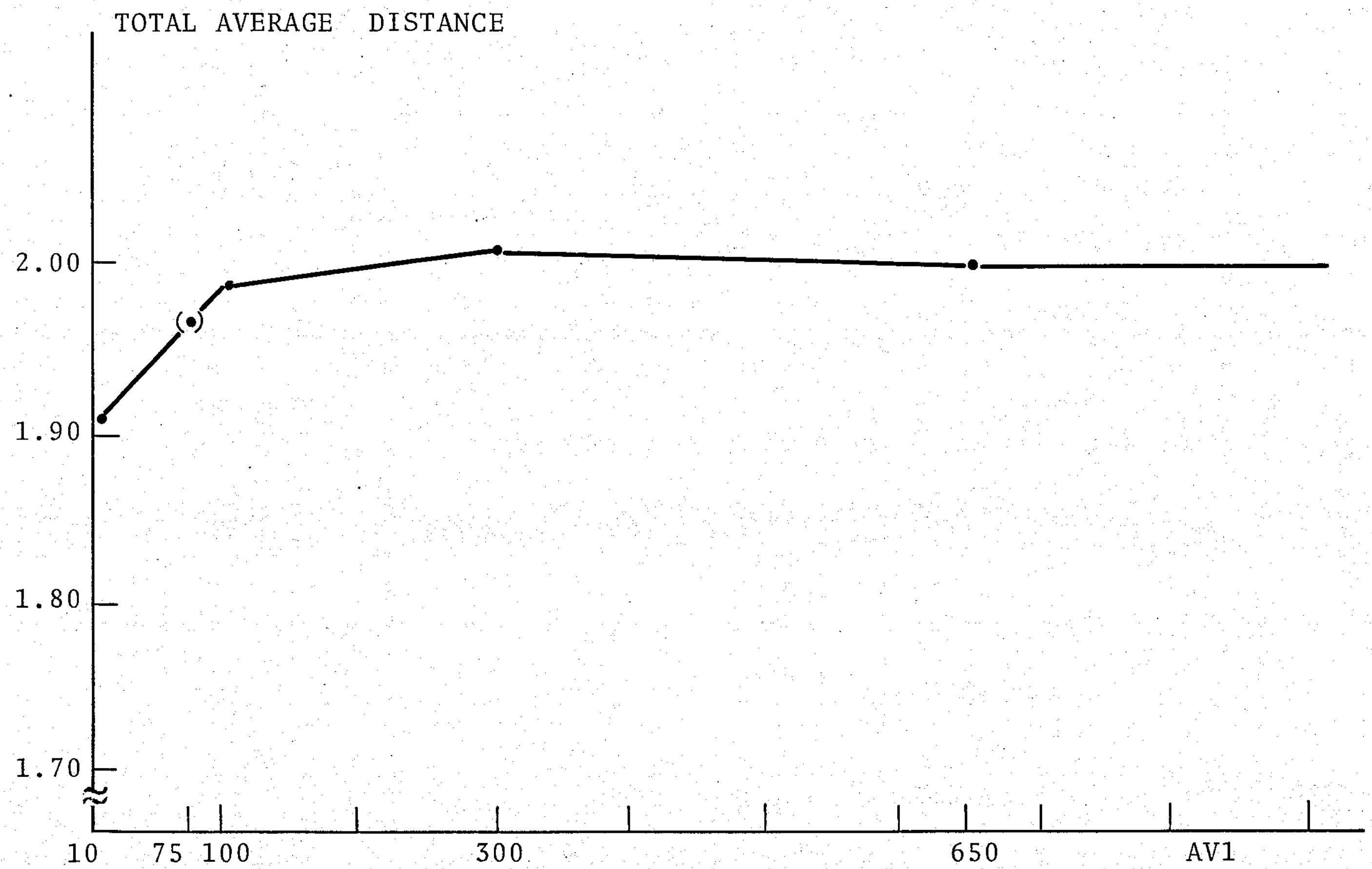


FIGURE 5-8a. AVERAGE DISTANCE OF THE TOTAL POPULATION FROM THE CENTER AS A FUNCTION OF THE REPULSION-ATTRACTION PARAMETER

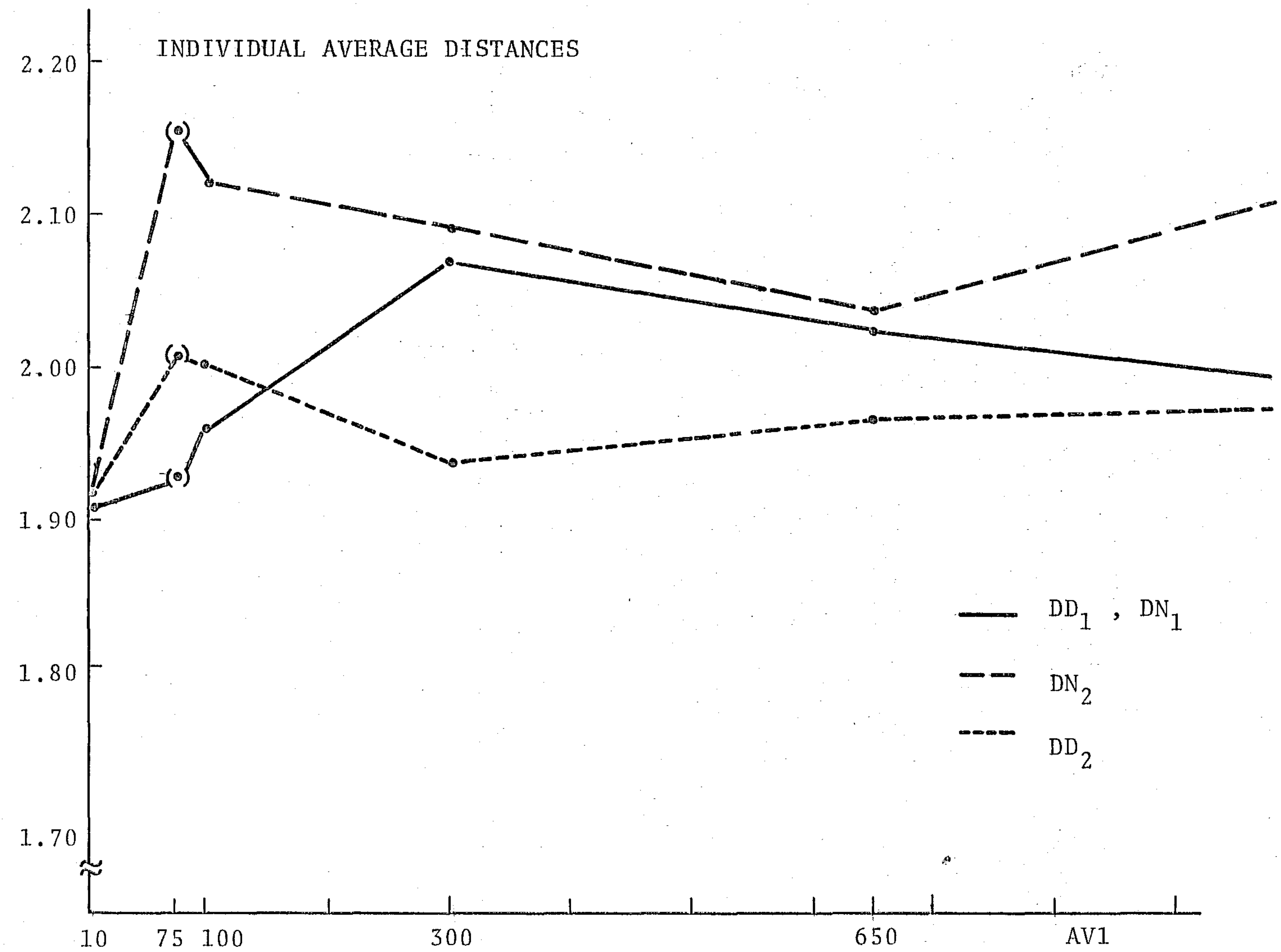


FIGURE 5-8b. AVERAGE DISTANCES FROM THE CENTER AS A FUNCTION OF THE REPULSION-ATTRACTION PARAMETER (SEE EQUATION 5-11, 5-12, FOR DEFINITION OF AVERAGE DISTANCES, DD, DN)

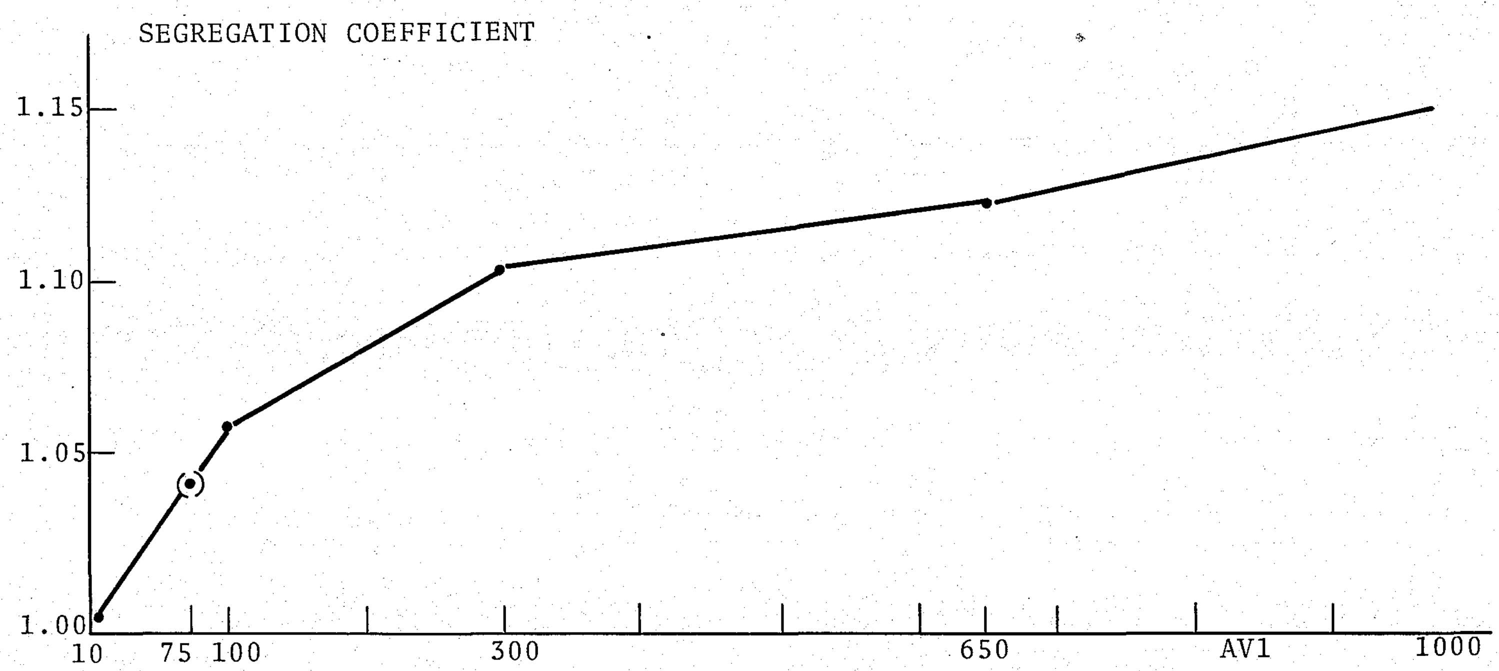


FIGURE 5-8c. SEGREGATION COEFFICIENT AS A FUNCTION OF THE REPULSION-ATTRACTION PARAMETER

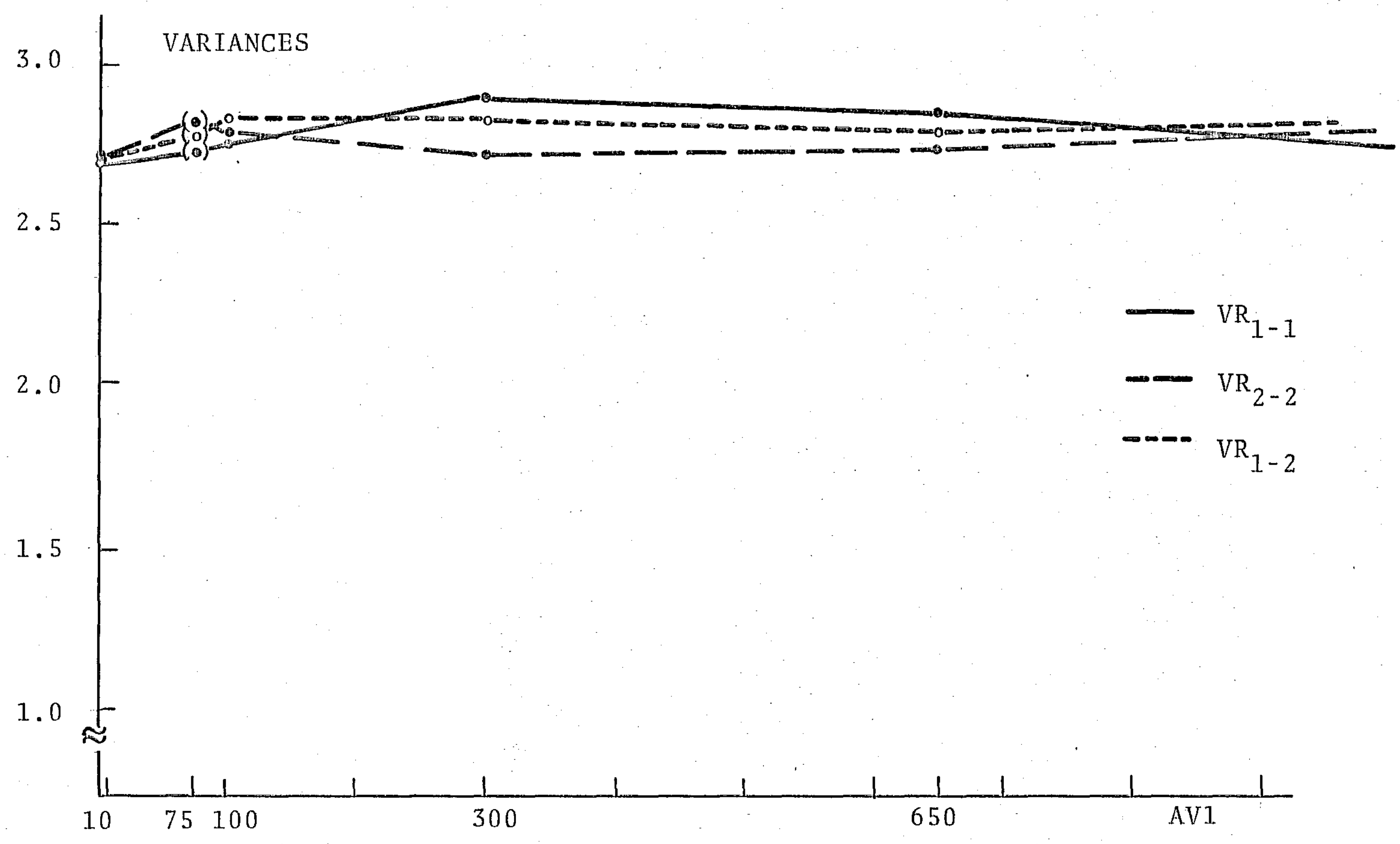
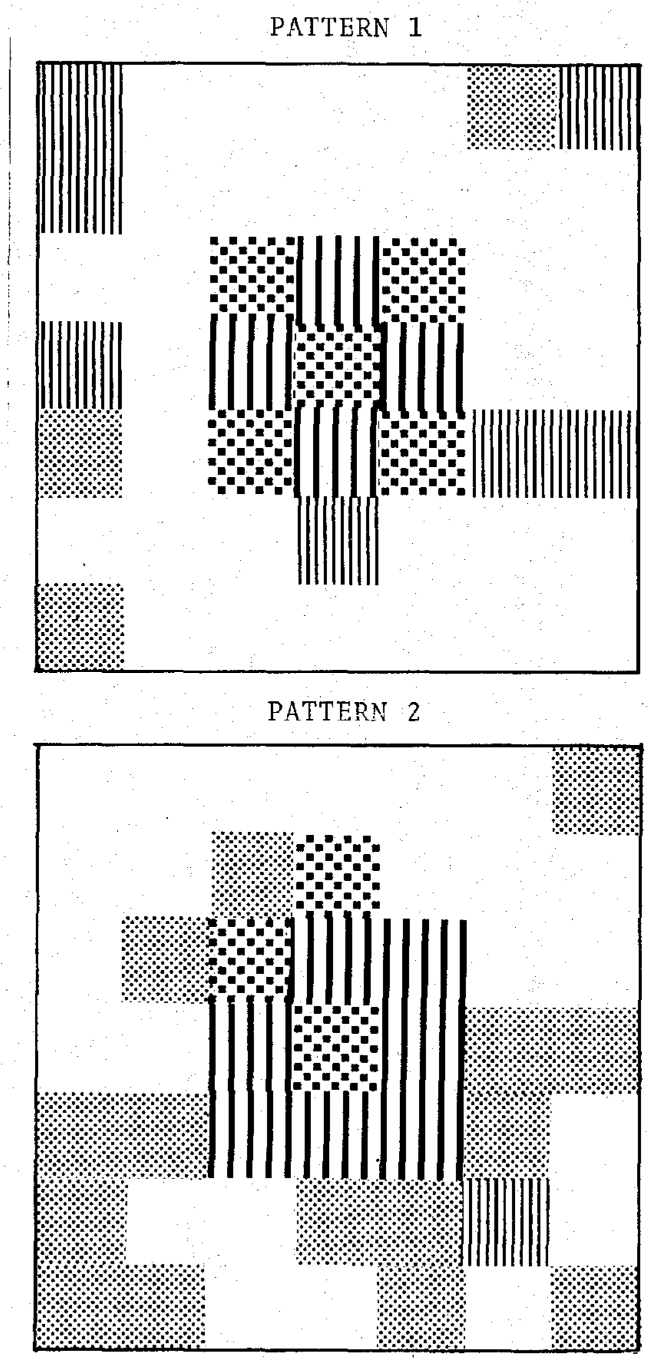


FIGURE 5-8d. INDIVIDUAL AND BETWEEN GROUP DISTANCE VARIATIONS



$A_{12} = 0.1$

$A_{13} = 0.01$

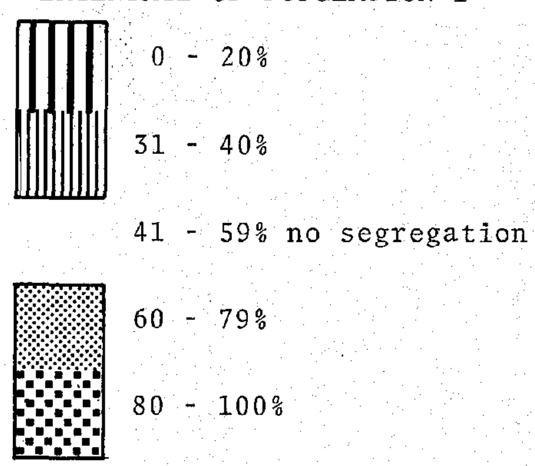
$AZ_1 = AZ_2 = 30$

$CL_1 = 30$

$CL_2 = 5$

$AVI = 300$

PERCENTAGE OF POPULATION 1



Population 1 (the poorer group) predominates in the dotted areas; population 2 predominates in the areas shown with vertical lines.

FIGURE 5-9. SPATIAL POPULATION DENSITY DISTRIBUTION WITH A RELATIVELY HIGH REPULSION-ATTRACTION PARAMETER, AVI = 300 AND RELATIVELY STRONG DESIRES FOR WITHIN-GROUP INTERACTIONS, $A_{12} = 0.1$, $A_{13} = 0.01$; SENSITIVITY OF POPULATION 1 TO TRANSPORT COSTS RELATIVELY LARGE COMPARED WITH THAT OF POPULATION 2 ($CL_1 = 6\ CL_2$)

With some imagination, one can make some analogy between this pattern and the classical case of the American city where low class urban people are located in the center and higher class people in the periphery, the whole being surrounded by a ring of lower class rural people (see Appendix B, Figures 13 and 14).

2. The second pattern, different than the first because of different chance events or history, with a similar segregation coefficient has this time, however, the larger part of its space subject to residential segregation. Higher class people (population 2) occupy a very large part of the high density areas, leaving to others only the center and a small area to the northwest (Figure 5-9). The low class people are pushed away to the southern periphery where they form two vast areas of domination.

In the same spirit as the above analogy, one can conjecture here about the resemblance of this pattern to the case of a French city almost entirely occupied by priviledged people (maybe because of the architectural quality of the center) and pushing away low class people into vast suburbs. Another analogy which this brings to mind is the case of a Third World city facing the problem of important immigration that cannot be absorbed by the old central neighborhoods. These "urban villagers" locate thus in large peripheral squatter zones.

5.3 URBAN TRANSPORT AND POPULATION DISTRIBUTION

Following an idea of Bussiere* we have compared the effects of changing the coefficients of our distance function (transport effects) with his observations of several cities, particularly Paris and London.

The radial distribution of population density from the center to the periphery follows an exponential law:

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*Private communication.

$$P(d) = P_c \exp(- \beta d) \tag{5-21}$$

where d is the distance from the center, P_c the population density at the center, and β the slope of the curve. It is easy to show that $2/\beta$ is the average distance of inhabitants from the center. ($2/\beta$ = DD). The function P_c (β) (Figure 5-10) shows brutal discontinuities which separate long periods of steady evolution. The disruption may be explained by the introduction of new transportation modes. For example, discontinuities which are observed in 1852 in London's curve may be attributed to the development of the railway sytem; the one observed in 1901 in London's and Paris' curves may be related to the development of public transport and cars.

In order to see the effect of the introduction of a new transportation system on the spatial distribution of an urban population, we have studied a growing urban system which undergoes an abrupt change in the values of its transport parameters, in the course of the simulation. With the system of equations described in Section 3, let us choose a simple form for the location function:

$$F_i = \frac{1}{(A + B P_i + C d_{ic}^2)} \tag{5-22}$$

$$d_{ii} = 0.522$$

$$\frac{dP_i}{dt} = (D \frac{F_i}{\sum_{j=1}^{n} F_j} - P_i) \ . \tag{5-23}$$

The location function F_i does not take into account cooperative processes; it contains only inhibiting processes: a population density effect ($B P_i$) on the one hand and distance from the center ($C d_{ic}$) on the other hand. With this location function, Equation 5-23 describes the evolution of the urban population. The Figure 5-11 represents the function $P_c(\beta)$ for different simulations.

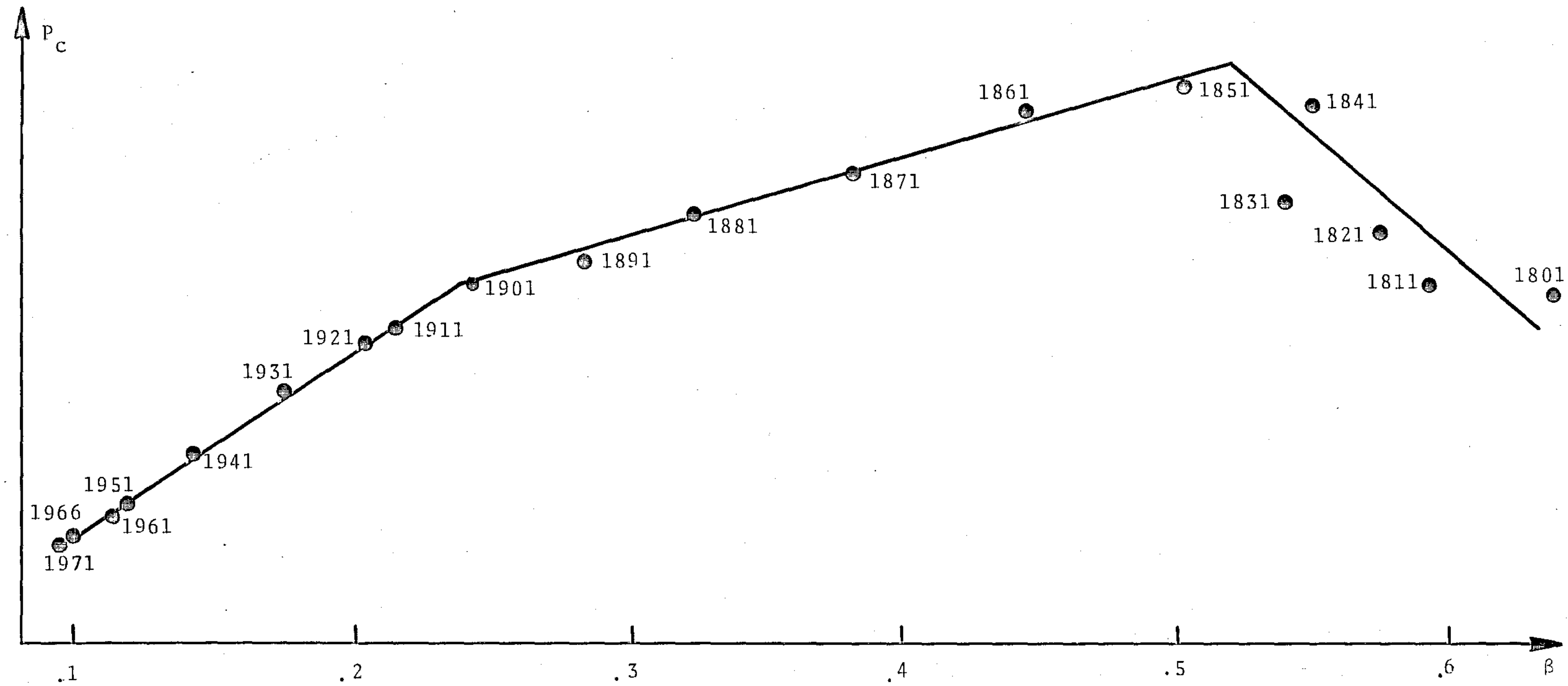

FIGURE 5-10a. POPULATION DENSITY AS A FUNCTION OF β FOR DIFFERENT YEARS, LONDON*

*From R. Bussière

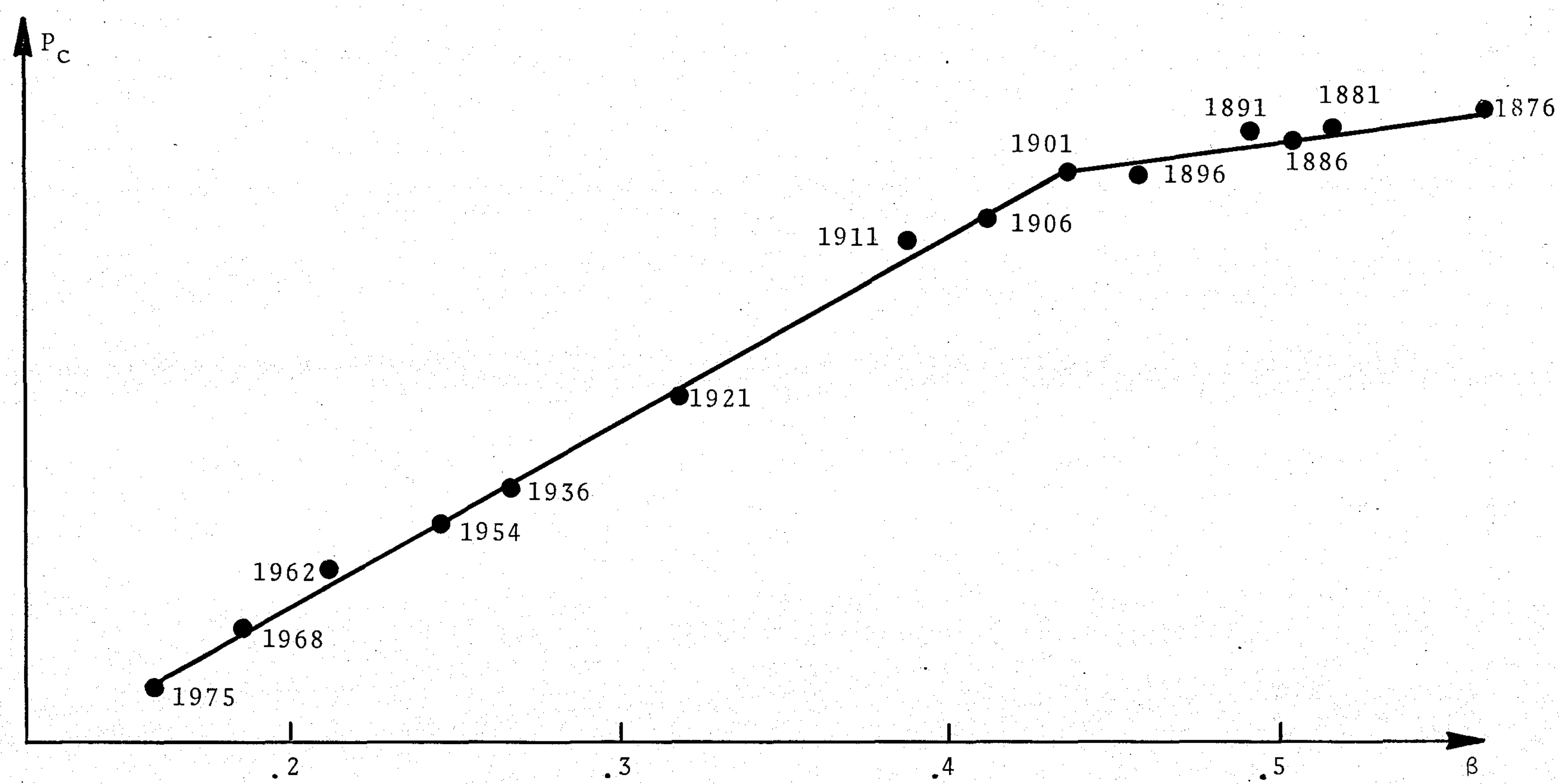

FIGURE 5-10b. POPULATION DENSITY AS A FUNCTION OF β FOR DIFFERENT YEARS, PARIS*

*From R. Bussière

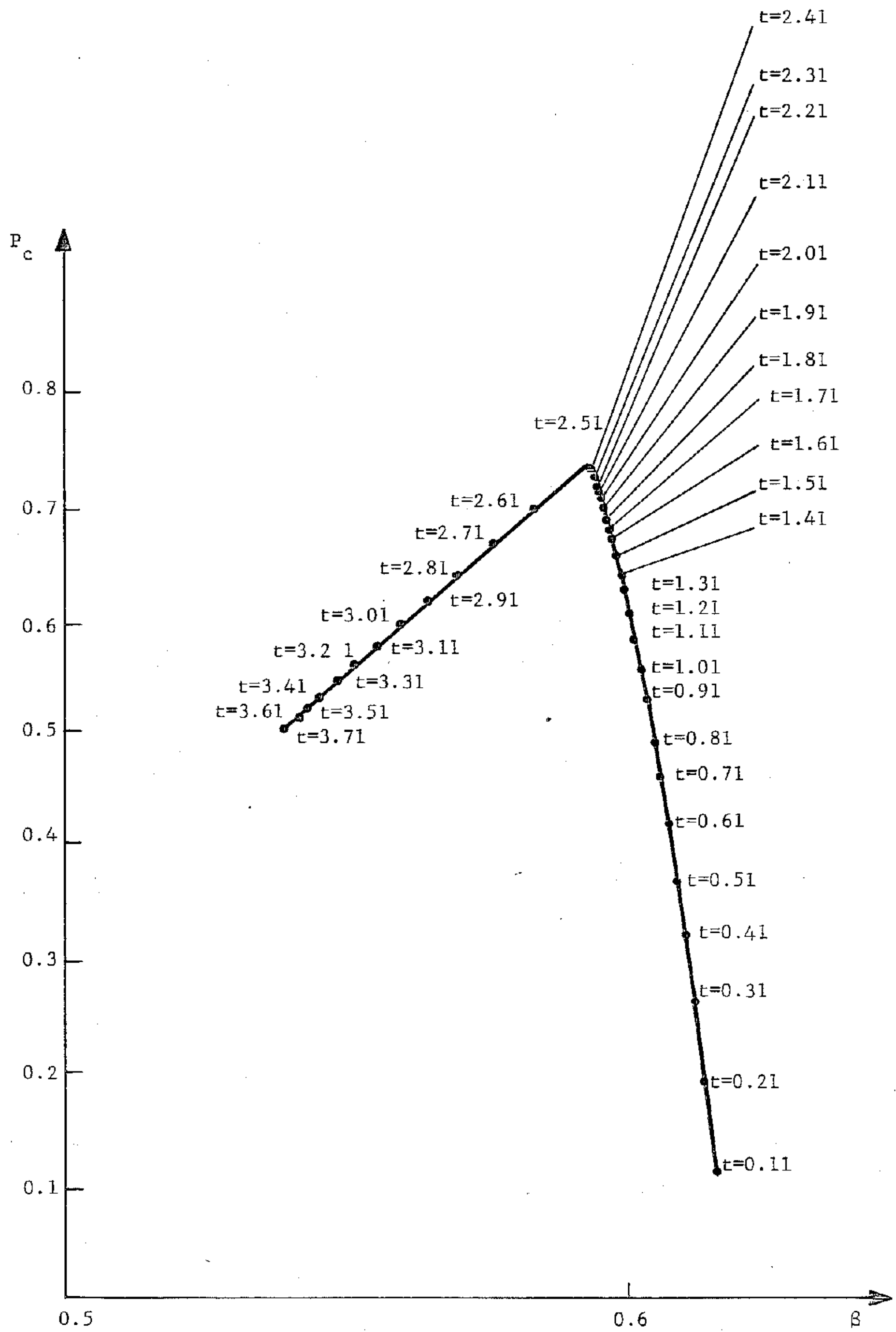

FIGURE 5-11a. CENTRAL CITY DENSITY P_c AS A FUNCTION OF β FOR DIFFERENT TIMES IN THE CITY'S GROWTH

At time t=2.5, a new easier form of transportation has been introduced.

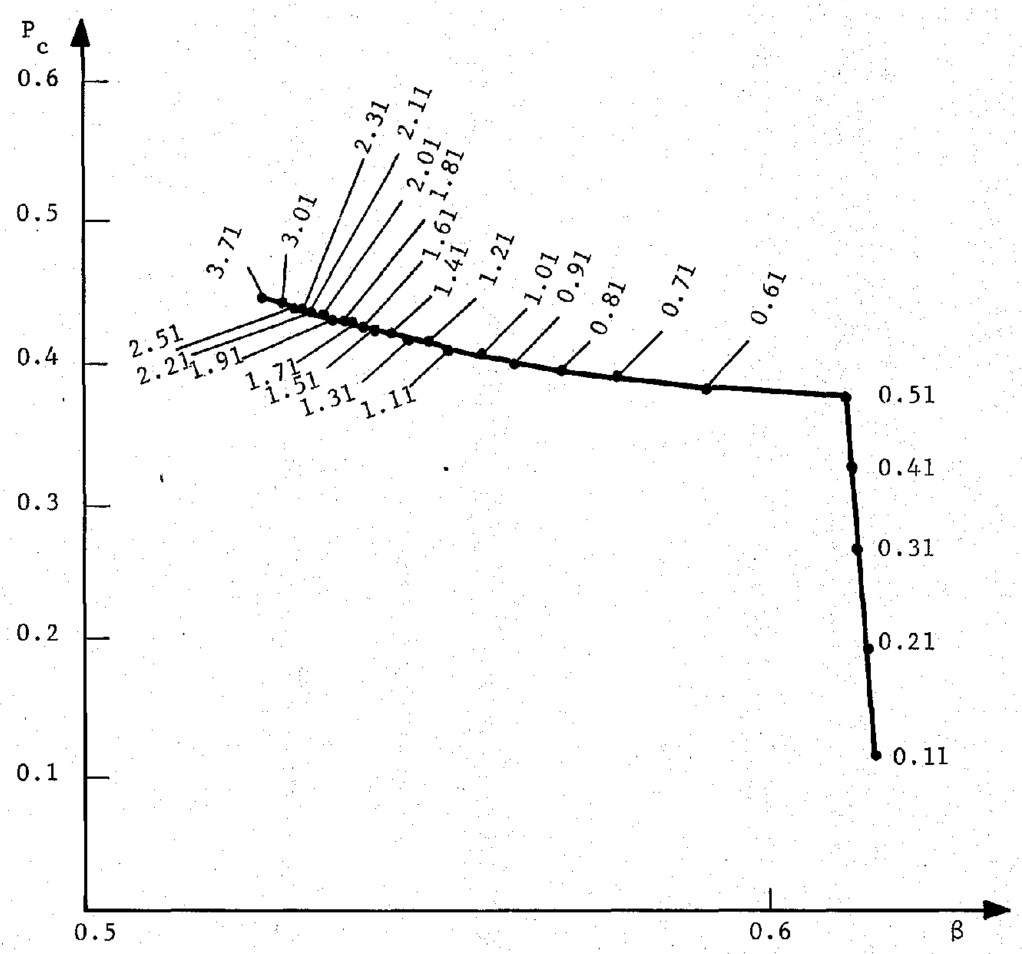

FIGURE 5-11b. CENTRAL CITY POPULATION DENSITY AS A FUNCTION OF β FOR DIFFERENT TIMES IN THE GROWTH OF THE CITY

At time t=0.5 a new easier transportation system has been introduced.

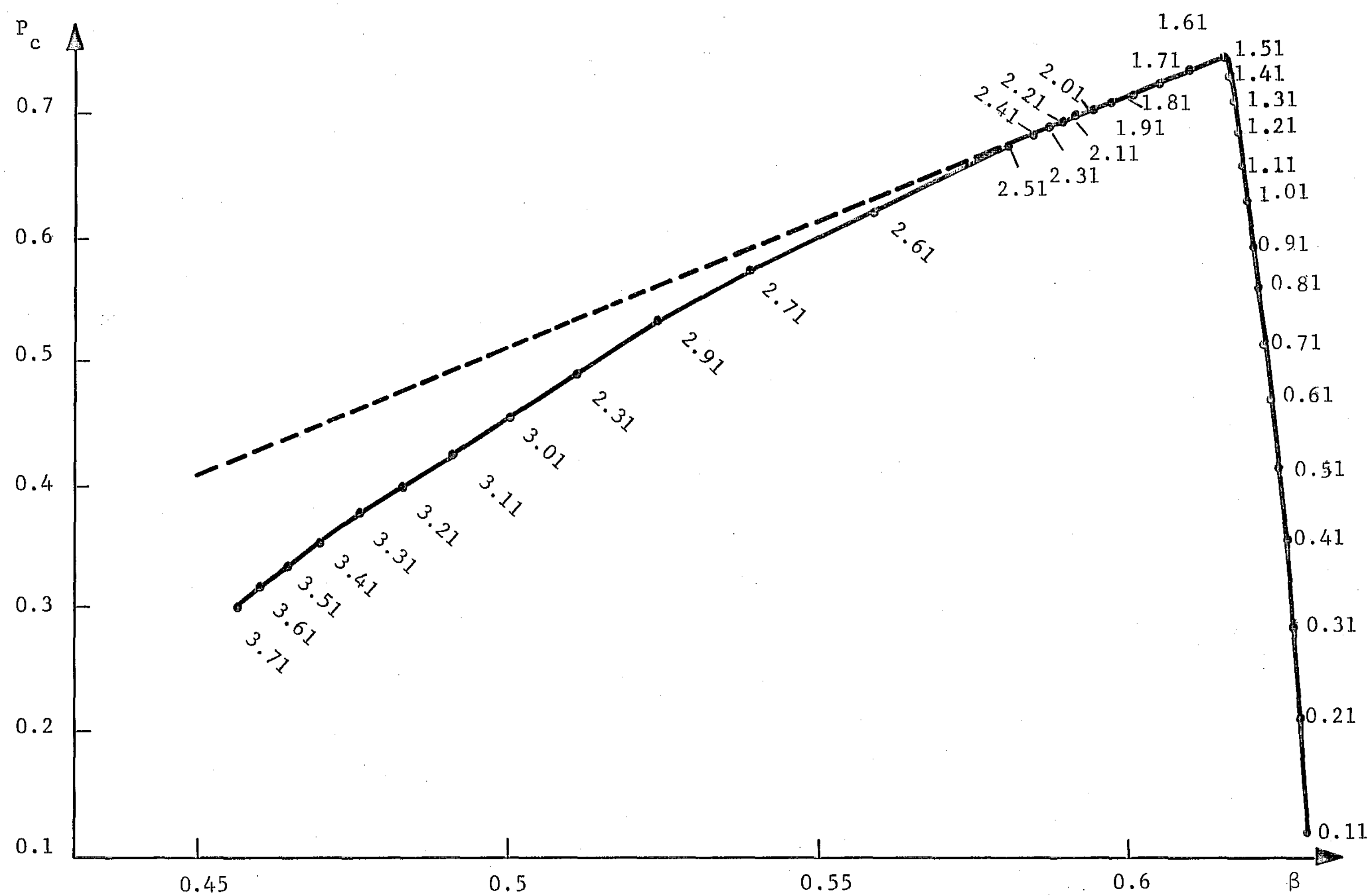

FIGURE 5-11c. CENTRAL CITY POPULATION DENSITY AS A FUNCTION OF β FOR DIFFERENT TIMES IN THE GROWTH OF THE CITY

At times t=1.51 and t=2.5 new easier modes of transportation have been introduced.

For Figure 5-11a the values of the parameters of Equations 5-22, 23 are the following: A=0.5; B=1, C=5; D=5. At time t=2.5, the value of the transport coefficient C=5 is changed to a lower value C=1 (producing easier transportation). At this moment, a discontinuity appears in the function $P_c(\beta)$ and the population becomes more dispersed, the density at the center, P_c, falling considerably.

In Figure 5-11b the simulation has the same values of the parameters as above, however, the value of C is changed at the earlier time t=0.5. While the slope of $P_c(\beta)$ is always negative (indicating increasing center densities), the orders of magnitude are very different before and after the change in the value of the transport coefficient. After the change to easier transport, the rapid increase in center densities all but ceases.

Figure 5-11c has the same values of the parameters except for the transport coefficient C and the time at which the change takes place. The simulation begins with: C=9 and at time t=1.51 the value of C is reduced to 3. A second change is imposed at t=2.5, C=0.1. The first change has a definite impact on the curve causing central city densities to decline rapidly; the second change is less visible, although the slopes are different before and after this second change. There is some delay between the change and the moment the change affects the result, (the modification appears at time t=2.71), further reducing the central city densities.

5.4 MODIFICATION OF THE EXPONENTIAL DISTRIBUTION BY THE DEVELOPMENT OF CENTRAL CRATERS

In the previous section the possible competition between inhabitants and jobs was not considered. This, however, may be done by explicitly allowing for such competition in the location function. The equation of evolution of an urban population is given by

$$\frac{dP_i}{dt} = \theta\left(D \frac{F_i}{\sum_{j=1}^{n} F_j} - P_i\right) \tag{5-24}$$

where the location function F_i is now:

$$F_i = \frac{1}{(A + B\ P_i + E\ X_i + C\ d_{ic}^2)} \tag{5-25}$$

and where $E\ X_i$ represents the employment effect. For the employment, the equations of evolution are:

$$\frac{dX_i}{dt} = \theta(D - X_i) \qquad i=c \tag{5-26a}$$

$$\frac{dX_i}{dt} = 0 \qquad i \neq c \tag{5-26b}$$

which prescribe an employment that is confined to the central city (i=c).

Figure 5-12a is obtained with the following values for the parameters:

$A = 0.5$, $B = 1$, $C = 1$, $E = 0.2$, $D = 5$, $\theta = 0.2$.

The central residential density is not the maximum here since employment centers must occupy some of this central city land. By increasing E we increase the competitive edge of employment needs for space and hence, the residental crater becomes more and more marked.

Figures 5-12 b and c were obtained with the same values of the parameters as 5-12 a except that E has been increased to 0.5 and 1 respectively (thus giving employment needs a greater competitive edge, and thereby leaving less downtown space for residential purposes).

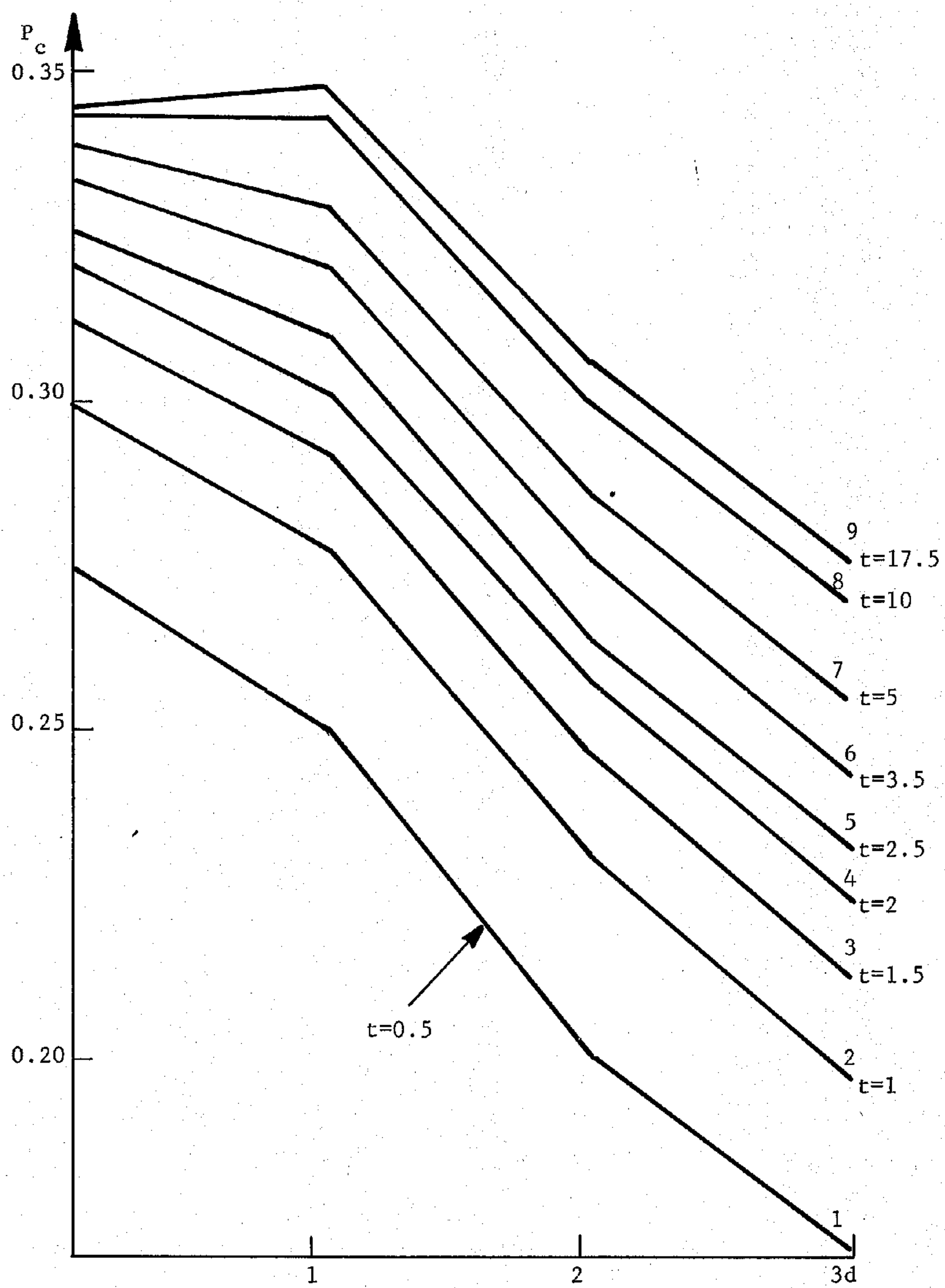

FIGURE 5-12a. POPULATION DENSITY AS A FUNCTION OF DISTANCE FROM THE CENTER OF THE CITY FOR DIFFERENT TIMES IN THE EVOLUTION OF THE CITY SHOWING THE EFFECT OF COMPETITION FOR CENTER CITY LAND BETWEEN RESIDENCES AND JOBS

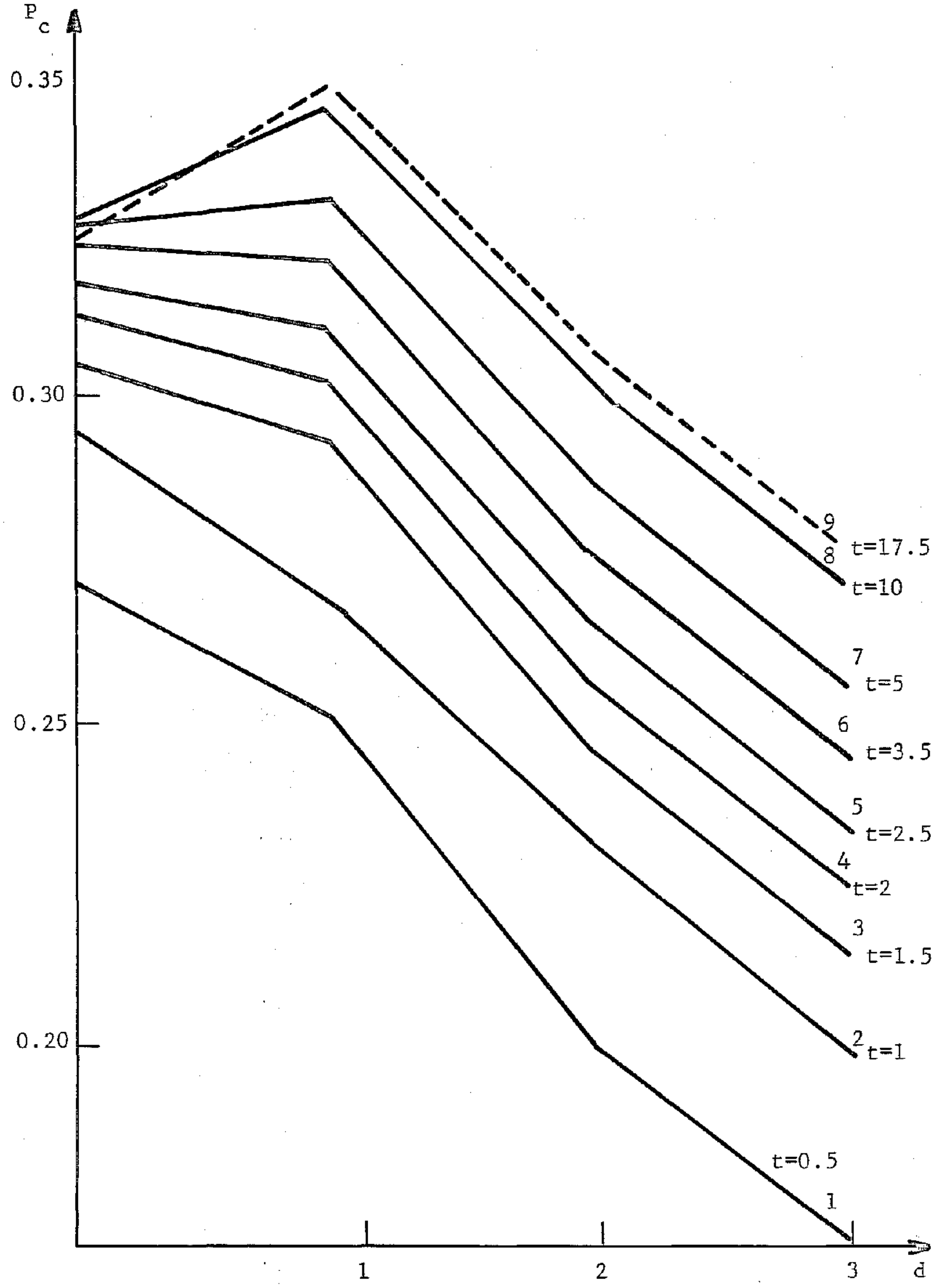

FIGURE 5-12b. TIME EVOLUTION OF URBAN RESIDENTIAL POPULATION DENSITY AS A FUNCTION OF DISTANCE FROM THE CENTER SHOWING EFFECT OF THE COMPETITIVE ADVANTAGE OF EMPLOYMENT NEEDS FOR CENTER CITY LAND

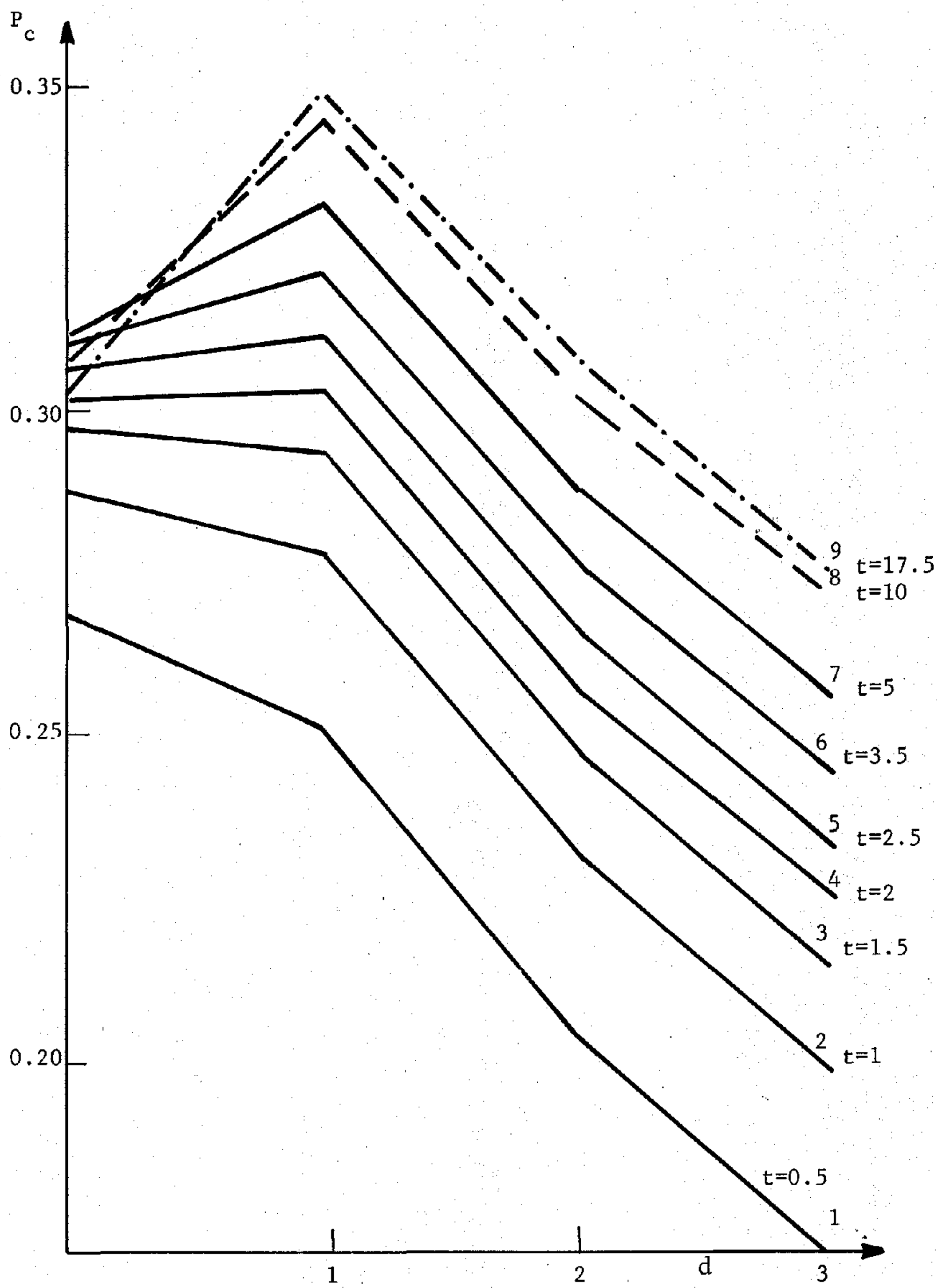

FIGURE 5-12c. TIME EVOLUTION OF URBAN RESIDENTIAL POPULATION DENSITY AS A FUNCTION OF DISTANCE FROM THE CENTER SHOWING THE EFFECT OF THE COMPETITIVE ADVANTAGE OF INDUSTRY FOR CENTER CITY SPACE

It should be noted that these results compare well with the results for the evolution of the distribution of Chicago's population density distribution shown in Figure 1-2.

APPENDIX A

SOME PROPERTIES OF THE EQUATION $\frac{dX_i}{dt} = \left(\frac{DF_i}{A} - X_i\right)$

Here, we shall develop in detail a very simple example of the type of equation that we have discussed in this report, stressing, in particular, the idea of a threshold or a critical amplitude for a fluctuation.

Let us consider a system which is homogeneous except at two points which are the seat of identical economic functions. The two sites are therefore in competition. We have:

$$\frac{dX_i}{dt} = \left(\frac{DF_i}{A} - X_i\right) \qquad i = 1,2$$

where $A = F_1 + F_2$

for the equations describing the evolution of the number of agents at each of the sites considered. Several forms are possible:

a) Suppose that $F_1 = \alpha_1$ and $F_2 = \alpha_2$

where α_1 and α_2 are constants. In this case there is a single stationary state:

$$X_1 = D/(1 + \alpha_2/\alpha_1), \qquad X_2 = D - X_1;$$

and this state is always stable.

b) Suppose that $F_1 = \beta_1 X_1$ and $F_2 = \beta_2 X_2$. There are two stationary states which are possible:

$$X_1 = D; \; X_2 = 0$$

which is stable if $\beta_1 > \beta_2$

and unstable for the inverse condition, $\beta_1 < \beta_2$

and the state:

$$X_1 = 0; \; X_2 = D$$

which has the opposite stability condition, that is, it is stable if $\beta_2 > \beta_1$ and unstable if $\beta_2 < \beta_1$. Clearly the stability of one state implies the instability of the other, and therefore for any particular values of β_1 and β_2 there will only be one state which is possible.

c) Suppose, finally, relations of the type:

$$F_1 = \beta_1 X_1 + \gamma_1 X_1^2 \text{ and } F_2 = \beta_2 X_2 + \gamma_2 X_2^2.$$

The stationary states possible for such a system are:

$X_1 = D$ $X_2 = 0$, which is stable if $\beta_2 < \beta_1 + \gamma_1 D$

and unstable if $\beta_2 > \beta_1 + \gamma_1 D$.

or the state $X_1 = 0$; $X_2 = D$ which is stable if $\beta_1 < \beta_2 + \gamma_2 D$

and unstable if $\beta_1 > \beta_2 + \gamma_2 D$.

Finally, we have the state $X_1 \neq 0$; $X_2 \neq 0$.

This is always unstable if it exists, and its conditions of existence correspond to the condition that one of the other states is stable. The following table describes these various possibilities.

$X_1 = D$; $X_2 = 0$	$X_1 \neq 0$; $X_2 \neq 0$	$X_1 = 0$; $X_2 = D$
STABLE	NONEXISTENT	UNSTABLE
Conditions		
$\beta_2 < \beta_1 + \gamma_1 D$		$\beta_1 > \beta_2 + \gamma_2 D$
UNSTABLE	NONEXISTENT	STABLE
Conditions		
$\beta_2 > \beta_1 + \gamma_1 D$		$\beta_1 < \beta_2 + \gamma_2 D$
STABLE	UNSTABLE	STABLE
Condtions		
$\beta_2 < \beta_1 + \gamma_1 D$		$\beta_1 < \beta_2 + \gamma_2 D$

This example illustrates the idea of a critical size of fluctuation. The state $X_1 = D$; $X_2 = 0$ is, according to linear stability theory, stable, which means that very small amplitude fluctuations are damped. However, the existence of the unstable state with both X_1 and X_2 non-zero as well as the stable state $X_2 = D$ $X_1 = 0$, tells us that a very large perturbation of X_2 from zero may carry the system beyond the zone of stability of the state $X_1 = D$, $X_2 = 0$ and into the region where the state $X_1 = 0$, $X_2 = D$ is stable. From this we see that a change of state of the system may occur only if perturbations which exceed a certain critical amplitude occur. We should stress that this idea of a critical size has nothing whatsoever to do with that of a minimum size necessary for the functioning of a unit of production.

A practical example of this concept of critical size is that of a shopping center. If a small shopping center containing relatively few types of commerce is introduced at some point in the urban tissue, then it may not succeed in developing, and in the long run will disappear. If, on the contrary, a large center has been introduced, then it may develop and grow.

APPENDIX B
ANALYSIS OF SOME DATA FROM THE "URBAN DATA BOOK," REF. 19

The analysis of some of the data included in the "Urban Data Book" provides an image of the variation of global characteristics as well as of the internal structure of the cities as a function of their size - for cities larger than 700,000 inhabitants. In the absence of experimental data on city evolution with time, this image of the American urban system in 1970 gives us the opportunity to imagine the growth of a theoretical city on the basis of the ergodicity hypothesis. The natural log of the city sizes then can be used as the time variable under the condition of a constant population growth rate.

Total employment growth

At any time, the employment density decays from the CBD to the periphery (Figure B-1). During the development of the city, its total employment density increases (Figure B-2), but this change takes place in a differentiated manner in the urban space. Whereas there is indication that the CBD employment grows linearly (Figure B-3), its share of the Urbanized Area total employment (Figure B-4) seems first to decrease and then to increase above a city size of about 4 million. Consequently, the spatial dynamics of employment growth apparently take the form of a "rocking" in time, of successive centralization and decentralization.

Total population growth

As is well known in the literature, the population density decays exponentially from the center to the periphery (Figure B-5). Some cities show a decrease in density in the center, yet always compensated by a higher density in the next ring so that the average density calculated over the first two miles around the center conforms to the values observed in cities which do not have such density craters (Figures B-6a and 6b). On the other hand, the population density calculated on the basis of complete rings around the center leads to very comparable values whether or not

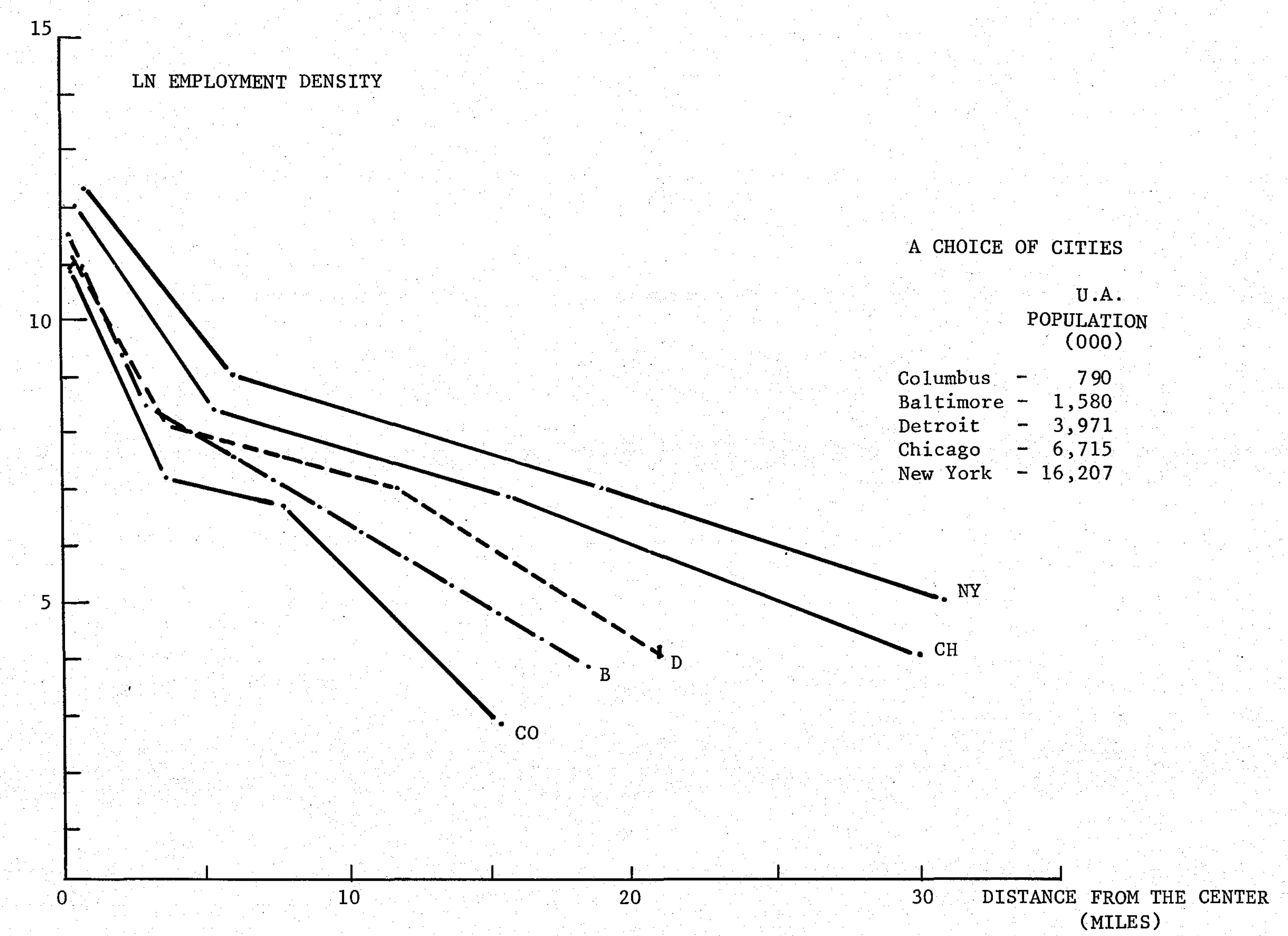

FIGURE B-1. SPATIAL DISTRIBUTION OF EMPLOYMENT DENSITY
(FROM TABLE 2 OF THE "URBAN DATA BOOK")

Notes

(1) 5 SMSA's have been eliminated because of their highly irregular shape: Dallas-Fort Worth, Minneapolis-St. Paul, San Diego, San Franciso-Oakland, Tampa-St. Petersburg.

(2) Average family income = $\dfrac{\sum_{n=1}^{11} \bar{I}_n \cdot F_n}{\sum_{n=1}^{11} F_n}$

n = number of rings (a total of 11)

$\bar{I}_n$ = mean family income of a ring

F_n = number of families of a ring

(3) The poor people are defined as the people living in the rings whose mean family income is lower than the general average family income, calculated over the 20 mile radius.

(4) Average distance of poor people =

$$\sum_{n=1}^{i} \left[r_{(n-1)} + \frac{r_{(n)} - r_{(n-1)}}{2} \right] \cdot P_n \Big/ \sum_{n=1}^{11} P_n$$

i = number of rings of poor people in a row

r = radius of the ring

P_n = population of the ring

If there are two groups of poor people, there will be two average distances.

(5) Same definition applied to the people living in the rings whose mean family income is higher than the general family income.

(6) Average total distance =

$$\sum_{n=1}^{11} \left[r_{(n-1)} + \frac{r_{(n)} - r_{(n-1)}}{2} \right] \cdot P_n \Big/ \sum_{n=1}^{11} P_n$$

(7) $$\bar{d}_{CBD} = \sqrt{\frac{S_{CBD}}{\frac{\pi}{2}}}$$, S_{CBD} = CBD area

(8) $$\bar{d}_m = r_{(m-1)} + \frac{\sqrt{\frac{S_m}{\pi}} - \sqrt{\frac{S_{(m-1)}}{\pi}}}{2}$$ m = number of zones

(9) $$\bar{d}_w = \frac{\sum_{m=1}^{4} W_m \cdot \bar{d}_m}{\sum_{m=1}^{4} W_m}$$ W_m = Number of workers living in a zone

(10) $$\bar{d}_e = \frac{\sum_{m=1}^{4} E_m \cdot \bar{d}_m}{\sum_{m=1}^{4} E_m}$$ E_m = Employment in a zone

(11) δ_c = population density of the center (0-1 mile)
δ_r = population density of the first ring (1-2 miles).

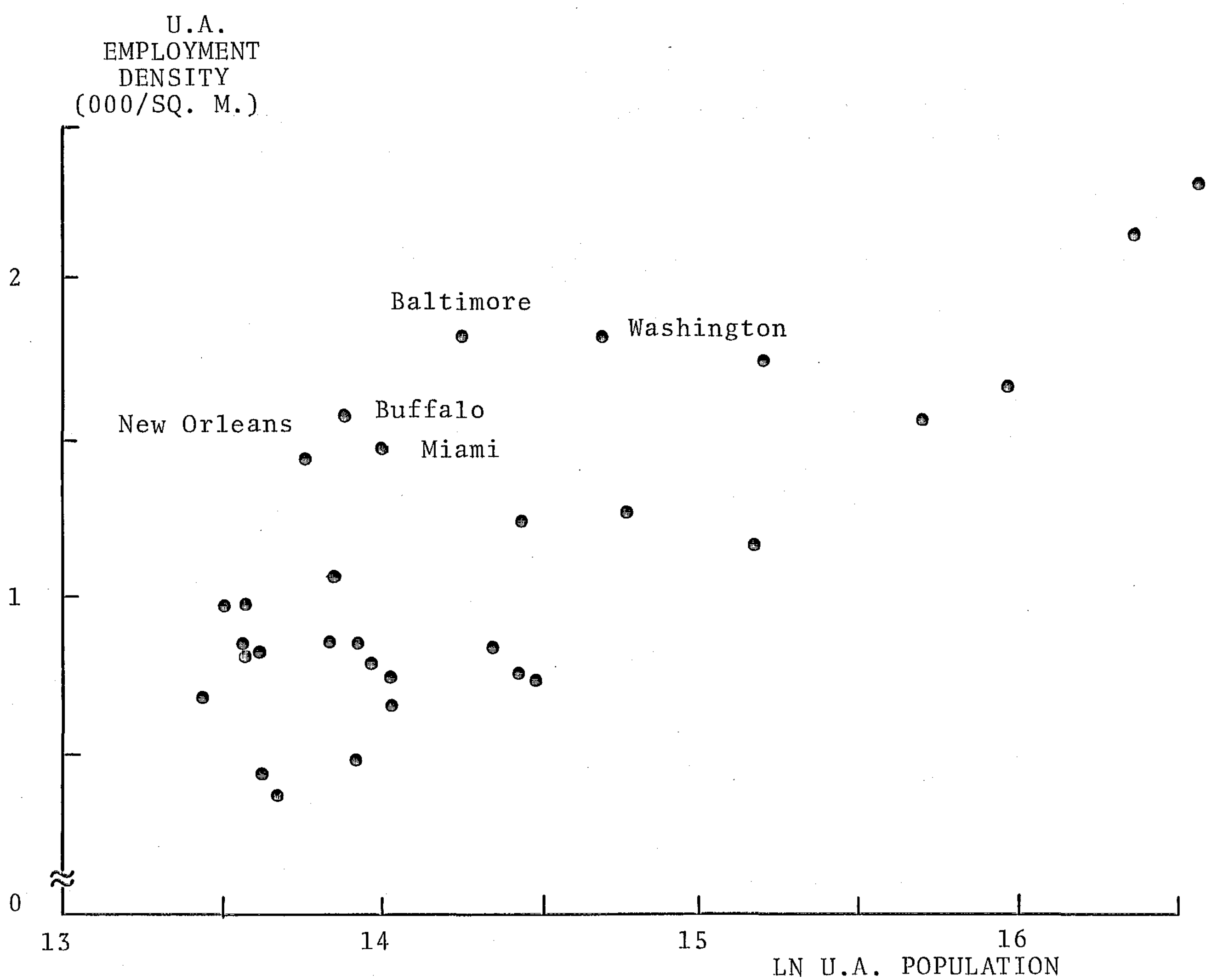

FIGURE B-2. EMPLOYMENT DENSITY IN THE URBANIZED AREA (FROM TABLE 2 OF THE "URBAN DATA BOOK")

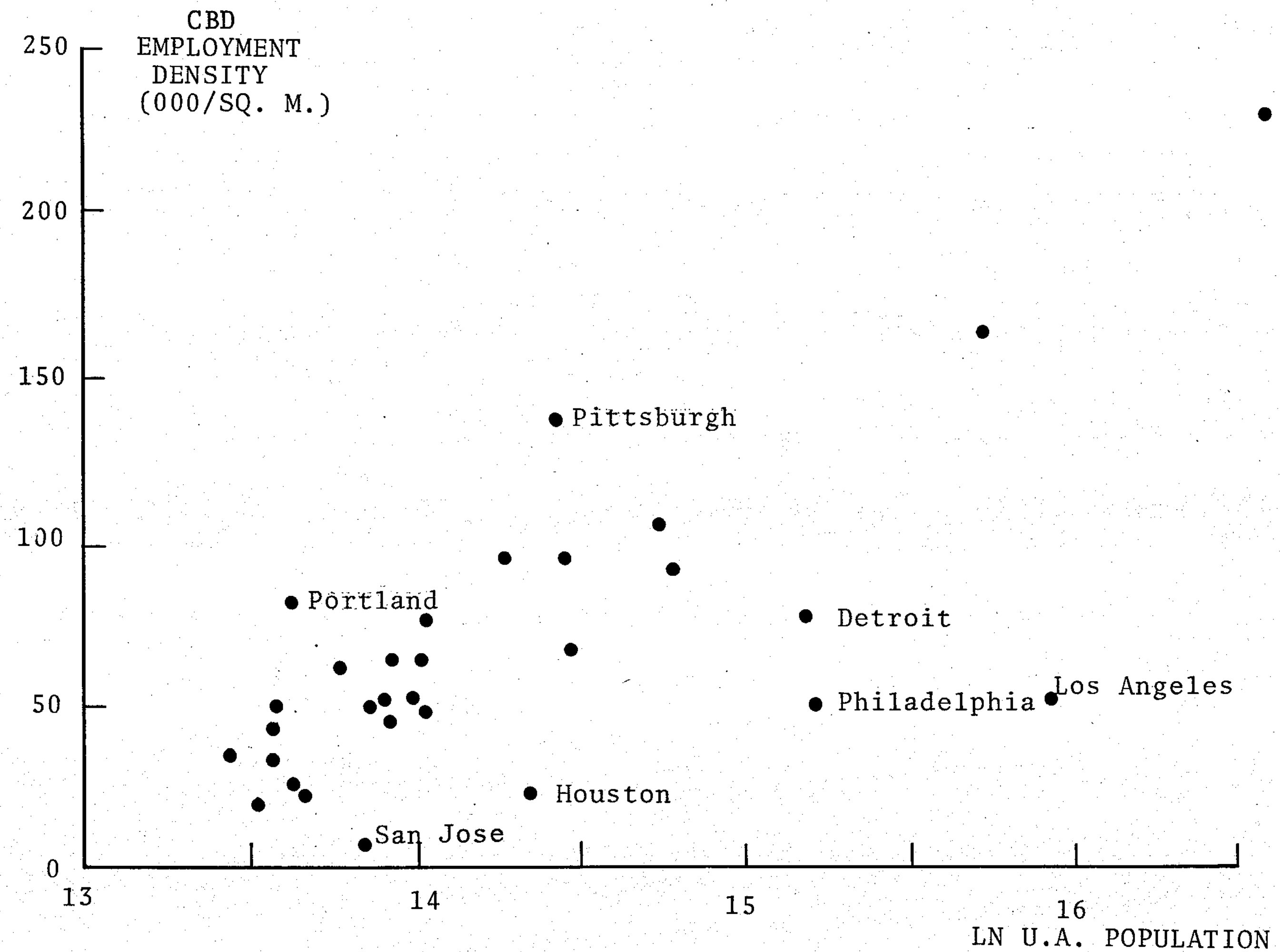

FIGURE B-3. EMPLOYMENT DENSITY IN THE CBD (FROM TABLE 2 OF THE "URBAN DATA BOOK")

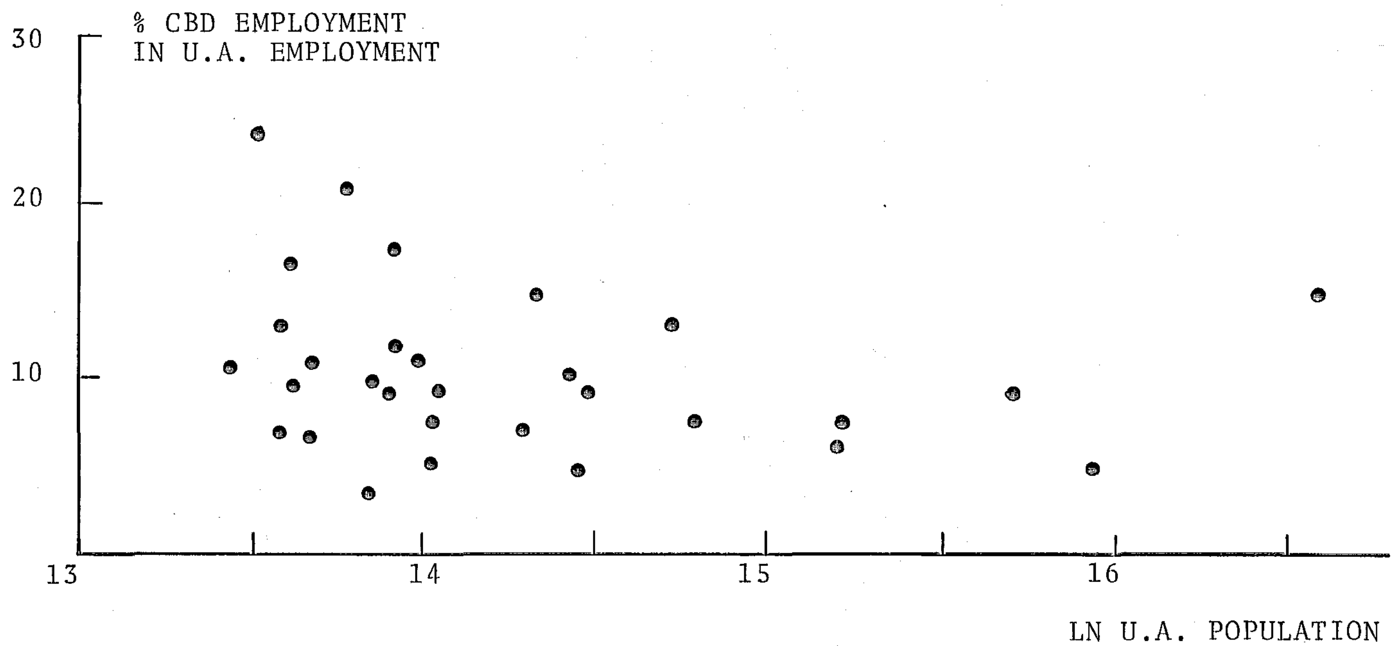

FIGURE B-4. CBD EMPLOYMENT SHARE OF THE TOTAL URBANIZED AREA EMPLOYMENT (FROM TABLE 2 OF THE "URBAN DATA BOOK")

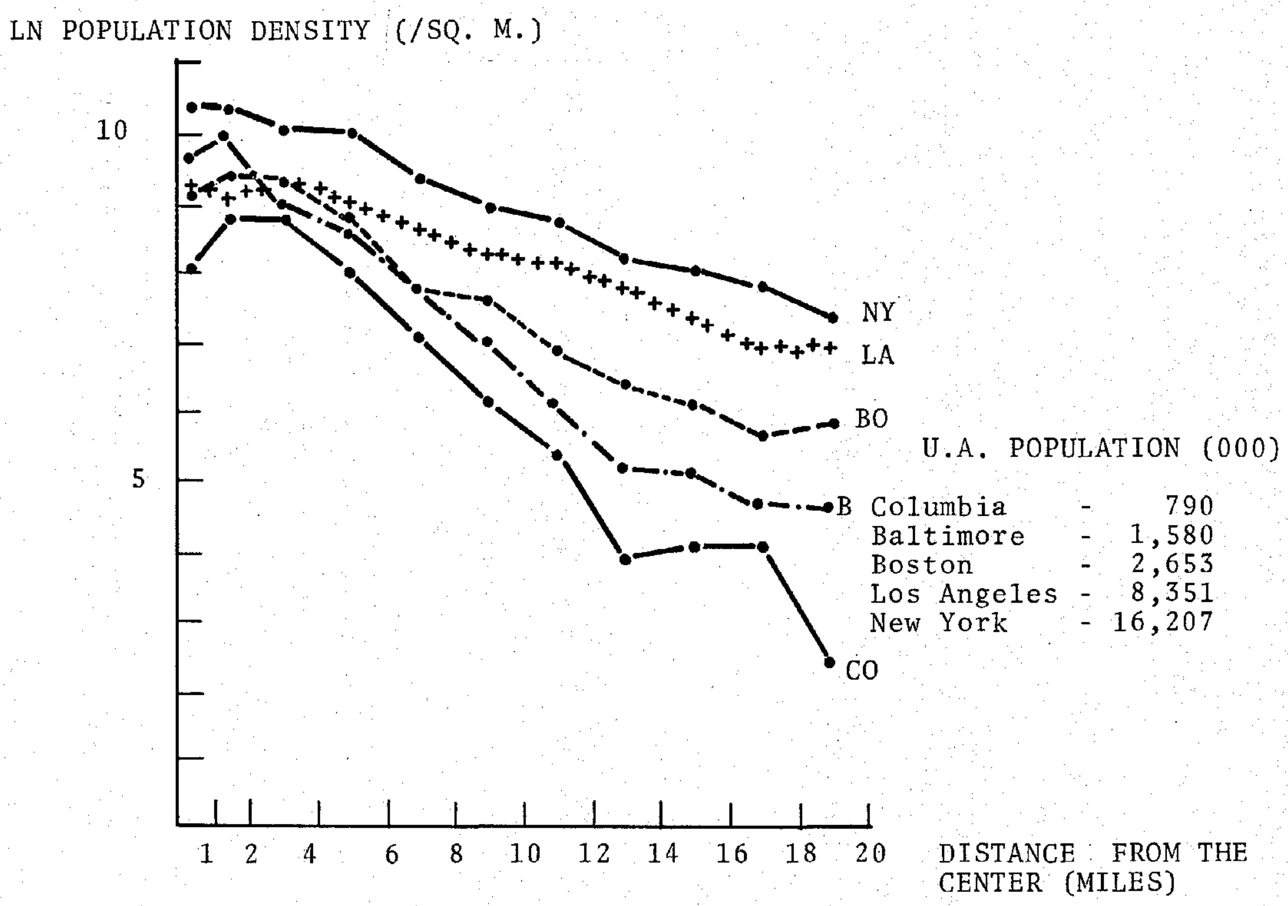

FIGURE B-5. SPATIAL DISTRIBUTION OF POPULATION DENSITY (FROM TABLE 1 OF THE "URBAN DATA BOOK")

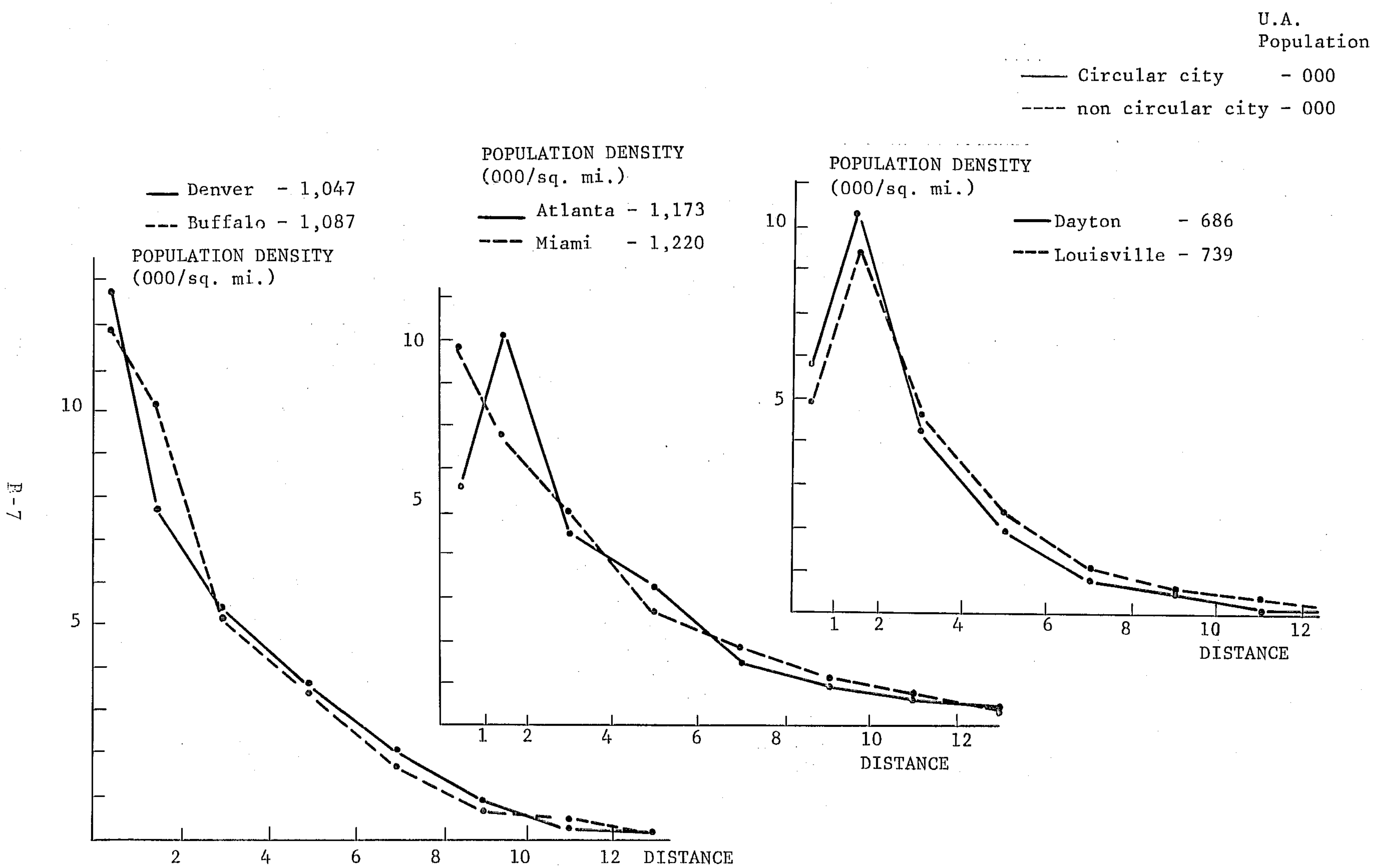

FIGURE B-6a. POPULATION DENSITIES (FROM TABLE 1 OF THE "URBAN DATA BOOK")

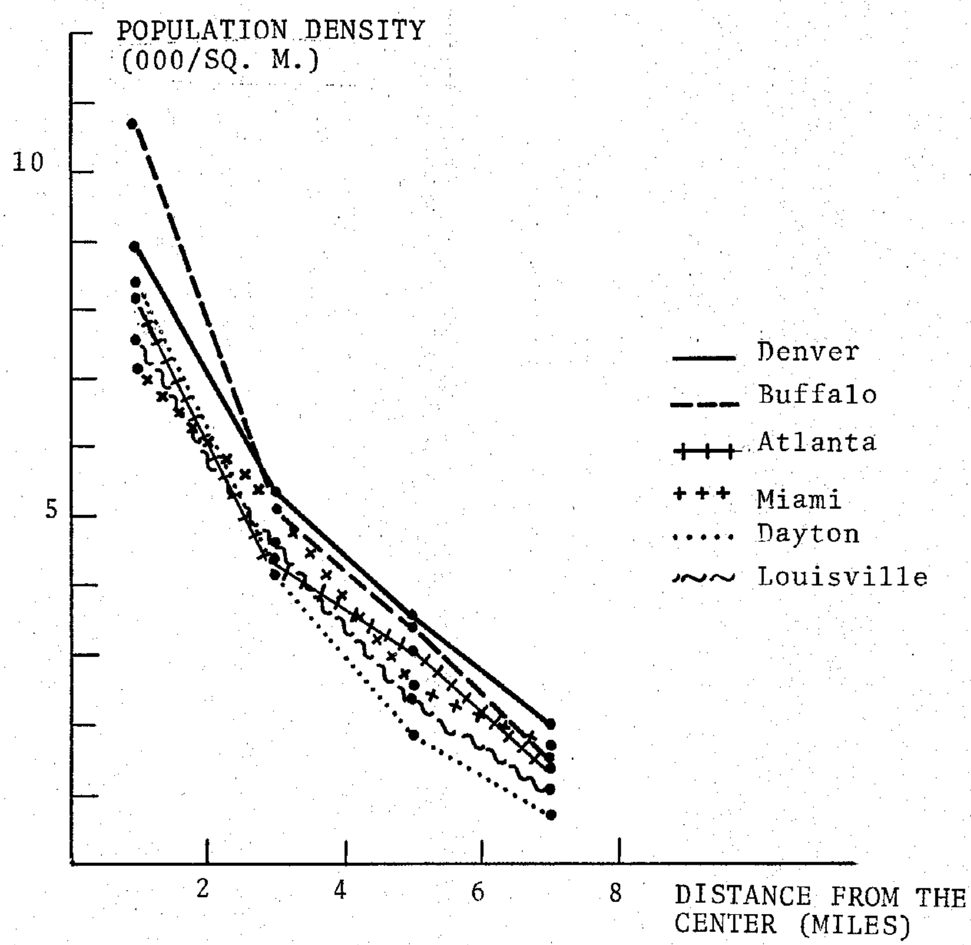

FIGURE B-6b. POPULATION DENSITIES
AVERAGE DENSITY OF THE TWO FIRST
MILES AROUND THE CENTER (FROM
TABLE 1 OF THE "URBAN DATA BOOK")

the city has a circular shape (Figure B-6a and B-6b). That is to say, the total demand of residential location at a certain distance from the center seems to be constant in spite of local physical or economic irregularities. During its development, the city shows a regular increase of its urbanized area (Figure B-7) as well as of its total density (Figure B-8), but this again takes place differently in space. The slope of the density line (Figure B-5) decreases when the city becomes bigger which shows that the density growth rate increases with distance from the center; that is why the average distance from the center of residential location increases with the size of the city (Figure B-9). As for the existence of a density crater, in the center, it does not seem to be related to the size of the city (Figure B-10).

Employment - Workers balance

The comparison of the employment and workers spatial distributions (Figure B-11) clearly shows that the employment is always the most spatially centralized. This is expressed by the fact that the ratio of the average distance of workers to the average distance of employment is always bigger than 1 (Figure B-12). However, the city seems to modify the value of this ratio quite strongly in the course of its development: it first increases as long as the city does not exceed a size of about 1.5 million, then decreases for a size between 1.5 and about 6 million, after which it seems to start going up again. This would indicate that the centrifugal forces that characterize the big city first act on the population and only later on the employment. Finally, it could be possible to see the employment centralize again in the CBD in a later phase (Figure B-4 and B-12).

Residential segregation

The only type of residential segregation we have tried to analyze is the one based on income. The spatial distribution of the mean family income is quite systematic, and its evolution with the size of the city is rather clear (Figures B-13a and 13b). In medium size cities, the lowest mean family income is found in the center, the oldest part of the urbanized area. Income

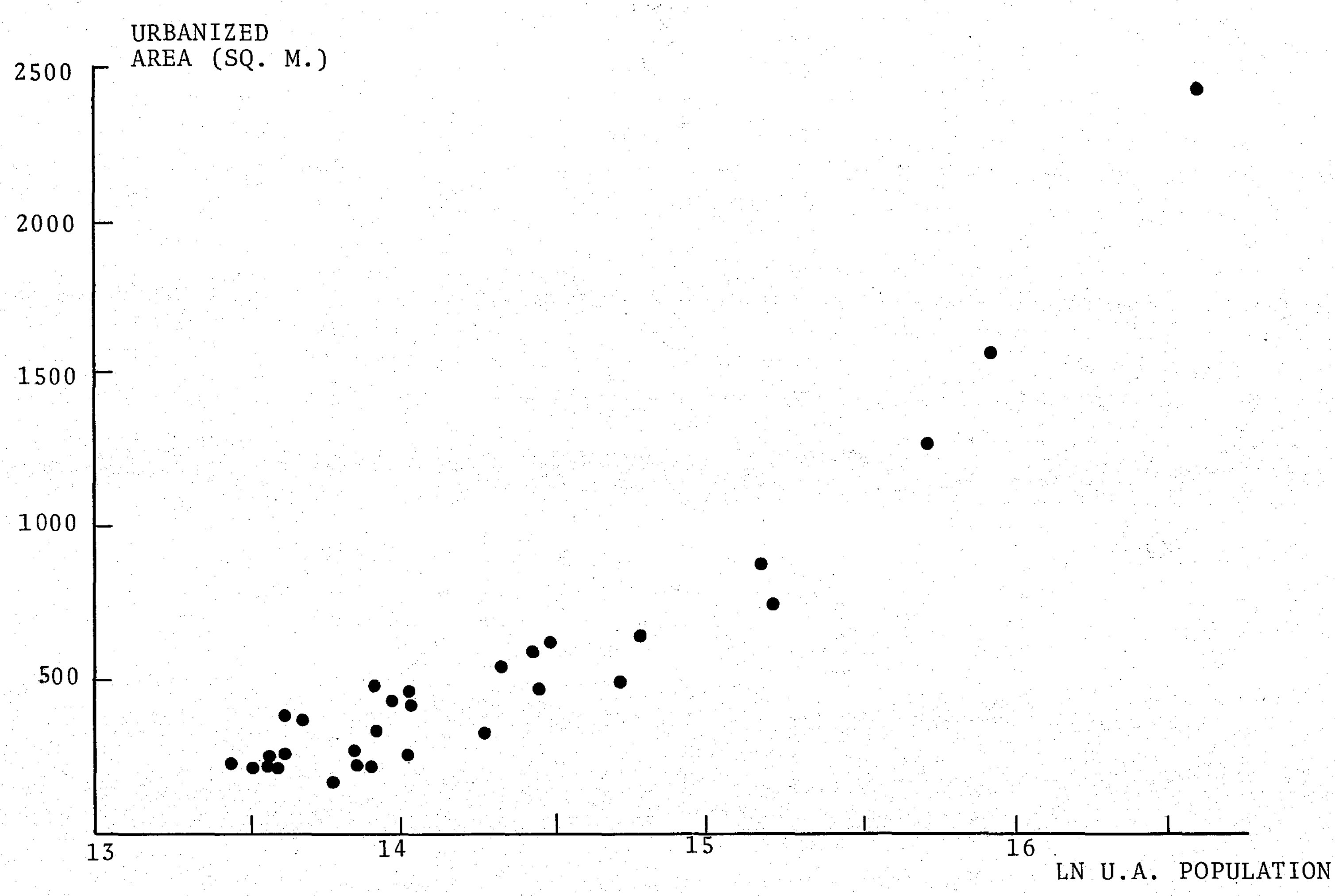

FIGURE B-7. GROWTH OF THE URBANIZED AREA WITH ITS POPULATION SIZE (FROM TABLE 1 OF THE "URBAN DATA BOOK")

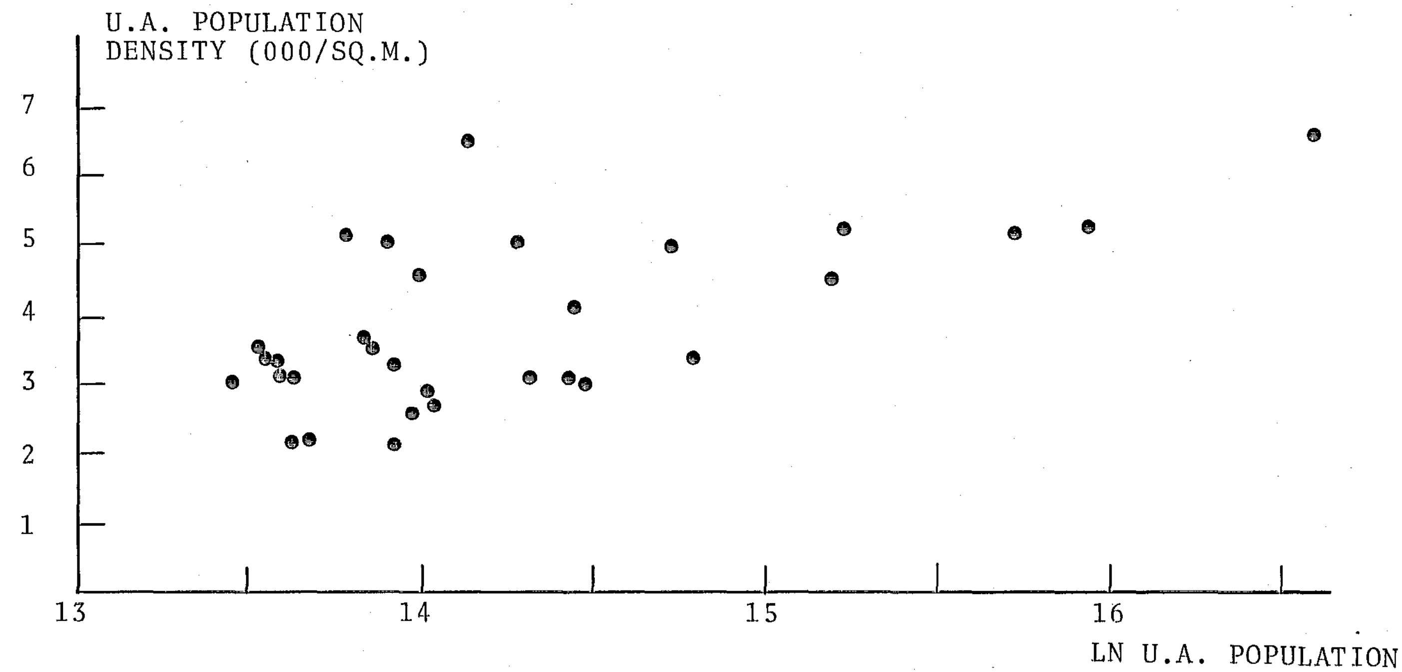

FIGURE B-8. GROWTH OF THE TOTAL URBANIZED AREA DENSITY
(FROM TABLE 1 OF THE "URBAN DATA BOOK")

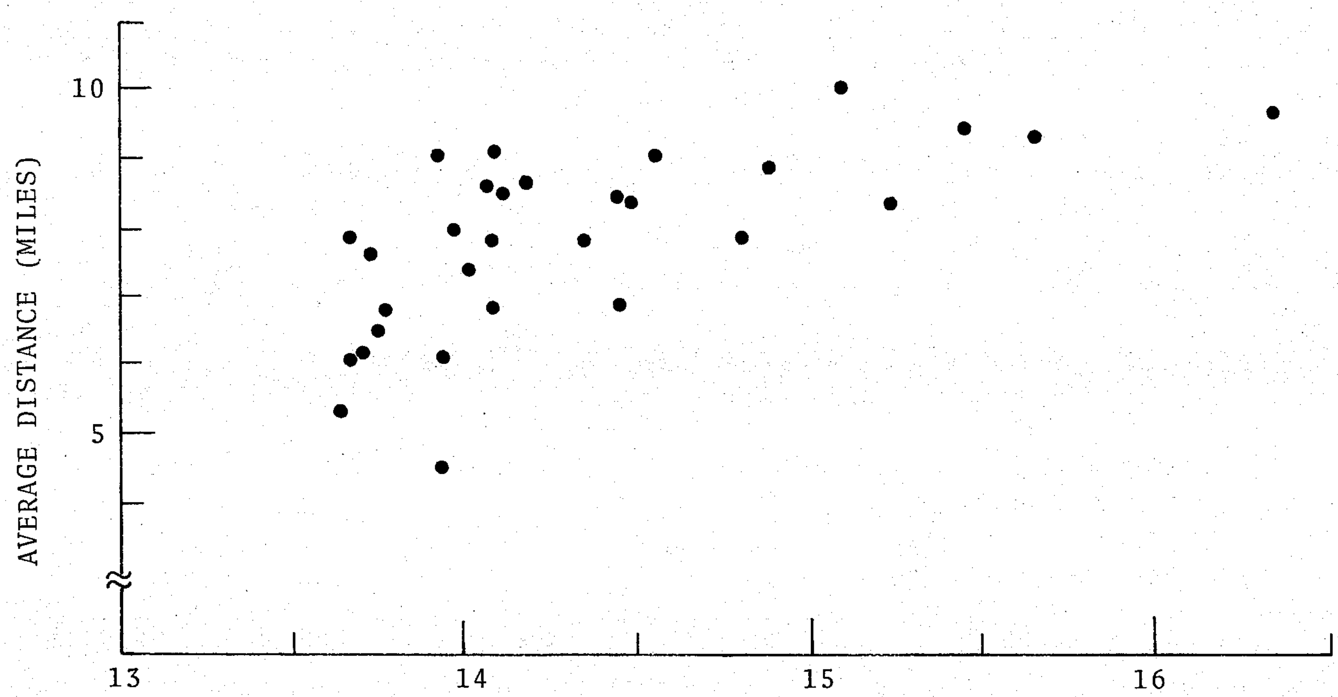

FIGURE B-9. AVERAGE DISTANCE FROM THE CENTER OF RESIDENTIAL LOCATION (FROM TABLE 1 OF THE "URBAN DATA BOOK")

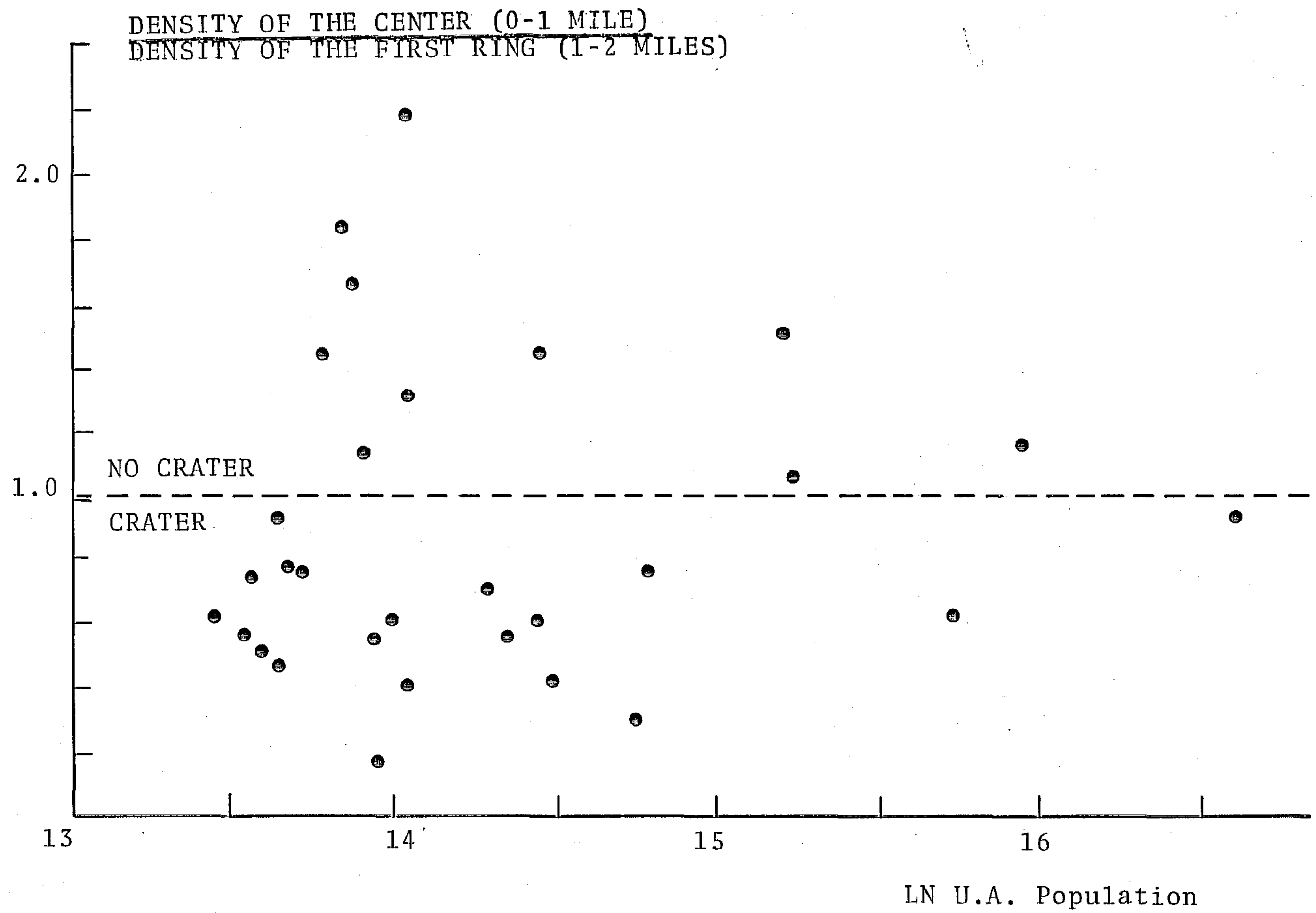

FIGURE B-10. POPULATION DENSITY CRATERS
(FROM TABLE 1 OF THE "URBAN DATA BOOK")

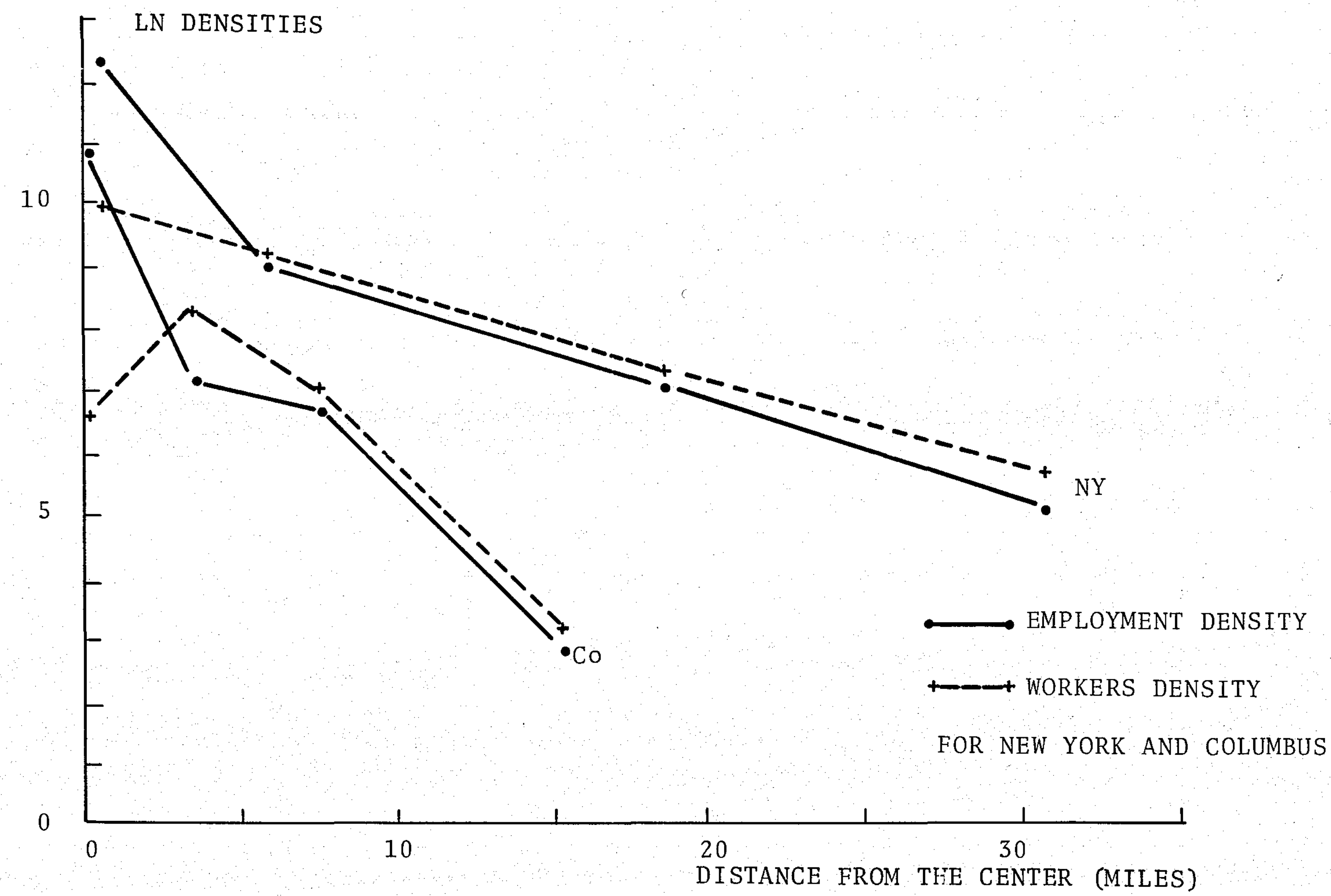

FIGURE B-11. SPATIAL DISTRIBUTION OF EMPLOYMENT AND WORKERS DENSITIES FOR TWO CITIES (FROM TABLE 2 OF THE "URBAN DATA BOOK")

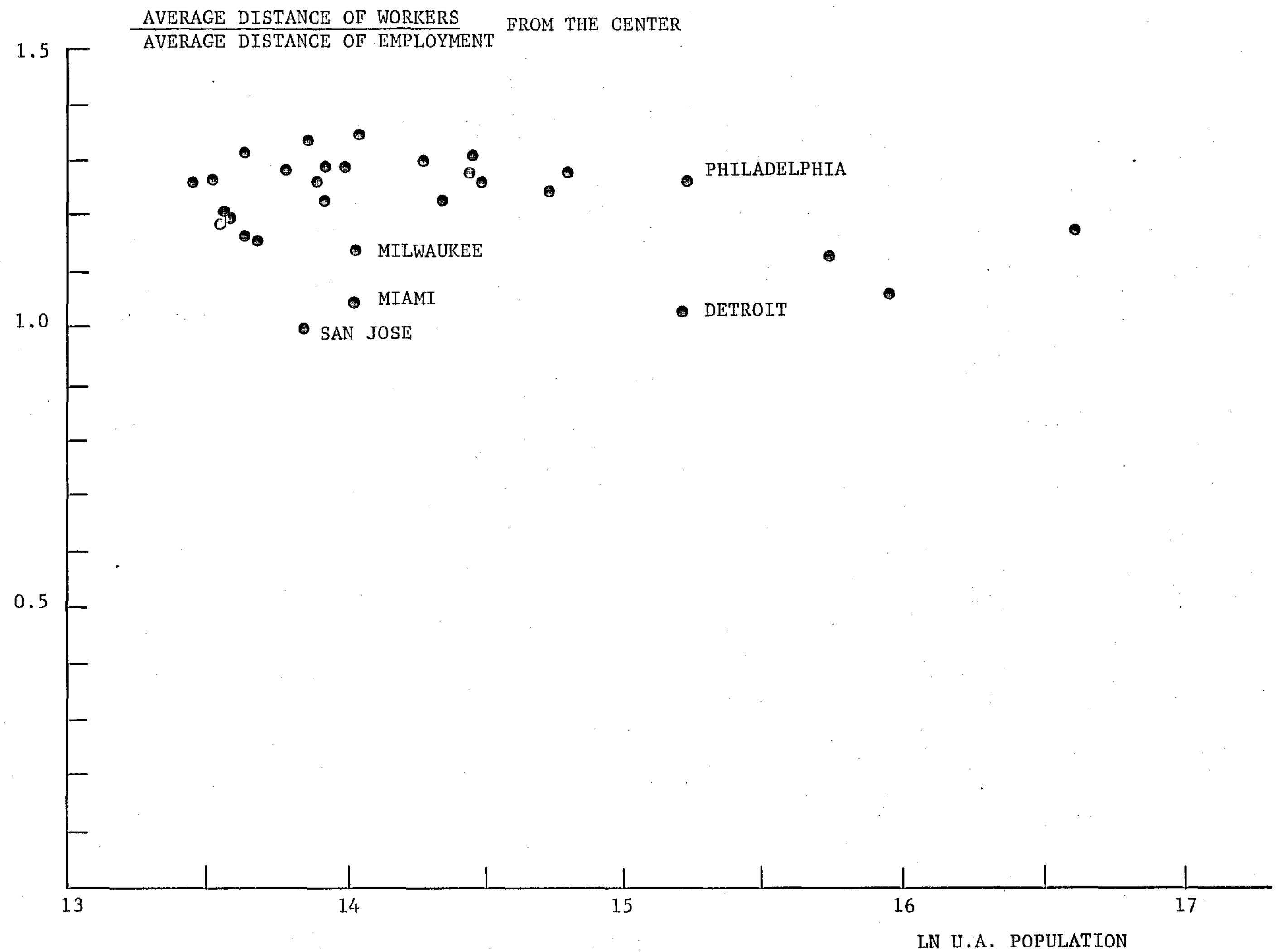

FIGURE B-12. EMPLOYMENT CENTRALIZATION (FROM TABLE 2 OF THE "URBAN DATA BOOK") rises with distance from the center and reaches a maximum in the urban fringe. Far away, in rural areas, it decreases again. In the absence of more detailed data, we have taken the average family income, calculated over 20 miles radius, as the limit between poor people and rich people. This arbitrary limit changes from city to city but is reasonable if economic segregation is to be considered as a more relative than absolute phenomenon. According to this definition, the percentage of poor people lies, in the majority of the cases, between 30 percent and 43 percent, the overall average being 45 percent.

When the city gets bigger, each group expands inside the density envelope. Rich people consume more space than poor people so that the average distance of residential location of each group increases at different rates. This is expressed in Figures B-14a and 14b where we can see a rather linear growth of each of these average distances, with different slopes however. Income declines in neighborhoods close to the center and strongly rises in the periphery.

In big cities, urban renewal introduces rich people in the center. This deeply disturbs the income profile. Poor people are driven away and more distant neighborhoods see their income decline. On the other hand, areas which were previously rural are absorbed in the expanding urbanized area; their mean family income rises consequently. However, in some old cities, the wealthy families have never left the center, and the present income profile expresses the survival of the archaic structure of the pre-industrial city.

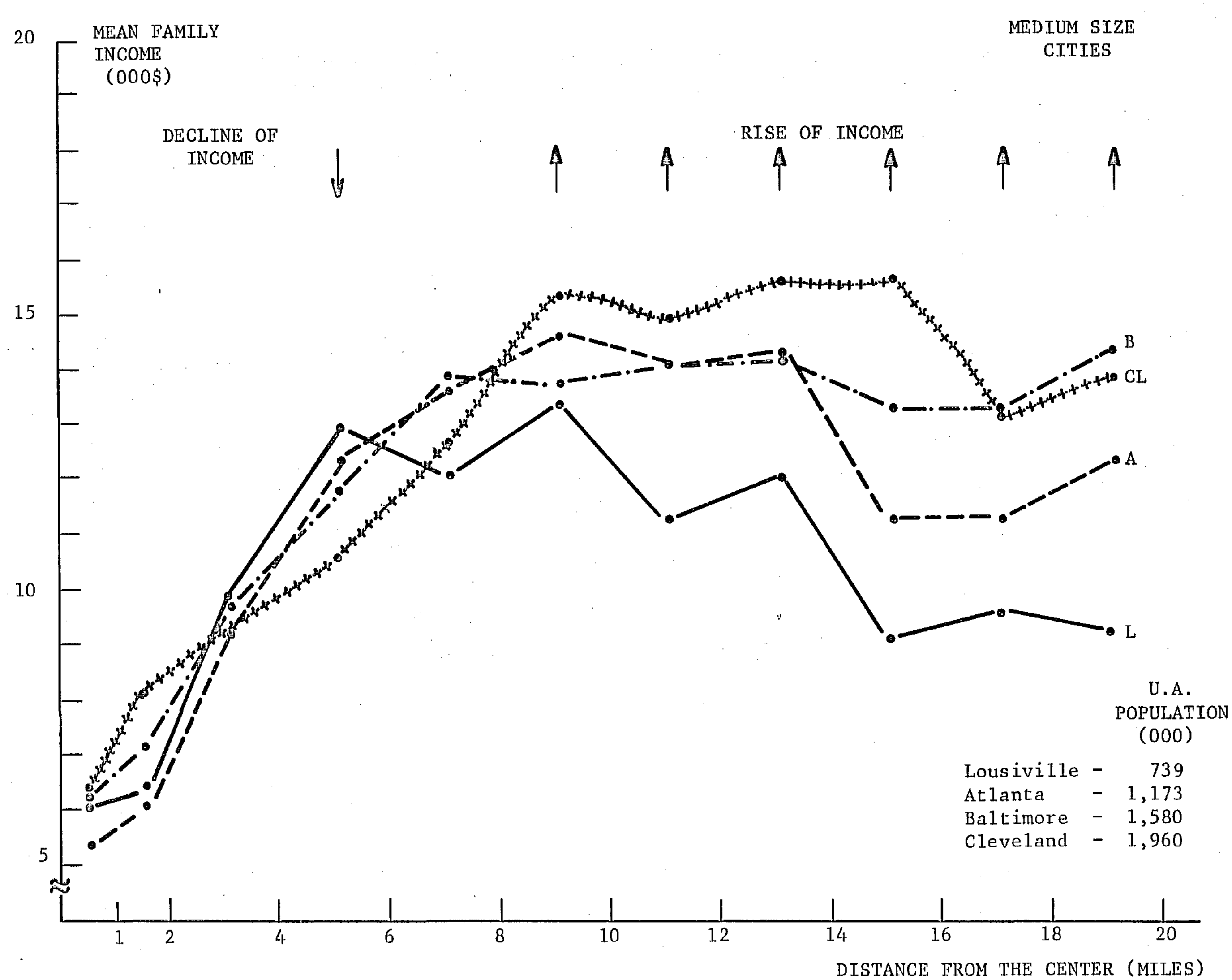

FIGURE B-13a. SPATIAL DISTRIBUTION OF INCOME (FROM TABLE 1 OF THE "URBAN DATA BOOK")

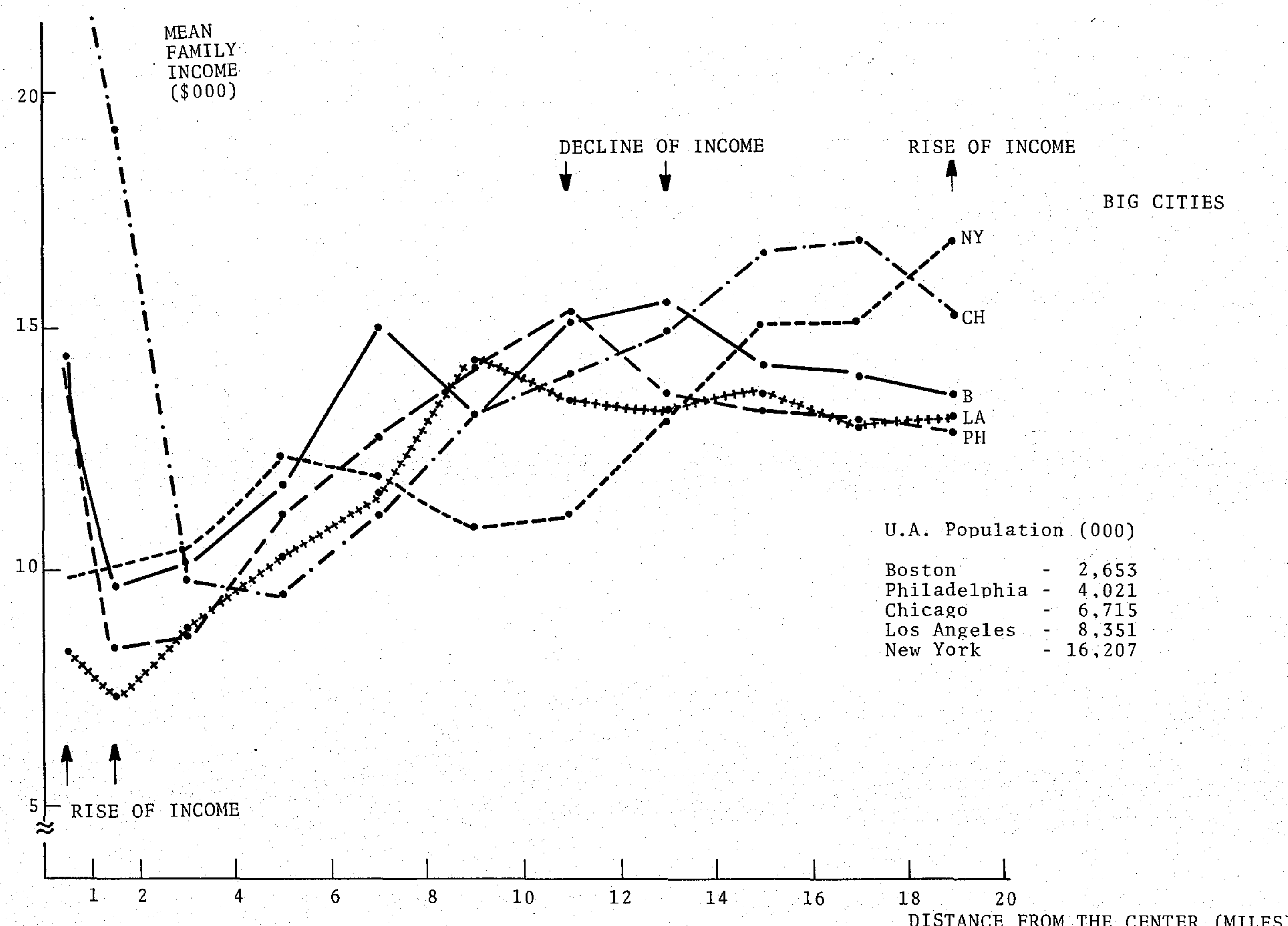

FIGURE B-13b. SPATIAL DISTRIBUTION OF INCOME
(FROM TABLE 1 OF THE "URBAN DATA BOOK")

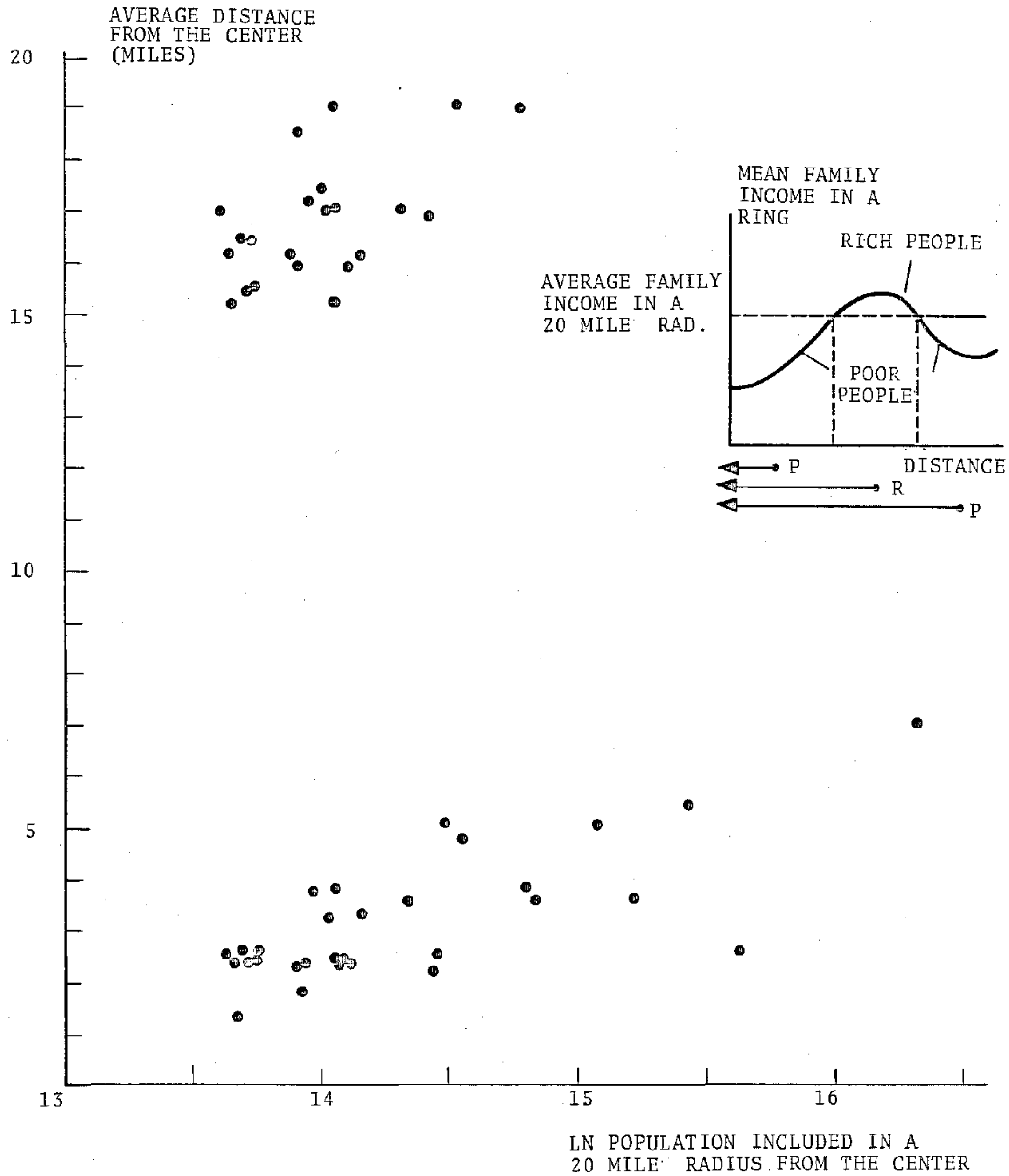

FIGURE B-14a. AVERAGE DISTANCE FROM THE CENTER OF RESIDENTIAL LOCATION OF POOR PEOPLE (FROM TABLE 1 OF THE "URBAN DATA BOOK")

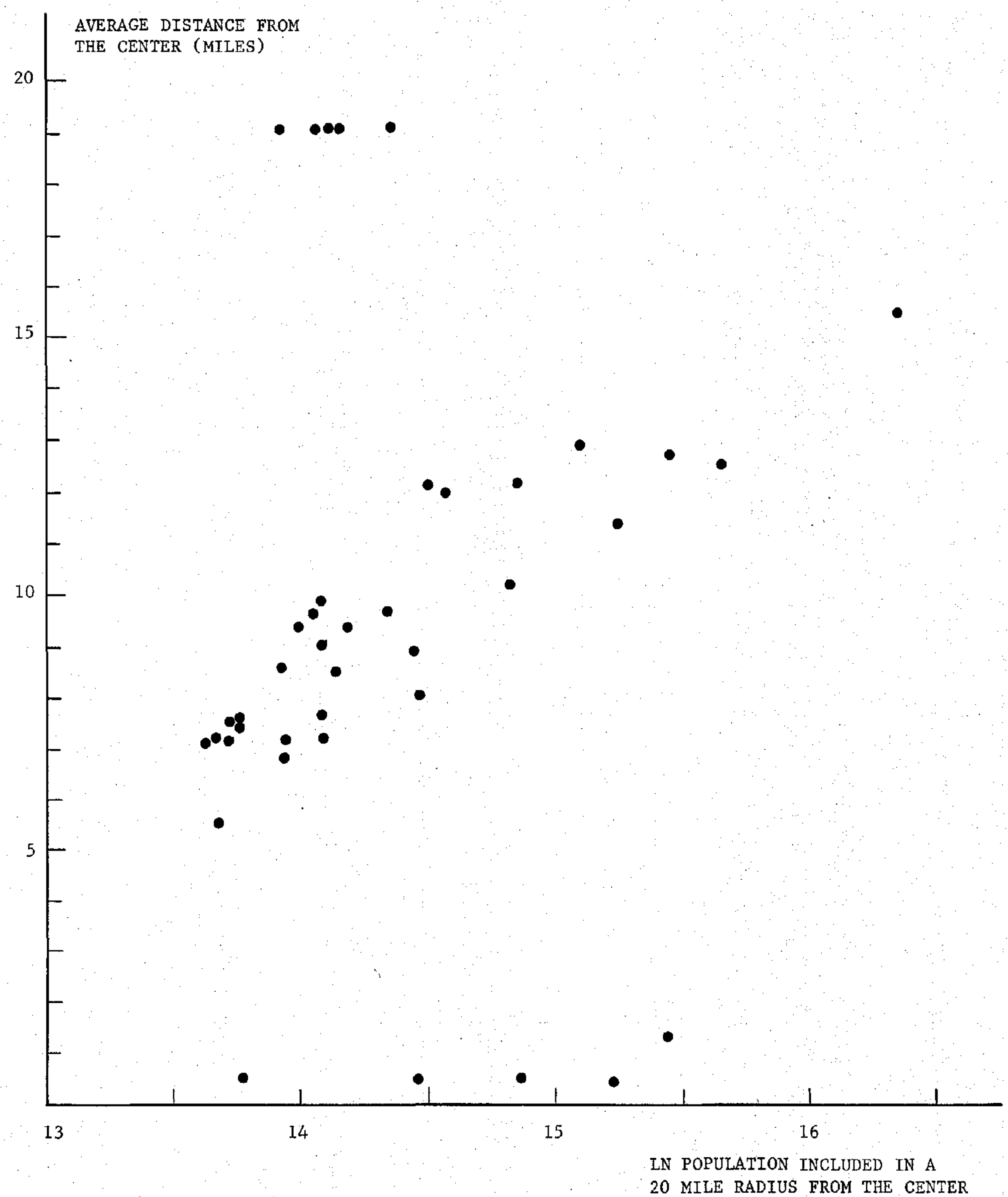

FIGURE B-14b. AVERAGE DISTANCE FROM THE CENTER OF RESIDENTIAL LOCATION OF RICH PEOPLE (FROM TABLE 1 OF THE "URBAN DATA BOOK")

DATA CALCULATED FROM TABLE 1 OF THE "URBAN DATA BOOK"

Cities (1)	Rank	Total population in 20m. rad.	Average family income ($) (2)	(%) poor people (3)	Average Distances: Poor people (miles) (4)	Average Distances: Rich people (miles) (5)	Average Distances: Total (miles) (6)
New York	1	12,129,002	12,456	68.2	7.0	15.5	9.7
Los Angeles	2	6,164,330	12,569	31.2	2.7	12.5	9.4
Chicago	3	5,074,672	12,996	41.1	5.4	1.3-12.7	9.5
Philadelphia	4	4,037,763	12,228	36.2	3.6	0.5-11.4	8.4
Detroit	5	3,591,709	13,427	36.2	5.1	12.9	10.1
Boston	7	2,850,255	13,180	36.5	3.7	0.5-12.1	8.9
Washington	8	2,675,726	14,901	42.0	3.8-19.0	10.2	7.9
Cleveland	9	1,930,345	12,977	52.4	5.1	12.1	8.4
St. Louis	10	2,066,050	11,895	53.6	4.8-19.0	12.0	9.1
Pittsburgh	11	1,866,696	11,535	40.8	2.6-16.9	0.5- 8.0	8.5
Houston	13	1,670,776	11,914	36.4	3.6-17.0	9.7-19.0	7.8
Baltimore	14	1,857,457	11,963	30.5	2.3	8.9	6.9
Milwaukee	16	1,309,623	12,594	36.2	2.5-17.0	7.7-19.0	6.8
Seattle	17	1,307,954	13,166	48.0	2.5-15.2	7.7	9.2
Miami	18	1,420,807	11,483	53.2	3.4-16.2	9.4	8.7
Atlanta	20	1,342,473	12,235	34.3	2.4-15.9	8.5-19.0	8.5
Cincinnati	21	1,314,325	11,037	30.8	2.4-19.0	9.0	7.9
Kansas City	22	1,173,280	12,077	45.1	3.8-17.2	9.4	8.0
Buffalo	23	1,221,486	12,030	57.2	3.4-17.4	9.6	7.4
Denver	24	1,105,376	12,168	32.7	2.4-18.5	7.2	6.1
San Juan	25	1,267,206	13,781	46.8	3.8-17.0	9.9-19.0	8.6
New Orleans	26	1,100,232	10,217	56.4	1.9-15.9	6.8	4.5
Phoenix	27	919,430	11,442	38.7	2.4-15.4	7.5	7.7
Portland	28	948,607	11,882	41.8	2.5-15.5	0.5- 7.5	6.8
Indianapolis	29	941,436	12,289	40.0	2.6-16.4	7.4	6.5
Providence	30	1,156,691	11,001	50.5	2.4-16.1	8.6-19.0	9.1
Columbus	31	894,234	11,958	42.4	2.6-16.4	7.1	6.2
San Antonio	32	834,386	9,577	42.3	2.6-17.0	7.1	5.3
Louisville	33	859,010	11,104	39.1	2.4-16.2	7.1	6.1
Dayton	34	878,373	12,492	42.2	1.3-15.2	5.5	7.9

DATA CALCULATED FROM TABLE 2 OF THE "URBAN DATA BOOK"

Cities	Average distances by zone. Total average distance							$\frac{\delta_c}{\delta_r}$
	CBD (miles) (7)	Other (miles) (8)	U.A. (miles) (8)	RαSU (miles) (8)	$\bar{d}_w$ (miles) (9)	$\bar{d}_e$ (miles) (10)	$\frac{\bar{d}_w}{\bar{d}_e}$	(%) (11)
New York	0.6	5.9	18.6	30.9	12.5	10.6	1.18	0.93
Los Angeles	0.5	7.3	17.8	30.6	13.7	12.8	1.07	1.15
Chicago	0.4	5.2	15.5	29.9	11.3	10.0	1.13	0.64
Philadelphia	0.5	3.6	10.9	24.6	10.3	8.2	1.27	1.07
Detroit	0.3	3.6	11.6	20.8	9.2	8.7	1.06	1.50
Boston	0.3	2.2	9.1	16.0	7.7	6.0	1.28	0.79
Washington	0.3	2.6	8.5	20.0	8.2	6.6	1.25	0.31
Cleveland	0.3	2.8	9.7	18.1	7.7	6.1	1.26	0.43
St. Louis	0.2	2.4	8.3	24.2	9.9	7.5	1.31	1.24
Pittsburgh	0.2	2.3	9.0	22.5	10.5	8.2	1.28	0.61
Houston	0.6	5.6	12.1	28.9	10.1	8.2	1.23	0.57
Baltimore	0.2	2.9	7.5	18.4	8.2	6.3	1.30	0.72
Milwaukee	0.3	3.0	8.8	16.8	6.6	5.8	1.14	0.52
Seattle	0.2	3.2	8.7	24.1	8.0	5.9	1.35	2.19
Miami	0.2	1.8	6.2	17.3	4.8	4.6	1.05	1.31
Atlanta	0.3	3.5	9.1	17.6	8.4	6.5	1.29	0.61
Cincinnati	0.3	2.7	7.7	18.2	8.0	6.2	1.29	0.56
Kansas City	0.3	4.6	10.6	21.1	9.3	7.6	1.23	0.19
Buffalo	0.2	2.1	5.9	15.4	6.4	5.1	1.26	1.17
Denver	0.3	3.0	6.9	21.2	7.2	5.4	1.34	1.65
San Jose	0.3	3.4	7.7	14.9	6.2	6.2	1.00	1.81
New Orleans	0.3	2.9	6.4	16.4	5.1	4.0	1.28	1.46
Phoenix	0.3	4.7	10.0	32.7	8.8	7.6	1.16	0.78
Portland	0.2	2.9	7.3	21.6	7.9	6.0	1.32	0.94
Indianapolis	0.4	5.7	10.8	21.2	9.9	8.5	1.17	0.49
Providence	0.2	2.4	6.6	11.8	5.7	4.7	1.20	0.79
Columbus	0.3	3.5	7.6	15.3	5.8	4.8	1.21	0.52
San Antonio	0.3	4.1	8.1	16.7	6.1	5.1	1.19	0.76
Louisville	0.4	2.6	6.3	12.6	5.2	4.1	1.27	0.59
Dayton	0.3	2.0	6.0	15.9	7.7	6.1	1.26	0.61

APPENDIX C - REPORT OF INVENTIONS

The work performed under this contract, while leading to no invention, has produced a first dynamic model of the evolution of the spatial distribution of urban populations based on the concept of "order by fluctuation," which is an improvement over previous urban models. It was shown that fluctuations play a vital role in the evolutionary process of urban growth (Section 2). The evolution of a complex system cannot be known simply by studying the deterministic equations describing its internal dynamics. It is necessary, in addition, to study the effects of fluctuations or historical accident which can drive the system to new modes of behavior. Taking account of both the deterministic elements of urban growth and the appearance of innovations at chance locations in an economic region, a dynamic model of the evolution of spatial organization of urban centers was developed in Sections 3 and 4 and simulations produced in Section 5.

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