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COMPARISON OF SOLID STATE INVERTERS FOR AC  
INDUCTION MOTOR TRACTION PROPULSION SYSTEMS

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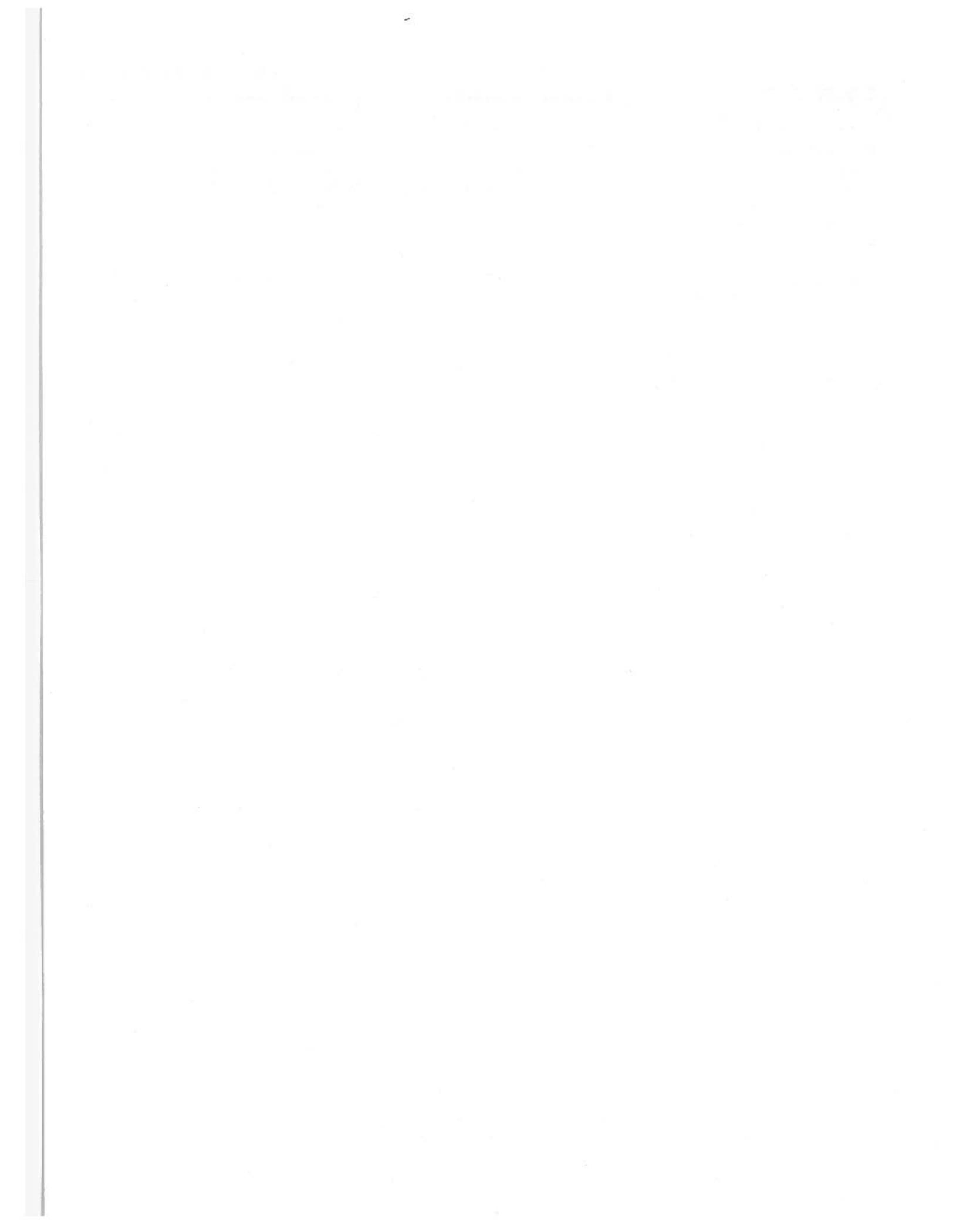
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16. Abstract <p>This report is one of a series concerned with the application of ac machines as traction motors for railroad motive power. It presents results of a laboratory evaluation and computer analysis of different inverter systems. Three inverter systems, sine wave, pulse-width modulated, and parallel capacitor commutated, were selected as being representative of inverter types applicable to traction use and were evaluated under similar operating conditions with asynchronous ac motors. The report includes descriptions of operating theory for each inverter type studied, descriptions of test circuitry, test procedures, and analysis of test results. Efficiency, power factor, distortion, regeneration, device utilization, and their effects on system characteristics and influence on system selection are discussed.</p>					
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## PREFACE

The work described in this report was performed in the Electrical Power and Propulsion Laboratory of the Transportation Systems Center (TSC) under sponsorship of the Federal Railroad Administration, Office of Research and Development (FRA, ORD). The objective of this work was to determine the power characteristics of representative low power, three-phase inverter systems powering induction motors. The results are intended to assist the FRA in evaluating the characteristics and trade-offs that must be considered in selecting the type of power conditioning unit incorporated into a full-scale system design.

The authors acknowledge the FRA program sponsor, Matthew Guarino, for his support and guidance through the duration of the program.

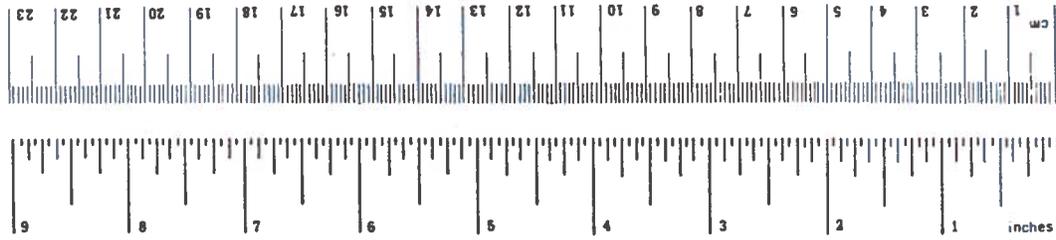
## METRIC CONVERSION FACTORS

### Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	<b>LENGTH</b>			
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	<b>AREA</b>			
in <sup>2</sup>	square inches	6.5	square centimeters	cm <sup>2</sup>
ft <sup>2</sup>	square feet	0.09	square meters	m <sup>2</sup>
yd <sup>2</sup>	square yards	0.8	square meters	m <sup>2</sup>
mi <sup>2</sup>	square miles	2.6	square kilometers	km <sup>2</sup>
	acres	0.4	hectares	ha
	<b>MASS (weight)</b>			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
	<b>VOLUME</b>			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
cup	cups	0.24	liters	l
qt	quarts	0.47	liters	l
gal	gallons	0.95	liters	l
ft <sup>3</sup>	cubic feet	3.8	liters	l
yd <sup>3</sup>	cubic yards	0.03	cubic meters	m <sup>3</sup>
		0.76	cubic meters	m <sup>3</sup>
	<b>TEMPERATURE (exact)</b>			
	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

### Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	<b>LENGTH</b>			
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
	<b>AREA</b>			
cm <sup>2</sup>	square centimeters	0.16	square inches	in <sup>2</sup>
m <sup>2</sup>	square meters	1.2	square yards	yd <sup>2</sup>
km <sup>2</sup>	square kilometers	0.4	square miles	mi <sup>2</sup>
ha	hectares (10,000 m <sup>2</sup> )	2.5	acres	acres
	<b>MASS (weight)</b>			
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	short tons
	<b>VOLUME</b>			
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m <sup>3</sup>	cubic meters	35	cubic feet	ft <sup>3</sup>
m <sup>3</sup>	cubic meters	1.3	cubic yards	yd <sup>3</sup>
	<b>TEMPERATURE (exact)</b>			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



\* 1 in = 2.54 (exact). For other metric conversions and more detailed tables, see NBS Misc. Publ. 286, Units of Weights and Measures, Price \$2.25, SD Catalog No. C 1.1.10.286.

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## SYMBOLS AND ABBREVIATIONS

ac	-	alternating current
amp (A)	-	ampere
BART	-	Bay Area Rapid Transit
cci	-	constant current inverter
cm	-	centimeter
db	-	decibel
dc	-	direct current
DOT	-	Department of Transportation
FRA, ORD	-	Federal Railroad Administration, Office of Research and Development
ft-lb	-	foot-pound
GE	-	General Electric Company
hp	-	horsepower
hr	-	hour
Hz	-	Hertz
kHz	-	kiloHertz
kva	-	kilovolt ampere
kvar	-	kilovolt ampere, reactive
kW	-	kiloWatt
LRV	-	Light Rail Vehicle
MARTA	-	Metropolitan Atlanta Rapid Transit Authority
max	-	maximum
min	-	minute
mph	-	miles per hour
msec	-	milli second
mut	-	motor under test
MW	-	MegaWatt
nci	-	natural commutated inverter
NEMA	-	National Electrical Manufacturer Association
pdr	-	phase delay rectifier
pf	-	power factor
pf <sub>DISP</sub>	-	displacement power factor
rms	-	root mean square
rpm	-	revolutions per minute

sec	-	second
TSC	-	Transportation System Center
usec ( $\mu$ sec)	-	micro second
V	-	volt
vac	-	volts, alternating current
vdc	-	volts, direct current
v <sub>rms</sub>	-	volts, root mean square
w	-	watts
WABCO	-	Westinghouse Air Brake Company
$\alpha$	-	delay angle
$\phi$	-	conduction delay angle
$\delta$	-	commutation half-angle in radians = $\pi/30$

## EXECUTIVE SUMMARY

The Federal Railroad Administration (FRA) Office of Research and Development has monitored the worldwide progress of three phase ac traction systems, and has sponsored the development of full scale ac power conditioning systems for the control of linear induction and synchronous motors. The FRA is now planning a full scale three-phase ac traction motor development program which will transfer the linear motor technology to rotary ac machines, and which will assess the suitability for U.S. railroad operations of ac motor control systems developed for industrial and foreign railroad operations.

To provide the FRA with the technical information required to evaluate candidate ac traction power conditioning systems, TSC has been conducting a series of projects consisting of laboratory studies, computer studies, industrial contracts, and university research studies to provide information relating to the power and performance characteristics of three-phase, alternating current (ac) power conditioning systems applied to the control of traction motors.

This report presents test results obtained from a study which included laboratory measurements and computer analysis of three small scale inverter systems that power asynchronous (induction), three-phase ac motors. Each inverter has different characteristics, but each is a variable voltage and variable frequency drive system. The inverters included one natural commutated inverter and two forced commutated inverters -- a pulse-width modulated inverter and a complementary, parallel capacitor, commutated inverter. Test data taken for these inverters were restricted to operation with asynchronous motors. Each inverter was instrumented for measurement of power characteristic and performance data while powering static loads and ac induction motors coupled to the TSC component dynamometer. Computer simulations of the inverter characteristics were used to correlate test data in the analysis of test results.

The natural commutated inverter incorporates a design stratagem which involves series and parallel combinations of thyristors to generate the three-phase output. The inverter is a self-protecting and self-limiting design which enhances its reliability. A resonant circuit capacitor in series with the conducting thyristors and the load blocks direct current (dc) flow and insures thyristor commutation; thus, there is minimal danger of commutation failure.

The power factor of the inverter with the present input rectifier scheme is influenced principally by distortion rather than displacement parameters. At low current levels, high peak to root means square (rms) current ratios generate a high distortion component of reactive power and, consequently, low power factor. As the load increases, the conduction periods of the diodes also increase and the distortion reduces. Thus, the power factor improves with increased inverter loading.

Harmonic distortion in the output of the inverter is low because the waveform approaches an ideal sinusoid.

The pulse-width modulated inverter provides variable voltage and variable frequency power from an ac source using a diode rectifier. Both voltage and frequency are controlled with a single set of power switches by controlling the firing sequences and the conduction times of the switches. The commutation components are in parallel with the load switches. They carry currents with measured peak values five times the maximum load current and must be sized for those high peak to rms current ratios.

The use of pulse-width modulation results in an output voltage waveform with high distortion. The output current amplitude for each harmonic is low because of the high impedance of the lagging power factor load. This type of inverter is suited for applications where the harmonic distribution of the output is not objectionable. Its principal advantages are:

1. control of output voltages and frequency with only one set of semiconductor switches
2. lower losses because the commutation components are not in series with the load

The constant current inverter regulates the dc link current, and limits the voltage and drive frequencies to maintain the dc link current at or below a preset limit.

The semiconductor and passive components used in the inverter can be sized for torque limit currents rather than for higher pulse level currents.

Commutation by capacitive coupling of the commutation pulse provides a large commutation margin. Therefore, the inverter thyristors can be rectifier grade thyristors rather than more costly inverter grade thyristors.

The inverter uses a phase delay rectifier circuit, which has low power factor at low output voltage, to control dc link voltage. The low power factor will result in high apparent power demand and may impact utility supply service costs.

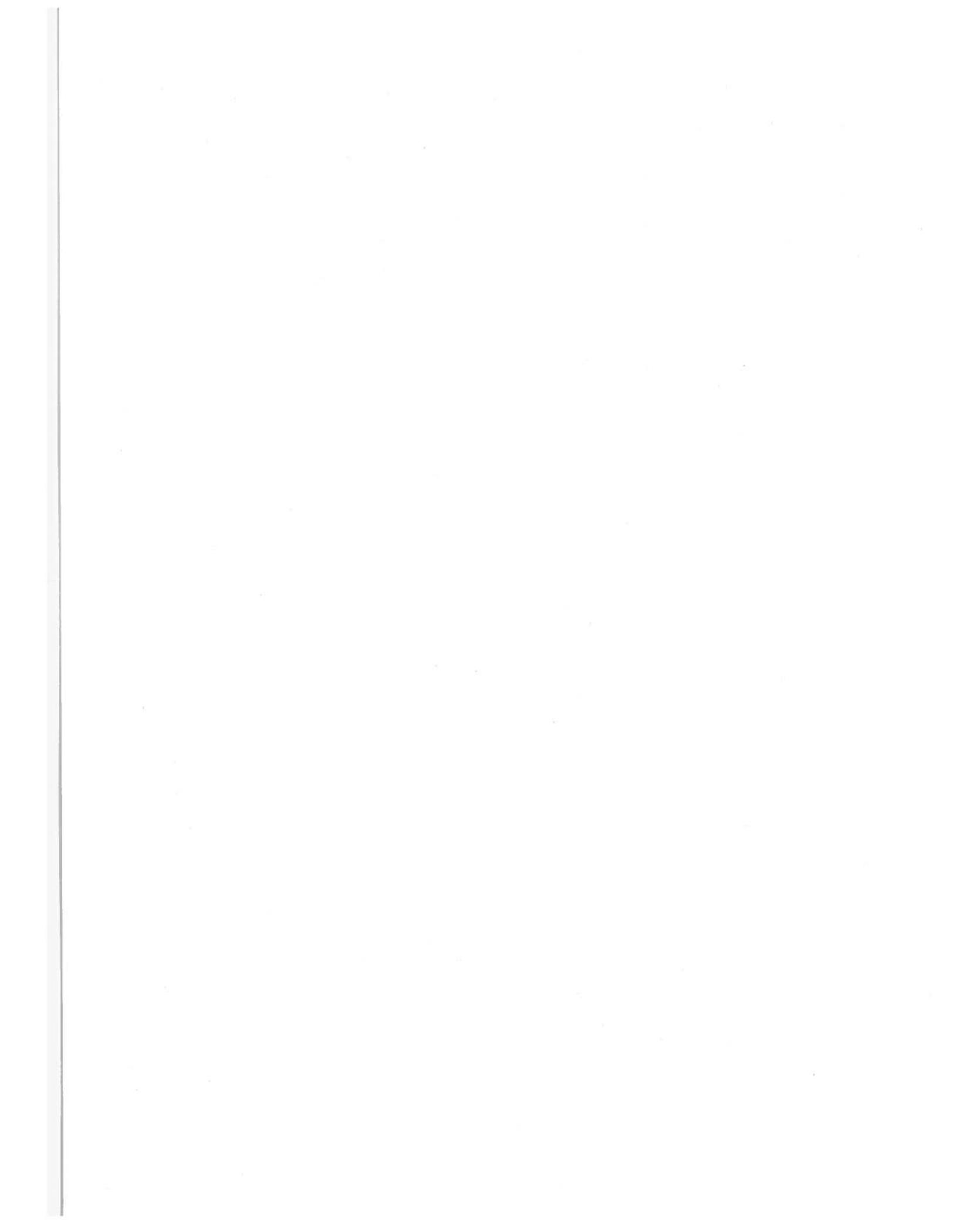
Of the three circuits tested, the input current harmonics in the constant current inverter are the least severe. Since the unit is a current regulated device, the input and output currents are nearly equal in magnitude, have similar wave shapes, and have harmonic distributions which closely follow a  $1/n$  distribution.

Despite the disparities in wave shapes of the inverter outputs, the effects on motor performance are not severe. The sinusoidal output voltage of the natural commutated inverter presents the fewest number of complications from harmonic torque and interference standpoints. Pulse-width modulation produces voltage harmonics which extend to high frequencies, but at these frequencies the motor impedances are high and the harmonic currents are insignificant. The significant harmonics are those which result in motor currents and torques. For both the pulse-width modulated inverter and the constant current inverter, these harmonics are the fifth and seventh, and for both inverters they are the same proportion of the fundamental value.

The results of the laboratory tests and the model simulations indicate that either of the three inverter types can be used for an ac traction system, and that they can control motor torque and speed from zero speed to the speed corresponding to rated output of the inverter. Although each system utilizes different types of input and output power circuitry, overall efficiency and motor performance are similar.

The results of the TSC study indicate that the choice of inverter type selected for a specific application will be influenced by trade-offs of the inverter system's salient characteristics, their impacts on the power distribution network, and the requirements of the application. They also indicate that further development efforts are required on a full scale level to obtain the necessary data to demonstrate acceptable levels of durability, performance and efficiency and to evaluate parameters which cannot be scaled from laboratory scale to full scale hardware.

Further laboratory level research efforts are also indicated. These should concentrate on the adaptation of the inverter types to single phase ac power distribution as it exists on electrified railroads, and also on power factor and regeneration control.



# 1. INTRODUCTION

The traction motor selected for electric rail vehicle propulsion in the past has been the series wound dc motor, a component whose development has progressed to the point where further refinements will not significantly improve performance characteristics. Despite its ruggedness and the suitability of its torque-speed characteristics to traction applications, the series wound dc motor suffers from two major shortcomings:

1. low power to weight ratio
2. maintenance requirements.

Each of these factors is attributable to the presence of brushes and a commutator which require periodic maintenance and which add significantly to motor size and weight. It is generally agreed that for locomotive applications, the series wound dc motor has reached its power limit of 1000 to 1200 horsepower (hp) per axle. Also, greater motor power will force increases in motor weight, which will increase detrimental, weight-induced mechanical effects on roadbed wear and truck dynamics.

## 1.1 AC MOTOR PROPULSION SYSTEMS

The ac motor, particularly the squirrel cage induction motor, has long been recognized for its high power to weight ratio and, most notably, for its high reliability and low maintenance requirements. Previous application of the motor has been in industrial drive systems where speed regulation was not a critical parameter. For those applications where speed control was critical, the shunt wound dc motor was used. The squirrel cage induction motor, however, is replacing the dc motor in industrial applications because of the availability of reliable, variable voltage, variable frequency, power conditioning systems for accurate torque, and speed control. The advent of high speed, high voltage, and high current thyristors, and the development of three-phase, variable voltage, variable frequency drives for large industrial applications

have made possible the design of reliable and efficient power conditioners for ac motor control. The adaptations of these motor power conditioners to the rail vehicle environment will probably be the first step in the development of an ac traction motor system.

## 1.2 TRACTION SYSTEM EXPERIENCE

Inverter, ac motor, propulsion systems have already been used for experimental and regular service on rapid transit cars,<sup>1,2</sup> diesel-electric locomotives<sup>3</sup>, and electric locomotives.<sup>4</sup> Brown Boveri has proceeded from testing a 200 kilowatt (kW) system in 1974, to the delivery of five prototype, electric locomotives in early 1979, that are rated at 1.4 megawatts (MW) per inverter system, 5.6 MW total per locomotive.<sup>5</sup> The company has also built 12 bimode, diesel-engine/600 volts dc, catenary, inverter-drive, industrial locomotives.

Compared to the developmental programs of the European electric locomotive builders, there has been little comparable work on locomotive propulsion systems in the United States. Work has been done by Westinghouse Air Brake Company (WABCO) and General Electric Company (GE) for rapid transit cars, and by Garrett Air Research for air-cushion vehicles. The active, rapid transit, propulsion system work in the United States has been directed toward choppers with dc traction motors, as typified by Bay Area Rapid Transit (BART), Metropolitan Atlanta Rapid Transit Authority (MARTA), and the Light Rail Vehicle (LRV) program.

## 1.3 TYPES OF SYSTEMS

Adjustable-speed, ac drive systems utilize inverters which are classified by the method used to commutate or shut off the main thyristors during each cycle of operation. These classifications are:

1. forced commutated inverters
2. natural commutated inverters
3. machine commutated inverters.

Forced commutation involves discharging the energy stored in a commutating capacitor to reverse the voltage across a conducting thyristor to shut off that thyristor. The discharge is usually effected by an auxiliary commutating thyristor. The forced commutated inverter is used where the dc bus voltage is fixed and where the output voltage must be pulse-width modulated. The current source inverter is a special type in which the firing of the next thyristor discharges the commutating capacitor and shuts off the previous thyristor. The forced commutated inverter is both the most compact and most complex of the inverters, and can drive induction motors, linear motors, and synchronous motors, usually up to 1000 hp.

Natural commutation requires a series-resonant, output circuit. When a main thyristor is fired, its current "rings" at the resonant frequency of the output circuit and it self-commutates at a current zero or a voltage peak. The natural commutated inverter is used to generate high-frequency power (into the 20-to-30 kilohertz (kHz) range) for induction heating, ultrasonics, and lighting. The inverter also can be configured to synthesize lower-frequency waveforms from trains of high-frequency pulses, using pulse position and pulse polarity modulation. Machine commutation requires a synchronous motor as the inverter load. The system is made up of two bridge converters interconnected by a dc link. One converter operates as a rectifier; the second as an inverter, using the rotationally induced motor voltage as the reference bus. These systems have been built up to about 20,000 hp for large blowers and pumps, and for starting large synchronous motors.

#### 1.4 SCOPE OF REPORT

When resources for the development of a three-phase, traction system become available, the Department of Transportation (DOT) will select a system which shows a high probability of success. In an effort to accumulate the technical information which can be used as a guide in the selection process, TSC has been conducting a series of programs consisting of computer studies, laboratory demonstrations and measurements, industrial contracts, and

university research studies to provide information relating to the power and performance characteristics of three-phase, ac power, conditioning systems. This report provides test results obtained from a program which included laboratory measurements and computer analysis of three inverter systems, each a variable voltage and variable frequency drive system, but each with different characteristics. The inverters selected for the test program are representative of types which are applicable to traction service and which are available on the commercial market. Differences in output voltage waveform, in output voltage control, in dc link control, and in commutation technique provide a variety of input-output system characteristics which can be used in evaluating the applicability of an inverter type to a particular traction system application.

The results are not intended to identify the prime candidate system for full scale development, nor to evaluate the product of any specific manufacturer. Rather, they are meant to serve as an aid in evaluating the trade-offs that must be considered in selecting the type of power conditioning unit incorporated into a full scale design.

## 2. AC TRACTION PROPULSION SYSTEM REQUIREMENTS

### 2.1 VEHICLE REQUIREMENTS

Each electrically driven vehicle, whether it be a rapid transit car or an electric locomotive, must undergo a cycle of acceleration, coast, brake, and dwell, (as shown in Figure 2-1) The vehicles differ in the rates of acceleration and braking, and the times that they spend in each step of the cycle. The propulsion system must be capable of providing the motoring and braking torques required for the cycle, as well as the ability to maintain top speed. The torque requirements are calculated from the speed profile along the route and the parameters of grade, wind, train weight, and friction.

### 2.2 PROPULSION SYSTEM REQUIREMENTS

A typical torque versus speed power profile for an electric traction system is shown in Figure 2-2. The principle characteristics of that profile are:

1. maximum tractive effort (shown as acceleration) available from zero speed to motor base speed,  $V_1$ . (Base speed is that speed where the motor delivers rated horsepower at rated torque.)
2. constant horsepower operation from motor base speed,  $V_1$ , to speed,  $V_2$
3. operation from speed,  $V_2$ , to maximum speed with torque determined by the limitations of the motor
4. dynamic braking operation from maximum speed to speed,  $V_3$ , with torque limited by the adhesion properties of the wheel and rail
5. dynamic braking operation from speed,  $V_3$ , to standstill at maximum braking torque.

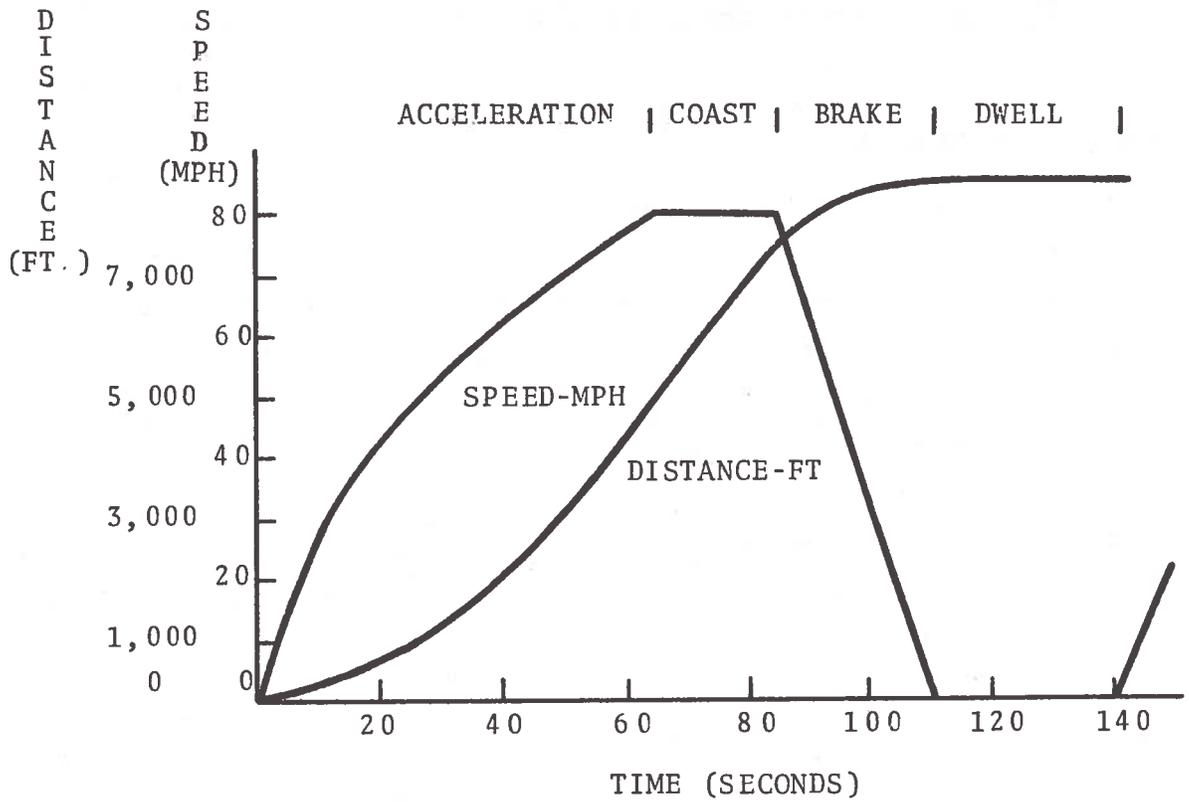


FIGURE 2-1. SAMPLE DUTY CYCLE

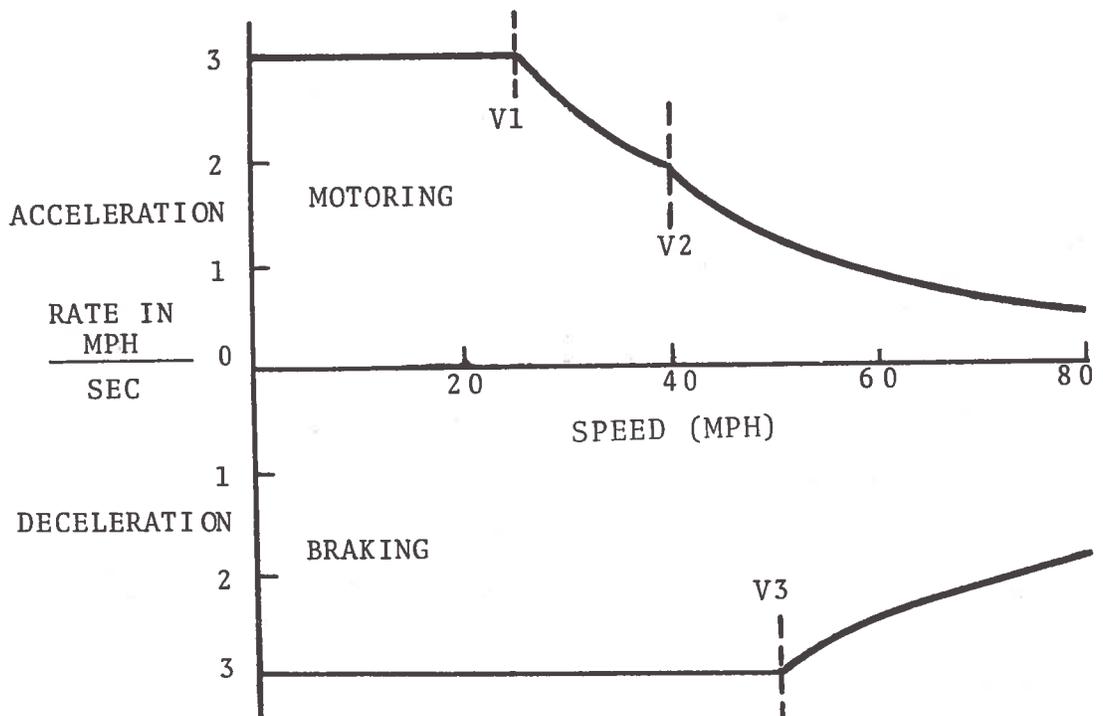


FIGURE 2-2. CAR PERFORMANCE CURVES

The loci shown in Figure 2-2 represent the maximum torque capability of the propulsion system at each speed. Obviously, the propulsion system must be capable of operating at all motoring and braking torques below those levels.

### 2.3 AC PROPULSION SYSTEMS

The inverter-induction motor drive system can provide the torques of Figure 2-2 by adjusting the frequency and voltage of the motor at each speed point as shown in Figure 2-3, which shows the characteristics of an ac traction system. For the ac machine, speed is proportional to applied frequency; therefore, an efficient, ac drive system must be a variable frequency system. The inverter must also be a variable voltage source because as the frequency increases, the air gap flux density will decrease and reduce the torque capability of the motor. To maintain a constant air gap flux density, the inverter must provide a constant, volt-per-Hertz output ratio. By maintaining this ratio, the motor can be made to operate near maximum rated torque from zero speed to maximum speed.

For example, the frequency of the inverter is raised from  $f_0$  to  $f_1$  to cover the motoring speed range from zero to  $V_1$ . Over this range, the motor voltage is adjusted proportionally to frequency to maintain constant air-gap flux density. The frequency is further raised to  $f_2$  to cover the speed range to  $V_2$ , but the voltage is maintained constant. In the motoring mode, the frequency must be kept higher than the frequency corresponding to the actual motor speed by the increment called the slip speed. In the braking mode, the frequency must be kept lower at each speed point to force the motor to present negative torque to the wheels and to regenerate the braking energy back through the inverter. An important feature of induction motor drive systems is that the maximum speed at which the motor can operate at any frequency is the synchronous speed for that frequency. In the event of load shedding, such as a wheel slip, the motor will accelerate along the tractive effort line for the particular applied frequency to a point near the zero of the tractive effort curve, a speed change

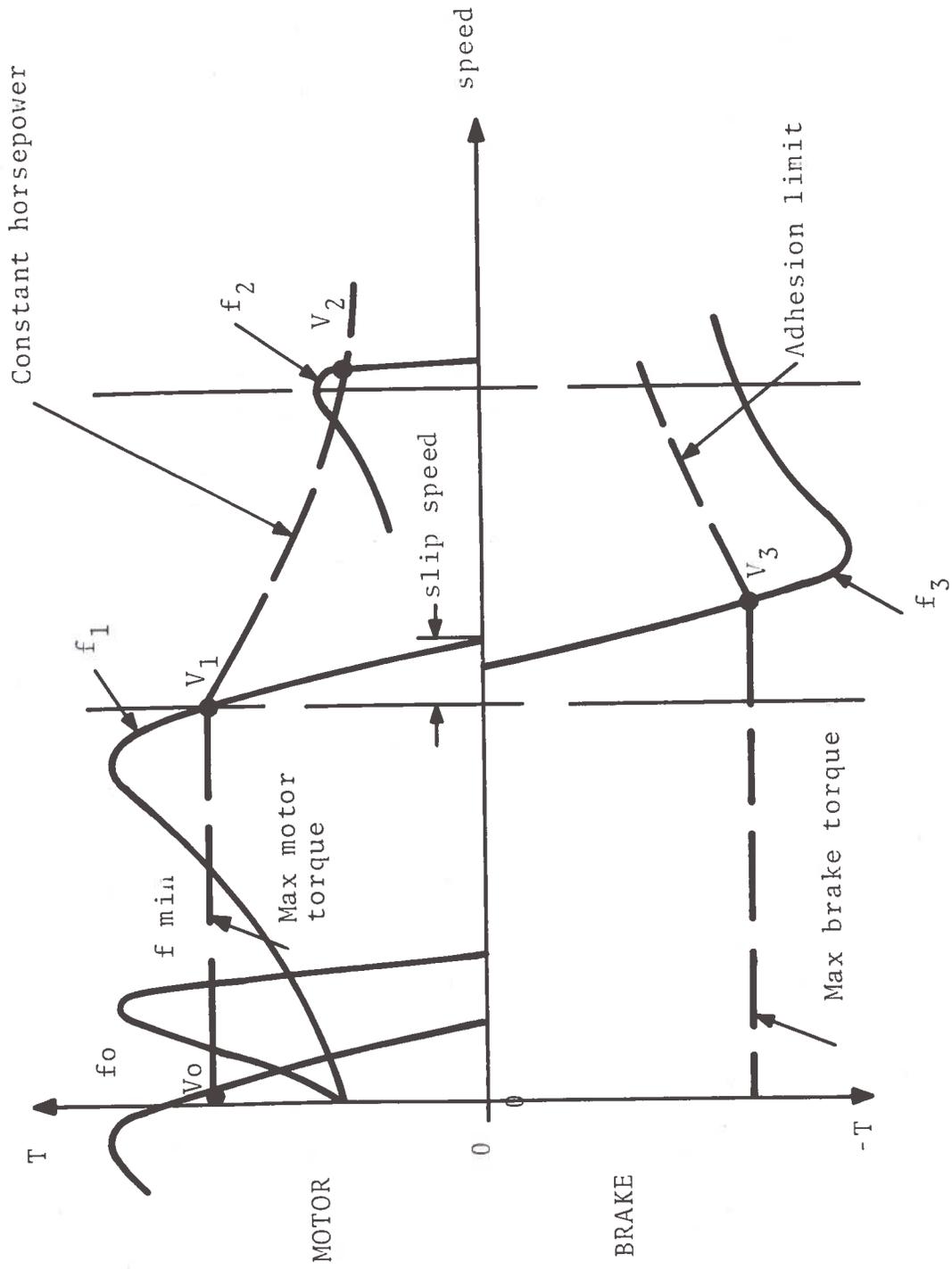


FIGURE 2-3. INDUCTION MOTOR CHARACTERISTICS SUPERIMPOSED ON PROPULSION SYSTEM REQUIREMENTS

generally less than 10 percent. Despite the apparent advantages, utilization of ac motors for traction applications lacks industrial application for two reasons:

1. The development of the series wound dc motor has proceeded to the point where the state of the art is far ahead of the ac motor for traction applications.
2. Power conditioner designs have been directed to industrial rather than rail vehicle environments.

#### 2.4 HORSEPOWER REQUIREMENTS

To provide perspective on the application of inverter-induction motor drive systems for traction applications, the typical traction requirements will be compared with the state of the art in industrial drive systems. Rapid transit and self-propelled intercity cars use four traction motors rated in the range of 140 to 180 hp per motor. The peak power during acceleration is about 1000 kW per car. Electric locomotives are rated to about 6000 hp in actual service and to about 10,000 hp on an experimental basis. They use six 1000 hp traction motors. A single inverter with a short-time 1000 kW rating can meet the requirements for a self-propelled car. A 1200 kW inverter for each motor can meet the requirements of a locomotive. Industrial drives with modern devices can achieve up to 200 kW per single-phase bridge. The traction requirements can be met with six bridges suitably phase-shifted to obtain the required three-phase output. Traction inverter systems rated at 1400 kW have been successfully tested.

#### 2.5 CONSTRAINTS ON AC PROPULSION SYSTEMS

The test program described in this report focuses on three candidate inverters for traction duty. However, the constraints of such duty may restrict the choice to one type regardless of small differences in its performance and/or cost relative to the others. Typical constraints for various applications include the following items:

1. operation directly from dc wayside power
2. minimum levels of electromagnetic interference
3. ability to regenerate energy to the supply line
4. individual wheel-slip control
5. sustained low-speed operation.

The test program will disclose measurable features of the actual inverter systems selected for the program. These test results will provide information on whether the candidate inverters can be used for specific traction applications.

### 3. TEST PROGRAM

The test program involved laboratory measurements of three inverter systems, powering static, fixed loads and driving an induction motor with load controlled by a dynamometer. Computer models and simulations were used in conjunction with measurements to analyze test results.

#### 3.1 INVERTERS IN TEST PROGRAM

The three inverter systems tested are:

1. A natural commutated inverter manufactured by the Bose Corporation, rated at 10 kW output with a 3 $\phi$  input voltage of 325 volts. The inverter provides 3 $\phi$  output voltage, variable from 0 to 208 volts line to line rms, at output frequencies from 0 to 400 Hertz (Hz). Pulse position modulation is used to synthesize the output waveform.
2. A forced commutated inverter manufactured by General Electric, rated at 15 hp output with a 3 $\phi$  input voltage of 230 volts. The inverter provides 3 $\phi$  output voltage, variable from 0 to 208 volts line to line rms at output frequencies from 0 to 400 Hz. The basic inverter circuit is the "McMurray" circuit<sup>7</sup> and pulse-width modulation is used to control the output voltage.
3. A parallel capacitor commutated inverter manufactured by the Louis-Allis Company, rated at 20 hp output with a 3 $\phi$  input voltage of 460 volts. The inverter provides 3 $\phi$  output voltage, variable from 0 to 460 volts at output frequencies of 0 to 120 Hz. The inverter control circuitry regulates dc current, thus the inverter provides a constant current excitation to a motor load, and it will be referred to as the constant current inverter throughout this report.

The inverters used in the test program were chosen from available industrial products based on variety in input circuitry, wave-

form generation, and output wave shape.

The inverters can each be segmented into three sections as shown in Figure 3-1:

1. rectifier
2. dc link
3. inverter.

The rectifier circuit converts input 3 $\phi$  ac power to dc. The dc link, which includes filtering, serves as a buffer to isolate the inverter from the rectifier. It prevents source-load interaction, and makes it possible to operate either of the inverters from presently available wayside power sources. The inverter converts the dc input to variable voltage, variable frequency, 3 $\phi$  ac power for the test load.

The rectifier circuit for both the natural commutated inverter and the pulse-width modulated inverter is a full-wave, six diode, rectifier bridge. The natural commutated inverter employs a capacitive input filter while the pulse-width modulated inverter incorporates an inductive input filter.

The constant current inverter is supplied by a six thyristor, full-wave, phase delay rectifier with an inductive input filter. Output voltage magnitude is maintained by controlling the point in the input voltage cycle that the bridge thyristors are gated on. Each input bridge presents different load characteristics to the ac source.

## 3.2 DESCRIPTION OF OPERATION

A brief description of the inverter operation is given below; more detailed descriptions are given in Appendix B. In depth analysis can be found in references 7 through 12.

### 3.2.1 Natural Commutated Inverter

The natural commutated inverter, with the capacitive input filter, appears as a low impedance to the source during the

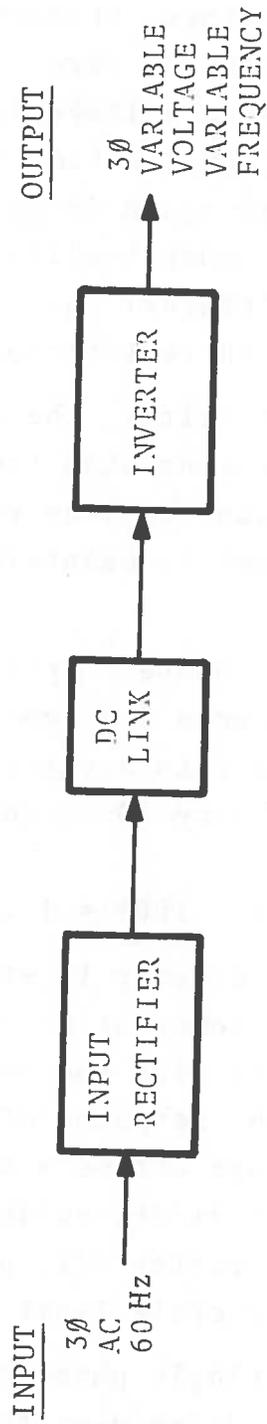


FIGURE 3-1. INVERTER SYSTEM COMPONENTS

rectifier conduction interval. The currents during these intervals are high in amplitude, but narrow in width, which results in high peak to rms ratios.

In a rectifier circuit, any diodes with forward bias on them will conduct. The capacitors, because of their low impedance, quickly charge to the instantaneous level of the ac source voltage and then continue to the peak ac voltage level. For light loads, the diodes will not conduct from the time the input ac voltage peaks until the filter voltage again drops below the instantaneous, line to line voltage level. Under heavily loaded conditions, the diodes conduct for the full interval that they are forward biased (1/3 cycle), but peak to rms current ratios remain high.

With the capacitive input filter, the dc voltage can vary from  $E_d = \sqrt{2} V_\ell$  (for no load conditions when the capacitors charge to the peak value of the input line to line voltage) to  $E_d = 3\sqrt{2} V_\ell$  (where the load is heavy enough to maintain continuous diode conduction).

A 360 Hz ripple present in the output voltage waveform is a distortion component which lowers the average dc voltage from the peak value of the input voltage to a value of  $3/\pi$  times that peak voltage. Thus, for the full wave  $3\phi$  circuit, no load to full load regulation is

$$(1 - 3/\pi) \quad 100\% = 4.5\%.$$

The natural commutated inverter is similar to a series inverter where the thyristor commutation components, an inductor and a capacitor, are in series with the load. Commutation is accomplished when the resonant response of those commutation components reverses the voltage across a conducting thyristor. These components also provide  $di/dt$ ,  $dv/dt$ , and fault protection. With a faulted output, the inverter will go into a current limit mode of operation at the duty cycle limit of the inverter bridge.

The basic circuit of a single phase of a three-phase system is shown in Figure 3-2. The drive uses three individual inverters to provide a three-phase variable voltage and frequency output which approximates in waveform the three-phase reference input signals. The pulse-widths of the series resonant pulses are

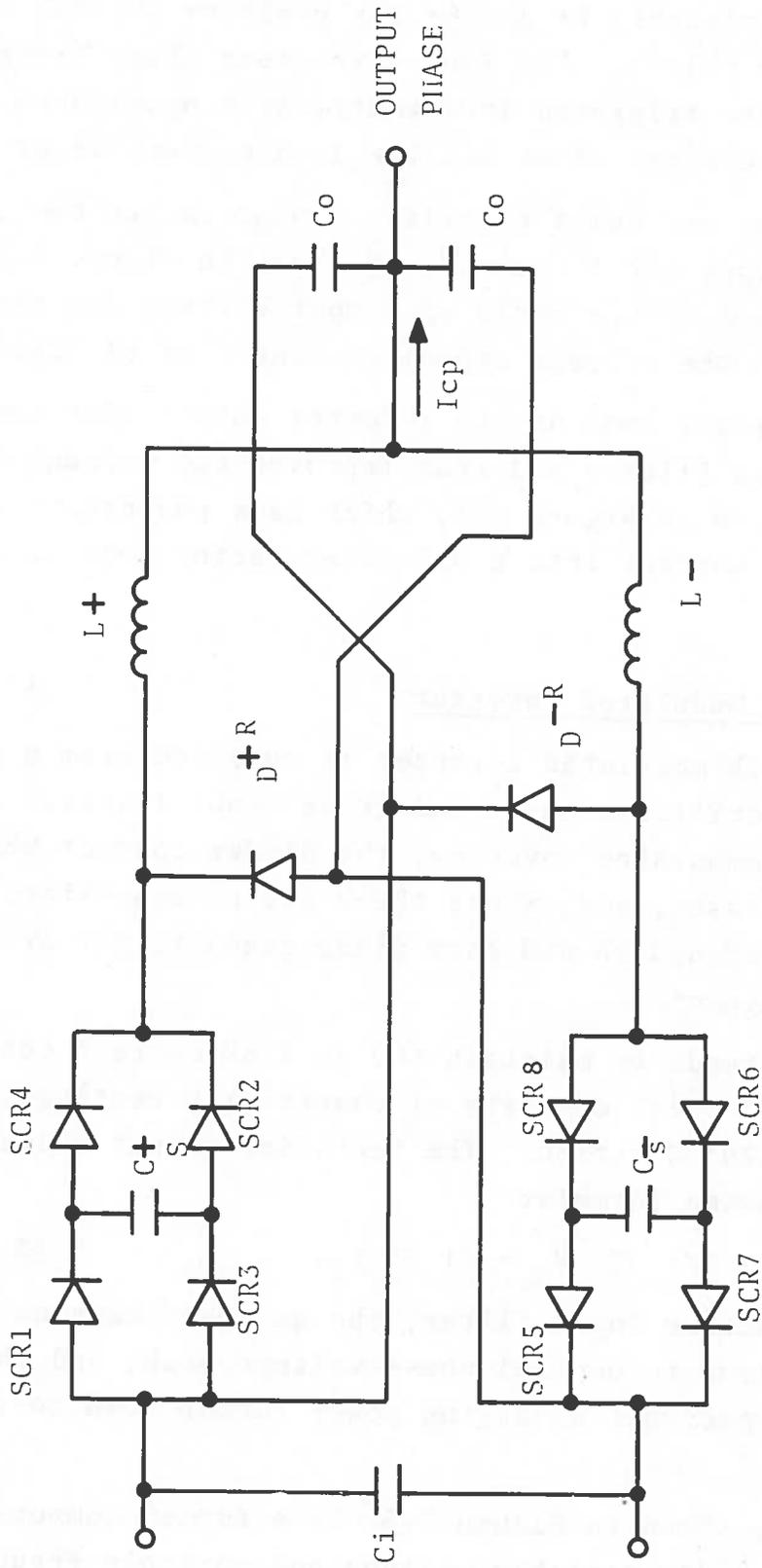


FIGURE 3-2. NATURAL COMMUTATED INVERTER BASIC POWER STAGE

determined by the respective inductors and capacitors in series with conducting thyristors: L+ and C+ for positive pulses; L- and C- for negative pulses. The four thyristors which bridge each of the capacitors are triggered in diametrically opposed pairs so that the capacitor current is ac but the load current is dc.

Pulse frequency and pulse polarity modulation are combined to control output voltage and frequency, as shown in Figure 3-3, which is a photograph of one cycle of output voltage and the charge current pulses into the storage capacitor bank (Icp of Figure 3-2).

An induction motor load on the inverter output adds inductive energy storage which filters and thus improves the current wave shape. This is shown in Figure 3-4, which is a photograph of the output voltage and current into a 0.7 power factor load at a frequency of 60 Hz.

### 3.2.2 Pulse-Width Modulated Inverter

The pulse-width modulated inverter is supplied from a full-wave, six diode rectifier with an inductive input filter. As with the natural commutated inverter, the diodes conduct when they are forward biased, and, since there are no capacitors, diode current does not extinguish and each diode conducts for 1/3 (120°) of the input voltage cycle.

The inductor tends to maintain the dc link current constant so that the diode current consists of essentially rectangular shaped pulses of 120° duration. The rectifier output voltage is given by the following formula:

$$E_d = \frac{3}{\pi} \sqrt{2} V_\ell = 1.35 V_\ell. \quad (3-1)$$

With the inductive input filter, the ac phase current peak occurs after the line to neutral phase voltage peak, and the rectifier circuit presents a lagging power factor load to the ac source.

The inverter, shown in Figure 3-5, is a forced commutated inverter which provides variable voltage and variable frequency power for driving ac motors. Output current flows from the dc

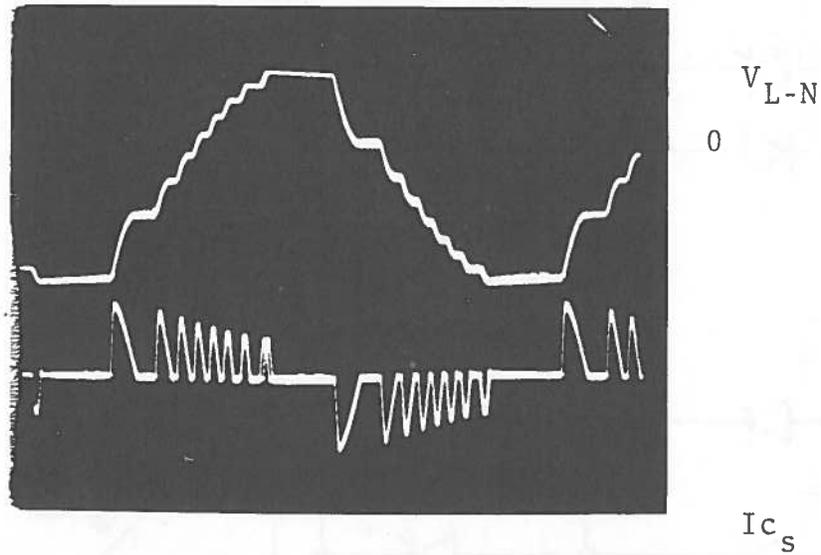


FIGURE 3-3. LINE TO NEUTRAL OUTPUT VOLTAGE AND CHARGE CURRENT PULSES

PF = 0.7  
 I = 20 AMPS  
 f = 60 Hz

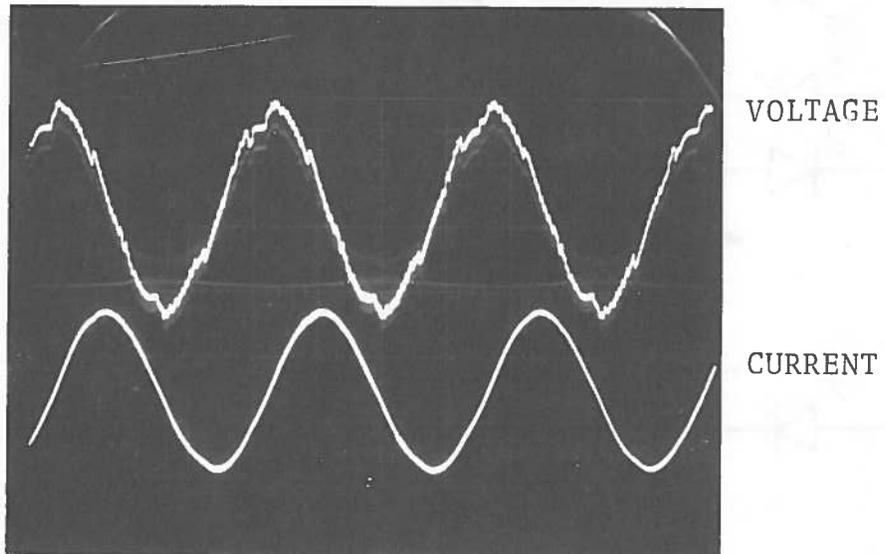


FIGURE 3-4. OUTPUT VOLTAGE AND CURRENT WITH LAGGING POWER FACTOR LOAD



source to the load through the main power thyristor switches, which, with the exception of snubbers and current sensing circuits, are the only elements in series with the load. Once turned on, these conducting switches must be turned off, or "commutated", by reducing the current through them to a level below their holding current. Several methods for force commutating thyristors have been developed. The inverter tested in the TSC program was an auxiliary impulse commutated inverter, commonly called the "McMurray" circuit, whose commutation pulses are generated by a separate commutation thyristor and a series resonant tank circuit. Voltage and frequency control is achieved by controlling the conduction intervals and sequences of the main power switches. Both voltage and frequency are continuously adjustable over their operating ranges: voltage from 0 to 208 volts, line to line, and frequency from 4 to 400 Hz.

The voltage, supplied from the dc link is varied by using pulse-width modulation. The frequency is controlled by varying the rate at which the load is connected to the positive and negative sides of the dc bus. Phasing of the output waveform is obtained by internal timing of the thyristor firing pulses to maintain correct phase angle and rotation. The output waveform is similar to a six pulse waveform; however it is chopped or notched, so that each cycle consists of a series of equal amplitude, but varied width pulses. The magnitude of the resultant output voltage is maintained by controlling the width of the notches between the output pulses. The modulation and thyristor recovery considerations limit the maximum duty cycle to approximately 90 percent. Thus, the inverter cannot deliver full power from its total volt-ampere capability.

In the event of a load fault, the conducting thyristors must be commutated before the fault current exceeds the commutation limit. Failure to commutate under these conditions can result in catastrophic failure of the device.

Pulse-width modulation allows both frequency and output voltage magnitude to be controlled with one set of switching

semiconductors. The use of an uncontrolled, diode, input rectifier bridge also eliminates the displacement component of reactive power; thus, the system operates at power factors close to unity.

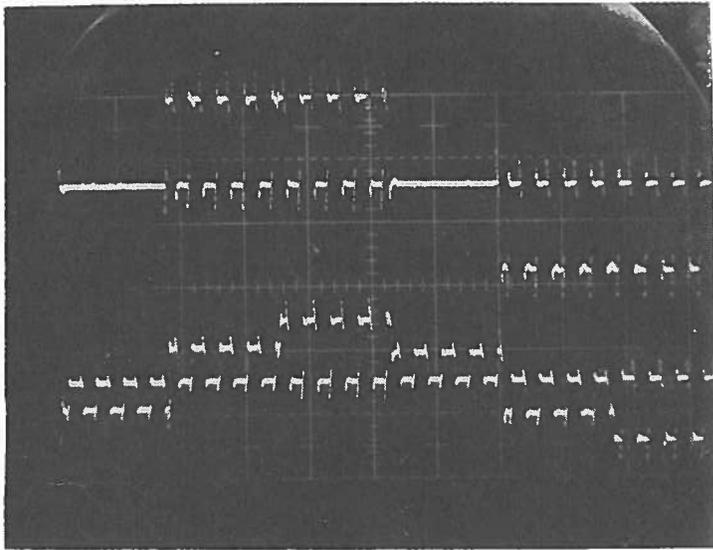
The resulting waveforms, at one frequency in each of the three frequency ranges, are shown in Figure 3-6.

The top traces in the output, voltage waveform photos are the line to line voltages. The line to neutral voltages are shown in the bottom traces. The change in the number of chops as frequency increases is evident in the photos. As the frequency in any range increases, the switching transients per second and the total switching loss increase. At 120 Hz, for instance, there are 2160 commutations per second. At that point, the waveform is changed to one with four notches, and the commutations per second are reduced to 1200 with a corresponding reduction in switching loss. Similarly, at 213 Hz the waveform is again modified to one with two notches and the switching losses are again reduced.

This change in waveform as a function of frequency results in discontinuities in the inverter characteristics at each of the break points. The effects are noticeable in the power characteristics and in the harmonic data. The harmonic ratios not only change for each waveform as the number of notches change, but also as the width of the notches varies with output voltage control.

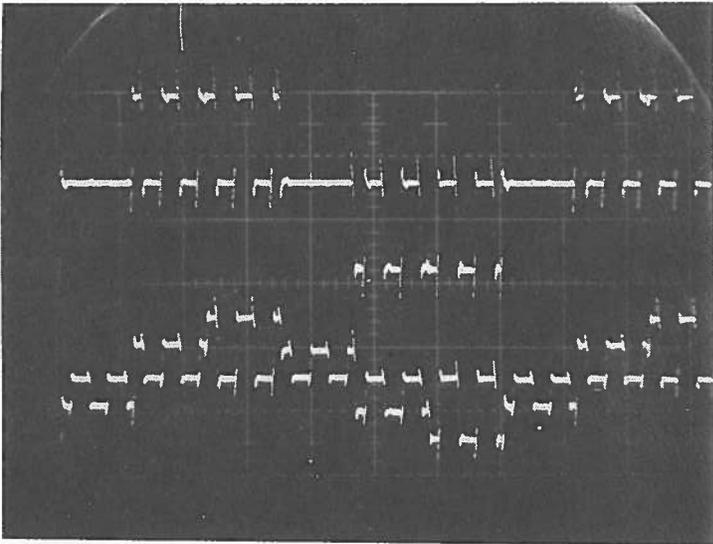
### 3.2.3 Constant Current Complementary Commutated Inverter

The constant current inverter is supplied by a six thyristor, full-wave, phase delay rectifier with an inductive input filter. DC link voltage magnitude is managed by controlling the point in the input voltage cycle that the bridge thyristors are gated on. When the thyristors are gated on without any delay, the circuit operation is identical to the six diode bridge with inductive filtering which supplies the pulse-width modulated inverter. Output voltage is at its maximum,  $E_d = 3/\pi \sqrt{2} V_\ell$ , and the power factor is maximum,  $3/\pi$ , lagging.



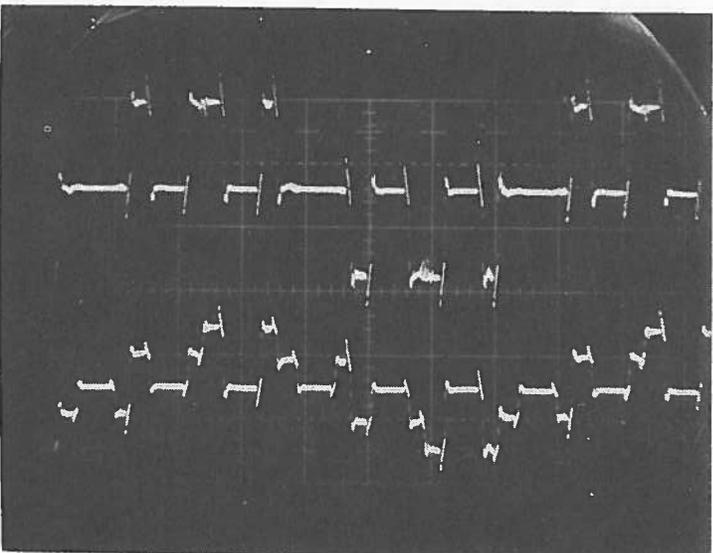
a) EIGHT NOTCH  
OUTPUT

100 Hz



b) FOUR NOTCH  
OUTPUT

150 Hz



c) TWO NOTCH  
OUTPUT

300 Hz

FIGURE 3-6. OUTPUT VOLTAGE WAVEFORMS

As the thyristor turn-on gating pulses are delayed, the output voltage and input power factor are reduced. For the case where the dc link current magnitude is constant, it can be shown that the phase delay rectifier output voltage is given by:

$$E_d = E_{do} \cos \alpha, \quad (3-2)$$

where  $E_{do}$  is the maximum dc output voltage of the rectifier with no delay, and  $\alpha$  is the angle, in electrical degrees, that thyristor conduction is delayed from the point where it first becomes forward biased.

As the delay angle is increased, the phase angle between the ac line current and line to neutral phase voltage also increases and the input power factor decreases. The power factor of the phase delay rectifier is related to the phase delay angle by the following relationship:

$$\text{pf} = 3/\pi \cos \alpha. \quad (3-3)$$

This relationship is valid for the conditions of constant dc current, which produces rectangular-shaped, line current pulses and fixed, harmonic to fundamental, current ratios, a condition maintained by the control circuitry of the constant current inverter. The maximum power factor, at zero delay angle, is:

$$\text{pf} = 3/\pi = 0.955. \quad (3-4)$$

Although the delay angle is zero and the displacement between the phase currents and voltages are zero, there is a distortion component of reactive power which limits the maximum system power factor to 0.955. As the delay angle increases, the distortion factor remains 0.955, but the displacement factor increases and power factor decreases. At delay angles greater than  $58.4^\circ$ , the system power factor falls below 0.5.

The constant current inverter circuit utilizes a three-phase inverter circuit commonly called a parallel capacitor commutated, or complementary commutated, inverter circuit. Commutation of conducting thyristors is accomplished by capacitively coupling a commutation transient from an on-going thyristor to a conducting

thyristor. The resulting reverse voltage bias across the conducting thyristor forces the anode current to zero and commutates the thyristor.

A schematic of the power circuit for the complementary commutated inverter is shown in Figure 3-7. The phase delay rectifier (pdr) converts the constant voltage 460 v, 3 $\emptyset$  input voltage to variable voltage dc for the inverter. The dc choke (L) filters the pdr output and maintains a constant current into the inverter terminals. The inverter control systems regulates the dc link current, thus providing inherent fault protection. With a faulted output, the pdr phases back until the output voltage is low enough to apply only the inverter limit current. In the motoring mode, the dc output voltage of the pdr is positive in the direction shown and the dc current flows in the direction shown. During regeneration, the polarity of the dc voltage is reversed, as shown in the Figure, by action of the inverter but the direction of the dc current remains unchanged. During regeneration, the motor becomes the source of the ac power, the inverter functions as a rectifier, converting the motor ac voltage to a dc voltage, and the pdr functions as an inverter, converting the dc voltage to 60 Hz, 3 $\emptyset$  ac.

### 3.3 OUTPUT VOLTAGE PROFILE

Each inverter's power handling capability is limited by thyristor current considerations. Peak currents through the inverter are limited to approximately 1.5 times rated output current, considerably lower than the five to six times rated drawn by a typical induction motor in across-the-line starting. Since the inverters are variable frequency drive, peak motor torques are developed from standstill to rated speed at lower current levels by supplying the motor with a constant volts/Hz excitation.

The natural commutated inverter develops a quasi sine wave output waveform and it follows fairly closely to a constant volt/Hz excitation profile except for an adjustable, low voltage offset at low frequencies as shown in Figure 3-8. The offset is necessary

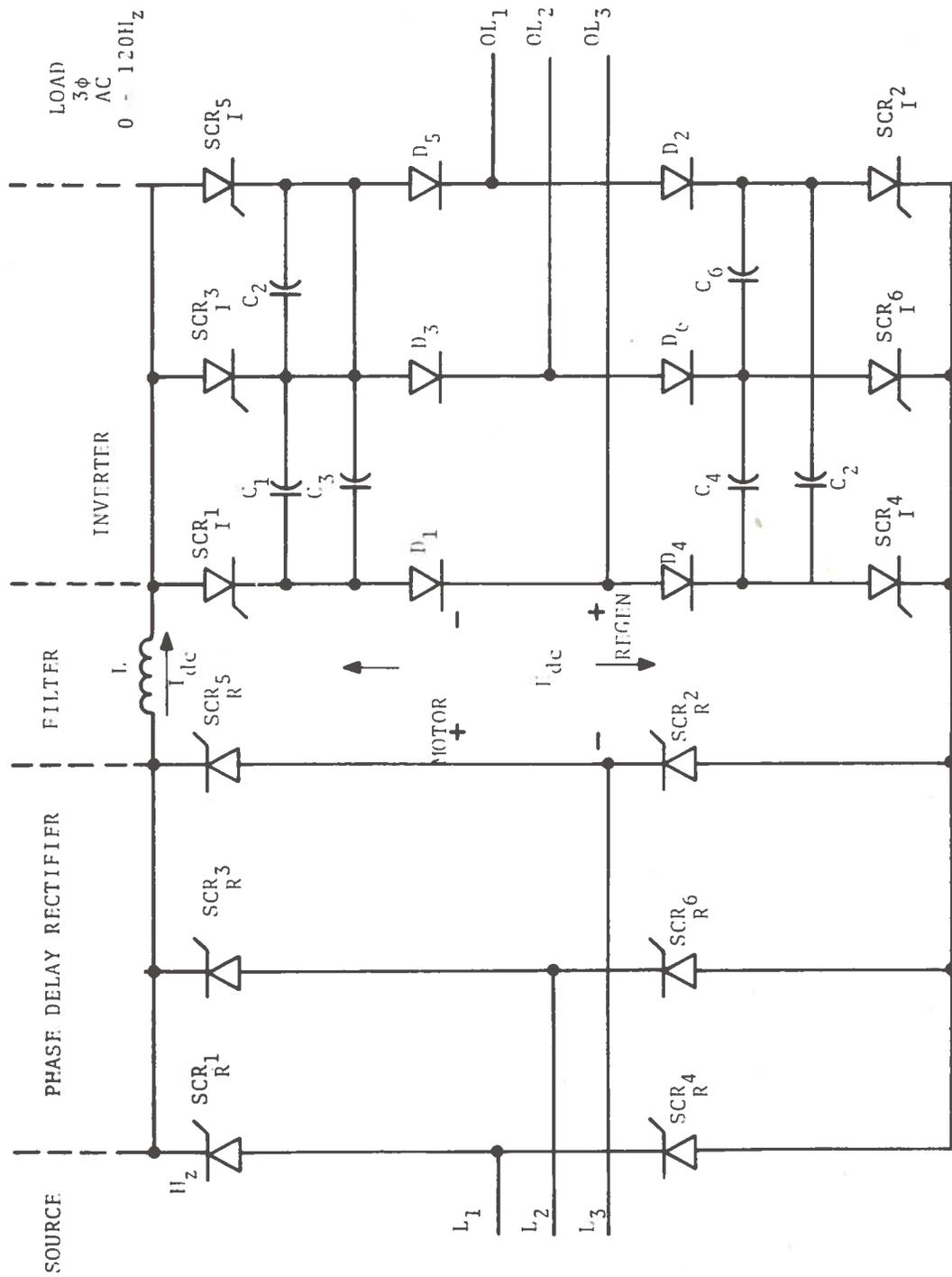


FIGURE 3-7. CONSTANT CURRENT INVERTER POWER CIRCUIT SCHEMATIC

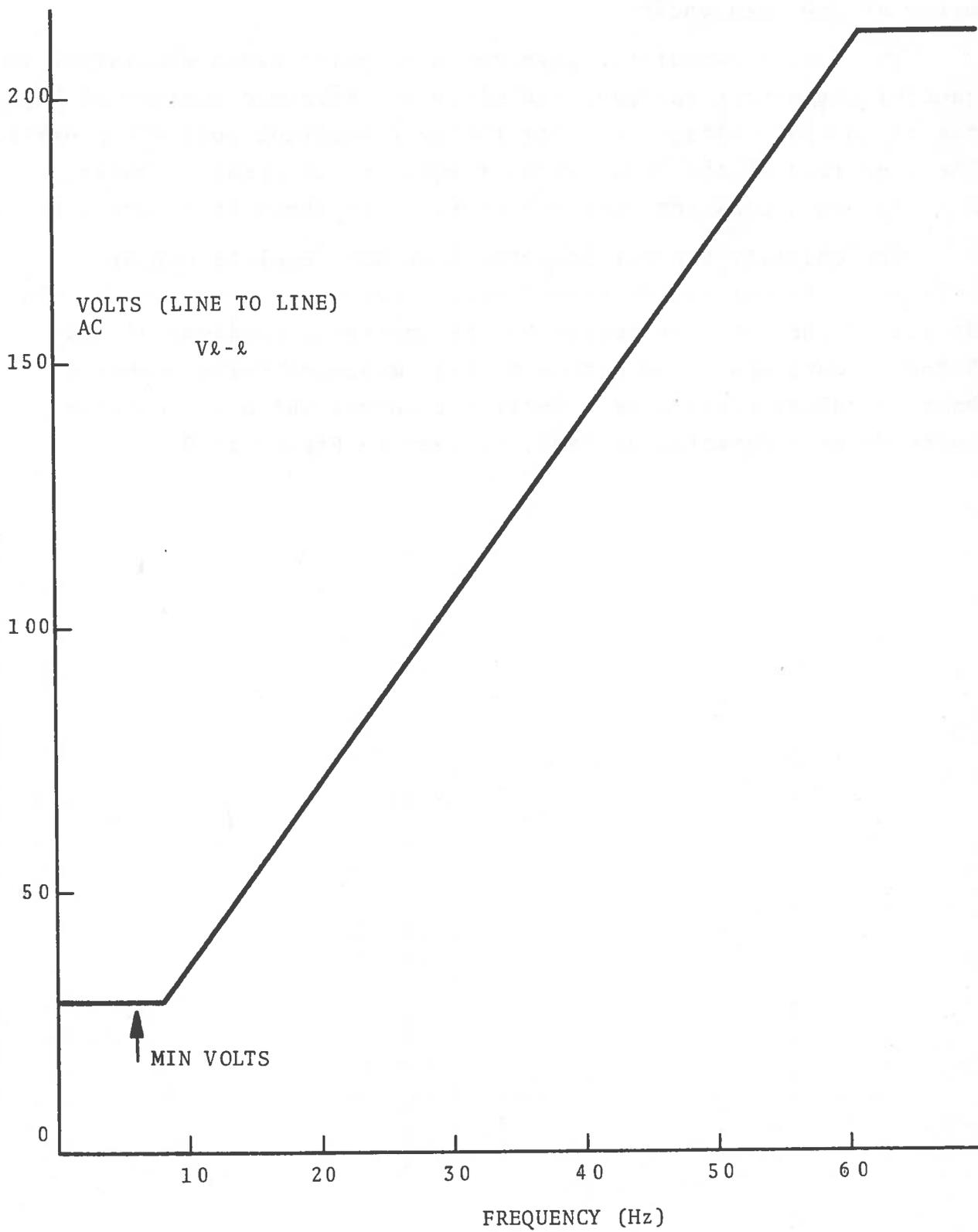


FIGURE 3-8. NATURAL COMMUTATED INVERTER VOLTS/Hz PROFILE

to overcome winding resistance in order to adequately flux the motor at low frequencies.

The forced commutated inverter uses pulse-width modulation to control the output voltage, and since its harmonic content is high, the rms output voltage does not follow a constant volts/Hz profile. The magnitude of the fundamental frequency component, however, does follow a constant volts/Hz profile, as shown in Figure 3-9.

The constant current inverter does not regulate output voltage. The voltage developed across the motor terminals is the result of the motor responses to the impressed fundamental and harmonic currents. The actual voltage measured falls within a band of values containing a family of curves which are constant volts/Hz as a function of load, as seen in Figure 3-10.

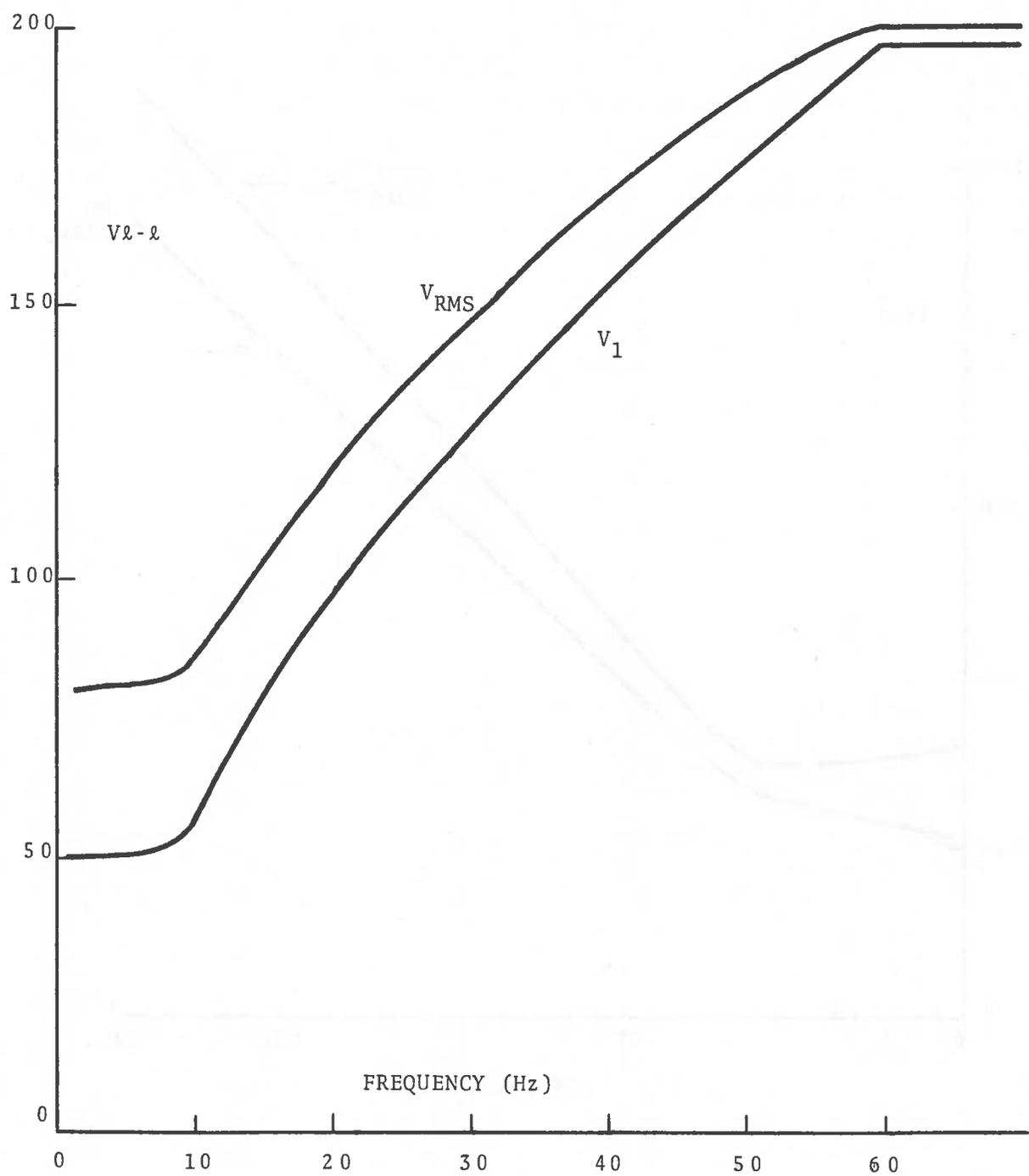


FIGURE 3-9. PULSE-WIDTH MODULATED INVERTER VOLTS/Hz PROFILE

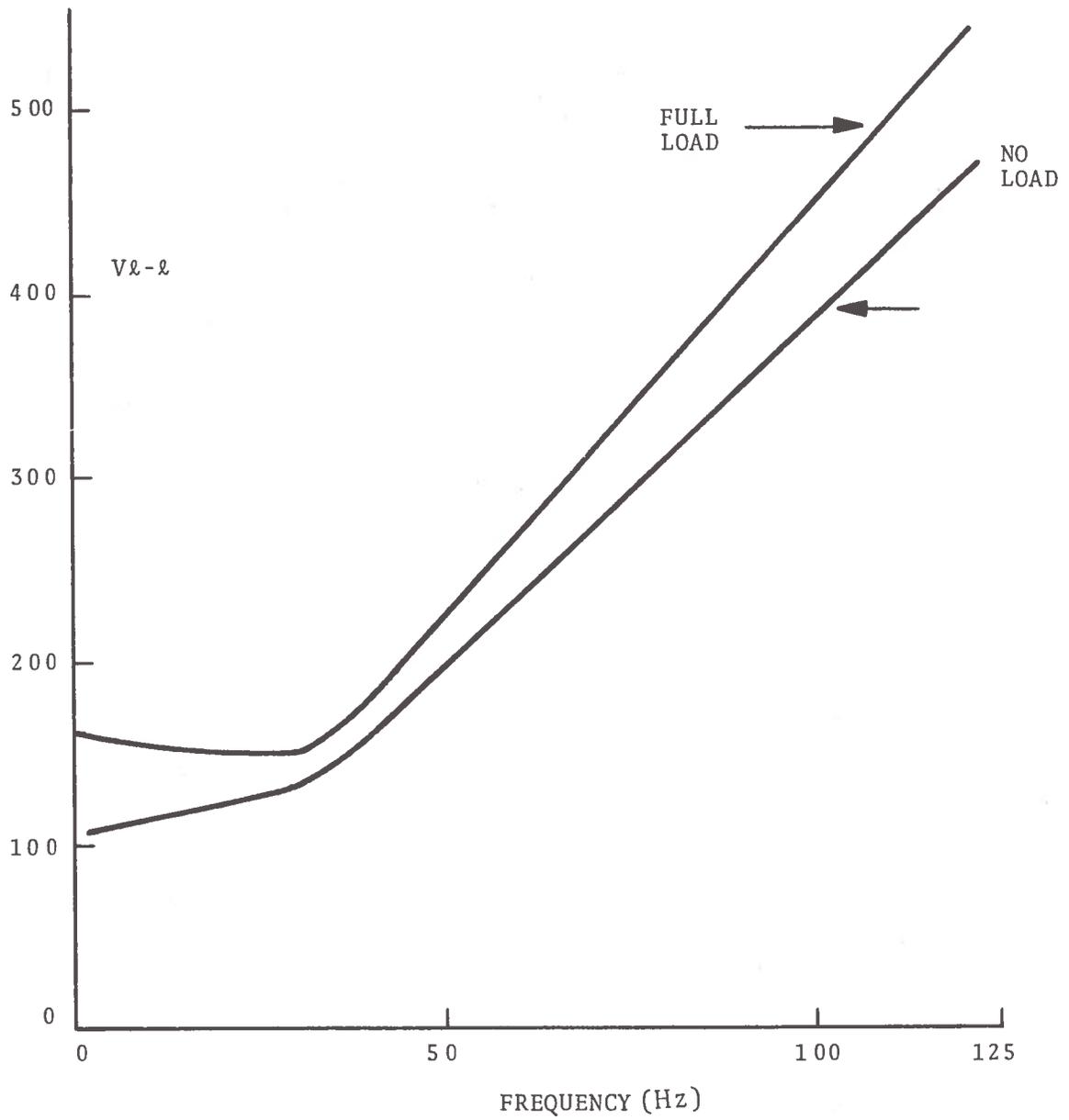


FIGURE 3-10. CONSTANT CURRENT INVERTER VOLTS/Hz PROFILE

## 4. TEST DESCRIPTION

### 4.1 DESCRIPTION OF FACILITY

The inverter testing was performed in the TSC Electrical Power and Propulsion Test Facility which is designed to perform power and harmonic measurements on power conditioning systems and rotating equipment to 100 hp. Equipment available includes precision voltage, current and power meters, and transducers, whose frequency ranges extend beyond the range of the power conditioning equipment and components; also appropriate spectrum analyzers; frequency counters; and recorders. In addition to the laboratory equipment, portable systems suitable for field measurements are available. These, in concert with potential and current transducers, can be used to make field power measurements on railroad equipment presently in service. The facility also includes an ample array of standard test equipment, such as power supplies, oscilloscopes, function generators, and meters.

Several dc and asynchronous ac motors, ranging in size from 10 to 100 hp and controlled by a component dynamometer, are available as test loads for power conditioning systems.

### 4.2 TEST DYNAMOMETER

The component dynamometer used for the inverter testing is shown in Figure 4-1. It is an oil-floated, cradled dc dynamometer, capable of absorbing 65 hp and generating 62 hp at a base speed of 5000 revolutions per minute (rpm); maximum speed is 8000 rpm. Overload capabilities are:

1. 150 percent of rated load for 15 min at 1 hr intervals
2. 200 percent of rated load for 1 min at 10 min intervals
3. 300 percent of rated load for 5 sec at 5 min intervals.

The dynamometer is a four quadrant drive, capable of delivering or absorbing power in either direction of rotation. It can be

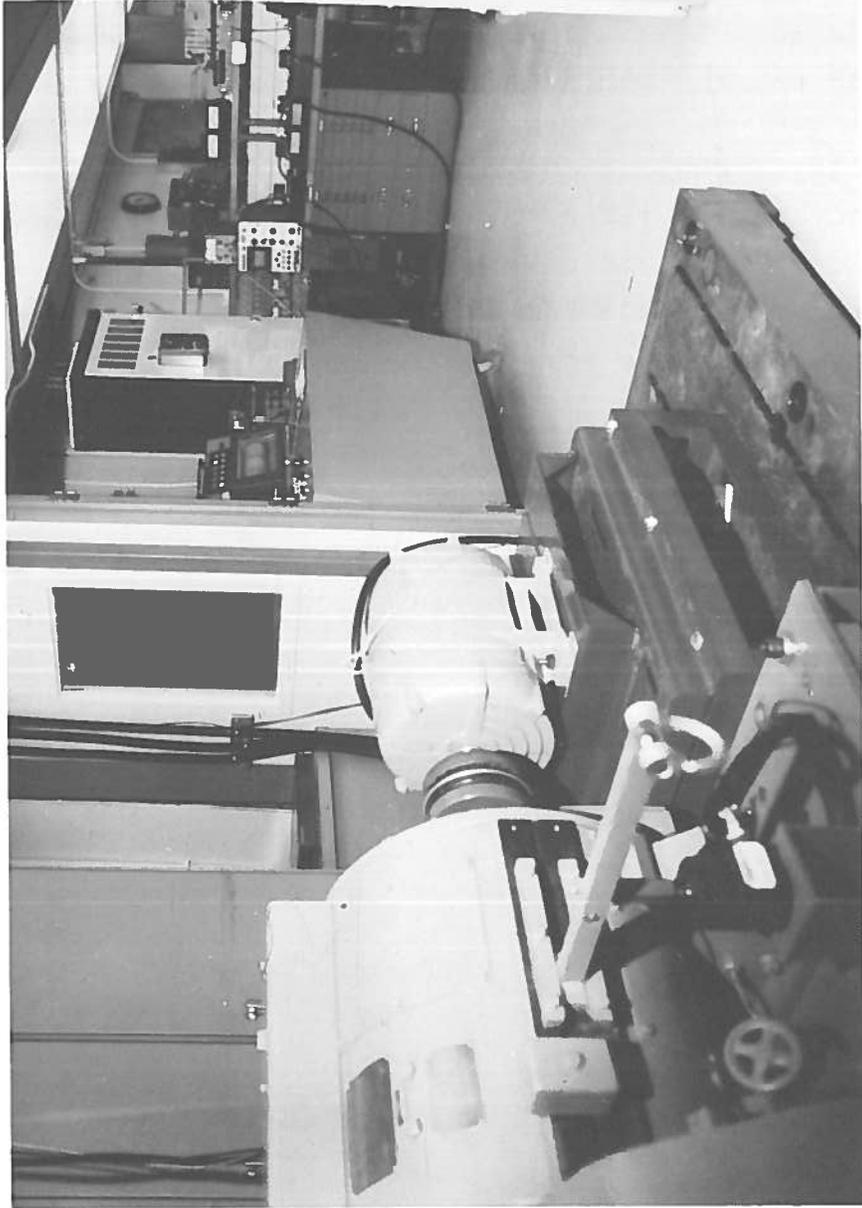


FIGURE 4-1. TRANSPORTATION SYSTEMS CENTER DYNAMOMETER

operated in either a torque control or speed control mode; manually at the control console or automatically from a chart reading programmer or other remote terminal. Torque and speed are regulated to 0.5 percent under steady state conditions. Torque is measured with a BLH type U3 G1, load cell, mounted 18 inches from the axial center line of the dynamometer shaft. The load cell output is fed to an amplifier whose output voltage is proportional to dynamometer shaft torque. This signal is then used in the torque control loop of the dynamometer, and is displayed on a digital voltmeter mounted on the dynamometer control console. The dynamometer has two shaft-mounted speed sensors. One is a belt driven dc generator whose output voltage is proportional to speed, and whose polarity is an indication of direction of rotation. This output is used in the speed control loop of the dynamometer, and it is available for closed loop speed control of inverters, although this speed signal was not used for inverter speed control in this program. The second speed sensor is used for the dynamometer speed visual readout. It consists of a 60-tooth gear mounted on the shaft, a magnetic pick-up, and a digital counter with a 1 second sample gate time. The number of pulses counted during the 1 second sampling interval is equal to the shaft speed in rpm, and this output is displayed on an indicator mounted on the control console.

#### 4.3 TEST CIRCUIT

To provide comparative data for inverter evaluation, each inverter was tested while driving a 20 hp, 3 $\phi$ , 460 volt, 1800 rpm, 120 Hz, NEMA\* class B, induction machine. Since the natural commutated inverter and the forced commutated inverter are rated for 208 volts maximum output at 60 Hz, the motor was underexcited by these inverters at higher frequencies. The constant current inverter is rated for full output power at 120 Hz and 460 volts, therefore this inverter was the only one of the three which could develop the full 20 hp output of the machine. In making power

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\* NEMA - National Electrical Manufacturers Association

characteristic measurements and in collecting harmonic data, the inverter output voltage and frequency were established. Load torque was then applied through control of the component dynamometer. The applied load was incrementally increased at the rate of 5 foot-pounds (ft-lb) per step until either rated inverter current was reached, or the motor began to stall and lose speed. In an efficient load control application, the traction motor will operate in the low slip region of its torque-speed characteristic from zero speed to maximum speed. Therefore, data taken at speeds lower than that of maximum torque are not relevant to this study.

The power, response, and harmonic characteristics for the natural commutated inverter and pulse-width modulated inverter were measured using the motor as a load where possible. At voltages and frequencies where the motor was an inappropriate load, static inductors and resistors were used to simulate motor loads. These static loads were made up of a series combinations of resistors and inductors connected in a Y configuration. The inductors used have an impedance of 10 ohms at 60 Hz, and are tapped at 1.0 ohm intervals. They have linear impedance characteristics in the test frequency range and are rated for a continuous duty of 50 amperes (amps). For each load run where static components were used, resistance was held constant; frequency and the inductor tap setting were changed to set the desired impedance and power factor.

Static load configurations for the inverter testing represented typical load conditions as seen in motor drive applications. They include a load power factor of 0.5, which is similar to a linear induction motor, and a load power factor of 0.8, which is similar to a rotary motor. Data was also taken with a resistive load unity power factor, where such measurements were feasible. The inverters investigated in the TSC program were instrumented as shown in Figure 4-2.

The load indicated in the Figure consists of a three-phase induction motor coupled to the TSC dynamometer. This was the configuration used for testing the three inverters when operated with motor loads. Static loads were substituted for the motor as required when the motor could not be used.

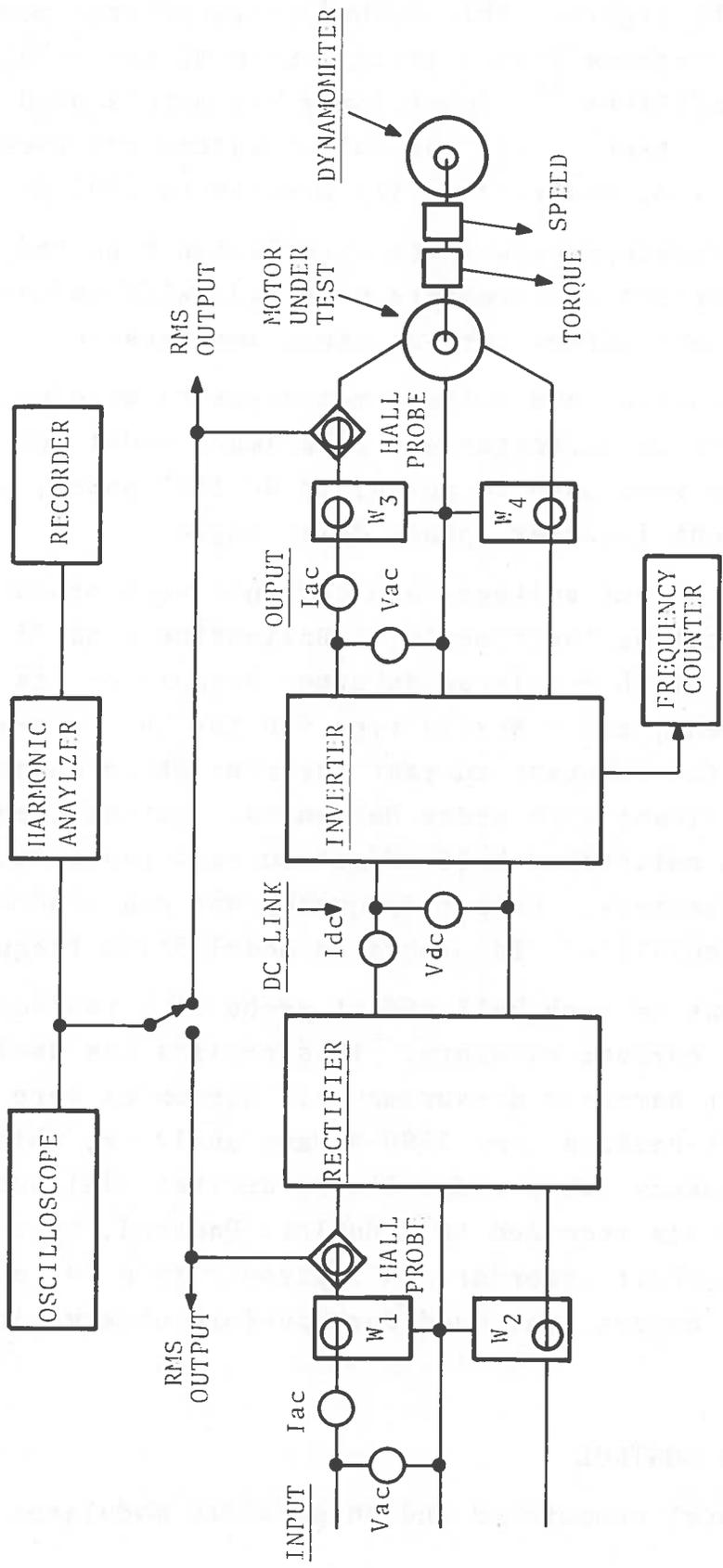


FIGURE 4-2. INVERTER TEST CIRCUIT

Power measurements were made using the two wattmeter method, as shown in the Figure. This method provides true power measurement in a three-phase system irrespective of any unbalance or distortion conditions.<sup>13</sup> Input power wattmeters used were Hallmark model HWP, type 2, and the output wattmeters were Weston model 310, form 3, accurate to 1/2 percent to 2500 Hz.

Voltage readings were taken with Weston type 960 ac voltmeters, and current measurements were made with calibrated, true rms reading, hall effect current probe ammeters built at TSC.

DC link current and voltage measurements were made using a Weston type 901 dc voltmeter and a Hallmark model hdc dc ammeter. These readings were used to calculate dc link power, and, for the constant current inverter, phase delay angle.

Inverter output voltages and currents were measured using true rms indicating instruments, a Ballantine type 323 voltmeter for the pulse-width modulated inverter because of its high voltage harmonic content, and a Weston type 960 for the natural commutated inverter and the constant current inverter whose outputs did not contain significant high order harmonics. Output currents were measured with calibrated hall effect current probes and Weston type 904 ac ammeters. Output frequency was measured using a Transistor Specialties, Incorporated model 361-R frequency counter.

One output of each hall effect probe is a replica of the instantaneous current waveform. This replica was used for photographs and for harmonic measurements. Harmonics were measured with a Hewlett-Packard type 3590-A wave analyzer, which has an internal frequency sweep and a linear decibel (db) output. Harmonic data was recorded on a Hewlett-Packard, type 7414A, four channel strip chart recorder. A Tektronix type 547 oscilloscope, fitted with a camera, was used for waveform observations and photographs.

#### 4.4 INVERTER CONTROL

The natural commutated and pulse-width modulated inverters

tested at TSC are open loop systems which regulate output voltage and frequency, but do not have any speed regulation in the control loop. The test motor thus operated at the equilibrium point of its torque-speed characteristic where motor slip speed varied as a function of the motor load. Since the inverters as delivered did not include closed loop control, control system evaluation could not be made.

The constant current inverter is a closed loop system which regulates motor speed using a tachometer feedback signal. Speed error signals modulate the inverter frequency and current level so that the motor operates at command speed with motor excitation being varied to maintain that speed.

Since the power characteristics of the inverters and their impact on distribution systems and motor operating parameters were being studied, control systems were not incorporated to perform closed loop tests. It is felt that each of the three systems tested can meet operational performance criteria if a closed loop tachometer feedback control system is incorporated in the inverter volts-per-Hertz control.

All three inverter circuits are basically capable of regeneration. However, the natural commutated inverter and pulse-width modulated inverter were not operable in that mode because of the diode input bridge rectifier circuits. These will only allow current flow in a single direction and will not operate with a reversal of output polarity; therefore, regenerative operation was not possible. Whenever the inverter was forced into regeneration by action of the dynamometer, the dc link voltage rose to the overvoltage trip-off point and de-energized the inverter.

The constant current inverter circuit and the phase delay rectifier input circuit are both capable of regenerative operation and the system was tested in the regenerative mode.

#### 4.5 TEST DATA

Power characteristic data, measured and calculated from

measurements, are tabulated in Tables A-1 through A-66, and included as Appendix A. A description of the measurement and calculation procedures used to obtain the power characteristics data is given in Appendix C. Data includes power characteristics data for the natural commutated inverter and the pulse-width modulated inverter powering static loads of 0.5, 0.8 lagging power factor, a power factor of 1.0, and an induction motor load from 10 Hz to 120 Hz. The static load measurements are included because each inverter is rated to 400 Hz and the TSC facility does not have 400 Hz load motor capability. Comparisons between static loads and motor loads are favorable and it is felt that static data can be extrapolated to include motor data at higher frequencies than the 120 Hz achieved in the TSC tests.

The constant current inverter incorporates closed loop tachometer speed control; therefore, only induction motor loads to maximum rated frequency of 120 Hz were included in the inverter tests. This inverter is the only one of those configured for regenerative operation. Although some regenerative data is included in Appendix A, it is not dealt with in depth because it is not possible to make comparisons with either of the other inverters.

## 5. INPUT POWER FACTOR

Input power factor is the ratio of total input power (P) to total apparent input power ( $S_T$ ) of the inverter system. Since each inverter system in the test program is supplied by a three-phase, ac source, the system power factor can be calculated from measurements made at the inverter's ac line terminals using the following formula:

$$\text{pf} = \frac{P}{S_T}. \quad (5-1)$$

Each inverter incorporates a different scheme for rectifying the ac input to supply the dc voltage to the inverter:

1. the natural commutated inverter uses a full-wave, unidirectional bridge rectifier with a capacitive input filter
2. the pulse-width modulated inverter uses a full-wave, unidirectional bridge rectifier with an inductive input filter
3. the constant current inverter uses a full-wave, bidirectional phase delay rectifier with an inductive input filter.

The power factor of the full-wave bridges which supply the inverter systems is a function of real power, displacement reactive power, and distortion reactive power.

There is a power factor associated with rectifier circuits even though there may be no reactive components in the load. This power factor exists because the chopping action of rectifier circuits generate harmonic currents which distort the current waveform and produce a distortion component of reactive power.

In addition, the delay of thyristor turn-on in the phase delay rectifier circuit causes the fundamental components of the current and voltage to be out of phase, resulting in a displacement component of reactive power.

The dc voltage output of a three-phase, full-wave, rectifier

bridge circuit is:

$$E_d = \frac{3}{\pi} \sqrt{2} V_{\ell}, \quad (5-2)$$

where  $V_{\ell}$  is the rms value of the line to line ac input voltage. There is a 360 Hz ripple component in the output which introduces distortion. Therefore, total apparent power is higher than total real power and the maximum input power factor obtainable can be shown to be  $3/\pi$  or 0.955. Measurements differ slightly from this value because of single phase loads which are separate loads from the full-wave bridge, but which enter into system measurements.

Power factors of the three inverters are shown in Figures 5-1 through 5-4. For the natural commutated inverter and the pulse-width modulated inverter, the input power factor is a function of the output power level; but for the constant current inverter, power factor is a function of the delay angle,  $\alpha$ , and the dc link voltage, which vary as functions of both speed and load.

### 5.1 NATURAL COMMUTATED INVERTER

The input power factor of the rectifier circuit which powers the natural commutated inverter is shown in Figure 5-1.

The internal current in the inverter proper consists of a series of high energy discontinuous pulses with high peak to rms ratios. These pulses must be supplied from a low impedance source. Therefore, a bank of storage capacitors is used as the filter. The capacitors, however, also present a low impedance to the source, and the current into them is also characterized by high peak to rms ratios. The conduction intervals of the bridge diodes are limited to the time required to charge the storage capacitor bank to the peak value of the input line voltage. The result is an input of discrete current pulses and high harmonic content, which causes a high distortion reactive power and lower system input power factor.

As load increases, the current pulses widen and the harmonic ratios decrease. However, the current pulse waveform does not become a quasi-square wave as with an inductively filtered load. Rather, it is still characterized by a high peak to rms ratio with

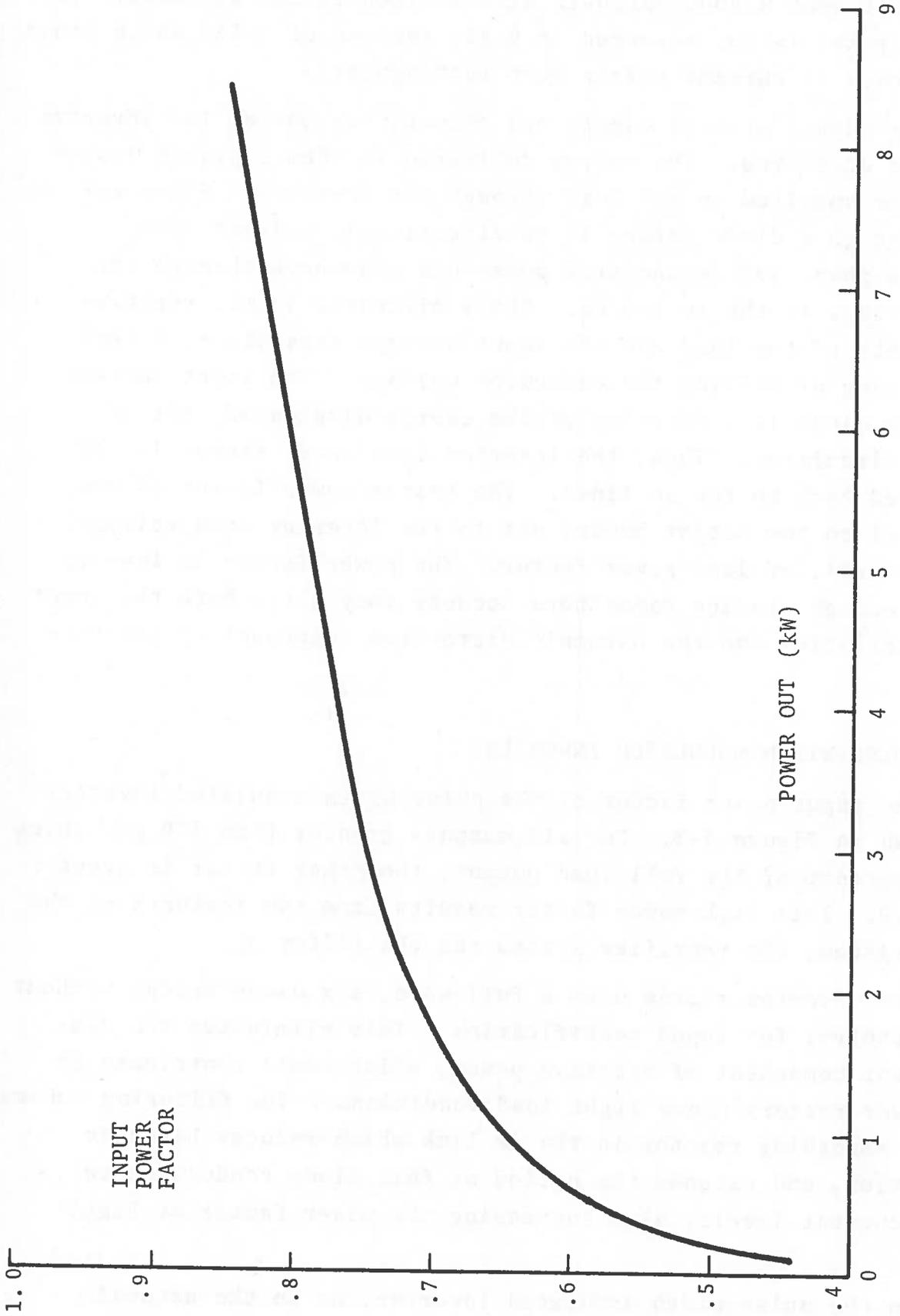


FIGURE 5-1. INPUT POWER FACTOR VERSUS POWER OUT;  
NATURAL COMMUTATED INVERTER

the result that a good harmonic distribution is not achieved. The maximum power factor measured is 0.82, instead of 0.955 which would be the case if current pulses were rectangular.

The energy storage capacitors effectively buffer the inverter from the dc source. The energy delivered to them replaces losses and power supplied to the load through the inverter. Since current flow through a diode bridge is unidirectional, neither load reactive power nor regenerated power can flow back through the diode bridge to the ac source. These circulate in the reactive components of the load and the input storage capacitors, either maintaining or raising the capacitor voltage. The input current from the diode is a function of the energy dissipated, not of energy circulated. Thus, the inverter load power factor is not reflected back to the ac lines. The system power factor is proportional to the output power, not to the inverter load voltage, load current, or load power factor. The power factor is lowered by the energy storage capacitors because they alter both the input characteristics and the harmonic distortion component of reactive power.

## 5.2 PULSE-WIDTH MODULATED INVERTER

The input power factor of the pulse-width modulated inverter is shown in Figure 5-2. For all outputs greater than 3.0 kW, which is 20 percent of the full load output, the power factor is greater than 0.9. This high power factor results from two features of the input system, the rectifier system and the filtering.

The inverter system uses a full-wave, six diode bridge without phase control for input rectification. This eliminates the displacement component of reactive power, which would contribute to low power factors under light load conditions. The filtering scheme uses a smoothing reactor in the dc link which reduces harmonic distortion, and extends the period of full diode conduction to lower current levels, also increasing the power factor at light load.

In the pulse-width modulated inverter, as in the natural

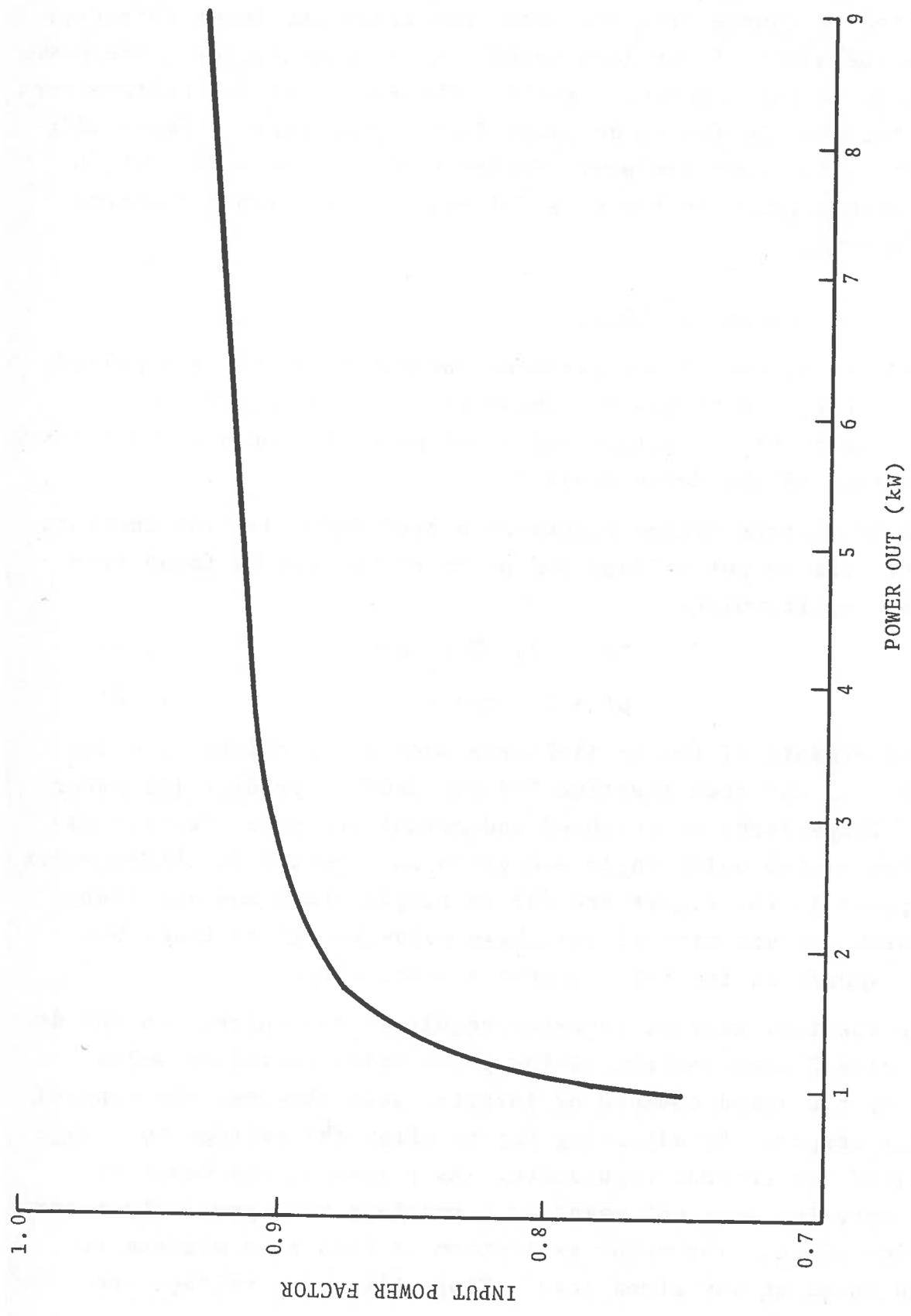


FIGURE 5-2. PULSE-WIDTH MODULATED INVERTER, INPUT POWER FACTOR VERSUS POWER OUTPUT

commutated inverter, the use of a well filtered dc link effectively buffers the ac source from the load, and makes the input characteristics independent of the load magnitude or power factor. The power dissipation in the magnetic circuits, the fans, and the transformers is also included in the input power factor parameters. These additional circuits lower the power factor to 0.92 from 0.955, which is the maximum power factor of a full-wave, inductively filtered, six diode bridge.

### 5.3 CONSTANT CURRENT INVERTER

DC link voltage of the constant current inverter is supplied by a full-wave, six thyristor, phase delay bridge rectifier circuit. Both output voltage and input power factor are functions of the cosine of the delay angle ( $\alpha$ ).

The regulation scheme maintains a continuous dc link current; therefore, the output voltage and power factor can be found from the following formulas:

$$E_d = 3/\pi \sqrt{2} V_L \cos \alpha \quad (5-3)$$

$$\text{pf} = 3/\pi \cos \alpha. \quad (5-4)$$

Measurements of the dc link were used to calculate  $\alpha$ , using equation 5-3, and then equation 5-4 was used to predict the power factor. Comparisons of measured and calculated power factors as a function of the delay angle are given in Figure 5-3. Differences which appear in the Figure are due to single phase and auxiliary loads which are not part of the phase delay rectifier loads but which do appear in the total system measurements.

The constant current inverter regulates the current in the dc link by closed loop control of the phase delay rectifier delay angle. As the speed command or inverter load changes, the control circuitry responds by adjusting ( $\alpha$ ) to allow the voltage to change as required for current regulation. As a result, the constant current inverter does not maintain a constant volt-per-hertz motor excitation slope. The motor excitation is held at a minimum to maintain speed at any given load. Thus, the motor voltage and

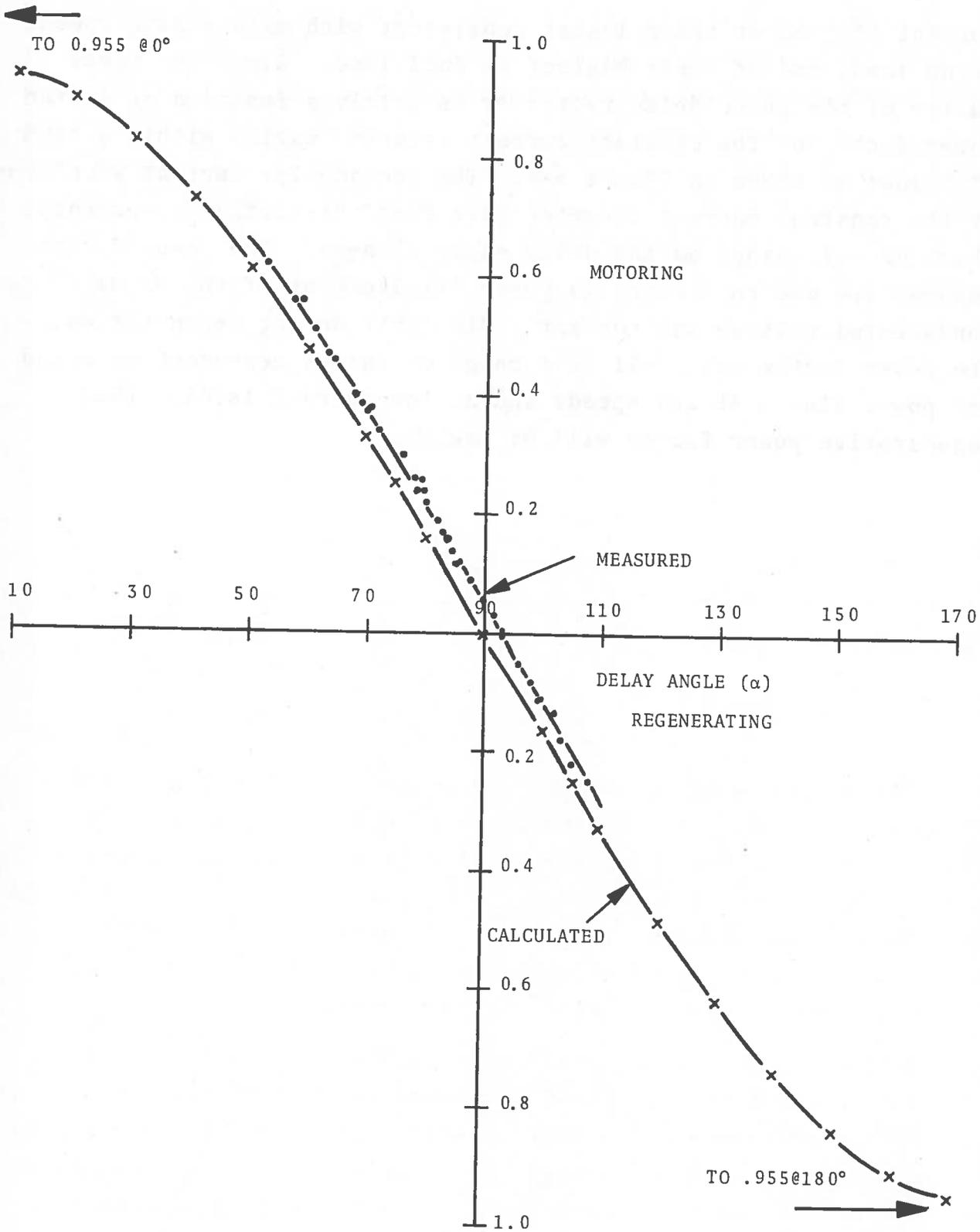


FIGURE 5-3. CONSTANT CURRENT INVERTER POWER FACTOR VERSUS DELAY ANGLE ( $\alpha$ )

current will be at their lowest consistent with maintaining speed, at no load, and at their highest at full load. Since the power factor of the phase delay rectifier is solely a function of  $\alpha$ , the power factor of the constant current inverter varies within a band of values as shown in Figure 5-4. The rectangular current waveforms of the constant current inverter have fixed distortion components, which do not change as the delay angle changes. The power factor changes are due to changes in phase displacement of the 60 Hz fundamental voltage and current. Similarly during regeneration, the power factor will fall in a range of values dependent on speed and power flow. At low speeds and at low current levels, the regenerative power factor will be low.

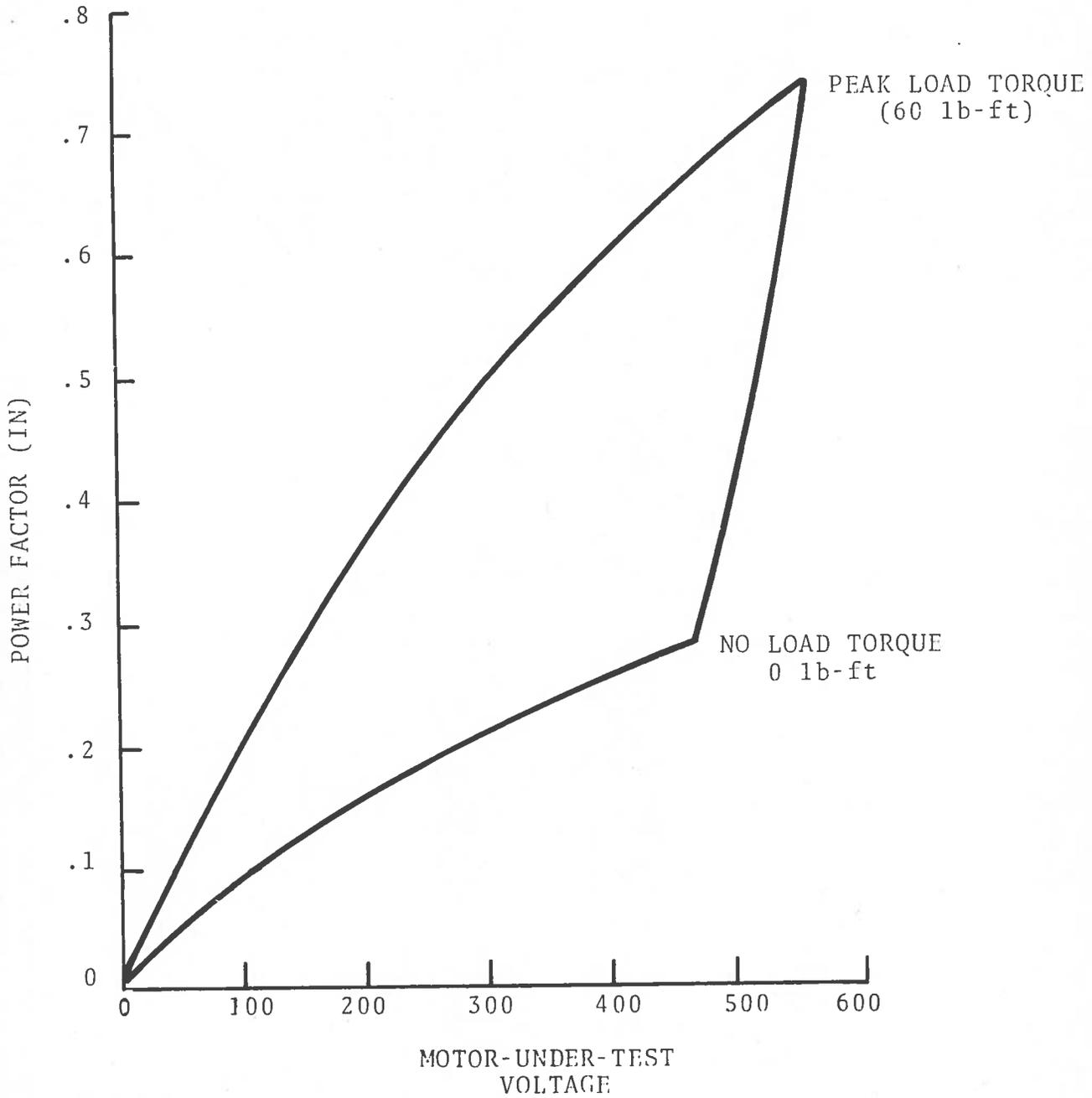
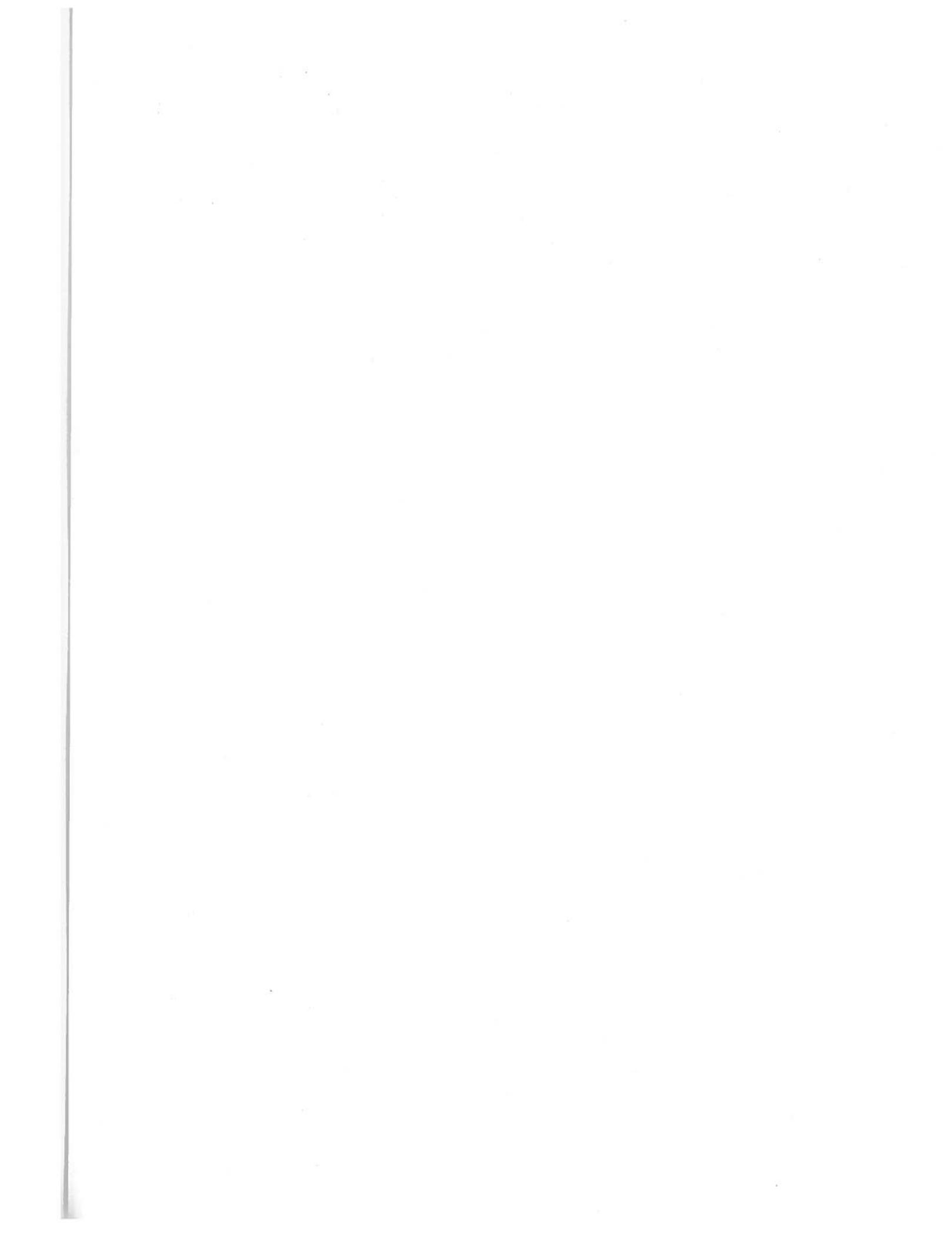


FIGURE 5-4. CONSTANT CURRENT INVERTER INPUT POWER FACTOR RANGE VERSUS MUT VOLTAGE



## 6. INPUT HARMONICS

In three-phase rectifier circuits, current flows through each phase line in one direction for 1/3 of the voltage cycle, in the alternate direction for another 1/3 of the cycle, and is zero for the remaining 1/3 of a cycle. This rectifier chopping action generates distortion currents consisting of harmonics of the fundamental 60 Hz input current. These have been measured in the laboratory with a harmonic wave analyzer, and the relative magnitude (in db) displayed on a strip chart recorder.

Not all harmonics are present in a three-phase, six pulse system. In particular, the third harmonic and its multiples are all of equal magnitude and they are in phase. Therefore, the net contribution to the output of each of the triplen harmonics is zero. The even harmonics are also absent from the spectra because of the zero baseline symmetry. Even harmonics would only appear if there was a dc offset in the analyzed waveform. Accounting for the symmetry and the triplen cancellations, the first harmonic present is the fifth, followed by the seventh, eleventh, thirteenth, etc.

The current harmonics contribute to the rms line current vectorially:

$$I_{\ell} = \sqrt{\sum_{m=1} I_m^2} \quad (6-1)$$

Since this rms line current is used in the determination of apparent power and power factor, the presence of the harmonics results in a lower power factor for the system.

### 6.1 NATURAL COMMUTATED INVERTER

In the natural commutated inverter, energy storage capacitors on the input deliver power between conduction intervals of the input rectifiers. At light loads the capacitors do not discharge significantly between these conduction intervals, so the input current waveform consists of a series of discontinuous current pulses. The amplitudes of these pulses are high because the input

capacitors do not limit current and a high peak to rms ratio with high harmonic distortion results. At heavy loads the current becomes continuous but it does not become a quasi-square wave and peak to rms ratios remain high. Current harmonic data are shown in Figures 6-1 to 6-3. Figure 6-1 is taken with the inverter lightly loaded at a dc link current of 3.5 amps; Figure 6-2, at the onset of continuous conduction; and Figure 6-3, a heavily loaded case where dc link current is 15 amps. The harmonic ratios associated with these figures are listed in Table 6-1 and plotted in Figure 6-4.

The ratio of any harmonic to the fundamental is higher for the light load cases than for the heavier loaded cases; therefore, the distortion component of input rms current is also high. As the load increases, the harmonic ratios decrease and their contribution to rms line current is reduced, but the input line current remains a multipulse waveform, as shown in the Figures.

At the high current levels of full scale traction equipment, these harmonics can cause losses in the series line impedance of the input lines, excess current in bypass capacitors, counter torques in rotating equipment, and interference in telecommunications and radio equipment.

The harmonic currents are also only distortion currents which do not contribute to any real power. This can be shown by computing the integral of the instantaneous power over a full cycle as described in Appendix C.

In a direct rectification system, such as that just described for the natural commutated inverter, the diodes conduct when they are forward biased. There is no displacement between the fundamental current and voltage waveforms; hence, the displacement power factor does not vary. The harmonic distortion of the input waveform, however, does decrease as the input current level increases, becoming less severe at higher levels. The rms distortion current is given by:

$$I_d = \sqrt{(I_d)^2 - (I_1)^2} \quad (6-2)$$

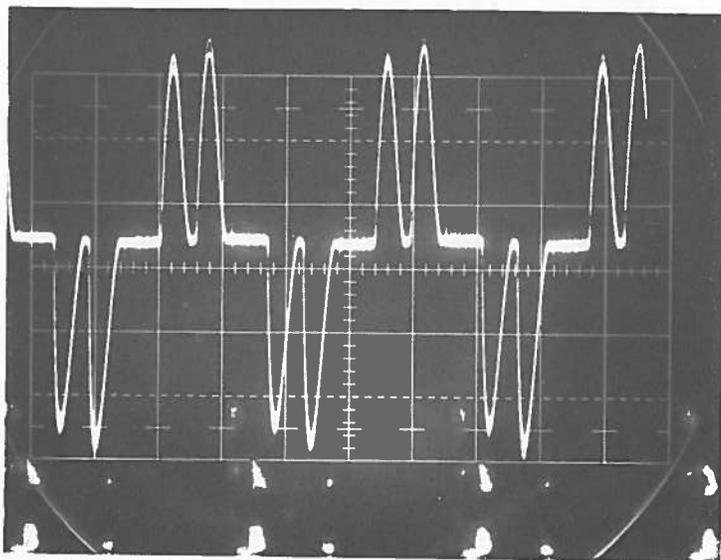
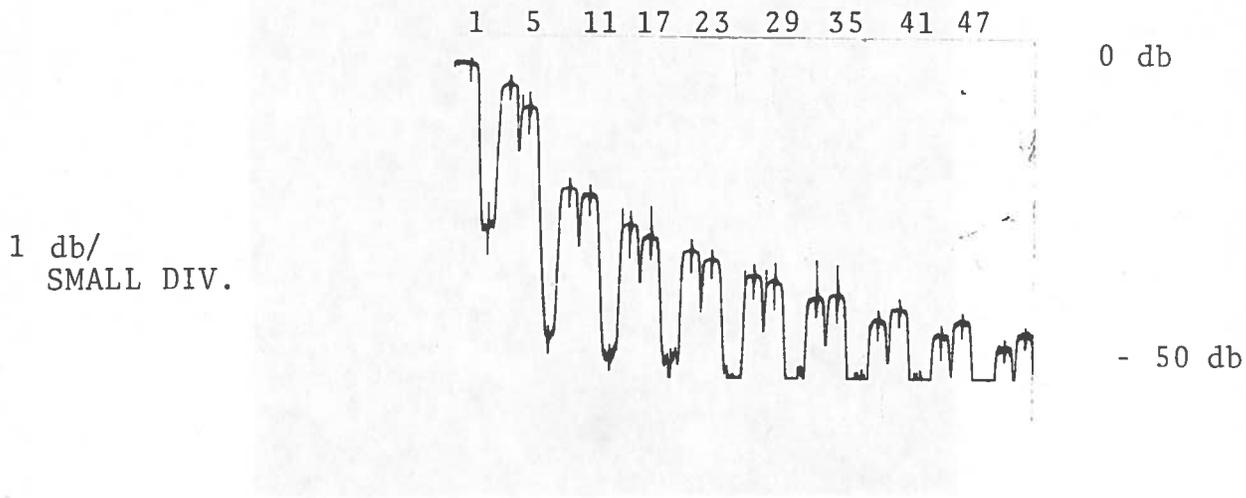


FIGURE 6-1. NATURAL COMMUTATED INVERTER, INPUT CURRENT HARMONICS,  
 $I_{in} = 3.5$  AMPS DC, DISCONTINUOUS CONDUCTION

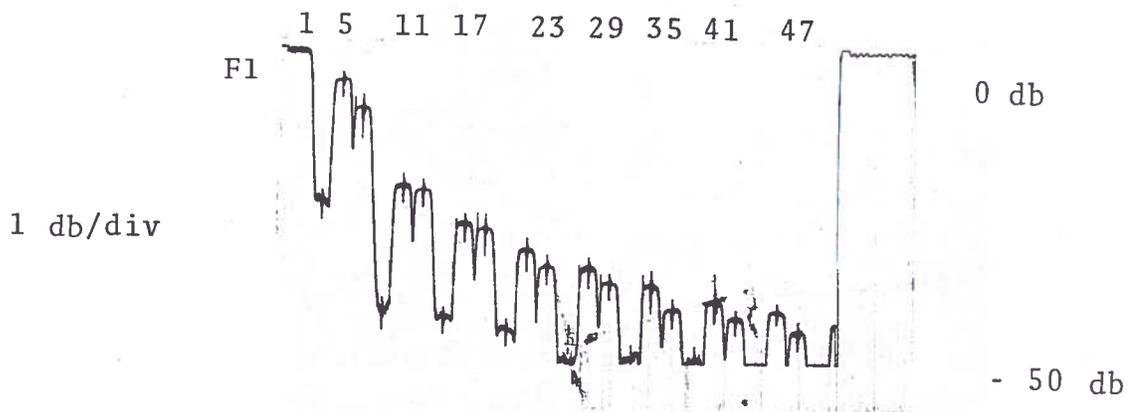
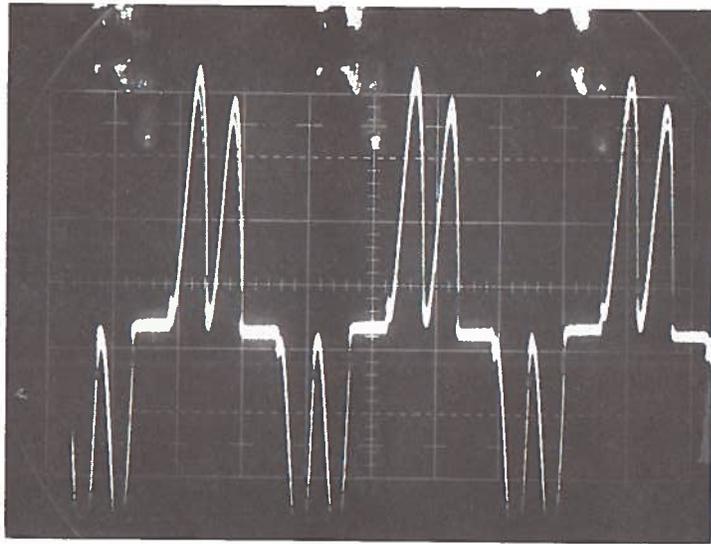
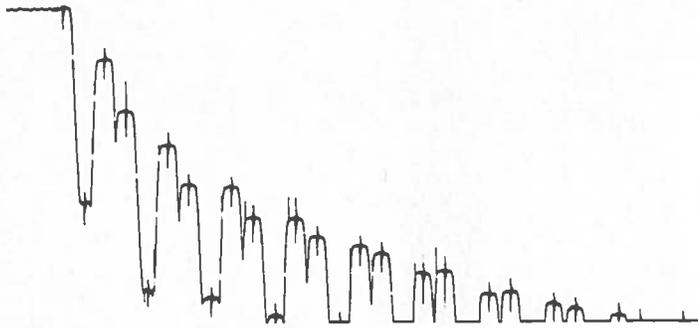


FIGURE 6-2. NATURAL COMMUTATED INVERTER, INPUT CURRENT HARMONICS,  $I_{in} = 4.8$  AMPS DC, INITIAL CONTINUOUS CONDUCTION

1 5 11 17 23 29 35 41

1 db/div



0 db

- 50 db

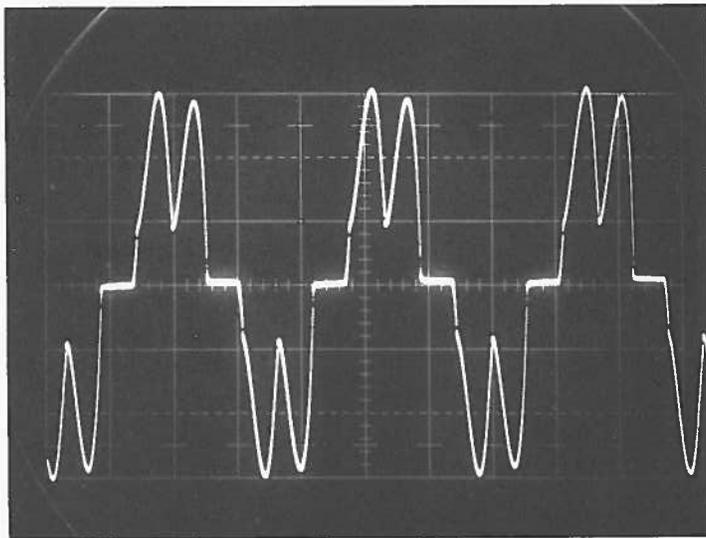


FIGURE 6-3. NATURAL COMMUTATED INVERTER, INPUT CURRENT HARMONICS,  
 $I_{in} = 15$  AMPS DC, CONTINUOUS CONDUCTION

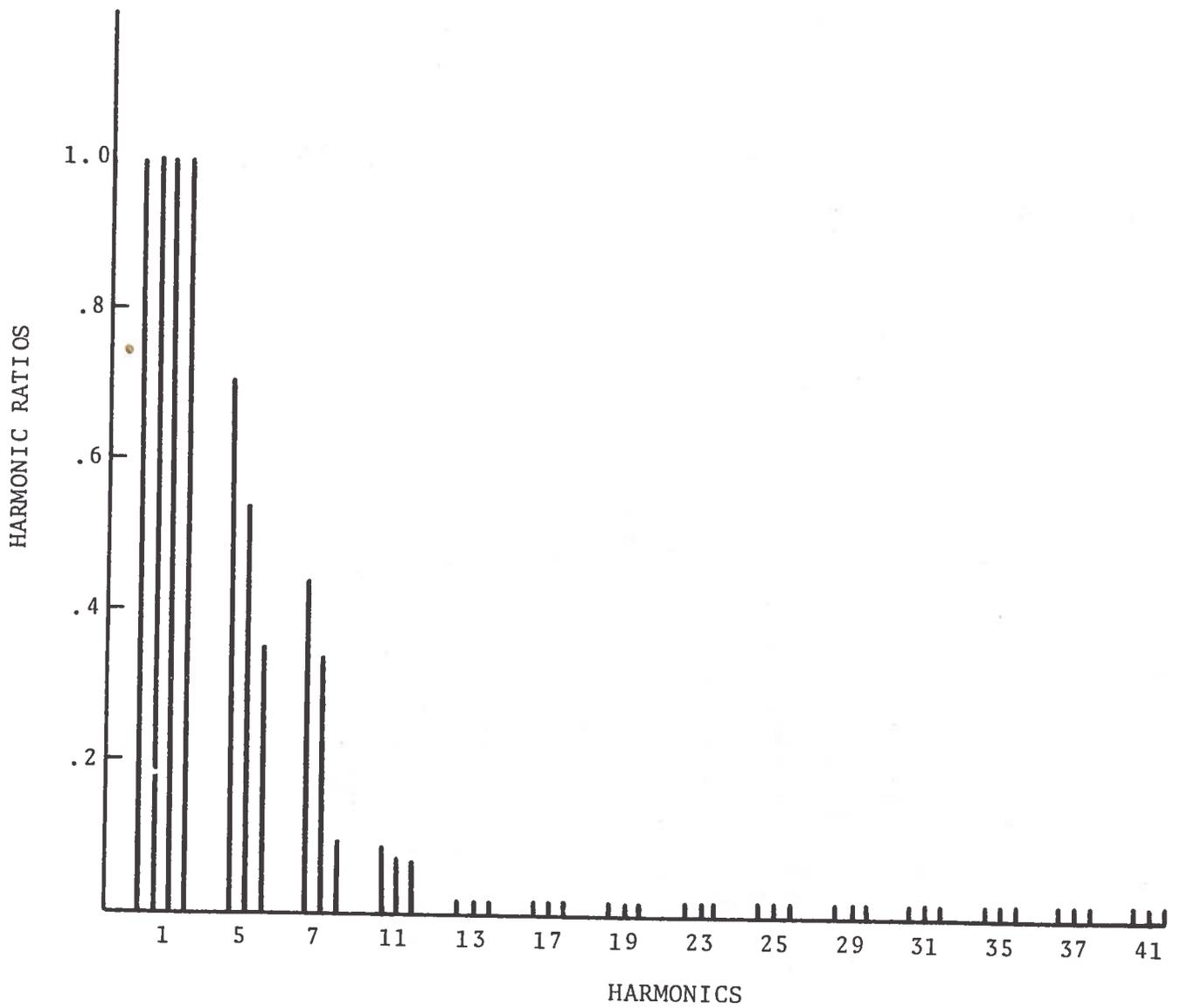


FIGURE 6-4. NATURAL COMMUTATED INVERTER INPUT HARMONIC CURRENT RATIOS

TABLE 6-1. NATURAL COMMUTATED INVERTER INPUT CURRENT HARMONIC DISTRIBUTION

HARMONIC NUMBER	IRMS = 3.5A		IRMS = 4.8A		IRMS = 15.0A	
	$\Delta db$	$I_N/I_1$	$\Delta db$	$I_N/I_1$	$\Delta db$	$I_N/I_1$
1	0	1.0	0	1.0	0	1.0
5	3	.71	5	.56	9	.35
7	7	.45	9	.35	20	.10
11	21	.089	22	.080	22	.08
13	22	.080	23	.071	30	.032
17	26	.050	28	.040	31	.028
19	28	.040	29	.035	33	.022
23	31	.028	33	.022	37	.014
25	32	.025	36	.016	38	.013
29	35	.018	36	.016	40	.010
31	36	.016	38	.013		
35	38	.013	38	.013		
37	37	.014	42	.008		
41	42	.008				
43	40	.010				

and the distortion factor is:

$$df = \frac{I_d}{I_\ell} \quad (6-3)$$

For the three cases described in this section the harmonic ratios and distortion factors at the three current levels are shown in Table 6-2.

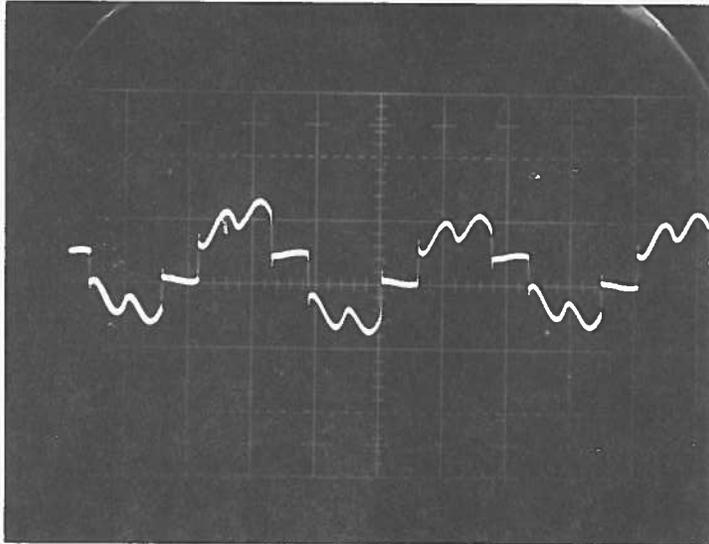
Idc	$I_\ell/I_\ell$	$I_d/I_\ell$	d.f.
3.5A	1.313	0.843	0.64
4.8A	1.21	0.681	0.56
15.0A	1.085	0.42	0.39

These formulas and current values describe the input power characteristics of the inverter with energy storage capacitors on the input. Because of the energy storage buffer between the source and the inverter proper, load configuration does not alter the input characteristics. The input power factor is a measure of harmonic distortion, and is related to the input current purity, which improves as the rms line current increases.

## 6.2 PULSE-WIDTH MODULATED INVERTER

The pulse-width modulated inverter also has a six diode full-wave, three-phase rectifier bridge to convert the input ac to a dc link. Unlike the natural commutated inverter, the pulse-width modulated inverter has an inductive input filter which affects the harmonic distribution of the currents by attenuating harmonics and reducing the peak to rms ratios. For significant current levels the dc current is constant and the input current becomes a quasi-square wave current whose wave shape is characteristic of three-phase, inductively filtered, rectifier circuits. The harmonics for this waveform follow a 1/n distribution, and represent the minimum harmonic distribution achievable without incorporating harmonic filtering.

Input harmonics for two current levels into the pulse-width modulated inverter are shown in Figures 6-5 and 6-6, and listed in



Input Line

Current

$I_L = 4A$  RMS

$f = 60$  Hz

$f_1 = 60$  Hz

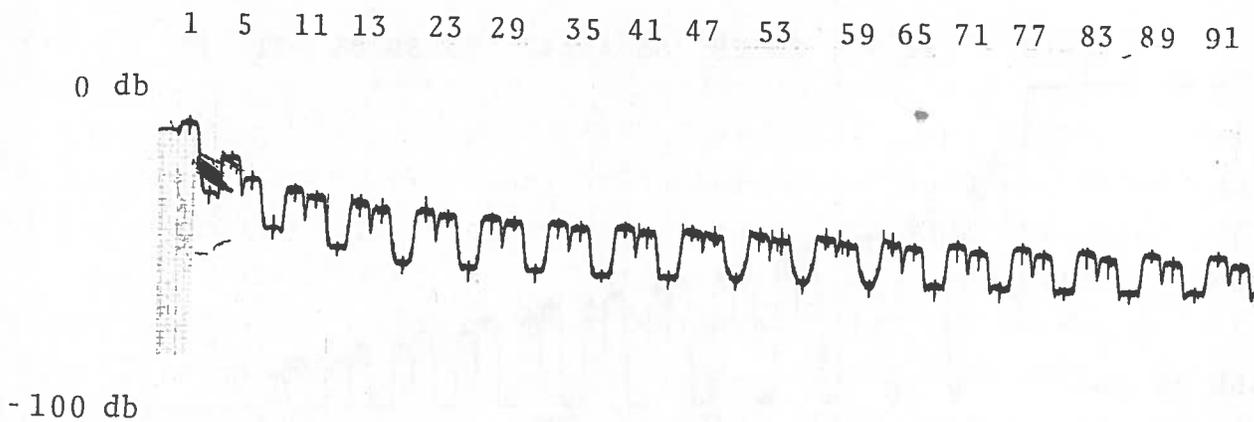
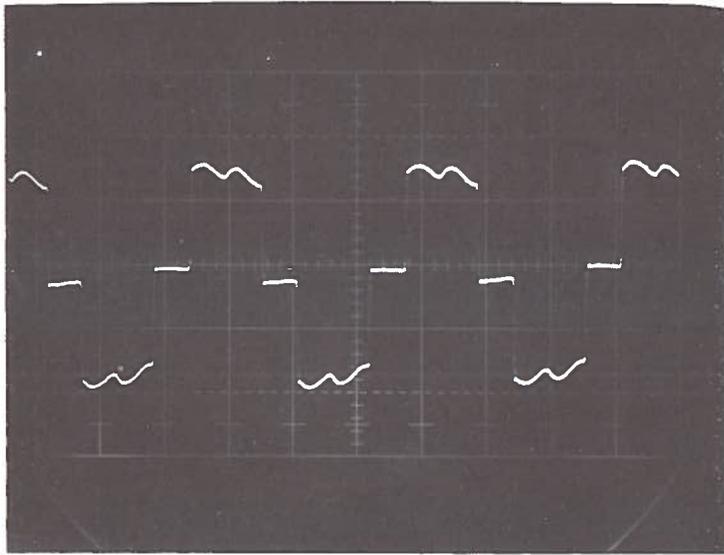


FIGURE 6-5. PULSE-WIDTH MODULATED INVERTER, INPUT CURRENT HARMONIC SPECTRUM AT  $I_{in} = 4$  AMPS RMS



INPUT LINE CURRENT

$I_L = 20$  AMPS

$f = 60$  Hz

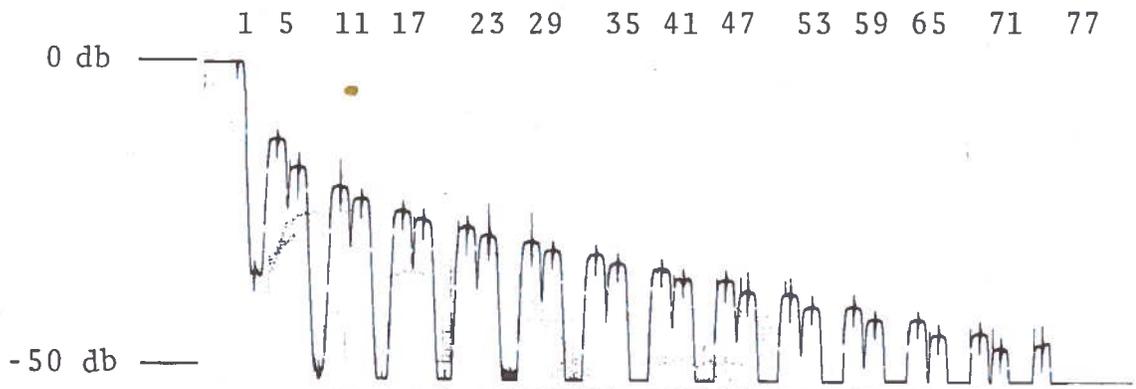


FIGURE 6-6. PULSE-WIDTH MODULATED INVERTER, INPUT CURRENT HARMONIC SPECTRUM AT  $I_{in} = 20$  AMPS RMS

Table 6-3. There are two components of input line current which flow into the inverter system. One is the rectifier power supply current for the inverter proper, and the second is the current for the auxiliary electronics such as relay, logic, control, and cooling. The current for the auxiliary is a single phase current which is supplied from a transformer connected across two of the input phases. This single phase current is included in the following input current analysis.

The current for the auxiliaries, which is supplied through the stepdown transformer, is a 60 Hz, sinusoidal component. It is discernible as an apparent displacement of the zero axis in the Figures. Since it is a constant magnitude, it is less significant as the input current level to the inverter proper increases. The filter removes a significant portion of the distortion and the harmonic ratios approach the  $1/n$  distribution common to currents in rectified, three-phase, inductively filtered circuits.

Figure 6-5 shows the waveform and harmonic distribution for an input current of 4 amps. At this level the 60 Hz auxiliary current is a significant proportion of the input and the baseline distortion is evident. Figure 6-6 shows the waveform and harmonic distribution for an input current of 20 amps. In this case the 60 Hz component is a lesser proportion of the input current. The harmonic ratios of these waveforms are listed in Table 6-3. The 60 Hz auxiliary current component added to the 60 Hz component of the six pulse rectifier input increases the apparent fundamental component and decreases the apparent harmonic ratios. The measured harmonic ratios, however, do not differ significantly from those obtained with quasi-square wave currents. Since the auxiliary loads are connected across two of the three input lines, an unbalanced condition exists, but this unbalance does not impact power characteristic measurements.

### 6.3 CONSTANT CURRENT INVERTER

The control loop of the constant current inverter controls the delay angle to vary the output voltage of the phase delay

TABLE 6-3. INPUT HARMONICS

HARMONIC NUMBER	$I_{in} = 4 \text{ A}$		$I_{in} = 20 \text{ A}$	
	db	$\frac{I_n}{I_1}$	db	$\frac{I_n}{I_1}$
1	-12	1	0	1
5	-23	0.28	-13	0.22
7	-31	0.11	-18	0.125
11	-33	0.088	-20	0.10
13	-37	0.056	-22	0.03
17	-39	0.042	-24	0.063
19	-40	0.04	-26	0.05
23	-41	0.036	-26	0.05
25	-42	0.033	-28	0.04
29	-43	0.028	-29	0.035
31	-44	0.025	-31	0.028
35	-44	0.025	-32	0.025
37	-45	0.022	-33	0.022
41	-45	0.022	-32	0.025
43	-47	0.018	-34	0.020
47	-47	0.018	-34	0.020
49	-48	0.016	-36	0.016
53	-48	0.016	-36	0.016
55	-49	0.014	-38	0.012
			-38	0.012
			-40	0.01

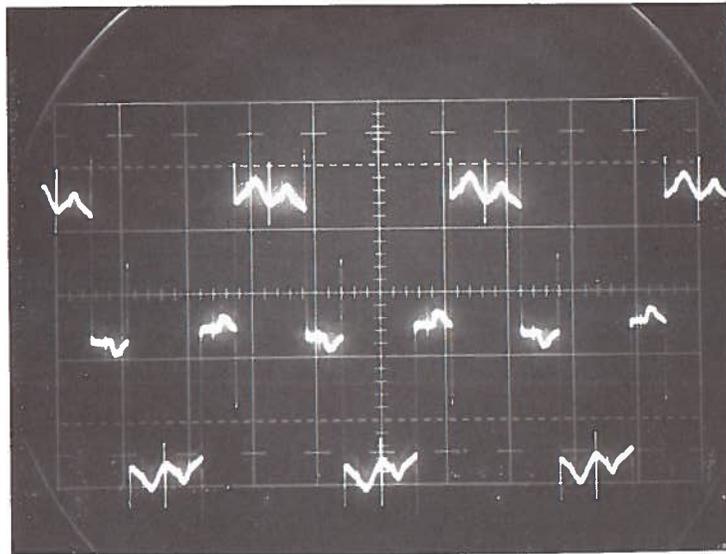
rectifier to regulate the dc link current. The large reactor and the control scheme maintain a continuous current in the dc link and the input line current is a quasi-square wave for all conditions. The harmonic content in that case does not vary from full pdr output voltages at  $\alpha = 0$ , through zero volts at  $\alpha = 90^\circ$ , and into regeneration as  $\alpha$  becomes greater than  $90^\circ$ .

An input current waveform and the harmonic spectrum of that waveform are shown in Figure 6-7. Harmonic ratios are listed in Table 6-4. Note that for three-phase systems with  $120^\circ$  rectangular current waveforms, the harmonic distributions follow a  $1/n$  pattern and the distortion factor is at a minimum value of 0.29.

The constant current inverter, operating with a motor load, supplies a minimum of 20 amps motor excitation at all operating points. The current therefore only varies over a range of 20 amps at no load to 44 amps at full loads. Even at the lower level, the single phase load currents are too low to significantly offset the  $1/n$  harmonic distribution pattern.

The inverter input current wave shape is the same at all current levels; therefore, the distortion factor is constant. The measured value of 0.29 agrees with the values calculated from equation 6-3.

Input distortion factors measured for the three inverters are shown in Figure 6-8. The natural commutated inverter and the pulse-width modulated inverter each have high distortion at low current levels with distortion falling asymptotically toward 0.29, the level of minimum distortion in three-phase rectifier systems.



$f_1 = 60 \text{ Hz}$

$f/f_1 = 1 \ 5 \ 11 \ 17 \ 23 \ 29$

0 db

-50 db

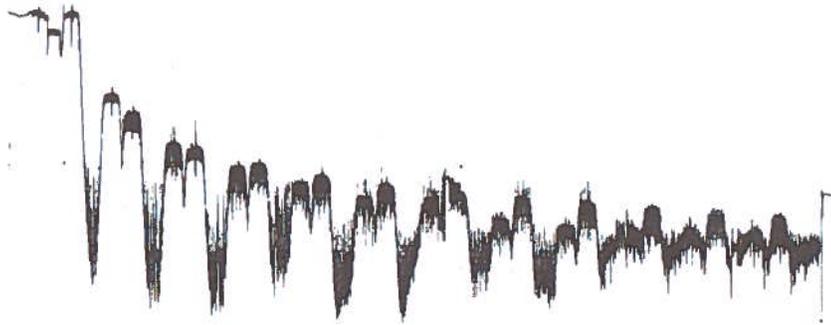


FIGURE 6-7. CONSTANT CURRENT INVERTER, INPUT CURRENT HARMONICS AT  $I_{in} = 22 \text{ AMPS}$

TABLE 6-4. CONSTANT CURRENT INVERTER INPUT HARMONIC DISTRIBUTION

Harmonic	$I_1 - I_N$ $\Delta db$	$I_N/I_1$
1	0	1.0
5	14	0.2
7	18	0.126
11	24	0.063
13	24	0.063

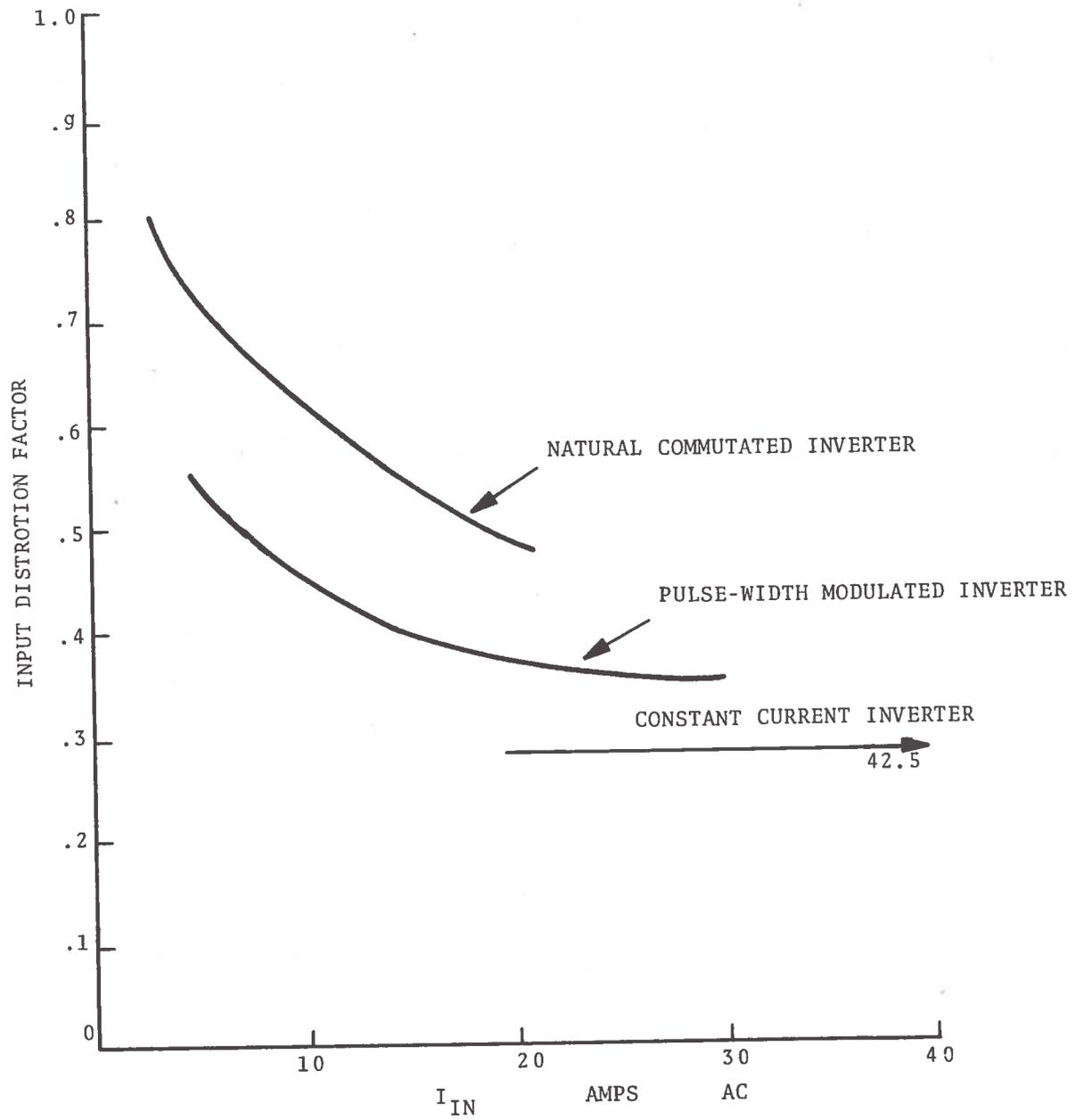


FIGURE 6-8. INPUT DISTORTION FACTOR VERSUS INPUT CURRENT

## 7. OUTPUT HARMONICS

Since the inverters each incorporate different methods of producing the output voltage, they have different harmonic patterns in that output. The natural commutated inverter compares the output to a reference sine wave, and, by means of pulse repetition rate modulation, adds or removes charge in the output filter capacitors. The output voltage developed in this manner has a strong fundamental component and few harmonics; thus, it is sinusoidal in shape for all but insignificantly light loads.

The pulse-width modulated inverter varies the on-to-off ratios of the inverter thyristors so that the fundamental component of the output voltage is proportional to a reference voltage. The output developed in this manner consists of a fixed quantity of variable width rectangular pulses which produce the fundamental and high harmonic voltage levels at multiples of the modulation frequency. The resultant current harmonics are dependent on the load impedance at the harmonic frequency; and, since the harmonic voltages are high, the current levels can be significant.

The constant current inverter delivers quasi-square waves of current to the load by virtue of the fact that the dc link current is regulated. The output voltage is the composite of the voltages of each harmonic current and the load impedance at the harmonic frequency. The harmonic current magnitude decreases as a function of  $1/n$ , and the voltages developed by them becomes less significant as the harmonic order increases. The output voltage of the inverter into a motor load is close to sinusoidal with some distortion evident.

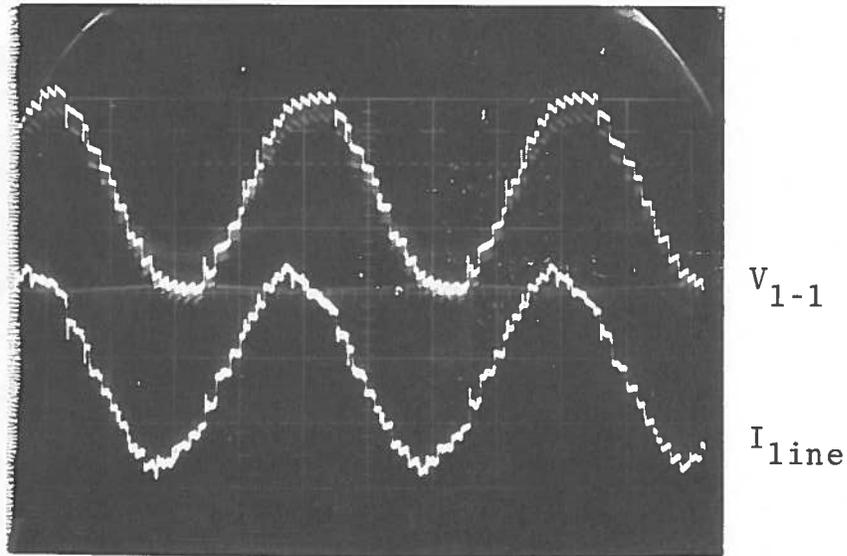
Output harmonic data for the three inverters at various load levels are shown in Figures 7-1 through 7-16. As with the input harmonics, the output of the harmonic wave analyzer was displayed on a strip chart recorder in a db versus frequency format. The photographs show the waveforms analyzed, while the harmonic content of the waveform is shown in the strip chart records. For

resistive load cases, the current and voltage waveform harmonic distributions are the same. For the lagging power factor load cases, the harmonic distributions shown are for the current waveform only.

## 7.1 NATURAL COMMUTATED INVERTER

The harmonic distribution in the output of the natural commutated inverter is low. In particular, with a lagging power factor static or motor load, the output current wave shape approaches that of a pure sinusoid. The reference for the output is a sine wave voltage at the output frequency and proportional to the required output amplitude. The control circuitry samples the output and controls thyristor firing time so that the output voltage approaches the sinusoidal reference. The result, as shown in Figures 7-1 through 7-5, is a waveform with very low harmonic to fundamental ratios. This is true for all outputs where the discharge time constant of the output storage capacitors is low compared to the reference frequency. Thus, waveform purity is better at low frequencies and at heavier loads.

At 25 Hz, with an unfiltered resistive load at a current of 1 amp, (Figure 7-1) the magnitude of the highest harmonic, the fifth, is discernible in the inverter noise at -33 db, 2.2 percent of the fundamental magnitude. Under the same conditions, but at 400 Hz, (Figure 7-2), the fifth harmonic is 26 db down, a magnitude of 5 percent of the fundamental. With such low harmonic ratios, even with unfiltered loads, it is not surprising that the addition of inductance in the load (lagging power factor) further reduces harmonic content and improves waveform purity. Figure 7-3, harmonic spectra for static loads simulating an induction motor at 60 Hz and a 0.8 power factor, shows that there are no discernible harmonics in the current waveform. Similarly Figure 7-4, output current harmonic spectra for the inverter driving a motor load at 60 Hz, and at conditions similar to those in Figure 7-3, shows the same purity of waveform. The distortion apparent in Figure 7-4 results from the fact that the motor actually operates at a higher



$F_1 = 25 \text{ Hz}$

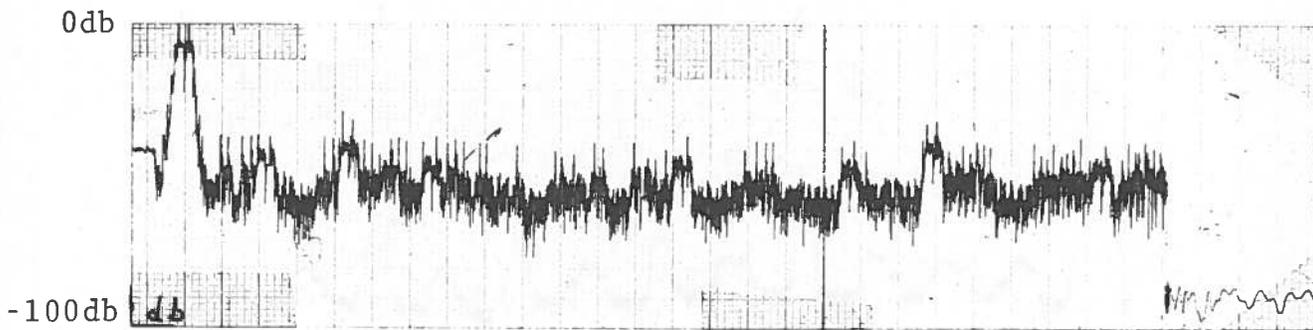
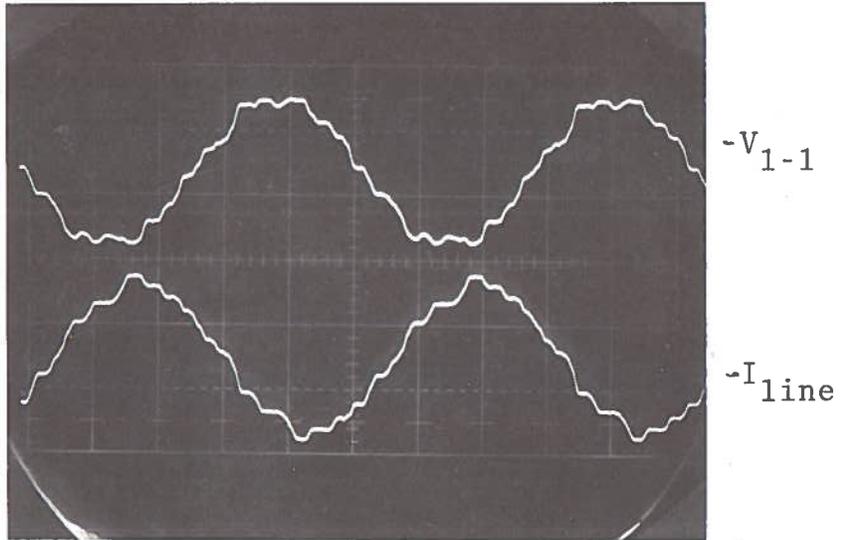


FIGURE 7-1. NATURAL COMMUTATED INVERTER,  
 OUTPUT CURRENT HARMONIC SPECTRA AT 25 Hz  
 $V_{out} = 120 \text{ VOLTS RMS}$ ,  $I_{out} = 1 \text{ AMP RMS}$   
 RESISTANCE LOAD



$F_1 = 400 \text{ Hz}$

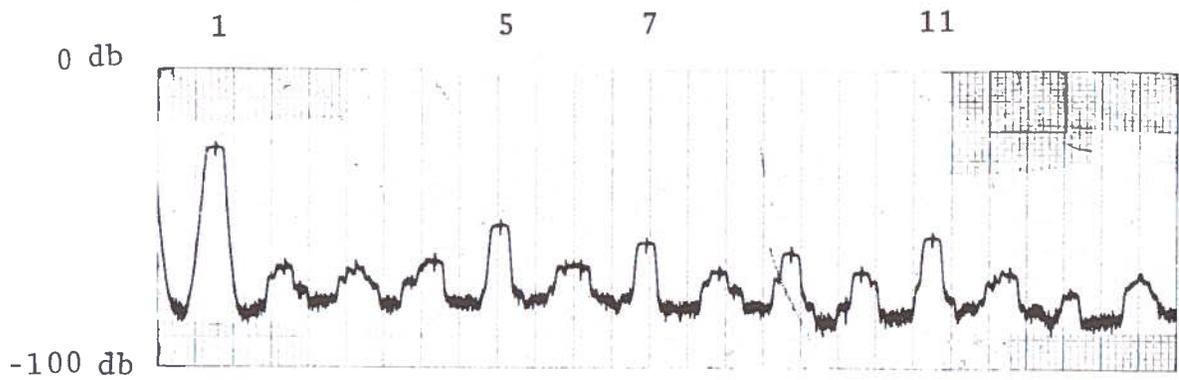
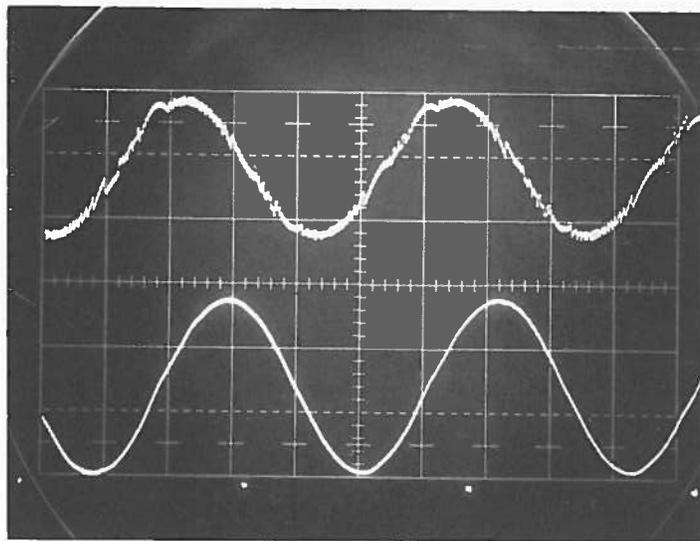


FIGURE 7-2. NATURAL COMMUTATED INVERTER,  
 OUTPUT CURRENT HARMONIC SPECTRA AT 400 Hz  
 $V_{\text{out}} = 120 \text{ VOLTS RMS}$ ,  $I_{\text{out}} = 1 \text{ AMP RMS}$   
 RESISTANCE LOAD



$V_{1-1}$

$I_{LINE}$

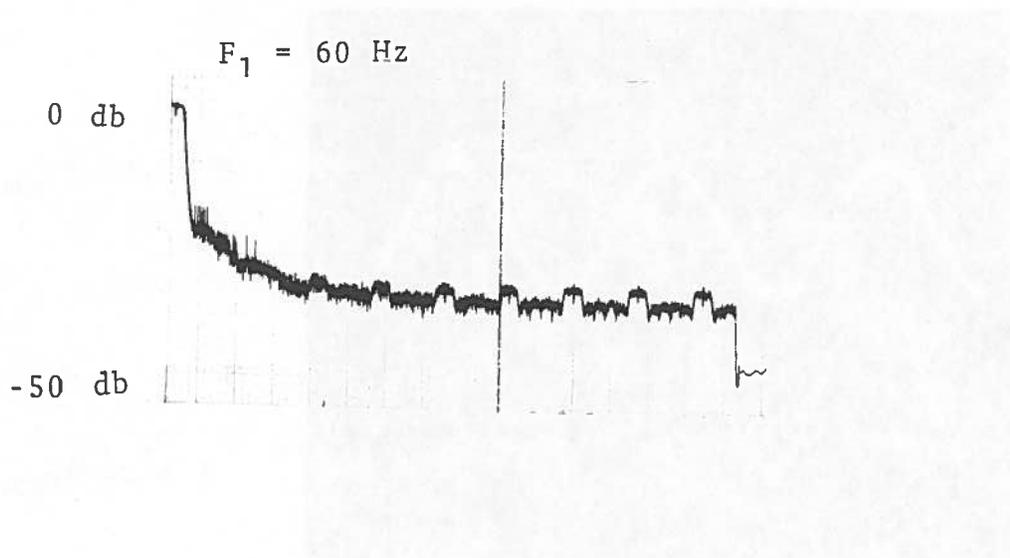
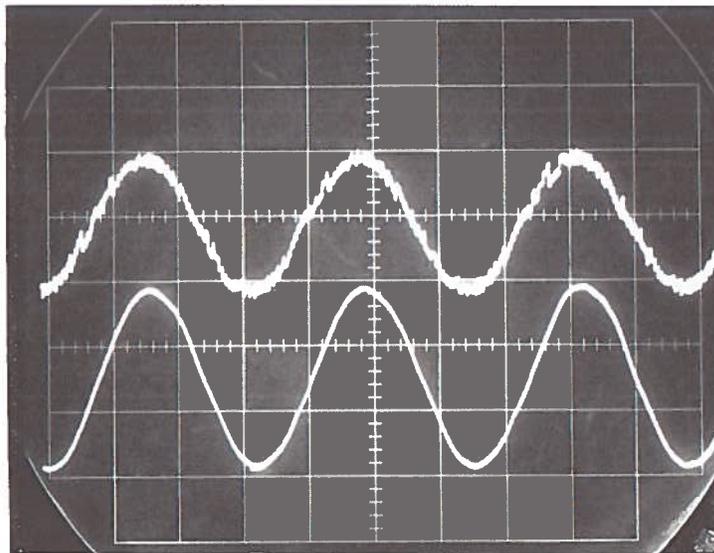
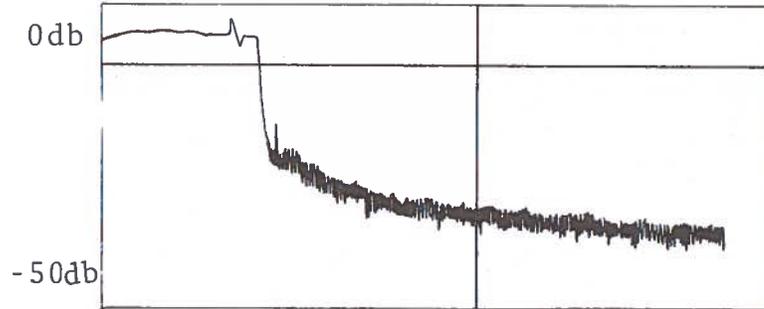


FIGURE 7-3. NATURAL COMMUTATED INVERTER,  
 OUTPUT CURRENT HARMONIC SPECTRA AT 60 Hz  
 $V_{out} = 148 V_{1-1}$ ,  $I_{out} = 17.5$  AMPS  
 PF = 0.8

$F_1 = 60 \text{ Hz}$



$V = 200 \text{ V/cm}$

$I = 20 \text{ A/cm}$

FIGURE 7-4. NATURAL COMMUTATED INVERTER,  
OUTPUT CURRENT HARMONIC SPECTRA AT 60 Hz  
 $V_{\text{out}} = 148 \text{ V}$ ,  $I_{\text{out}} = 17.5 \text{ AMPS}$   
INDUCTION MOTOR LOAD

power factor than 0.8, and therefore, there is less output current filtering. Data for the resistive load cases of Figures 7-1 and 7-2, the lagging power factor cases, static load of Figure 7-3, and motor load of Figure 7-4 indicates that for all practical inverter loads the natural commutated inverter can be treated as a sine wave generator.

Figure 7-5 shows the output current, output voltage, and the harmonic spectral sweep for the inverter under overload or current limit conditions, recognizable by the distortion at the waveform peaks. When the inverter enters this current limited operation, the dwell time between consecutive power pulses is controlled by thyristor recovery considerations. At this time there is a periodic component to the current pulses, and two distinct harmonic patterns are present. One is due to the distortion of the sinusoidal waveform caused by current limiting action. The other is due to the pulse repetition frequency, which is at its maximum during the output voltage peak periods. The lower order harmonics (5 through 17) are caused by the distortion at the peaks, while the higher order harmonics (greater than 3 kHz) are caused by the pulse repetition frequency.

## 7.2 PULSE-WIDTH MODULATED INVERTER

The output of the pulse-width modulated inverter is not filtered by energy storage capacitors; thus, the waveform contains significant harmonic distortion components which affect the output characteristics. Output harmonic data for this inverter are shown in Figures 7-6 through 7-16. Included are figures showing the harmonics distributions for the inverter driving a motor load and driving static load elements at a 0.8 power factor. The static load data was taken at three frequencies, each in a different pulse number group: 100 Hz for the eight-notch waveform; 150 Hz for the four-notch waveform; and 300 Hz for the two-notch waveform. The waveform of the voltage at a particular frequency does not vary as a function of the output current. Therefore, the waveform at 150 volts output is the same whether the output current is 1 amp or 44 amps, and whether the power factor is 1.0 or a lagging

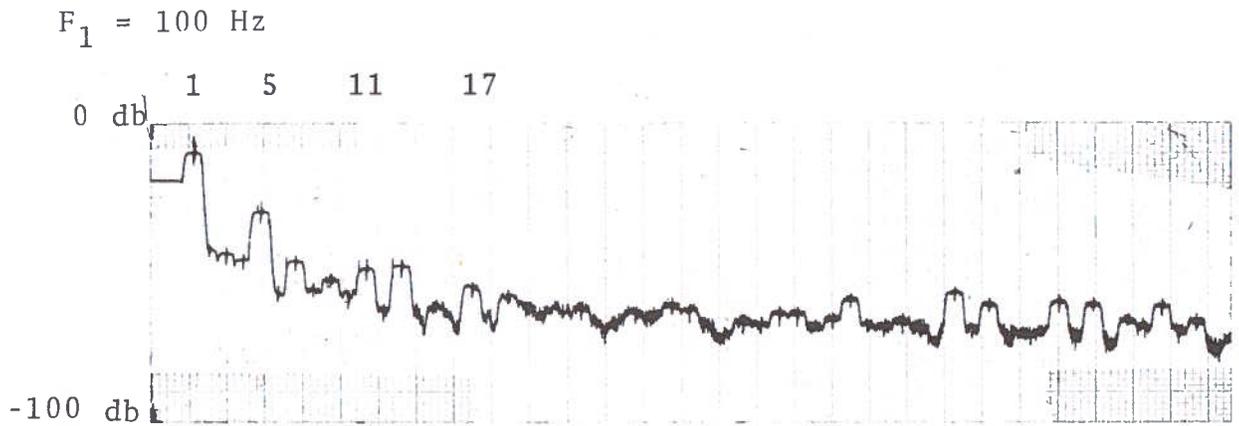
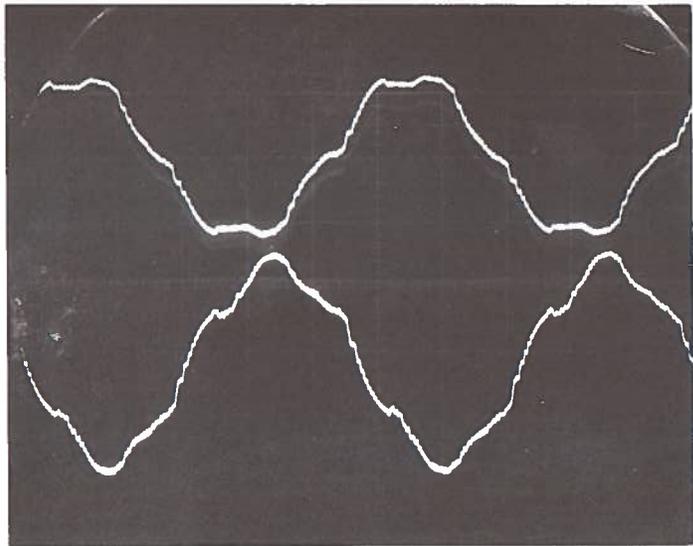


FIGURE 7-5. NATURAL COMMUTATED INVERTER, OUTPUT CURRENT HARMONIC SPECTRA AT 100 Hz, INVERTER OVERLOADED

0.5. The voltage harmonic distribution is a function of the number of pulses; the harmonic magnitudes are proportional to the pulse-widths or duty cycle ( $T$ ). Figures 7-6 and 7-7 show voltage harmonic spectra for four duty cycle conditions,  $T = 0.25, 0.5, 0.87,$  and  $1.0$ ;  $0.87$  is the maximum duty cycle for the test inverter. The  $1.0$  case is an unmodulated, maximum voltage case included for comparison.

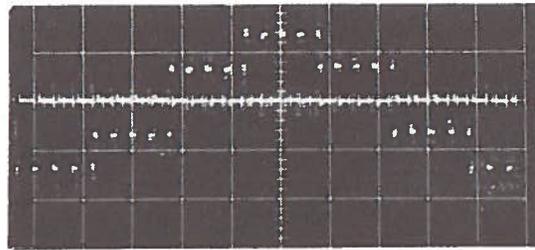
As the duty cycle increases, the harmonic magnitudes decrease. However, they persist to high orders for all cases except for the unmodulated waveform,  $T = 1.0$ . Ratios of the magnitude of harmonic voltages to the fundamental voltage are listed in Table 7-1. The dominant harmonics are the twenty-third and twenty-fifth, which, at a duty cycle of  $0.25$ , are nearly equal in magnitude to the fundamental. At a duty cycle of  $0.5$  they are reduced by  $3$  db; ( $30$  percent); at maximum duty cycle they are further reduced by  $15$  db ( $80$  percent). But for all modulated cases, harmonics within  $10$  percent of the fundamental exist to harmonic orders higher than the seventy-third. By comparison, for the unmodulated wave the eleventh harmonic is less than  $10$  percent of the fundamental,  $9.09$  percent.

The voltage harmonics appear in clusters at multiples of the modulation frequency, which, for the eight-notch waveform is  $24$  times the fundamental. Thus, the harmonics peak on each side of the twenty-fourth, forty-eighth, seventy-second, etc. harmonic.

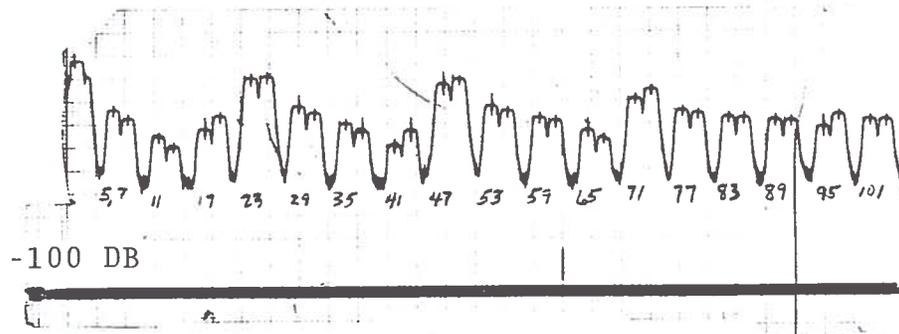
Figures 7-8 and 7-9 show voltage harmonic spectra for the two other waveforms of the pulse-width modulated inverter, the four-notch and two-notch. These waveforms have modulation frequencies of  $12$  and six times the fundamental respectively, and the harmonic clusters appear at these frequency multiples.

The current harmonic spectra, when the load has a lagging power factor, do not exhibit the same harmonic distributions as the voltage harmonics because of filtering by the motor reactance. As frequency increases, the impedance increases proportionally and the current resulting from the harmonic voltage is attenuated. Current harmonic spectra for the inverter driving a motor load at

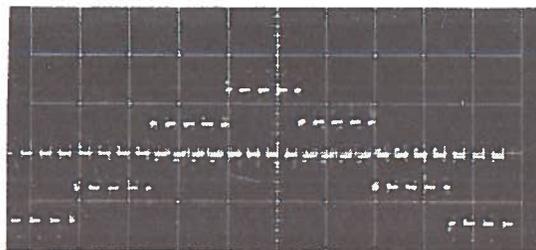
T = 0.25



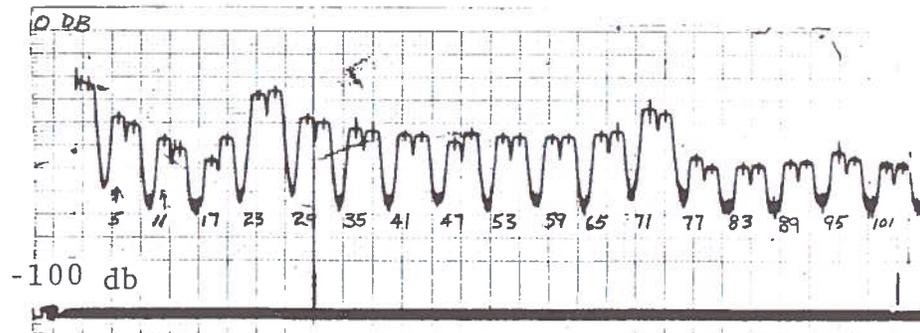
VOLTS  
LINE - NEUTRAL



T = 0.5



VOLTS  
LINES - NEUTRAL



$$F_1 = 60 \text{ Hz}$$

FIGURE 7-6. PULSE-WIDTH MODULATED INVERTER  
VOLTAGE HARMONIC SPECTRA

208 V RMS

I = 21 A ac

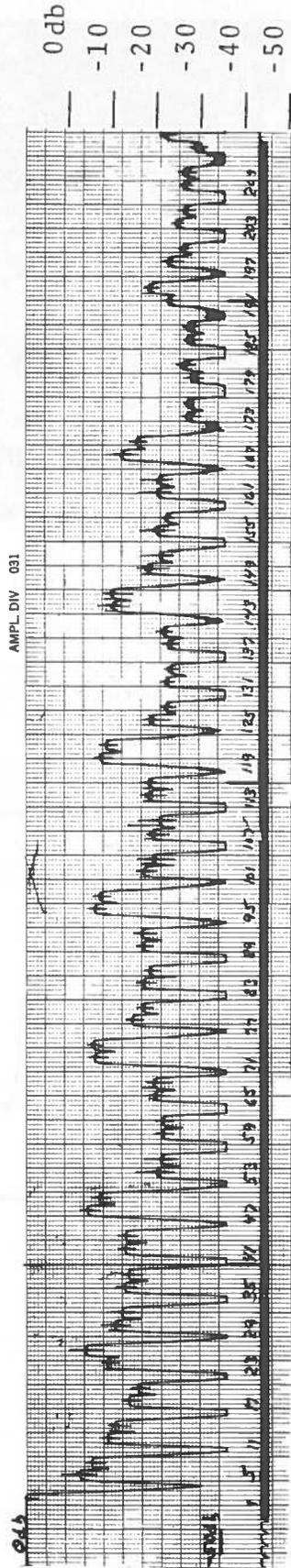
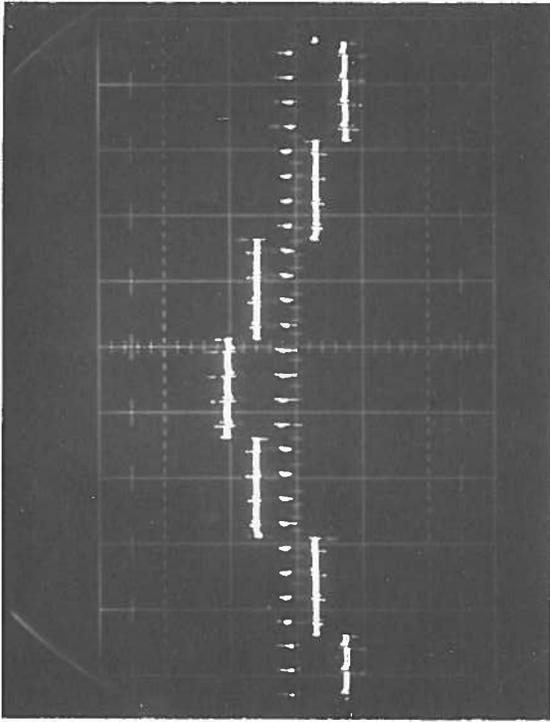
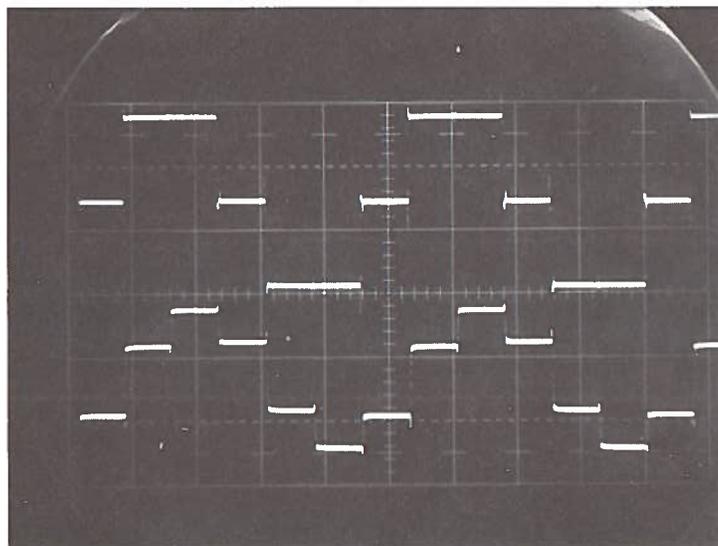


FIGURE 7-7a. PULSE-WIDTH MODULATED INVERTER, VOLTAGE HARMONIC SPECTRA

$F_1 = 60 \text{ Hz}$

$\tau = \tau_{\max} = 0.87$



$f_1 = 120 \text{ Hz}$

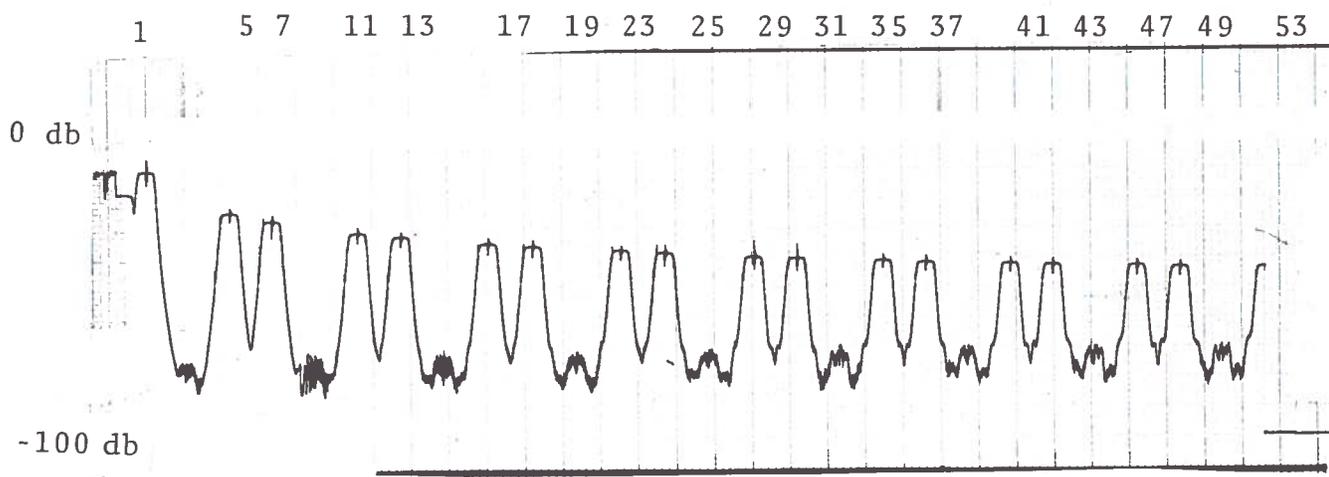


FIGURE 7-7b. SIX STEP (QUASI-SQUARE) WAVEFORM,  
HARMONIC DISTRIBUTION

TABLE 7-1. PULSE-WIDTH MODULATED INVERTER, VOLTAGE HARMONIC VALUES

	$\tau = 0.25$		$\tau = 0.5$		$\tau = 0.87$		$\tau = 1.0$		4 Notch, F = 150 Hz		2 Notch, F = 300 Hz	
	0	1.0	0	1.0	0	1.0	0	1.0	0	1.0	0	1.0
1	0	1.0	0	1.0	0	1.0	0	1.0	0	1.0	0	1.0
5	14	.20	14	.20	14	.20	14	.20	16	.16	6	.50
7	17	.14	16	.16	17	.14	17	.14	25	.056	0	1.0
11	24	.063	22	.08	21	.089	21	.089	3	.71	17	.14
13	28	.040	26	.05	23	.071	23	.071	0	1.0	6	.50
17	22	.080	32	.025	27	.045	25	.056	12	.25	7	.45
19	16	.16	27	.08	29	.035	26	.050	16	.16	21	.089
23	0	1.0	4	.63	21	.089	27	.045	16	.16	11	.28
25	0	1.0	2	.80	15.5	.17	28	.040	6	.50	14	.20
29	12	.25	14	.20	23	.071	30	.032	14	.20	20	.10
31	14	.20	16	.16	25	.056	30	.032	15	.18	20	.10
35	20	.10	18	.126	36	.05	31	.028	11	.28	16	.16
37	22	.080	19	.11	27	.045	32	.025	26	.050	16	.16
41	29	.035	20	.10	26	.05	32	.025				
43	22	.080	20	.10	26	.05	32	.025				
47	2	.80	24	.063	16	.16	33	.022				
49	0	1.0	20	.10	19	.11	34	.020				
53	12	.25	22	.08	34	.020						
55	14	.20	27	.08	35	.018						
59	18	.126	27	.08	36	.016						
61	19	.11	22	.08	36	.016						
65	22	.080	20	.10	34	.020						
67	26	.050	19	.11	33	.022						
71	8	.40	10	.32	18	.126						
73	4	.63	12	.25	18	.126						

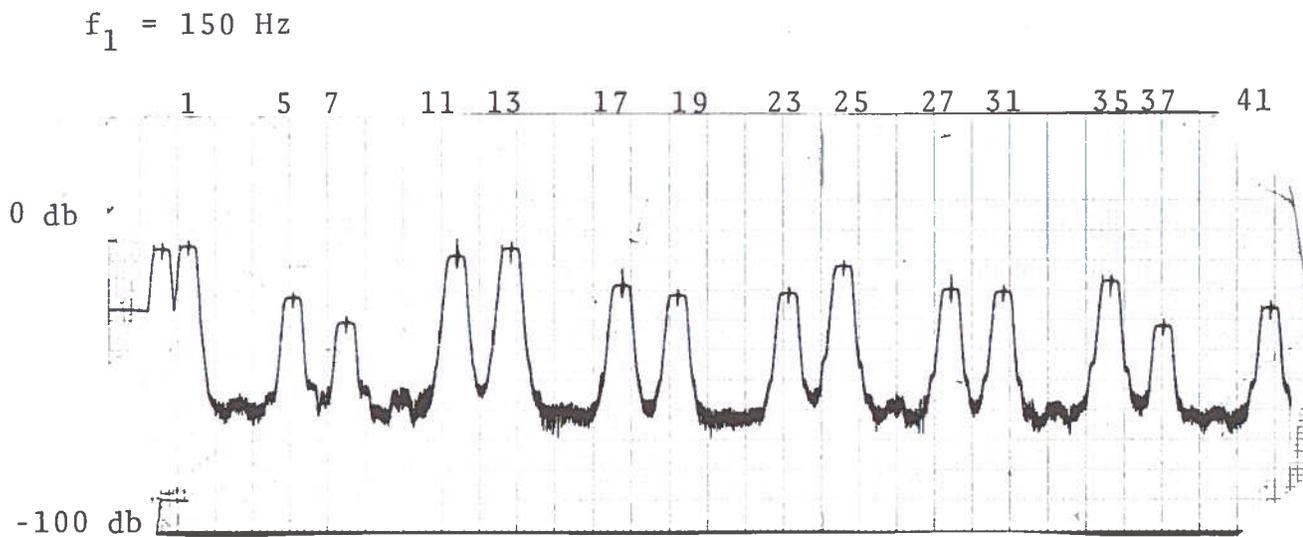
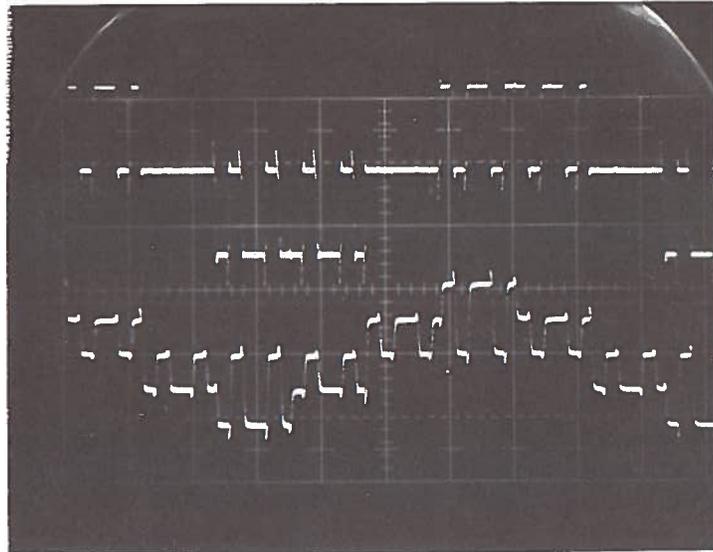
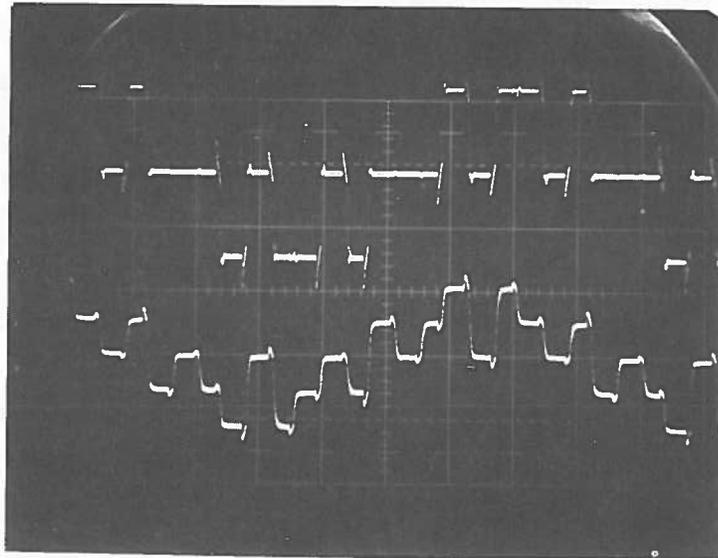


FIGURE 7-8. PULSE-WIDTH MODULATED INVERTER, FOUR NOTCH OUTPUT VOLTAGE HARMONIC SPECTRUM,  $E_0 = 150$  VOLTS,  $F = 150 \text{ Hz}$

$F_1 = 300 \text{ Hz}$



$V_{L-L}$

$V_{L-N}$

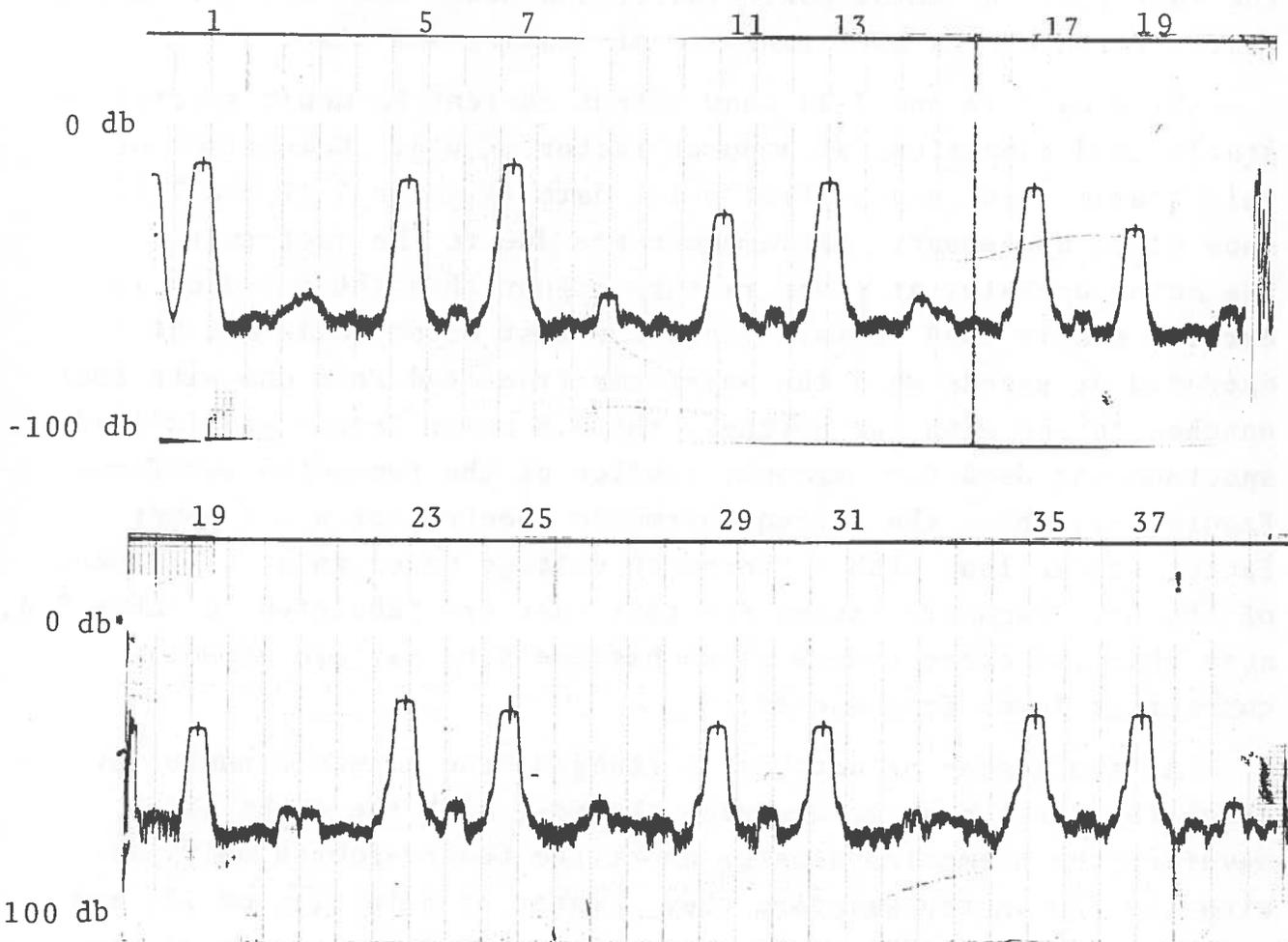
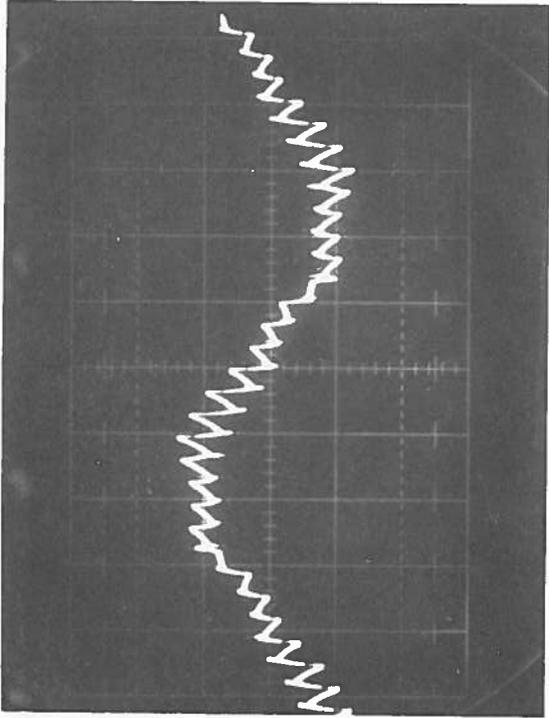


FIGURE 7-9. PULSE-WIDTH MODULATED INVERTER, OUTPUT VOLTAGE HARMONIC SPECTRUM, TWO NOTCH,  $E_0 = 150 \text{ VOLTS}$ ,  $F = 300 \text{ Hz}$

a duty cycle of 0.25 with an eight pulse-modulated wave is shown in Figure 7-10. The harmonic ratios are listed in Table 7-2. For this case, as described earlier, the magnitudes of the twenty-third and twenty-fifth voltage harmonics were equal to the fundamental voltage, but the current harmonics are down approximately 20 db, an order of magnitude difference. Figures 7-11 and 7-12 show current harmonic spectra for the inverter driving a motor load, with a pulse modulation producing a four notch wave at two current levels. Harmonic ratios are listed in Table 7-3. In each case the highest harmonic is the thirteenth. For the lower current, 9.0 amp case, the thirteenth harmonic magnitude is 0.282 of the fundamental, and for the 20 amp case the thirteenth harmonic is 0.126 of the fundamental. These differences are brought about by the fact that the motor power factor and hence the harmonic current distribution varies as a function of loading and slip.

Figures 7-13 and 7-14 show output current harmonic spectra for static load conditions at a power factor of 0.8. Comparison of this static load data to test motor data (Figures 7-11 and 7-12) show close agreement. Differences are due to the fact that the motor operates at power factors higher than the 0.8 used in earlier static load tests. Since the test motor could not be operated at speeds when the waveforms transitioned from one with four notches to one with two notches, the 0.8 power factor static load spectrum was used for harmonic studies of the two-notch waveform. Figure 7-15 shows the current harmonic spectra for a 0.8 power factor static load with a two-notch voltage waveform at a frequency of 300 Hz. Harmonic ratios for this case are tabulated in Table 7-4, note that the current wave shape has the same pattern as motor current at lower frequencies.

As the number of notches is changed, the harmonic number at which the first peak occurs also changes; with the eight-notch waveform the harmonics cluster about the twenty-fourth multiple; with the four-notch waveform they cluster at multiples of 12; and with the two-notch the clusters are centered on multiples of six. The actual frequency range of the harmonics, however, remains fairly constant as inverter frequency changes. For each waveform



$I_L$

15.2 AMP RMS

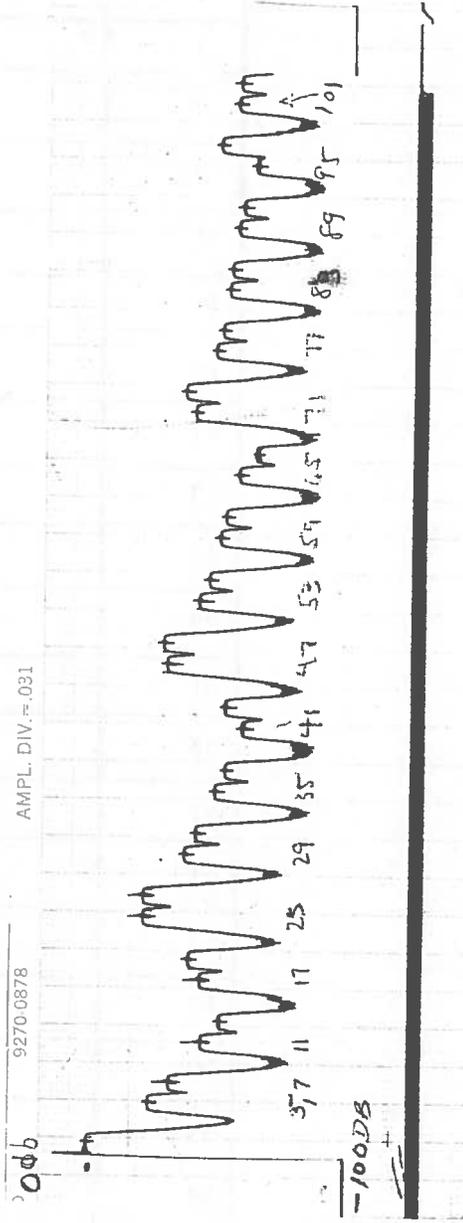
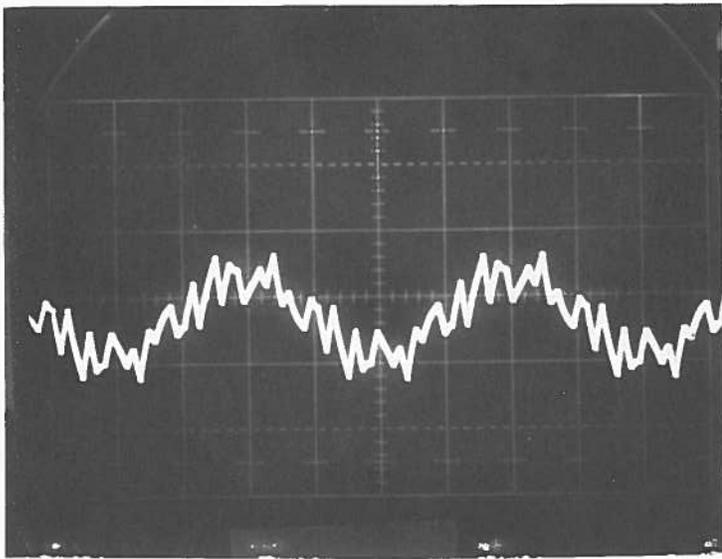


FIGURE 7-10. PULSE-WIDTH MODULATED INVERTER, OUTPUT CURRENT HARMONIC SPECTRUM,  $\tau = 0.25$ ,  $I = 15.2$  AMPS RMS

TABLE 7-2. PULSE WIDTH MODULATED INVERTER OUTPUT CURRENT HARMONIC SPECTRA

$T = 0.25$ ,  $E_o = 108 V_{L-L}$ , Motor Phase Current = 15.2A rms

N	$\Delta db$	$I_N/I_1$
1	0	1.0
5	-20	0.1
7	-28	.04
11	-38	.013
13	-44	.006
17	-37	.014
19	-34	.02
23	-18	.13
25	-18	.13
29	-32	.025
31	-35	.018
35	-42	.008
37	-45	.006
41		
43		
47		
49		
53		
55		
59		
61		



I = 9.0 AMPS RMS

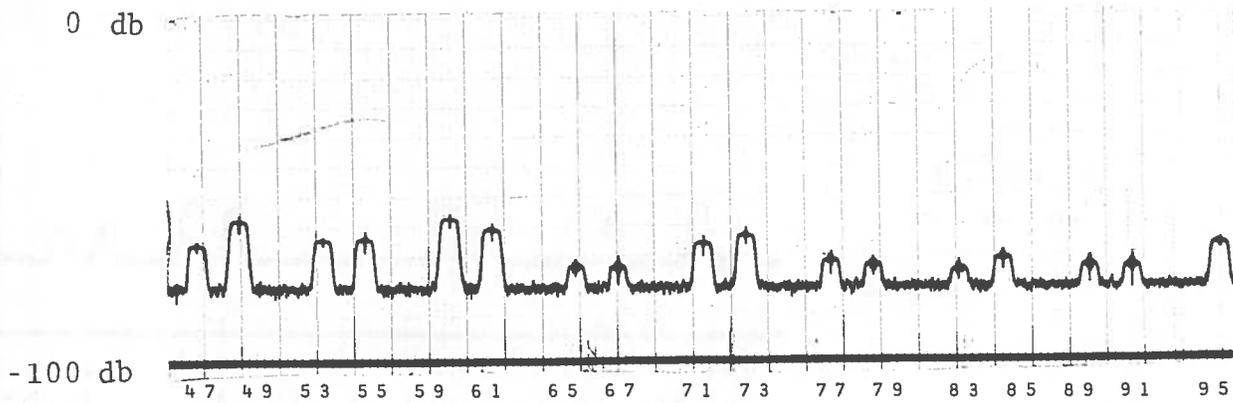
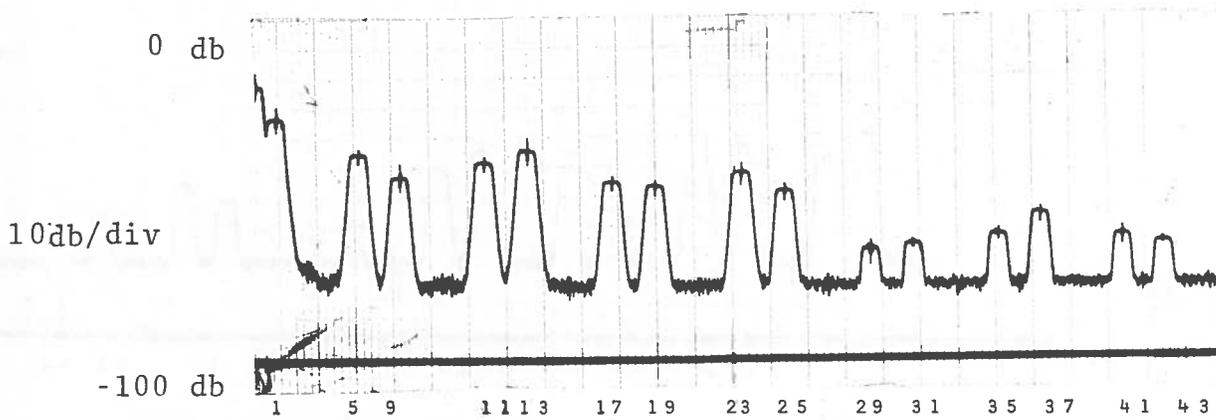
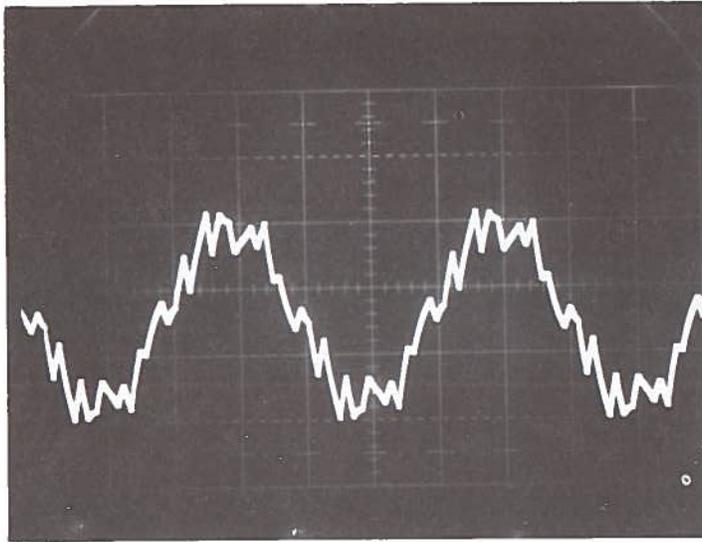


FIGURE 7-11. PULSE-WIDTH MODULATED INVERTER,  
 OUTPUT CURRENT HARMONIC SPECTRUM,  $E_0 = 162$  VOLTS,  
 $F = 124$  Hz, WAVEFORM = FOUR NOTCH



I = 20 AMPS RMS

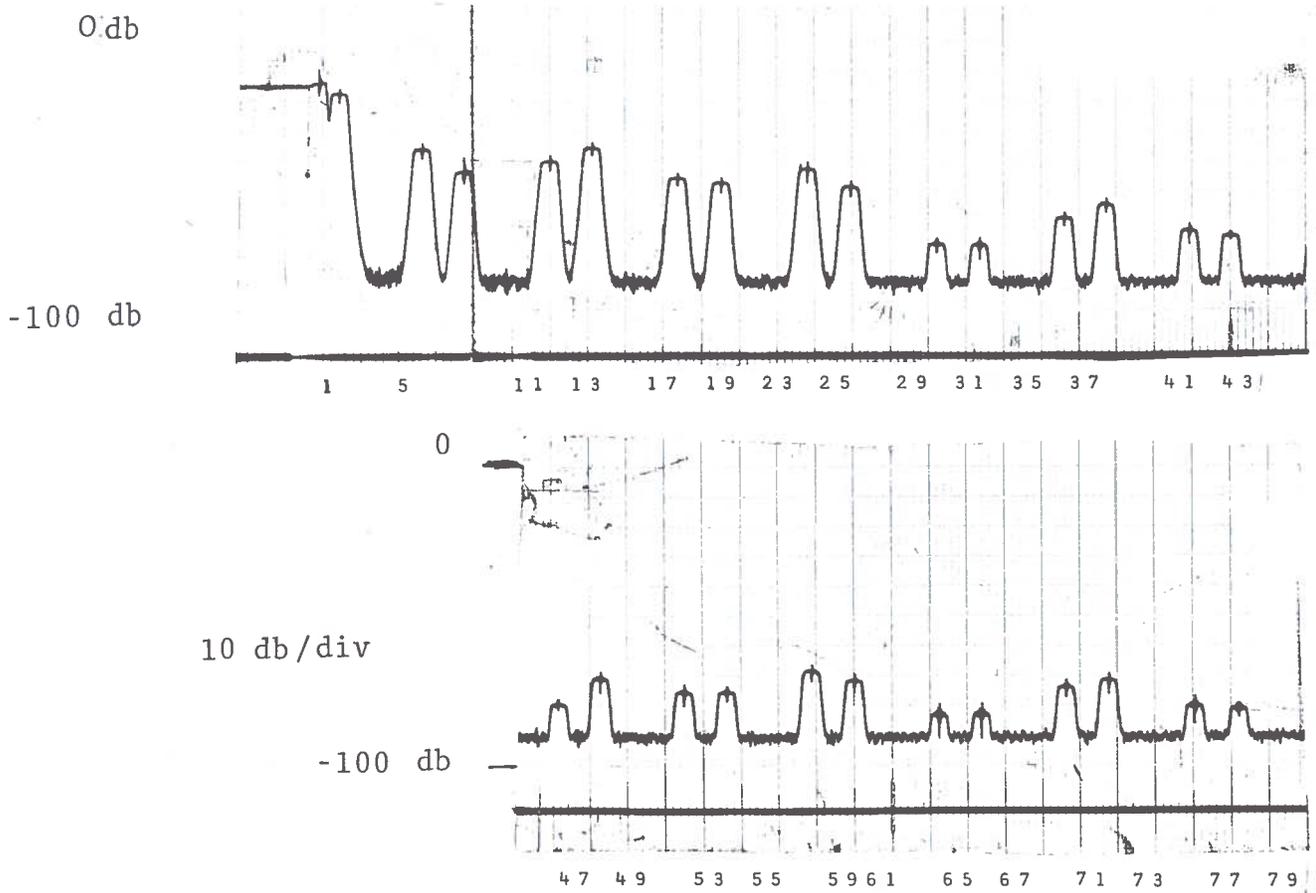


FIGURE 7-12. PULSE-WIDTH MODULATED INVERTER,  
 OUTPUT CURRENT HARMONIC SPECTRUM, E = 162 VOLTS,  
 F = 124 Hz, WAVEFORM = FOUR NOTCH

TABLE 7-3. PULSE-WIDTH MODULATED INVERTER, OUTPUT CURRENT HARMONIC SPECTRA

4 NOTCH 124 Hz				
IL =	9A rms		20A rms	
N	$\Delta$ db	IN/I <sub>1</sub>	$\Delta$ db	IN/I <sub>1</sub>
1	0	1.0	0	1.0
5	-12	.25	-19	.112
7	-21	.089	-26	.05
11	-15	.178	-23	.07
13	-11	.282	-18	.13
17	-22	.08	-29	.04
19	-23	.07	-30	.03
23	-19	.11	-25	.06
25	-26	.05	-31	.03
29	-45	.007	-50	.003
31				
35				
37				
41				
43				
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67				

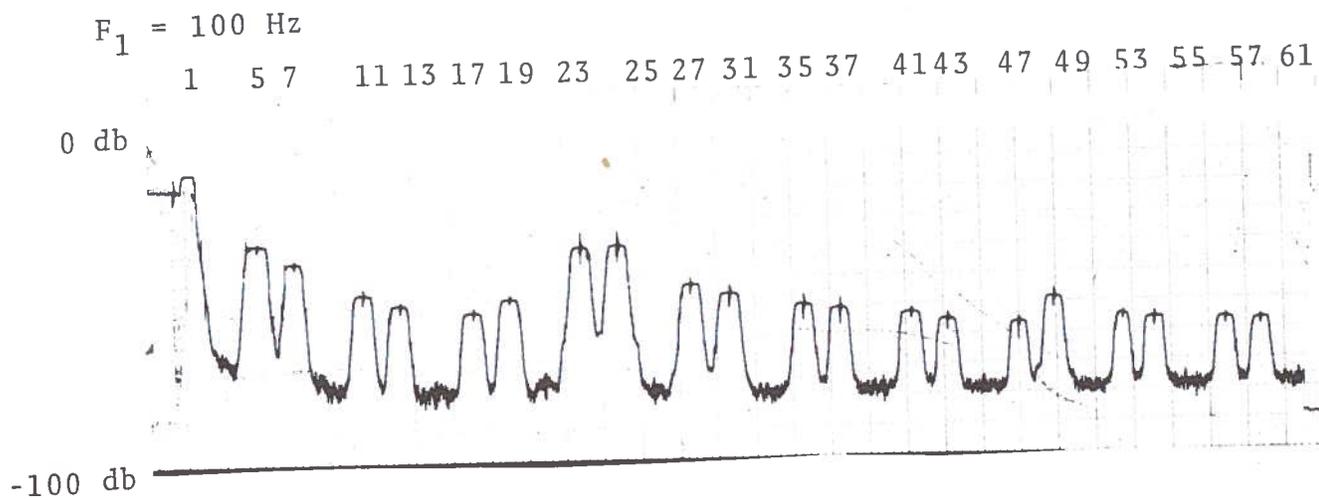
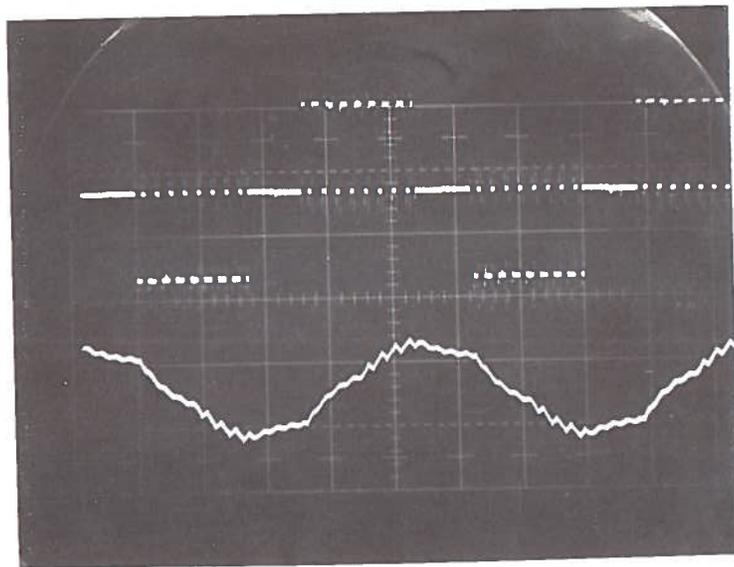
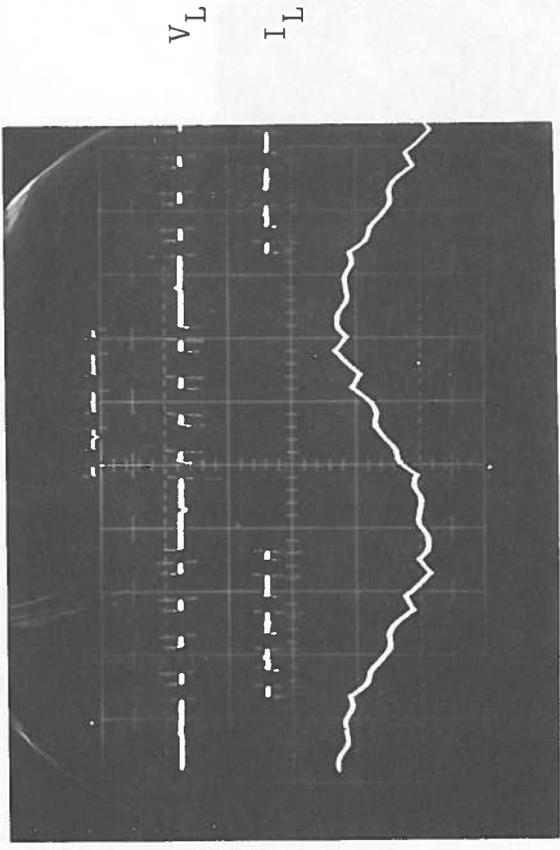


FIGURE 7-13. OUTPUT HARMONIC SPECTRUM,  
 $Z = 4.3 \text{ OHMS}$ ,  $E_0 = 150 \text{ VOLTS}$ ,  
 $F = 100 \text{ Hz}$ ,  $\text{PF} = 0.8$



$F_1 = 150 \text{ Hz}$

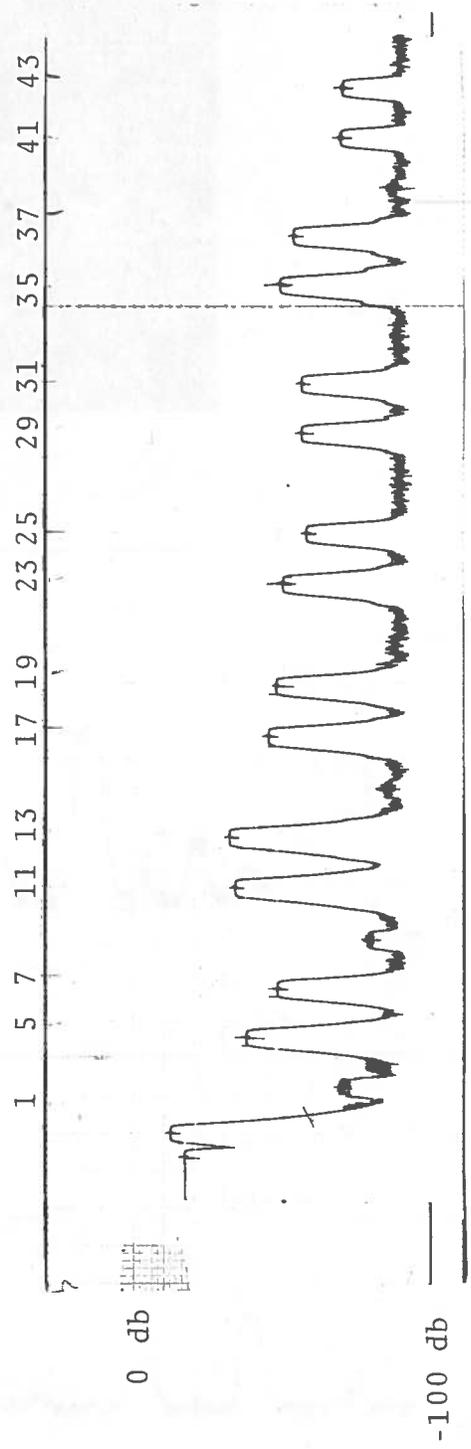


FIGURE 7-14. OUTPUT HARMONIC SPECTRUM,  $Z = 4.3 \text{ OHMS}$ ,  $E = 150 \text{ VOLTS}$ ,  $F = 150 \text{ Hz}$ ,  $PF = 0.8$

$F_1 = 300 \text{ Hz}$

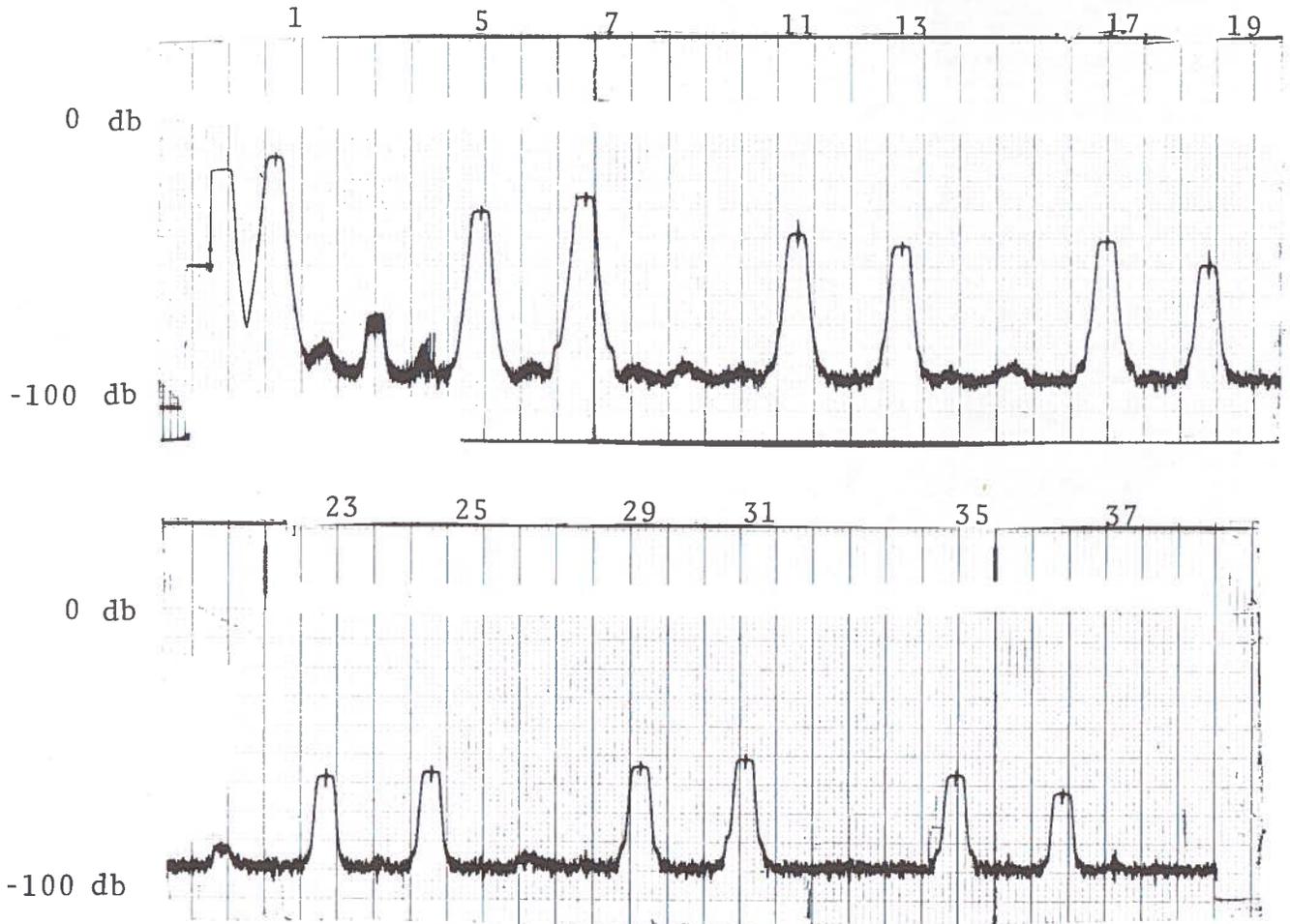
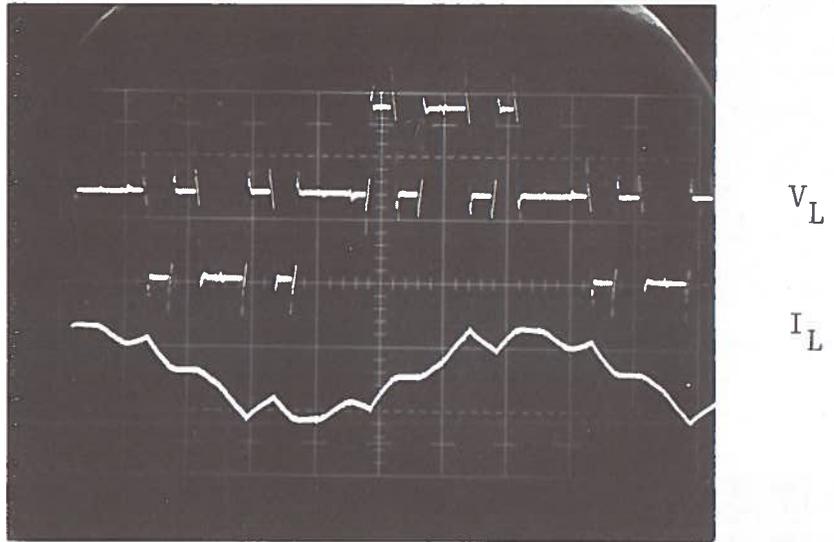


FIGURE 7-15. OUTPUT HARMONIC SPECTRUM,  
 $Z = 4.3 \text{ OHMS}$ ,  $E_0 = 150 \text{ VOLTS}$ ,  
 $F = 300 \text{ Hz}$ ,  $\text{PF} = 0.8$

TABLE 7-4. PULSE-WIDTH MODULATED INVERTER STATIC LOAD CURRENT HARMONIC SPECTRA

Z = 4.3 ohms      Pf = 0.8

N	F = 100 Hz		F = 150 Hz		F = 300 Hz	
	8 Notch		4 Notch		2 Notch	
	A	B	A	B	A	B
1	0	1.0	0	1.0	0	1.0
5	25	.06	26	.05	19	.11
7	31	.03	36	.016	14	.20
11	42	.008	22	.08	26	.05
13	47	.005	20	.1	31	.028
17	48	.004	33	.022	29	.035
19	44	.006	35	.018	38	.013
23	27	.045	38	.013	46	.005
25	27	.045	40	.01		
29	40	.01				
31						
35						
37						
41						
43						
47						
49						
53						
55						

A = Δ db

B =  $\frac{IN}{I_1}$

the range of center frequency for the first cluster is:

eight notches 4 to 120 Hz	=	96 Hz to 2.88 kHz
four notches 120 to 200 Hz	=	1.44 kHz to 2.40 kHz
two notches 200 to 400 Hz	=	1.2 kHz to 2.40 kHz

For this particular set of operating frequencies, the range of harmonics extends further with the eight-notch waveform than with the four-notch or the two-notch waveforms.

As the number of notches is changed and the harmonics clusters move closer to the fundamental frequency, the harmonic currents increase. Under identical static load conditions of 150 volts and at a load power factor of 0.8 the highest harmonic current magnitude is  $.04I_{\ell}$  for the eight-notch output,  $0.11I_{\ell}$  for the four-notch output and  $0.21I_{\ell}$  for the two-notch output. The effects of these harmonics on the motor operating characteristics have not been fully analyzed. However, a reduction in efficiency can be anticipated because the contributions of the harmonics become more severe as the number of waveform notches is increased.

High distortion components present in the output voltage waveform indicate misleading results when comparing measured output characteristics to calculated values using the laboratory setup. Measured characteristics indicate a lower power factor than calculated for the output rms voltage. When harmonic data is used to isolate the components of current and voltage, and the impedance of the load is recalculated for each harmonic frequency the measurements and calculations are in agreement. The results of such a calculation for static load test, are shown in Table 7-5. This table lists the harmonic magnitudes of current and voltage in the output waveforms for the following conditions: output frequency = 100 Hz; output voltage = 150 volts rms; load impedance = 4.33 ohms; and output power factor = 0.5 lagging. Also included in the listing is the calculated impedance of the load at the harmonic frequencies, calculated harmonic current, and calculated power for the highest harmonics.

TABLE 7-5. HARMONIC COMPONENTS

$f = 100 \text{ Hz}$   
 $V_{1-1} = 150V_{\text{RMS}}$   
 $V_{\phi} = 86.15V_{\text{RMS}}$   
 $P.f = 0.5$   
 $I_0 = 13.5A$   
 $P_0 = 1.19 \text{ kW}$   
 $P_{\phi} = 397 \text{ W}$

Measured			Calculated		
Harmonic	Phase Voltage Volts AC	Phase Current Amps AC	Impedance Ohms	Phase Current Amps AC	Phase Power Watts
1	57.3	13.4	4.33	13.4	359.0
5	13.2	.54	18.5	.72	1.1
7	7.6	.27	26.2	.29	.2
11	5.4	.08	41.2	.18	
13	3.4	.05	48.6	.07	
17	2.6	.02	63.5	.04	
19	5.4	.05	71	.07	
23	42.3	.405	86	.49	.5
25	53.8	.405	98.3	.57	.7
29	13.2	.09	108	.12	
31	10.6	.07	116	.09	

Calculated

Phase Power 391.5 W

Measured

Phase Power 397 W

The results of the harmonic analyses indicate that for an inverter which uses pulse-width modulation to regulate the output voltage, the output power and power factor are related to output frequency, the number of pulses in the output, and the pulse-widths of the output pulse train. The data indicates that the inverter is better suited for operation with low power factor loads than with high power factor loads. Because of the high harmonic content in the voltage waveform, their effects become more significant as power factor increases and the filtering action of the load is lost. As the number of notches and the ratio of the chopping frequency to the fundamental decrease, the filtering action of the load is more important in limiting harmonic currents; thus, reinforcing the dependence of the inverter on a low power factor load for harmonic current limiting. Under static operation, the load power factor can be set and the harmonic spectra measured and displayed, but under dynamic conditions load power factor is a function of the motor slip. Thus, harmonic ratios are likely to vary as a function of motor operating parameters. Nevertheless, harmonic patterns are distinctive, and although they may change in magnitude, the pattern of the distribution is unchanged. A complete description of harmonic current behavior with the pulse-width modulated inverter in any full scale system will require extensive computation and analysis, using the equivalent circuit parameters of the particular motor being driven and the modulation characteristics of the inverter output. The feasibility of this approach has been demonstrated in the simulation modeling done as part of this program.

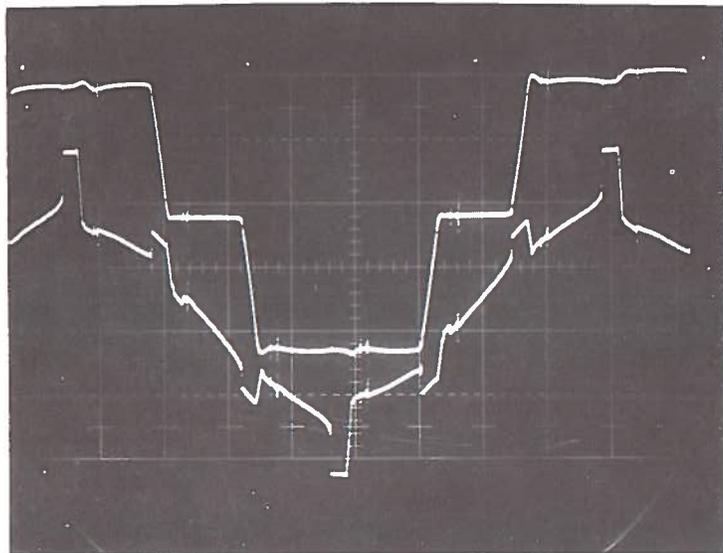
A detailed description of harmonic influence in three-phase induction machines is given in reference 14, where it is shown that the harmonics produce effects in the motor torque-speed characteristics at speeds corresponding to high slip points. The fifth harmonic, for example, produces negative torques at slip speeds of 1.251. The seventh produces positive torque from zero speed to one-seventh of synchronous speed, and negative torques, diminishing in magnitude, from one-seventh synchronous speed to synchronous speed. The variable frequency, power conditioning

systems used to supply the inverters operate the motor near synchronous speed at all points, therefore the effects of the harmonics on the motor performance are not distinguishable. Harmonic impact on communication and control systems will have to be investigated on an individual basis, since the severity of any interference on any system cannot be predicted.

### 7.3 CONSTANT CURRENT INVERTER

Harmonic studies of waveforms associated with the constant current inverter are less complex than those of the pulse-width modulated inverter because the constant current inverter delivers a six pulse, quasi-square wave of current to the motor load. These wave shapes have well defined, harmonic components whose magnitude is inversely proportional to the harmonic number; identical to those of the phase delay, input bridge discussed in the section dealing with input harmonics. Even numbered and triplen (3, 9, 15...) harmonics do not appear in the output. Figure 7-16 shows waveform photos and harmonic spectra for the constant current inverter's current waveform. Harmonic values are listed in Table 7-6. Similarity to the waveform of Figure 6-7 is evident, the principle difference being in the slower rise and fall times of the output current waveform. Harmonic distributions follow the  $1/n$  pattern characteristic of this waveform with a cluster near the forty-second multiple, due to the linear ramp leading and trailing edges. Constraining the current to a fixed wave shape forces the voltage to vary as a function of the load impedance. Because the motor presents a lagging power factor load to the constant current inverter, the output voltage has a wave shape which has a strong fundamental component, but some harmonic distortion pattern is evident.

A characteristic of constant current inverters is the series of voltage pulses which occur at the transition points where current transfers from one line to another. This is a result of

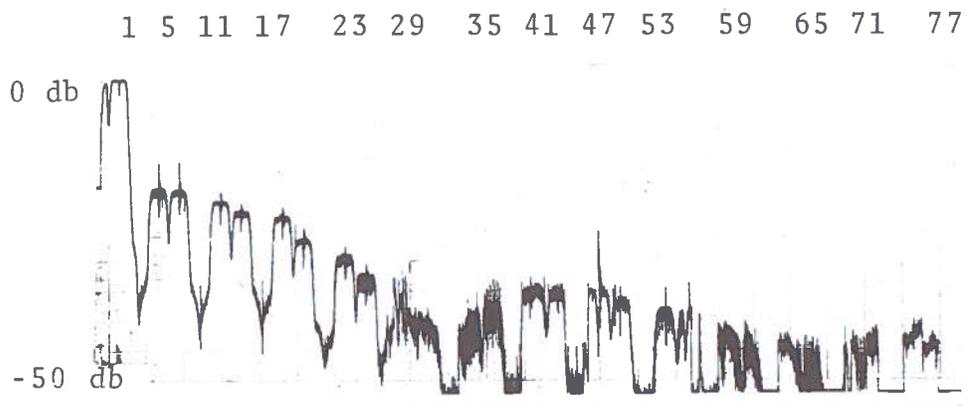


TOP TRACE

LINE CURRENT

BOTTOM TRACE

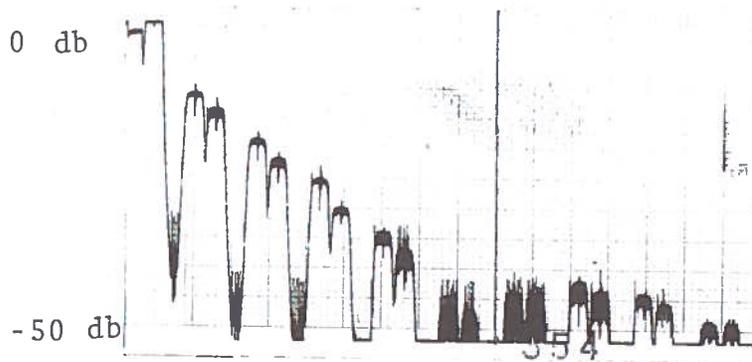
LINE TO LINE  
VOLTAGE



VOLTAGE

V = 244 V AC

60 Hz



CURRENT

I = 23.5 AMP AC

60 Hz

FIGURE 7-16. CONSTANT CURRENT INVERTER,  
OUTPUT HARMONIC SPECTRA

TABLE 7-6. CONSTANT CURRENT INVERTER OUTPUT HARMONIC DATA

23.5A

N	Vdb	VN/V1	Idb	IN/I1	Vdb
1	0	1.0	0	1.0	0
5	18	.126	14	.20	14
7	18	.126	17	.14	17
11	20	.10	22	.08	22
13	23	.071	25	.056	24
17	24	.063	28	.040	28
19	28	.040	33	.022	32
23	30	.032	37	.014	35
25	33	.022	41	.009	40
29	-	-			47
31	45	.066			50
35	39	.011			
37	37	.014			
41	35	.018			
43	36	.016			
47	35	.018			
49	37	.014			
53	38	.013			
59	39	.011			
61					
65					
67					

coupling the turn-on transient of an on-going thyristor to commutate a conducting thyristor. The effects of these pulses on the harmonic distribution of the voltage waveforms can be seen in the figures, where for the no load, lower voltage case, harmonic component distribution patterns extend to high order harmonics. The motor currents associated with these higher order voltage harmonic are greater than 40 db below the fundamental, a magnitude of less than 1 percent, and therefore not significant.

## 8. LOSSES AND EFFICIENCY

Efficiency is the proportion of total real power input which is delivered as real power to the load. The remainder is either used within the inverter for auxiliary power or lost in the power conditioning process. For the three inverter systems tested, the measured auxiliary losses are as listed.

### 8.1 AUXILIARY LOSSES

1. Natural commutated inverter = 500 watts
2. Pulse-width modulated inverter = 220 watts
3. Constant current inverter = 300 watts

These losses are essentially fixed losses which do not vary significantly with load or frequency. They occur principally in the electronics sections and include waveform generation, current and voltage controls, power supplies, metering circuits, and fans. They are least significant at high output levels, but at light loads they constitute a major portion of the output power and account for the poor efficiency at those levels.

It should be noted that these losses are not linearly related to inverter power handling capability and in full scale equipment would be of little importance.

### 8.2 POWER CONDITIONING LOSSES

#### 8.2.1 Switching Loss

The power processing losses are functions of the total number of switching transients per second and the current levels within the inverter. Switching losses result from the high, instantaneous power dissipated in some of the semiconductor devices during turn-on or recovery. Because they are proportional to the current being switched, the switching losses are not easily isolated from the current level or conduction losses. However, an attempt has been made to separate them so that a coarse analysis can be done. Losses

due to inverter switching transients have been plotted in Figure 8-1. Comparisons of the curves show the importance of limiting the number of switching transients per second to minimize these losses.

For the natural commutated inverter, switching losses increase as frequency increases, and as output voltage increases. In the first case, as frequency increases more polarity changes per second are required and there are more switching transients per second. A proportional increase in switching losses results. In the second case, for a given load the output current will increase as the voltage across the load increases. Since the natural commutated inverter maintains the output by supplying charge impulses to the output filter capacitors, more such impulses and thus more switching transients are required to maintain the output voltage level.

The pulse-width modulated inverter varies the output voltage by controlling the width of fixed numbers of pulses. The total number of switching transients per cycle of output voltage is constant, therefore switching losses do not vary as a function of output voltage. The number of commutations per second, however, is proportional to frequency, thus the switching loss component is dependent on output frequency.

At frequencies where switching losses approach the dissipation limits of the components, the output waveform is changed to reduce the number of commutations per cycle, and per second, so that the frequency range of the inverter can be extended. There are two waveform changes made in the pulse-width modulated inverter, one at 120 Hz where the output transits from 18 pulse (18 commutations per thyristor) to 10 pulses, and the second at 200 Hz, where the 10 pulse waveform is reduced to six pulses. The number of pulses refers to the total pulses per phase per cycle of output voltage.

The switching losses in the natural commutated inverter and in the pulse-width modulated inverter are comparable. In each case the current through the commutation loop components does not vary significantly, as it is the loop current of a series

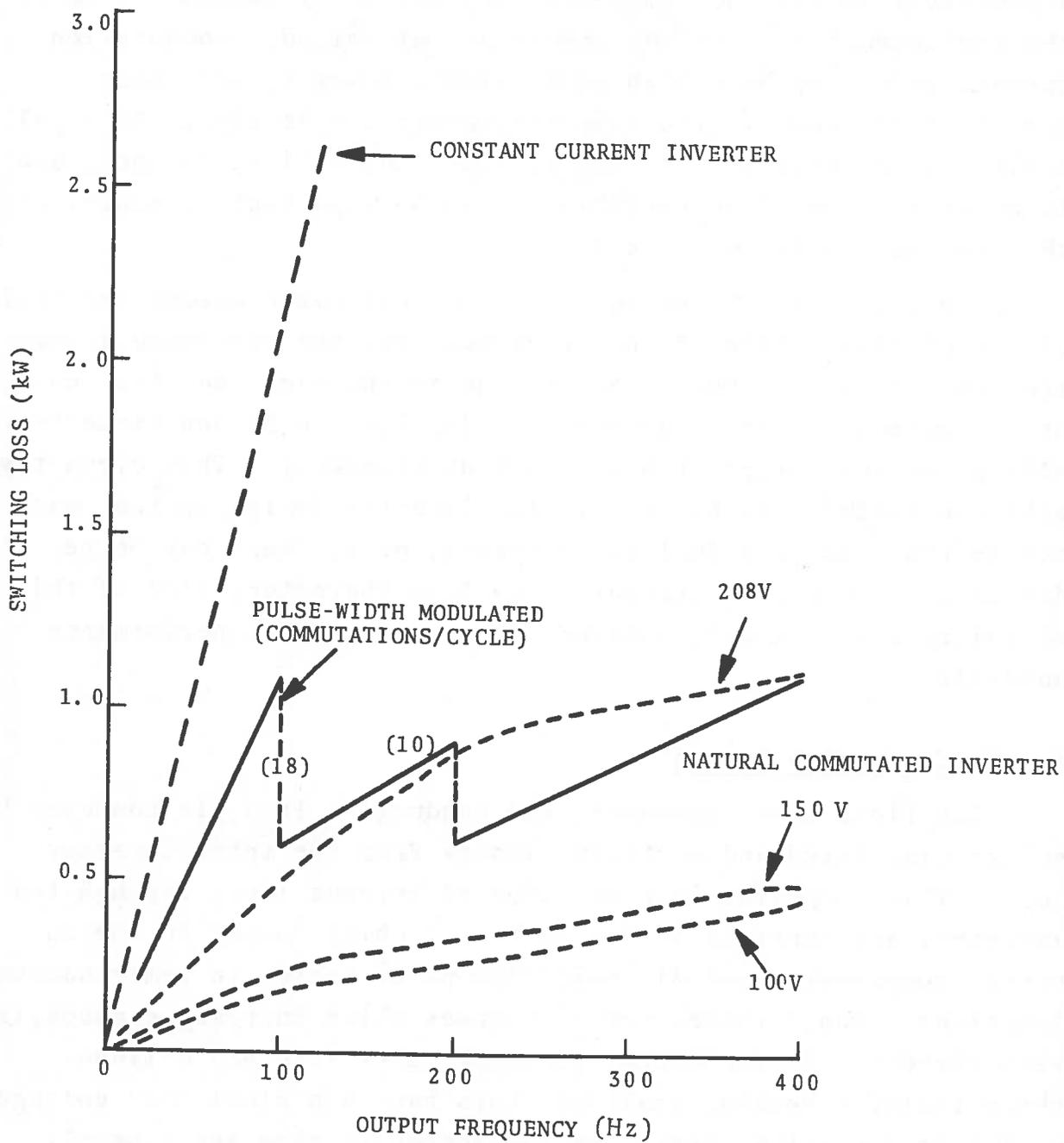


FIGURE 8-1. SWITCHING LOSSES VERSUS OUTPUT FREQUENCY

resonant LC network, and the current amplitude is determined principally by  $\sqrt{L/C}$ . The commutation current waveforms are half sine wave pulses which begin at zero current and do not rise appreciably before the semiconductors are fully turned on; thus, the semiconductor switching losses are minimized. Commutation current pulses do have high peak values, however, and ohmic losses in the commutation loop components can be high. In a full scale system, component sizing to lower internal resistance, and location to lower lead resistance will be important in minimizing the commutation circuit losses.

The constant current inverter has less commutations per cycle of output than either of the other two, yet the switching losses are considerably higher. This is due to the fact that the constant current inverter incorporates dissipative burden circuitry with power loss proportional to output frequency. This circuitry, although included in the particular inverter design tested, may not be required in a full scale system, or at least may be redesigned to be more efficient. The loss characteristics of this circuitry are included, however, in overall system performance analysis.

### 8.2.2 Conduction Losses

The final loss component, the conduction loss, is computed by subtracting fixed and switching losses from the total inverter loss. This component is a function of current level through the inverter, and consists of two parts: 1) ohmic losses in the inverter components, and 2) conduction power losses in semiconductor junctions. Ohmic losses are  $I^2R$  losses which increase exponentially with current. Semiconductor junction power loss has a linear characteristic because semiconductors have a nominal "on" voltage of 1.0 to 2.0 volts, therefore the losses in them are linearly proportional to current. The combination of the two components gives the conduction loss a log plus linear slope which is evident in the plots of conduction losses in the three inverters, shown in Figure 8-2.

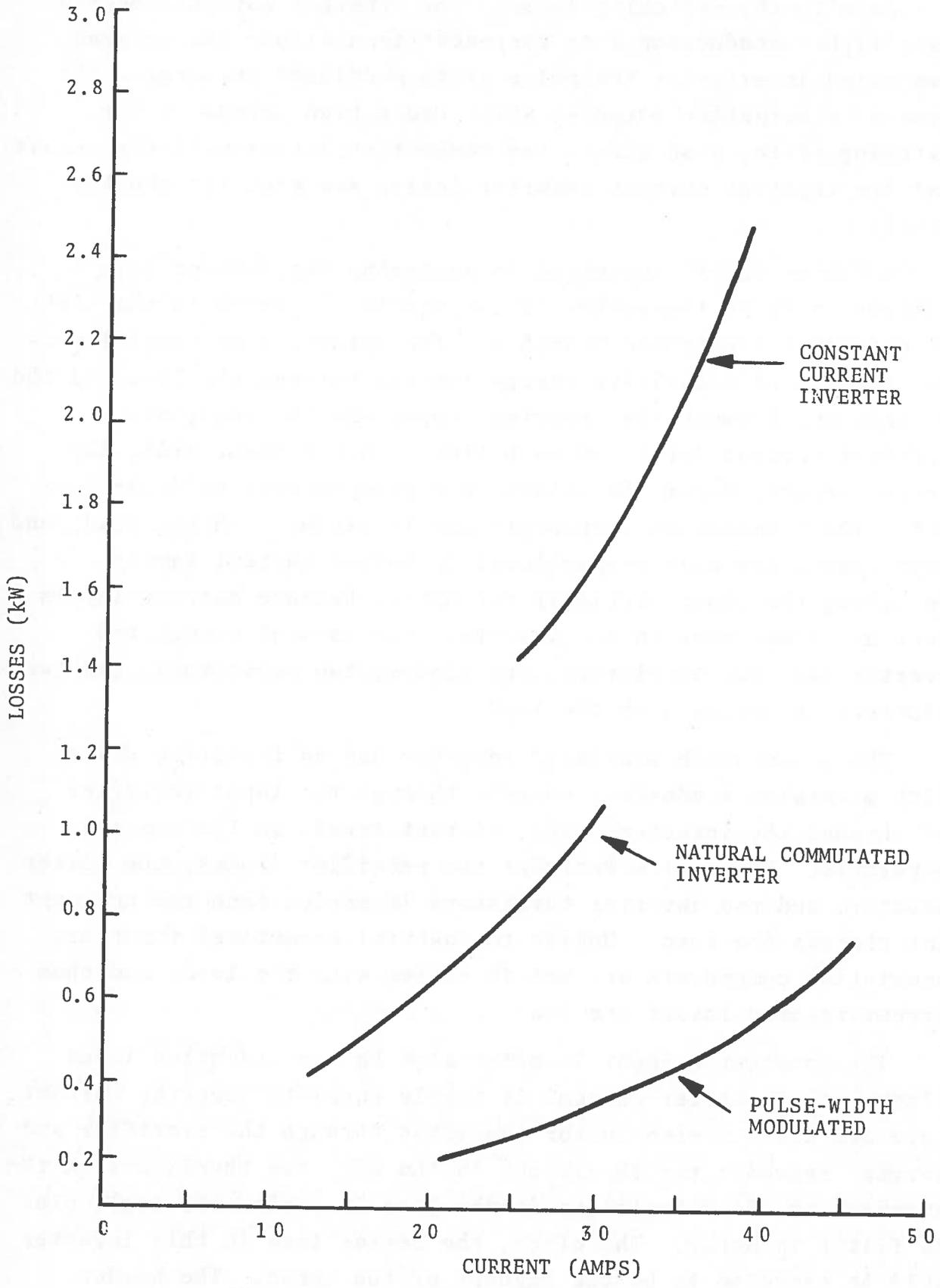


FIGURE 8-2. CURRENT RELATED LOSSES

As with the switching losses, the constant current inverter has a higher conduction loss component than either the natural commutated inverter or the pulse-width modulated inverter. The internal dissipative burdens, which cause high losses in the switching state, also affect the conduction losses with the result that the constant current inverter losses are high for the test article.

A factor to be considered in analyzing the current level-related losses is the number of components in series in the high current power electronic branches. The natural commutated inverter, because of capacitive energy storage between the line and the dc link, and between the inverter proper and the load, has different current levels in each link. On the input side, the current levels, hence the losses, are proportional to diode current. The commutation components are in series with the load, and their losses are also proportional to output current levels. Neglecting the input rectifier circuitry, because current levels there are lower than in the inverter, the natural commutated inverter has four thyristors, two diodes, two capacitors, and two inductors in series with the load.

The pulse-width modulated inverter has an inductive filter which maintains a constant current through the input rectifier and through the inverter; thus, current levels in the two are comparable. The system includes two rectifier diodes, the filter inductor, and two inverter thyristors in series from the ac input line through the load. Unlike the natural commutated inverter, commutation components are not in series with the load, and thus current-related losses are low.

The constant current inverter also has an inductive input filter, and rectifier current is nearly equal to inverter current. There are eight semiconductor junctions through the rectifier and inverter segment: two thyristors in the pdr; two thyristors in the inverter; and four isolation diodes (two in series per leg), plus the filter inductor. Therefore, the series loss in this inverter would be expected to be the highest of the three. The burden

circuitry incorporated in the inverter also adds to the steady state losses, making the constant current inverter the least efficient of the three inverters tested.

### 8.3 EFFICIENCY

The effects of losses on inverter system performance can be seen in Figures 8-3 through 8-5, which are plots of inverter efficiency versus power out. As one would expect, efficiency at light loads is characteristically low because the bulk of the power is used to supply fixed loads and is not processed to the output. As the power output increases, the ratio of output to input increases and quickly approaches the level where the fixed losses are no longer significant, approximately 3 kW of output power. From that point on, switching and steady state losses become the limiting factors.

Of the three inverters tested, the pulse-width modulated inverter operated most efficiently, attaining efficiencies greater than 90 percent at full load output. The natural commutated inverter reached efficiencies of 82 percent at comparable power levels and operating frequencies. The constant current inverter, because of the resistive burden circuit losses, reached a peak efficiency of 74 percent at full load output power, but only 68 percent at outputs comparable to the other two inverters.

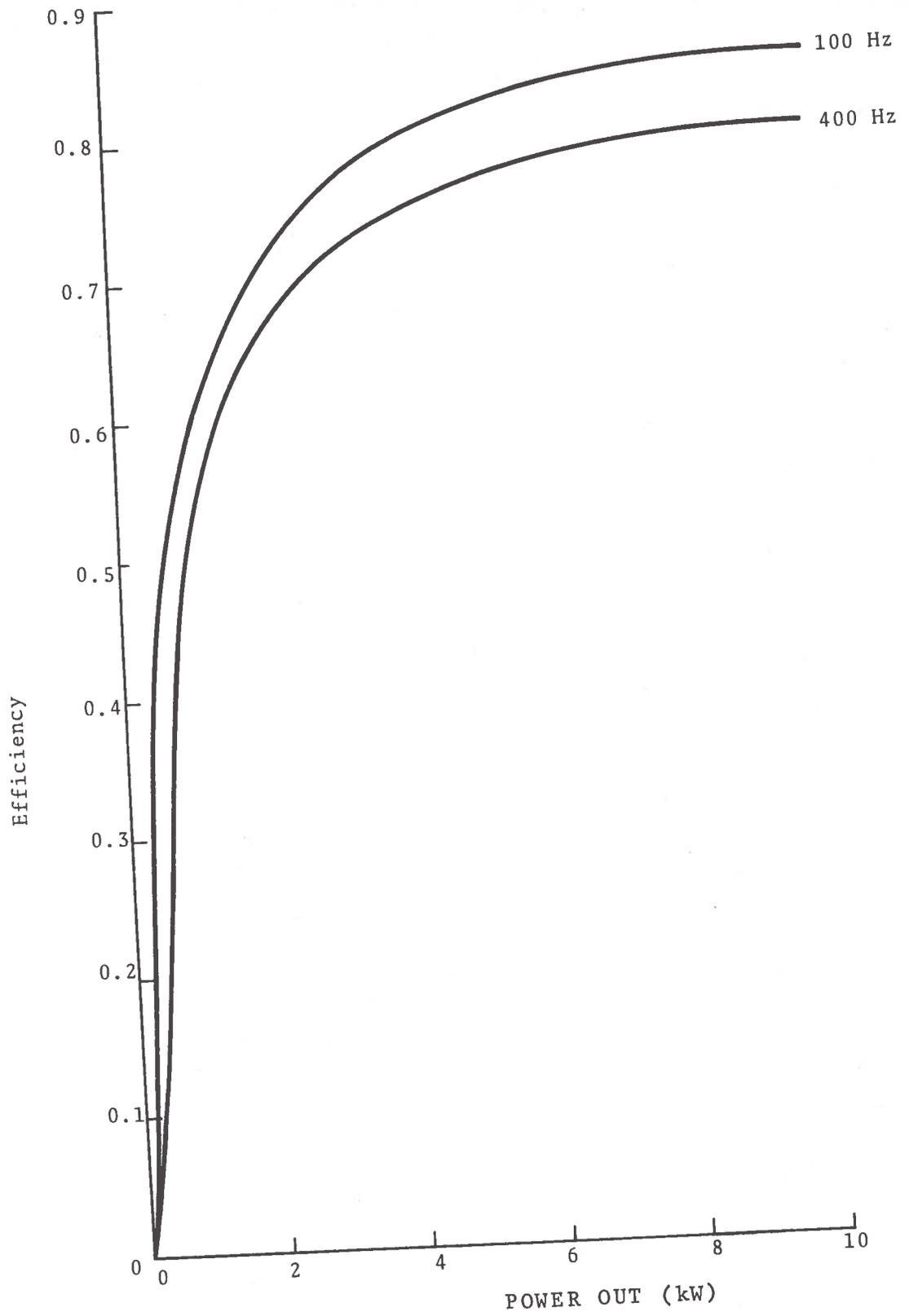


FIGURE 8-3. NATURAL COMMUTATED INVERTER EFFICIENCY VERSUS POWER OUTPUT AT 100 Hz AND 400 Hz

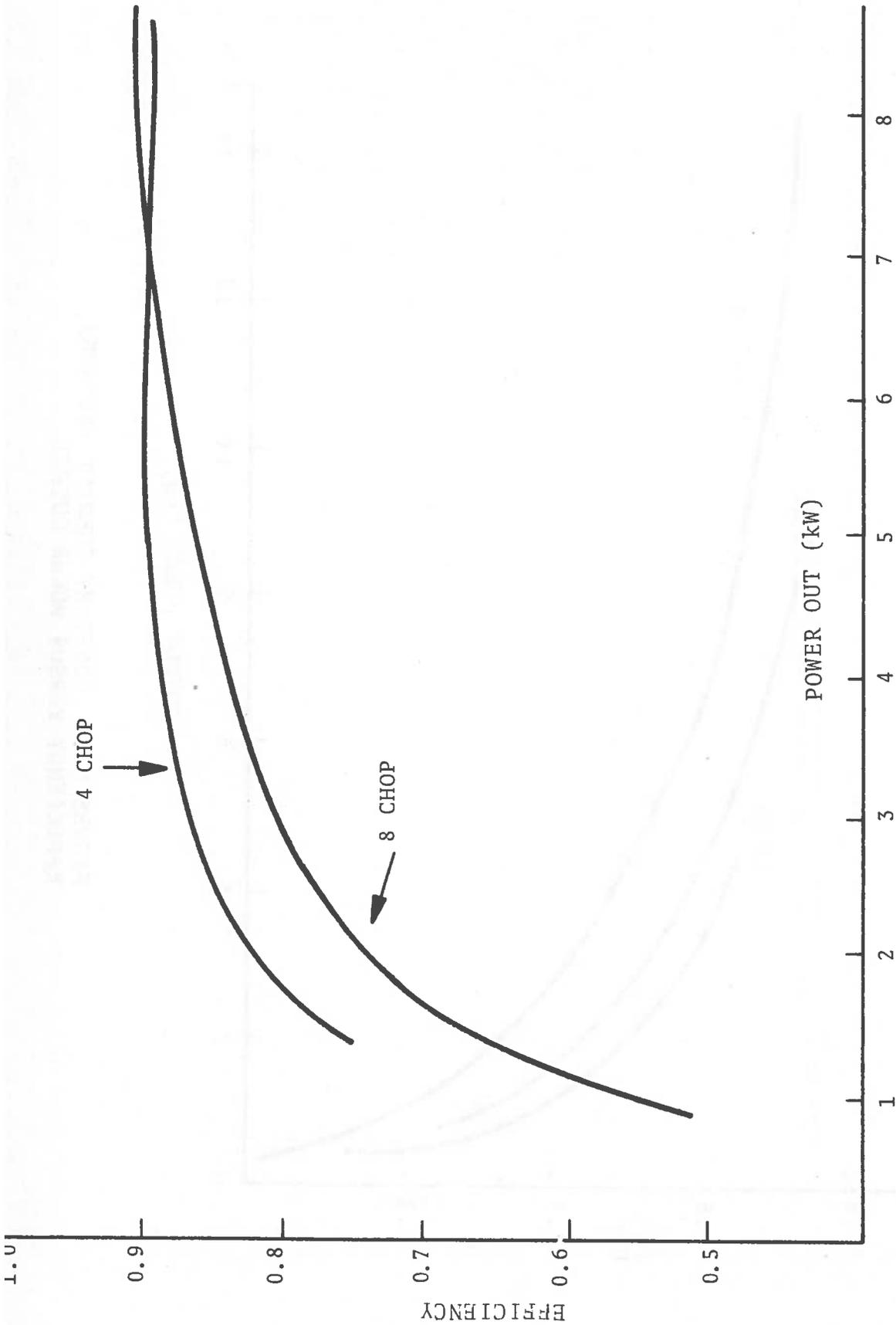


FIGURE 8-4. PULSE-WIDTH MODULATED INVERTER, EFFICIENCY VERSUS POWER OUTPUT

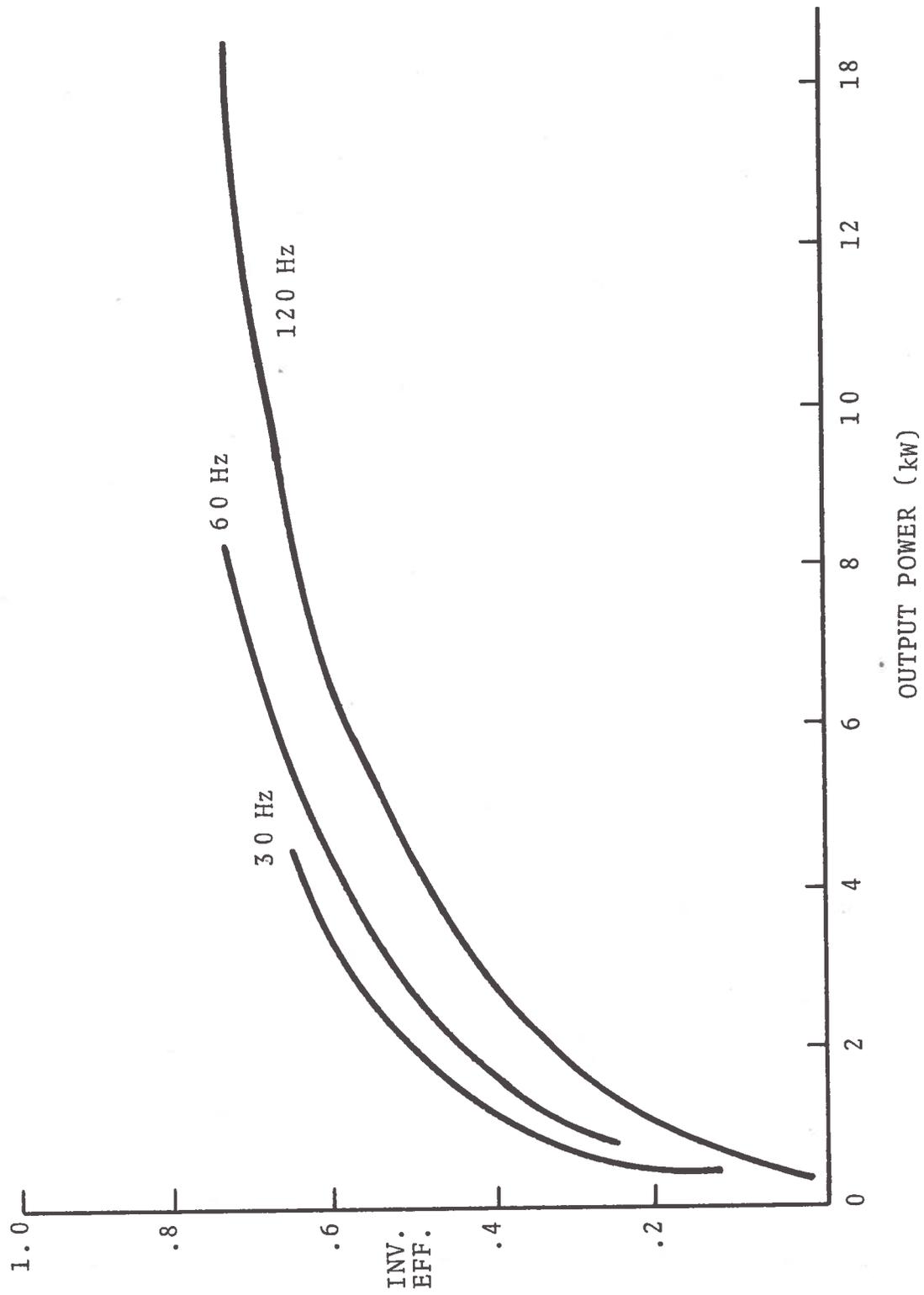


FIGURE 8-5. CONSTANT CURRENT INVERTER, EFFICIENCY VERSUS POWER OUTPUT

## 9. COMPUTER SIMULATION OF AC INDUCTION MOTOR AND INVERTER DRIVES

The computer simulation of the natural commutated inverter pulse-width modulated, and constant current inverter drives plus ac induction motor offers a convenient means for comparing the idealized drive characteristics with those measured in the laboratory. This technique allows one to examine the motor-drive characteristics under a variety of conditions. It also gives an insight into the limitations of the various drives, which can be particularly helpful in system design or in the selection of a motor-drive for use in a given propulsion application.

This simulation study consists of two separate investigations: the first is the modeling of a 120 Hz, 20 hp, ac induction machine designed for a rated speed of 1785 rpm; the second is the modeling of three inverter drives (natural, pulse-width modulated, and constant current), used to control speed of the ac induction machine.

The simulation of three inverters requires models which generate the current-voltage waveform characteristics of each inverter. Since these waveforms can be described by Fourier expansions, one method of checking the model's validity is to compare Fourier harmonic amplitudes with those measured in the laboratory. This test provides a check on the validity of the inverter model, independent of the model used for the induction machine. Once the validity of the inverter models is established, these models are used in conjunction with the induction motor model to predict the combined motor-drive characteristics. This puts particular importance on the validity of the motor model, since errors in the machine simulation lead to increased errors in the combined motor-drive simulation. Particular attention is therefore given to the selection of induction motor parameters which yield a good representation of motor performance over a wide range of operating conditions. This includes both motoring and regenerative motor modes.

## 9.1 INDUCTION MACHINE MODEL

The nameplate data for the ac induction machine studied in this program is given in Table 9-1.

Manufacturer:	Louis Allis
Rated Power:	20 hp at 120 Hz
No. of Electrical poles:	8
Rated Speed:	1785 rpm
Terminal Voltage:	460 V
NEMA Type:	B

Figure 9-1 shows a sketch of the equivalent circuit used to describe the induction machine. Seven circuit parameters are required; two for the primary, four for the secondary, and one to describe the magnetization reactance. Four circuit parameters are required to model the secondary (rotor) due to the speed dependent secondary impedance of NEMA type B machines.

The values of the equivalent parameters were determined from laboratory tests of the ac induction motor. The initial tests were conducted with a 60 Hz variac-drive to eliminate complications which might otherwise arise from the higher harmonic outputs of the inverter drives. At 60 Hz, rated motor speed is 892 rpm and rated output power is 10 hp.

The methods for determining the equivalent circuit parameters follows standard test procedures, which include:

1. dc primary resistance measurement
2. no load impedance measurement
3. locked rotor impedance measurement
4. locked rotor torque measurement
5. rotational loss measurement (windage, friction).

A further tests is required to determine the rotor circuit parameters.

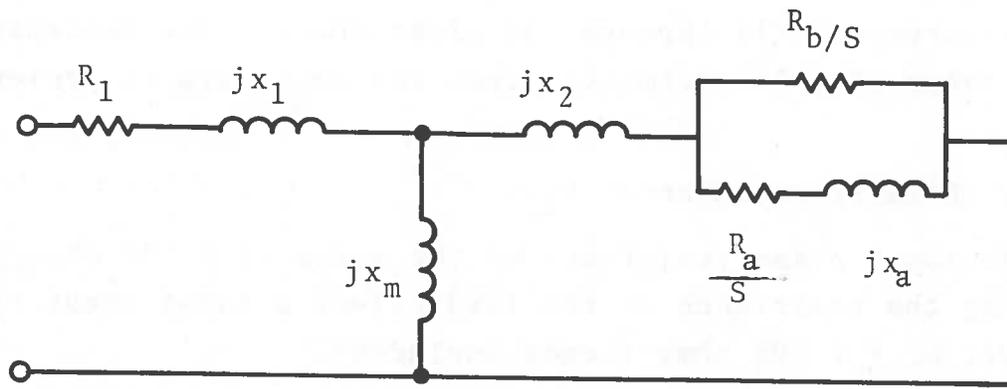


FIGURE 9-1. INDUCTION MOTOR EQUIVALENT CIRCUIT

6. torque speed characteristics at fixed motor terminal voltage.

Test 6 provides data on stall torque and its related slip, which is useful in checking the consistency of the parameter choice.

Appendix D contains a detailed description of the motor test data and the calculations required to determine the equivalent circuit parameters. The following section summarizes the significant steps in the determination of these circuit parameters.

### 9.1.1 Determination of Motor Equivalent Circuit Parameters

The equivalent circuit parameters were determined from the test measurements (1) through (6) given above. The determination of the motor circuit parameters from the test data is presented below.

1. Primary resistance,  $R_1$ .

The input phase resistance of the motor is 0.089 ohms/phase. Including the resistance of the leads gives a total input resistance of,  $R_1 = 0.205$  ohms (leads included).

2. Primary Leakage Reactance,  $X_1$ .

Blocked motor test data yields an input reactance of 0.91 ohms. Assuming that the primary leakage reactance is one-half the input reactance, then,  $X_1 = 0.455$  ohms (60 Hz).

3. Magnetization Reactance,  $X_m$ .

No load test data yields an average input reactance of 6.319 ohms. Subtracting the primary leakage reactance,  $X_1$ , from the input reactance gives a magnetization reactance,  $X_m = 5.955$  ohms (60 Hz).

4. Secondary Circuit Parameters:  $R_a$ ,  $R_b$ ,  $X_2$ ,  $X_a$ .

The secondary circuit parameters are determined by empirically fitting the computed input motor impedance to measured input impedance at three values of motor slip ( $S=0.054$ ,  $S=0.50$ ,  $S=1.0$ ).

The choice of parameters is restricted by the shape of the thrust-speed characteristic as described by the so-called "m" parameter (14) where

$$m = \frac{R_b - \frac{R_a R_b}{R_a + R_b}}{X_A \left[ \frac{R_b}{R_a + R_b} \right]^2} \quad (9-1)$$

Assigning  $m = 1.0$ , the values of the secondary circuit parameters, which yield a "best fit" to the measured motor impedance, are  $R_a = 0.1$ ,  $R_b = 0.5$ ,  $X_2 = 0.35$ , and  $X_a = 0.5$  ohms.

The motor circuit parameters for 60 Hz excitation are listed in Table 9-2. The reactive impedances at other frequencies are found by multiplying the values in Table 9-2 by excitation frequency/60 Hz.

TABLE 9-2. INDUCTION MOTOR EQUIVALENT CIRCUIT PARAMETERS AT 60 HZ	
Induction Motor Equivalent Circuit Parameters (60 Hz)	
R1	= 0.205
X1	= 0.450
Xm	= 5.93
X2	= 0.35
Xa	= 0.50
Ra	= 0.10
Rb	= 0.503

Figure 9-2 shows the input impedance of the motor for 60 Hz excitation in the region near zero slip as computed using the value of motor parameters given in Table 9-2.

### 9.1.2 Induction Motor Torque, Current-Speed Characteristics

Figure 9-3 shows the motor torque and input circuit characteristics for a fixed motor terminal voltage of 75 volts 1-1 and 60 Hz drive frequency. The solid curves give the computed

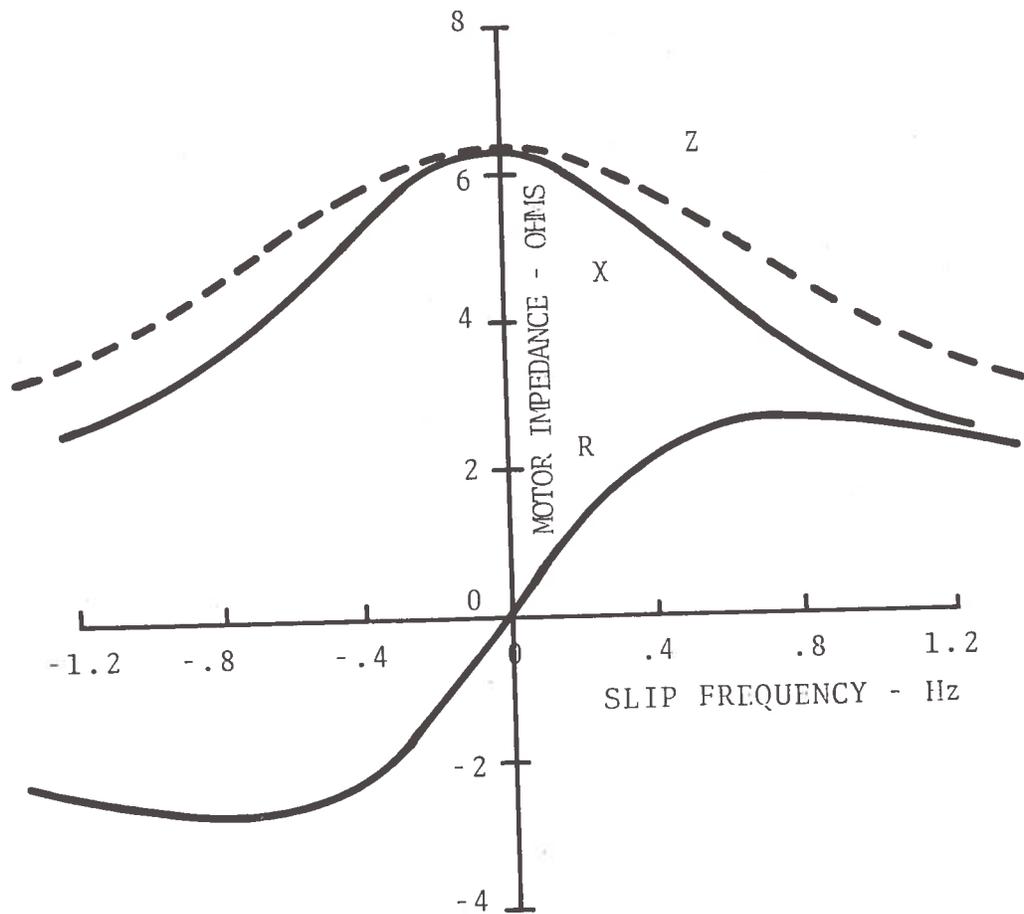


FIGURE 9-2. COMPUTED INPUT IMPEDANCE OF INDUCTION MOTOR NEAR SYNCHRONOUS SPEED (NEGLECTING PRIMARY RESISTANCE OF 0.205 OHM)

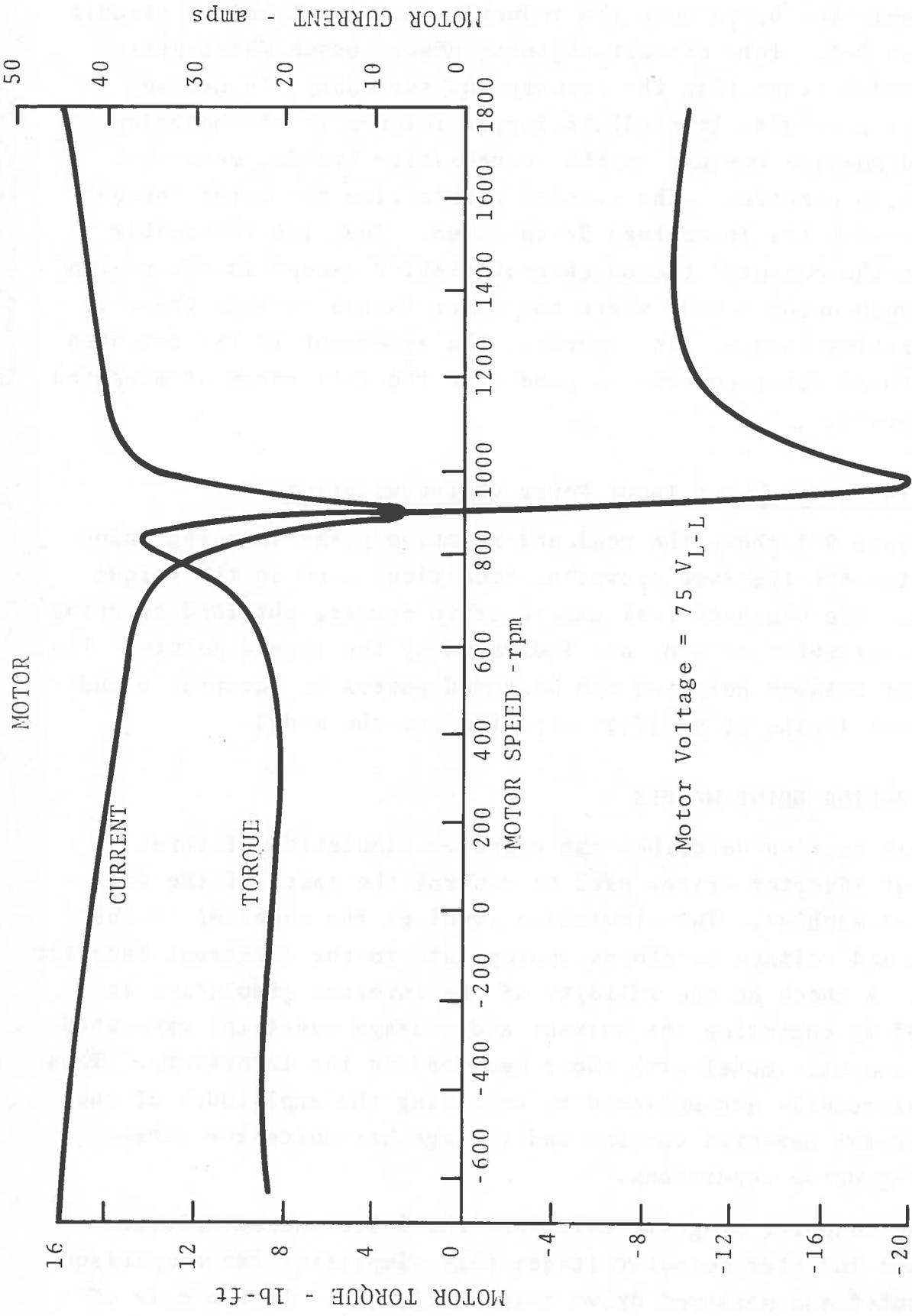


FIGURE 9-3. TORQUE-CURRENT CHARACTERISTIC OF INDUCTION MACHINE

characteristics based upon the induction motor equivalent circuit of Figure 9-1. This circuit neglects power losses which arise from sources other than the primary and secondary  $I^2R$  heating loss. As a result, it predicts larger torques in the motoring mode and smaller torques in the regenerative braking mode than observed in practice. The circled points give the motor torque measured with the laboratory dynamometer. They lie reasonably close to the computed torque characteristics except in the region above synchronous speed, where the motor torque exceeds the computed braking torque. In contrast, the agreement in the computed and measured motor current is good over the full range of measured motor speeds.

### 9.1.3 Induction Motor Input Power Characteristics

Figure 9-4 shows the real and reactive power into the induction motor for the same operating conditions used in the torque studies. The measured real and reactive powers, obtained by using the two wattmeter method, are indicated by the legend points. The agreement between computed and measured powers is acceptable and within the limits of validity expected for the model.

## 9.2 INVERTER DRIVE MODELS

This section describes the computer simulation of three different inverter drives used to control the speed of the ac induction machine. The simulation requires the modeling of the current and voltage waveforms appropriate to the different inverter drives. A check on the validity of the inverter simulation is obtained by comparing the current and voltage waveforms generated by the computer model with those measured in the laboratory. This was most readily accomplished by comparing the amplitudes of the computed and measured current and voltage harmonics for various operating drive conditions.

The computer programs calculate the drive characteristics for fixed inverter output voltage; this simplifies the comparison of computed and measured drive characteristics. In the case of

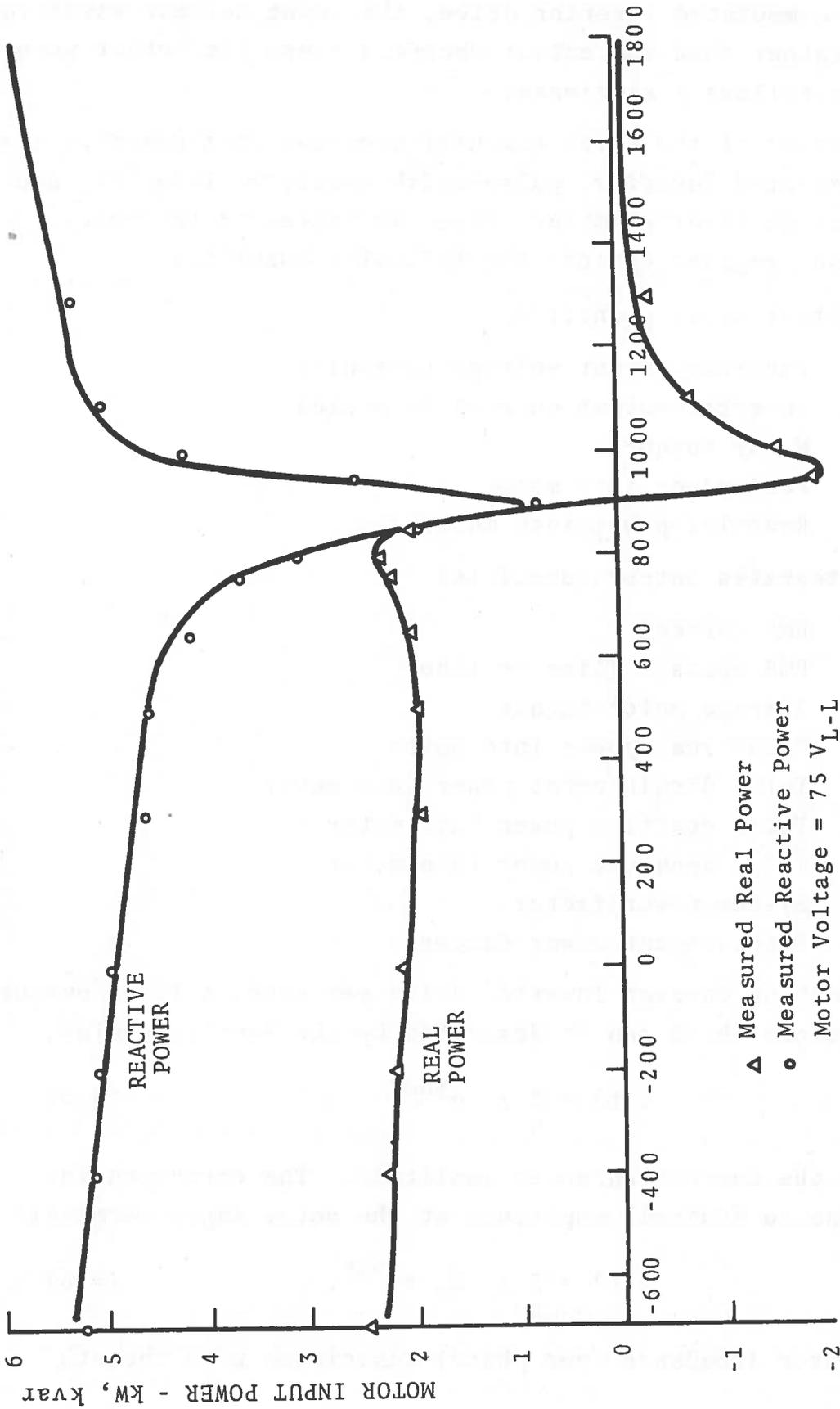


FIGURE 9-4. INPUT POWER CHARACTERISTICS OF INDUCTION MACHINE

the natural commutated inverter drive, the input current waveform is modeled rather than the output waveform since the output waveform closely follows a sine-wave.

The listing of the three computer programs that describe the natural commutated inverter, pulse-width modulated inverter, and constant current inverter motor drives is contained in Appendix E. These programs compute the following quantities:

1. Instantaneous quantities
  - a. Inverter output voltage harmonics
  - b. Inverter output current harmonics
  - c. Motor torque
  - d. Real power into motor
  - e. Reactive power into motor
2. Integrated output quantities
  - a. RMS current
  - b. RMS voltage (line to line)
  - c. Average motor torque
  - d. Total real power into motor
  - e. Total displacement power into motor
  - f. Total reactive power into motor
  - g. Total apparent power into motor
  - h. System power factor
  - i. Displacement power factor.

The constant current inverter drive generates a fixed output current waveform which can be described by the Fourier series,

$$i(t) = \sum_n I_n e^{jn\omega t} \quad (9-2)$$

where  $I_n$  is the current harmonic amplitude. The corresponding voltage (line to neutral) amplitude at the motor input terminals is

$$v(t) = \sum_n I_n Z_n e^{jn\omega t}. \quad (9-3)$$

$Z_n$  is the motor impedance (per phase) associated with the  $n$ th harmonic.

The pulse-width modulated inverter and the natural commutated inverter drives are voltage drives in contrast to the constant current inverter drive. Their instantaneous output voltages (line to neutral) can be described by

$$v(t) = \sum_n V_n e^{jn\omega t}, \quad (9-4)$$

where  $V_n$  is the voltage harmonic amplitude (line to neutral). The instantaneous motor phase current is then

$$i(t) = \sum_n (V_n/Z_n) e^{jn\omega t}. \quad (9-5)$$

The output (real) power of the three-phase drive is

$$P = \frac{3}{2} \sum_n V_n \cdot I_n \cos \theta_n, \quad (9-6)$$

where  $\theta_n$  refers to the angle between the current and voltage harmonic phasors. The output apparent power  $S$  of the three-phase drive is

$$S = \frac{3}{2} \sqrt{\sum_n V_n^2 \cdot \sum_n I_n^2}. \quad (9-7)$$

Total reactive power  $Q_T$  is

$$Q_T = \sqrt{S^2 - P^2}. \quad (9-8)$$

The displacement reactive power  $Q_p$  is computed according to

$$Q_p = \frac{3}{2} \sum_n V_n \cdot I_n \sin \theta_n. \quad (9-9)$$

The following sections describe the computer simulation of the natural commutated inverter, pulse-width modulated inverter, and constant current inverter drives.

### 9.2.1 Natural Commutated Inverter Model

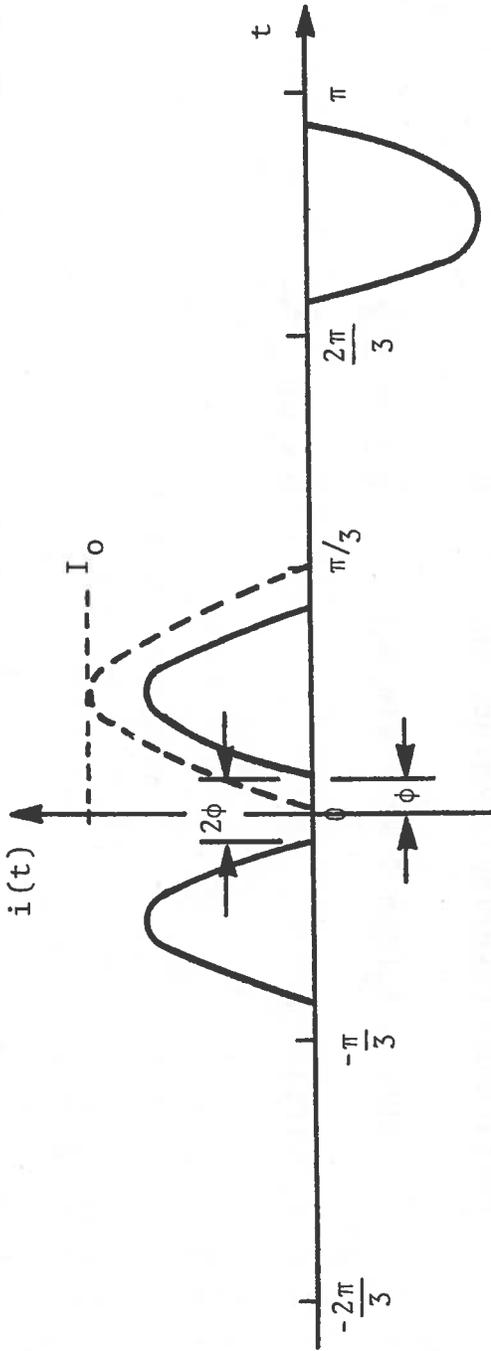
The characteristics of the natural commutated inverter drive are summarized in Table 9-3. The drive utilizes three thyristor inverters to provide variable amplitude and frequency output. The voltage output waveform closely follows the input sine wave reference signal; hence, the model of the output voltage is trivial.

The simulation exercise is confined to the description of the non-sinusoidal input currents to the drive.

TABLE 9-3. NATURAL COMMUTATED INVERTER DRIVE	
Input:	323 vac 3 phase, 60 Hz
Output:	50 amp max. 0-2 kHz 3.5 kva/phase 10 kW

The large capacitor across the output of the rectifier stage causes severe distortion in the input current waveform to the drive. The charge required to maintain the capacitor voltage is supplied in the form of short current pulses during the moments of peak ac line voltage. As the current level increases, the width and amplitude of the input current pulses increase until, in the limit of high input current levels, the current waveform approaches 120 degree conduction pulses characteristic of the constant current inverter output current waveform.

9.2.1.1 Inverter Input Current - Figure 9-5 shows the input current waveform used in modeling the natural commutated inverter drive. The angle  $\phi$  denotes the onset of the current referenced to the midpoint of symmetry as shown. As input current increases,  $\phi$  decreases until finally it becomes zero at the point where the current pulse functions overlap. The expressions for the current in the "discontinuous" and "continuous" conduction modes are indicated in the Figure. The presence of inductive elements in the rectifier circuit causes a phase delay in the current. The resulting lag in the current relative to the line voltage is taken into account by introducing an empirical displacement power factor of 0.974 into the computer model.



Discontinuous Conduction:

$$i(t) = I_0 (\sin 3\omega t - \sin 3\phi)$$

$$i(t) = 0$$

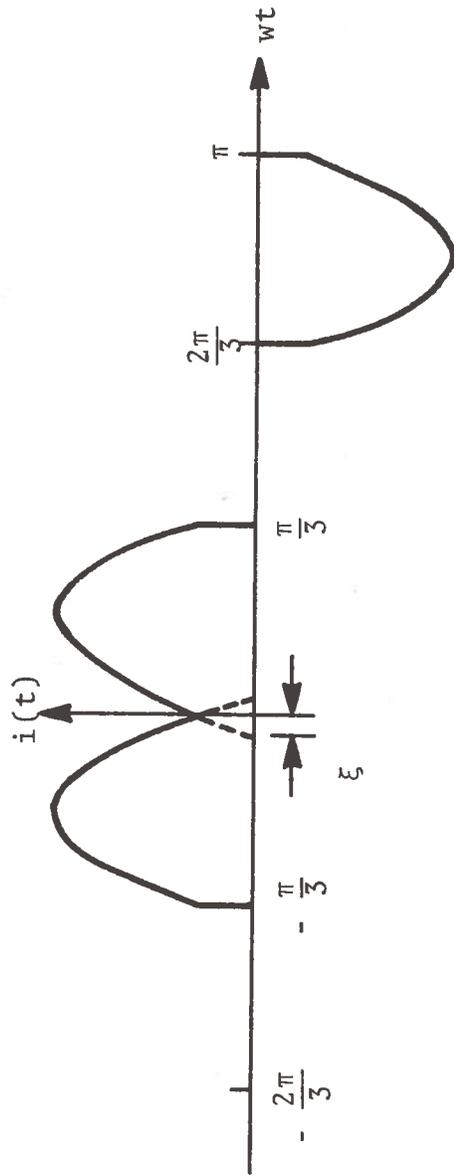
$$i(t) = -I_0 (\sin 3\omega t - \sin 3\phi)$$

$$\phi \leq \omega t \leq \frac{\pi}{3} - \phi$$

$$\frac{\pi}{3} - \phi \leq \omega t \leq \frac{2\pi}{3} + \phi$$

$$\frac{2\pi}{3} + \phi \leq \omega t \leq \pi - \phi$$

FIGURE 9-5. IDEALIZED CURRENT WAVEFORM FOR SIMULATING INPUT CURRENT TO NATURAL COMMUTATED INVERTER DRIVE



Continuous (Overlap) Conduction:  $\xi \leq 0$

$$i(t) = I_0 (\sin 3wt - \sin 3\xi) \quad -\frac{\pi}{3} \leq wt \leq \frac{\pi}{3}$$

$$i(t) = 0 \quad \frac{\pi}{3} \leq wt \leq \frac{2\pi}{3}$$

FIGURE 9-5. (CONT.) IDEALIZED CURRENT WAVEFORM FOR SIMULATING INPUT CURRENT TO NATURAL COMMUTATED INVERTER DRIVE

The input inverter current is described by equation 9-2. Assuming the zero-time reference as given in Figure 9-5, the harmonic current amplitudes for the discontinuous and continuous conduction modes are given by

Discontinuous mode:  $0 \leq \phi \leq \pi/6,$

$$I_n = \frac{I_o}{\pi} (1 - \cos n\pi) \left\{ \frac{\cos(3-n)\phi - \cos(3-n)(\frac{\pi}{3} - \phi)}{3 - n} \right. \quad (9-10)$$

$$+ \frac{\cos(3+n)\phi - \cos(3+n)(\frac{\pi}{3} - \phi)}{3 + n}$$

$$\left. - \frac{2 \sin 3\phi (\sin n(\frac{\pi}{3} - \phi) - \sin n\phi)}{n} \right\}$$

Continuous mode:  $-\frac{\pi}{6} \leq \xi \leq 0$

$$I_n = \frac{I_o}{\pi} (1 - \cos n\pi) \left\{ \frac{(1 - \cos(3-n)\pi/3)}{3 - n} + \frac{(1 - \cos(3+n)\pi/3)}{3 + n} \right.$$

$$\left. - \frac{2 \sin 3\xi \sin n\pi/3}{n} \right\} \quad (9-11)$$

$I_o$  denotes the peak amplitude of the current waveform at zero onset angle as shown in Figure 9-5. The parameter,  $\xi$ , adjusts the degree of overlap of the current waveform. See Appendix E, derivation of natural commutated inverter computer model.

The rms input line currents for the two modes are

Discontinuous mode:

$$I_{RMS} = \sqrt{\frac{2}{\pi}} I_o \sqrt{\frac{(\pi - 6 + \sin 6\phi)}{6} - \frac{2 \sin 6\phi}{3}} \quad (9-12)$$

$$\sqrt{+ (\frac{\pi}{3} - 2\phi) \sin^2 \frac{2}{3}\phi}$$

Continuous mode:

$$I_{RMS} = \sqrt{\frac{2}{\pi}} I_o \sqrt{\frac{\pi}{6} + \frac{4 \sin 3|\xi|}{3} + \frac{\pi}{3} \sin^2 3\xi}. \quad (9-13)$$

9.2.1.2 Natural Commutated Inverter Harmonic Input Spectra - The input current harmonic spectra are tabulated in Table 9-4 for three values of input line current. Included in the table are the measured harmonic current amplitudes and the corresponding amplitudes computed using equation 9-10.

The current parameter  $I_o$  was determined from equation 9-12 using the measured rms current at the onset of current overlap. The conduction delay angle,  $\phi$ , was similarly found by setting equation 9-11 equal to the measured line currents for discontinuous conduction. (Note that the measured current waveform was in the discontinuous mode.) The agreement in measured and computed input current harmonics is considered satisfactory.

TABLE 9-4. NATURAL COMMUTATED INVERTER CURRENT HARMONICS						
$I_o = 34.6 \text{ ARMS}$						
$I_{\text{rms}} = 5.7\text{A}$		$I_{\text{rms}} = 10.2\text{A}$		$I_{\text{rms}} = 15.8\text{A}$		
$\phi=13^\circ$		$\phi=8^\circ$		$\phi=3^\circ$		
n	Meas.	Comp.	Meas.	Comp.	Meas.	Comp.
	In	In	In	In	In	In
1	Odb	0	Odb	0	Odb	0
5	2.5	2.1	3	2.9	4	4.8
3	5	4.4	6	6.1	8	10.8
11	12	12.8	17	20.3	21	25.7
13	17	21.8	20	33.2	20	22.7
17	24	23.0	24	22.6	28	42.2
19	27	21.9	25	27.4	30	30.4

The data obtained from the study of the input harmonics spectra is used to determine the ratio of fundamental to total rms input line current to the inverter. Figure 9-6 shows the ratio as a function of the rms line current as determined from

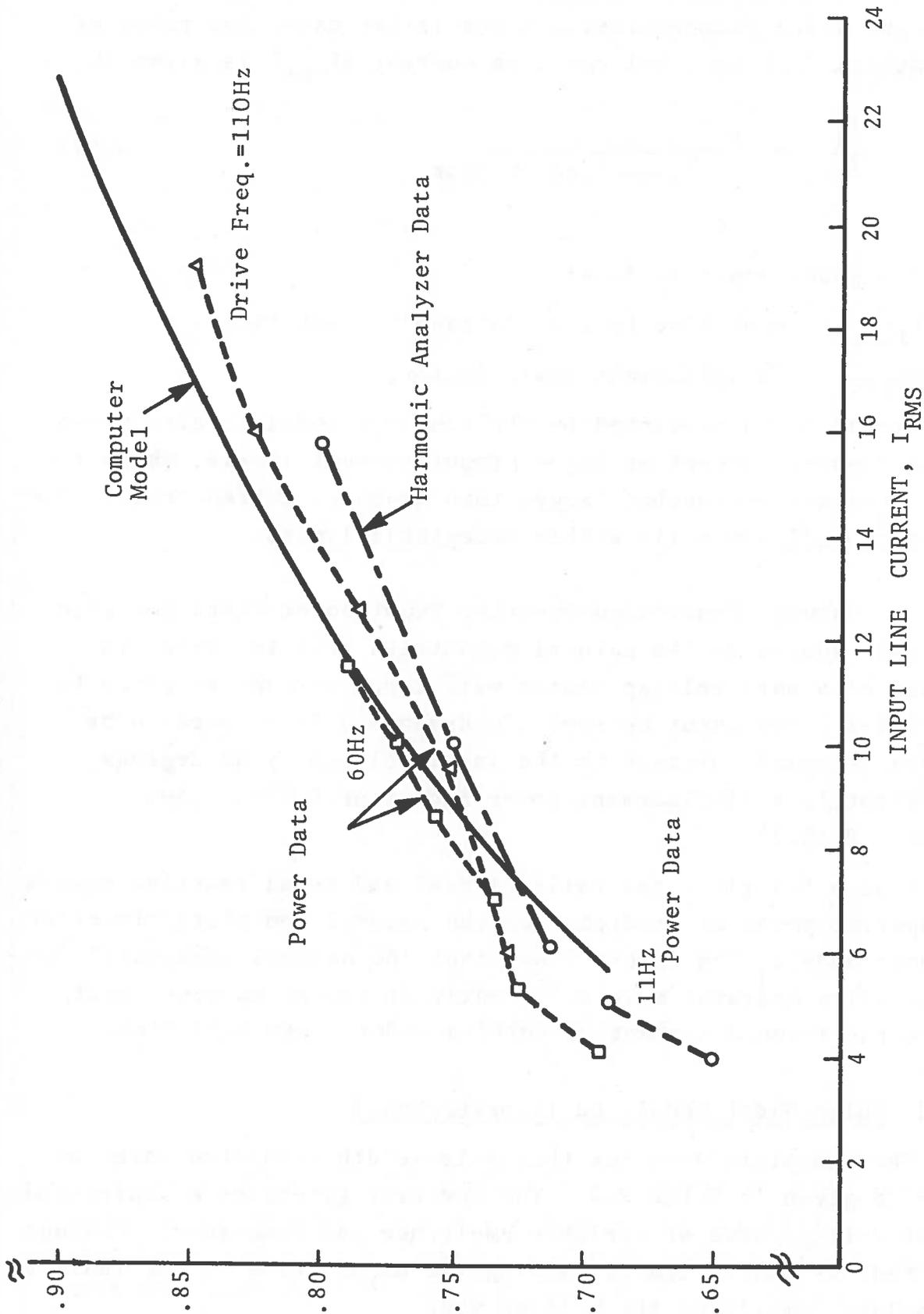


FIGURE 9-6. RELATIVE AMPLITUDE OF FUNDAMENTAL INPUT CURRENT TO NATURAL COMMUTATED INVERTER DRIVE

harmonic analyzer data and from measured input power data at different drive frequencies. In the latter case, the ratio of fundamental ( $I_1$ ) to total rms line current ( $I_{rms}$ ) is given by,

$$\frac{I_1}{I_{rms}} = \frac{P}{\sqrt{3} V_{line} I_{rms} \cdot pf_{DISP}}, \quad (9-14)$$

where

$P$  = power input to drive

$V_{line}$  = input line to line voltage to rectifier,

$PF_{DISP.}$  = displacement power factor.

The current ratio predicted by the computer model is also shown in the figure. Except at higher input current levels, where the model predicts a somewhat larger than measured current ratio, the computer predictions lie within acceptable limits.

9.2.1.3 Natural Commutated Inverter Input Power Characteristic - The power source to the natural commutated inverter drive is modeled as a hard voltage source with input current as given by Figure 9-5. The input current (fundamental) is assumed to be delayed in phase relative to the input voltage by 13 degrees, equivalent to a displacement power factor of 0.974. (See equation 9-14.)

Figure 9-7 gives the ratio of real and total reactive powers to apparent power as predicted by the natural commutated inverter computer model. The figure shows that the natural commutated inverter drive operates more efficiently at higher current levels due to the reduced current distortion under these conditions.

### 9.2.2 Pulse-Width Modulated Inverter Model

The nameplate data for this pulse-width modulated inverter drive is given in Table 9-5. The inverter generates a controlled output voltage wave of variable amplitude and frequency. Voltage amplitude is controlled by varying the on-off time of the individual pulses comprising the voltage wave.

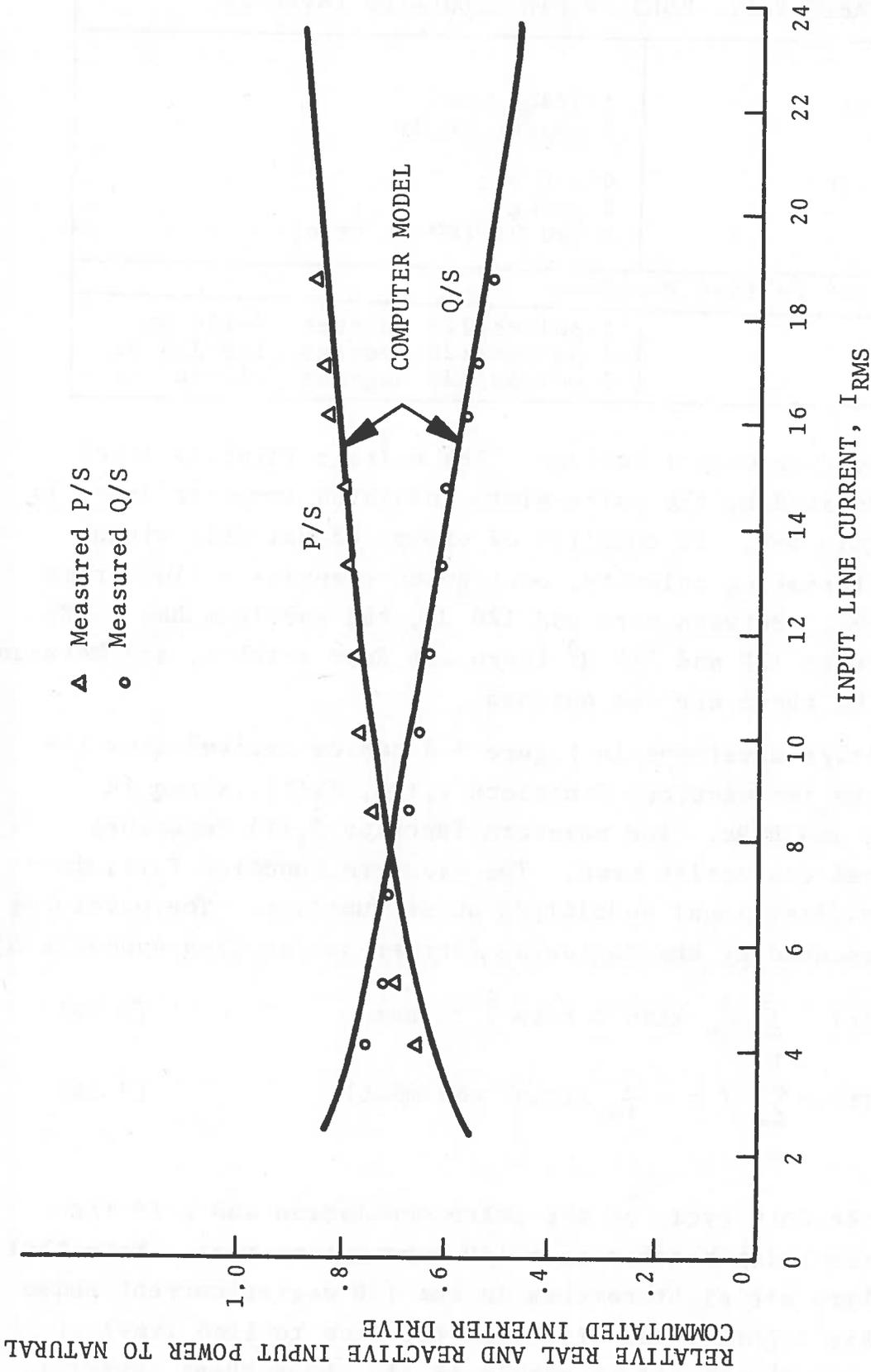


FIGURE 9-7. NATURAL COMMUTATED INVERTER DRIVE INPUT POWER CHARACTERISTICS

TABLE 9-5. PULSE-WIDTH MODULATED INVERTER	
Input:	230/460 vac 3 phase, 60 Hz
Output:	0-230 vac 3 phase 4-400 Hz (60 Hz base)
Output Voltage Waveform:	
	8 pulses/120 degrees 0-120 Hz
	4 pulses/120 degrees 120-213 Hz
	2 pulses/120 degrees 213-400 Hz

9.2.2.1 - Inverter Output Voltage - The voltage (line to line) waveform generated by the pulse-width modulated inverter drive is shown in Figure 9-8. It consists of groups of variable width pulses of alternating polarity, each group spanning a time frame of 120 degrees. Between zero and 120 Hz, the waveform has eight notches, between 120 and 213 Hz there are four notches, and between 213 and 400 Hz there are two notches.

The voltage waveforms in Figure 9-8 can be derived from the product of the two waveform functions  $f_1(t)$ ,  $f_2(t)$ , shown in Figures 9-9b and 9-9c. The waveform function  $f_1(t)$  describes the 120 degree conduction wave. The waveform function  $f_2(t)$  describes a unidirectional modulation pulse function. The waveforms can be represented by the following Fourier series (see Appendix E).

$$f_1(t) = \sum_n \frac{4}{\pi n} \sin n \frac{\pi}{2} \cos n \frac{\pi}{6} \cos n \omega t \quad (9-15)$$

$$f_2(t) = \sum_m \left( \tau + \frac{2}{\pi m} \sin m \pi \tau \cos 2m p \omega t \right) \quad (9-16)$$

where  $\tau$  is the duty cycle of the pulse modulation and  $p$  is the number of modulation notches in a 180 degree time span. Note that for  $p=12$ , there are eight notches in the 120 degree current pulse function. See Figure 9-9a. If  $V_o$  is the line to line (rms) voltage seen at the output terminals of the three-phase inverter,

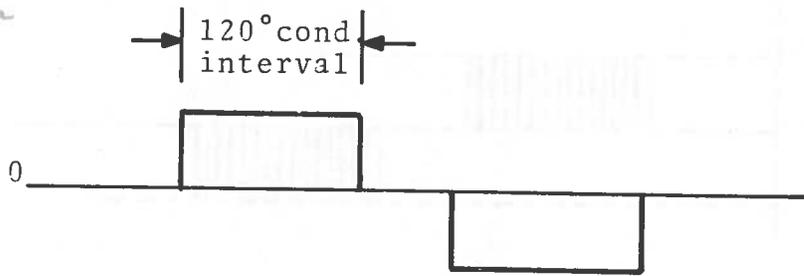


FIGURE 9-8a.

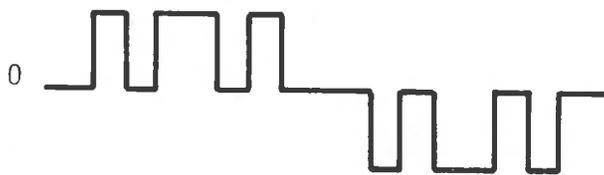


FIGURE 9-8b.  $213 \leq f \leq 400$

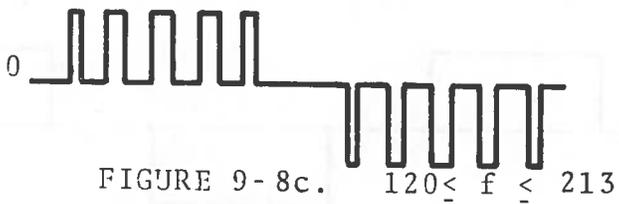


FIGURE 9-8c.  $120 \leq f \leq 213$

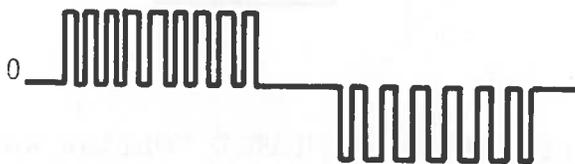


FIGURE 9-8d.  $0 \leq f \leq 120$

FIGURE 9-8. PULSE-WIDTH MODULATED VOLTAGE WAVEFORMS AT DIFFERENT OPERATING FREQUENCIES

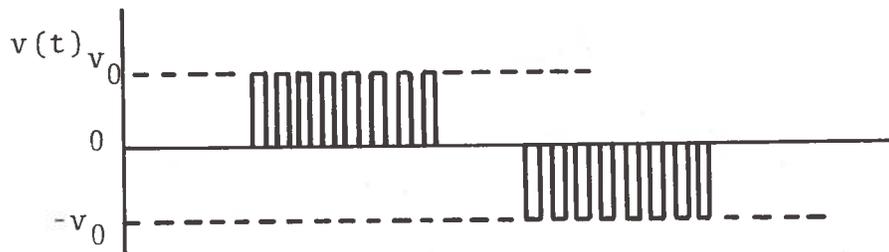


FIGURE 9-9a. PULSE-WIDTH MODULATED OUTPUT VOLTAGE WAVEFORM

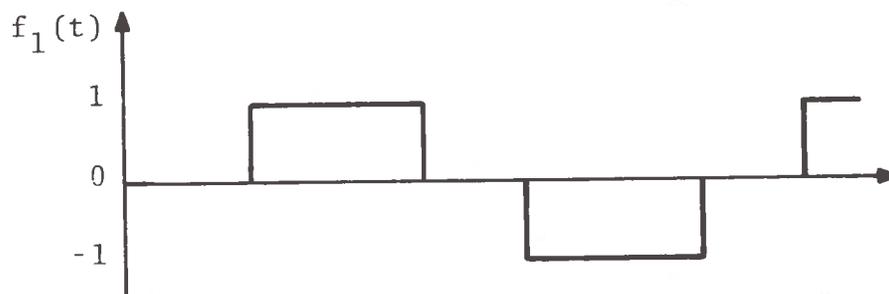


FIGURE 9-9b. PULSE-WIDTH MODULATED VOLTAGE WAVEFORM OMITTING PULSE MODULATION

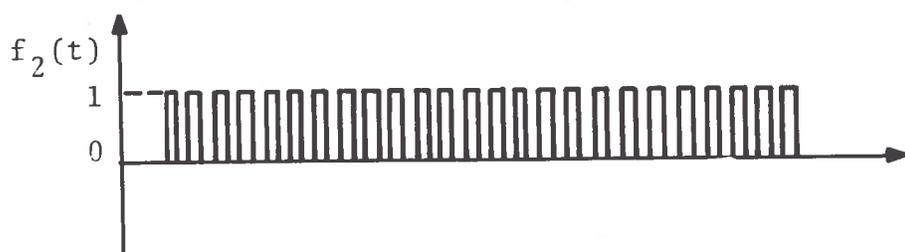


FIGURE 9-9c. PULSE-WIDTH MODULATED VOLTAGE WAVEFORM PULSE MODULATION

the instantaneous line to line inverter voltage is given by

$$v(t) = \frac{V_o}{\sqrt{\frac{2\tau}{3}}} f_1(t) \cdot f_2(t) \quad (9-17)$$

Expressing  $v(t)$  as the Fourier series,

$$v_\ell(t) = \sum_n V_n \cos n\omega t. \quad (9-18)$$

The harmonic coefficients,  $V_n$ , can be found by equating like terms in the Fourier expansion of equations 9-17 and 9-18. The final expression for the harmonic amplitude coefficient is

$$V_n = \frac{2\sqrt{6\tau}}{\pi} V_o \frac{\sin n\pi}{2} \frac{\cos n\pi}{6} \left\{ \frac{1}{n} - \sum_m \frac{2n}{(2mp)^2 - n^2} \frac{\sin m\pi\tau}{m\pi\tau} \right\}. \quad (9-19)$$

The pulse function  $p$  is restricted to multiples of three in the above expression. For most test runs,  $p$  is equal to 12, corresponding to eight notches in the voltage waveform (See Figure 9-9a).

The current harmonic amplitudes are found by dividing the respective voltage harmonic amplitudes by the motor impedance at the harmonic frequency.

9.2.2.2 Pulse-Width Modulated Inverter Harmonic Output Spectra - The pulse-width modulated computer model described in the previous section is next used to determine the inverter output current and voltage harmonics when the inverter is driving the ac induction machine. Tables 9-6 and 9-7 present the computed as well as measured harmonics (db relative to fundamental) for pulse duty width;  $\tau = 0.25$ , and motor slips of .055 and 0.2. The pulse number,  $p$ , is 12 corresponding to eight notches in a 120 degree conduction period.

The integrated harmonic quantities are summarized at the bottom of the table. The fundamental voltage harmonics amplitude

TABLE 9-6. PULSE-WIDTH MODULATION CALCULATIONS

TAU= 0.25		VP=104.0		P= 12		Computed		Measured	
FREQ= 60.		SLIP= 0.055		RPM= 851.		UN	IN	VN	IN
N	VN V	IN A	T LB-FT	P KW	Q KVAR	DB	DB	DB	DB
1.	49.7895	15.0254	6.1425	0.9241	0.9083	0.00	0.00	0.0	0.0
5.	10.6294	1.4967	-0.0269	0.0048	0.0271	13.41	20.03	14	18
7.	8.1210	0.8377	0.0082	0.0015	0.0117	15.75	25.07	17	26
11.	6.4144	0.4274	-0.0021	0.0004	0.0047	17.80	30.92	24	36
13.	6.3589	0.3595	0.0015	0.0003	0.0040	17.88	32.42	28	42
17.	7.7754	0.3371	-0.0013	0.0002	0.0045	16.13	32.98	22	36
19.	10.0009	0.3882	0.0017	0.0003	0.0067	13.94	31.75	16	32
23.	45.2258	1.4517	-0.0231	0.0042	0.1136	0.84	20.30	0	16
25.	-44.4285	-1.3124	0.0188	0.0035	0.1009	0.99	21.18	0	16
29.	-9.1480	-0.2331	-0.0006	0.0001	0.0037	14.72	36.19	12	30
31.	-6.8612	-0.1635	0.0003	0.0001	0.0019	17.21	39.26	14	34
35.	-5.2170	-0.1102	-0.0001	0.0000	0.0010	19.59	42.70	20	40
37.	-5.0710	-0.1013	0.0001	0.0000	0.0009	19.84	43.42	22	43
41.	-5.9507	-0.1073	-0.0001	0.0000	0.0011	18.45	42.92	29	48
43.	-7.4905	-0.1288	0.0002	0.0000	0.0017	16.45	41.34	22	42
47.	-32.3643	-0.5091	-0.0028	0.0005	0.0285	3.74	29.40	2	24
49.	31.0345	0.4683	0.0024	0.0004	0.0252	4.11	30.13	0	22
53.	6.0689	0.0847	-0.0001	0.0000	0.0009	18.28	44.98	12	34
55.	4.4279	0.0595	0.0000	0.0000	0.0005	21.02	48.04	14	37
59.	3.1723	0.0398	-0.0000	0.0000	0.0002	23.92	51.55	18	40
61.	2.9856	0.0362	0.0000	0.0000	0.0002	24.44	52.36	19	42
65.	3.2646	0.0371	-0.0000	0.0000	0.0002	23.67	52.14	22	46
67.	3.9519	0.0436	0.0000	0.0000	0.0003	22.01	50.74	26	52
71.	15.6361	0.1629	-0.0003	0.0001	0.0044	10.06	39.30	8	32
73.	-14.2572	-0.1444	0.0002	0.0000	0.0034	10.86	40.34	3	
77.	-2.4759	-0.0238	-0.0000	0.0000	0.0001	26.07	56.01		
79.	-1.6815	-0.0157	0.0000	0.0000	0.0000	29.43	59.60		
83.	-1.0046	-0.0090	-0.0000	0.0000	0.0000	33.90	64.50		
85.	-0.8389	-0.0073	0.0000	0.0000	0.0000	35.47	66.27		
89.	-0.6480	-0.0054	-0.0000	0.0000	0.0000	37.71	68.91		

INTEGRATED OUTPUT QUANTITIES  
 VRMS= 98.82100 VOLTS,RMS (LINE-LINE)  
 V1(FUND)= 49.7895VOLTS  
 IRMS= 15.29010 AMPS,RMS  
 I1(FUND)= 15.0254AMPS  
 TORQUE= 6.11863 LB-FT  
 MOTOR EFF.= 0.78583  
 REAL POWER= 0.94059 KW  
 REACTIVE POWER(TOTAL)= 2.58867KVAR QX/S= 0.93988  
 DISPLACEMENT POWER(FUND)= 0.90835KVAR  
 DISPLACEMENT POWER(TOTAL)= 1.25609 KVAR  
 APPARENT POWER = 2.75426 KVA  
 POWER FACTOR(FUND)= 0.71315  
 POWER FACTOR(SYSTEM)= 0.34151

TABLE 9-7. PULSE-WIDTH MODULATION CALCULATIONS

TAU= 0.25 UP=104.0 P= 12  
 FREQ= 60. SLIP= 0.200 RPM= 720.

N	VN V	IN A	T LB-FT	P KW	Q KVAR	Computed		Measured	
						VN DB	IN DB	VN	IN
1.	49.7895	23.1986	4.7534	0.9386	1.7667	0.00	0.00	0	0
5.	10.6294	1.5007	-0.0262	0.0047	0.0272	13.41	23.78	14	24
7.	8.1210	0.8385	0.0081	0.0015	0.0117	15.75	28.84	17	32
11.	6.4144	0.4275	-0.0020	0.0004	0.0047	17.80	34.69	24	42
13.	6.3589	0.3595	0.0014	0.0003	0.0040	17.88	36.19	28	46
17.	7.7754	0.3371	-0.0012	0.0002	0.0045	16.13	36.75	22	42
19.	10.0009	0.3882	0.0016	0.0003	0.0067	13.94	35.53	16	37
23.	45.2258	1.4517	-0.0229	0.0042	0.1136	0.84	24.07	0	22
25.	-44.4285	-1.3124	0.0187	0.0034	0.1009	0.99	24.95	0	22
29.	-9.1480	-0.2331	-0.0006	0.0001	0.0037	14.72	39.96	12	35
31.	-6.8612	-0.1635	0.0003	0.0001	0.0019	17.21	43.04	14	39
35.	-5.2170	-0.1102	-0.0001	0.0000	0.0010	19.59	46.47	20	45
37.	-5.0710	-0.1013	0.0001	0.0000	0.0009	19.84	47.20	22	48
41.	-5.9507	-0.1073	-0.0001	0.0000	0.0011	18.45	46.70	29	53
43.	-7.4905	-0.1288	0.0002	0.0000	0.0017	16.45	45.11	22	48
47.	-32.3643	-0.5091	-0.0028	0.0005	0.0285	3.74	33.17	2	28
49.	31.0345	0.4683	0.0023	0.0004	0.0252	4.11	33.90	0	27
53.	6.0689	0.0847	-0.0001	0.0000	0.0009	18.28	48.75	12	40
55.	4.4279	0.0595	0.0000	0.0000	0.0005	21.02	51.81	14	43
59.	3.1723	0.0398	-0.0000	0.0000	0.0002	23.92	55.32	18	46
61.	2.9856	0.0362	0.0000	0.0000	0.0002	24.44	56.14	19	48
65.	3.2646	0.0371	-0.0000	0.0000	0.0002	23.67	55.91	22	52
67.	3.9519	0.0436	0.0000	0.0000	0.0003	22.01	54.52	26	57
71.	15.6361	0.1629	-0.0003	0.0001	0.0044	10.06	43.07	8	38
73.	-14.2572	-0.1444	0.0002	0.0000	0.0036	10.86	44.12	4	34
77.	-2.4759	-0.0238	-0.0000	0.0000	0.0001	26.07	59.78		
79.	-1.6815	-0.0157	0.0000	0.0000	0.0000	29.43	63.37		
83.	-1.0046	-0.0090	-0.0000	0.0000	0.0000	33.90	68.27		
85.	-0.8389	-0.0073	0.0000	0.0000	0.0000	35.47	70.04		
89.	-0.6480	-0.0054	-0.0000	0.0000	0.0000	37.71	72.69		

INTEGRATED OUTPUT QUANTITIES  
 VRMS= 98.82100 VOLTS,RMS (LINE-LINE)  
 V1(FUND)= 49.7895VOLTS  
 IRMS= 23.37113 AMPS,RMS  
 I1(FUND)= 23.1986AMPS  
 TORQUE= 4.73003 LB-FT  
 MOTOR EFF.= 0.50652  
 REAL POWER= 0.95500 KW  
 REACTIVE POWER(TOTAL)= 4.10017KVAR QX/S= 0.97393  
 DISPLACEMENT POWER(FUND)= 1.76674KVAR  
 DISPLACEMENT POWER(TOTAL)= 2.11460 KVAR  
 APPARENT POWER = 4.20992 KVA  
 POWER FACTOR(FUND)= 0.46917  
 POWER FACTOR(SYSTEM)= 0.22685

(rms) are only about one-half the integrated total rms voltage. Thus the inverter must develop considerably higher output voltages to produce the required fundamental output voltage.

The computed real and reactive power harmonics,  $P_n$  and  $Q_n$ , are also shown in Tables 9-6 and 9-7 for the two values of motor slip. While the higher harmonics contribute little to the real power (two to three percent), they contribute substantially to the total reactive power (20 to 25 percent) delivered to the motor.

The voltage and current harmonic spectra are shown plotted in Figures 9-10 and 9-11 for the two motor slips. The dashed lines showing the computed harmonic amplitudes agree reasonably well with the measured harmonic data. Of particular interest is the good correlation in the amplitude of the dominant voltage harmonics which occur near harmonics multiples of 24. (Note that pulse modulation function has 24 pulses per cycle.) Also of interest is the relatively small amplitude of the higher order current harmonics.

### 9.2.2.3 Pulse-Width Modulated Inverter Output Power Characteristics

The computer model is next applied to predict the inverter output characteristics when used to drive the induction machine.

Figure 9-12 shows the inverter output power for motor slips in the range of -1 to +2, (-900 rpm to 1800 rpm) and constant drive voltage of 117 vrms (60 Hz). The measured inverter output power is indicated in the Figure. The triangular points give the displacement power measured with the two wattmeter method and include the higher harmonic reactive contributions. The dashed curve shows the displacement reactive power predicted by the computer model using only the fundamental voltage and current. The agreement with data is fair in this case.

### 9.2.2.4 Pulse-Width Modulated Inverter Drive-Motor Characteristics

The motor-drive characteristics predicted by the pulse-width modulated induction motor model are shown in Figure 9-13 along with the measured torque and current characteristics. The inverter drive model predicts somewhat higher torques and motor input

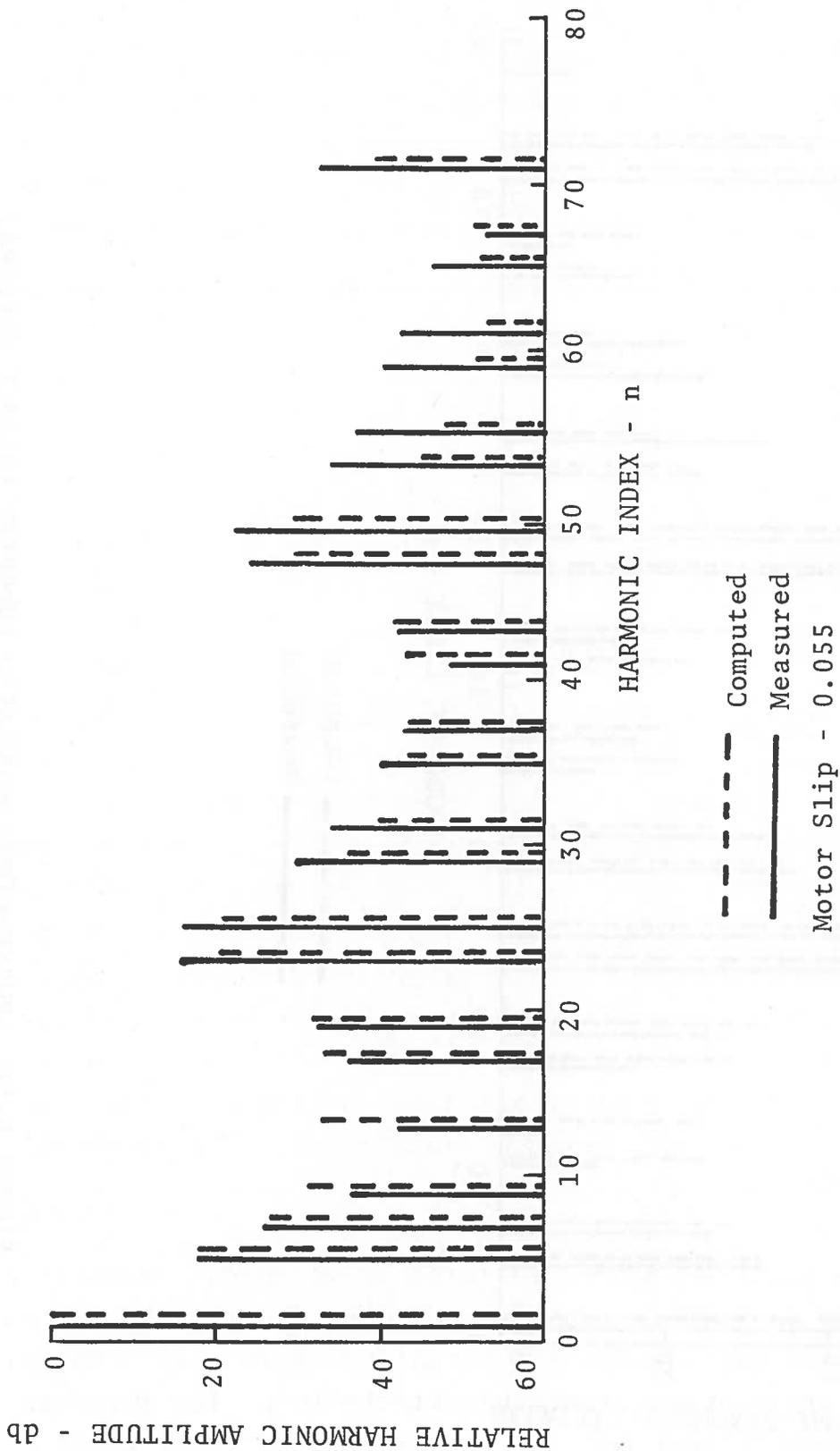


FIGURE 9-10. PULSE-WIDTH MODULATED INVERTER CURRENT SPECTRA.  
 DUTY CYCLE = 0.25, PULSE MODULATION INDEX,  $p = 12$

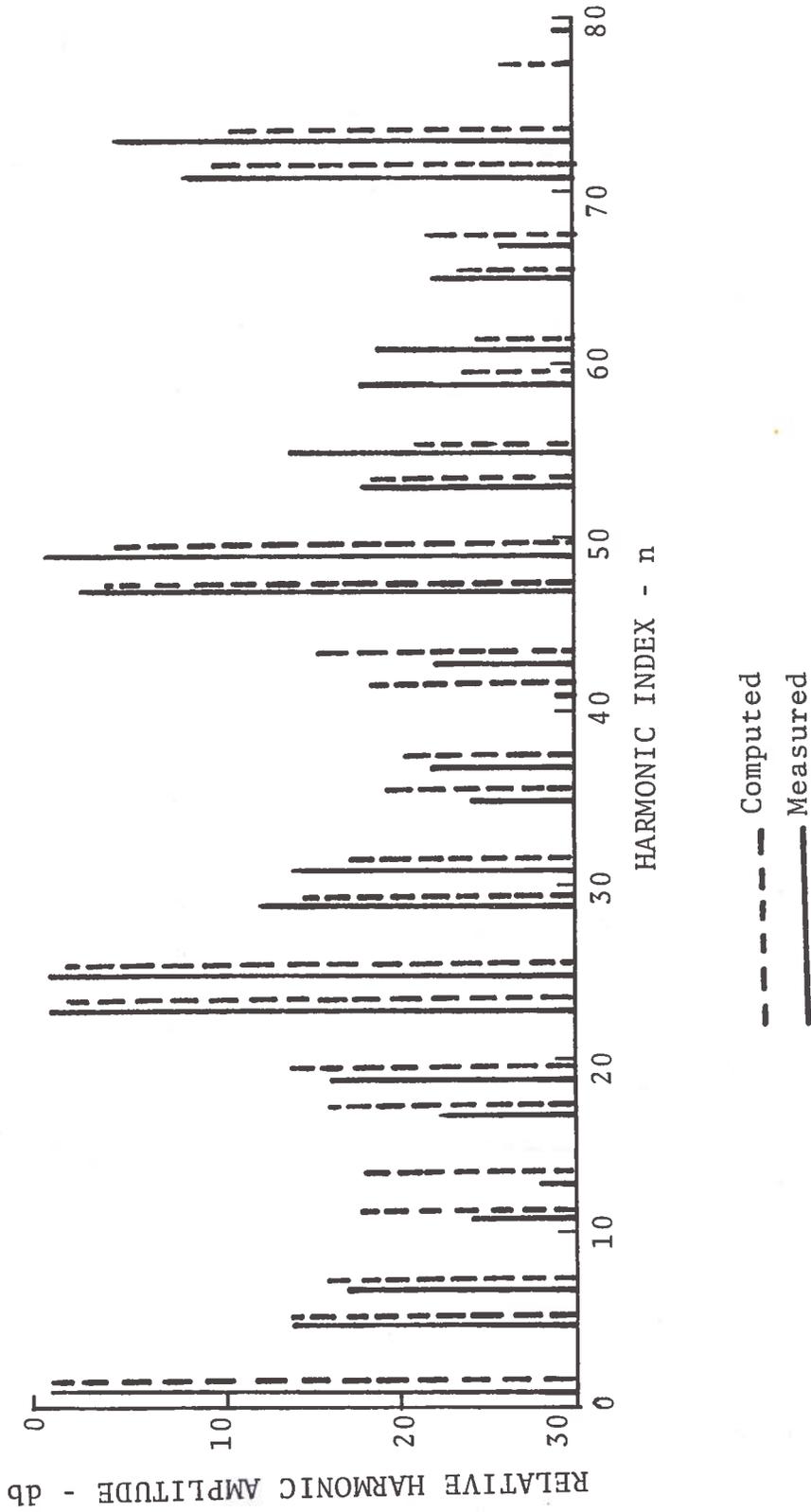


FIGURE 9-11. PULSE-WIDTH MODULATED INVERTER VOLTAGE SPECTRA.  
 DUTY CYCLE = 0.25, PULSE MODULATION INDEX,  $p = 12$

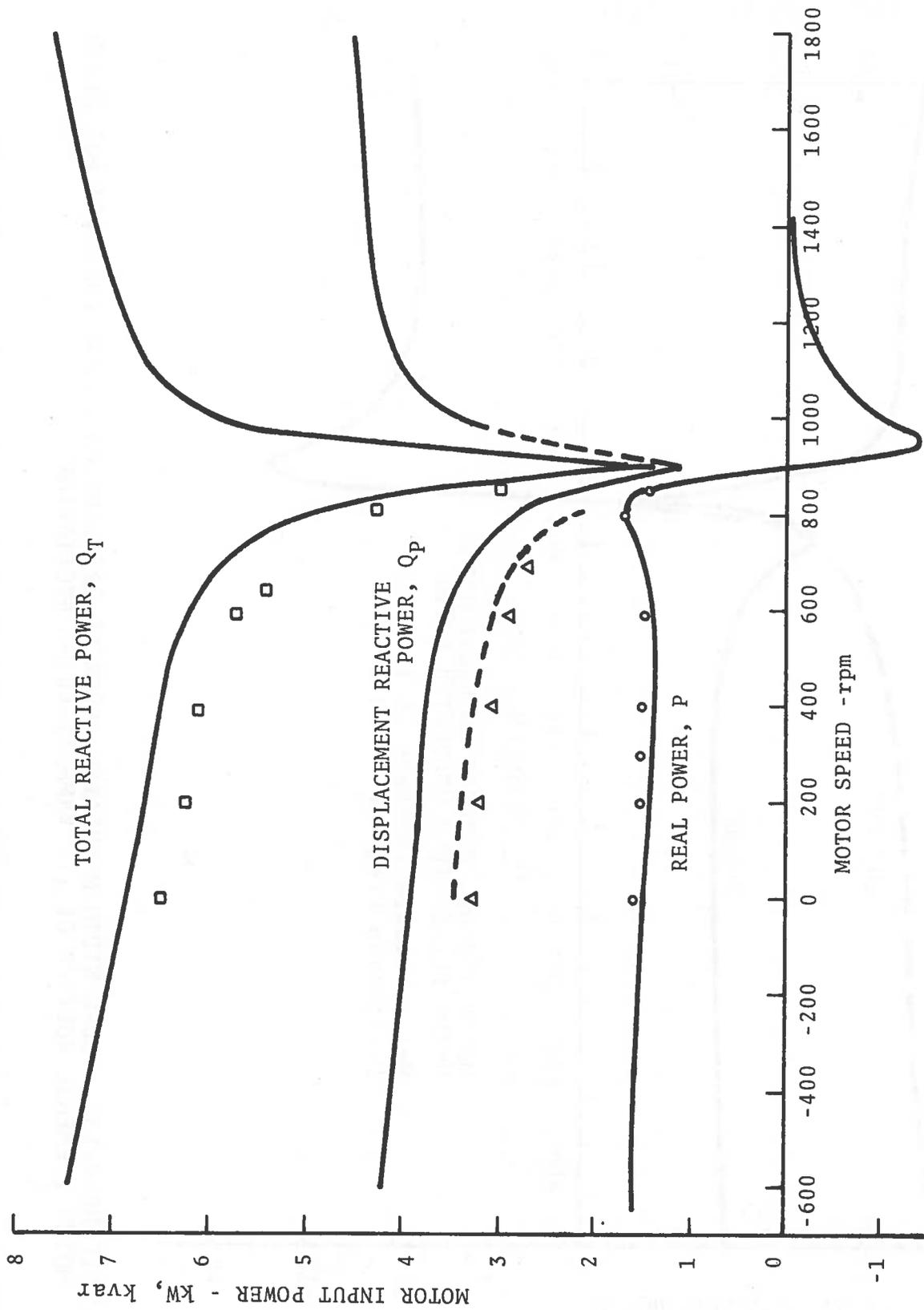
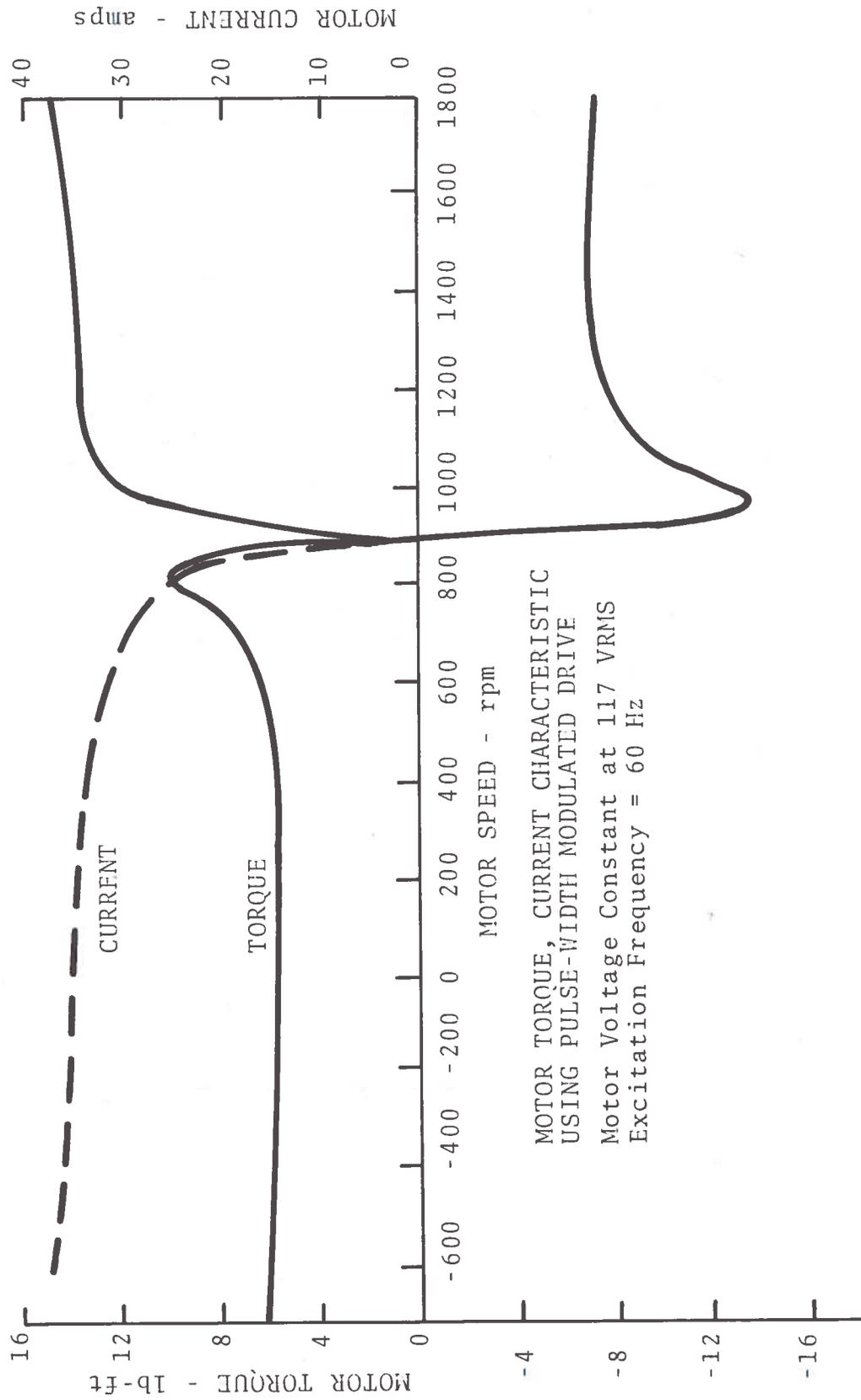


FIGURE 9-12. PULSE-WIDTH MODULATED INVERTER OUTPUT POWER CHARACTERISTICS. OUTPUT VOLTAGE FIXED AT 117 VRMS, 60 HZ

PULSE-WIDTH MODULATED



MOTOR TORQUE, CURRENT CHARACTERISTIC  
USING PULSE-WIDTH MODULATED DRIVE

Motor Voltage Constant at 117 VRMS  
Excitation Frequency = 60 Hz

FIGURE 9-13. PULSE-WIDTH MODULATED INVERTER DRIVE-MOTOR CHARACTERISTICS FOR FIXED MOTOR TERMINAL VOLTAGE OF 117 VRMS AT 60 Hz EXCITATION

currents than measured. Figure 9-14 shows the computed and measured motor torque and current in the region of zero slip. The computed characteristics assume a fixed motor voltage of 207 vrms (line to line) while the data points reflect motor voltages that vary from 207 vrms at zero slip to 202 vrms at a slip of 0.02.

### 9.2.3 Constant Current Inverter Model

The nameplate data for the constant current inverter drive is given in Table 9-8. The drive comprised a phase delayed rectifier, dc link, and ac inverter. Output current amplitude and frequency are controlled through closed-loop feedback within the drive.

TABLE 9-8. CONSTANT CURRENT INVERTER DRIVE	
Input:	460 vac 3 phase, 60 Hz
Output:	0-460 vac 3 phase 0-120 Hz

9.2.3.1 Inverter Output Current - Figure 9-15 shows the idealized current waveform used to model the constant current inverter drive. Conduction takes place over 120 electrical degree intervals. The commutation interval is indicated by the linear slope of current at the leading and trailing edges of the current pulse. The commutation angle, as measured from the beginning of commutation to zero current, is 12 degrees.

The constant current inverter current waveform in Figure 9-15a can be described by the superposition of two functions  $f_1(t)$  and  $f_2(t)$  shown in Figure 9-15b and 9-15c respectively.

$$i(t) = f_1(t) + f_2(t) = \sum_n I_n \cos n\omega t \quad (9-20)$$

If the zero-time reference is chosen at the midpoint of

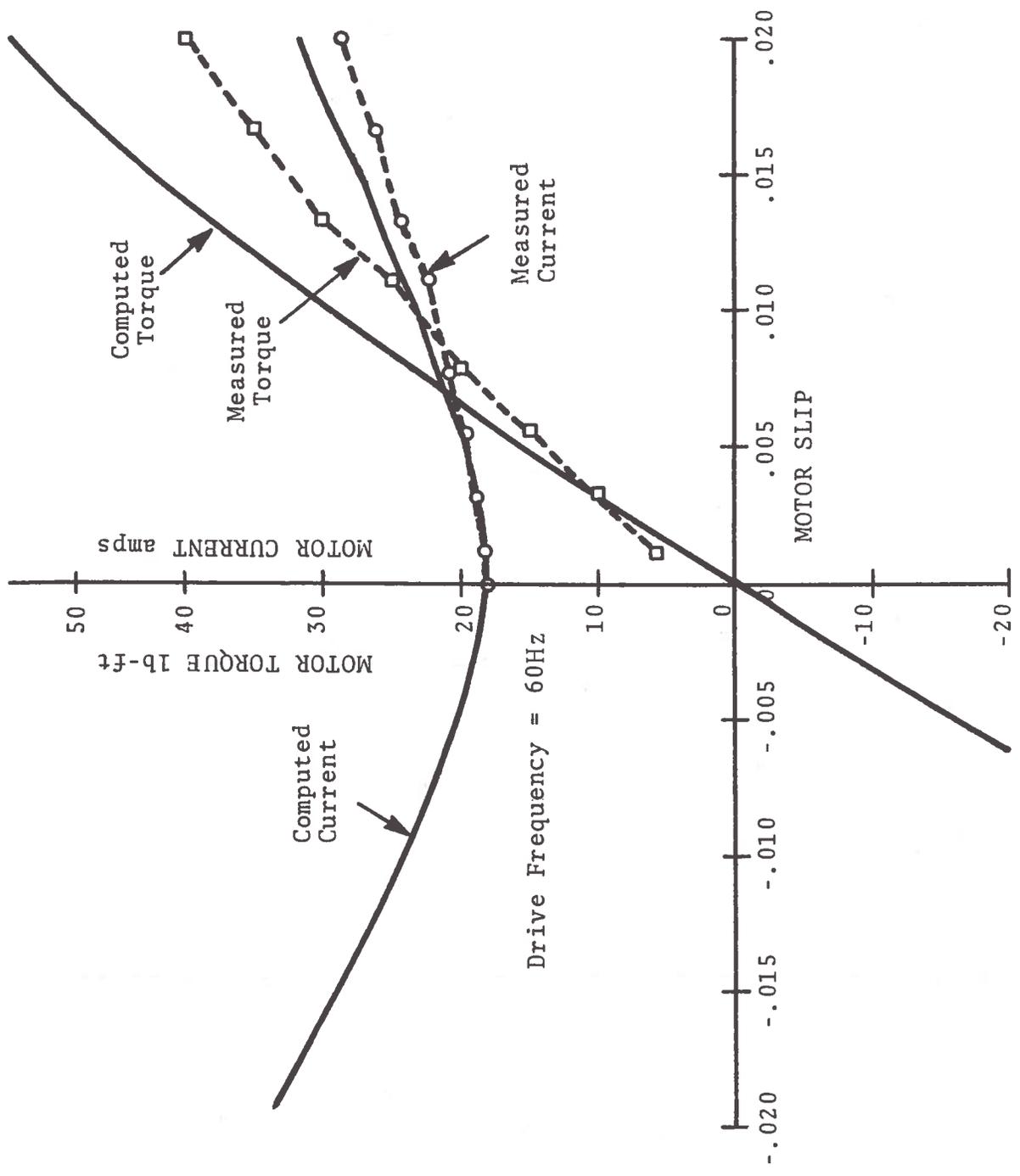


FIGURE 9-14. MOTOR TORQUE, CURRENT CHARACTERISTIC NEAR ZERO SLIP USING PULSE-WIDTH MODULATED DRIVE, 60Hz EXCITATION

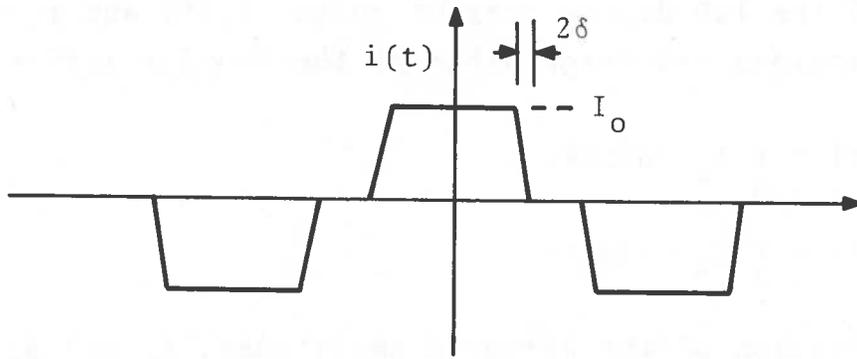


FIGURE 9-15a. CONSTANT CURRENT INVERTER OUTPUT CURRENT WAVEFORM

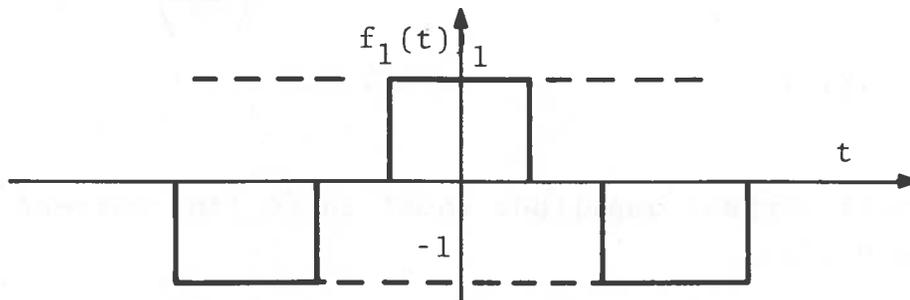


FIGURE 9-15b. CONSTANT CURRENT INVERTER WAVEFORM OMITTING COMMUTATION

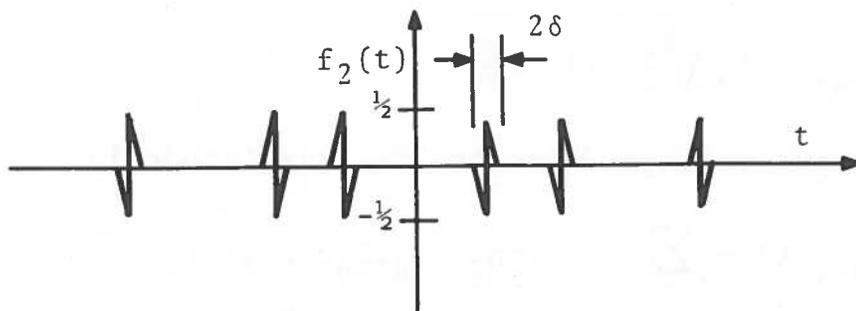


FIGURE 9-15c. CONSTANT CURRENT INVERTER WAVEFORM:  
COMMUTATION CONTRIBUTION

conduction of the 120 degree current pulse,  $f_1(t)$  and  $f_2(t)$  are symmetric functions and describable by the Fourier series,

$$f_1(t) = \sum_n A_n \cos n\omega t, \quad (9-21)$$

$$f_2(t) = \sum_m B_m \cos m\omega t. \quad (9-22)$$

The derivation of the harmonic amplitudes,  $A_n$  and  $B_m$ , is given in Appendix E. It consists of describing the commutation current by a saw-tooth function, and collecting terms common to a given harmonic component. The final result for the current harmonic amplitude is

$$I_n = \frac{4}{\pi} I_o \frac{\sin n\pi}{2} \frac{\cos n\pi}{6} \left[ \frac{1}{n} + \frac{2\sin n\delta}{\delta} \sum_m \frac{\cos m\pi}{\left(\frac{\pi m}{\delta}\right)^2 - n^2} \right] \quad (9-23)$$

$$n = 1, 5, 7, 11, \dots$$

$$m = 1, 2, 3, \dots$$

where

$I_o$  = peak current amplitude equal to dc link current  $I_d$   
(see Figure 9-15a).

$\delta$  = commutation half-angle in radians =  $\pi/30$ .

The amplitude,  $I_n$ , decreases as  $1/n$  initially but exhibits peaks at higher orders which are centered about harmonic multiples of  $\pi/\delta$ .

The current amplitude,  $I_o$ , is related to the rms line current according to

$$I_{rms} = I_o \sqrt{\frac{2}{3} \left(1 - \frac{\delta}{\pi}\right)}. \quad (9-24)$$

The output voltage (line to line) amplitude is

$$v_{1-1}(t) = \sum_n 2 \frac{\cos n\pi}{3} I_n \cdot Z_n \cdot \cos n\omega t, \quad (9-25)$$

$$n = 1, 5, 7, \dots$$

where

$z_n$  = phase impedance of motor winding.

9.2.3.2 Constant Current Inverter Harmonic Spectra - The computer model is used to predict the constant current inverter output current and voltage spectra for typical drive conditions. Table 9-9 shows the computed current, voltage harmonics for the test motor operating at a slip of .0083 (893 rpm) with 60 Hz drive frequency. The corresponding harmonics measured with a spectrum analyzer are shown in the two columns at the right. The good correlation between measured and computed harmonic amplitudes demonstrates that the computer model accurately describes constant current inverter drive performance.

The computer model is next used to examine the effect of motor speed on motor input voltage waveform distortion. Table 9-10 gives the computed harmonics for a motor slip of 0.2 (720 rpm). The large harmonic content in the input voltage waveform (to the motor) shows that the constant current inverter drive generates large distortion power when used with inductive loads. This is discussed in more detail in the next section.

Figures 9-16a, 9-16b, and 9-17 present the voltage and current harmonics as a function of harmonic order. The dashed lines give the computed harmonic amplitudes as predicted by the constant current inverter model.

9.2.3.3 Constant Current Inverter Motor-Drive Characteristics - The motor-drive characteristics computed for a fixed excitation frequency of 60 Hz are shown in Figure 9-18. A comparison of these motor-drive characteristics with the pulse-width modulated motor-drive characteristics shows that the constant current inverter drive produces larger peak motor torques than does the pulse-width modulated drive for the same input voltage to the motor.

The motor-drive characteristics near zero motor slip were next considered. Figure 9-19 gives the measured and computed torque and current characteristics in the region of small motor slips at 60 Hz.

TABLE 9-9. CONSTANT CURRENT INVERTER CALCULATIONS

FREQ= 60. SLIP= 0.0083 RPM= 893. FLAG= 0.00										
N	VN V	IN A	T LB-FT	P KW	Q KVAR	VN DB	IN DB	VN DB	IN DB	
1.	105.6312	11.3288	7.4302	1.0288	1.7994	0.00	0.00	0	0	
5.	15.8643	-2.1657	-0.0726	0.0122	0.0583	16.47	14.37	14	14	
7.	-14.5626	1.4775	0.0339	0.0057	0.0368	17.21	17.69	16	16	
11.	-12.3090	-0.8147	-0.0101	0.0017	0.0173	18.67	22.86	21	22	
13.	11.0805	0.6235	0.0059	0.0010	0.0119	19.58	25.19	23	24	
17.	8.3955	-0.3630	-0.0020	0.0003	0.0053	21.99	29.89	27	27	
19.	-6.9879	0.2707	0.0011	0.0002	0.0033	23.59	32.43	31	32	
23.	-4.1944	-0.1344	-0.0003	0.0000	0.0010	28.02	38.51	34	36	
25.	2.8715	0.0847	0.0001	0.0000	0.0004	31.31	42.52	38	40	
29.	0.5123	-0.0130	-0.0000	0.0000	0.0000	46.29	58.78	44	50	
31.	0.4768	-0.0114	0.0000	0.0000	0.0000	46.91	59.98	48	52	
35.	2.0000	0.0422	-0.0000	0.0000	0.0001	34.46	48.58	50	47	
37.	-2.5161	-0.0502	0.0000	0.0000	0.0002	32.46	47.06	52	47	
41.	-3.0589	0.0551	-0.0000	0.0000	0.0003	30.76	46.26	46	44	
43.	3.0995	-0.0533	0.0000	0.0000	0.0003	30.65	46.55	48	46	
47.	2.7899	0.0439	-0.0000	0.0000	0.0002	31.56	48.24	46	46	
49.	-2.4766	-0.0374	0.0000	0.0000	0.0002	32.60	49.64	48	48	
53.	-1.6445	0.0229	-0.0000	0.0000	0.0001	36.16	53.87	48	50	
55.	1.1711	-0.0157	0.0000	0.0000	0.0000	39.10	57.14	52	50	
59.	0.2238	0.0028	-0.0000	0.0000	0.0000	53.48	72.13			
61.	0.2137	0.0026	0.0000	0.0000	0.0000	53.88	72.82			
65.	0.9271	-0.0105	-0.0000	0.0000	0.0000	41.13	60.62			
67.	-1.1849	0.0131	0.0000	0.0000	0.0000	39.00	58.75			
71.	-1.4722	-0.0153	-0.0000	0.0000	0.0000	37.12	57.37			
73.	1.5036	0.0152	0.0000	0.0000	0.0000	36.93	57.43			
77.	1.3630	-0.0131	-0.0000	0.0000	0.0000	37.79	58.74			
79.	-1.2109	0.0113	0.0000	0.0000	0.0000	38.81	60.00			
83.	-0.7976	-0.0071	-0.0000	0.0000	0.0000	42.44	64.05			
85.	0.5643	0.0049	0.0000	0.0000	0.0000	45.45	67.26			
89.	0.1033	-0.0009	-0.0000	0.0000	0.0000	60.20	82.41			

INTEGRATED OUTPUT QUANTITIES  
 VRMS= 110.00000 VOLTS,RMS (LINE-LINE)  
 V1(FUND)= 105.63124 VOLTS  
 IRMS= 11.68386 AMPS,RMS  
 I1(FUND)= 11.32877 AMPS  
 TORQUE= 7.38629 LB-FT  
 MOTOR EFF.= 0.89186  
 REAL POWER= 1.04993 KW  
 REACTIVE POWER= 1.96292KVAR  
 DISPLACEMENT POWER= 1.93522 KVAR  
 APPARENT POWER = 2.22608 KVA  
 POWER FACTOR(FUND)= 0.49634  
 POWER FACTOR(SYSTEM)= 0.47165

TABLE 9-10. CONSTANT CURRENT INVERTER CALCULATIONS

FREQ= 60.		SLIP= 0.200		RPM= 720.		FLAG= 0.00	
N	UN V	IN A	T LB-FT	P KW	Q KVAR	UN DB	IN DB
1.	159.7887	74.4507	48.9576	9.6674	18.1965	0.00	0.00
5.	100.8119	-14.2328	-2.3564	0.4258	2.4485	4.00	14.37
7.	-94.0402	9.7096	1.0817	0.1963	1.5693	4.60	17.69
11.	-80.3390	-5.3542	-0.3212	0.0587	0.7427	5.97	22.86
13.	72.4680	4.0974	0.1866	0.0342	0.5132	6.87	25.19
17.	55.0222	-2.3855	-0.0625	0.0115	0.2271	9.26	29.89
19.	-45.8231	1.7789	0.0346	0.0064	0.1410	10.85	32.43
23.	-27.5243	-0.8835	-0.0085	0.0016	0.0421	15.28	38.51
25.	18.8479	0.5568	0.0034	0.0006	0.0182	18.57	42.52
29.	3.3637	-0.0857	-0.0001	0.0000	0.0005	33.53	58.78
31.	3.1308	-0.0746	0.0001	0.0000	0.0004	34.16	59.98
35.	13.1353	0.2774	-0.0008	0.0002	0.0063	21.70	48.58
37.	-16.5266	-0.3302	0.0012	0.0002	0.0094	19.71	47.06
41.	-20.0934	0.3623	-0.0014	0.0003	0.0126	18.01	46.26
43.	20.3609	-0.3501	0.0013	0.0002	0.0123	17.89	46.55
47.	18.3287	0.2883	-0.0009	0.0002	0.0092	18.81	48.24
49.	-16.2708	-0.2455	0.0006	0.0001	0.0069	19.84	49.64
53.	-10.8047	0.1508	-0.0002	0.0000	0.0028	23.40	53.87
55.	7.6942	-0.1034	0.0001	0.0000	0.0014	26.35	57.14
59.	1.4706	0.0184	-0.0000	0.0000	0.0000	40.72	72.13
61.	1.4041	0.0170	0.0000	0.0000	0.0000	41.12	72.82
65.	6.0917	-0.0693	-0.0001	0.0000	0.0007	28.38	60.62
67.	-7.7860	0.0859	0.0001	0.0000	0.0012	26.24	58.75
71.	-9.6740	-0.1008	-0.0001	0.0000	0.0017	24.36	57.37
73.	9.8802	0.1001	0.0001	0.0000	0.0017	24.18	57.43
77.	8.9563	-0.0860	-0.0001	0.0000	0.0013	25.03	58.74
79.	-7.9571	0.0745	0.0001	0.0000	0.0010	26.06	60.00
83.	-5.2410	-0.0467	-0.0000	0.0000	0.0004	29.68	64.05
85.	3.7080	0.0323	0.0000	0.0000	0.0002	32.69	67.26
89.	0.6786	-0.0056	-0.0000	0.0000	0.0000	47.44	82.41

INTEGRATED OUTPUT QUANTITIES  
 VRMS= 255.00000 VOLTS,RMS (LINE-LINE)  
 V1(FUND)= 159.78868 VOLTS  
 IRMS= 76.78439 AMPS,RMS  
 I1(FUND)= 74.45075 AMPS  
 TORQUE= 47.51520 LB-FT  
 MOTOR EFF.= 0.46707  
 REAL POWER= 10.40368 KW  
 REACTIVE POWER(TOTAL)= 32.27840KVAR  
 DISPLACEMENT POWER(FUND)= 18.19654KVAR  
 DISPLACEMENT POWER(TOTAL)= 23.96879 KVAR  
 APPARENT POWER = 33.91359 KVA  
 POWER FACTOR(FUND)= 0.46917  
 POWER FACTOR(SYSTEM)= 0.30677

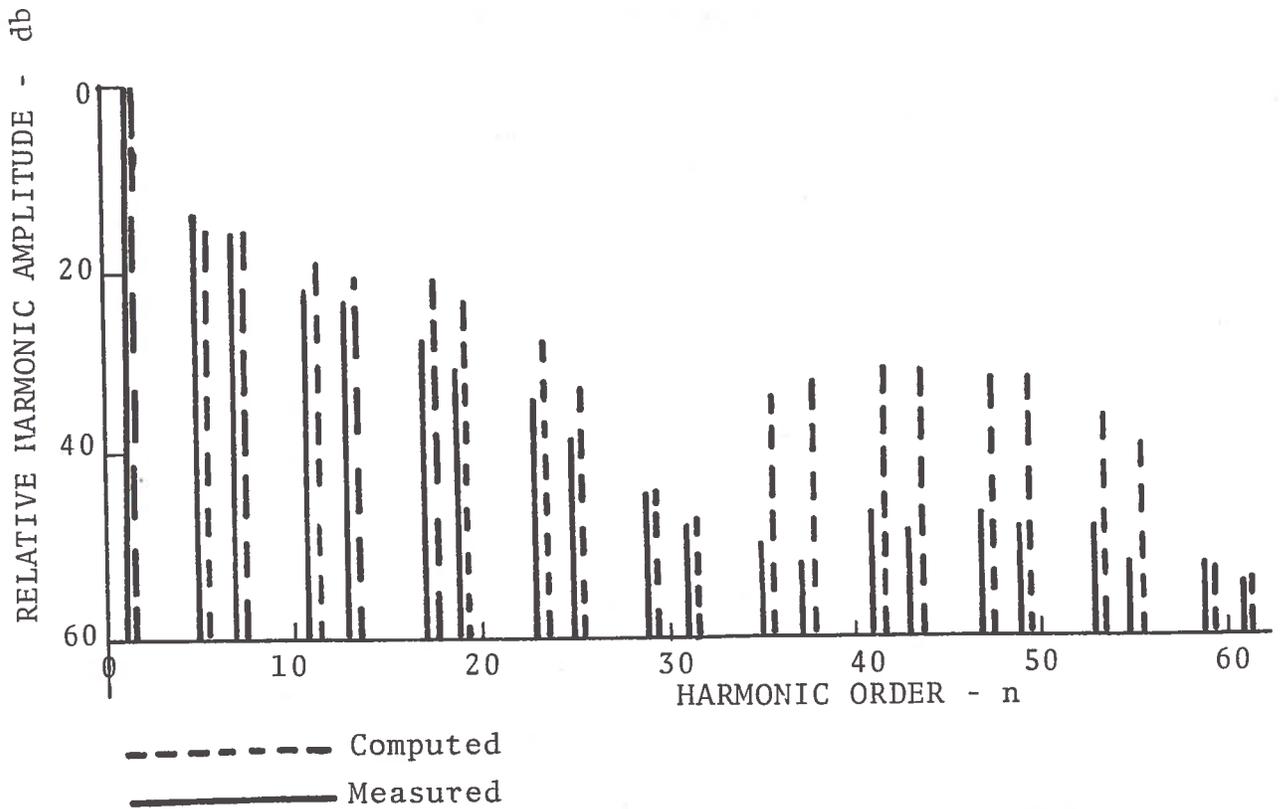
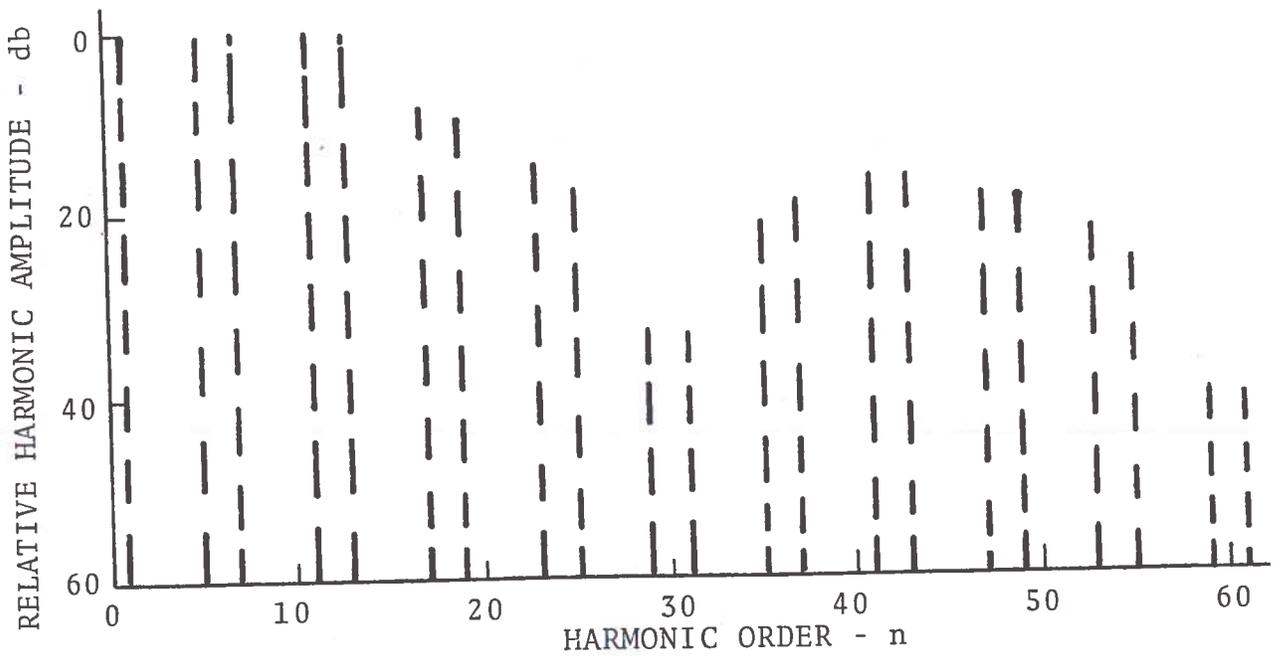


FIGURE 9-16a. VOLTAGE HARMONIC SPECTRUM OF CONSTANT CURRENT INVERTER OPERATING AT A SLIP OF 0.0083



Motor Slip = 0.20

FIGURE 9-16b. VOLTAGE HARMONIC SPECTRUM OF CONSTANT CURRENT INVERTER DRIVE FOR MOTOR OPERATING AT A SLIP OF 0.20

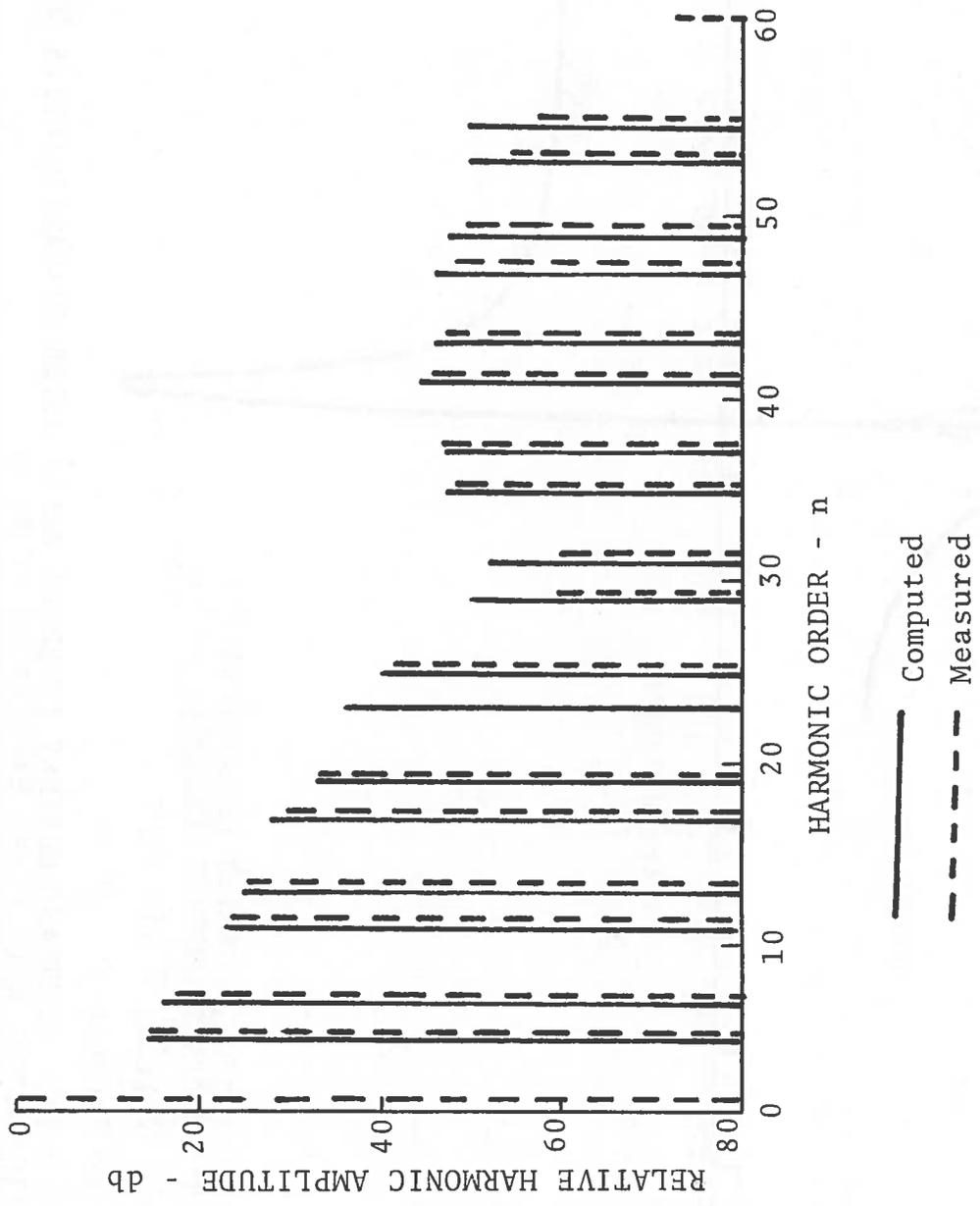
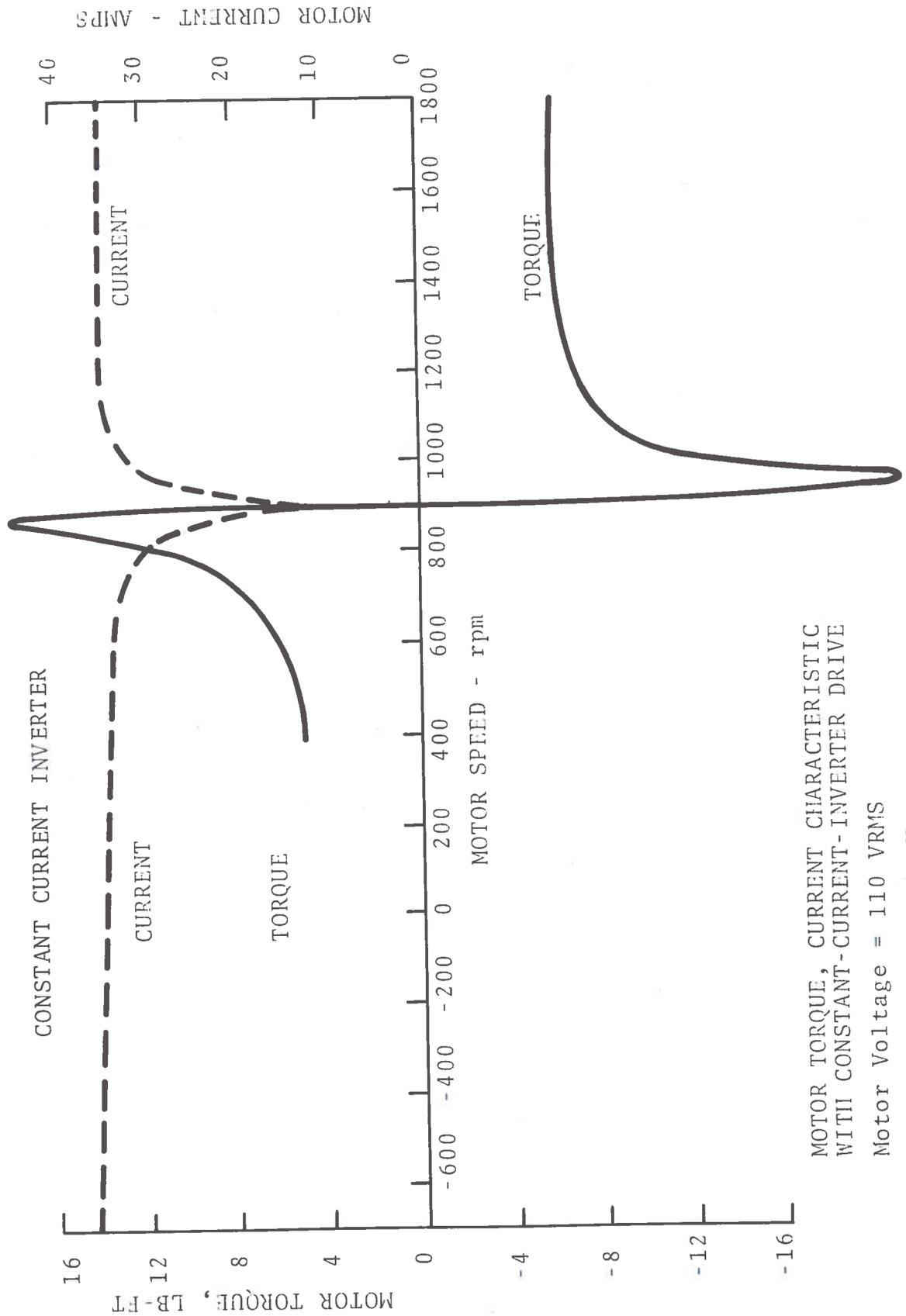


FIGURE 9-17. CURRENT HARMONIC SPECTRUM OF CONSTANT CURRENT INVERTER DRIVE



MOTOR TORQUE, CURRENT CHARACTERISTIC WITH CONSTANT-CURRENT-INVERTER DRIVE

Motor Voltage = 110 VRMS

Drive Frequency = 60 Hz

FIGURE 9-18. CONSTANT CURRENT INVERTER DRIVE-MOTOR CHARACTERISTICS USING INDUCTION MOTOR WITH FIXED EXCITATION OF 60 Hz

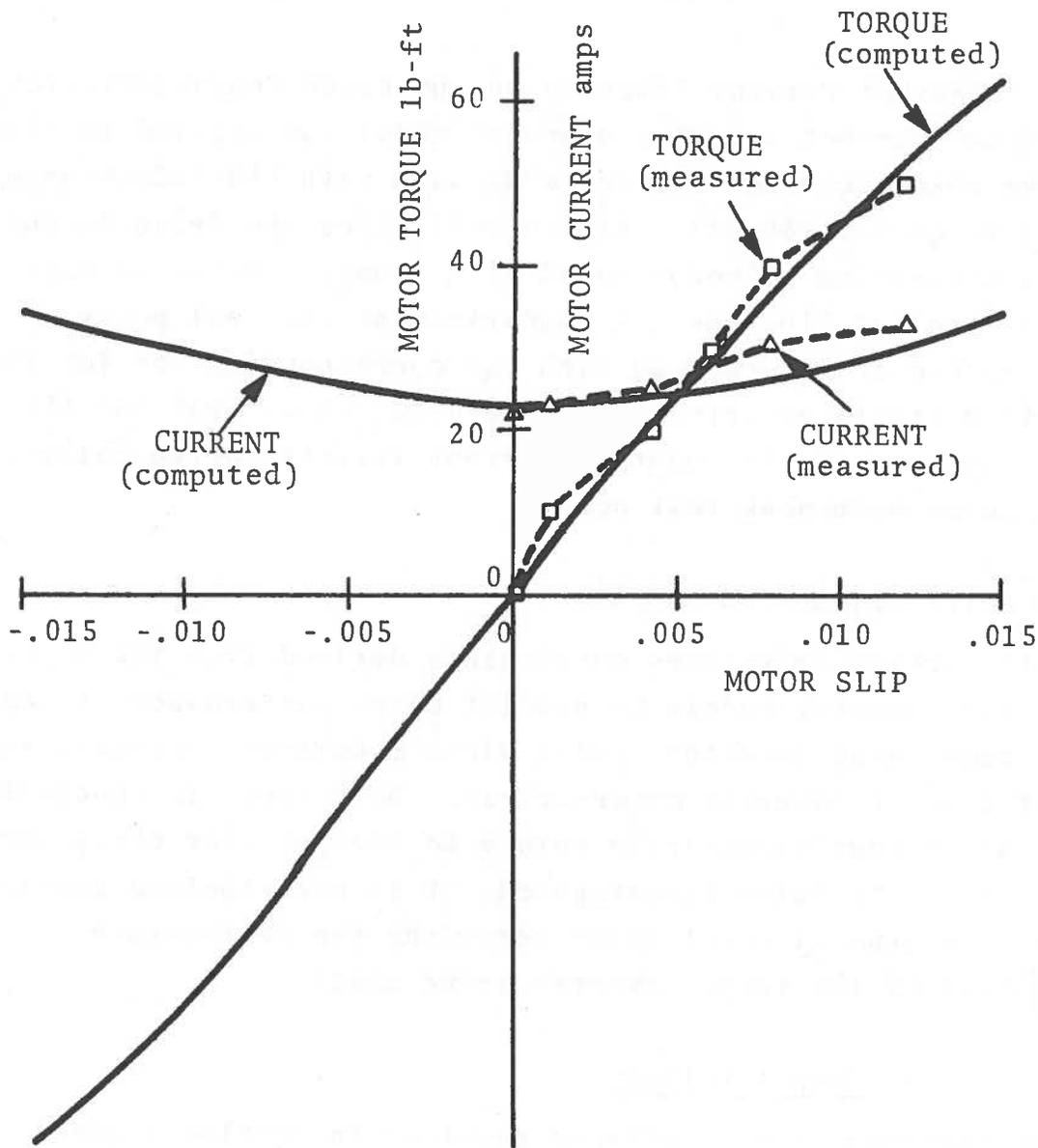


FIGURE 9-19. CONSTANT CURRENT INVERTER DRIVE-MOTOR CHARACTERISTICS IN THE REGION NEAR ZERO SLIP. DRIVE VOLTAGE = 245 VRMS  
DRIVE FREQUENCY = 60 Hz

The computed characteristics assume fixed inverter output voltage of  $245 V_{rms}$ . The good agreement between the measured and computed motor characteristics further confirms the validity of the computer model.

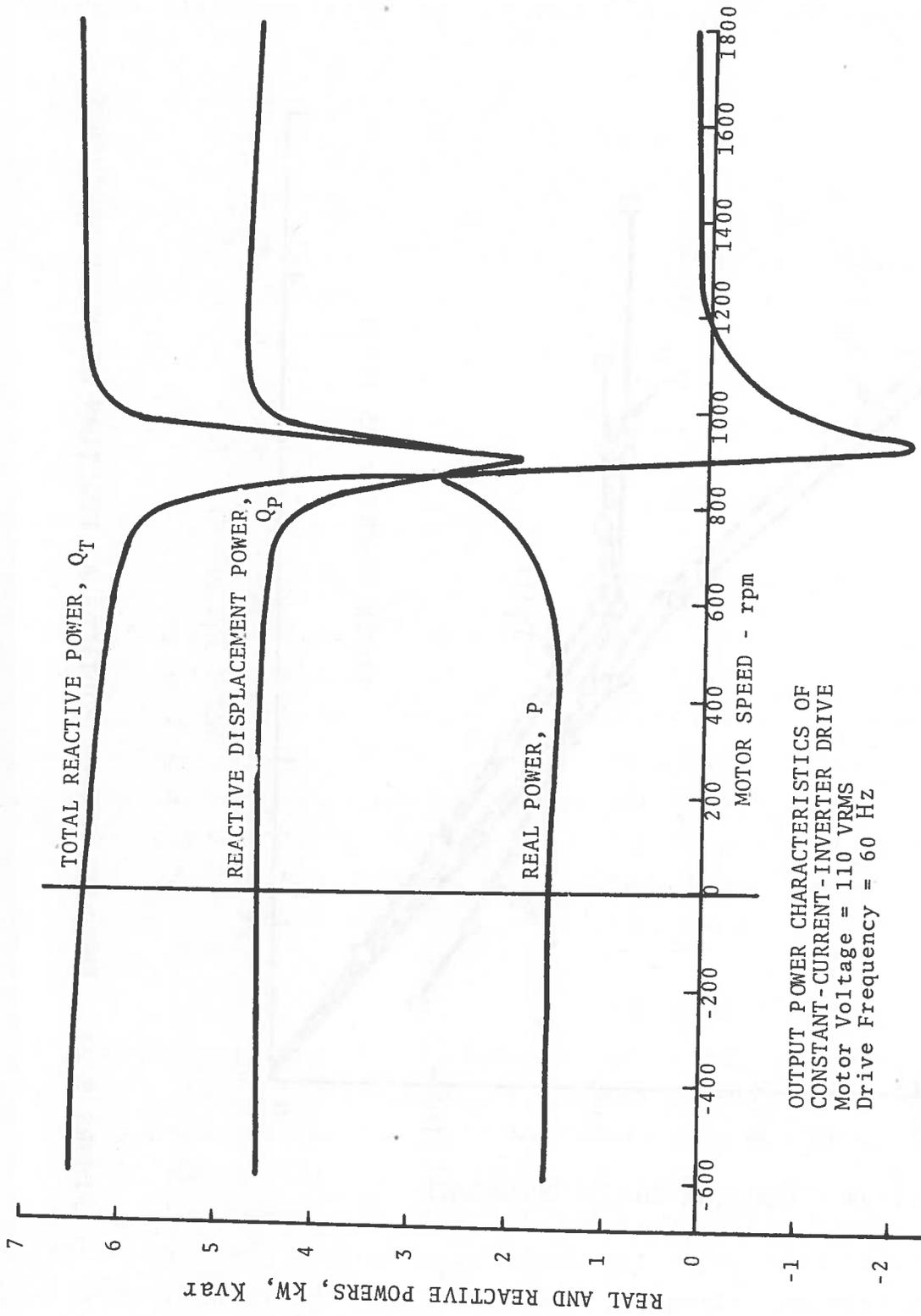
9.2.3.4 Constant Current Inverter Output Power Characteristics - The constant current inverter computer model was applied to predict the drive output characteristics when used with the induction motor at fixed frequency (60 Hz). Figure 9-20 gives the drive output power as a function of motor speed at a constant motor voltage (line to line) of  $110 V_{rms}$ . A comparison of the real power characteristic in Figure 9-20 with the corresponding one for the pulse-width modulated drive in Figure 9-12, shows that for the example considered, the constant-current inverter drive delivers substantially more peak real power.

### 9.3 COMPUTER SIMULATION RESULTS

This section summarizes conclusions derived from the application of the computer models to predict motor performance of the natural commutated inverter, pulse-width modulated inverter, and constant current inverter motor-drives. While certain conclusions on this study must necessarily relate to the peculiar characteristics of the units being investigated, it is nevertheless possible to draw some general conclusions regarding the performance capabilities of the three inverter drive units.

#### 9.3.1 Inverter Output Voltage

The inverter output voltages required to develop a given motor torque are different for the three inverter drives. This is illustrated in Figure 9-21 where inverter rms output voltage is shown as a function of drive frequency for a motor torque of 25 lb-ft. The solid curves give the measured rms output voltage of the three inverter drives; the dashed curves show the fundamental component of output voltage as computed using the inverter models. Three facts are apparent from the Figure: first, the natural



CONSTANT-CURRENT-INVERTER

FIGURE 9-20. CONSTANT CURRENT INVERTER OUTPUT POWER CHARACTERISTICS WITH INDUCTION MOTOR AT FIXED DRIVE FREQUENCY OF 60Hz

Q

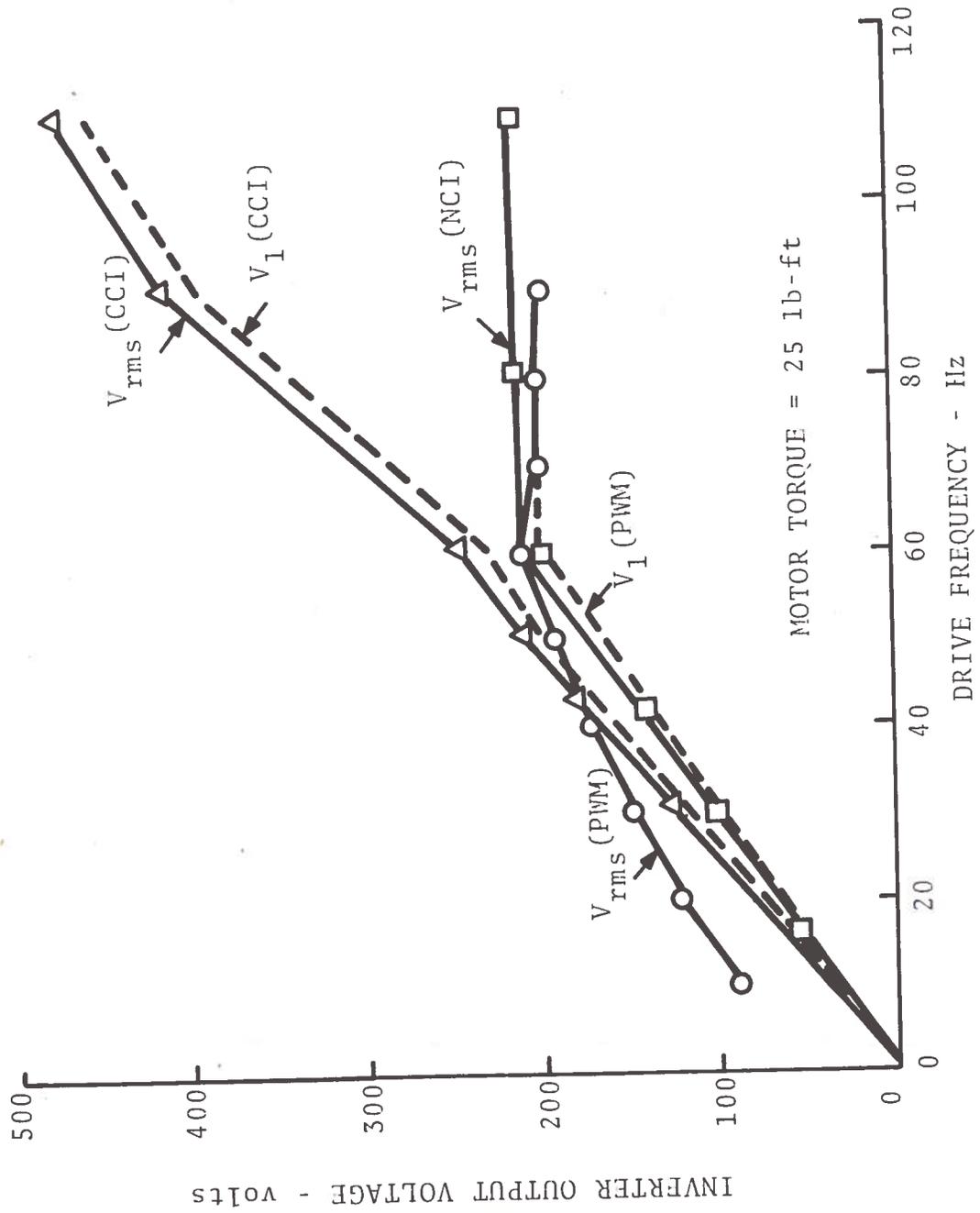


FIGURE 9-21. INVERTER OUTPUT VOLTAGES AS FUNCTION OF DRIVE FREQUENCY

commutated inverter and pulse-width modulated inverter are voltage-limited at 60 Hz drive frequency in contrast to the constant current inverter; second, the constant current inverter develops a higher (fundamental) volts-per-Hertz output than do the natural commutated and pulse-width inverters; and third, the volts-per-Hertz output of the pulse-width modulated inverter is not constant. The first two facts pertain to the basic design of the units. The constant current inverter was designed to drive the test machine at rated drive frequency of 120 Hz. The non-linear volts-versus-drive frequency characteristic of the pulse-width modulated inverter is a result of the larger harmonic content in the inverter output voltage. The increased pulse-width modulated inverter drive voltage required for a given motor torque results in increased harmonic distortion power fed into the motor. Since this distortion power is comprised of higher harmonics having harmonic orders considerably above that of the fundamental, the effect on motor performance is generally quite small.

### 9.3.2 Choice of Constant Current Inverter Versus Pulse-Width Modulated Inverter Drive for Optimum System Performance

The computer models are exercised to study the output characteristics of the constant current inverter and pulse-width modulated inverter driving the ac induction test machine. Both inverters have similar output characteristics when operating with resistive loads at rated load. However, with reactive or complex loads, as in the case of the induction machine, considerable differences can exist in their output drive characteristics.

The constant current inverter drive delivers a fixed current waveform to the induction motor load. Since the harmonic content in the waveform remains constant at all drive frequencies, the voltage harmonics appearing across reactive loads can be quite high as evidenced by the large distortion present in the output voltage waveforms. The effect of this on drive performance is illustrated in Figure 9-22 where motor torque is shown as a function of motor speed for constant current inverter and pulse-width modulated inverter output voltages. Inverter output

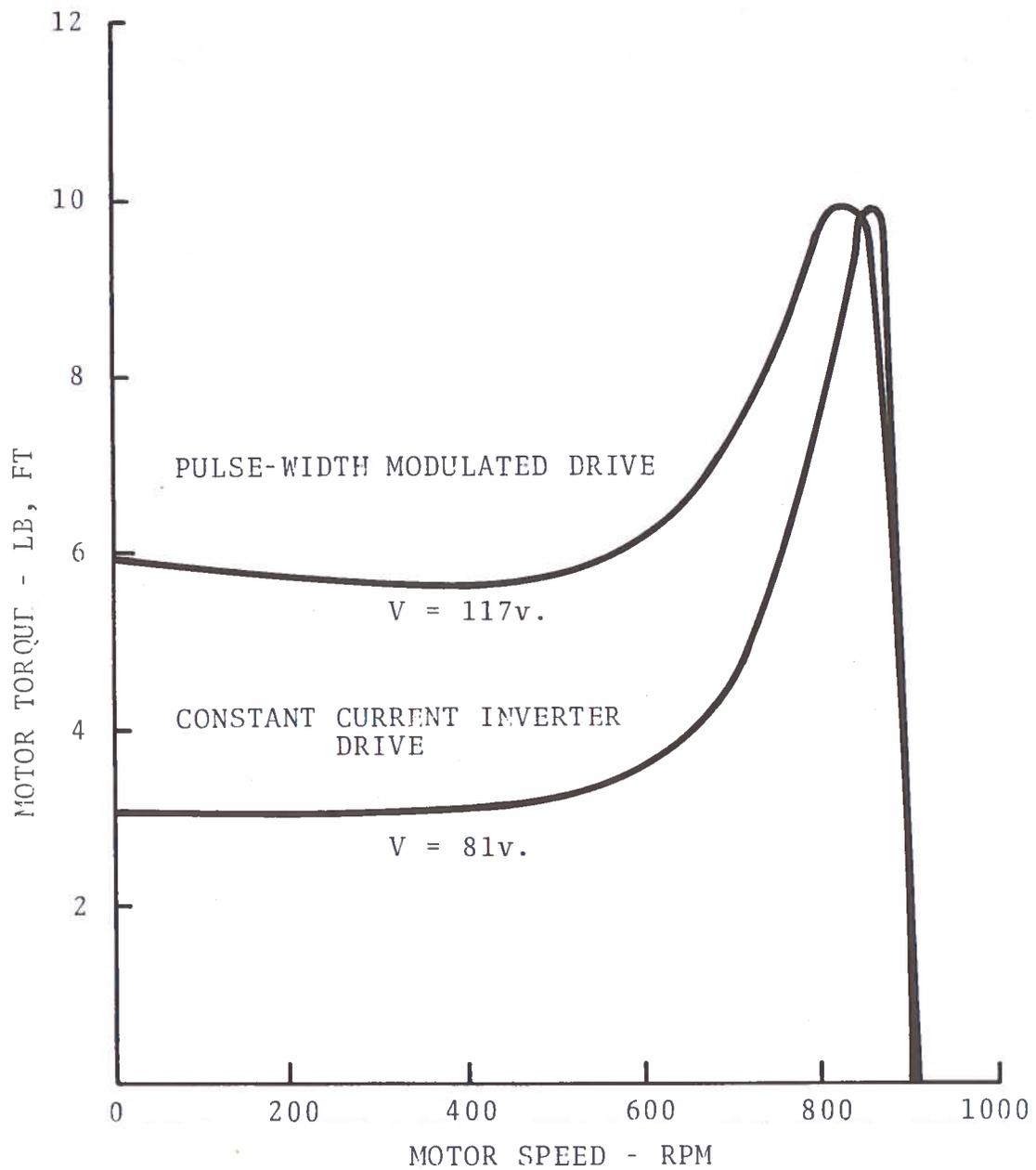


FIGURE 9-22. MOTOR TORQUE DEVELOPED WITH PULSE-WIDTH MODULATED AND CONSTANT CURRENT INVERTER DRIVES AT INVERTER FREQUENCY=60 Hz

voltages were chosen to give equal peak motor torques. The Figure shows two interesting results. First, the motor torque developed with the constant current inverter drive falls off more rapidly at higher motor slips than does the corresponding torque using the pulse-width modulated inverter drive. Secondly, the peak motor torque occurs at a smaller motor slip with the constant current inverter drive. Both these effects have their origin in the increased distortion voltage developed in the output of the constant current inverter drive at higher motor slip frequencies. The increased distortion voltage reduces the fundamental voltage component and hence the fundamental motor input current. It should be mentioned that the drive voltages for the two examples are different; if equal drive voltages had been assumed for the two motor drives, the constant current inverter drive would have produced a higher motor torque than would the pulse-width modulated inverter drive over the operating motor speed range.

#### 9.4 COMPUTER STUDY CONCLUSION

The following conclusions are reached from the computer study of constant current inverter and pulse-width modulated inverter drive performances:

1. The constant current inverter drive operates most effectively at small motor slips or with a constant volts/Hertz corresponding to small motor slip frequencies. Generally, less output rms voltage is required to drive the motor for a given torque than with the pulse-width modulated inverter drive when the motor is operated at reduced output levels. If the motor is operated at high slip frequencies, large distortion can be expected in the output waveform resulting in increased reactive distortion powers. The effect of the fifth harmonic in the output current on motor performance is more serious in the case of the constant current inverter drive than for the pulse-width modulated inverter drive. The control of constant current inverter output current by the phase delay rectifier unit results in large input displacement powers under some conditions.

2. The pulse-width modulated inverter drive operates most effectively at higher output voltages and increased motor speeds. Under these conditions the pulse duty factor tends toward unity and voltage distortion is reduced. At lower drive output voltages and/or lower motor speeds, relatively higher output voltages are required to develop a given motor torque as compared with the constant current inverter drive.

## 10. CONCLUSIONS

The results of the laboratory tests and the model simulations indicate that either of the three inverter types can be used for an ac traction system and that they can control motor torque and speed from zero speed to the speed corresponding to rated output of the inverter. Although each system utilizes different types of input and output power circuitry, over all efficiency and motor performance are similar. A summary of inverter characteristics is listed in Table 10-1.

### 10.1 NATURAL COMMUTATED INVERTER

The natural commutated inverter incorporates a design stratagem which involves series and parallel combinations of thyristors to generate the three-phase output. Reliability is stressed through features of the basic circuit which limit thyristor stresses and output current levels. At turn-on, the sinusoidal current rise is held within the  $di/dt$  limit by the choice of the series resonant components. At turn-off, the thyristors self-extinguish when the voltage passes through zero, and the subsequent reapplication of forward voltage is constrained within the  $dv/dt$  limits by the sinusoidal waveform, again a result of the series resonant circuit action. Switching losses are minimized because the relatively slow rise of the sinusoidal current pulse through the thyristor allows the thyristor to turn on completely before that current is significant. Overloading the inverter results in a decrease in output voltage as the inverter enters current limited operation. Thus, the inverter is a self-protecting and self-limiting design which enhances its reliability. The series resonant circuit places a capacitor in series with the conducting thyristors and the load. This capacitor blocks direct current flow and insures thyristor commutation; thus, there is minimal danger of commutation failure.

The per phase requirements of the natural commutated inverter are:

TABLE 10-1. INVERTER CHARACTERISTICS

	NATURAL COMMUTATED INVERTER	PULSE WIDTH MODULATED INVERTER	PARALLEL CAPACITOR COMMUTATED INVERTER
INVERTER GRADE THYRISTORS	24	12	0
RECTIFIER GRADE THYRISTORS	0	0	12
RECTIFIER DIODES	6	6	0
HIGH SPEED DIODES	6	6	6
COMMUTATION CAPACITORS	6	3	6
COMMUTATION INDUCTORS	6	3	0
DC LINK FILTER	CAPACITOR BANK	SMALL REACTOR	LARGE REACTOR
OUTPUT FILTER	CAPACITOR BANK	NONE	NONE
REGENERATION	REQUIRES ADDITIONAL LINE COMMUTATED INVERTER		YES
DISTORTION FACTOR	HIGH	LOW	LOW
DISPLACEMENT POWER FACTOR	HIGH	HIGH	LOW
OUTPUT DISTORTION	LOW	HIGH	LOW
LOSSES	LOW	LOW	HIGH
FAULT PROTECTION	GOOD	POOR	GOOD

- a. eight inverter grade thyristor
- b. two resonating inductors
- c. two resonating capacitors
- d. two clamp diodes
- e. two output storage capacitors.

The component count to achieve a three-phase system and the number of components in series with the load are high. In addition, the inverter requires capacitive energy storage on the input to provide the high amplitude current pulses of the pulse repetition rate modulation scheme. These considerations contribute to a low power density in the natural commutated inverter and will have to be compared to the volume availability, reliability, and waveform requirements of a deployed system.

The power factor of the inverter with the present input rectifier scheme is influenced principally by distortion rather than displacement parameters. The input storage capacitors are connected directly across the diode bridge; thus the conduction intervals of the diodes are limited to the time required to charge the capacitors to the peak value of the input voltage. At low current levels, this results in high peak to rms current ratios, high distortion component of reactive power, and consequently, low power factor. As the load increases, the conduction periods of the diodes also increase and the distortion reduces. The power factor thus improves with increased inverter loading.

The harmonics generated by the natural commutated inverter in the input lines are high because of the low impedance presented to the higher frequencies by the input storage capacitors. The high peak to rms ratios of the input current are indicative of high distortion power and high harmonic content.

Efficiency is a function of the power transferred through the inverter to the load. It is low at low power levels where losses make up the bulk of the input power, but it increases rapidly to greater than 75 percent at 400 Hz and 40 percent of rated output power.

Harmonic distortion in the output of the inverter is low; in particular, with a lagging power factor load the output approaches an ideal sinusoid. The harmonic distribution in the output of the inverter was not objectionable for any of the load cases studied.

The natural commutated inverter is excellently suited for applications where reliability and purity of waveform are considerations. The output generation scheme results in very good waveform tracking capability under most load conditions and results in minimum harmonic distortion of output voltage and current.

## 10.2 PULSE-WIDTH MODULATED INVERTER

The pulse-width modulated inverter provides variable voltage and variable frequency power from a dc source without using a chopper, or from an ac source using a diode rectifier instead of a phase controlled rectifier. Both voltage and frequency are controlled with a single set of power switches by controlling the firing sequences and the conduction times of the switches. Operation in this mode results in an inverter system with high efficiency, high power factor, and reasonable input harmonics. Pulse-width modulation, however, produces high harmonic distortion of the output and lowers the output power factor. The output waveform changes, which result from the pulse-width variations as the output voltage changes, cause the harmonic magnitudes to vary with voltage. The reduction in the number of notches at discrete frequencies as the output frequency increases also results in discontinuities in the harmonic distributions at these break points.

The commutation components are in parallel with the load switches; thus, although they do not carry full load current, they do carry currents with measured peak values five times maximum load current. These components must be sized for these high peak to rms current ratios.

The use of a full-wave diode bridge, coupled with inductive input filtering, effectively buffers the inverter proper and load from the ac source. Its input characteristics are dependent on

the power transferred, not the load configuration.

Inverter input harmonics are close to theoretical limits, again because of inductive filtering and full-wave rectification without phase control. The input current waveform is essentially a quasi-square wave for all outputs in excess of 3.0 kW. The low input harmonic distortion and the absence of a displacement component of reactive power result in a power factor greater than 0.9 for all loads greater than 3.0 kW.

The efficiency of the inverter is characteristically low at low output, but is above 80 percent for outputs over 4.0 kW for most frequencies. Switching losses have the greatest impact on efficiency because of the large number of switching transients generated by the chopping scheme used for voltage control. These losses are minimized, however, by reducing the number of notches in the output waveform as the frequency is increased.

The use of pulse-width modulation, with modulation being controlled by notching the output waveform in addition to modifying pulse widths, results in an output waveform with varying degrees of harmonic distortion. The output power and power factor are not simply related to load magnitude and power factor; instead they are functions of the output frequency, the number of pulses in the output and the pulse-widths of the output pulse train. The harmonic distortion is primarily voltage distortion; the current level for each harmonic is low because of the high impedance of the lagging power factor load.

This type of inverter is suited for applications where the harmonic distribution of the output is not objectionable. Its principal advantages are:

1. Control of output voltages and frequency with only one set of semiconductor switches;
2. Commutation components not in series with the load;
3. Good input power characteristics.

The drive type is capable of regeneration, although not

as the test unit is presently configured. Regenerated energy raises the level of the dc bus voltage, since terminal voltage polarity does not reverse, a line commutated inverter must be added to redirect regenerated energy back to the ac source.

### 10.3 CONSTANT CURRENT INVERTER

The constant current inverter regulates the dc link current, limiting the voltage and drive frequencies to maintain the dc link current at or below a preset limit. The semiconductor and passive components used in the inverter can therefore be sized for torque limit conditions rather than for pulse level conditions as with the natural commutated and pulse-width modulated inverters. Commutation by capacitive coupling of the commutation pulse provides a large commutation margin, therefore the inverter thyristors can be rectifier grade thyristors rather than more costly inverter grade thyristors.

The inverter has dissipative internal burden circuitry which impairs its efficiency figures, resulting in peak efficiencies less than 75 percent, while the natural commutated and pulse-width modulated inverters operate closer to 90 percent efficiency. In a full scale system, operating efficiencies can be expected to be higher because auxiliary losses do not increase proportionally with kva rating. Efficiency may be increased by redesign of some circuits, but this may be accomplished at the expense of some other characteristics. The inverter also uses a phase delay rectifier circuit to control dc link voltage, and this circuit has low power factor at low output voltage. The combination of low efficiency and low power factor will result in high apparent power demand and may impact utility supply service costs. The low power factor is especially critical when full scale traction systems are considered. A traction system must deliver high acceleration power from zero speed to cruise speed. At low speeds, where voltage is low but acceleration current is high, the total apparent power demand may make use of a phase delay rectifier input circuit unfeasible.

The harmonics in the constant current inverter are least

severe of the three circuits tested. Since the unit is a current regulated device, the input and output currents are nearly equal in magnitude, with input current being slightly higher because of the inverter auxiliary power loads. Also, since both input and output are three-phase ac circuits, both currents have similar wave shapes and hence, the same harmonic distributions. The quasi-square wave shape harmonics closely follow a  $1/n$  distribution.

The choice of inverter type selected for full scale development must be strongly influenced by the application. The use of other than a variable voltage variable frequency drive is precluded from both efficiency and performance considerations. A sinusoidal output voltage generating inverter presents the least complications from harmonic torque and interference standpoints. The pulse-width modulated inverter appears more suitable for powering loads with low lagging power factors because the inductive filtering action eliminates harmonic currents which could be present because of high levels of voltage harmonics. The constant current inverter constrains the current waveform to be rectangular and therefore operates most effectively into high power factor loads where impedances, and hence motor voltages, at the harmonic frequencies are low.

Despite the disparities in wave shapes of the inverter outputs, the effects on motor performance are not severe. Pulse-width modulation produces voltage harmonics which extend to high frequencies, but at these frequencies the motor impedances are high and the harmonic currents are insignificant. The significant harmonics are those which result in motor currents and torques. For both the pulse-width modulated inverter and the constant current inverter, these are the fifth and seventh, and for both inverters they are the same proportion of the fundamental value.

A detailed description of harmonic influence in three-phase induction machines is given in reference 14, where it is shown that the harmonics produce effects in the motor torque-speed characteristics at speeds corresponding to high slip points. The fifth harmonic, for example, produces negative torques peaking at

slip speeds of 1.251, and the seventh produces positive torque from zero speed to one-seventh of synchronous speed and negative torques, diminishing in magnitude, from one-seventh synchronous speed to synchronous speed. The variable frequency power conditioning systems used to supply the inverters operate the motor near synchronous speed at all points, therefore the effects of the harmonics on the motor performance are not distinguishable. Harmonic impact on communication and control systems will have to be investigated on an individual basis, since the severity of any interference on any system cannot be predicted.

Results of the computer study show that:

1. The constant current inverter drive operates most effectively at small motor slips or with a constant volts/Hertz corresponding to small motor slip frequencies. Using the constant current inverter drive less output rms voltage is required to drive the motor (for a given torque) than with the pulse-width modulated inverter drive. If the motor is operated at high slip frequencies, large distortion can be expected in the output waveform which results in increased reactive distortion powers. The control of constant current inverter output current via the phase delay rectifier can result in large input displacement reactive powers.

2. The pulse-width modulated inverter drive operates most effectively at high output power condition. At low output power levels, relatively larger output voltages are required to develop a given motor torque compared with constant current inverter drives. The control of voltage output amplitude by pulse-width control eliminates the problem of increased reactive power in the input experienced by the constant current inverter drive.

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APPENDIX A  
TABULATED INVERTER TEST DATA

TABLE A-1. NATURAL COMMUTATED INVERTER, TABULATED DATA FOR LOAD = 24Ω, AT LOAD PF = 1.0

E <sub>o</sub>	LOAD Z = 24Ω LOAD Pf = 1.0												
	FREQ.	INPUT VAC	INPUT IAC A	PIN KW	OUTPUT VAC	OUTPUT IAC A	P <sub>OUT</sub> KW	St KVA	EFF.	Pf	Losses KW	PIN + FIXED P <sub>LOSS</sub> KW	TOTAL EFF.
100V	25	325	1.22	.42	100	1.82	.360	.688	.857	.610	.060	.92	.390
	50	324	1.22	.45	100	1.82	.340	.688	.756	.654	.110	.95	.358
	100	324	1.37	.50	97	1.82	.320	.772	.640	.68	.180	1.0	.320
	150	324	1.83	.625	99	1.82	.360	1.03	.577	.607	.265	1.125	.320
	200	324	1.97	.70	103	1.91	.380	1.11	.544	.630	.320	1.20	.317
	250	324	2.13	.80	104	1.91	.390	1.20	.487	.666	.410	1.30	.300
	300	324	2.44	.85	98	1.82	.345	1.38	.406	.616	.505	1.35	.256
	350	324	2.49	.90	98	1.82	.360	1.40	.400	.645	.540	1.40	.257
	400	324	2.75	1.00	98	1.82	.360	1.55	.400	.645	.640	1.50	.240
	25	325	2.75	1.00	150	2.91	.88	1.55	.880	.645	.120	1.50	.586
	50	324	2.75	1.03	151	3.00	.88	1.55	.854	.665	.150	1.53	.575
	100	325	2.90	1.03	146	2.91	.81	1.58	.770	.665	.240	1.53	.529
150	325	3.35	1.25	150	3.09	.88	1.88	.705	.665	.370	1.75	.503	
200	324	3.65	1.40	152	3.09	.90	2.05	.642	.683	.500	1.90	.474	
250	324	3.95	1.50	150	3.09	.88	2.22	.587	.678	.620	2.00	.440	
300	324	3.95	1.55	147	2.91	.83	2.22	.536	.698	.720	2.05	.405	
350	325	4.26	1.60	145	2.91	.80	2.39	.500	.670	.800	2.10	.381	
400	325	4.60	1.70	145	2.91	.81	2.59	.475	.657	.890	2.20	.368	
25	324	4.88	1.83	204	4.36	1.68	2.75	.920	.665	.150	2.33	.721	
50	323	4.88	1.93	207	4.45	1.69	2.75	.874	.702	.240	2.43	.695	
100	323	5.63	2.20	210	4.45	1.78	3.16	.808	.696	.420	2.70	.659	
150	323	6.09	2.40	209	4.45	1.76	3.43	.734	.700	.640	2.90	.607	
200	324	6.40	2.45	208	4.45	1.74	3.60	.712	.680	.710	2.95	.599	
250	324	6.71	2.63	210	4.54	1.76	3.78	.670	.695	.870	3.13	.562	
300	324	7.02	2.80	210	4.45	1.76	3.94	.628	.710	1.04	3.30	.533	
350	323	7.32	2.95	210	4.45	1.74	4.12	.590	.717	1.21	3.45	.504	
400	323	7.62	3.10	210	4.45	1.73	4.28	.558	.724	1.37	3.60	.481	

TABLE A-2. NATURAL COMMUTATED INVERTER,  
TABULATED DATA FOR LOAD =  $6\Omega$ , AT LOAD PF = 1.0

LOAD Z =  $6\Omega$       LOAD Pf = 1.0

$E_o$	FREQ.	INPUT VAC	INPUT IACA	PIN KW	OUTPUT VAC	OUTPUT VACA	POUT KW	St KVA	EFF.	P. f	LOSSES KW	PIN + FIXED P. KW.	TOTAL EFF.
100V	25	324	4.88	1.8	98	8.81	1.545	2.74	.86	.656	.255	2.3	.671
	50	323	4.57	1.8	98	8.92	1.53	2.56	.85	.702	.270	2.3	.665
	100	323	4.88	1.83	99	8.92	1.535	2.74	.838	.668	.295	2.33	.658
	150	323	4.88	1.85	99	8.92	1.525	2.74	.823	.676	.325	2.35	.649
	200	323	5.18	2.0	100	9.00	1.54	2.91	.767	.688	.460	2.50	.616
	250	323	5.48	2.07	100	9.10	1.605	3.07	.775	.676	.465	2.57	.625
	300	323	5.94	2.27	102	9.28	1.695	3.33	.746	.684	.575	2.77	.612
	350	322	5.94	2.3	102	9.28	1.655	3.33	.720	.690	.645	2.80	.591
	400	324	5.48	2.05	95	8.72	1.485	3.07	.724	.668	.565	2.55	.582
	25	322	9.76	4.0	148	13.65	3.53	5.46	.883	.731	.470	4.50	.784
50	322	9.76	4.0	148	13.75	3.53	5.46	.883	.731	.470	4.50	.784	
100	322	10.1	4.1	149	13.8	3.57	5.65	.872	.727	.530	4.60	.776	
150	322	10.4	4.2	150	13.8	3.58	5.82	.853	.722	.620	4.7	.762	
200	322	10.7	4.4	150	13.9	3.66	5.98	.831	.735	.740	4.9	.747	
250	322	11.0	4.5	150	13.9	3.65	6.15	.812	.732	.850	5.0	.730	
300	322	11.0	4.5	150	13.85	3.65	6.15	.812	.732	.850	5.0	.730	
350	322	11.3	4.7	151	13.9	3.66	6.32	.778	.742	1.04	5.2	.704	
400	322	11.7	4.83	152	14.0	3.70	6.54	.766	.738	1.13	5.33	.694	
25	318	17.7	7.67	206	19.45	6.89	9.82	.90	.785	.780	8.17	.843	
50	318	17.85	7.7	207	19.6	6.96	9.92	.905	.777	.740	8.20	.849	
100	318	18.0	7.75	207	19.6	6.87	9.98	.888	.777	.880	8.25	.823	
150	318	18.3	7.95	209	19.6	6.97	10.15	.889	.78	.988	8.45	.825	
200	318	18.6	8.10	209	19.8	7.02	10.3	.868	.786	1.08	8.60	.816	
250	318	19.1	8.30	209	19.8	7.05	10.6	.848	.782	1.25	8.80	.801	
300	318	19.2	8.40	209	19.8	7.05	10.65	.838	.790	1.35	8.90	.792	
350	318	19.2	8.30	206	19.5	6.81	10.65	.833	.782	1.49	8.80	.774	
400	318	18.6	8.00	200	19.0	6.42	10.3	.803	.777	1.58	8.50	.755	

TABLE A-3. NATURAL COMMUTATED INVERTER,  
TABULATED DATA FOR LOAD =  $4.37\Omega$ , AT LOAD PF = 1.0

LOAD Z =  $4.37\Omega$     LOAD PF = 1.0

FREQ	INPUT VAC	INPUT IACA	PIN KW	OUTPUT VAC	OUTPUT IACA	POUT KW	St KVA	EFF.	P. f	LOSSES KW	PIN + FIXED P KW.	TOTAL EFF.
25	322	6.71	2.575	99	12.45	2.17	3.73	.844	.694	.405	3.075	.705
50	324	6.71	2.575	99	12.50	2.16	3.73	.840	.694	.415	3.075	.702
100	323	6.71	2.60	99.5	12.45	2.18	3.73	.837	.696	.420	3.100	.703
150	323	6.87	2.60	100	12.65	2.23	3.82	.857	.68	.370	3.100	.719
200	323	6.87	2.65	99	12.45	2.16	3.82	.837	.693	.490	3.150	.686
250	323	7.32	2.85	102	12.65	2.28	4.07	.800	.700	.570	3.350	.680
300	323	7.02	2.75	100	12.55	2.18	3.91	.794	.703	.570	3.250	.670
350	323	7.32	2.9	101	12.55	2.24	4.07	.772	.714	.660	3.450	.649
400	323	7.67	3.1	102	12.8	2.28	4.27	.737	.727	.820	3.650	.625
25	322	13.1	5.45	147	18.9	4.76	7.29	.875	.75	.690	5.950	.800
50	322	13.1	5.45	147	18.9	4.78	7.29	.878	.75	.670	5.950	.803
100	322	13.1	5.5	147	18.9	4.77	7.29	.868	.755	.730	6.00	.795
150	322	13.4	5.65	148	19.1	4.84	7.45	.857	.758	.810	6.150	.787
200	322	13.7	5.70	148	19.1	4.78	7.62	.838	.748	.920	6.200	.770
250	322	13.7	5.80	148	19.3	4.85	7.62	.836	.76	.950	6.300	.769
300	322	14.0	5.90	148	19.1	4.86	7.78	.823	.758	1.04	6.40	.759
350	322	14.3	6.05	149	19.3	4.88	7.95	.808	.76	1.17	6.550	.745
400	322	14.6	6.10	148	19.3	4.87	8.12	.798	.752	1.23	6.600	.734
25	315	24.4	10.80	207	27.5	9.72	13.4	.90	.807	1.080	11.30	.860
50	315	24.4	10.80	208	27.6	9.73	13.4	.90	.807	1.07	11.30	.861
100	315	24.6	10.80	208	27.6	9.72	13.5	.90	.803	1.08	11.30	.860
150	315	24.7	10.95	209	27.6	9.77	13.6	.893	.807	1.18	11.450	.853
200	315	24.7	11.0	208	27.6	9.69	13.6	.88	.810	1.31	11.50	.843
250	315	24.1	10.90	207	27.5	9.55	13.6	.875	.806	1.35	11.40	.837
300	315	25.2	10.55	203	27.0	9.10	13.2	.863	.798	1.45	11.050	.823
350	315	25.2	11.20	206	27.3	9.41	13.8	.842	.812	1.79	11.700	.804
400	318	23.8	10.30	196	26.2	8.52	13.0	.878	.794	1.78	10.80	.788

TABLE A-4. NATURAL COMMUTATED INVERTER,  
TABULATED DATA FOR LOAD = 24.0Ω, AT LOAD PF = 0.8

LOAD Z = 24.0Ω, LOAD Pf = 0.8

E <sub>o</sub>	FREQ.	INPUT		PIN KW	OUTPUT		P <sub>out</sub> KW	St KVA	EFF	P. f.	Losses KW	PIN + FIXED KW	TOTAL EFF
		V <sub>ac</sub>	A <sub>ac</sub>		V <sub>ac</sub>	A <sub>ac</sub>							
100V	43	325	0.92	0.35	99	1.82	0.275	0.517	.786	.677	.075	0.85	.323
	110	325	1.22	0.40	99	1.82	0.235	0.686	.588	.583	.165	0.90	.261
	173	325	1.22	0.55	99	1.82	0.275	0.686	.500	.802	.275	1.05	.262
	210	323	1.83	0.65	104	2.00	0.280	1.023	.431	.636	.370	1.15	.243
	288	324	2.14	0.75	102	2.00	0.300	1.200	.400	.625	.450	1.25	.240
	344	324	2.14	0.85	100	1.82	0.285	1.200	.335	.709	.565	1.35	.211
	398	324	2.44	0.85	99	1.91	0.275	1.368	.324	.622	.575	1.35	.204
150V	43	325	2.14	0.85	149	3.28	0.735	1.203	.865	.706	.115	1.35	.544
	110	325	2.44	0.90	150	3.09	0.690	1.372	.766	.656	.210	1.40	.493
	173	325	2.75	1.00	147	2.91	0.665	1.546	.665	.647	.335	1.50	.443
	210	323	3.05	1.10	148	3.09	0.690	1.704	.627	.645	.410	1.60	.431
	288	324	3.36	1.30	148	3.09	0.685	1.883	.527	.690	.615	1.80	.380
	344	324	3.66	1.45	150	3.09	0.695	2.052	.479	.707	.755	1.95	.356
	398	323	4.27	1.55	149	3.09	0.710	2.386	.458	.650	.840	2.05	.346
208V	43	324	4.27	1.60	209	4.55	1.445	2.393	.903	.668	.155	2.10	.688
	110	324	4.27	1.70	204	4.37	1.305	2.395	.768	.710	.395	2.20	.593
	173	324	4.88	1.90	204	4.37	1.300	2.735	.684	.694	.600	2.40	.542
	210	323	5.19	2.00	204	4.37	1.325	2.900	.663	.690	.675	2.50	.530
	288	323	5.80	2.20	206	4.37	1.345	3.241	.611	.679	.855	2.70	.498
	344	324	6.25	2.45	209	4.37	1.395	3.503	.569	.699	1.055	2.95	.473
	398	323	6.71	2.50	202	4.28	1.555	3.749	.622	.667	.945	3.00	.518

TABLE A-5. NATURAL COMMUTATED INVERTER,  
TABULATED DATA FOR LOAD =  $6\Omega$ , AT LOAD PF = 0.8

LOAD Z = $6\Omega$ LOAD PF = 0.8												
FREQ	INPUT VAC	INPUT IAC A	PIN KW	OUTPUT VAC	OUTPUT IAC A	POUT KW	St KVA	EFF.	P.f INPUT	LOSSES KW	PIN + FIXED P. KW	TOTAL EFF
43	324	3.97	1.60	99	9.1	1.28	2.23	.800	.718	.320	2.10	.609
108	324	3.97	1.60	98	9.1	1.27	2.23	.793	.718	.330	2.10	.605
134	324	3.66	1.45	97	8.92	1.20	2.05	.828	.707	.250	1.95	.615
202	325	4.27	1.60	100	9.1	1.275	2.40	.797	.666	.325	2.10	.607
259	325	4.42	1.75	103	9.46	1.39	2.50	.794	.703	.360	2.25	.618
315	322	4.27	1.70	99	9.1	1.23	2.40	.723	.714	.470	2.20	.559
404	324	4.58	1.85	97	8.92	1.18	2.57	.635	.720	.625	2.35	.502
43	322	8.54	3.60	152	14.38	3.06	4.76	.850	.756	.540	4.10	.746
108	323	7.93	3.35	147	14.01	2.93	4.44	.875	.750	.420	3.85	.761
134	323	7.93	3.25	148	13.83	2.83	4.44	.871	.734	.420	3.75	.755
202	323	8.24	3.35	148	13.83	2.88	4.61	.858	.726	.475	3.85	.748
259	323	8.54	3.55	149	14.01	2.90	4.78	.817	.743	.650	4.05	.716
315	322	9.15	3.80	153	14.38	3.02	5.10	.795	.745	.780	4.30	.702
404	323	9.15	3.80	146	13.88	2.88	5.12	.758	.742	.920	4.30	.669
43	320	14.95	6.50	205	19.47	5.75	8.28	.885	.784	.750	7.00	.821
108	320	15.25	6.65	212	20.20	5.95	8.45	.895	.787	.700	7.15	.832
134	320	14.64	6.35	206	19.47	5.64	8.11	.888	.783	.710	6.85	.823
202	320	14.95	6.45	207	19.66	5.66	8.28	.878	.778	.790	6.95	.814
259	321	15.40	6.80	209	20.02	5.86	8.56	.862	.794	.940	7.30	.803
315	320	15.86	7.05	210	20.02	5.92	8.79	.840	.802	1.130	7.55	.784
404	322	15.86	7.00	205	19.66	5.71	8.84	.816	.791	1.290	7.50	.761

TABLE A-6. NATURAL COMMUTATED INVERTER,  
 TABULATED DATA FOR LOAD =  $4.37\Omega$ , AT LOAD PF = 0.8

LOAD  $Z = 4.37$       LOAD PF = 0.8

FREQ	INPUT VAC	INPUT IAC A	PIN KW	OUTPUT VAC	OUTPUT IAC A	P <sub>OUT</sub> KW	St KVA	EFF	P. f INPUT	LOSSES KW	PIN + FIXED, KW	TOTAL P <sub>EFF.</sub>
52.3	323	5.19	2.0	98	12.4	1.62	2.9	.81	.690	.380	2.5	.648
102.5	325	5.19	2.05	100	12.6	1.65	2.92	.805	.703	.400	2.55	.647
157	324	5.19	2.0	99	12.4	1.67	2.91	.835	.687	.330	2.5	.668
229	323	5.19	2.0	99	12.6	1.64	2.90	.820	.690	.360	2.5	.656
293	322	5.49	2.1	98	12.4	1.63	3.06	.776	.687	.470	2.6	.627
377	324	5.80	2.3	101	12.6	1.71	3.25	.743	.707	.590	2.8	.611
52.3	322	10.98	4.55	149	19.29	3.86	6.12	.848	.743	.690	5.15	.749
102.5	323	10.68	4.40	149	19.11	3.84	5.97	.873	.737	.560	4.90	.784
157	323	10.68	4.45	149	19.11	3.85	5.97	.865	.746	.600	4.95	.777
229	322	10.98	4.60	151	19.47	3.94	6.12	.857	.752	.660	5.10	.773
293	320	11.29	4.70	151	19.66	3.91	6.42	.832	.733	.790	5.20	.752
377	322	11.59	4.85	152	19.47	4.00	6.46	.824	.751	.850	5.35	.748
52.3	318	18.91	8.25	204	26.39	7.32	10.4	.887	.793	.930	8.75	.836
102.5	320	18.45	8.10	204	26.57	7.22	10.21	.891	.793	.880	8.60	.839
157	320	18.91	8.25	206	26.75	7.35	10.47	.890	.788	.900	8.75	.840
229	320	18.91	8.35	206	26.93	7.43	10.47	.889	.798	.920	8.85	.839
293	320	19.22	8.40	203	26.57	7.39	10.64	.880	.789	1.01	8.90	.830
377	318	19.52	8.60	205	26.57	7.34	10.74	.853	.801	1.26	9.10	.806

TABLE A-7. NATURAL COMMUTATED INVERTER,  
 TABULATED DATA FOR LOAD =  $24\Omega$ , AT LOAD PF = 0.5

LOAD Z = 24      LOAD PF = 0.5

$E_o$	FREQ	INPUT VAC	INPUT IAC <sub>A</sub>	PIN KW	OUTPUT VAC	OUTPUT IAC <sub>A</sub>	POUT KW	St KVA	EFF.	P.f. INPUT	LOSSES KW	PIN + FIXED KM	TOTAL P <sub>EFF</sub>
100V	62.5	324	.61	.250	102	2.00	.200	.342	.800	.731	.050	.750	.266
	96.2	324	.92	.300	102	2.00	.190	.516	.633	.582	.110	.800	.240
	156	325	.92	.350	99	2.00	.185	.517	.529	.677	.165	.850	.218
	208	325	1.22	.450	102	2.00	.195	.687	.433	.655	.255	.950	.205
	249	324	1.22	.400	98	2.00	.155	.685	.387	.584	.245	.900	.172
	312	320	1.83	.700	104	2.18	.205	1.013	.293	.690	.495	1.20	.170
	373	323	1.83	.650	96	1.82	.170	1.023	.262	.635	.480	1.15	.147
	408	325	1.83	.700	96	1.82	.170	1.029	.243	.680	.530	1.20	.146
150V	62.5	323	1.53	.550	149	3.28	.480	.855	.873	.643	.070	1.050	.457
	96.2	324	1.83	.600	148	3.09	.465	1.026	.775	.585	.135	1.10	.423
	156	325	2.44	.800	150	3.28	.475	1.372	.594	.583	.325	1.30	.365
	208	325	2.44	.900	153	3.28	.465	1.372	.516	.656	.435	1.40	.332
	249	324	2.44	.975	149	3.09	.455	1.368	.466	.713	.520	1.475	.308
	312	320	3.05	1.10	147	3.28	.455	1.688	.414	.651	.645	1.61	.283
	373	324	3.36	1.25	147	3.28	.460	1.883	.368	.664	.790	1.75	.263
	408	324	3.66	1.35	149	3.28	.490	2.052	.363	.658	.860	1.85	.265
208V	62.5	323	2.75	1.05	206	4.46	.930	1.537	.886	.683	.120	1.55	.600
	96.2	324	3.05	1.15	206	4.37	.915	1.678	.709	.685	.235	1.65	.555
	156	325	3.66	1.40	205	4.46	.895	2.058	.639	.680	.505	1.90	.471
	208	324	4.27	1.60	207	4.46	.705	2.394	.566	.668	.695	2.10	.431
	249	324	4.58	1.725	209	4.55	.925	2.567	.536	.672	.800	2.225	.416
	312	320	4.58	1.80	203	4.46	.895	2.535	.497	.710	.905	2.30	.389
	373	323	5.19	2.025	205	4.55	.930	2.900	.459	.698	1.095	2.52	.368
	408	325	5.34	2.10	200	4.55	.900	3.002	.428	.699	1.200	2.60	.346



TABLE A-9. NATURAL COMMUTATED INVERTER,  
TABULATED DATA FOR LOAD =  $4.37\Omega$ , AT LOAD PF = 0.5

LOAD Z = 4.37    LOAD PF = 0.5

$E_o$	FREQ	INPUT VAC	INPUT IAC <sub>A</sub>	PIN KW	OUTPUT VAC	OUTPUT IAC <sub>A</sub>	P <sub>OUT</sub> KW	St KVA	EFF.	P.F. INPUT	LOSSES KW	PIN + FIXED P <sub>CM</sub>	TOTAL EFF.
100V	49	324	3.35	1.50	95	12.56	1.0	1.88	.769	.692	.300	1.80	.555
	108	325	3.35	1.50	99	12.74	1.08	1.88	.831	.690	.220	1.80	.600
	150	325	3.35	1.50	100	13.10	1.10	1.88	.846	.690	.200	1.80	.611
	212	325	3.35	1.25	100	12.92	1.08	1.88	.864	.663	.170	1.75	.617
	266	324	3.66	1.45	99	13.47	1.23	2.05	.848	.707	.220	1.95	.630
	333	325	3.96	1.55	103	13.47	1.21	2.23	.781	.696	.340	2.05	.590
	390	324	3.66	1.45	93	12.01	1.06	2.05	.731	.707	.390	1.95	.544
150V	49	322	7.63	3.05	148	19.84	2.46	4.25	.807	.718	.590	3.55	.693
	108	325	7.02	2.80	148	19.47	2.41	3.95	.861	.709	.390	3.30	.730
	150	323	7.32	2.90	149	20.20	2.58	4.09	.890	.709	.320	3.40	.759
	212	324	7.02	2.80	149	19.84	2.51	3.93	.896	.712	.290	3.30	.761
	266	323	7.93	3.20	150	20.93	2.77	4.44	.822	.720	.430	3.70	.749
	333	323	7.63	3.10	152	20.20	2.62	4.26	.845	.727	.480	3.60	.728
	390	324	7.93	3.15	150	20.20	2.62	4.44	.832	.709	.530	3.65	.718
208V	49	320	14.03	5.90	210	28.76	5.07	7.77	.859	.760	.830	6.40	.792
	108	322	12.81	5.45	208	27.85	4.83	7.14	.886	.764	.620	6.95	.695
	150	321	13.12	5.55	208	28.21	4.92	7.29	.886	.762	.630	6.15	.800
	212	322	12.50	5.20	205	27.48	4.71	6.96	.906	.747	.490	5.70	.826
	266	320	13.73	5.80	204	28.76	5.17	7.60	.891	.763	.630	6.30	.820
	333	320	13.57	5.80	210	28.21	5.10	7.51	.879	.772	.700	6.30	.809
	390	320	14.03	6.00	211	28.29	5.15	7.77	.858	.772	.850	6.50	.792

TABLE A-10. INVERTER POWER CHARACTERISTICS

INVERTER:		NATURAL COMMUTATED INVERTER													
NOMINAL FREQUENCY (Hz)	11	INPUT							OUTPUT						
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	df	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff		
		327	2.4	1.36	.90	.66	.74	38	18.5	1.22	.32	.36	.05		
		327	2.7	1.53	.98	.64	.74	38	18.9	1.24	.48	.49	.24		
		326	3.1	1.75	1.12	.64	.76	38	19.0	1.25	.64	.57	.36		
		325	3.7	2.08	1.2	.58	0.80	38	20.0	1.32	.76	.63	.44		
		325	4.0	2.25	1.42	.63	.76	38	21.0	1.38	.92	.65	.48		
		325	4.5	2.53	1.6	.63	.76	38	22.5	1.48	1.08	.68	.51		
		324	5.1	2.86	1.92	.67	.72	38	25.0	1.65	1.24	.65	.51		

TABLE A-11. INVERTER POWER CHARACTERISTICS

INVERTER:		NATURAL COMMUTATED INVERTER										
		INPUT						OUTPUT				
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	PF	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff	
15	326	2.7	1.52	.98	.64	52	19.0	1.71	.36	.37	.05	
	326	3.0	1.69	1.04	.61	52	19.0	1.71	.48	.46	.33	
	326	3.5	1.98	1.14	.58	52	19.3	1.74	.72	.63	.44	
	326	4.1	2.31	1.42	.61	52	20.0	1.8	.92	.65	.51	
	326	4.6	2.60	1.70	.66	52	21.0	1.89	1.16	.68	.53	
	326	5.1	2.88	1.92	.67	52	22.5	2.03	1.32	.69	.58	
	326	5.8	3.27	2.22	.68	52	24.5	2.21	1.56	.70	.58	
	325	6.4	3.60	2.44	.68	52	26.0	2.34	1.76	.72	.60	
	325	7.1	4.00	2.82	.71	52	28.5	2.57	2.0	.71	.59	

TABLE A-12. INVERTER POWER CHARACTERISTICS

INVERTER:		NATURAL COMMUTATED INVERTER												
NOMINAL FREQUENCY (Hz)	20	INPUT						OUTPUT						
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	df	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff	
		327	2.8	1.59	1.02	.64	.73	69	18.5	2.21	.44	.43	0	
		327	3.5	1.98	1.14	.58	.81	69	19.0	2.27	.64	.56	.34	
		327	4.1	2.32	1.42	.61	.79	69	19.3	2.31	.80	.56	.53	
		327	4.7	2.66	1.74	.65	.74	69	20.0	2.39	1.12	.64	.56	
		327	5.3	3.00	2.0	.67	.74	69	21.1	2.52	1.36	.68	.61	
		326	6.0	3.39	2.3	.68	.72	69	22.5	2.69	1.68	.73	.62	
		325	6.7	3.77	2.66	.71	.69	69	24.2	2.89	2.0	.75	.61	
		325	7.4	4.17	3.0	.72	.67	69	26.5	3.17	2.2	.83	.65	
		325	8.1	4.56	3.42	.75	.63	69	28.0	3.35	2.48	.73	.65	
		325	8.9	5.01	3.7	.74	.65	69	30.1	3.60	2.76	.75	.66	
		324	9.8	5.50	4.22	.77	.62	69	32.5	3.88	3.12	.74	.64	

TABLE A-13. INVERTER POWER CHARACTERISTICS

INVERTER:		NATURAL COMMUTATED INVERTER													
NOMINAL FREQUENCY (Hz)	25	INPUT							OUTPUT						
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff			
	329	3.0	1.71	1.06	.62	87	18.8	2.83	.48	.45	.13				
	329	3.6	2.05	1.4	.68	87	18.9	2.85	.68	.49	.39				
	329	4.4	2.51	1.6	.64	87	19.5	2.94	1.0	.63	.53				
	329	5.1	2.91	1.9	.65	87	20.0	3.01	1.32	.70	.60				
	329	5.9	3.36	2.3	.68	87	21.2	3.19	1.72	.75	.61				
	327	6.6	3.74	2.7	.72	87	22.5	3.39	2.04	.76	.64				
	327	7.4	4.19	3.0	.72	87	24.0	3.62	2.32	.77	.67				
	326	8.2	4.63	3.5	.76	87	25.6	3.86	2.64	.75	.68				
	327	9.0	5.10	3.9	.77	87	27.5	4.14	2.96	.76	.69				
	326	9.9	5.59	4.3	.77	87	29.5	4.45	3.28	.76	.70				
	326	10.8	6.10	4.66	.76	87	31.9	4.81	3.68	.79	.69				

INVERTER:		NATURAL COMMUTATED INVERTER											
NOMINAL FREQUENCY (Hz)	30	INPUT						OUTPUT					
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	PF	df	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT EFF
	327	3.2	1.81	1.06	.56	.80	104	18.5	3.33	.56	.53	.15	
	327	3.8	2.15	1.4	.65	.74	104	18.9	3.40	.76	.54	.42	
	327	4.7	2.66	1.7	.64	.75	104	19.3	3.48	1.16	.68	.55	
	326	5.5	3.11	2.12	.68	.71	104	20.0	3.60	1.56	.74	.61	
	326	6.4	3.61	2.6	.72	.67	104	21.0	3.78	1.88	.72	.67	
	325	7.2	4.05	3.0	.74	.65	104	22.2	4.00	2.28	.76	.69	
	325	8.2	4.62	3.5	.76	.62	104	24.0	4.32	2.72	.78	.69	
	325	9.2	5.18	3.9	.75	.64	104	25.5	4.59	3.08	.79	.71	
	325	10.1	5.69	4.34	.76	.63	104	27.4	4.94	3.40	.78	.73	
	325	11.1	6.25	4.8	.77	.62	104	29.5	5.31	3.84	.80	.72	
	325	12.0	6.75	5.3	.79	.59	104	31.5	5.67	4.28	.81	.71	

TABLE A-15. INVERTER POWER CHARACTERISTICS

INVERTER:		NATURAL COMMUTATED INVERTER													
NOMINAL FREQUENCY (Hz)	35	INPUT							OUTPUT						
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff			
		328	3.4	1.93	1.2	.62	121	18.5	3.88	.56	.47	.20			
		327	4.1	2.32	1.42	.61	121	18.8	3.94	.88	.62	.42			
		327	5.1	2.89	1.9	.66	121	19.2	4.02	1.28	.67	.58			
		326	6.1	3.44	2.4	.70	121	20.0	4.19	1.72	.72	.64			
		325	7.1	4.00	2.82	.71	121	21.0	4.4	2.16	.77	.68			
		325	8.1	4.56	3.32	.73	121	22.3	4.67	2.64	.80	.70			
		325	9.1	5.12	3.9	.76	121	23.8	4.99	3.08	.79	.71			
		325	10.1	5.69	4.32	.76	121	25.5	5.34	3.52	.82	.73			
		325	11.2	6.30	4.88	.77	121	27.3	5.72	3.96	.81	.73			
		325	12.2	6.87	5.3	.77	121	29.2	6.12	4.4	.83	.74			
		325	13.3	7.49	6.0	.80	121	31.4	6.58	4.88	.81	.74			

TABLE A-16. INVERTER POWER CHARACTERISTICS

INVERTER:		NATURAL COMMUTATED INVERTER											
NOMINAL FREQUENCY (Hz)	40	INPUT						OUTPUT					
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	df	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff
		328	3.5	1.99	1.36	.68	.71	139	18.8	4.53	.64	.47	.20
		327	4.4	2.49	1.6	.64	.75	139	18.9	4.55	.96	.60	.44
		327	5.5	3.12	2.1	.67	.72	139	19.5	4.69	1.44	.69	.59
		327	6.6	3.74	2.6	.70	.69	139	20.3	4.89	1.84	.71	.69
		326	7.7	4.35	3.18	.73	.66	139	21.0	5.06	2.4	.76	.70
		325	8.8	4.95	3.7	.75	.64	139	22.5	5.42	3.0	.81	.70
		326	9.9	5.59	4.24	.76	.63	139	23.7	5.71	3.48	.82	.72
		325	11.1	6.25	4.8	.77	.62	139	25.2	6.07	3.96	.83	.74
		325	12.1	6.81	5.3	.78	.60	139	26.9	6.48	4.4	.74	.75
		325	13.2	7.4	5.92	.80	.58	139	28.8	6.93	4.92	.83	.76
		325	14.5	8.16	6.5	.80	.57	139	30.8	7.42	5.44	.84	.76

TABLE A-17. INVERTER POWER CHARACTERISTICS

INVERTER: NOMINAL FREQUENCY (Hz)		NATURAL COMMUTATED INVERTER									
		INPUT					OUTPUT				
V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	df
Volts	Amps	KVA	KW		Volts	Amps	KVA	KW			
326	3.7	2.09	1.4	.67	156	18.6	5.03	.72	.51	.20	.68
326	4.6	2.60	1.78	.69	156	19.0	5.13	1.12	.63	.43	.62
325	5.8	3.26	2.30	.71	156	19.3	5.21	1.6	.70	.60	.7
325	7.1	4.00	2.80	.70	156	20.0	5.40	2.2	.79	.65	.6
325	8.2	4.62	3.50	.76	156	21.0	5.67	2.72	.78	.70	.68
325	9.5	5.35	4.0	.76	156	22.4	6.05	3.36	.84	.70	.64
325	10.8	6.08	4.62	.76	156	23.7	6.40	3.88	.84	.73	.74
325	12.0	6.75	5.3	.79	156	25.5	6.89	4.4	.83	.75	.9
325	13.2	7.43	5.88	.79	156	27.0	7.30	4.96	.84	.76	.92
324	14.5	8.14	6.48	.80	156	29.0	7.84	5.52	.85	.76	.92
324	15.7	8.81	7.14	.81	156	30.7	8.29	6.04	.85	.77	1.1

TABLE A-18. INVERTER POWER CHARACTERISTICS

INVERTER:		NATURAL COMMUTATED INVERTER													
NOMINAL FREQUENCY (Hz)	50	INPUT							OUTPUT						
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	dF	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT EFF		
		326	3.8	2.15	1.4	.65	.74	174	18.5	5.58	.8	.57	.13		
		326	4.9	2.77	1.8	.65	.75	174	18.9	5.70	1.28	.71	.41		
		326	6.2	3.5	2.42	.69	.70	174	19.2	5.79	1.88	.78	.56		
		325	7.6	4.28	3.12	.73	.66	174	20.0	6.03	2.44	.78	.65		
		325	8.9	5.01	3.82	.76	.62	174	21.1	6.36	3.04	.80	.69		
		325	10.2	5.74	4.5	.78	.60	174	22.3	6.72	3.72	.83	.71		
		325	11.5	6.47	5.06	.78	.60	174	23.8	7.17	4.28	.85	.74		
		325	12.9	7.26	5.70	.79	.59	174	25.3	7.62	4.88	.86	.75		
		324	14.2	7.97	6.36	.80	.57	174	26.9	8.11	5.44	.86	.77		
		324	15.6	8.75	7.04	.80	.56	174	28.8	8.68	6.08	.86	.77		
		324	16.9	9.48	7.78	.82	.54	175	30.6	9.27	6.72	.86	.77		

TABLE A-19. INVERTER POWER CHARACTERISTICS

INVERTER:		NATURAL COMMUTATED INVERTER																
		INPUT							OUTPUT									
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub> Volts	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	V <sub>AC</sub> Volts	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	
		Amps	KVA	KW		Volts	Amps	KVA	KW					Volts	Amps	KVA	KW	
55	329	4.0	2.28	1.42	.62	190	18.5	6.09	.88	.62	.20	190	18.5	6.09	.88	.62	.20	
	329	5.0	2.85	1.9	.67	190	18.5	6.09	1.36	.72	.43	190	18.5	6.09	1.36	.72	.43	
	327	6.5	3.68	2.54	.69	190	19.0	6.25	2.0	.79	.59	190	19.0	6.25	2.0	.79	.59	
	326	8.0	4.52	3.32	.74	190	20.0	6.58	2.64	.80	.67	190	20.0	6.58	2.64	.80	.67	
	326	9.6	5.42	4.08	.75	190	21.0	6.91	3.32	.81	.70	190	21.0	6.91	3.32	.81	.70	
	325	11.0	6.19	4.8	.78	190	22.3	7.34	4.04	.84	.72	190	22.3	7.34	4.04	.84	.72	
	325	12.4	6.98	5.5	.79	190	23.5	7.73	4.72	.86	.74	190	23.5	7.73	4.72	.86	.74	
	325	14.0	7.88	6.26	.79	189	25.0	8.18	5.4	.86	.75	189	25.0	8.18	5.4	.86	.75	
	325	15.4	8.67	7.0	.81	189	27.0	8.84	6.04	.86	.77	189	27.0	8.84	6.04	.86	.77	
	324	16.7	9.37	7.7	.82	190	29.0	9.54	6.68	.87	.78	190	29.0	9.54	6.68	.87	.78	
	324	18.0	10.1	8.42	.83	190	31.0	10.2	7.36	.87	.79	190	31.0	10.2	7.36	.87	.79	

TABLE A-20. INVERTER POWER CHARACTERISTICS

INVERTER:		NATURAL COMMUTATED INVERTER												
NOMINAL FREQUENCY (Hz)	60	INPUT						OUTPUT						
		V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	
		Volts	Amps	KVA	KW			Volts	Amps	KVA	KW			
		327	4.1	2.32	1.52	.66	.67	209	18.5	6.70	1.0	.66	.09	
		327	5.3	3.0	2.1	.70	.69	207	18.5	6.63	1.56	.74	.41	
		326	7.0	3.95	2.8	.71	.68	208	19.3	6.95	2.2	.79	.58	
		325	8.6	4.84	3.58	.74	.65	209	20.1	7.28	3.0	.84	.64	
		325	10.1	5.69	4.3	.76	.63	207	20.8	7.46	3.64	.85	.70	
		325	11.6	6.53	5.04	.77	.61	208	22.0	7.92	4.32	.86	.74	
		325	13.3	7.49	5.9	.79	.58	208	23.5	8.47	5.08	.86	.75	
		325	14.8	8.33	6.52	.80	.57	208	25.0	9.01	5.84	.88	.76	
		324	16.2	9.09	7.50	.83	.53	209	27.0	9.77	6.56	.88	.77	
		324	17.6	9.88	8.28	.84	.50	208	28.5	10.3	7.24	.87	.78	
		323	18.8	10.5	9.02	.86	.46	208	30.5	11.0	7.96	.88	.79	

TABLE A-21. INVERTER POWER CHARACTERISTICS

INVERTER:		NATURAL COMMUTATED INVERTER													
NOMINAL FREQUENCY (Hz)	80	INPUT							OUTPUT						
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff			
		326	3.4	1.92	1.2	.63	210	13.9	5.06	1.0	.83	.07			
		325	5.5	3.10	2.1	.68	210	14.5	5.27	1.84	.88	.46			
		324	7.5	4.21	2.8	.67	210	15.5	5.64	2.76	.99	.61			
		324	10.1	5.67	4.2	.74	210	17.3	6.29	3.76	.90	.67			
		324	12.3	6.9	5.28	.77	210	19.5	7.09	5.12	.97	.66			
		324	14.3	8.02	6.3	.79	210	21.5	7.82	5.6	.89	.75			
		323	16.4	9.17	7.4	.81	210	24.0	8.73	6.56	.89	.77			
		320	18.4	10.2	8.44	.83	210	27.0	9.82	7.56	.90	.77			
		320	20.0	11.1	9.68	.87	210	30.5	11.1	8.56	.88	.78			
		320	22.6	12.5	10.9	.87	210	34.0	12.3	9.64	.88	.78			

TABLE A-22. INVERTER POWER CHARACTERISTICS

INVERTER:	NATURAL COMMUTATED INVERTER													
	INPUT							OUTPUT						
	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff	df		
100	326	2.5	1.41	1.14	.81	210	11.3	4.11	1.08	.95	.12	.06		
	326	6.0	3.39	2.3	.68	210	12.4	4.51	2.12	.92	.50	.18		
	325	8.9	5.01	3.7	.74	210	14.5	5.27	3.36	.91	.63	.34		
	325	11.8	6.64	5.02	.76	210	17.3	6.29	4.6	.92	.69	.42		
	324	14.5	8.14	6.42	.79	210	20.7	7.53	5.72	.89	.73	.70		
	324	17.5	9.82	7.76	.79	210	24.0	8.73	6.8	.88	.77	.86		
	321	19.9	11.1	9.22	.83	210	29.3	10.7	8.24	.89	.76	.98		

TABLE A-23. INVERTER POWER CHARACTERISTICS

INVERTER: NOMINAL FREQUENCY (Hz)	NATURAL COMMUTATED INVERTER													
	INPUT							OUTPUT						
	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff			
110	326	2.5	1.41	1.15	.82	210	10.5	3.82	1.2	-	.18			
	326	6.0	3.39	2.4	.71	210	11.6	4.22	2.32	.97	.51			
	326	9.5	5.36	3.94	.74	210	14.5	5.27	3.64	.92	.64			
	325	12.7	7.15	5.5	.77	210	18.0	6.55	5.04	.92	.69			
	324	16.0	8.98	7.2	.80	210	22.6	8.22	6.44	.89	.72			
	323	19.1	10.7	8.82	.83	210	28.5	10.4	7.88	.89	.73			



TABLE A-25. PULSE-WIDTH MODULATED INVERTER,  
POWER CHARACTERISTICS AT LOAD =  $24\Omega$ , PF = 1.0

1	2	3	4	5	6	7	8	9	10
Output Volts RMS	Output Freq Hz	Input Current A <sub>RMS</sub>	Input Power Watts	Output Current A <sub>RMS</sub>	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power VA
50	25	2.34	485	1.1	105	812	.60	21.7	95
	50	2.68	660	1.1	105	929	.71	17.4	95
	100	3.13	800	1.1	105	1685	.74	14.4	95
	150	3.63	980	1.1	105	1250	.78	11.7	99
	200	4.15	1135	1.2	105	1935	.79	10.6	110
	250	3.88	1045	1.1	105	1340	.78	11.0	103
	300	4.32	1210	1.0	105	1495	.81	9.5	87
	350	4.82	1370	1.1	105	1670	.82	8.8	101
400	5.23	1530	1.1	105	1810	.85	6.9	96	
100	25	3.26	835	2.4	417	1130	.74	46.7	416
	50	3.91	1055	2.4	417	1356	.78	37.0	416
	100	5.22	1510	2.3	417	1810	.84	25.8	398
	150	4.75	1355	2.3	417	1640	.83	28.8	398
	200	5.51	1610	2.3	417	1910	.84	24.2	398
	250	4.82	1380	2.3	417	1665	.84	28.2	398
	300	5.14	1490	2.3	417	1780	.84	25.2	398
	350	5.56	1630	2.3	417	1920	.85	23.9	398
400	6.10	1805	2.3	417	2110	.86	21.6	398	
100	25	4.82	1395	3.6	935	1670	.84	66.2	935
	50	5.45	1620	3.6	935	1890	.86	57.0	935
	100	6.83	2080	3.6	935	2360	.88	44.4	935
	150	6.31	1940	3.6	935	2185	.89	47.6	935
	200	7.07	2175	3.6	935	2450	.89	42.4	935
	250	6.32	1935	3.6	935	2190	.89	47.7	935
	300	7.42	2080	3.6	935	2570	.81	44.5	935
	350	7.23	2230	3.6	935	2510	.89	41.5	935
400	7.70	2380	3.6	935	2670	.89	38.8	935	

TABLE A-26. PULSE-WIDTH MODULATED INVERTER,  
POWER CHARACTERISTICS AT LOAD =  $6\Omega$ , PF = 1.0

1	2	3	4	5	6	7	8	9	10
Output Volts RMS	Output Freq Hz	Input Current A <sub>RMS</sub>	Input Power Watts	Output Current A <sub>RMS</sub>	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power VA
50	25	3.32	810	4.5	440	1130	.72	59.2	382
	50	3.69	980	4.7	430	1260	.78	43.8	399
	100	4.13	1150	4.7	430	1410	.82	37.4	399
	150	4.84	1340	4.8	470	1660	.81	35.0	424
	200	5.16	1500	4.8	475	1760	.85	31.5	424
	250	4.87	1350	4.3	480	1650	.82	35.5	358
	300	5.78	1600	4.8	535	1970	.81	33.4	441
	400	6.44	1735	4.6	535	2220	.79	30.8	414
100	25	7.22	2150	9.8	1730	2460	.88	80.7	1698
	50	7.73	2340	9.6	1705	2640	.89	73.0	1663
	100	9.07	2745	9.3	1785	3090	.89	65.0	1611
	150	8.62	2600	9.4	1805	2940	.89	69.9	1628
	200	9.20	2805	9.1	1760	3140	.89	62.6	1576
	250	8.52	2575	9.2	1780	2910	.89	69.2	1593
	300	8.87	2690	9.1	1785	3030	.89	66.4	1576
	400	9.22	2800	9.0	1750	3150	.89	62.6	1559
150	25	9.62	2960	9.0	1750	3280	.90	59.3	1559
	50	14.1	4460	15.0	3910	4810	.93	87.7	3897
	100	14.9	4690	15.0	3940	5080	.92	83.8	3897
	150	16.3	5125	14.9	4150	5550	.92	80.9	3891
	200	15.8	4950	14.8	3960	5380	.92	80.0	3794
	250	16.6	5185	14.8	3990	5660	.92	77.0	3845
	300	15.8	4935	14.8	4100	5380	.92	83.1	3845
	400	16.2	5100	14.6	3980	5520	.92	78.0	3793
150	25	16.6	5245	14.6	4150	5650	.93	79.0	3793
	50	17.05	5365	14.5	4140	5810	.92	77.2	3767

TABLE A-27. PULSE-WIDTH MODULATED INVERTER,  
POWER CHARACTERISTICS AT LOAD =  $2.72\Omega$ , PF = 1.0

1	2	3	4	5	6	7	8	9	10
Output Volts RMS	Output Freq Hz	Input Current A <sub>RMS</sub>	Input Power Watts	Output Current A <sub>RMS</sub>	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power VA
50V	25	4.55	1220	9.16	880	1560	.78	72.2	793
	50	5.27	1440	9.47	910	1810	.80	63.2	820
	100	5.92	1620	9.47	900	2020	.80	55.5	820
	150	6.72	1890	9.78	1010	2300	.82	53.4	847
	200	7.10	1990	9.78	980	2430	.82	49.2	847
	250	6.72	1790	8.86	920	2300	.78	51.4	767
	300	7.82	2130	9.17	920	2680	.80	43.0	794
	400	8.73	2120	8.55	920	2990	.71	43.3	740
100V	25	9.47	2330	9.17	970	3250	.72	41.7	794
	50	12.9	4100	20.1	3570	4420	.93	87.5	3481
	100	13.65	4300	19.8	3630	4670	.93	84.5	3429
	150	14.8	4620	19.5	3670	5050	.92	79.8	3377
	200	15.3	4770	20.1	3900	5240	.91	82.0	3481
	250	15.5	4880	19.5	3870	5310	.92	79.0	3377
	300	14.8	4570	19.8	3770	5050	.90	82.3	3429
	400	15.3	4770	19.5	3910	5240	.91	82.2	3377
150V	25	16.0	5070	19.8	3990	5480	.93	78.8	3429
	50	16.0	5100	19.5	3930	5480	.93	77.2	3377
	100	28.0	8820	31.1	8030	9600	.92	91.1	8080
	150	28.7	9000	31.1	8060	9850	.91	89.5	8080
	200	30.0	9420	30.9	8090	10250	.92	85.7	8028
	250	29.8	9290	31.1	8120	10200	.91	87.5	8080
	300	30.6	9520	31.0	8160	10450	.91	85.7	8054
	400	29.8	9320	31.1	8120	10200	.91	87.2	8080
	200	30.2	9100	31.1	8160	10300	.89	89.0	8080
	350	32.4	10150	31.8	8640	11100	.92	85.0	8261
	400	31.7	990	31.4	8250	10800	.92	83.2	8157

TABLE A-28. PULSE-WIDTH MODULATED INVERTER,  
POWER CHARACTERISTICS AT LOAD -  $24\Omega$ , PF = 0.8

1	2	3	4	5	6	7	8	9	10	11	
Output Volts RMS	Output Freq. Hz	Input Current Arms	Input Power Watts	Output Current Arms	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power VA	Apparent Load Power Factor	
50	43	2.6	565	1.1	10	910	.62	.2	95		
	58	2.7	620	1.1	5	945	.66	.1	95		
	110	3.15	810	1.2	10	1102	.73	.1	103		
	130	3.45	875	1.25	15	1207	.73	.1	108		
	173	3.85	1020	1.30	10	1347	.76	.1	113		
	210	4.2	1165	1.40	10	1469	.79	.1	126		
	216	3.5	925	1.30	5	1225	.76	.1	117		
	288	4.2	1165	1.30	10	1470	.79	.1	122		
	344										
	398										
100	43	2.82	655	1.1	13	987	.66	1.9	190	.06	
	58	3.45	835	1.2	14	1207	.69	1.7	208	.07	
	110	4.6	1310	2.0	30	1609	.81	2.3	346	.09	
	130	3.7	965	1.3	40	1295	.75	4.2	225	.17	
	173	4.3	1205	1.5	60	1505	.80	4.9	260	.23	
	210	4.9	1390	1.62	50	1214	.81	3.5	281	.18	
	216	3.7	965	1.3	45	1297	.75	4.6	225	.20	
	288	4.2	1125	1.4	45	1476	.79	3.8	243	.19	
	344	4.7	1345	1.5	40	1644	.82	2.9	260	.15	
	398	5.28	1540	1.5	50	1847	.83	3.2	260	.19	
150	43	3.7	965	2.38	310	1295	.75	32.1	618	0.50	
	58	4.22	1165	2.50	335	1475	.79	28.8	650	0.52	
	110	5.53	1615	2.50	330	1935	.83	20.4	650	0.51	
	130	4.58	1290	2.50	330	1602	.87	25.6	650	0.51	
	173	5.28	1530	2.62	350	1842	.83	22.8	681	0.52	
	210	5.93	1720	2.62	340	2075	.83	19.8	681	0.50	
	216	4.72	1305	2.62	360	1662	.79	27.5	681	0.53	
	288	5.15	1408	2.38	310	1802	.82	20.9	618	0.50	
	344	5.6	1645	2.38	325	1959	.84	19.7	618	0.53	
	398	6.1	1835	2.45	335	2134	.86	18.2	637	0.53	

TABLE A-29. PULSE-WIDTH MODULATED INVERTER,  
POWER CHARACTERISTICS AT LOAD =  $6\Omega$ , PF = 0.8

1	2	3	4	5	6	7	8	9	10	11
Output Volts RMS	Output Freq Hz	Input Current A <sub>RMS</sub>	Input Power Watts	Output Current A <sub>RMS</sub>	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power VA	Apparent 10 AP Power Factor
100	43	3.65	950	4.68	255	1277	.74	26.8	810	.32
	64	4.06	1135	4.83	275	1420	.80	24.2	836	.33
	108	5.62	1520	4.68	265	1966	.77	17.4	810	.33
	134	4.48	1235	4.83	285	1567	.79	23.0	836	.34
	180	4.84	1430	4.40	245	1697	.84	17.1	761	.32
	202	5.65	1700	5.58	395	1977	.86	23.2	965	.41
	216	4.27	1215	4.83	315	1494	.81	25.9	836	.38
	259	4.65	1362	4.68	305	1627	.84	22.4	810	.38
	315	5.63	1530	4.9	330	1970	.78	21.4	848	.39
	404	6.18	1850	4.87	295	2162	.86	15.6	842	.35
150	43	6.58	1990	10.15	1250	2302	.86	63.0	2640	.47
	64	5.95	2475	11.20	1550	2756	.90	62.6	2912	.53
	108	8.48	2615	10.30	1305	2967	.88	49.9	2679	.49
	134	8.27	2535	11.10	1540	2893	.88	60.7	2886	.53
	180	8.8	2725	11.00	1570	3079	.89	55.4	2860	.53
	202	9.44	2980	11.40	1645	3303	.90	55.2	2964	.56
	216	8.03	2505	11.00	1545	2809	.89	60.4	2860	.54
	259	8.15	2545	10.65	1470	2851	.89	57.7	2769	.53
	314	8.73	2735	10.60	1480	3054	.90	54.1	2756	.54
	404	9.84	3060	10.78	1570	3443	.89	51.3	2803	.56

TABLE A-30. PULSE-WIDTH MODULATED INVERTER, POWER CHARACTERISTICS AT LOAD =  $2.72\Omega$ , PF = 0.8

1	2	3	4	5	6	7	8	9	10	11
Output Volts RMS	Output Freq Hz	Output Current A <sub>RMS</sub>	Input Power Watts	Output Current A <sub>RMS</sub>	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power VA	Apparent Load Power Factor
100	50	4.30	1220	9.00	470	1496	.82	38.5	1559	.30
	63	5.07	1480	10.32	600	1764	.89	40.5	1787	.34
	125	6.20	1930	9.72	530	2178	.89	27.6	1683	.31
	143	5.75	1690	10.40	580	2001	.85	34.3	1801	.32
	167	5.94	1800	10.00	620	2067	.87	34.4	1730	.36
	189	6.57	2020	10.80	720	2216	.89	35.6	1870	.38
	234	5.63	1660	10.29	690	1959	.85	41.6	1780	.39
	268	5.75	1700	9.61	600	2001	.85	35.3	1664	.36
	320	6.26	1920	9.80	650	2178	.88	33.0	1699	.38
	383	6.88	2170	9.92	680	2394	.91	31.3	1718	.40
150	50	12.05	3820	22.00	2920	4193	.91	76.4	5716	.51
	63	17.54	4400	23.60	3410	4570	.93	77.5	6131	.56
	125	17.52	5350	25.08	3920	6097	.88	73.2	6515	.60
	143	13.61	4360	22.80	3222	4736	.92	73.8	5923	.53
	167	13.77	4420	22.40	3170	4792	.92	71.7	5819	.55
	189	14.55	4660	22.80	3300	5063	.92	70.8	5923	.56
	234	13.14	4230	22.28	3120	4572	.93	73.7	5788	.54
	268	13.80	4450	22.40	3260	4802	.93	73.2	5819	.56
	320	14.52	4690	22.40	3360	5053	.93	71.6	5819	.58
	383	15.14	4890	23.08	3380	5269	.93	71.2	5996	.56

TABLE A-31. PULSE-WIDTH MODULATED INVERTER,  
POWER CHARACTERISTICS AT LOAD =  $24.0\Omega$ , PF = 0.5

1	2	3	4	5	6	7	8	9	10	11
Output Volts RMS	Output Freq Hz	Input Current A <sub>RMS</sub>	Input Power Watts	Output Current A <sub>RMS</sub>	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power VA	Apparent Load Power Factor
100V	42	2.75	640	1.73	25	963	.665	3.9	300	.09
	63	3.20	860	2.37	40	1120	.768	4.6	410	.10
	113	3.19	1315	3.28	35	1817	.724	3.4	190	.08
	125	3.36	935	2.28	45	1176	.795	4.8	395	.11
	179	4.27	1220	2.73	45	1495	.816	5.3	473	.14
	208	5.04	1340	2.91	30	1764	.760	3.7	504	.10
	249	3.97	1050	2.46	40	1390	.756	5.7	426	.14
	312	4.27	1235	2.64	30	1495	.826	5.7	457	.15
	373	4.88	1410	2.73	40	1708	.826	4.3	473	.13
	408	5.19	1530	2.91	40	1817	.842	3.9	504	.12
150V	42	3.05	815	2.55	170	1068	.763	20.8	663	.26
	63	3.66	1075	2.91	225	1281	.840	21.8	756	.32
	113	5.19	1520	3.09	205	1817	.836	14.8	803	.28
	125	3.81	1110	2.91	200	1334	.832	18.9	756	.28
	179	4.88	1410	3.28	230	1708	.826	17.7	852	.29
	208	5.19	1535	3.46	225	1817	.845	16.6	899	.28
	249	4.27	1235	3.00	195	1495	.826	18.2	779	.28
	312	4.88	1470	3.28	200	1708	.861	18.4	852	.32
	373	5.34	1610	3.09	210	1869	.861	18.9	803	.30
	408	5.80	1730	3.28	185	2030	.852	13.6	852	.28

TABLE A-32. PULSE-WIDTH MODULATED INVERTER,  
POWER CHARACTERISTICS AT LOAD =  $6\Omega$ , PF = 0.5

1	2	3	4	5	6	7	8	9	10	11
Output Volts RMS	Output Freq HZ	Input Current A <sub>RMS</sub>	Input Power Watts	Output Current A <sub>RMS</sub>	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power VA	Apparent Load Power Factor
100V	31	2.75	680	3.91	140	963	.71	20.6	667	.21
	67	3.97	1000	4.63	170	1390	.72	17.0	801	.21
	115	5.80	1490	5.09	155	2630	.73	10.4	881	.18
	124	3.66	1055	4.19	165	1281	.82	15.6	726	.23
	156	4.58	1245	4.55	170	1603	.78	13.7	788	.22
	206	5.49	1540	5.09	225	1922	.80	14.6	881	.26
	232	3.97	1075	3.82	120	1390	.77	11.2	661	.18
	292	4.58	1300	4.00	160	1603	.81	12.3	642	.23
	305	5.49	1560	4.50	200	1922	.81	12.8	788	.25
	150	31	4.73	1395	9.28	820	1656	.84	58.8	2410
67		6.41	1945	10.47	1020	2244	.87	52.4	2720	.38
115		6.86	2110	8.92	800	2401	.88	38.0	2317	.35
124		6.41	1885	10.01	1005	2244	.84	53.3	2600	.39
156		7.02	2130	10.56	1050	2457	.87	49.3	2743	.38
206		7.32	2270	9.83	935	2562	.89	41.2	2553	.37
232		5.19	1535	8.01	600	1816	.85	39.0	2080	.29
292		6.71	2040	9.10	855	2349	.87	41.9	2364	.36
365		7.63	2330	9.65	950	2671	.87	40.7	2507	.38

TABLE A-33. PULSE-WIDTH MODULATED INVERTER,  
POWER CHARACTERISTICS AT LOAD =  $2.72\Omega$ , PF = 0.5

1	2	3	4	5	6	7	8	9	10	
Output Volts RMS	Output Freq Hz	Input Current A <sub>RMS</sub>	Input Power Watts	Output Current A <sub>RMS</sub>	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power VA	Apparent Load Power Factor
100	92	3.66	1000	9.46	350	1279	.78	35.0	1639	.21
	71	4.88	1320	10.01	380	1705	.77	28.8	1735	.22
	118	7.02	1950	12.14	510	2453	.80	26.1	2107	.24
	132	4.58	1300	8.55	300	1599	.81	23.0	1477	.20
	165	5.19	1470	9.46	320	1812	.81	21.8	1633	.20
	207	5.80	1510	7.10	250	2025	.75	16.6	1225	.20
	299	4.73	1370	8.92	340	1652	.79	25.9	1595	.22
	275	5.34	1550	10.56	470	1860	.83	30.2	1828	.26
	342	5.03	1360	7.64	270	1759	.77	19.8	1323	.20
	405	7.02	2020	11.10	520	2453	.82	25.7	1926	.27
150	42	7.93	2480	21.48	1700	2771	.90	68.5	5592	.30
	71	9.76	3120	23.48	2090	3411	.92	67.0	6239	.34
	118	14.64	4660	28.76	3030	5116	.91	66.4	7481	.41
	132	8.85	2770	21.48	1800	3091	.90	65.0	5590	.32
	165	9.76	3110	22.20	1880	3410	.91	60.5	5767	.33
	207	7.93	2450	15.65	1170	2771	.88	47.7	4062	.29
	244	8.99	2760	20.93	1730	3144	.88	62.7	5440	.31
	275	10.37	3250	22.93	2080	3624	.90	64.0	5960	.35
	342	7.93	2460	17.11	1360	2771	.89	55.2	4444	.31
	405	11.29	3540	22.39	2020	3944	.90	57.6	5821	.35

TABLE A-34. PULSE-WIDTH MODULATED INVERTER,  
POWER CHARACTERISTICS AT LOAD =  $24\Omega$ , PF = AS LISTED

1	2	3	4	5	6	7	8	9	10
Output Volts RMS	Output Freq Hz	Input Current A <sub>RMS</sub>	Input Power Watts	Output Current A <sub>RMS</sub>	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power VA
208V @ PF=1.0	25	7.34	2265	5.3	1975	2540	.89	87.0	1920
	50	7.44	2285	5.3	1975	2580	.89	86.0	1920
	100	7.62	2325	5.3	1975	2640	.88	85.0	1920
	150	7.77	2385	5.3	1975	2690	.88	82.7	1920
	200	7.89	2430	5.3	1975	2730	.89	81.2	1920
	250	8.07	2480	5.3	1975	2790	.89	79.5	1920
	300	8.23	2525	5.3	1975	2850	.89	78.2	1920
	350	8.38	2590	5.3	1975	2900	.89	76.2	1920
	400	8.54	2650	5.3	1975	2960	.89	74.5	1920
	208V @ PF=0.8	43	6.05	1825	5.08	1580	2117	.86	81.0
58		6.42	1935	5.34	1553	2246	.86	80.2	1942
110		6.58	1945	5.34	1535	2302	.85	78.9	1942
130		6.60	1980	5.40	1555	2309	.86	78.5	1955
173		6.44	1960	5.20	1475	2253	.87	75.2	1891
210		7.00	2110	5.50	1600	2449	.86	75.8	2000
216		7.05	1935	5.50	1605	2467	.86	82.9	1972
288		6.56	1985	4.90	1375	2295	.87	69.2	1757
344		7.03	2020	5.15	1510	2460	.84	72.9	1846
398		7.20	2215	5.15	1510	2520	.88	68.1	1846
208V @ PF=0.5	42	4.27	1215	4.55	865	1495	.81	72.8	1592
	63	4.27	1275	4.64	980	1495	.85	72.9	1623
	113	4.58	1375	4.73	915	1603	.86	70.2	1655
	125	4.58	1385	4.82	925	1603	.86	71.1	1687
	179	4.88	1435	4.73	910	1708	.84	68.3	1654
	208	4.88	1450	4.73	910	1708	.85	66.2	1655
	249	5.03	1500	4.73	905	1361	.85	65.0	1655
	312	5.80	1725	5.49	745	2031	.85	65.2	1921
373	5.49	1675	4.73	900	1922	.87	60.2	1655	
408	5.49	1685	4.73	825	1922	.88	58.4	1655	

TABLE A-35. PULSE-WIDTH MODULATED INVERTER, LISTED  
 POWER CHARACTERISTICS AT LOAD =  $6\Omega$ , PF = AS LISTED

1	2	3	4	5	6	7	8	9	10
Output Volts RMS	Output Freq Hz	Input Current A <sub>RMS</sub>	Input Power Watts	Output Current A <sub>RMS</sub>	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power VA
208V @ PF=1.0	25	26.2	8140	20.8	7610	8940	.91	93.7	7349
	50	26.0	8175	20.9	7620	8870	.92	93.2	7385
	100	27.2	8225	21.0	7620	9280	.84	92.8	7420
	150	27.6	8280	21.0	7620	9470	.88	92.1	7420
	200	27.8	8325	21.1	7610	9470	.88	91.7	7455
	250	26.6	8380	21.1	7620	9080	.92	91.0	7455
	300	26.9	8460	21.1	7610	9170	.92	90.1	7455
	350	27.1	8500	21.1	7630	9250	.92	89.7	7455
	400	27.2	8550	21.2	7630	9280	.92	89.2	7490
	208V @ PF=0.8	43	21.4	6465	20.6	5915	7427	.87	91.5
64		20.8	6635	21.0	6030	7191	.92	90.8	7815
108		20.7	6550	21.2	5900	7126	.92	90.1	7770
134		21.0	6565	21.2	5915	7157	.92	90.1	7810
180		20.8	6680	21.4	5990	7264	.92	89.7	7865
202		21.0	6115	21.4	5940	7191	.92	89.7	7810
216		21.0	6700	21.5	5995	7264	.92	89.5	7865
259		21.0	6710	21.8	5945	7264	.92	88.6	7740
315		21.2	6785	21.5	5905	7340	.92	88.0	7725
404		21.9	6975	21.8	6055	7569	.92	87.1	7740
208V @ PF=0.5	31	13.0	4100	20.6	3625	4536	.90	90.6	7411
	67	13.1	4170	20.6	3655	4592	.91	87.6	7411
	115	13.4	4200	20.4	3640	4697	.89	86.6	7342
	124	13.1	4100	20.2	3560	4597	.89	86.8	7242
	156	13.1	4215	20.6	3630	4697	.90	86.1	7411
	206	13.4	4305	20.4	3665	4697	.92	85.1	7342
	232	8.5	2660	16.8	2075	2958	.88	78.0	5715
	292	13.7	4310	20.0	3580	4806	.90	83.1	7143
	365	15.0	4795	21.1	3990	5233	.92	83.2	7422

TABLE A-36. PULSE-WIDTH MODULATED INVERTER,  
POWER CHARACTERISTICS AT LOAD =  $2.72\Omega$ , PF = AS LISTED

1	2	3	4	5	6	7	8	9	10
Output Volts RMS	Output Freq Hz	Input Currents A <sub>RMS</sub>	Input Power Watts	Output Current A <sub>RMS</sub>	Output Power Watts	Input Apparent Power VA	Input Power Factor	Efficiency %	Output Apparent Power
208V @ PF=1.0	25	50.0	15,760	43.20	14,530	17,100	.92	92.4	15,114
	50	50.2	15,780	43.70	14,500	17,200	.92	91.8	15,289
	100	50.5	15,890	43.70	14,530	17,300	.92	91.8	15,114
	150	50.5	15,890	43.20	14,570	17,300	.92	91.8	15,114
	200	50.2	15,900	43.20	14,570	17,200	.92	91.7	15,114
	250	50.6	15,980	43.20	14,570	17,300	.92	91.0	15,114
	300	50.6	16,010	43.20	14,560	17,300	.93	91.0	15,114
	350	50.8	16,050	43.20	14,560	17,400	.92	90.8	15,114
	400	52.7	16,070	43.20	14,600	18,100	.89	90.9	15,114
	208V @ PF=0.8	50	42.1	13,330	43.68	12,080	14,418	.93	90.0
63		41.8	13,170	43.68	12,020	14,459	.91	91.2	15,408
125		43.9	13,930	44.98	12,650	15,196	.92	90.8	15,892
143		42.1	13,360	44.28	12,110	17,563	.92	90.6	15,617
167		42.7	13,360	43.68	12,090	14,774	.90	90.5	15,357
189		45.1	14,260	45.44	12,910	15,618	.91	90.6	15,976
234		41.8	13,300	44.20	11,970	14,459	.92	90.0	15,540
268		44.2	14,000	44.98	12,620	15,304	.92	90.0	15,581
320		42.4	13,380	43.34	11,970	14,670	.91	90.2	14,862
383		40.3	12,790	43.52	11,370	13,930	.92	88.9	14,321
208V @ PF=0.5	42	25.9	8260	45.32	8160	9060	.91	98.7	16,170
	71	25.9	8260	44.95	7320	9060	.91	89.0	16,115
	133	25.6	8200	44.77	7240	8953	.92	88.3	16,051
	165	28.4	9050	47.32	8190	9912	.91	90.5	16,965
	207	15.9	5080	32.76	4280	5542	.92	84.3	11,745
	244	25.9	8270	44.77	7280	9060	.91	88.0	16,051
	275	29.9	9560	48.11	8370	10,445	.92	87.5	17,356
	342	18.9	6080	36.04	3810	6608	.92	62.6	12,921
	405	27.5	8900	46.03	7700	9700	.92	86.5	15,552

TABLE A-37. INVERTER POWER CHARACTERISTICS

INVERTER:		PULSE WIDTH MODULATED													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	df		
	Volts	Amps	KVA	KW			Volts	Amps	KVA	KW	%	%			
	203	5.0	1.76	1.33	.76	.628	91.3	20.2	3.2	.96	.72	.23	.92		
	203	5.3	1.86	1.40	.75	.643	91.3	21.3	3.37	1.12	.80	.29	.92		
	203	5.7	2.00	1.67	.84	.53	91.8	23.0	3.66	1.28	.77	.34	.92		
	203	6.3	2.22	1.90	.86	.51	91.8	26.0	4.14	1.44	.76	.37	.88		
	202	7.1	2.50	2.19	.88	.47	91.8	29.2	4.64	1.70	.77	.36	.91		
	201	8.3	2.92	2.58	.88	.46	91.8	35.1	5.58	2.04	.79	.31	.91		

TABLE A-38. INVERTER POWER CHARACTERISTICS

INVERTER:		PULSE WIDTH MODULATED													
NOMINAL FREQUENCY (Hz)	15	INPUT							OUTPUT						
		V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	PF	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff		
		Volts	Amps	KVA	KW			Volts	Amps	KVA	KW	%	%		
		202	-	-	1.17	-	-	107	19.6	3.63	0.80	.68	-		
		202	5.0	1.76	1.39	.80	.60	107	19.8	3.67	0.90	.65	.17		
		202	5.5	1.93	1.61	.83	.52	108	20.6	3.81	1.16	.72	.28		
		202	6.1	2.14	1.83	.86	.50	107	21.6	4.0	1.36	.74	.34		
		202	6.7	2.36	2.03	.86	.49	107	23.2	4.3	1.56	.77	.39		
		202	7.3	2.57	2.30	.89	.44	107	25.0	4.63	1.78	.77	.42		
		202	8.2	2.88	2.52	.88	.47	107	27.2	5.03	2.0	.79	.45		
		302	9.1	3.20	2.80	.88	.47	107	30.2	5.59	2.24	.80	.45		
		202	10.2	3.59	3.17	.88	.46	107	34.3	6.35	2.56	.81	.44		
		201	11.7	4.11	3.68	.90	.44	107	40.4	7.48	3.04	.83	.39		

TABLE A-39. INVERTER POWER CHARACTERISTICS

INVERTER:		PULSE WIDTH MODULATED													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	df	V <sub>AC</sub>	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff %	MUT Eff %	df		
20	203	4.8	1.69	1.33	.79	.59	124	19.5	4.19	.84	.63	-			
	203	5.4	1.90	1.58	.83	.53	124	19.8	4.25	1.02	.65	.20			
	203	6.2	2.18	1.86	.85	.50	124	20.4	4.38	1.34	.72	.31	.88		
	203	6.8	2.39	2.17	.91	.40	124	21.3	4.58	1.56	.72	.40	.88		
	203	7.7	2.71	2.41	.89	.44	124	22.8	4.90	1.88	.78	.44	.87		
	203	8.7	3.06	2.70	.88	.46	124	24.6	5.29	2.16	.80	.47	.86		
	202	9.6	3.38	2.97	.88	.46	124	26.6	5.72	2.40	.81	.50	.85		
	202	10.5	3.69	3.24	.88	.47	124	29.1	6.26	2.70	.83	.51	.85		
	202	11.6	4.08	3.66	.90	.43	124	31.8	6.33	3.02	.83	.52	.84		
	202	12.8	4.50	4.05	.90	.43	124	35.3	6.84	3.38	.83	.51	.80		
	202	14.1	4.96	4.53	.91	.40	124	39.8	8.56	3.76	.83	.49	.84		

TABLE A-40. INVERTER POWER CHARACTERISTICS

INVERTER:		PULSE WIDTH MODULATED													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff			
25	Volts	Amps	KVA	KW			Volts	Amps	KVA	KW					
	202	5.1	1.79	1.45	.81	.56	138	19.6	4.69	.88	.61	.05			
	202	6.0	2.1	1.8	.86	.49	138	20.0	4.69	1.2	.67	.22			
	202	7.0	2.45	2.15	.88	.47	138	20.1	4.82	1.5	.70	.35			
	202	8.0	2.8	2.42	.86	.49	138	20.9	5.00	1.84	.76	.43			
	204	8.9	3.11	2.79	.90	.43	138	22.1	5.29	2.16	.77	.48			
	204	10.0	3.50	3.07	.88	.47	138	23.6	5.65	2.48	.81	.52			
	204	10.9	3.81	3.40	.89	.44	138	25.5	6.1	2.8	.82	.55			
	204	12.0	4.20	3.80	.90	.42	138	27.6	6.61	3.16	.83	.58			
	202	13.1	4.59	4.2	.92	.39	138	30.6	7.32	3.48	.83	.58			
	202	14.4	5.04	4.6	.91	.40	138	33.5	8.02	3.85	.84	.58			
	202	15.7	5.50	5.02	.91	.40	138	37.0	8.86	4.32	.86	.57			

TABLE A-41. INVERTER POWER CHARACTERISTICS

INVERTER: NOMINAL FREQUENCY (Hz)		PULSE WIDTH MODULATED													
		INPUT							OUTPUT						
V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	df			
Volts	Amps	KVA	KW			Volts	Amps	KVA	KW	%	%				
205	5.5	1.93	1.6	.83	.52	153	19.9	5.28	0.9	.56	.09	.82			
205	6.6	2.31	2.06	.89	.44	152	20.5	5.41	1.38	.67	.23	.81			
204	7.7	2.73	2.41	.88	.46	152	21	5.54	1.76	.73	.36	.81			
204	9.0	3.15	2.8	.89	.44	152	22	5.78	2.16	.77	.44	.80			
204	10.2	3.57	3.2	.90	.43	151	23.4	6.12	2.52	.79	.50	.79			
203	11.1	3.88	3.54	.91	.41	151	24.6	6.45	2.88	.81	.54	.79			
204	12.4	4.34	3.97	.91	.40	151	26.3	6.89	3.26	.82	.57	.78			
203	13.6	4.76	4.4	.92	.38	151	28.4	7.44	3.68	.84	.59	.77			
203	15.0	5.26	4.8	.91	.40	151	30.6	8.01	4.02	.84	.61	.76			
203	16.2	5.67	5.22	.92	.38	151	33.3	8.72	4.44	.85	.61	.76			
203	17.8	6.23	5.77	.92	.38	151	36.3	9.50	4.92	.86	.61	.75			

TABLE A-42. INVERTER POWER CHARACTERISTICS

INVERTER: NOMINAL FREQUENCY (Hz)	PULSE WIDTH MODULATED													
	INPUT							OUTPUT						
	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	df	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff		
35	204	5.6	1.98	1.67	.84	.51	165	19.5	5.58	.94	.56	.05		
	204	7.0	2.47	2.13	.87	.48	164	19.9	5.66	1.44	.68	.26		
	204	8.2	2.90	2.58	.89	.44	164	20.5	5.83	1.88	.73	.39		
	204	9.5	3.36	3.0	.89	.44	164	21.5	6.11	2.32	.77	.48		
	204	11.0	3.89	3.44	.89	.46	163	22.8	6.44	2.72	.79	.54		
	203	12.2	4.29	3.9	.91	.41	163	24.4	6.89	3.16	.81	.58		
	203	13.6	4.78	4.4	.92	.38	163	26.0	7.34	3.6	.82	.61		
	202	15.0	5.25	4.8	.92	.40	163	28.0	7.88	4.0	.83	.63		
	202	16.5	5.77	5.3	.92	.39	162	30.2	8.48	4.48	.85	.64		
	202	18.0	6.30	5.81	.92	.38	162	32.6	9.15	5.0	.86	.64		
	202	19.6	6.86	6.39	.93	.36	162	35.4	9.94	5.48	.86	.65		

TABLE A-43. INVERTER POWER CHARACTERISTICS

INVERTER:		PULSE WIDTH MODULATED													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	df		
40	Volts	Amps	KVA	KW			Volts	Amps	KVA	KW					
	204	5.9	2.08	1.79	.86	.49	175	19.5	5.91	.98	.55	.02	.75		
	204	7.2	2.54	2.26	.89	.51	175	19.8	6.00	1.44	.64	.29	.75		
	204	8.6	3.04	2.74	.90	.42	175	20.5	6.21	1.96	.72	.43	.73		
	204	10.2	3.60	3.19	.89	.46	175	21.4	6.48	2.44	.77	.52	.72		
	204	11.6	4.10	3.70	.90	.42	174	22.6	6.80	2.96	.8	.57	.71		
	203	13.2	4.64	4.22	.91	.41	174	24.1	7.26	3.44	.82	.61	.72		
	203	14.7	5.17	4.76	.92	.38	174	25.8	7.77	3.92	.82	.64	.70		
	203	16.2	5.70	5.23	.92	.39	173	27.6	8.26	4.44	.85	.65	.67		
	203	17.9	6.29	5.81	.92	.38	173	29.8	8.92	4.96	.85	.67	.66		
	203	19.6	6.89	6.29	.91	.41	173	32.1	9.61	5.6	.89	.66	.66		
	202	21.3	7.45	6.94	.93	.36	173	34.8	10.4	5.96	.86	.69	.66		

TABLE A-44. INVERTER POWER CHARACTERISTICS

INVERTER: NOMINAL FREQUENCY (Hz)	PULSE WIDTH MODULATED													
	INPUT							OUTPUT						
	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	df	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff		
45	204	6.0	2.12	1.81	.85	.49	185	19.2	6.15	1.04	.58	.006		
	204	7.5	2.65	2.32	.88	.47	185	19.6	6.27	1.48	.64	.32		
	204	9.1	3.21	2.82	.88	.47	185	20.3	6.49	2.04	.73	.47		
	204	10.9	3.85	3.4	.88	.46	184	21.1	6.71	2.62	.77	.55		
	203	12.5	4.40	4.0	.91	.40	183	22.4	7.08	3.2	.8	.59		
	203	14.2	4.99	4.58	.92	.39	183	23.9	7.56	3.72	.81	.63		
	202	15.9	5.56	5.14	.92	.38	182	25.5	8.04	4.28	.83	.66		
	202	17.7	6.19	5.73	.93	.37	181	27.5	8.64	4.84	.85	.68		
	202	19.5	6.82	6.33	.93	.37	181	29.5	9.27	5.36	.85	.70		
	202	21.4	7.49	6.91	.92	.38	181	31.8	9.99	5.96	.86	.70		
	201.5	23.4	8.17	7.47	.92	.40	181	34.4	10.8	6.56	.88	.70		

TABLE A-45. INVERTER POWER CHARACTERISTICS

INVERTER: NOMINAL FREQUENCY (Hz)	PULSE WIDTH MODULATED													
	INPUT							OUTPUT						
	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	df	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff %	MUT Eff %	df	
50	204	6.3	2.23	1.9	.85	.50	194	19.1	6.43	1.08	.57	.06	.68	
	204	8.0	2.83	2.51	.89	.45	193	19.6	6.57	1.68	.70	.32	.68	
	203	10.0	3.52	3.1	.88	.47	193	20.8	6.97	2.25	.73	.47	.67	
	203	11.7	4.11	3.78	.92	.38	192	21.2	7.06	2.88	.76	.55	.64	
	203	13.5	4.75	4.38	.92	.38	192	22.5	7.50	3.48	.80	.60	.63	
	202	15.4	5.39	4.93	.92	.40	192	23.9	7.96	4.08	.83	.64	.62	
	202	17.1	5.98	5.57	.93	.36	191	25.5	8.45	4.64	.83	.68	.60	
	202	19.1	6.68	6.30	.94	.33	191	27.3	9.04	5.16	.82	.71	.57	
	202	21.0	7.35	6.80	.93	.38	191	29.4	9.73	5.84	.86	.71	.56	
	202	23.0	8.05	7.46	.93	.37	190	31.5	10.4	6.48	.87	.72	.56	
	201	25.1	8.74	8.18	.94	.35	190	34.0	11.2	7.12	.87	.72	.54	

TABLE A-46. INVERTER POWER CHARACTERISTICS

INVERTER:		PULSE WIDTH MODULATED													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff			
55	Volts	Amps	KVA	KW			Volts	Amps	KVA	KW	%	%			
	203	6.6	2.32	2.01	.87	.48	206	19.5	6.97	1.08	.54	0			
	203	8.5	2.99	2.62	.88	.47	205	19.9	7.06	1.76	.67	.33			
	203	10.6	3.73	3.29	.88	.46	205	20.5	7.29	2.38	.72	.49			
	203	12.5	4.40	3.99	.91	.41	205	21.5	7.65	3.02	.76	.58			
	202	14.7	5.14	4.66	.91	.42	205	22.6	8.02	3.76	.81	.62			
	202	16.6	5.81	5.36	.92	.38	205	24.1	8.56	4.37	.82	.67			
	202	18.8	6.58	6.04	.92	.39	204	25.6	9.05	-	-	-			
	201	20.8	7.24	6.71	.93	.37	204	27.4	9.69	5.68	.85	.71			
	201	22.9	7.97	7.42	.93	.36	204	29.4	10.4	6.36	.86	.73			
	201	25.2	8.77	8.12	.93	.37	203	31.5	11.1	7.01	.86	.74			
	201	27.5	9.57	8.90	.93	.36	203	34.0	12.0	7.72	.87	.74			

TABLE A-47. INVERTER POWER CHARACTERISTICS

INVERTER:		PULSE WIDTH MODULATED													
NOMINAL FREQUENCY (Hz)	60	INPUT							OUTPUT						
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	df	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff %	MUT Eff %	df	
		203	6.6	2.32	2.01	.87	.48	210	18.1	6.57	1.04	.52	0	.64	
		203	8.7	3.06	2.67	.87	.48	208	18.5	6.67	1.76	.66	.36	.65	
		202	10.9	3.81	3.40	.89	.44	207	19.2	6.90	2.48	.73	.52	.63	
		202	12.0	4.20	3.80	.91	.42	233	22.3	9.01	3.28	.86	.58	.60	
		202	14.2	4.97	4.51	.91	.41	232	23.4	9.41	4.04	.90	.63	.59	
		201	16.4	5.71	5.24	.92	.41	231	24.5	9.81	4.72	.90	.67	.54	
		201	18.6	6.48	5.99	.93	.39	231	25.9	10.4	5.44	.91	.70	.56	
		201	20.7	7.21	6.67	.93	.38	230	27.4	10.9	6.14	.92	.72	.53	
		200	23.0	7.97	7.42	.93	.37	228	29.1	11.5	6.84	.92	.73	.52	
		200	26.0	9.01	8.20	.91	.41	227	31.0	12.2	7.56	.92	.75	.51	
		199	27.9	9.62	8.97	.93	.36	226	33.4	13.1	8.28	.92	.76	.49	
		198	30.5	10.46	9.80	.94	.35	225	35.6	13.9	9.00	.92	.76	.48	

TABLE A-48. INVERTER POWER CHARACTERISTICS

INVERTER: NOMINAL FREQUENCY (Hz)	PULSE WIDTH MODULATED													
	INPUT							OUTPUT						
	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	df	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff %	MUT Eff %	df	
80	203	5.5	1.93	1.59	.82	.54	238	15.5	6.21	1.18	.74	.03	.65	
	203	8.0	2.81	2.51	.89	.44	235	16.1	6.57	2.04	.81	.42	-	
	202	11.0	3.85	3.45	.90	.44	233	17.4	7.03	3.04	.88	.56	.60	
	202	14.0	4.90	4.42	.90	.42	232	19.0	7.65	3.96	.90	.64	.58	
	202	17.0	5.95	5.43	.91	.40	231	21.3	8.53	4.94	.91	.68	.57	
	202	20.2	7.07	6.49	.92	.39	230	24.0	9.56	5.94	.92	.70	.55	
	201	23.5	8.18	7.55	.92	.38	229	27.0	10.7	6.92	.92	.72	.54	
	201	26.5	9.23	8.56	.93	.37	227	30.4	11.9	7.92	.93	.74	.51	
	200	30.0	10.39	9.72	.94	.35	227	34.6	13.6	8.96	.92	.74	.50	

TABLE A-49. INVERTER POWER CHARACTERISTICS

INVERTER:		PULSE WIDTH MODULATED													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	df		
100	Volts	Amps	KVA	KW			Volts	Amps	KVA	KW	%	%			
	203	6.0	2.11	1.8	.85	.49	235	12.5	5.1	1.36	.76	.17	.68		
	203	9.0	3.16	2.82	.89	.44	233	13.9	5.62	2.44	.87	.43	.67		
	202	12.8	4.48	4.07	.91	.41	232	16.2	6.51	3.64	.89	.58	.66		
	202	16.9	5.91	5.4	.91	.40	230	19.5	7.77	4.92	.91	.64	.58		
	202	20.9	7.31	6.74	.92	.38	229	23.4	9.28	6.16	.91	.68	.56		
	201	25.1	8.74	8.11	.93	.37	228	28.5	11.2	7.4	.91	.70	-		
	201	30.1	10.48	9.76	.93	.36	227	36.9	14.5	9.0	.92	.68	-		

TABLE A-50. INVERTER POWER CHARACTERISTICS

INVERTER:		PULSE WIDTH MODULATED													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)		V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	df	
120		Volts	Amps	KVA	KW			Volts	Amps	KVA	KW	%	%		
		203	6.5	2.29	1.99	.87	.47	234	10.9	4.43	1.52	.76	.22	.61	
		202	10.5	3.67	3.27	.89	.44	232	13.2	5.31	2.88	.88	.44	.58	
		202	15.1	5.28	4.81	.91	.41	230	17.0	6.77	4.32	.90	.58	.56	
		202	20.1	7.03	6.45	.92	.39	229	22.4	8.89	5.88	.91	.64	.70	
		202	26.0	9.10	8.44	.93	.37	228	32.0	12.6	7.68	.91	.65	-	

TABLE A-51. INVERTER POWER CHARACTERISTICS

INVERTER:	CONSTANT-CURRENT INVERTER													
	INPUT							OUTPUT						
	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff			
Volts	Amps	KVA	KW		Volts	Amps	KVA	KW						
10	465	23	18.5	2.0	.11	114	22.2	4.38	.4	.2	0			
	465	22.8	18.4	2.0	.11	115	22.2	4.42	.48	.24	.22			
	465	22.8	18.4	2.2	.12	117	22.2	4.50	.64	.29	.33			
	465	23.0	18.5	2.3	.12	118	22.2	4.54	.78	.34	.41			
	462	23.7	19.0	2.4	.13	120	22.2	4.61	.88	.37	.48			
	462	24.8	19.8	2.7	.14	124	24.0	5.15	1.08	.40	.49			
	462	26.2	21.0	2.9	.14	128	25.8	5.72	1.2	.41	.53			
	462	27.7	22.2	3.1	.14	134	26.7	6.20	1.4	.45	.53			
	462	29.2	23.4	3.5	.15	138	28.2	6.74	1.56	.45	.55			
	462	30.7	24.6	3.7	.15	143	29.7	7.36	1.8	.49	.53			
	462	32.4	25.9	4.0	.15	148	31.2	8.00	1.96	.49	.54			
	462	34.1	27.3	4.3	.16	153	32.7	8.67	2.16	.50	.54			
	462	35.7	28.6	4.6	.16	157	34.3	9.33	2.36	.51	.54			

TABLE A-52. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT-CURRENT INVERTER									
		INPUT					OUTPUT				
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff
20	Volts	Amps	KVA	KW		Volts	Amps	KVA	KW		
	462	23.2	18.6	2.0	.11	210	22.2	8.07	.44	.22	0
	462	23.0	18.4	2.4	.13	211	22.2	8.11	.64	.27	.33
	462	23.0	18.4	2.6	.14	212	22.2	8.15	.88	.34	.48
	463	23.2	18.6	2.8	.15	212	22.3	8.19	1.16	.41	.55
	463	24.2	19.4	3.1	.16	216	23.2	8.68	1.4	.45	.61
	463	25.4	20.4	3.5	.17	222	24.5	9.42	1.68	.48	.63
	463	26.8	21.5	3.8	.18	228	25.8	10.2	2.0	.53	.64
	462	28.3	22.6	4.2	.19	235	27.3	11.1	2.24	.53	.67
	463	29.7	23.8	4.6	.19	240	28.8	12.0	2.56	.56	.67
	463	31.3	25.1	5.0	.20	246	30.3	12.9	2.88	.58	.67
	462	32.8	26.2	5.4	.21	252	31.7	13.8	3.2	.59	.67
	362	34.4	27.6	5.8	.21	258	33.2	14.8	3.46	.60	.68
	362	36.1	28.9	6.3	.22	263	34.8	15.9	3.76	.60	.68

TABLE A-53. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT-CURRENT INVERTER													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	df		
30	Volts	Amps	KVA	KW			Volts	Amps	KVA	KW					
	470	22.8	18.6	2.2	.12	.27	132	22.0	5.03	.52	.24	0	.37		
	470	22.8	18.6	2.6	.14	.27	133	22.0	5.07	.92	.35	.39	.42		
	470	22.8	18.6	2.9	.16	.27	133	22.0	5.07	1.28	.44	.56	.41		
	470	22.8	18.6	3.3	.18	.27	135	22.0	5.14	1.72	.52	.62	.47		
	470	23.2	18.9	3.8	.20	.27	130	22.3	5.02	2.08	.55	.68	.43		
	470	24.4	19.9	4.3	.22	.29	130	23.5	5.29	2.48	.58	.72	.42		
	470	25.8	21.0	4.8	.23	.29	133	25.0	5.76	2.96	.62	.72	.42		
	470	27.5	22.4	5.3	.24	.30	136	26.5	6.24	3.40	.64	.73	.43		
	468	29.2	23.7	5.8	.25	.28	138	28.0	6.69	3.84	.66	.74	.42		
	468	30.7	24.9	6.5	.26	.26	142	29.7	7.3	4.28	.66	.75	.43		
	467	32.6	26.4	7.0	.27	.29	145	31.3	7.86	4.72	.67	.75	.46		
	467	34.4	27.8	7.6	.27	.30	149	33.1	8.54	5.22	.69	.75	.48		
	467	36.2	29.3	8.2	.28	.30	150	35.0	9.09	5.72	.70	.75	.47		

TABLE A-54. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT-CURRENT INVERTER										
		INPUT					OUTPUT					
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	
40	Volts	Amps	KVA	KW		Volts	Amps	KVA	KW			
	470	22.5	18.3	2.4	.13	158	22.2	6.08	0.6	.25	0	
	470	22.8	18.6	2.8	.15	158	22.2	6.08	1.0	.36	.43	
	468	22.4	18.2	3.2	.18	158	22.2	6.08	1.48	.46	.57	
	468	23.0	18.6	3.8	.20	158	22.5	6.16	1.96	.52	.65	
	468	24.3	19.7	4.3	.22	158	23.4	6.40	2.48	.58	.69	
	467	25.7	20.8	5.0	.24	162	24.6	6.90	2.96	.59	.72	
	468	27.0	21.9	5.5	.25	165	26.0	7.43	3.48	.63	.73	
	468	28.5	23.1	6.2	.27	168	27.5	8.00	4.00	.65	.74	
	468	29.8	24.2	6.7	.28	172	28.8	8.58	4.52	.67	.75	
	468	31.4	25.5	7.4	.29	174	30.3	9.13	5.04	.68	.76	
	468	33.1	26.8	8.0	.30	177	31.8	9.75	5.56	.70	.76	
	467	34.8	28.1	8.7	.31	180	33.4	10.4	6.12	.70	.76	
	468	36.3	29.4	9.4	.32	182	35.2	11.1	6.68	.71	.76	

TABLE A-55. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT-CURRENT INVERTER												
NOMINAL FREQUENCY (Hz)	50	INPUT						OUTPUT						
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff		
	467	22.5	18.2	2.5	.14	197	22.3	7.61	0.6	.24	0			
	467	22.4	18.1	3.2	.18	198	22.3	7.65	1.2	.38	.44			
	468	23.7	19.2	3.8	.20	198	22.7	7.78	1.8	.47	.59			
	467	24.6	19.9	4.5	.23	202	23.7	8.29	2.4	.54	.65			
	468	25.7	20.8	5.2	.25	206	24.8	8.85	3.04	.58	.70			
	467	27.2	22.0	6.0	.27	210	26.1	9.49	3.68	.61	.72			
	467	28.6	23.1	6.7	.29	213	27.4	10.1	4.36	.65	.73			
	467	29.8	24.1	7.5	.31	218	28.8	10.9	5.00	.66	.75			
	467	31.3	25.3	8.1	.32	221	30.2	11.6	5.56	.68	.77			
	468	32.8	26.6	8.9	.34	225	31.6	12.3	6.24	.70	.77			
	467	34.2	27.7	9.7	.35	227	33.0	13.0	6.84	.71	.78			
	468	35.7	28.9	10.4	.36	230	34.4	13.7	7.48	.72	.78			
	467	37.3	30.2	11.2	.37	233	35.8	14.4	8.16	.73	.78			

TABLE A-56. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT-CURRENT INVERTER													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	df		
60	Volts	Amps	KVA	KW			Volts	Amps	KVA	KW					
	472	23.0	18.8	3.0	.16	.287	235	22.2	9.04	0.7	.23	0	.33		
	472	23.3	19.0	3.7	.19	.33	235	22.3	9.08	1.4	.38	.46	.37		
	472	23.7	19.4	4.5	.23	.31	235	22.5	9.16	2.1	.47	.61	.30		
	472	24.1	19.7	5.2	.27	.26	236	23.0	9.40	2.8	.54	.68	.25		
	472	25.1	20.5	6.0	.29	.27	238	24.0	9.89	3.6	.60	.71	.32		
	472	26.2	21.4	6.8	.32	.28	242	25.0	10.5	4.3	.63	.74	.35		
	472	27.3	22.3	7.6	.34	.26	245	26.2	11.1	5.0	.66	.77	.38		
	472	28.7	23.5	8.4	.36	.25	248	27.6	11.9	5.7	.68	.78	.35		
	472	30.2	24.7	9.3	.38	.28	252	28.9	12.6	6.5	.70	.79	.35		
	472	31.7	25.9	10.1	.39	.33	255	30.3	13.4	7.2	.71	.80	.38		
	472	33.1	27.1	10.9	.40	.31	257	31.8	14.2	8.0	.73	.80	.38		
	472	35.0	28.6	12.0	.42	.32	260	33.3	15.0	8.75	.80	.80	.38		
	470	36.3	29.5	12.8	.43	.28	263	35.0	15.9	9.5	.74	.81	.38		

TABLE A-57. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT-CURRENT INVERTER												
NOMINAL FREQUENCY (Hz)	70	INPUT						OUTPUT						
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff	MUT Eff		
		472	22.8	18.6	3.3	.18	273	21.9	10.6	0.8	.24	0		
		472	22.8	18.6	4.1	.22	273	21.8	10.3	1.55	.38	.49		
		472	23.1	18.9	4.9	.26	272	22.1	10.4	2.3	.47	.65		
		472	23.7	19.4	5.8	.30	273	22.6	10.7	3.2	.55	.71		
		472	24.8	20.3	6.6	.33	278	23.7	11.4	4.0	.61	.75		
		470	26.1	21.2	7.7	.36	283	25.1	12.3	5.0	.65	.75		
		468	27.7	22.5	8.7	.39	288	26.4	13.2	5.9	.68	.77		
		468	29.0	23.5	9.7	.41	292	27.7	14.0	6.6	.68	.80		
		470	30.4	24.7	10.6	.43	296	29.2	15.0	7.5	.71	.80		
		468	32.0	25.9	11.6	.45	299	30.6	15.8	8.4	.72	.81		
		468	33.5	27.2	12.5	.46	304	32.1	16.9	9.3	.74	.81		
		468	35.2	28.5	13.8	.48	307	33.7	17.9	10.2	.74	.81		
		468	36.8	29.8	14.7	.49	311	35.7	19.0	11.1	.76	.81		

TABLE A-58. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT-CURRENT INVERTER																
		INPUT							OUTPUT									
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	
80	Volts	Amps	KVA	KW		Volts	Amps	KVA	KW			Volts	Amps	KVA	KW			
	468	23.5	19.0	3.8	.20	310	21.8	11.7	0.8	.21	0							
	468	23.0	18.6	4.6	.25	309	22.0	11.8	1.75	.38	.49							
	467	24.0	19.4	5.4	.28	312	22.5	12.2	2.7	.50	.63							
	468	24.7	20.0	6.5	.33	317	23.5	12.9	3.7	.57	.69							
	468	25.8	20.9	7.6	.36	323	24.7	13.8	4.6	.61	.74							
	468	27.2	22.0	8.7	.40	328	26.0	14.8	5.6	.64	.76							
	468	28.5	23.1	9.8	.42	333	27.2	15.7	6.6	.67	.77							
	468	30.0	24.3	10.9	.45	337	28.7	16.8	7.6	.70	.79							
	468	31.3	25.4	11.9	.47	343	30.2	17.9	8.6	.72	.80							
	467	32.8	26.5	13.1	.49	347	31.4	18.9	9.5	.73	.81							
	468	34.2	27.7	14.1	.51	350	32.8	19.9	10.4	.74	.82							
	467	36.0	29.1	15.2	.52	354	34.3	21.0	11.5	.76	.81							
	463	37.2	29.5	16.4	.55	358	35.8	-	-	-	-							

TABLE A-59. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT-CURRENT INVERTER													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	df		
90	Volts	Amps	KVA	KW			Volts	Amps	KVA	KW					
	465	22.7	18.3	3.9	.21	.29	340	21.8	12.8	0.9	.23	0	.35		
	465	22.7	18.3	4.9	.27	.29	340	21.5	12.7	1.9	.39	.50	.28		
	465	22.7	18.3	5.8	.32	.30	335	21.5	12.5	3.0	.52	.64	.32		
	465	22.8	18.4	6.8	.37	.28	330	21.8	12.5	4.0	.59	.72	.32		
	465	23.7	19.1	7.9	.41	.29	330	22.5	12.9	5.0	.63	.77	.31		
	465	25.0	20.1	9.0	.45	.28	335	23.8	13.8	6.1	.68	.79	.32		
	465	26.5	21.3	10.1	.48	.27	340	25.3	14.9	7.2	.71	.80	.32		
	465	28.2	22.7	11.5	.51	.29	345	26.7	16.0	8.2	.71	.82	.31		
	462	29.8	23.8	12.6	.53	.28	350	28.3	17.2	9.3	.74	.82	.32		
	462	31.5	25.2	13.8	.55	.29	356	29.8	18.4	10.5	.76	.82	.31		
	462	33.0	26.4	14.8	.56	.30	360	31.5	19.6	11.5	.78	.83	.31		
	462	34.8	27.8	16.4	.59	.31	366	33.2	21.0	12.6	.77	.84	.35		
	460	36.7	29.2	17.6	.60	.28	371	34.8	22.4	13.7	.78	.84	.33		

TABLE A-60. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT-CURRENT INVERTER										
		INPUT					OUTPUT					
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf			S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	
100	Volts	Amps	KVA	KW				KVA	KW			
	462	22.7	18.2	4.3	.24			14.2	1.0	.23	0	
	463	23	18.4	5.3	.29			14.3	2.0	.38	.53	
	462	24	19.2	6.9	.36			14.8	3.3	.48	.64	
	463	-	-	-	-			15.6	4.5	-	.71	
	462	25	20.0	9.0	.45			16.7	5.7	.63	.74	
	462	-	-	-	-			18.2	6.8	-	.78	
	462	-	-	-	-			19.3	8.2	-	.77	
	462	31	24.8	13.5	.54			20.8	9.4	.70	.79	
	462	33.5	26.8	14.5	.54			22.0	10.5	.72	.81	
	462	33.5	26.8	15.9	.59			23.4	11.7	.74	.82	
	462	34.8	27.8	17.6	.63			24.8	12.9	.73	.82	
	462	36.3	29	18.4	.64			26.2	14.1	.77	.83	
	460	38.0	30.3	19.9	.66			27.6	15.3	.77	.83	

TABLE A-61. INVERTER POWER CHARACTERISTICS

INVERTER: NOMINAL FREQUENCY (Hz)	CONSTANT-CURRENT INVERTER										
	INPUT						OUTPUT				
	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff %	MUT Eff %
110	462	23.2	18.6	4.8	.259	430	22.0	16.4	1.05	22	0
	462	23.7	19.0	6.1	.321	435	22.5	17.0	2.4	39	50
	462	24.7	19.8	7.7	.388	444	23.5	18.1	3.7	48	64
	462	26.0	20.8	9.2	.440	455	24.7	19.5	5.1	55	70
	462	27.7	22.2	10.6	.478	462	26.2	20.1	6.6	62	72
	462	28.8	23.0	12.1	.527	470	27.5	22.4	7.9	65	75
	462	30.3	24.2	13.6	.563	478	29.0	24.0	9.2	68	78
	462	31.7	25.4	15.1	.595	482	30.3	25.3	10.5	70	79
	462	33.5	26.8	16.6	.619	488	31.7	26.8	11.9	72	80
	462	34.5	27.6	18.0	.652	495	33.1	28.4	13.3	74	81
	462	36.2	29.2	19.6	.677	498	34.6	39.8	14.6	74	82
	462	37.7	30.2	21.1	.699	503	35.8	31.2	16.0	76	82
	462	39.3	31.4	22.5	.715	508	37.3	32.8	17.4	77	82

TABLE A-62. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT-CURRENT INVERTER													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	df	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	df		
120	Volts	Amps	KVA	KW			Volts	Amps	KVA	KW					
	462	25.0	20.0	5.6	.28	.24	460	23.7	18.9	1.3	.23	0	.33		
	462	26.2	21.0	7.2	.34	.23	492	24.7	21.0	2.7	.38	.47	.32		
	462	27.5	22.0	9.0	.41	.24	505	26.5	23.2	4.2	.47	.61	.33		
	462	28.5	22.8	10.6	.47	.09	515	27.5	24.5	5.8	.55	.66	.26		
	462	30.5	24.4	12.3	.50	.24	522	29.0	26.2	7.1	.58	.72	.28		
	462	31.8	25.4	13.7	.54	.31	527	30.2	27.6	8.8	.64	.72	.28		
	462	33	26.4	15.5	.59	.26	533	31.2	28.8	10.1	.65	.76	.28		
	462	35.0	28.0	17.0	.61	.32	542	33	31.0	11.8	.69	.76	.26		
	463	36.0	28.9	19.2	.67	.20	547	34.4	32.6	13.1	.68	.78	.30		
	462	37.5	30.0	20.2	.67	.34	553	35.7	34.2	14.5	.72	.79	.27		
	462	39.0	31.2	21.8	.70	.33	558	37.2	36.0	16.0	.73	.80	.29		
	462	40.8	32.6	23.4	.72	.38	562	38.5	37.5	17.5	.75	.80	.29		
	462	42.5	34.0	24.8	.73	.41	-	-	-	-	-	-	-		

TABLE A-63. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT CURRENT INVERTER - REGENERATION DATA													
		INPUT							OUTPUT						
NOMINAL FREQUENCY (Hz)		V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>IN</sub>	Pf	V <sub>AC</sub>	I <sub>AC</sub>	S <sub>T</sub>	P <sub>OUT</sub>	Eff	MUT Eff	P <sub>MUT</sub>		
30		Volts	Amps	KVA	KW		Volts	Amps	KVA	KW	%	%	KW		
		464	22.8	18.32	+2.22	+ .12	128	22.0	4.88	0.5	-		0		
		464	22.8	18.32	+1.6	.09	118	22.0	4.50	-0.2	-	31.3	- .64		
		460	23.5	18.72	+1.0	.05	115	22.5	4.48	-0.7	-	54.7	-1.28		
		460	25.5	20.32	+ .58	.03	115	25	4.98	-1.3	-	68.4	-1.9		
		460	28.3	22.55	+ .12	.01	118	27.5	5.62	-1.8	-	12.7	-2.55		
		460	31.2	24.81	- .10	0	120	30.3	6.30	-2.3	43	72.1	-3.19		

TABLE A-64. INVERTER POWER CHARACTERISTICS

INVERTER:		CONSTANT CURRENT INVERTER -- REGENERATION DATA													
NOMINAL FREQUENCY (Hz)	60	INPUT							OUTPUT						
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	df	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff %	MUT Eff %	P <sub>MUT</sub> KW	
		463	23	18.4	3.1	.17		233	22	8.88	.8	-	-	0	
		463	23	18.4	1.76	.1		230	22	8.76	-.5	-	39	-1.28	
		460	23.4	18.6	.3	.02		222	22.5	8.65	-1.9	-	74	-2.56	
		460	25.0	19.9	-.98	-.05		220	24.0	9.14	-3.0	33	78	-3.84	
		460	27.8	22.2	-1.88	-.08		225	27.0	10.52	-4.2	45	82	-5.12	
		460	30.8	24.5	-2.7	-.11		230	29.8	11.87	-5.31	51	83	-6.7	

TABLE A-65. INVERTER POWER CHARACTERISTICS

INVERTER: NOMINAL FREQUENCY (Hz)	CONSTANT CURRENT INVERTER - REGENERATION DATA													
	INPUT							OUTPUT						
	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff %	MUT Eff %	P <sub>MUT</sub> KW		
90	460	24.4	19.4	4.5	.23	358	23.0	14.3	1.3	-	-	0		
	460	23.2	18.5	2.2	.12	345	22.0	13.2	-.8	-	42	-1.91		
	460	23.4	18.6	0	0	331	22.4	12.8	-2.9	0	76	-3.82		
	460	24.1	19.2	-2.14	-.11	321	23.2	12.9	-4.9	44	86	-5.73		
	460	27.0	21.5	-3.8	-.18	312	26.0	14.0	-6.6	58	86	-7.65		
	460	30.4	24.2	-5.28	-.22	320	29.2	16.2	-8.3	64	87	-9.56		

TABLE A-66. INVERTER POWER CHARACTERISTICS

INVERTER: NOMINAL FREQUENCY (Hz)		CONSTANT CURRENT INVERTER - REGENERATION DATA											
		INPUT						OUTPUT					
		V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>IN</sub> KW	Pf	V <sub>AC</sub> Volts	I <sub>AC</sub> Amps	S <sub>T</sub> KVA	P <sub>OUT</sub> KW	Eff %	MUT EFF %	P <sub>MUT</sub>
110		460	26.9	21.4	6.1	.28	465	25.7	20.7	1.9	-	0	0
		460	24.5	19.5	2.9	.15	440	23.5	17.9	-0.9	-	38	-2.36
		460	23.5	18.7	0	0	412	22.5	16.1	-3.5	0	74	-4.71
		460	24.0	19.1	-2.4	-.13	390	23.3	15.7	-5.9	41	83	-7.07
		460	26.5	21.1	-5.22	-.25	360	25.5	15.9	-8.2	63	87	-9.42
		460	30.0	23.9	-7.0	-.29	370	29.0	18.6	-10.2	69	87	-11.78

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## APPENDIX B

### INVERTER THEORY OF OPERATION

The theory of operation for each inverter has been described in depth in several papers and reports; a brief description and review is presented in this appendix.

#### B.1 NATURAL COMMUTATED INVERTER

The natural commutated inverter is a series inverter with a resonant L-C circuit in series with the load. Thyristor commutation is accomplished when the resonant response of those components reverses the voltage across a conducting thyristor.

The basic circuit of a single phase of a three phase system is shown in Figure B-1. The drive uses three individual inverters to provide a three-phase variable voltage and frequency output which approximates in waveform the three-phase reference input signals. The pulse-widths of the series resonant pulses are determined by the respective inductors and capacitors in series with conducting thyristors: L+ and C+ for positive pulses; L- and C- for negative pulses. The four thyristors which bridge each of the capacitors are triggered in diametrically opposed pairs so that the capacitor current is ac but the load current is dc.

Pulse frequency and pulse polarity modulation are combined to control output voltage and frequency. The two capacitors, labeled  $C_0$ , are charged by the resonant current pulses, which have peak magnitudes up to 200 amps.; these capacitors supply power to the load between thyristor conduction intervals. The stored charge on the resonant circuit capacitors reverses polarity during each firing interval. When the voltage peaks in the opposite polarity the thyristors commutate and the reverse diodes D+ and D- begin to conduct. These diodes allow the energy stored in the inductor to dissipate in the load. Since the diodes clamp the

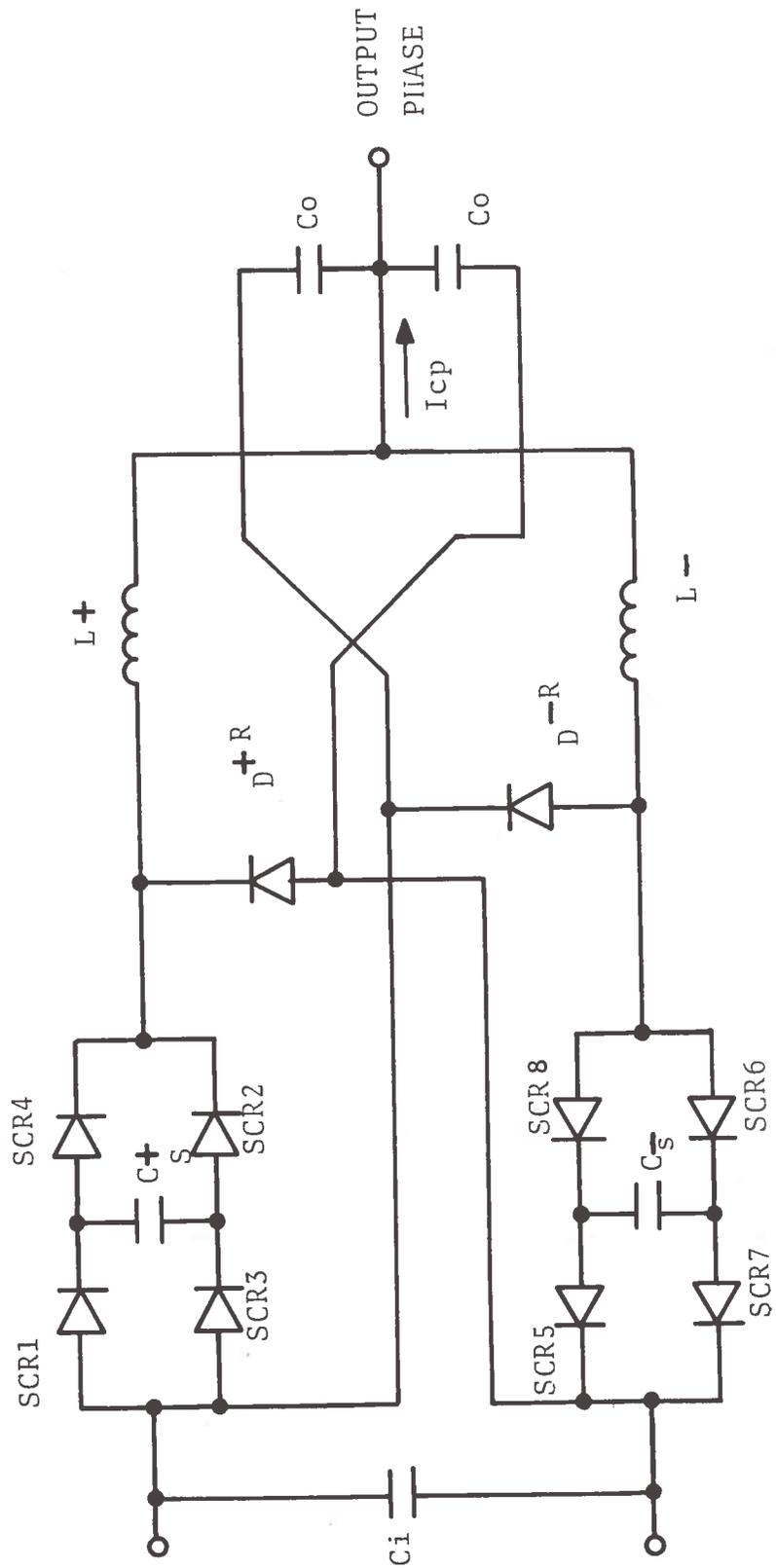


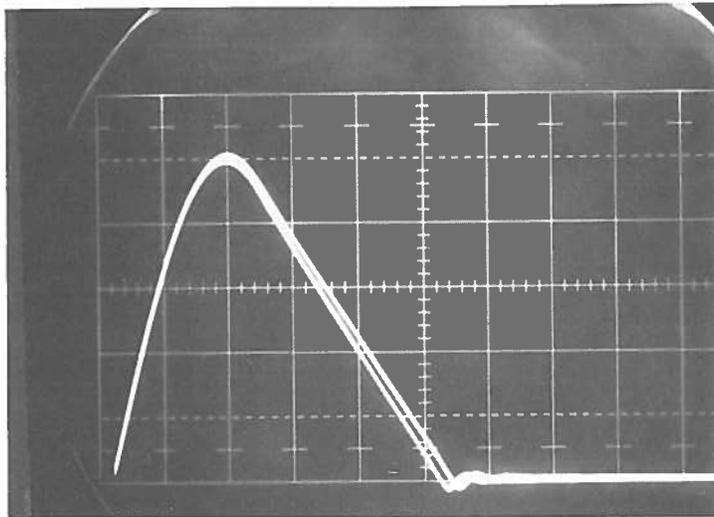
FIGURE B-1. NATURAL COMMUTATED INVERTER BASIC POWER STAGE

output voltage across the circuit, the decay of current in the inductor is a linear ramp with a slope of  $E/L$ , where  $E$  is the output voltage across the storage capacitors,  $C_o$ .

Figure B-2 is a photograph of the current which flows through the inductor to the storage capacitors,  $C_o$ . It is composed of two segments, the resonant sinusoidal current in the thyristor and the linear current through the reverse diode. Figure B-3 is a photograph of the thyristor current, which rises sinusoidally from zero to its peak value. As the voltage across the series resonant capacitor,  $C_s$ , peaks, the current begins to decay; the induced voltage in the inductor changes polarity and the thyristors commutate. This commutation is evident in the sharp decrease in the thyristor current. The energy stored in the inductor maintains a current in the reverse diode, as shown in Figure B-4. At the initiation of diode current the voltage across the circuit is clamped to the level of the voltage across the storage capacitors, and the current decays with a slope of  $E/L$ . The decay time is inversely proportional to the output voltage level, as shown in Figure B-5, which is a photograph of one cycle of output voltage and the charge current pulses into the storage capacitor bank ( $I_{cp}$  of Figure B-1). Four characteristics of the output waveform generation are evident in Figure B-5:

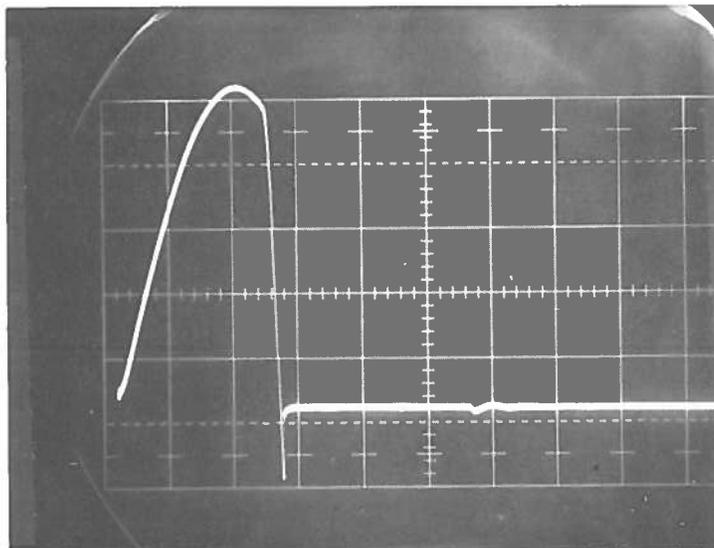
1. The change in slope of the charge pulse current decay time as a function of output voltage magnitude.
2. The pulse frequency and pulse polarity modulation employed to generate the output.
3. The output voltage which results from integration of the storage capacitors.
4. The reduction of charge pulse current amplitude as a function of  $E_{in}/E_o$ .

Both the slope of the linear portion and the peak amplitude of the current pulse are functions of the output voltage across the storage capacitors. This is due to the fact that the total voltage across the circuit is the sum of the input voltage across



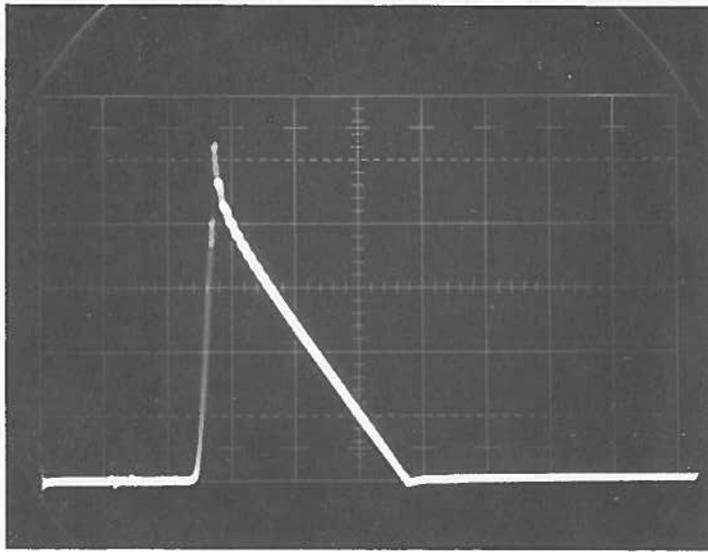
20  $\mu$ SEC/CM  
50 A/CM

FIGURE B-2.  $I_L$  OUTPUT CURRENT BEFORE FILTER



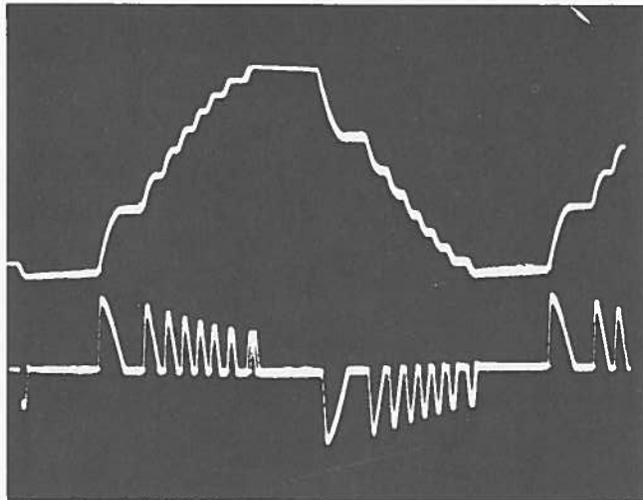
20  $\mu$ SEC/CM  
50 A/CM

FIGURE B-3. SCR CURRENT



20  $\mu$ SEC/CM  
50 A/CM

FIGURE B-4. DIODE CURRENT



0  $V_{L-N}$

$I_{C_S}$

FIGURE B-5. LINE TO NEUTRAL OUTPUT VOLTAGE AND CHARGE CURRENT PULSES

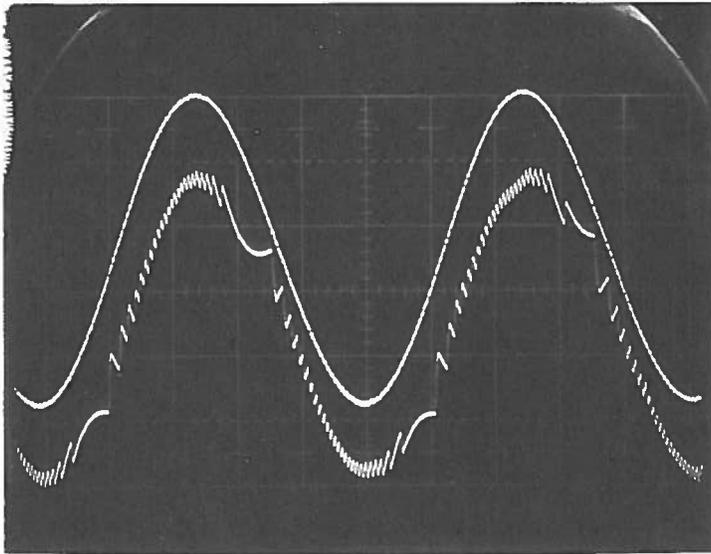
the series capacitor and the voltage across the output capacitor. Thus, the stored energy in the series capacitor available for transfer is a function of the resultant loop voltage. Since the source voltage and the series capacitor voltage are essentially constant, the only variable is the output capacitor voltage. It can be seen from Figure B-5 that the magnitude of the charge current pulse is proportional to the difference of the instantaneous output and the source voltage.

Each current pulse delivers a charge to the storage capacitors,  $C_o$ , which increases the output voltage. At the conclusion of the conduction interval, the stored charge dissipates in the load and the voltage decays. The output voltage is compared to a reference, and when it falls below that reference another conduction period is initiated. The output voltage generated by this multi-pulse scheme is shown in Figure B-6. The top trace is the reference voltage, and the bottom trace is the voltage across a resistive load. An induction motor load on the inverter output adds inductive energy storage which filters and thus improves the current waveshape. This is shown in Figure B-7, which is a photograph of the output voltage and current into a 0.7 power factor load at a frequency of 60 Hz.

## B.2 PULSE-WIDTH MODULATED INVERTER

The pulse-width modulated inverter is a forced commutated inverter which provides variable voltage and variable frequency power for driving ac motors. Output current flows from the dc source to the load through the main power thyristor switches, which, with the exception of snubbers and current sensing circuits, are the only elements in series with the load. Once turned on, these conducting switches must be turned off, or "commutated," by reducing the current through them to a level below their holding current. Several methods for commutating thyristors have been developed; the inverter tested in the TSC program was an auxiliary impulse commutated inverter, commonly called the "McMurray"

$f = 100 \text{ Hz}$   
 $I_0 = 10 \text{ AMPS}$   
 $V_R = 1 \text{ V/CM}$   
 $V_0 = 50 \text{ V/CM}$

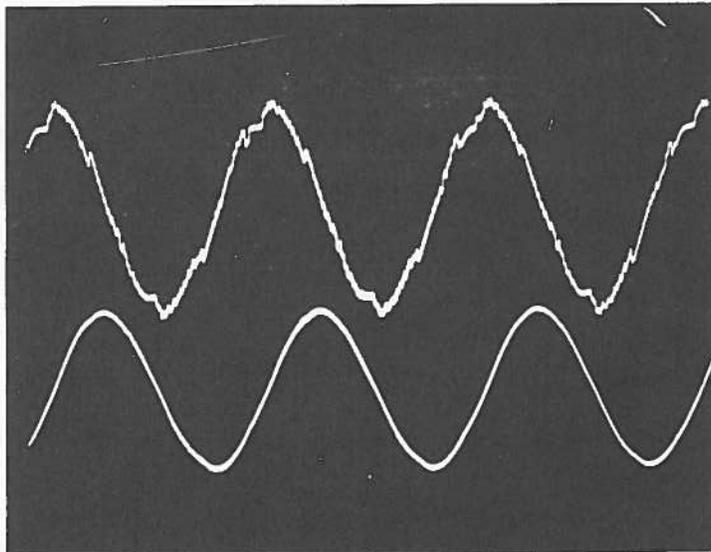


REFERENCE  
VOLTAGE

OUTPUT  
VOLTAGE

FIGURE B-6. REFERENCE AND OUTPUT VOLTAGE WAVEFORMS

$PF = 0.7$   
 $I = 20 \text{ AMPS}$   
 $f = 60 \text{ Hz}$



VOLTAGE

CURRENT

FIGURE B-7. OUTPUT VOLTAGE AND CURRENT WITH  
LAGGING POWER FACTOR LOAD

circuit, whose commutation pulses are generated by a separate commutation thyristor and a series resonant tank circuit. Voltage and frequency control is achieved by controlling the conduction intervals and sequences of the main power switches. Both voltage and frequency are continuously adjustable over their operating ranges: voltage from 0 to 208 volts, line to line, and frequency from 4 to 400 Hz.

The voltage, supplied from the dc link, is varied by using pulse-width modulation. The frequency is controlled by varying the rate at which the load is connected to the positive and negative sides of the dc bus. Phasing of the output waveform is obtained by internal timing of the thyristor firing pulses to maintain correct phase angle and rotation. The output waveform is similar to a six pulse waveform; however it is chopped or notched, so that each cycle consists of a series of equal amplitude, but varied width pulses. The magnitude of the resultant output voltage is controlled by controlling the width of the notches between the output pulses.

Pulse-width modulation allows both frequency and output voltage magnitude to be controlled with one set of switching semi-conductors. The use of an uncontrolled diode input rectifier bridge also eliminates the displacement component of reactive power; thus the system operates at power factors close to unity.

A schematic of the forced commutated inverter circuit is shown in Figure B-8. The main power switches are SCR1 through SCR6, and the commutation switches are SCR7 through SCR12. The inductors L1 through L3 and capacitors C1 through C3 form the resonant circuits for commutation. Diodes D1 through D6 provide feed-back paths for reactive power, regenerated power and resonant commutation power.

A single leg of a forced, commutated, three-phase bridge is shown in Figure B-9. Components for thyristor commutation are shown, but protection, gating and current sensing circuits are omitted.

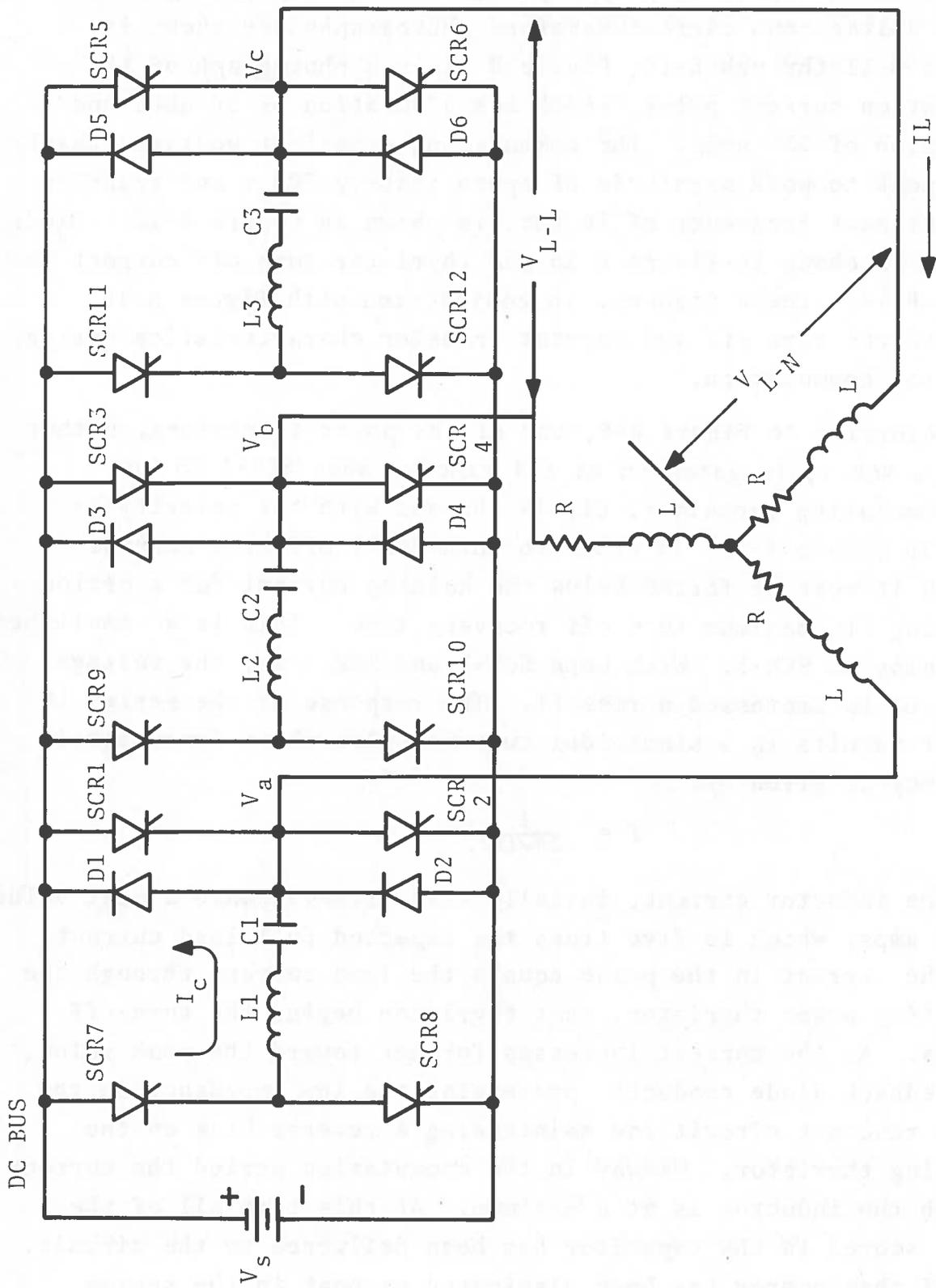


FIGURE B-8. FORCED COMMUTATED INVERTER SCHEMATIC

Voltage and current waveforms of the power and commutation components are shown in proper phase relationship in Figure B-10. Actual voltage and current waveform photographs are shown in Figures B-11 through B-14, Figure B-11 is a photograph of the commutation current pulse, which has a duration of 50 usec and a peak value of 230 amps. The commutating capacitor voltage, which has a peak to peak magnitude of approximately 700 v and transits at a resonant frequency of 10 kHz, is shown in Figure B-12. Diode current is shown in Figure B-13 and thyristor turn-off current in Figure B-14. These figures, in conjunction with Figure B-10, indicate the turn-off and current transfer characteristics during thyristor commutation.

Referring to Figure B-9, one of the power thyristors, either SCR-1 or SCR-2, is gated on at all times. When SCR-1 is on, the commutating capacitor, C1, is charged with the polarity as shown in Figure B-9. In order to turn SCR-1 off, the current through it must be forced below the holding current for a period exceeding its maximum turn-off recovery time. This is accomplished by turning on SCR-3. With both SCR-1 and SCR-3 on, the voltage across C1 is impressed across L1. The response of the series LC circuit results in a sinusoidal current pulse whose fundamental frequency is given by:

$$f = \frac{1}{2\pi\sqrt{LC}}.$$

The inductor current, initially zero, rises toward a peak value of 230 amps, which is five times the expected full load current. When the current in the pulse equals the load current through the conducting power thyristor, that thyristor begins the turn-off process. As the current increases further toward the peak value, the feedback diode conducts, preserving the low impedance in the series resonant circuit and maintaining a reverse bias on the off-going thyristor. Midway in the commutation period the current through the inductor is at a maximum. At this time all of the energy stored in the capacitor has been delivered to the circuit. Some of that energy has been dissipated as heat in the series components, but most of it has been stored by the inductor as a

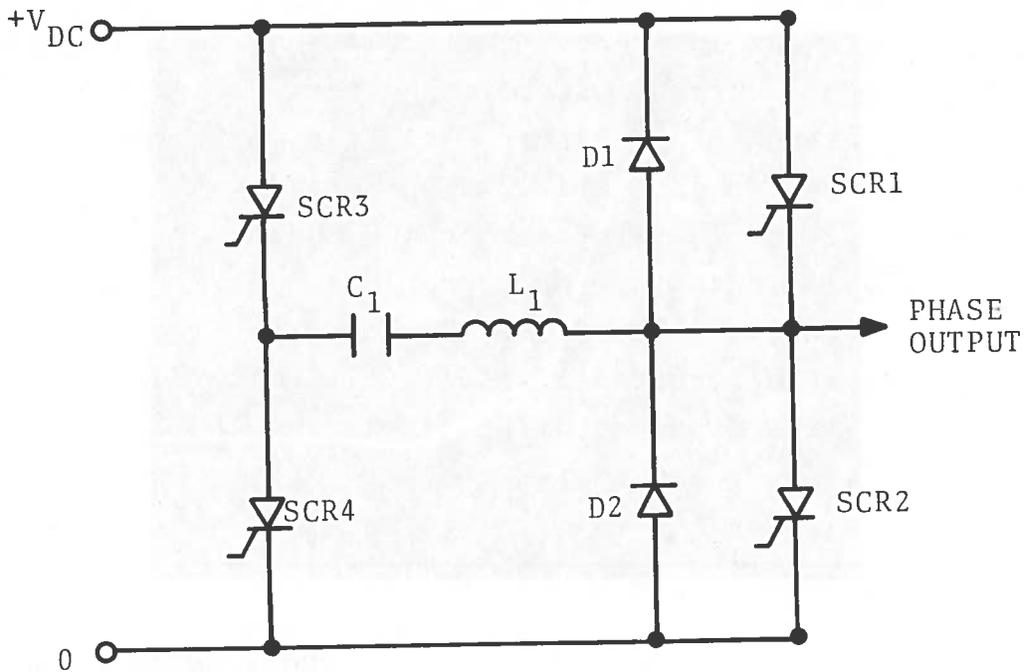


FIGURE B-9. FORCED COMMUTATED INVERTER PHASE MODULE

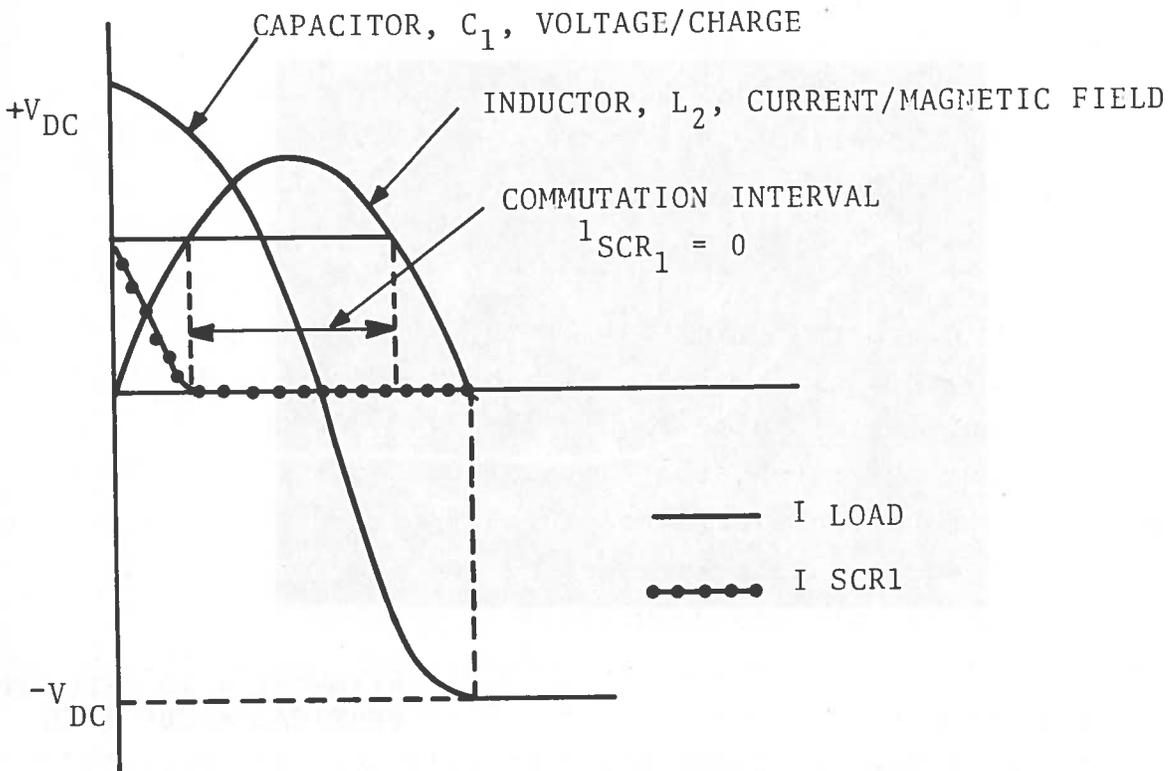
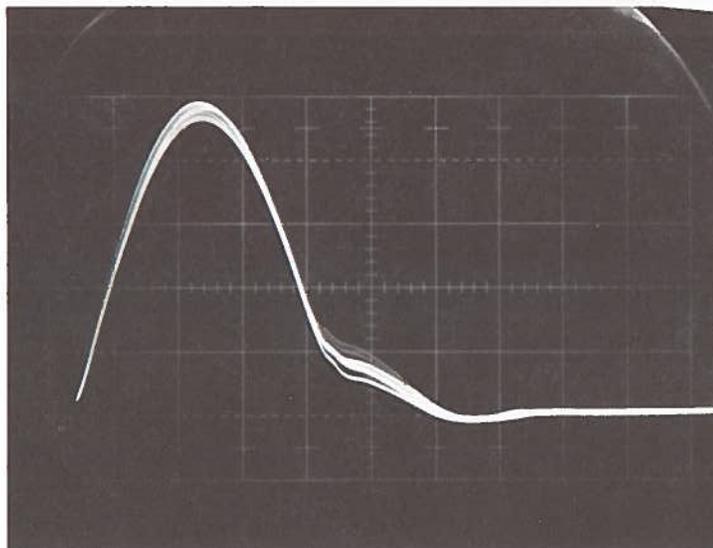
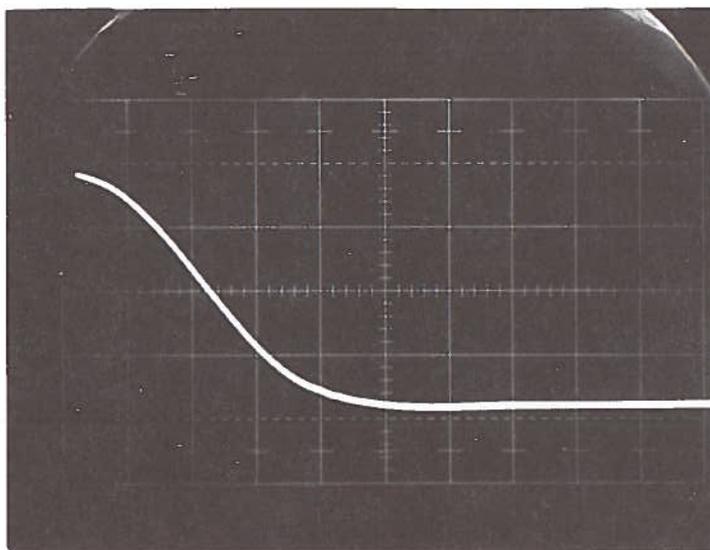


FIGURE B-10. COMMUTATION COMPONENT DYNAMIC WAVEFORMS



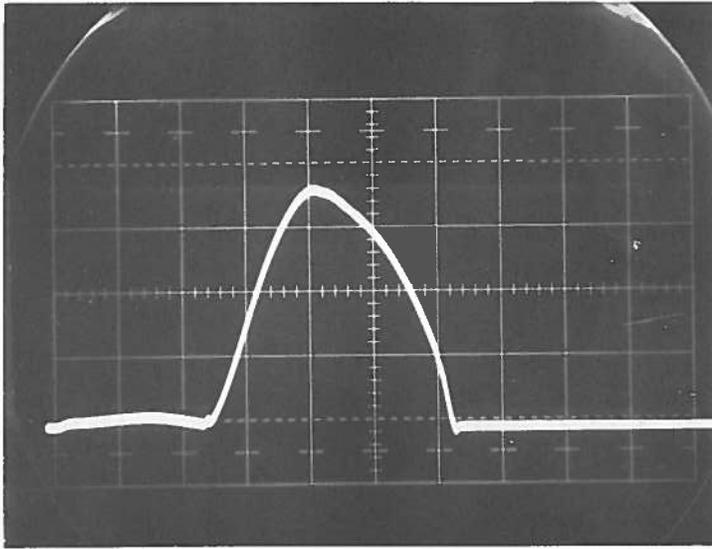
HORIZONTAL = 10  $\mu$ SEC/CM  
VERTICAL = 50 A/CM

FIGURE B-11. COMMUTATION CURRENT PULSE



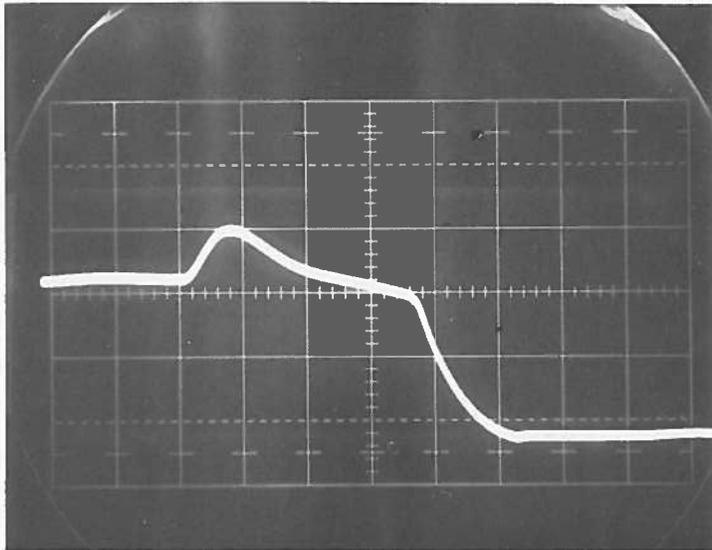
HORIZONTAL = 10  $\mu$ SEC/CM  
VERTICAL = 200 V/CM

FIGURE B-12. COMMUTATING CAPACITOR VOLTAGE



HORIZONTAL = 10  $\mu$ SEC/CM  
 VERTICAL = 50 A/CM

FIGURE B-13. DIODE CURRENT



HORIZONTAL = 10  $\mu$ SEC/CM  
 VERTICAL = 20 A/CM

FIGURE B-14. INVERTER THYRISTOR CURRENT  
 TURN-OFF TRANSIENT

magnetic field, which begins to collapse. This collapsing field supports the current in the circuit, maintains a forward bias on the feedback diode and a reverse bias on the off-going thyristor, and charges the capacitor to peak voltage in the opposite polarity. When the resonant current in the pulse decreases to zero, the commutating thyristor turns off, and the circuit is conditioned for the next commutation operation. The inductor current is zero, and the capacitor is charged with the proper polarity to commutate SCR-2, the next power thyristor to be turned on.

The commutation components for the power thyristors consists of the commutating thyristor, the series resonant LC network and the feedback diode. Although the rms currents in these commutation circuits are low, the peak currents are high, and all of the devices must be selected on the basis of their repetitive peak current capabilities.

The commutating thyristor and the feedback diode must have low forward impedance to minimize losses during the commutation period. In addition, the feedback diode provides a current path for reactive load current and regenerative current. An important characteristic necessary in the feedback diode is a fast reverse recovery time. The reversal of voltage across the diode after the commutation interval is very fast; if the diode is slow in recovering to its blocking state, the instantaneous device power dissipation may exceed its capability and destroy it.

The commutating networks of the inverter inherently protect the commutating thyristors from  $dv/dt$  and  $di/dt$  transients, because the sinusoidal current and voltage waveforms can be constrained by design to be within limits. This facet of the inverter is shown in Figure B-11, the commutation current pulse, which is also the current through the commutation thyristor, and in Figure B-12, which is the voltage across the commutating capacitor. This capacitor voltage is also the voltage across the commutating thyristor. These figures show that  $dv/dt$  transient across the commutation thyristors is below 30 volts/usec, which is within the device rating, and the  $di/dt$  transient is also well below the

device rating.

The inverter thyristors, however, lacking inherent built-in protection for either transient, must be protected by additional snubbers. These snubbers limit the transients within manufacturer's specification. Figure B-15 shows the reverse voltage recovery of an inverter thyristor. The snubber circuit limits  $dv/dt$  to 100 volts/usec, which is less than 50 percent of rating. The snubbing inductor in series with the thyristor limits  $di/dt$  to approximately 30 amps/usec, which is also less than 50 percent of rating; the turn-on current pulse is shown in Figure B-16.

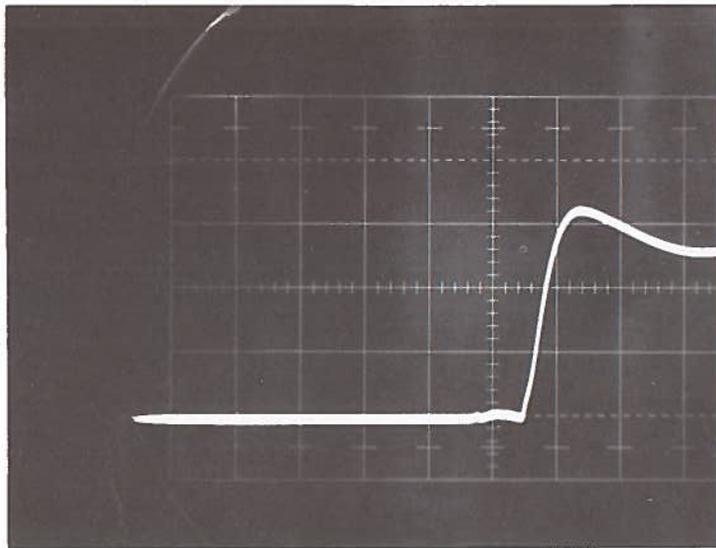
Switching considerations in the power and commutation loops of the inverter dictate the frequency limits of the modulation scheme. The number of notches per cycle is reduced at specific frequencies to keep the number of switching transients, and hence the switching losses, below a selected maximum. The number of notches per half-cycle and the frequency ranges that they cover are:

eight notches:	4 Hz to 120 Hz
four notches:	120 Hz to 213 Hz
two notches:	213 Hz to 400 Hz

The resulting waveforms, at one frequency in each of the three frequency ranges, are shown in Figure B-17.

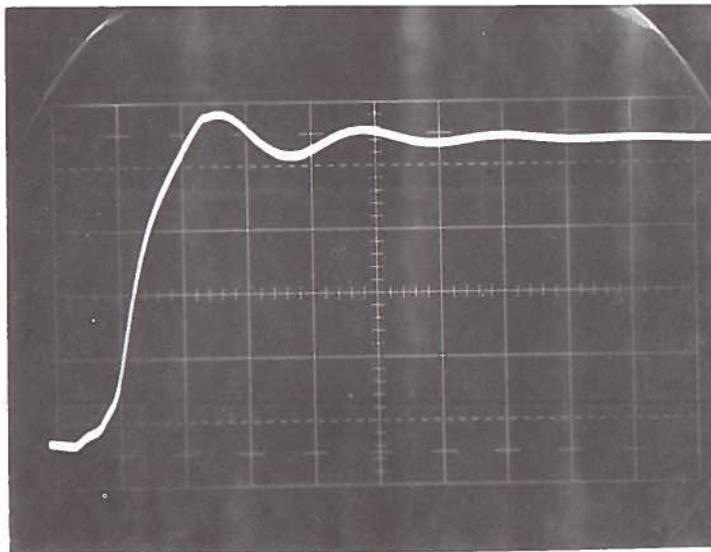
The top traces in the output voltage waveform photos are the line to line voltages. The line to neutral voltages are shown in the bottom traces. The change in the number of chops as frequency increases is evident in the photos. As the frequency in any range increases, the switching transients per second and the total switching loss increase. At 120 Hz, for instance, there are 2160 commutations per second. At that point the waveform is changed to one with four notches, and the commutations per second are reduced to 1200, with a corresponding reduction in switching loss. Similarly, at 213 Hz the waveform is again modified to one with two notches, and the switching losses are again reduced.

This change in waveform as a function of frequency results in



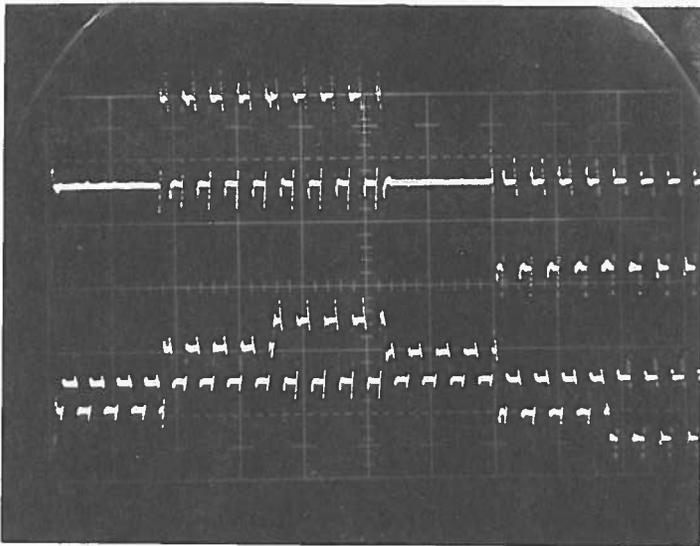
HORIZONTAL = 5  $\mu$ SEC/CM  
VERTICAL = 100 V/CM

FIGURE B-15. INVERTER THYRISTOR VOLTAGE TURN-OFF TRANSIENT

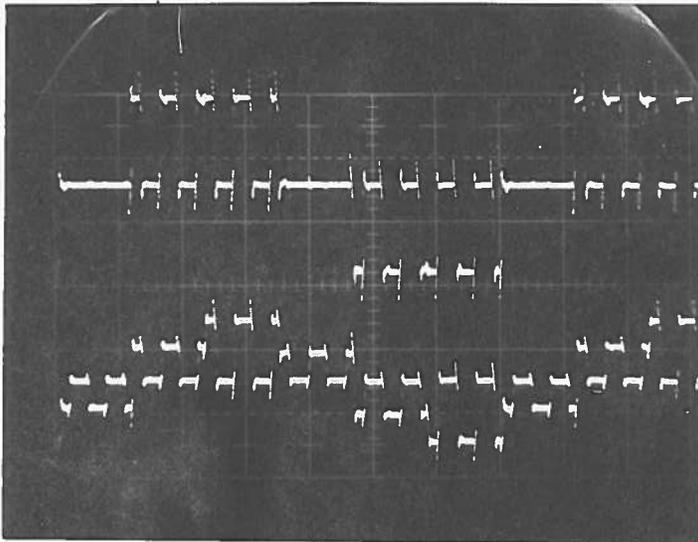


HORIZONTAL = 10  $\mu$ SEC/CM  
VERTICAL = 10 AMP/CM

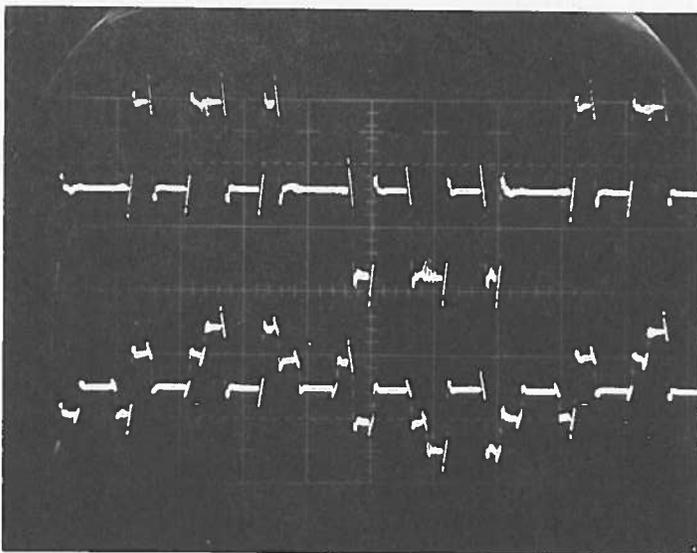
FIGURE B-16. INVERTER CURRENT TURN-OFF TRANSIENT



a) EIGHT NOTCH  
OUTPUT 100 Hz



b) FOUR NOTCH  
OUTPUT 150 Hz



c) TWO NOTCH  
OUTPUT 300 Hz

FIGURE B-17. OUTPUT VOLTAGE WAVEFORMS

discontinuities in the inverter characteristics at each of the break points. The effects are noticeable in the power characteristics and in the harmonic data. The harmonic ratios not only change for each waveform as the number of notches change, but also as the width of the notches vary with output voltage control.

### B.3 CONSTANT CURRENT COMPLEMENTARY COMMUTATED INVERTER

The constant current inverter circuit utilizes a three-phase inverter circuit commonly called a parallel capacitor commutated, or complementary commutated, inverter circuit. Commutation of conducting thyristors is accomplished by capacitively coupling a commutation transient from an on-going thyristor to a conducting thyristor. The resulting reverse voltage bias across the conducting thyristor forces the anode current to zero and commutates the thyristor.

A schematic of the power circuit for the complementary commutated inverter is shown in Figure B-18. The phase delay rectifier (pdr) converts the constant voltage 460 v, 3 $\emptyset$  input voltage to variable voltage dc for the inverter. The dc choke (L) filters the pdr output and maintains a constant current into the inverter terminals. In the motoring mode, the dc output voltage of the pdr is positive in the direction shown and the dc current flows in the direction shown. During regeneration, the polarity of the dc voltage is reversed, as shown in the Figure, by action of the inverter but the direction of the dc current remains unchanged. During regeneration the motor becomes the source of ac power; the inverter functions as a rectifier, converting the motor ac voltage to a dc voltage; and the pdr functions as an inverter, converting the dc voltage to 60 Hz, 3 $\emptyset$  ac.

SCRs I1 through I6 are the six inverter thyristors. Only two of these, one positive and one negative, are on at any time, and they are gated on in the numbered sequence as shown in Figure

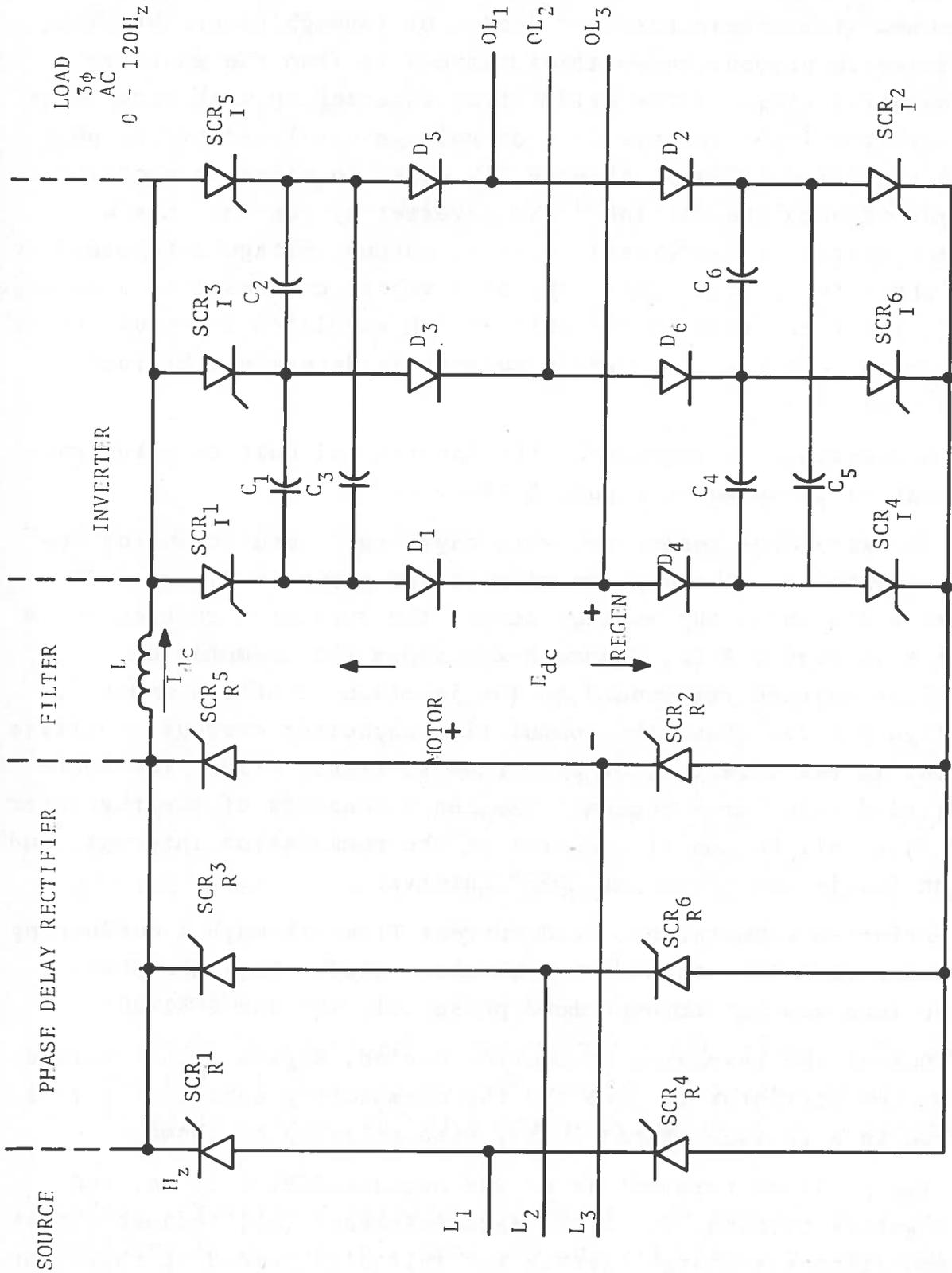


FIGURE B-18. CONSTANT CURRENT INVERTER POWER CIRCUIT SCHEMATIC

B-18. The gating sequence is reversed for reverse motor operation, but not for regenerative operation. Capacitors C1 through C6 are the commutation capacitors and diodes D1 through D6 are blocking diodes which prevent commutation components from the positive and negative sides of the bridge from interacting with each other. The inverter input voltage is a dc voltage developed by the phase delay rectifier (SCRs R1 through R6) gated to maintain a constant current through the dc link. The Inverter system thus has a current source characteristic with an output voltage determined by load characteristics. This type of inverter contrasts to a voltage source inverter, such as the pulse-width modulated inverter, whose voltage is held fixed and whose current is determined by load characteristics.

A diagram of a segment of the inverter circuit to illustrate commutation is shown in Figure B-19.

The waveforms associated with thyristor commutation for one output cycle of a three-phase circuit are shown in Figure B-20. Figure B-20a shows the voltage across the thyristor referenced to point Y in Figure B-19, Figure B-20b shows the commutation capacitor voltage referenced to the junction of SCRI-3 and D-3, and Figure B-20c shows the commutation capacitor current, positive current in the direction of the arrow in Figure B-19. The cycle is divided into three regions: Region I consists of the thyristor "ON" interval; Region II consists of the commutation interval; and Region III is the thyristor "OFF" interval.

Prior to commutation, load current flows through a conducting thyristor (SCR-11); through the blocking diode, D1; into phase I of the load and out through load phase III, (D2 and SCRI-2).

During the thyristor conduction period, Region I, the voltage across the thyristor is zero and the commutating capacitor (c) is charged to a voltage of  $+E_d$  volts, with polarity as shown.

The positive terminal is at  $+E_d$  because SCRI-1 is on, and the negative terminal is at the zero reference ( $E_y$ ) because either a commutation pre-charge circuit has initially forced it there, or previous commutation pulses have charged it to that level. Under

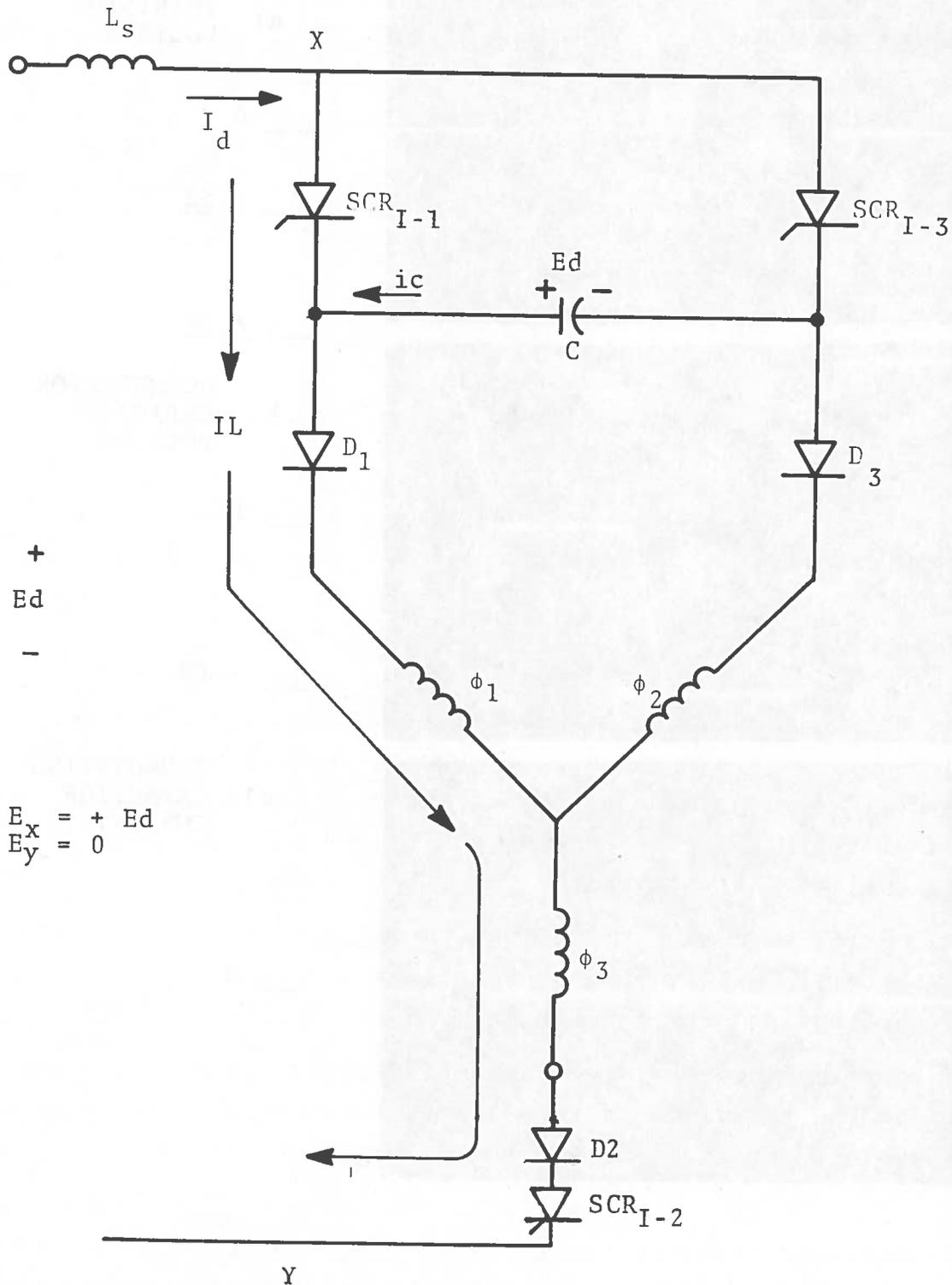
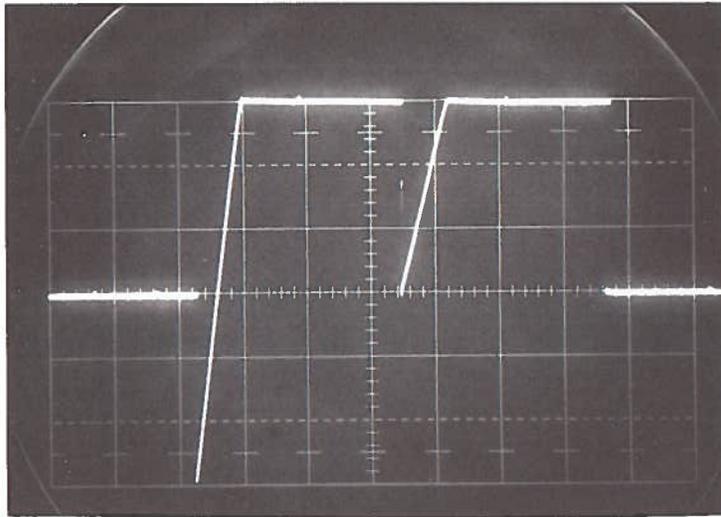


FIGURE B-19. COMMUTATION

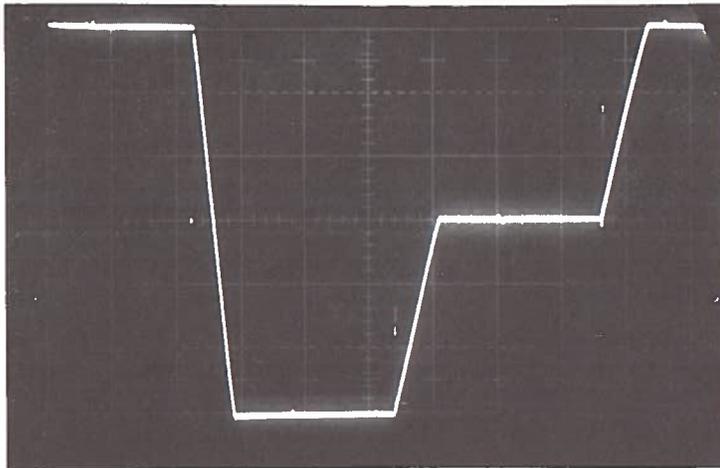


— +  $E_d$

a) THYRISTOR  
VOLTAGE

— 0

— -  $E_d$

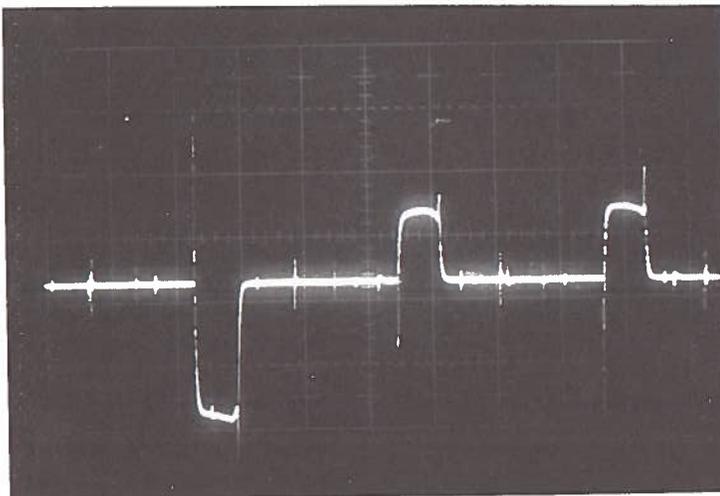


— +  $E_d$

b) COMMUTATION  
CAPACITOR  
VOLTAGE

— 0

— -  $E_d$



c) COMMUTATION  
CAPACITOR  
CURRENT

— 0

FIGURE B-20. THYRISTOR COMMUTATION WAVEFORMS  
IN THREE-PHASE CIRCUITS

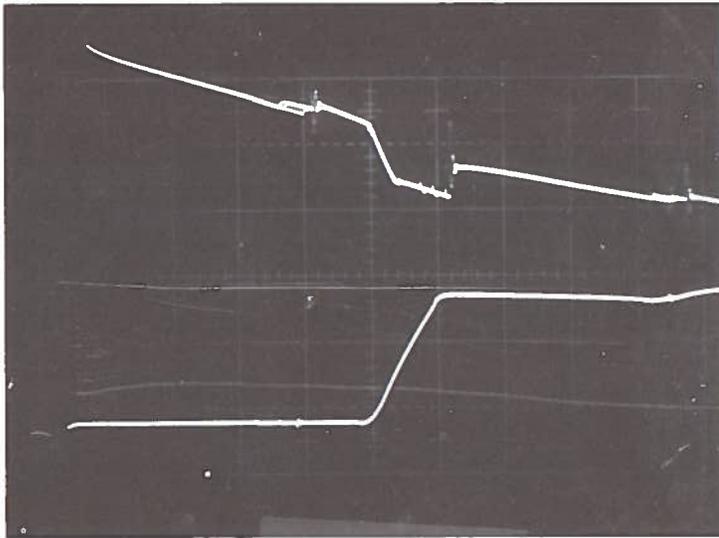
these conditions D3 is reverse biased and in the blocking mode, D3 prevents the capacitor from discharging, and thus maintains the conditions required for successful commutation of SCRI-1.

When SCRI-3 is triggered on to initiate the commutation of SCRI-1 and transfer load current to  $\emptyset 2$ , the voltage at point x is pulled down to the capacitor negative terminal voltage ( $E_y$ ). Because of the high impedance presented by the series line reactor ( $L_s$ ),  $E_d$  is not a hard voltage and is able to change. The voltage across the capacitor, however, can not change instantaneously, and thus a reverse voltage is impressed across SCRI-1, forcing it to commutate. Load current continues to flow through  $\emptyset 1$ , via SCRI-3, C and D1 as the commutating capacitor discharges.  $L_s$  enforces the constraint that the circuit current be essentially constant thus the capacitor voltage change is linear as shown in Figure B-20b and the discharge current is nearly a constant amplitude as shown in Figure B-20c. When the capacitor has discharged to the point that the voltage at  $\emptyset 2$  equals the voltage at  $\emptyset 1$ , the current transfers from  $\emptyset 1$  to  $\emptyset 2$  at a rate determined by the commutating reactance as shown in Figure B-21.

At the point that the voltage at  $\emptyset 1$  equals the voltage at  $\emptyset 2$ , the capacitor is fully discharged. The transfer of the energy stored in the commutating reactance to the capacitor continues the capacitor charge transfer until the voltage across the capacitor is equal in magnitude but opposite in polarity to that shown in Figure B-19.

Line to line voltage is boosted to  $E_d$  during that interval, and excess commutation energy is dissipated in burden circuitry to prevent over-charging.

Midway through Region III a commutation transient is generated by the third positive branch transition. The current associated with this transient discharges the capacitor so that there is no voltage across it. The capacitor is then recharged to the polarity shown in Figure B-19, the voltage required to commutate SCRI-1, when SCRI-1 is turned on at the start of Region I.



VOLTS

200 V/CM

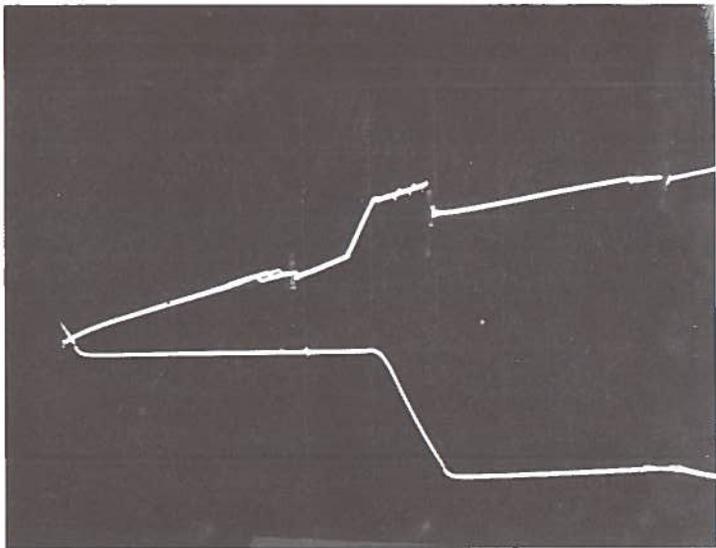
AMPS

20 A/CM

HORIZONTAL =

5  $\mu$ SEC/CM

POSITIVE TRANSITION



VOLTS

200 V/CM

AMPS

20 A/CM

HORIZONTAL =

5  $\mu$ SEC/CM

NEGATIVE TRANSITION

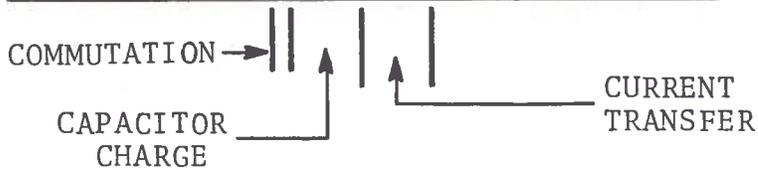
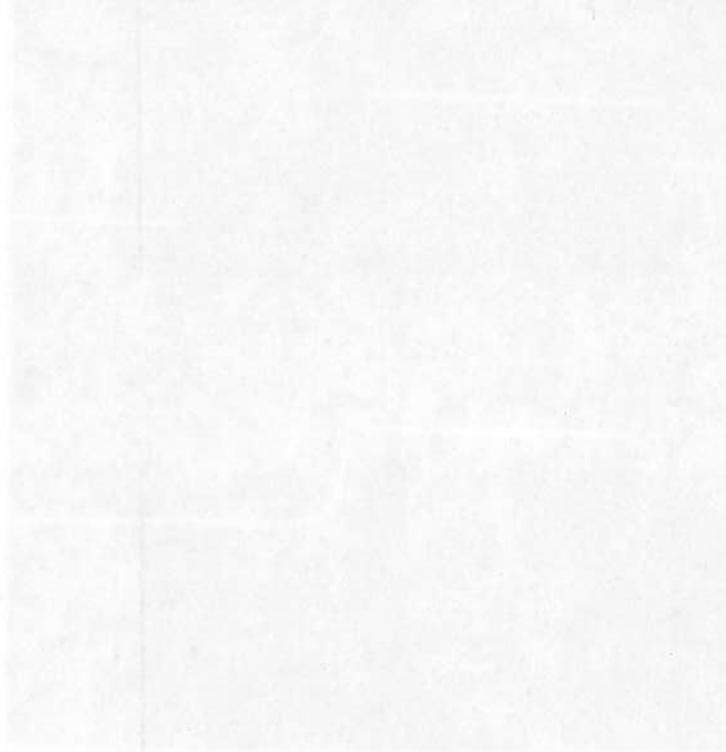


FIGURE B-21. CONSTANT CURRENT INVERTER, OUTPUT VOLTAGE AND CURRENT DURING COMMUTATION INTERVALS

It is important to note that in the commutation interval (Region II), the period of reverse bias across the thyristor is approximately one half of the total interval, but thyristor commutation is accomplished in a fraction of that time as shown in Figure B-22, which is a photograph of thyristor voltage and current at the start of Region I. This high turn-off time margin in the complementary commutated inverter makes it possible to employ rectifier grade thyristors instead of more costly inverter grade thyristors.



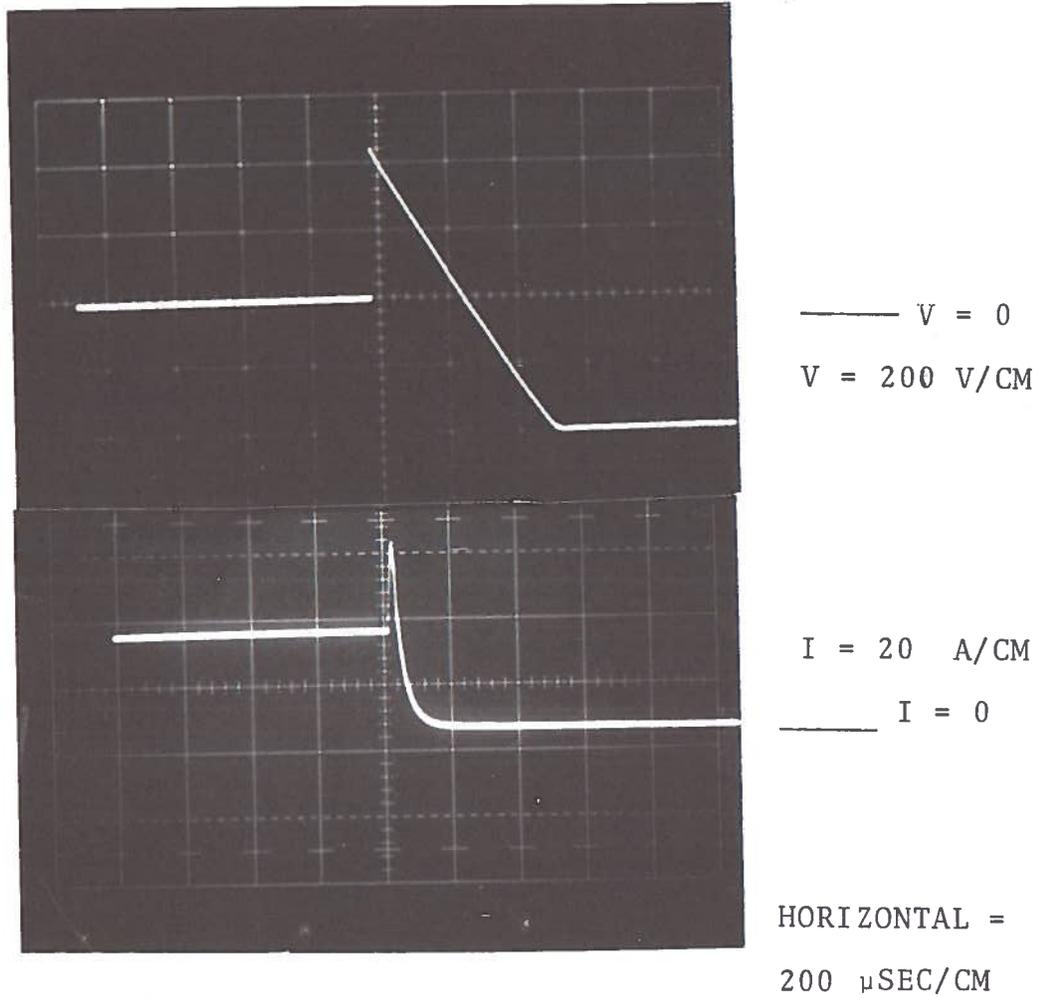


FIGURE B-22. CONSTANT CURRENT INVERTER, INVERTER THYRISTOR TURN-OFF

## APPENDIX C

### POWER CHARACTERISTICS AND MEASUREMENTS

The power flow in a three-phase system can be resolved into components according to the following criteria:

1. The apparent power ( $S_T$ ) is the total volt-ampere power which flows into the system. It is the sum of the products of rms phase voltage and rms phase current measured from each line to neutral,

$$S_T = 3 V_{1-n} \cdot I_{1-n} \quad (C-1)$$

Or in terms of rms line voltage and rms line currents:

$$S_T = \sqrt{3} V_L \cdot I_L \quad (C-2)$$

2. The active, real, or effective power (P) is that component of apparent power which is dissipated either in a load or as losses.
3. Reactive power (QT) is that component of apparent power which contributes to the total volt-amperes of a system but not to active power.
4. The system power factor (pf) is the ratio of the active power to the apparent power. It is also the cosine of the angle ( $\theta$ ) between the apparent power and the active power.

These criteria are illustrated in Figure C-1 which is a phasor diagram of a power system. In a balanced three-phase system, where currents and voltages are sinusoidal and of the same frequency, the power characteristics can be completely determined using the two wattmeter method.

Because the wattmeters are sensitive to the product of the current, the voltage, and the phase angle between them, the individual wattmeter outputs can be used to calculate active power,

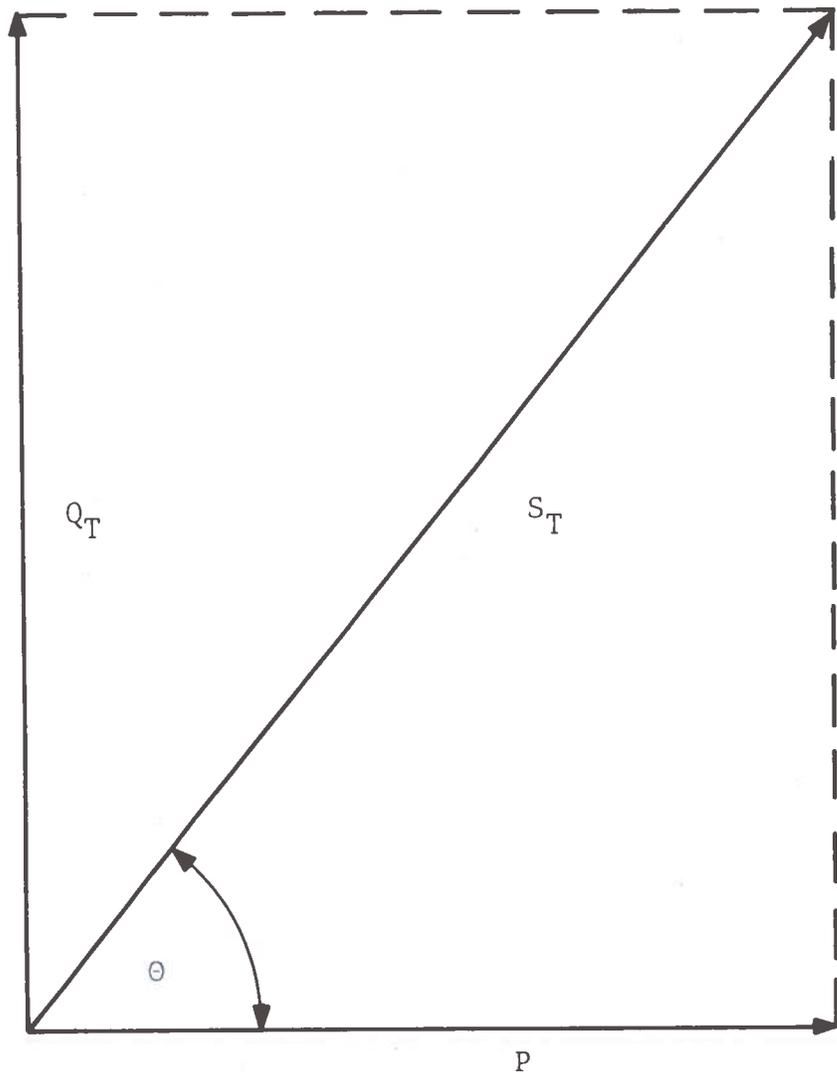


FIGURE C-1. PHASOR DIAGRAM OF TOTAL POWER COMPONENTS

reactive power, power factor, and apparent power using the following formulas:

$$P = W_1 + W_2$$
$$Q_1 = \sqrt{3} (W_1 - W_2)$$
$$\tan \theta = Q_1/P$$
$$\text{pf} = \cos \theta$$
$$S = P/\cos \theta$$

where

- $W_1, W_2$  = wattmeter readings,
- $P$  = active power,
- $Q$  = reactive power,
- $S$  = apparent power,
- $\text{pf}$  = system power factor,
- $\theta$  = system power factor angle,  
which is the angle between  
applied voltage and current.

Rectifier systems, in which the line currents are abruptly switched or chopped, contain harmonic distortion components of current and/or voltage. For these systems, significant measurement errors result if only wattmeters are used to measure the power characteristics. To characterize these systems, the following measurements are required: active power, apparent power, displacement reactive power, distortion reactive power, and system power factor. Some of these parameters are available through direct measurement, and the remainder can be determined through indirect measurements. The following formulas can be used to obtain the complete power characteristics of a three-phase system:

active power,  $P = W_1 + W_2$  (C-3)

total apparent power,  $S_T = \sqrt{3} V_L I_L$  (C-4)

system power factor,  $pf = P/S_T \cos\theta$  (C-5)

total reactive power,  $Q_T = S_T \sin\theta$  (C-6)

phase displacement power,  $Q_p = \sqrt{3} (W_1 - W_2)$  (C-7)

distortion reactive power,  $Q_d^2 = Q_T^2 - Q_p^2$ . (C-8)

Note also that the total reactive power is

$$Q_T = \sqrt{Q_p^2 + Q_d^2}. \quad (C-9)$$

The active power can be measured with two wattmeters, providing their bandwidth includes all of the harmonics that are present. The wattmeters give the solution to the integral

$$P = \frac{1}{T} \int_0^T v(t) \cdot i(t) dt \quad (C-10)$$

and give a true reading of power regardless of the degree of unbalance and distortion.

The rms voltage and rms current measurements must be made with true rms reading instruments.

There is a power factor associated with a rectifier or chopper system which is the result of either or both of two phenomena:

1. The delay of thyristor turn-on in phase delay rectifier systems results in a phase displacement of the current and voltage waveforms and, hence, a displacement component of power factor.
2. The chopping action of rectifier systems generates harmonic currents which distort the waveforms and produce a distortion component of power factor.

The current and voltage can be expressed in terms of the fundamental, its harmonics, and displacement angles,

$$v(t) = \sum_{n=1}^{\infty} E_{p_n} \sin n\omega t \quad (C-11)$$

$$i(t) = \sum_{n=1}^{\infty} I_{p_n} \sin (n\omega t + \psi_n) \quad (C-12)$$

and the integral for average power can be rewritten

$$P = \frac{1}{T} \int_0^T \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} E_{p_m} \sin m\omega t \cdot I_{p_n} \sin (n\omega t + \psi_n) d(\omega t) \quad (C-13).$$

This integral has a non-zero solution only for the cases where  $m=n$ .

Figure C-2 is a phasor diagram of the power components which can be constructed from data taken with two wattmeters. It does not completely describe the power characteristics of the system with harmonics. All of the components in Figure C-2 result from currents and voltages of the same frequencies. The apparent power,  $S_p$ , associated with these quantities can be resolved into its components; the active power,  $P$ , and a reactive power component,  $Q_p$ , which results from the phase displacement of the currents and voltages. The angle,  $\theta$ , is the displacement or phase angle between the active power and  $S_T$ .

For the case where the input voltage waveform contains only the fundamental frequency and no harmonics, the angle  $\theta$  is the angle between the active power of the fundamental and the apparent power of the fundamental. It is also the displacement angle between the fundamental voltage and the fundamental current. The cosine of this angle is the displacement power factor which is measured by watt hour and var hour meters. Note that the displacement power factor is the same as the system power factor only for sinusoidal conditions.

The displacement angle varies with the delay interval in phase delay rectifiers. Addition of inductive filtering to a rectifier output increases the current phase lag and results in a lower displacement power factor, but reduces harmonics.

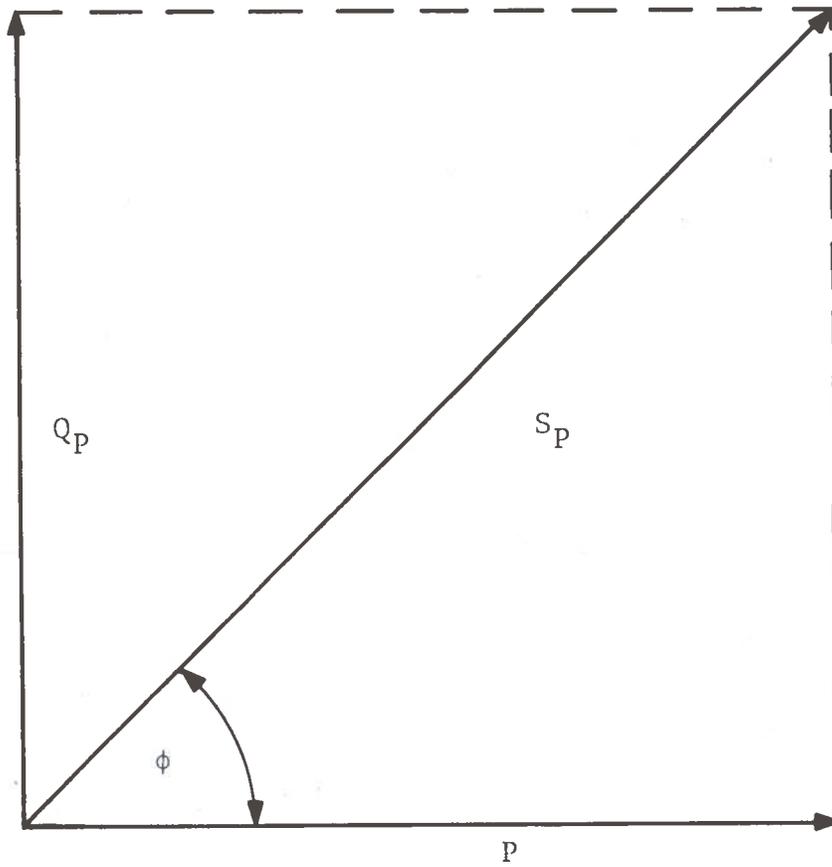


FIGURE C-2. PHASOR DIAGRAM OF POWER COMPONENTS FROM WATT METER READINGS

The presence of harmonics in the line current must also be considered in characterizing a power system. The harmonics do not contribute to active power, but they do contribute to apparent power, therefore, the harmonic or distortion component of power must be considered as a component of reactive power.

The rms quantities which are used to calculate total apparent power contain both fundamental and harmonic components. These components combine vectorially to produce the rms line currents and voltages, e.g.,

$$I_{\ell} = \sqrt{\sum_{n=1}^{\infty} I_n^2},$$

where

$$I_{\ell} = \text{rms line current,}$$

$$n = \text{order of harmonic,}$$

$$I_n = \text{rms value of nth harmonic.} \quad (C-14)$$

Since the harmonic distortion components contribute to the total rms value vectorially, the distortion components also contribute to total reactive power vectorially. This is illustrated in Figure C-3 which is a phasor diagram of the displacement and distortion components of reactive power. The resultant reactive power ( $Q_T$ ) is equal to system reactive power shown in the total power phasor diagram, Figure C-1.

A pictorial presentation of the power components and how they combine is presented in Figure C-4. The total system apparent power consists of three mutually orthogonal components, active power, phase displacement reactive power, and distortion reactive power. All three components must be known to completely characterize the system.

In the inverter test program, two wattmeters are used to measure the active power for all conditions.

The total system apparent power is determined from the true rms current and true rms voltage measurements.

The total power and the total apparent power measurements are used to determine the system power factor and total reactive power.

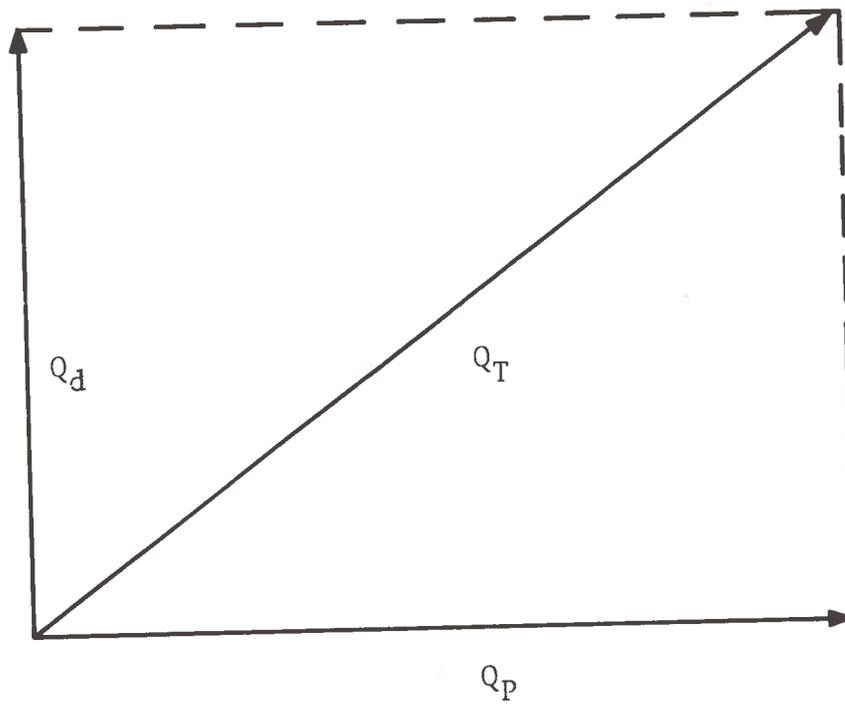


FIGURE C-3. PHASOR DIAGRAM OF REACTIVE POWER COMPONENTS

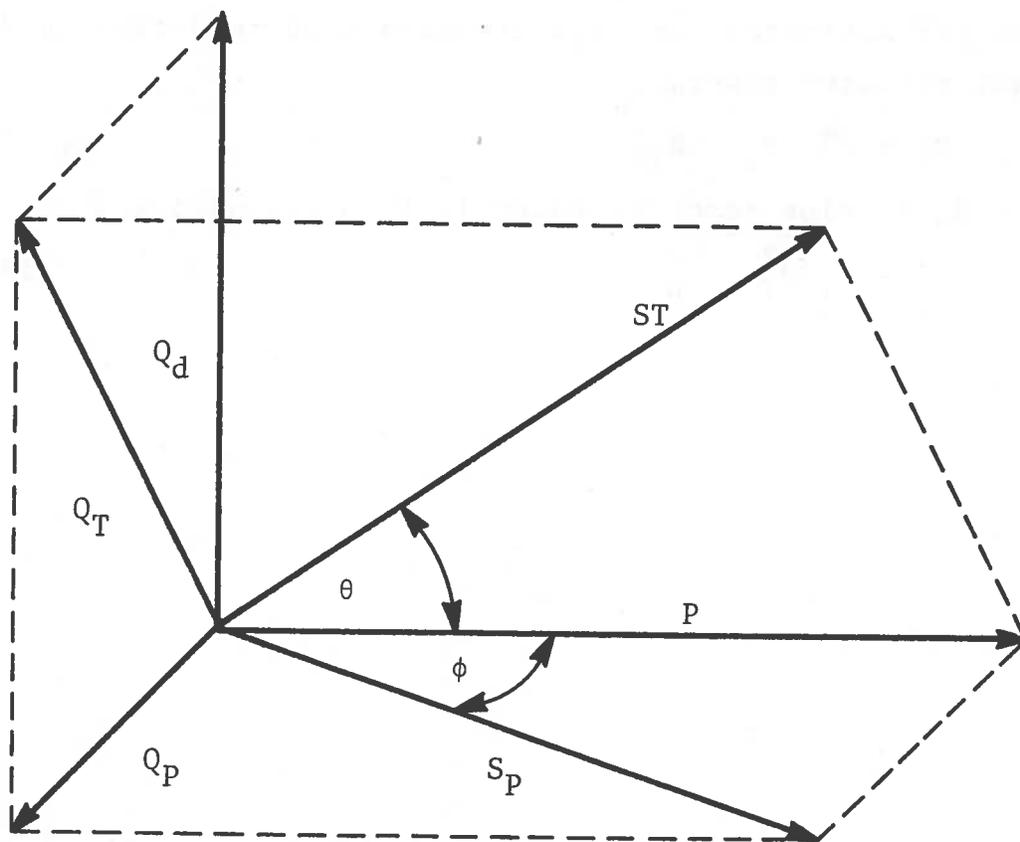


FIGURE C-4. PHASOR DIAGRAM OF TOTAL SYSTEM POWER COMPONENTS

$$\text{pf} = \frac{W_1 + W_2}{\sqrt{3} V_L \cdot I_L} = \cos\theta \quad (\text{C-15})$$

$$Q_T = \sqrt{3} V_L \cdot I_L \sin\theta . \quad (\text{C-16})$$

The two wattmeter readings are also used to determine displacement reactive power,

$$Q_p = \sqrt{3} (w_1 - w_2) \quad (\text{C-17})$$

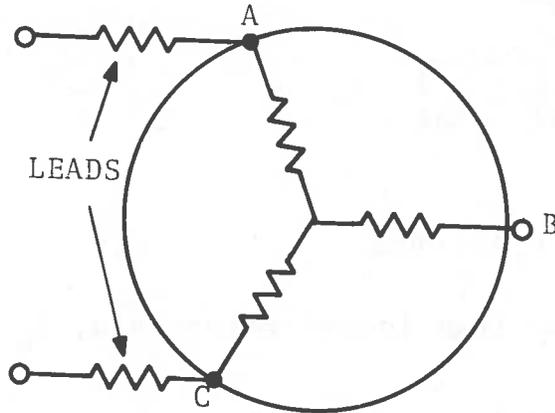
The distortion reactive power is then calculated from

$$Q_d = \sqrt{Q_T^2 - Q_p^2} . \quad (\text{C-18})$$

## APPENDIX D

### DETERMINATION OF MOTOR EQUIVALENT CIRCUIT PARAMETERS

a. DC Resistance Data:



Resistance measured at inverter output terminals to motor (includes lead resistance) = 0.41 ohms. Therefore  $R_1 = 0.205$  ohms phase. Note: Resistance measured at motor input terminals = 0.178 ohms.

b. No Load Data at 60 Hz

V	DATA		CALCULATED		
	I	P	$Z_{n\ell}$	$R_{n1}$	$X_{n1}$
volts	amps	watts	ohms	ohms	ohms
150	13.5	280	6.415	.5121	6.395
175	15.8	360	6.395	.4807	6.377
207	19.2	540	6.205	.4883	6.186

$$Z_{n\ell_{\text{average}}} = 6.338 \text{ ohms}$$

$$R_{n\ell_{\text{average}}} = .494 \text{ ohms}$$

$$X_{n\ell_{\text{average}}} = 6.319 \text{ ohms}$$

$$V_T = \text{motor terminal voltage, line-line} \quad (D-1)$$

$$P = \text{input power (two wattmeter method)} \quad (D-2)$$

$$I_1 = \text{motor phase current} \quad (D-3)$$

$$(Z_{nl}) = \frac{V_T}{\sqrt{3} I_1} \quad (D-4)$$

$$R_{nl} = \frac{P}{3 I_1^2} \quad (D-5)$$

$$X_{nl} = \sqrt{Z_{nl}^2 - R_{nl}^2} \quad (D-6)$$

Therefore:

$$X_1 + X_m = 6.319 \text{ ohms} \quad (D-7)$$

If  $X_1 = .365$  ohms from locked-rotor data,  $X_m = 5.955$  ohms.

c. locked rotor data

Input Power Data:					
I	P	R	X	T	Q
A				lb-ft	var
12.3	220	.485	.992	.65	450
19.9	520	.438	.991	2.05	901
25	780	.416	.961	3.05	1801
32.7	1420	.443	1.026	5.55	3291
50.5	3200	.418	.983	13.05	7517

$$R_{br} \text{ (blocked rotor)} = .44 \text{ ohms}$$

$$X_{br} \text{ (blocked rotor)} = .91 \text{ ohms}$$

$$I = \text{Motor phase current}$$

$$Q = \text{Input reactive power}$$

$$X_1 + X_{\text{secondary}} = .91 \text{ ohms} \quad (D-8)$$

Assume  $X_1 = 0.5 (X_1 + X_{\text{secondary}})$

$$X_1 = 0.455 \text{ ohms.}$$

d. Locked-Rotor Torque Measurements

I	T	T/I
A.	lb-ft	
12.3	.65	.004296
19.9	2.05	.005177
25	3.05	.00488
32.7	5.55	.00519
50.5	13.05	.005117

$$\left(\frac{T}{I}\right)_{\text{ave}} = .00493 \quad (D-9)$$

$$R_2^* = \frac{T\omega_s}{3I_2^2} = .2112 \text{ ohms} \quad (D-10)$$

where  $R_2^*$  is the equivalent secondary resistance.

e. Secondary Equivalent Circuit Parameters

$X_1$  and  $X_m$  are determined from no-load and blocked rotor tests.  $R_1$  is determined from the dc resistance test. Secondary circuit parameters are found by fitting measured motor impedance over the full motor speed range. Three slips were selected for parameter fitting from test data:  $S = .054, .50, \text{ and } 1.0$ .

Effective secondary impedance  $Z_2$  is given by

$$Z_2 = \frac{R_2^*}{S} + jX_2^* = \frac{1}{\frac{P+jQ}{3I_2^2} - (R_1+jX_1)} - \frac{1}{jX_m} \quad (D-11)$$

where:

$P+jQ$  = measured input complex power,

$R_1 = .205$  ohms

$X_1 = .455$  ohms

$X_m = 5.93$  ohms.

Tabulated Measured and Computed Motor Data

S	RPM	I A	P W	Q VAR	$\frac{R_2^*}{S}$	$X_2^*$
1.0	0	40.5	2140	4950	.285	.667
.5	450	37.3	1920	4573	.327	.777
.054	85.1	21.7	2020	1970	1.875	.554

This yields the following comparisons between "measured" and "modeled" effective secondary impedance.

S	"Measured" $R_2/S + jX_2^*$	"Modeled" $R_2/S + jX_2^*$
1.0	.285+j .667	.292+j .558
.5	.327+j .777	.333+j .683
.054	1.895+j .554	1.687+j .765.

The above choice of circuit parameters represents the "best fit" to motor impedance data as determined from input power test data; and yield for the characteristics parameter m, defined by Alger in reference 14,  $m = 1.0$ . The parameter m describes the type or shape of the torque-speed characteristic.<sup>(14)</sup>

The final choice of parameters used in the computer analysis of the induction machine drives is summarized in the Table D-1.

TABLE D-1. SUMMARY OF INDUCTION MOTOR EQUIVALENT CIRCUIT PARAMETERS (60 Hz)

R1 =	0.205 ohms
X1 =	0.450 ohms
Xm =	5.93 ohms
Xz =	0.35 ohms
Xa =	0.50 ohms
Ra =	0.10 ohms
Rb =	0.503 ohms

## APPENDIX E

### DERIVATION OF PULSE-WIDTH MODULATED INVERTER COMPUTER MODEL

The pulse-width modulated inverter is a voltage drive. The computer model simulates the output voltage waveform as given in Figure 9-16. Thus if  $f_1(t)$  describes the waveform in Figure 9-16b, and  $f_2(t)$  describes the waveform in Figure 9-16c, then the inverter output voltage is,

$$v(t) = f_1(t) \cdot f_2(t) \cdot V_o, \quad (E-1)$$

where

$$f_1(t) = \sum_{n=1}^{\infty} A_n \cos n\omega t \quad (E-2)$$

$$f_2(t) = \sum_{m=0}^{\infty} B_m \cos m\omega t. \quad (E-3)$$

The number of notches in a 180 degree time period is  $p$ . The model derivation requires the determination of the Fourier coefficients,  $A_n$  and  $B_m$ , and the final transformation of equation E-1 into the form,

$$v(t) = \sum_n^V V_n \cos n'\omega t. \quad (E-4)$$

The line-to-line voltage waveform is given by  $f_1(t)$ , where

$$\begin{aligned} A_n &= -\pi \frac{\int_{-\pi}^{\pi} f_1(t) \cos n\omega t \, d\omega t}{\int_{-\pi}^{\pi} \cos^2 n\omega t \, d\omega t} \\ &= \frac{4}{n\pi} \frac{\sin n\pi}{2} \frac{\cos n\pi}{6}. \end{aligned} \quad (E-5)$$

Similarly, the Fourier coefficient,  $B_m$ , is given by,

$$B_m = \frac{\int_{-\pi}^{\pi} f_2(t) \cos m 2p \omega t \, d\omega t}{\int_{-\pi}^{\pi} \cos^2 m 2p \omega t \, d\omega t} \quad (E-6)$$

$$B_m = \frac{\alpha_1}{\pi}, \quad m = 0$$

$$= \frac{2}{m\pi} \sin m \alpha_1, \quad m = 1, 2, 3, \dots \quad (E-7)$$

where  $\alpha_1$  is the half-width of the on-off modulation pulse expressed in radians.

The inverter output voltage,  $v(t)$ , then takes the form,

$$v(t) = V_o \sum_{n=1}^{\infty} \frac{4}{n\pi} \frac{\sin n \pi}{2} \frac{\cos n \pi}{6} \cos n \omega t \sum_{m=1}^{\infty} \left( \frac{\alpha_1}{\pi} + \frac{2}{m\pi} \sin m \alpha_1 \cos m 2p \omega t \right). \quad (E-8)$$

Next, terms common to a given harmonic order must be collected. Remembering that:

$$\cos \omega t \cos m 2p \omega t = 1/2 [\cos (m 2p + n) \omega t + \cos (m 2p - n) \omega t]. \quad (E-9)$$

Terms contributing to the amplitude of the  $n'$ -th harmonic include  $n = n'$  for the first term in the  $f_2(t)$  series and  $n = n' \pm 2mp$  for the remaining cosine product terms. The latter terms require special attention. To determine their contributions to the  $n'$ -th harmonic amplitude, the index transformation,  $n = n' \pm 2mp$  is made in equation (E-8). Summing their contributions to the  $n'$ -th harmonic gives

$$\frac{V_o 4}{\pi} \left[ \frac{\sin (n' - 2mp) \pi / 2}{(n' - 2mp)} \cos \frac{(n' - 2mp) \pi}{6} + \frac{\sin (n' + 2mp) \pi / 2}{(n' + 2mp)} \cos \frac{(n' + 2mp) \pi}{6} \right]^* \frac{\sin m \alpha_1}{m} \quad (E-10)$$

The above expression can be reduced by trigonometric expansion and observation that  $\sin m p \pi / 3 = 0$  for  $p$  equals to multiples of 3 to yield

$$v(t) = \frac{8}{\pi} V_o \sin n' \frac{\pi}{2} \cos n' \frac{\pi}{6} \frac{n' \sin m \alpha}{m [(2mp)^2 - (n')^2]}. \quad (E-11)$$

Finally, adding the  $n = n'$  contribution term to the above expression gives the amplitude of the  $n'$ -th voltage harmonic,

$$V_{n'} = \frac{4\alpha_1}{\pi^2} \sin n' \frac{\pi}{2} \cos \frac{n' \pi}{6} \left[ \frac{1}{n'} + \sum_m \frac{2n' \sin m \alpha_1}{\alpha_1 [n^2 - (2mp)^2]} \right]. \quad (E-12)$$

This can be put in the form given by equation 9-25 if  $\alpha_1$  is set equal to  $\alpha\tau$ , where  $\tau$  is the duty cycle of the on-off modulation pulse; and the amplitude coefficient is multiplied by  $\frac{\sqrt{3}V_o \text{ (RMS)}}{\sqrt{\tau^2}}$ , where  $V_o$  is the rms line-to-line output voltage of the inverter.

## E-2. DERIVATION OF THE CONSTANT CURRENT INVERTER COMPUTER MODEL

The constant current inverter drive develops a controlled current output having a waveform as shown in Figure 9-8. The simulation of the constant current inverter drive requires the derivation of an analytical expression to describe this waveform. The effect of commutation is to modify the square wave function at the leading and trailing edges of the current wave. This is simulated by superimposing on the current wave,  $f_1(t)$ , the current function,  $f_2(t)$ , shown in Figure 9-8c.

The inverter output current is given by

$$\begin{aligned} i(t) &= \sum_n I_n \cos n\omega t \\ &= I_o [f_1(t) + f_2(t)], \end{aligned} \quad (E-13)$$

where  $I_0$  is the peak value of the square wave current.

$$f_1(t) = \sum_n A_n \cos n\omega t \quad (\text{E-14})$$

$$A_n = \frac{\int_{-\pi}^{\pi} f_1(t) \cos n\omega t \, d\omega t}{\int_{-\pi}^{\pi} \cos^2 n\omega t \, d\omega t}$$

$$= \frac{4}{\pi n} \frac{\sin n\pi}{2} \frac{\cos n\pi}{6} \quad (\text{E-15})$$

and  $f_2(t)$  is the function shown in Figure E-1.

The Fourier expansion of the function  $f_2(t)$  begins by considering the saw-tooth function shown below.

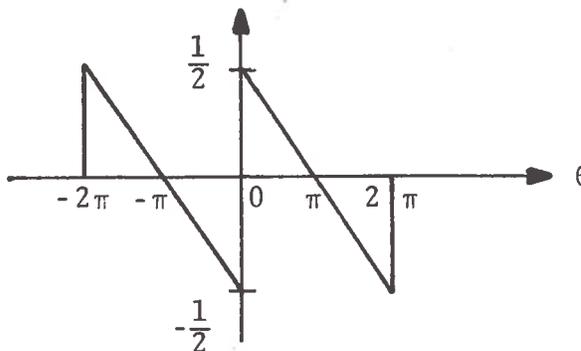


FIGURE E-1.

This function is described by the Fourier series,

$$f(\theta) = \sum_m \frac{1}{m\pi} \sin m\theta. \quad (\text{E-16})$$

Since the commutation takes place over a  $2\delta$  radian interval, the above series becomes

$$f_2(\theta) = \sum_m \frac{1}{m\pi} \sin m \frac{2\pi\theta}{2\delta} \quad -\delta \leq \theta \leq \delta. \quad (\text{E-17})$$

The contribution of  $f_2(t)$  to the current harmonic amplitude is found by evaluating the following integral,

$$\frac{2}{\pi} \int_{-\delta}^{\delta} \left( \cos\left(\theta + \frac{\pi}{3}\right) + \cos n \left(\theta + \frac{2\pi}{3}\right) \right) f_2(\theta) d\theta. \quad (E-18)$$

Now,

$$\cos n\left(\theta + \frac{\pi}{3}\right) + \cos n\left(\theta + \frac{2\pi}{3}\right) = 2 \cos \frac{n\pi}{6} \left[ \cos n\theta \cos \frac{n\pi}{2} - \sin n\theta \sin \frac{n\pi}{2} \right] \quad (E-19)$$

The first term on the right hand side gives zero contribution to the integral by symmetry. The second term gives

$$\begin{aligned} & \frac{-4}{\pi} \cos \frac{n\pi}{6} \sin \frac{n\pi}{2} \int_{-\delta}^{\delta} \sum_m \frac{1}{m\pi} \sin n\theta \sin \frac{m\pi\theta}{\delta} d\theta \\ &= \frac{-4}{\pi^2} \cos \frac{n\pi}{6} \sin \frac{n\pi}{2} \sum_m \frac{1}{m} \left( \frac{\sin(m\pi - n\delta)}{\frac{\pi}{\delta} m - n} - \frac{\sin(m\pi + n\delta)}{\frac{\pi}{\delta} m + n} \right). \end{aligned} \quad (E-20)$$

The term in the bracket can be rewritten as

$$\left( \right) = \frac{-2\pi m}{\left(\frac{m\pi}{\delta}\right)^2 - n^2} \cos m\pi \cdot \sin n\delta, \quad (E-21)$$

to yield

$$\begin{aligned} & \frac{2}{\pi} \int_{-\delta}^{\delta} \left[ \cos n\left(\theta + \frac{\pi}{3}\right) + \cos n\left(\theta + \frac{2\pi}{3}\right) \right] f_2(\theta) d\theta = \\ & \frac{8}{\pi\delta} \cos \frac{n\pi}{6} \sin \frac{n\pi}{2} \cdot \sin n\delta \sum_m \frac{\cos m\pi}{\left(\frac{m\pi}{\delta}\right)^2 - n^2}. \end{aligned} \quad (E-22)$$

Combining the above contribution with the  $A_n$  term gives the following expression for the current harmonic amplitude,

$$I_n = \frac{4}{\pi} I_0 \sin \frac{n\pi}{2} \cos \frac{n\pi}{6} \left[ \frac{1}{n} + 2 \frac{\sin n\delta}{\delta} \sum_m \frac{\cos m\pi}{\left(\frac{m\pi}{\delta}\right)^2 - n^2} \right] \quad (E-23)$$

$m=1, 2, 3, \dots$

This expression has non-zero values for  $n = 1, 5, 7, 11, \dots$

The line-to-line voltage is given by

$$\begin{aligned}
 v_{L-L}(t) &= \sum_n I_n \cdot Z_n [\cos n(\omega t - \frac{\pi}{3}) + \cos n(\omega t + \frac{\pi}{3})] \\
 &= \sum_n 2I_n \cdot Z_n \frac{\cos n\pi}{3} \cos n\omega t \quad (E-24) \\
 n &= 1, 5, 7, \dots
 \end{aligned}$$

### E-3. DERIVATION OF NATURAL COMMUTATED INVERTER COMPUTER MODEL

The natural commutated inverter drive delivers a controlled output voltage of variable amplitude and frequency. Except at low output power levels, the output voltage is a relatively good sine wave function; hence, the computer model neglects harmonics in the inverter output spectrum.

The input current to the drive experiences considerable distortion as a result of the capacitive loading of the input rectifier. The current waveforms given in Figure 9-5 are used as the basis for the model. Assuming the zero time reference as given in the Figure, the input current can be expressed by the Fourier series,

$$i(t) = \sum_n I_n \cos n\omega t, \quad (E-25)$$

where

$$I_n = \frac{\int_{-\pi}^{\pi} i(t) dt}{\pi} \quad (E-26)$$

For purposes of modeling,  $i(t)$  is assumed to be symmetric about the zero-time axis and to be given by the following functions over the time interval  $0 \leq \omega t \leq \pi$ .

Discontinuous Conduction:  $\phi \leq 0$

$$\begin{aligned}
 i(t) &= I_o (\sin 3\omega t - \sin 3\phi) & \phi \leq \omega t \leq \frac{\pi}{3} - \phi \\
 &= 0 & \frac{\pi}{3} - \phi \leq \omega t \leq \frac{2\pi}{3} + \phi \\
 &= -I_o (\sin 3\omega t - \sin 3\phi) & \frac{2\pi}{3} + \phi \leq \omega t \leq \pi - \phi. \quad (E-27)
 \end{aligned}$$

Overlapping (Continuous) Conduction:  $\xi < 0$

$$\begin{aligned} i(t) &= I_0 (\sin 3\omega t - \sin 3\xi) & 0 \leq \omega t \leq \pi/3 \\ &= 0 & \frac{\pi}{3} \leq \omega t \leq \pi. \end{aligned} \quad (E-28)$$

The parameter,  $\xi$ , modifies the degree of overlap.

The calculation of the harmonic amplitudes,  $I_n$ , proceeds in two steps. First, the time-dependent term is considered. It yields the following contribution to the harmonic amplitude,

$$\begin{aligned} & \frac{2I_0}{\pi} \left[ \int_{\phi}^{\frac{\pi}{3}-\phi} \sin 3\theta \cos n\theta d\theta + \int_{\frac{2\pi}{3}+\phi}^{\pi-\phi} \sin 3\phi \cos n\theta d\theta \right] \\ &= \frac{(1-\cos n\pi)}{\pi} \left[ \frac{\cos(3-n)\theta}{3-n} + \frac{\cos(3+n)\theta}{3+n} \right]_{\frac{\pi}{3}-\phi}^{\phi} \\ &= I_0 \frac{(1-\cos n\pi)}{\pi} \left[ \frac{\cos(3-n)\phi - \cos(3-n)(\frac{\pi}{3}-\phi)}{3-n} + \frac{\cos(3+n)\phi - \cos(3+n)(\frac{\pi}{3}-\phi)}{3+n} \right]. \end{aligned} \quad (E-29)$$

Second, the time-independent term is considered. It gives a contribution.

$$\begin{aligned} & \frac{2I_0}{\pi} \left[ \int_{\phi}^{\frac{\pi}{3}-\phi} -\sin 3\phi \cos n\theta d\theta + \int_{\frac{2\pi}{3}+\phi}^{\pi-\phi} \sin \phi \cos n\theta d\theta \right] \\ &= \frac{2I_0}{\pi n} \sin 3\phi (1-\cos n\pi) [\sin n\phi - \sin n(\frac{\pi}{3}-\phi)]. \end{aligned} \quad (E-30)$$

Combining the contributions from the time-dependent and time-independent terms gives for the harmonic amplitudes,

$$\begin{aligned}
 I_n = \frac{I_0(1-\cos n\pi)}{\pi} & \left[ \frac{\cos(3-n)\phi - \cos(3-n)\left(\frac{\pi}{3} - \phi\right)}{3-n} \right. \\
 & + \frac{\cos(3+n)\phi - \cos(3+n)\left(\frac{\pi}{3} - \phi\right)}{3+n} \\
 & \left. + \frac{2\sin 3\phi \cdot \sin n\phi - \sin n\left(\frac{\pi}{3} - \phi\right)}{n} \right] \quad (E-31)
 \end{aligned}$$

APPENDIX F  
COMPUTER MODEL  
PROGRAM LISTINGS

```

C      NCI.F4
C      COMPUTES CURRENT HARMONICS FOR NATURAL COMMUTATED INVERTER.
      REAL IN, IDB, IRMS, I(50)
      DIMENSION VN(50), PN(50), QN(50), TN(50)
      COMMON VP
90     FORMAT(1X, 8(1X, F7.2))
85     FORMAT(5F10.2)
80     FORMAT(1X, F5.0, 5(F10.4), 2(F8.2))
75     FORMAT(4X, 'N', 5X, 'VN', 9X, 'IN', 8X, 'T', 9X
1      , 'P', 9X, 'Q', 'VN', 5X, 'IN')
102    FORMAT(11X, 'V', 10X, 'A', 7X, 'LB-FT', 6X, 'KW', 7X, 'KVAR', 7X, 'DB', 5X,
1      'DB')
103    FORMAT(1X, 'VP=', F5.1, 5X, 'FREQ=', F5.0, 5X, 'SLIP=', F5.2, 'FLAG='
1      , F5.2)
115    FORMAT(1X, 'VRMS=', F10.5, 'VOLTS, RMS, LINE-LINE')
120    FORMAT(1X, 'IRMS=', F10.5, 'AMPS, RMS')
125    FORMAT(1X, 'TORQUE=', F10.5, 'LB-FT')
130    FORMAT(1X, 'REAL POWER=', F10.5, 'KW')
135    FORMAT(1X, 'DISPLACEMENT POWER=', F10.5, 'KVAR')
140    FORMAT(1X, 'APPARENT POWER=', F10.5, 'KVA')
145    FORMAT(1X, 'MOTOR EFF.=', F10.5)
150    FORMAT(1X, 'POWER FACTOR(FUND)=', F10.5)
155    FORMAT(1X, 'POWER FACTOR(SYSTEM)=', F10.5)
160    FORMAT(1X, 'REACTIVE POWER=', F10.5)
165    FORMAT(1X, 'RATIO=', F10.5)
95     FORMAT(1X, 'NATURAL COMMUTATED INVERTER')
100    FORMAT(1X, F5.0, 7(F10.4))
101    FORMAT(4X, 'N', 5X, 'I(N)', 6X, 'RATIO', 7X, 'IDB')
170    FORMAT(1X, 'IRMS=', F10.4, 3X, 'PHI=', F10.4, 3X, 'BETA=', F10.4)
70     FORMAT(1X, 'INTEGRATED CURRENT HARMONICS=', F10.4)
104    FORMAT(1X, 'DEL=', F10.4, 3X, 'THETA=', F10.4)
      PI=3.14159
      CON=29.4
      SQRT2=SQRT(2.)
      A=1.
      SLIP=0.1
      WRITE(3, 95)
      AN=1
      AL=N+1
      FREQ=60.
      AIN=1.0
      AL=2.
      VP=75.
      CN=VP
      SLIP=.09
      CALL INDM(AN, AIN, P, Q, T, AL, SLIP, CN, FREQ)
      AIN=SQRT(3.)*VP/1000./SQRT(P**2+Q**2)
      WRITE(3, 85)CN, AIN, P, Q
      BETA=0.0
      PHI=0.0
      CN=75.
2      CONTINUE
      DO 12 K=1, 3, 1
      DO 5 J=1, 5, 1

```

```

AA=-4.*COS(3.*PHI)*COS(3.*BETA)+3.*(PI/3.-2.*BETA)*SIN(6.*PHI)
BB=-1.+COS(6.*BETA)+4.*SIN(3.*PHI)*SIN(3.*BETA)-2*SIN(3.*PHI)**2
ARMS=(PI-6.*BETA+SIN(6.*BETA))/6.-4.*SIN(3.*PHI)*COS(3.*BETA)
1 /3.+(PI/3.-2.*BETA)*SIN(3.*PHI)**2
RMS=CON*SQRT(2.*ARMS/PI)
DELP=(AIN-RMS)/(A*BB+AA)/CON
PHI=PHI+DELP
IF(PHI.LE.0.0)A=0.0
IF(PHI.LE.0.0)BETA=0.0
IF(PHI.GT.0.0)BETA=PHI
WRITE(3,85)AIN,AA,BB,ARMS
5 CONTINUE
PHO=PHI*180./PI
BETO=BETA*180./PI
WRITE(3,170)RMS,PHO,BETO
BETA, PHI NOW DETERMINED
COMPUTE CURRENT HARMONICS
AN=-1.
DO 20 N=1,10,1
AL=N+1
AN=AN+3.-(-1.)*AL
AM=3.-AN
AJ=3.+AN
AN1=(COS(AM*BETA)-COS(AM*(PI/3.-BETA)))/AM
AN2=(COS(AJ*BETA)-COS(AJ*(PI/3.-BETA)))/AJ
AN3=-2./AM*SIN(3.*PHI)*(SIN(AM*(PI/3.-BETA))-SIN(AN*BETA))
IN=1E-7+CON*(1.-COS(AN*PI))/PI/SQRT2*(AN1+AN2+AN3)
CALL INDM(AN,IN,P,Q,T,AL,SLIP,CN,FREQ)
WRITE(3,85)BETA,PHI,IN,CN
AMPV=AMPV+CN**2
AMPI=AMPI+AIN**2
P1=P1+P
Q1=Q1+Q
VN(N)=CN
I(N)=AIN
PN(N)=P
QN(N)=Q
TN(N)=T
T1=T1+T
0 CONTINUE
VRMS=SQRT(AMPV)
RATIO=VP/VRMS/SQRT(3.)
WRITE(3,85)VRMS,RATIO
AIN=RATIO*SQRT(AMPI)
2 CONTINUE
T1=T1*RATIO**2
P1=P1*RATIO**2
Q1=Q1*RATIO**2
S=SQRT(3.)*VP*IRMS/1000.
5 CONTINUE
WRITE(3,75)
WRITE(3,102)
AN=-1.
DO 30 N=1,10,1

```

```

AL=N+1
AN=AN+3.-(-1.)**AL
VN(N)=RATIO*I(N)
PN(N)=PN(N)*RATIO**2
QN(N)=QN(N)*RATIO**2
TN(N)=TN(N)*RATIO**2
IF(N.EQ.1)V1=VN(1)
IF(N.EQ.1)AI1=I(1)
RAT1=V1/VN(N)
RAT2=AI1/I(N)
RAT1=ABS(RAT1)
RAT2=ABS(RAT2)
VDB=20.*ALOG10(RAT1)
IDB=20.*ALOG10(RAT2)
WRITE(3,80)AN,VN(N),I(N),TN(N),PN(N),QN(N),VDB,IDB
30 CONTINUE
PM=T1*FREQ*(1.-SLIP)*PI*746./2./550./1000.
EFF=PM/P1
RAT1=QN(1)/PN(1)
PF1=COS(ATAN(RAT1))
PF=P1/S
QX=SQRT(S**2-P1**2)
VRMS=RATIO*VRMS
WRITE(3,115)VRMS
WRITE(3,120)IRMS
WRITE(3,125)T1
WRITE(3,145)EFF
WRITE(3,130)P1
WRITE(3,160)QX
WRITE(3,135)Q1
WRITE(3,140)S
WRITE(3,150)PF1
WRITE(3,155)PF
WRITE(3,165)RATIO
40 CONTINUE
RETURN
END

```

```

C CCI,F4
C COMPUTES VOLTAGE, CURRENT HARMONICS FOR CCI INVERTER USING LOUIS ALLIS
C INDUCTION MOTOR. VOLTAGE HARMONICS ARE LINE-LINE VALUES.
C REAL IRMS,I(50),IDB
C DIMENSION VN(50),PN(50),QN(50),TN(50)
C COMMON VP
90 EORMAT(1X,8(1X,F7.2))
95 FFORMAT(6F10.2)
100 EORMAT(1X,F5.0,5(F10.4),2(F8.2))
101 FFORMAT(4X,'N',5X,'VN',9X,'IN',8X,'T',9X
    ,P',9X,'Q',9X,'VN',5X,'IN')
102 FFORMAT(11X,'V',10X,'A',7X,'LB-FT',6X,'KW',7X,'KVAR',7X,'DB',5X,
    'DB')
103 FFORMAT(1X,'TAU=',F5.2,5X,'VP=',F5.1,5X,'NO.OF PULSES=',F5.0)
104 EORMAT(1X,'FREQ=',F5.0,5X,'SLIP=',F6.3,5X,'RPM=',F5.0,5X,'FLAG=',
    F5.2)
110 EORMAT(1X,'INTEGRATED OUTPUT QUANTITIES')
115 FFORMAT(1X,'VRMS=',F10.5,' VOLTS,RMS (LINE-LINE)')
120 FFORMAT(1X,'IRMS=',F10.5,' AMPS,RMS')
125 FFORMAT(1X,'TORQUE=',F10.5,' LB-FT')
130 FFORMAT(1X,'REAL POWER=',F10.5,' KW')
135 FFORMAT(1X,'DISPLACEMENT POWER(TOTAL)=',F10.5,' KVAR')
140 FFORMAT(1X,'APPARENT POWER =',F10.5,' KVA')
145 FFORMAT(1X,'MOTOR EFF.=',F10.5)
150 FFORMAT(1X,'POWER FACTOR(FUND)=',F10.5)
155 FFORMAT(1X,'POWER FACTOR(SYSTEM)=',F10.5)
160 EORMAT(1X,'REACTIVE POWER(TOTAL)=',F10.5,'KVAR')
165 FFORMAT(1X,'RATIO=',F10.5)
170 FFORMAT(1X,'CONSTANT CURRENT INVERTER CALCULATIONS')
175 FFORMAT(1X,' ')
180 FFORMAT(1X,'I1(FUND)=',F10.5,1X,'AMPS')
185 FFORMAT(1X,'V1(FUND)=',F10.5,1X,'VOLTS')
190 FFORMAT(1X,'DISPLACEMENT POWER(FUND)=',F10.5,'KVAR')
195 FFORMAT(1X,'REACTIVE POWER(TOTAL)=',F10.5,'KAVR')
200 EORMAT(1X,'S=',F10.4,1X,'P=',F10.4,1X,'QP=',F10.4,1X,'QD=',
    F10.4)
1 OPEN(UNIT=1,FILE='FCCI')
READ(1,95)VP,SA,SS,SN,FREQ,FLAG
WRITE(3,170)

```

```

SLIP=SA
NS=SN
DO 40 J=1,NS,1
RPM=900./60.*FREQ*(1.-SLIP)
WRITE(3,175)
WRITE(3,104)FREQ,SLIP,RPM,FLAG
IF(FLAG.EQ.1.0)GO TO 5
WRITE(3,101)
WRITE(3,102)
CONTINUE
IMAX=48.
PI=3.14159
ANUM=30.
DEL=PI/ANUM
IRMS=0.0
PI=0.
Q1=0.0
T1=0.0
CN=0.0
X=0.0
AIMAX=IMAX
CONTINUE
AN=-1.0
AMPIEY.0
AMPL=0.0
DO 20 N=1,30,1
ACOM=0.0
CN=0.0
AL=N+1
AN=AN+3.-(-1.)**AL
DO 21 M=1,5,1
AM=M
B=2.*ANUM/((ANUM**AM)**2-AN**2)
A=2.*SQRT(3.)/PI**2*SIN(AN*DEL)
AJ=AM+1.
ACOM=ACOM+A*B*(-1.)**AJ
CONTINUE
AK=N
AIN=4./PI/AN*SIN(AN*PI/2.)*COS(AN*PI/6.)+(-1.)**AK*ACOM

```

5

10

21

```

GO TO 11
B=(PI/3.+DEL)/2./DEL*COS(AN*PI/3.)+(2.*PI/3.-DEL)/2.
/DEL*COS(AN*2.*PI/3.)
C=SIN(AN*DEL)/AN/DEL*(SIN(AN*PI/3.)+SIN(AN*2.*PI/3.))
D=1./AN/DEL*(AN*PI/3.*COS(AN*PI/3.)-AN*2.*PI/3.*COS(2.*AN*PI/3.))
11 CONTINUE
CALL INPDM(AN,AIN,P,Q,T,AL,SLIP,CN,FREQ)
CN=2.*COS(AN*PI/6.)*CN
AMPL=AMPL+CN**2
AMPI=AMPI+AIN**2
P1=PI+P
Q1=Q1+Q
VN(N)=CN
I(N)=AIN
PN(N)=P
QN(N)=Q
TN(N)=T
T1=T1+T
20 CONTINUE
VRMS=SQRT(AMPL)
RATIO=VP/VRMS
IRMS=RATIO*SQRT(AMPI)
T1=T1*RATIO**2
P1=P1*RATIO**2
Q1=Q1*RATIO**2
S=SQRT(3.)*VP*IRMS/1000.
25 CONTINUE
AN=-1.
DO 30 N=1,30,1
AL=N+1
AN=AN+3.-(-1.)*AL
VN(N)=RATIO*VN(N)
I(N)=RATIO*I(N)
PN(N)=PN(N)*RATIO**2
QN(N)=QN(N)*RATIO**2
TN(N)=TN(N)*RATIO**2
IF(N.EQ.1)V1=VN(1)
IF(N.EQ.1)A1=I(1)
IF(N.EQ.1)Q1=QN(1)

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RAT1=V1/VN(N)
RAT2=AI1/I(N)
RAT1=ABS(RAT1)
RAT2=ABS(RAT2)
VDB=20.*ALOG10(RAT1)
IDB=20.*ALOG10(RAT2)
IF(FLAG.EQ.1.0)GO TO 30
WRITE(3,100)AN,VN(N),I(N),TN(N),PN(N),QN(N),VDB,IDB
CONTINUE
PM=T1*FREQ*(1.-SLIP)*PI*746./2./550./1000.
EFF=PM/PI
RAT1=QN(1)/PN(1)
PE1=COS(ATAN(RAT1))
PF=PI/S
QX=SQRT(S**2-PI**2)
VRMS=RATIO*VRMS
WRITE(3,110)
WRITE(3,115)VRMS
WRITE(3,185)VI
WRITE(3,120)IRMS
WRITE(3,180)AI1
WRITE(3,125)TI
WRITE(3,145)EFF
WRITE(3,130)PI
WRITE(3,160)QX
WRITE(3,190)QM
WRITE(3,135)Q1
WRITE(3,140)S
WRITE(3,150)PF1
WRITE(3,155)PF
WRITE(3,165)RATIO
SIN=SQRT(3.)*465.*IRMS/1000.
PIN=2.5*PI
QD=0.244*SQRT(3.)*465.*IRMS/1000.
QP=SQRT(SIN**2-QD**2-PIN**2)
WRITE(3,200)SIN,PIN,QP,QD
SLIP=SLIP+SS
CONTINUE
RETURN
END

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C      PWM,F4
C      COMPUTES VOLTAGE, CURRENT HARMONICS FOR PWM WAVE USING LOUIS ALLIS
C      INDUCTION MOTOR. VOLTAGE HARMONICS ARE LINE-LINE VALUES.
C      DIMENSION DEL(81),AIR(50),AIX(50),CUR(81),AINN(81)
REAL IRMS,IDB
  90  FORMAT(1X,8(1X,F7.2))
  95  FORMAT(7F10.2)
 100  FORMAT(1X,F5.0,5(F10.4),2(F8.2))
 101  FORMAT(4X,N,5X,VN,9X,IN,8X,T,9X
1    ,P,9X,Q,9X,VN,5X,IN)
 102  FORMAT(11X,V,10X,A,7X,LB-FT,6X,KW,7X,KVAR,7X,DB,5X,
1    DB)
 103  FORMAT(1X,TAU=,F5.2,5X,VP=,F5.1,5X,NO.OF PULSES=,F5.0)
 104  FORMAT(1X,FREQ=,F5.0,5X,SLIP=,F6.3,5X,RPM=,F5.0)
 110  FORMAT(1X,INTEGRATED OUTPUT QUANTITIES)
 115  FORMAT(1X,VRMS=,F10.5, VOLTS,RMS (LINE-LINE))
 120  FORMAT(1X,IRMS=,F10.5, AMPS,RMS)
 125  FORMAT(1X,TORQUE=,F10.5, LB-FT)
 130  FORMAT(1X,REAL POWER=,F10.5, KW)
 135  FORMAT(1X,DISPLACEMENT POWER(TOTAL)=,F10.5, KVAR)
 140  FORMAT(1X,APPARENT POWER =,F10.5, KVA)
 145  FORMAT(1X,MOTOR EFF.=,F10.5)
 150  FORMAT(1X,POWER FACTOR(FUND)=,F10.5)
 155  FORMAT(1X,POWER FACTOR(SYSTEM)=,F10.5)
 160  FORMAT(1X,REACTIVE POWER(TOTAL)=,F10.5,KVAR,3X,QX/S=,F10.5)
 165  FORMAT(1X, )
 170  FORMAT(1X,PULSE WIDTH MODULATION CALCULATIONS)
 175  FORMAT(1X,V1(FUND)=,F10.4,VOLTS)
 180  FORMAT(1X,I1(FUND)=,F10.4,AMPS)
 185  FORMAT(1X,DISPLACEMENT POWER(FUND)=,F10.5,KVAR)
WRITE(3,170)
OPEN(UNIT=1,FILE=PPWM)
READ(1,95)VP,PULSE,SA,SS,SN,FREQ,FLAG
SLIP=SA
NS=SN
DO 60 K=1,NS,1
WRITE(3,165)
RPM=900./60.*FREQ*(1.-SLIP)
TAU=(VP/208.)*.2.

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WRITE(3,103)TAU,VP,PULSE
WRITE(3,104)FREQ,SLIP,RPM
IF(FLAG.EQ.1.)GO TO 5
WRITE(3,101)
WRITE(3,102)
CONTINUE
PI=3.14159
CON=3./PI*SQRT(TAU)
IRMS=0.0
P1=0.
Q1=0.0
T1=0.0
CN=0.0
X=0.0
IMAX=2.*PULSE
AIMAX=IMAX
CONTINUE
AN=1.0
AMPL=0.0
AMPL=0.0
DO 20 N=1,30,1
CN=0.0
AL=N+1
AN=AN+3.0*(-1.)**AL
DO 30 M=1,10,1
AM=M
A=SIN(AM*PI*TAU)/AM/PI/TAU
B=2.*AN/((AM*PULSE)**2-AN**2)
CN=CN+A*B*(-1.)**AM
CONTINUE
CN=CON*(1./AN-CN)*VP
AMPL=AMPL+CN**2
CALL MOTOR(AN,AIN,P,G,T,AL,SLIP,CN,FREQ)
AMPL=AMPL+AIN**2
P1=PI+P
Q1=Q1+Q
T1=T1+T
VRMS=SQRT(AMPL)
IRMS=SQRT(AMPL)

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S=SQRT(3.)*VP*IRMS/1000.
AINN(N)=AIN
RAT=Q/P
LEL(N)=ATAN(RAT)
IF(AN.GT.1.)GO TO 35
  CM=CN
  AIM=AIN
  QM=Q
  RAT1=Q/P
35 CONTINUE
  RATIO=ABS(CM/CN)
  VDB=20.*ALOG10(RATIO)
  RATIO=ABS(AIM/AIN)
  IDB=20.*ALOG10(RATIO)
  IF(FLAG.EQ.1.0)GO TO 20
  WRITE(3,100)AN,CN,AIN,T,P,Q,VDB,IDB
20 CONTINUE
50 CONTINUE
  PM=T1*FREQ*(1.-SLIP)*PI*746./2./550./1000.
  EFF=PM/P1
  PF1=COS(ATAN(RAT1))
  PF=P1/S
  QX=SQRT(S**2-P1**2)
  QS=QX/S
  WRITE(3,110)
  WRITE(3,115)VRMS
  WRITE(3,175)CM
  WRITE(3,120)IRMS
  WRITE(3,180)AIM
  WRITE(3,125)T1
  WRITE(3,145)EFF
  WRITE(3,130)P1
  WRITE(3,160)QX,QS
  WRITE(3,185)QM
  WRITE(3,135)Q1
  WRITE(3,140)S
  WRITE(3,150)PF1
  WRITE(3,155)PF
  SLIP=SLIP+SS
60 CONTINUE
  RETURN
  END

```

Spooler runtime 1 seconds, 10 KCS, 27 disk reads, 0 disk writes,  
8 pages

