REPORT NO. DOT-TSC-OST-78-19

JOINT COST, PRODUCTION TECHNOLOGY AND OUTPUT DISAGGREGATION IN REGULATED MOTOR CARRIERS

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NOVEMBER 1978

INTERIM REPORT

Prepared for

U.S. DEPARTMENT OF TRANSPORTATION
OFFICE OF THE SECRETARY
Office of the Assistant Secretary for
Policy, Plans and International Affairs
Washington DC 20590

Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
DOT-TSC-OST-78-19		

4. Title and Subtitle	5. Report Date
JOINT COST, PRODUCTION TECHNOLOGY AND OUTPUT DISAGGREGATION IN REGULATED MOTOR CARRIERS	November 1978
	6. Performing Organization Code
7. Author(s) Russell C. Cherry	8. Performing Organization Report No. DOT-TSC-OST-78-19
9. Performing Organization Name and Address U.S. Department of Transportation Research and Special Programs Administration Transportation Systems Center Washington DC 20590	10. Work Unit No. (TRAIS) OP812/R9805 11. Contract or Grant No.
12. Sponsoring Agency Name and Address U.S. Department of Transportation Office of the Secretary Office of the Assistant Secretary for Policy, Plans and International Affairs Washington DC 20590	13. Type of Report and Period Covered Interim Report 1965-1974 14. Sponsoring Agency Code

15.

16. Abstract This study uses a sample of 252 Class I Instruction 27 Motor Carriers (Instruction 27 carriers earned at least 75 percent of their revenues from intercity transportation of general commodities over a three year period, the ICC requires that these carriers report a set of supplemental data - such as TL and LTL revenues) of General Flight that existed continuously during the period 1965-1974 to estimate a long run cost function for the regular route, general freight section of the motor carrier industry. The functional form of the estimated equation belongs to the class of flexible, second order approximations to any cost function that are referred to as transcendental logarithmic or "translog" functions. This class of functions does not make any prejudgments about the proper functional form, or the nature of the economic technology that motor carriers use to produce output; the functions may be derived from a Taylor's series expansion. The output of the industry was disaggregated into four distinct types and inputs were disaggregated into nine classes. The outputs are: 1) truck load ton-miles; 2) less-than-truck load ton miles; 3) pick up and delivery tons per hour and 4) terminal-platform tons. The inputs for which prices were included in the cost function are: 1) labor-salaried, clerical and other; 2) labor-linehaul; 3) labor-pickup and delivery and terminal platform; 4) other imputs not elsewhere classified; 5) purchased transportation; 6) owner-operators; 7) materials; 8) fuel, and 9) capital. The estimated cost function shows that there are no economies of scale in the domains for which the function was estimated, and that the usual representation of cost, using a Cobb-Douglas or CES function, is a serious misspecification because the true underlying function is non-separable and therefore the composition of output is a function of the level of factor prices.

17. Key Words	18. Distribution Statement
Motor carrier cost	DOCUMENT IS AVAILABLE TO THE PUBLIC THROUGH THE NATIONAL TECHNICAL INFORMATION SERVICE, SPRINGFIELD, VIRGINIA 22161

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	138	

Form DOT F 1700.7 (8-72) Reproduction of completed page authorized

FOREWORD

This report consists of six sections, the first of which is a review of the theory of the firm that applies to this particular problem. Some of this material is sufficiently new that it has not yet found its way into the commonly used price theory textbooks, although it is taught in most major graduate schools. This section is intended to serve as a bridge between the professional literature in economics, and persons in government and industry to whom this material is potentially important. Those familiar with this material may wish to proceed to Sections 4. and 5.

This model of the supply side of the motor carrier industry will allow the government to evaluate the effects of policy on the industry and the firm with particular regard to rates and costs, capital and labor requirements and the characteristics of the shipments.

The cost model allows us to determine long run cost, the slope of the cost curve for economies of scale and evaluate the production technology faced by firms. The production technology allows us to evaluate the possibilities for instituting inputs for one another in response to a policy change or a macroeconomic disturbance-increased wages and fuel prices or reductions in industrial production, for example.

The cost model treats firm output as consisting of four parts: pickup and delivery, terminal and platform, truckload, and less-than-truckload line haul output. This is the first study to specify multiple outputs for the motor carrier firm, although it has long been recognized that this would be desirable.*

*I have benefited from numerous conversations with Prof. Ann F. Friedlaender of MIT and Prof. Richard H. Spady of Swarthmore and from suggestions made by numerous colleagues at TSC. In particular I am grateful to Robert Thibodeau, Mark Hollyer, Georgia Canellos and Ed Hymson.

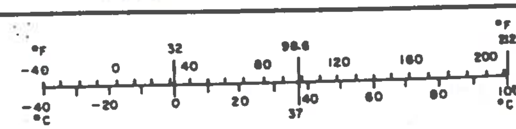
METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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1. THE THEORY OF THE FIRM

INTRODUCTION

The purpose of this study is to provide the government a tool for evaluating the effects of policy. The motor carriers industry is the single most important mode in the transportation industry, earning over 70 percent of the total revenues earned by all transportation modes. Motor carrier services are an input in all major industries. The prices of motor carrier services consequently affect commodity flows between cities and regions, the income and employment in regions, and the prices of products.

This section reviews those elements of the neoclassical theory of the firm that underlie this study. This review differs from most microtheory texts by placing more emphasis on the relationship between production and cost functions because we use a cost function to make inferences about the nature of production technology. The links between production and cost functions are called "duality theory." This section attempts to provide a non-technical guide to the use of the empirical results, so that many underlying mathematical details are omitted leaving the presentation less formal and terse than it would be if they were included.

1.1 PRODUCTION TECHNOLOGY

Neoclassical micro-theory usually introduces the theory of the firm by defining a production function with a single homogenous output Q, treated as an explicit* mathematical function of capital, K, and labor, L. This explicit representation assumes that

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* An explicit function is one written as an equation, such as y = f(x). An implicit function is one such as F(x,y) = 0. Implicit functions need have no solution in terms of either variable. The economic implications of solving an implicit function - quite apart from whether an equation is a solution mathematically - depend on the assumption of separability or homotheticity that is discussed below.

capital and labor are easily defined and measured and that output is a single homogenous entity. There are valid reasons to challenge these assumptions, some of which we will discuss below, but we first review the conventional wisdom and then show how this differs. A more standard treatment is found in Leftwich, Lancaster or Mansfield.* We assume a knowledge of calculus through partial differentiation, following the practice of texts such as Henderson and Quandt, Kogiku and Nicholson.

We begin in 1.1 with a discussion of the neoclassical production with two inputs and a single output written in explicit form. The restrictions on the partial derivatives of this function that are necessary to make the function "well behaved" are set out below. Using this function we develop the concept of an isoquant in 1.2 and derive and interpret its slope and curvature. The equilibrium of the firm is also defined. With the addition of an isoexpenditure line to the isoquant, the firms equilibrium can be shown. We show in 1.3 that an identical firm equilibrium results regardless of whether a firm maximizes profit, minimizes cost or maximizes output.

By 1.4 we have introduced all of the theory necessary to explain duality. In 1.5 we discuss how to exploit the concepts of duality to estimate cost relationships. In 1.6 we extend the notion of production and cost functions to multiple outputs. Multiple output production functions are called transformation functions or production possibilities curves, and we also discuss multiple output firm equilibria. In 1.7 we examine the hidden assumption in writing productions in explicit form and in 1.8 present the translog functional form, and discuss its use.

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*We adopt the style used in the American Economic Review for references: Works are listed using only the name of the author unless there are two works by the same author. If the works are in the same year the title is followed by Jones (1972a) or Jones (1972b); if in different years the author's name is followed by the year of publication

Technical Relationships

A neoclassical production function may be represented mathematically as

$$Q = f(K,L). \tag{1.1}$$

The production function represents only the technical nature which underly the input-output relationships of the firm, the output that results from a given quantity of inputs.

Marginal Products

The key technical relationships between inputs and output are the rates of change of output with respect to a small change in inputs. This is defined by the partial derivative of the production function with respect to either capital or labor; these are called the marginal products of the production function.*

$$\partial Q/\partial K = f_K > 0$$

and (1.2)

$$\partial Q/\partial L = f_L > 0 .$$

For notational convenience, we represent these derivatives as f_K and f_L respectively. Both derivatives are assumed to be positive; to do otherwise would mean that an additional unit of one input might reduce output as more and more units of that input were added. These functions also represent the demand schedules for the factors called derived demand functions. These derived demand

*Partial derivatives are required in taking derivatives of functions of several variables. The concept implies finding a rate of change of one variable with respect to another while leaving other variables in the function unchanged. In concept, a partial derivative is not essentially different from a derivative of a function of a single variable. It gives the rate of change of one variable with respect to another.

functions are assumed to be negatively sloped like all demand functions, or that the second partial derivatives are negative

$$\partial(f_K)/\partial K \equiv f_{KK},\ \partial(f_L)/\partial L \equiv f_{LL} < 0. \tag{1.2a}$$

The cross partial derivatives - the partial of the marginal product of capital with respect to labor, or the partial of the marginal product of labor with respect to capital, must be equal to each other* and positive

$$\partial(f_K)/\partial L \equiv \partial(f_L)/\partial K \equiv f_{KL} \equiv f_{LK} > 0. \tag{1.2b}$$

A production function with (mathematically) continuous first and second order partials of the proper sign is often said to be "well-behaved." These assumptions generate some empirical restrictions that we can use to our advantage in the estimation procedure to test the validity of the underlying assumptions.

1.2 ECONOMIC BEHAVIOR

Isoquants

The marginal products of the production function determine the technical rate at which factor inputs can be substituted for each other. The ratio of marginal products, $-f_K/f_L$, is called the marginal technical rate of substitution or just the marginal rate of substitution. The marginal rate of substitution has an appealing, geometric interpretation. To see it we must introduce the concept of an isoquant: a line representing all of the combinations of capital and labor that may be combined to produce

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*Young's theorem of differential calculus requires that second order partial derivatives be independent of the order of differentiation; the sign is assumed to be positive meaning that an increase in one factor input increases the marginal product of the other.

a fixed level of output.* A set of isoquants is shown in Figure 1.

Isoquants are smooth, negatively sloped lines in a geometric space with labor measured on the vertical axis and capital measured on the horizontal axis - which we will call L, K space - and they "bow in" toward the origin (are convex to the origin).

Mathematically, an isoquant is a locus of points represented by

$$Q^* = f(K,L) \tag{1.3}$$

where Q* is any fixed level of output.
The slope of an isoquant is the marginal rate of substitution. This can be shown by taking the total derivative of a production function for some constant level of output, Q*, and setting it equal to zero

$$dQ^* = f_K dK + f_L dL = 0. \tag{1.4}$$

The total derivative dQ* is equal to zero by definition since there is no change in output along an isoquant. The slope of the isoquant is $dL/dK = - f_K/f_L$, the marginal rate of substitution. Figure 1 shows only three isoquants Q_1, Q_2 and Q_3. Note that: 1) Isoquants further from the origin denote higher levels of output; 2) Isoquants exist for any conceivable level of output, not just Q_1, Q_2 and Q_3 but for all levels; lesser, between, and greater than. This property is referred to as that of being <u>everywhere dense</u>. 3) Isoquants may not intersect.

The <u>curvature</u> of an isoquant represents the degree to which inputs may be substituted for each other; input substitutability increases as the isoquant approaches a straight line and decreases

---

*The prefix "ISO" means constant and "quant" refers to the quantity of output. Isoquants are assumed to be smooth, although in practice they may have corners representing the fact that input choices may be discrete; for example, machines may come only in given capacities.

as it approaches a right angle bend. This concept is called the elasticity of substitution.

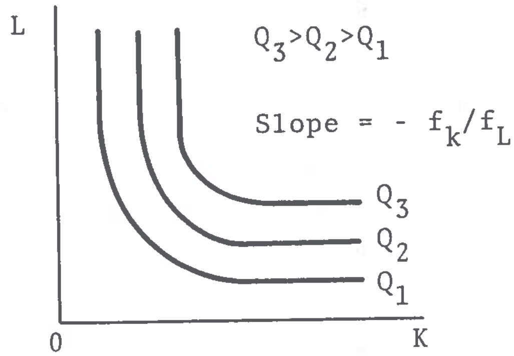

FIGURE 1. ISOQUANT MAP

Isoquants describe only the technical relationships that govern firm behavior; the economic elements of the problem are the market-determined prices of the factor inputs and the total outlay the firm allocates to production. Together, these elements determine the optimal economic behavior for the firm; the marginal rate of substitution determines the technical rate at which inputs can be substituted and the factor prices determine the allowable market rate of substitution between inputs.

Isoexpenditure Line

The link between the market and technical rates of factor substitution that determine the firm's optimal behavior is the behavioral objective of the firm. If the firm wishes to maximize the output that can be produced for a given expenditure on inputs - a fixed outlay, we can visualize the optimal firm behavior through the concept of an isoexpenditure line.* This line can be represented on the same graph as the isoquant. The isoexpenditure line is a straight, negatively sloped line in L,K space and represents all of the combinations of factor inputs that cost the same

*Most texts refer to the isoexpenditure line as the "isocost" line; since we use isocost to denote a locus of constant cost as inputs vary in factor price space (see Section 1.4 on Duality) we use isoexpenditure line to avoid any confusion that might result from using the term in two senses. See Varian's notes for more detail.

dollar amount. The slope of this line is the negative of the factor price ratio -rc/w where r is the price of capital services per unit time, c the acquisition cost of capital* and w the wage rate, as shown in Figure 2.

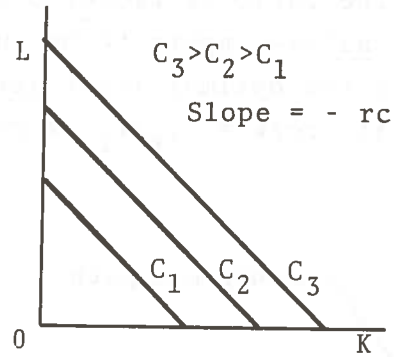

FIGURE 2. ISOEXPENDITURE LINE

Firm Equilibrium

Optimal economic behavior requires the firm to produce the maximum output possible with the inputs that the firm can afford. Geometrically, this means that the firm must equate the slope of an isocost line to the slope of the isoquant or

$$-rc/w = f_K/f_L \ . \tag{1.5}$$

*This formulation of capital cost takes into account the fact that both acquisition costs - the price of the physical capital such as a machine - and the cost of financial capital r determine the user cost of capital or the "price of capital services." The K in this specification represents units of physical capital- such as machines. It follows that cK is the dollar value of the capital stock, and r is an interest rate that must be greater than zero and less than 50 percent (probably). We assume that capital is utilized to capacity or that no additional "work" can be gotten out of the capital stock and, that there is no depreciation. Of course, there is depreciation in reality, but including it in this discussion does not add anything important and makes the notation more complex.

This condition alone does not imply the level of output that will be produced by the firm. That is determined by the firm's total fixed outlay or expenditure. The ratio of marginal products shows the rate that inputs can be substituted for each other based on purely technical grounds. The ratio of factor prices shows the rate that the factor market allows inputs to be substituted. Together, these ratios imply the optimal input levels, K* and L* shown in Figure 3. The ratio $-rc/w = -f_K/f_L$ is called the resource allocation ratio

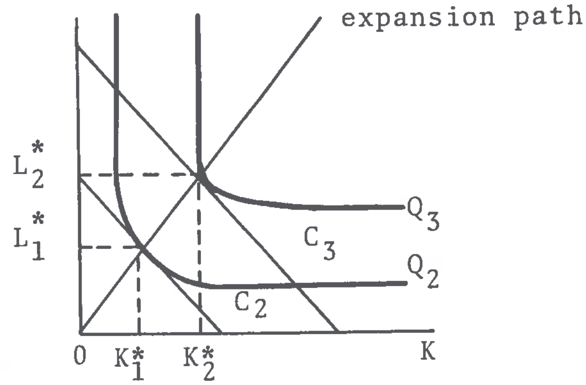

FIGURE 3. FIRM EQUILIBRIUM

A locus of points that satisfy the optimality or "necessary" conditions is called the expansion path. This locus occurs as the firm expands output by increasing its expenditures and moving to an isoexpenditure line further from the origin. If neither technology (the isoquant slope) nor factor prices (the isoexpenditure slope) change over time, the expansion path will be a straight line emanating from the origin such as the one shown in Figure 3. This defines the long run equilibrium of the firm because both inputs are freely variable.

The Firm's Planning Horizon

A key assumption here is that the firm is free to vary all of its inputs. Free variability of all inputs defines the economic long run. In the short run one factor is fixed.

1.3 OPTIMAL ECONOMIC BEHAVIOR

Optimal economic behavior was previously shown to mean producing the maximum output for a given expenditure. The same axiom, (that the firm will use factor inputs in quantities defined by the point at which the ratio of factor prices is equal to the ratio of marginal products, or equates the slope of the isoquant and iso-expenditure line), also applies to firms that:

1. Maximize profit;
2. Minimize the cost of producing a given output;
3. Maximize the output that can be produced for a given cost.

That is, the resource allocation ratio and expansion path for each different firm objective listed above is identical.

A demonstration that equivalent resource allocation ratios result from these seemingly-different firm objectives begins below.

1.3.1 Profit Maximization

$$\text{Maximize } \Pi(K,L) = PQ - wL - rcK \qquad (1.6)$$

where Π = profit

o Necessary Conditions:

$$\partial\Pi/\partial K = Pf_K - rc = 0$$
$$\partial\Pi/\partial L = Pf_L - w = 0 \qquad (1.7)$$

o Resource Allocation Ratio:

$$-rc/w = -f_K/f_L \ .$$

Equations 1.7 show that a competitive firm will equate the marginal revenue product, Pf_i $i = K,L$, of each factor to its price. Since price will equal marginal revenue (as it must under the assumption of perfect competition), this condition means that marginal revenue must equal marginal cost; $P = f_K/rc$ and $P = f_L/w$.

1.3.2 Production Cost Minimization

In this case the firm wishes to minimize the cost of producing

$$\text{Minimize } C(K,L) = wL + rcK$$
$$\text{Subject to } Q^* = f(K,L) \tag{1.8}$$

Form of Lagrange Expression:*

$$Z(K,L) = wL + rcK + \lambda)Q^* - f(K,L))$$

- Necessary Conditions:

$$\partial Z/\partial K = rc - \lambda f_K = 0$$
$$\partial Z/\partial L = w - \lambda f_L = 0 \tag{1.9}$$

- Resource Allocation Ratio:

$$-rc/w = -f_K/f_L$$

1.3.3 Output Maximization

$$\text{Maximize } Q^* = f(K,L) \tag{1.10}$$
$$\text{Subject to } C = wL - rcK$$

---

*When an expression is maximized or minimized subject to a side relation or constraint, as in the case of production cost minimization and output maximization, one way of solving the problem is to combine the objective function to be maximized or minimized into a single expression called a Lagrange expression using a new term called an unidentified Lagrange multiplier. The Lagrange expression may then be treated as the objective function and necessary conditions may be derived by taking derivatives of that expression. The undefined Lagrange multiplier (λ in 1.3.2, μ in 1.3.3) has the interpretation that it represents the change in the objective function due to a relaxation of the constraint. The maximization problem 1.3.3 is the primal to the dual of minimizing cost subject to an output constraint and it may be shown that the Lagrange multiplier for the primal is equal to the inverse of the dual Lagrange multiplier. See Henderson and Quandt or Kogiku for further discussion.

Form a Lagrange Expression

$$Z(K,L) = Q^* - f(K,L) + \mu(wL + rcK)$$

$$\partial Z/\partial K = -f_K + \mu rc = 0 \qquad (1.11)$$

$$\partial Z/\partial K = -f_L + \mu w = 0$$

Resource Allocation Ratio

$$-rc/w = -f_K/f_L \ .$$

1.4 DUALITY

Note these three different firm objectives result in identical resource allocation ratios. What significance does this have? It means the maximizing either profit or output for a given expenditure level produces the same factor input ratios as minimizing the cost of producing a given level of output so that the second objective is a dual operation to the primal one of profit maximization. See Diewert (1972) for a more advanced discussion of duality.

Duality also means that due to a unique set of correspondences between cost and production relations we can derive technical (production) relationships from the economic relationships found in the cost function. That is, the cost function allows us to infer values for the optimal input ratios, the marginal rate of substitution between inputs and the elasticity of substitution. This duality relationship was first proved by Shephard, (1953), and is called Shephard's Lemma; the key assumption underlying this correspondence is cost minimization.

The Geometry of Duality

The geometry of the dual cost relation is exactly analogous to the isoquant-isoexpenditure relationship we discussed above, but instead of analyzing the firm equilibrium in factor input space, we now analyze it in factor price space (w, rc space).

Isocost Curve

An isocost curve represents all possible combinations of the factor input capital and labor which cost exactly the same amount.* This curve is shown in Figure 4.

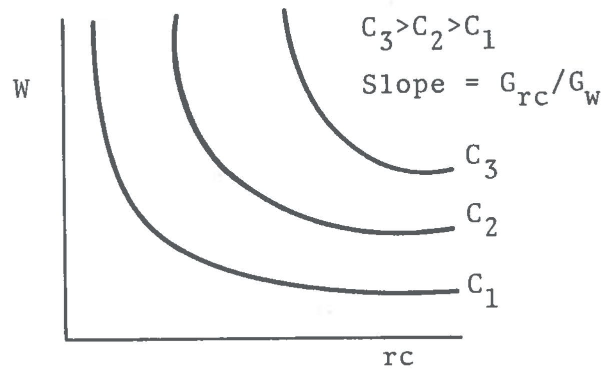

FIGURE 4. ISOCOST CURVE

Like isoquants, the isocost curves represent higher levels of expenditure (and hence output) as we move away from the origin. The regularity conditions imposed on the isoquants are analogous here. Let the cost function be

$$C = G(rc,w) \ . \tag{1.12}$$

The slope of the isocost curves is the ratio of derivatives of the cost function with respect to factor prices. If $\partial C/\partial rc - G_{rc}$ and $\partial C/\partial w = G_w$, both will be positive.

Transforming the isoexpenditure function into factor price space we note that the function has a slope of $-rc/w = -K^*/L^*$, where stars denote optimal levels of the inputs; this is shown in Figure 5. Optimal firm behavior viewed from the cost function approach, says that the firm must operate at the point where the isoexpenditure line is just tangent to the isocost curve associated with Q^*, that is $-K/L = -G_{rc}/G_w$. This also defines the firm's expansion path, but in factor price space.

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*This section follows Varian's Lecture Notes.

output for a given level of inputs with reasonable accuracy. It would not provide satisfactory estimates of the marginal products of either capital or labor because of the simultaneous determination of K and L, and the high correlation between K and L or "collinearity" as econometricians call it. These two are highly correlated because they are "complements" - increases in one lead to increases in the other - and because they are determined simultaneously with the level of output.

Assume that the production function to be estimated in

$$Q = f(K,L) + e \tag{1.13}$$

Q is the output of individual firms, K and L the levels of the factors employed, and e is an error term with mean zero and a variance that is constant across firms (homoscedastic).

This specification produces good estimates of the unknown parameters of the production function only if the independent variables (capital and labor) are not correlated with the error term. If any of the independent variables are correlated with the error term, the coefficient estimates will be "biased", in that their average value from repeated samples (expectation) will not equal the true parameters being estimated.

The error term embodies all of the effects of omitted variables (and our ignorance). In this case the variables that were left out are the price of output, the factor prices and inter-firm differences in marginal efficiency. Going back to the necessary conditions for a profit maximizing firm,

$$\begin{aligned} \partial\Pi/\partial K &= Pf_K - rc \\ \partial\Pi/\partial L &= Pf_L - w. \end{aligned} \tag{1.14}$$

By inspection of the necessary conditions it is clear that the omitted variables are P, rc and w. These omitted variables determine the size of output for P and the size of K and L for rc and w; clearly the independent variables will be correlated with the error term and so the estimates of production technology are

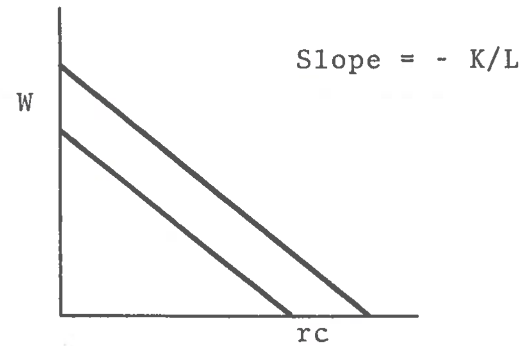

FIGURE 5. ISOEXPENDITURE LINES IN FACTOR PRICE SPACE

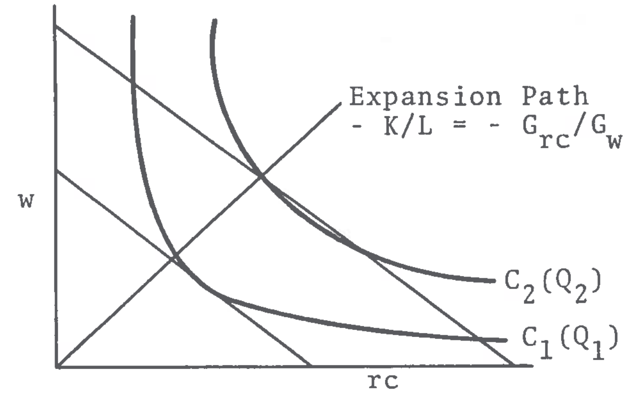

FIGURE 6. FIRM EQUILIBRIUM IN FACTOR PRICE SPACE

1.5 ESTIMATION AND DUALITY

If cost and production relations are equivalent, why bother with with cost functions? Because direct estimation of the production function often leads to estimates which are very poor econometrically. In particular, the assumption that factor inputs are homogenous and easily measured is usually false. In fact, the inputs are often subject to errors of measurement. Capital is particularly troublesome because it is often of different ages and efficiencies associated with intensities of use that further complicate accurate measurements. Furthermore, the book or accounting value of capital seldom accurately represents its market value.

If inputs could be measured in some satisfactory manner, how would the production relationships be estimated? If the production function were estimated directly, we could predict the level of output for a given level of inputs with reasonable accuracy. It would not provide satisfactory estimates of the marginal products of either capital or labor because of the simultaneous determination of K and L, and the high correlation between K and L or "collinearity" as econometricians call it. These two are highly correlated because they are "complements" - increases in one lead to increases in the other - and because they are determined simultaneously with the level of output.

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This specification produces good estimates of the unknown parameters of the production function only if the independent variables (capital and labor) are not correlated with the error term. If any of the independent variables are correlated with the error term, the coefficient estimates will be "biased", in that their average value from repeated samples (expectation) will not equal the true parameters being estimated.

The error term embodies all of the effects of omitted variables (and our ignorance). In this case the variables that were left out are the price of output, the factor prices and inter-firm differences in marginal efficiency. Going back to the necessary conditions for a profit maximizing firm,

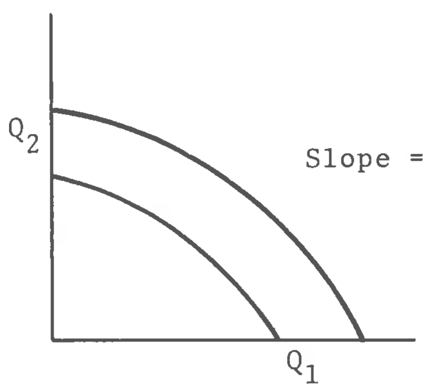
$$\begin{aligned} \partial\Pi/\partial K &= Pf_K - rc \\ \partial\Pi/\partial L &= Pf_L - w. \end{aligned} \tag{1.14}$$

By inspection of the necessary conditions it is clear that the omitted variables are P, rc and w. These omitted variables determine the size of output for P and the size of K and L for rc and w; clearly the independent variables will be correlated with the error term and so the estimates of production technology are biased. On the other hand, a properly specified cost function (without any known omitted variables) contains all of the requisite variables to insure that estimates of that relationship will be unbiased. Cost function estimates have better statistical properties than production function estimates but we are only required to measure their factor prices. This poses fewer problems than measuring the inputs themselves.

1.6 TRANSFORMATION FUNCTIONS

The conventional theory of the firm outlined above requires some extensions before we arrive at the functions that we actually estimated.

The first extension is to multiple outputs. A production possibilities or transformation curve represents all possible combinations of two goods that can be produced with fixed quantities of inputs. One such curve is shown in Figure 7. The transformation curve says that we cannot have more of both goods so long as the quantity of inputs or resources remain fixed. The concept displays the tradeoffs between production of two goods. The graph of the function in "output" space is concave or bows away from the origin.



$$\text{Slope} = -f_1/f_2 = -\frac{\partial Q_2/\partial X}{\partial Q_1/\partial X}$$

FIGURE 7. TWO OUTPUT TRANSFORMATION FUNCTIONS

The transformation function is

$$- f(Q_1,Q_2) + g(X) = 0 \qquad (1.15)$$

where X is a single, composite input. (If we assume two factor inputs, the analysis becomes much more complex).

Assume that the transformation function can be solved explicitly for X; the function gives the cost of producing all possible combinations of Q_1 and Q_2 with fixed input levels.

The slope of the transformation function is called the marginal rate of transformation between outputs, and is found by taking a total differential of the transformation function

$$- d_1 dQ_1 - f_2 dQ_2 + g_x dX = 0$$
$$\text{where } f_i = \partial/\partial Q_i \quad i = 1,2. \qquad (1.16)$$

Assuming that dX = 0 meaning no change in inputs along the transformation curve, the slope of the curve is

$$dQ_1/dQ_2 = - f_2/f_1 \;. \qquad (1.17)$$

The partial derivatives of the transformation function with respect to outputs are the marginal costs of each output. There is an inverse relationship between marginal cost and marginal product; if additional units of one factor add more to output than another, the marginal cost of the first is smaller than the second. The slope of the transformation function is

$$\frac{dQ_1}{dQ_2} = \frac{\partial Q_1/\partial X}{\partial Q_2/\partial X} \qquad (1.18)$$

(where $\partial Q_i/\partial X$ i = 1,2 is marginal product of outputs 1 and 2 respectively). The inverse relationship between marginal product and marginal cost should now be clear; that is $\partial Q_1/\partial X = 1/f_1$ and $\partial Q_2/\partial X = 1/f_2$, or marginal costs are equal to the inverse of marginal products.

The optimal output levels (for a profit maximizing firm) occur when the ratio of the marginal rates of transformation is equal to the ratio of product prices

$$\frac{\partial Q_2/\partial X}{\partial Q_1/\partial X} = \frac{P_1}{P_2} = \frac{f_2}{f_1} \; . \tag{1.19}$$

Graphically, a multiproduct equilibrium occurs at the tangency between the transformation function and a line representing all possible combinations of goods that produce a constant revenue - an isorevenue line. The equilibrium of a multiproduct firm is shown in Figure 8.

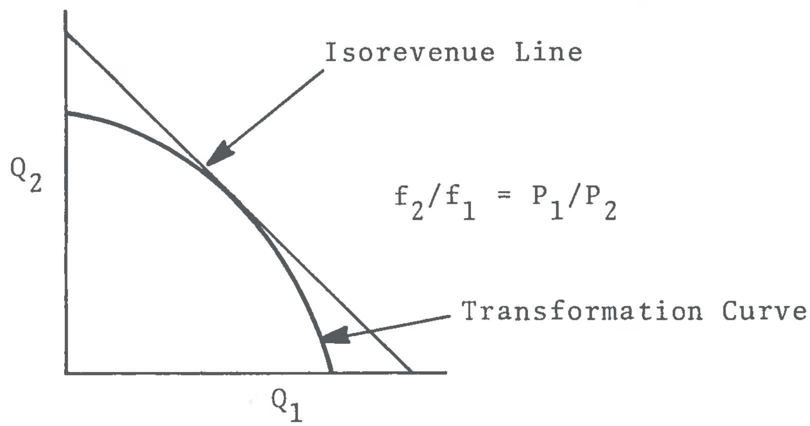


FIGURE 8. MULTIPLE OUTPUT EQUILIBRIUM FOR A PROFIT MAXIMIZING FIRM

1.7 SEPARABILITY AND JOINTNESS: A HIDDEN ASSUMPTION

Transformation functions are just multiple output production functions except that transformation functions do not assume that output can be written as an explicit function of inputs. There is no assumption that the function

$$- f(Q_1,Q_2) + g(X) = 0 \tag{1.20}$$

can be written in explicit form as

$$f(Q_1,Q_2) = g(X) \; . \tag{1.20a}$$

This seemingly-innocuous assumptions is made when ever production (or transformation) functions are written in explicit form. If the function can be written in explicit form it is called separable. See Friedlaender et al (1977) for more detail.

Separability means that if there are several distinct outputs they are all jointly produced and individual production functions, do not exist. We may not write

$$Q_1 = f(K,L) \tag{1.21}$$

and

$$Q_2 = f(K,L) \ .$$

Furthermore, under this circumstance, the specification of a function with multiple outputs is no more general than one with a single output. That is, we may just as well treat outputs Q_1 and Q_2 as being the same (aggregate across outputs) without losing any information about the underlying technology.

The neoclassical production function presented in 1.1 made the a priori assumption of separability, as do most commonly used empirical production functions such as the Cobb-Douglas or the more general constant elasticity of substitution (CES).

Table 1 offers several examples of products produced jointly from an intermediate product. Separability implies that none of the joint products have separate production functions even though we can write one for the intermediate products.

TABLE 1. EXAMPLES OF JOINT PRODUCTS

INTERMEDIATE PRODUCT	JOINT PRODUCTS
o Apple juice	o Cider, Applejack, Vinegar
o Grape juice	o Wine, Brandy, Sherry
o Sugar cane	o Molasses, Sugar, Rum,
o Milk	o Ice cream, Butter, Cheese
o Timber	o Paper, Lumber, Plywood
o Crude oil	o Fuel oil, Kerosene, Gasoline

If production is non-joint, each individual product does have an individual production function. There are no economies or diseconomies of jointness, that is, no advantages or disadvantages to the separate production of the products in a non-joint technology accrue.

The transformation function does not lend itself well to direct estimation because of the implicit assumption of separability embodied in writing it in explicit form. That is, in writing the function as

$$Q = f(K,L) \quad , \tag{1.22}$$

we prejudge input-output separability, rather than treat it as a testable hypothesis. The problem is, if we are to estimate the functions at all, there must be a dependent variable. Therefore, the usual functional forms assume separability and impose this assumption on the data. Furthermore, direct estimation of the transformation function has all of the problems associated with the direct estimation of any production process that we have previously discussed.

Just as before, the solution is to estimate the cost function and get back to the production technology by invoking Shephard's lemma. The function specified with multiple outputs is called a joint cost function, and examples are provided below.

1.8 THE TRANSLOG PRODUCTION FUNCTION

The most widely-used functional form to arise in the theoretical and empirical work on duality theory is the transcendental* logarithmic, or "translog" form which may be used to

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*A transcendental function is one that cannot be expressed as the root of an algebraic function with rational coefficients or one representing a trigonometric, exponential or logarithmic function not defined by the elementary operations of mathematics. It seems clear that the name was chosen because it has a nice ring as much as it clearly describes the functions. A logarithmic function is just one of the class of transcendental function.

specify either production or cost functions. The name translog may be a misnomer, because although the function is transcendental, it is also linear-quadratic in the logs of variables and could also have been accurately called the "quad log" function. It is a "flexible" functional form because it does not impose strong _a priori_ assumptions about the technology on the data, but allows tests of these assumptions.

A translog function may be interpreted as a second-order Taylor's Series expansion _in the variables_ around some arbitrary point of approximation. The seminal idea for the translog seems to have come from the use of various second order approximations to the CES production function (such as Kmenta's approximation*), or other linear-in-the-parameters numerical approximations made for the purpose of estimating production functions by Newton's and other similar methods. These approximations are second order Taylor's Series linearizations _in the parameters_, rather than the variables. From there it is a logical step - at least in retrospect - to the translog function. The logic of this progression does not diminish the credit due Christensen, Jorgensen

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*_Kmenta's CES Approximation_ - One logical step in the progression from more restructive functional forms was Kmenta's approximation to the CES production function.

The CES function with a multiplicative error term is

$$Q = A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-\mu/\rho}e^{u}.$$

Taking logs gives

$$\ln Q = \ln A - \mu/\rho \ln[\delta K^{-\rho} + (1-\delta)L^{-\rho}] + u.$$

The term in brackets cannot further be simplified using logs; Kmenta's solution is to take a Taylor's series expansion, in the parameters around the point, $\rho=0$;

$$\ln Q = \ln A + \mu\delta \ln K + \mu(1-\delta)\ln L - 1/2\rho\mu\delta(1-\delta)(\ln K - \ln L)^2 + u$$

$$= \beta_1 + \beta_2 \ln K + \beta_3 \ln L - 1/2\ \beta_4(\ln K - \ln L)^2 + u.$$

The first three terms of this function are identical to the Cobb Douglas function for which $\rho=0$; the remaining terms pertain to departures of ρ from zero. The parameters of the CES function are: $A = \exp \beta_1$; $\delta = \beta_2 + \beta_3$); $\delta = \beta_2 + \beta_3$; $\rho = -2\beta_4(\beta_2 + \beta_3)/\beta_2\beta_3$. If β_4 cannot be distinguished from zero the Cobb-Douglas function is appropriate.

The similarity of this approximation to the translog is even more clear when the penultimate term is expanded: $1/2(\beta_4 \ln K^2 - 2\beta_4 \ln K \ln L + \beta_4 \ln L^2)$.

and Lau for deriving the function and proving its relationship to the underlying duality theorems which relate it to production theory. Other functions have arisen from the recent interest in duality; production and cost functions developed by Diewert, Hall and others share the desirable theoretical properties of the translog function. These other functions, however, lack the analytical tractability, ease of estimation and interpretation of the translog and have not found such widespread acceptance.

Translog functions may be lengthy when they are specified with multiple inputs and outputs, so a translog cost and production function with fewer inputs and outputs than those actually estimated are shown below.

The Translog Production Function

Assume a single output Q, and two inputs K and L,

$$g(Q) - f(K,L) = 0 \tag{1.23}$$

capital K, labor L. The translog representation of this function (assuming separability) is

$$\begin{aligned} \ln Q = \alpha_o + \alpha_1 \ln(K-K') + \alpha_2 \ln(L-L') + 1/2\beta_1 \left(\ln(K-K')^2\right) \\ + 1/2\beta_2 \left(\ln(L-L')^2\right) + \gamma_1 \ln(K-K')\ln(L-L') \end{aligned} \tag{1.24}$$

where K' and L' represent the points of approximation, usually the variable means.

If the β and γ coefficients cannot be reliably distinguished from zero, the translog form collapses to the Cobb-Douglas; furthermore, if $\alpha_i + \alpha_2 = 1$, it is Cobb-Douglas with constant returns to scale. If the β and/or γ terms are reliably distinguishable from zero the functional form is not Cobb-Douglas.

Relationship to Taylor's Series Expansion

The Taylor's Series expansion for functions of several variables is

$$f(K,L) = f(K',L') + \frac{\partial f(K,L)}{\partial K}\,\frac{(K-K')}{1!} + \frac{\partial^2 f(K,L)}{\partial K^2}\,\frac{(K-K')^2}{2!}$$

$$+ \frac{\partial f(K,L)}{\partial L}\,\frac{(L-L')}{1!} + \frac{\partial f(K,L)}{\partial L^2}\,\frac{(L-L)^2}{2!} + \dots + \tag{1.25}$$

$$\dots + \frac{\partial^2 f(K,L)}{\partial K \partial L}\,\frac{(K-K')(L-L')}{2!} + \text{Higher Order Terms.}$$

The relationship between the Taylor's Series expansion and the translog functions should now be clear; the translog production function may be interpreted as a Taylor's Series expansion around lnQ = 0, where in K' = 0 and ln L' = 0, approximating <u>any</u> production function.*

The derivatives of the underlying function are equal to the coefficients of the translog function; that is, the parameters of the translog function are the partial derivatives of some arbitrary underlying function whose numerical value depends on the points of approximation. The parameters of the function may be interpreted as follows:

$$\alpha_1 = \frac{\partial \ln Q}{\partial \ln K} \quad = \text{elasticity of output with respect to capital;}$$

$$\alpha_2 = \frac{\partial \ln Q}{\partial \ln L} \quad = \text{elasticity of output with respect to labor;}$$

$$\beta_1 = \frac{\partial^2 \ln Q}{\partial \ln K^2} \quad = \text{slope of demand for capital function;}$$

$$\beta_2 = \frac{\partial^2 \ln Q}{\partial \ln L^2} \quad = \text{slope of demand for labor function;} \tag{1.26}$$

$$\gamma_1 = \frac{\partial^2 \ln Q}{\partial \ln K \partial \ln L} \quad = \text{tells whether capital and labor are complements of substitutes according as it is positive or negative.}$$

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*The translog function actually has two possible interpretations; either differential approximation or a numerical approximation. A function that is a Taylor's Series expansion around some arbitrary point may be considered both a numerical approximation and a differential approximation. In our discussions we have treated the translog as a differential approximation, and that imposes no special problems on its interpretation.

1.8.1 The Translog Factor Share Equations

The translog function may be also used to derive and estimate either factor demand functions or factor share equations. Factor demand functions are the demand schedules for the factor inputs derived from the necessary conditions for profit maximization. Factor share equations are just algebraic rearrangements of the factor demand equations. For example, profit is defined as:

$$\Pi = PQ - C \tag{1.27}$$

where

$Q = f(K,L)$ Production Technology

$C = wL + rcK$ Total Cost

P = output price (constant to the firm).

The first order conditions are:*

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*The assumption of perfect competition is embodied in this equation because price is treated as a constant to the firm. In the more general specification price varies at the firm level, and the derivatives of the profit function are

$$\frac{\partial \Pi}{\partial K} = \left(P + Q\frac{dP}{dQ}\right) f_K - rc = 0$$

$$\frac{\partial \Pi}{\partial K} = \left(P + Q\frac{dP}{dQ}\right) f_L - w = 0.$$

The term $\left(P + Q\frac{dP}{dQ}\right)$ is marginal revenue and can be written $P(1 + 1/e)f_K$ where e is the price elasticity of demand. This reduces to the competitive case when price elasticity is infinite (prefectly elastic to the firm) since

$$P(1 + 1/e)\ f_K - rc = Pf_K - rc \quad \text{iff } e = \infty.$$

The aggregate factor distribution relationships which result from the assumption of perfect competition are

$$f_K = \frac{rc}{P} \text{ and } f_L = \frac{w}{P}$$

where P is the price of output to the firm, or the price level for an economy.

$$Pf_K = rc \text{ and } Pf_L = w. \tag{1.28}$$

Since the translog function is a logarithmic function, the relationship between the ordinary (non-logarithmic) derivatives of the production function, f_K and f_L and the logarithmic derivatives of a translog function produce the share equations:

$$\frac{\partial \ln f(K,L)}{\partial \ln K} \frac{Q}{K} = f_K \quad . \tag{1.29}$$

This fundamental identity allows us to derive the factor share equations. Profit maximization requires that the marginal revenue products must equal factor prices:

$$Pf_K = rc \ .$$

Substituting from the identity between log and ordinary derivatives gives

$$\frac{\partial \ln f(K,L)}{\partial \ln K} \frac{PQ}{K} = rc \tag{1.30}$$

because the logarithmic derivative of a function <u>is</u> its elasticity. To obtain the ordinary derivative from the log derivative requires "reversing" the elasticity relationship.

Since Pf(K,L) is total revenue, the factor share of total revenue can be found if we rearrange the preceding equation

$$\frac{\partial \ln f(K,L)}{\partial \ln K} = \frac{rcK}{PQ} \tag{1.30a}$$

Let the share of factor payments of total revenue be M_i i = K,L; substituting the log derivative of the translog function into (1.30a) gives

$$M_K = \alpha_1 + \beta_1 \ln K + {}_1 \ln L \ .$$

1.8.2 Efficient Estimation Procedures

The most widely used estimation procedure for the translog function is to estimate the production function and one less than the total number of factor share equations* simultaneously. In this case we would estimate

$$\ln Q = \alpha_o + \alpha_1 \ln K + \alpha_2 \ln L + \frac{1}{2}\beta_1(\ln K)^2 + \frac{1}{2}\beta_3(\ln L)^2$$
$$+ \gamma_1 \ln K \ln L + e_1 \qquad (1.31)$$
$$M_K = \alpha_1 + \beta_1 \ln K + \gamma_1 \ln L + e_2.$$

To simplify notation, assume all variables use their own means as the points of approximation.

These two equations can conveniently be rewritten in matrix form as:

$$Y = X\beta + e \qquad (1.32)$$

where

$$Y = \begin{bmatrix} \ln Q \\ M_K \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & \ln K & \ln L & 1/2(\ln K)^2 & 1/2(\ln L)^2 & \ln K \ln L \\ 0 & 1 & 0 & \ln K & 0 & \ln L \end{bmatrix}$$

$$\beta' = [\alpha_o \alpha_1 \alpha_2 \beta_1 \beta_2 \gamma_1] \qquad e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

Displaying the equations as a linear system, the interdependence between the parameters of the two equations is emphasized: α_1

*We cannot estimate the production function and all of the factor share equations simultaneously because this would lead to a singular error matrix and parameter estimates could not be obtained; we must delete one factor share equation (labor in the example) and estimate one share equation plus the production function.

appears in both equations as does β_1 and γ_1 (the elements of β distribute across X column wise). This across-equation interdependence of coefficients allows us to use more information to estimate the parameters than if we estimated each equation separately. Simultaneous estimations lead to more efficient (smaller variance) estimates.

Some simultaneous estimation methods are sensitive (and give different parameter estimates) on which factor share equation is deleted. All least squares estimation methods are sensitive to the equation deleted <u>unless</u> they are run iteratively while maximum likelihood techniques are not; we use the full-information maximum likelihood (FIML) algorithm. See Barten for a discussion of this invariance property of maximum likelihood estimators.

<u>The Translog Joint Cost Function</u>

$$\begin{aligned} \ln C = {} & \alpha_o + \alpha_1 \ln Q_1 + \alpha_2 \ln Q_2 + \beta_1 \ln w_1 + \beta_2 \ln w_2 \\ & + 1/2\delta_{11} \ln Q_1 \ln Q_1 + 1/2\delta_{12} \ln Q_1 \ln Q_2 + 1/2\delta_{22} \ln Q_2 \ln Q_2 \\ & + 1/2\gamma_{11} \ln w_1 \ln w_1 + 1/2\gamma_{12} \ln w_1 \ln w_2 + 1/2\gamma_{22} \ln w_2 \ln w_2 \\ & + \rho_{11} \ln Q_1 \ln w_1 + \rho_{12} \ln Q_1 \ln w_2 + \rho_{21} \ln Q_2 \ln w_1 \\ & + \rho_{22} \ln Q_2 \ln w_2 . \end{aligned} \qquad (1.33)$$

The presence of the product factor price interaction terms ρ_{ij} indicates that separability (homotheticity) is not assumed, but that it can be tested using the likelihood ratio test.

The cost function may be written more compactly as

$$\ln C = \alpha_o + \sum_{i=1}^{m} \alpha_1 \ln Q_i + \sum_{j=1}^{n} \beta_j \ln w_j + 1/2 \sum_{j=1}^{m} \delta_{ij} \ln Q_i \ln w_j$$

$$+ \sum_{i=1}^{n} \sum_{j=1}^{n} \gamma_{ij} \ln w_i \ln w_j + \sum_{i=1}^{m} \sum_{j=1}^{n} \rho_{ij} \ln Q_i \ln w_j .$$

Using an identity relationship similar to the one used to derive the revenue factor shares equations, factor cost share equations may be derived. Since optimal factor employment equals the derivative of the cost function with respect to the price of that factor

$$X_i^* = \partial C/\partial w_i \ . \qquad (1.34)$$

The identity between logarithmic and non-logarithmic cost derivatives leads to the following factor cost share equations

$$\frac{\partial \ln C}{\partial \ln w_i} = \frac{\partial C}{\partial w_i}\frac{w_i}{C} \ . \qquad (1.34a)$$

$$X_i^* = \frac{\partial \ln C}{\partial \ln w_i}\frac{C}{w_i} \qquad (1.34b)$$

where $M_i = \frac{w_i X_i}{C}$ represents the factor cost shares.
Combining the logarithmic derivative of the cost function with the factor share definition the following factor share equation is given:

$$M_i = \beta_i + \gamma_{11}\ln w_i + \gamma_{12}\ln w_j + \rho_{11}\ln Q_1 + \rho_{12}\ln Q_2 \ . \qquad (1.35)$$

The derivation of the factor cost share equations is competely analogous to the production factor share equations in the production function case, except that it uses shares of total cost rather than total revenue. In macroeconomic work, the cost and revenue shares are identical (under an assumption of perfect competition). The dependent variables in the equations are interchangeable.

2. THE DEFINITION AND MEASUREMENT OF THE JOINT COST FUNCTION

2.1 COST MEASUREMENT

Cost functions in economics are based on the concept of opportunity and social cost. Opportunity cost means that decisions to produce one good involve foregoing some of another good because the total resources available to produce all goods are limited. The value of the product foregone to produce that product is the opportunity cost. It is the value the resources would have had if used in the next best alternative.

Opportunity cost identifies the economic cost to a firm by using a factor input in its production process. There is not an invariant congruence between opportunity and social cost. Society's interests and firm interests may not be coincident with society's if there are imperfections in the economic system (such as monopoly power) but, we can reconcile the two by defining economic cost as the payment necessary to maintain a resource in its current use.

There are several important measures of cost and one major dichotomy depends on whether we wish to deal with the long run or the short run. In the long run all factor inputs are variable and none are fixed; the long run is defined as that length of time necessary to vary all inputs in the production process. The short run is that period of time in which one or more inputs cannot be varied. These definitions do not depend on real or calendar time.

The firm's cost varies with the price of the factor inputs its uses in the production processes, the production technology - how inputs can be combined physically - and with the level of output the firm selects.

There are seven key measures of cost: 1) Total Cost; 2) Average Total Cost; 3) Marginal Cost; 4) Fixed Cost; 5) Average Fixed Cost; 6) Variable Cost; and 7) Average Variable Cost. There are two additional types of cost that are important in this study: joint cost and common cost.

Total Cost is the total cost of production, including any costs that are implicit, such as return to the owner for capital invested in an enterprise. By definition this must equal the sum of the expenditures on all of the inputs used in the production process.

Average Cost is total cost divided by the level of output associated with that cost.

Marginal Cost is the cost of producing an additional unit of output, or the rate of change of cost with respect to output. Incremental cost is a term often used by business men in industry, and it represents an approximation to marginal cost. It is the change in cost for a given change in output. As the change in output becomes smaller, the two become identical. The only reason that there is any difference between them is because incremental cost is discrete while marginal cost is not. There may be operational (numerical) differences but not conceptual differences between the two.

The cost Concepts may be defined in either the short run or the long run, but the ones heretofore defined are not essentially different in the long run or the short run. Those remaining measures of cost are exclusively short run concepts.

Variable Cost is the proportion of cost that changes as output changes. In the long run all costs change with output, therefore, variable cost is a short run concept.

Average Variable Cost is just variable cost divided by output.

In the short run the difference between variable cost and total cost is fixed cost, that is, cost that does not vary with output.

Average Fixed Cost is fixed cost divided by output.

The Measures of Cost

The cost function in generalized form is

$$C = G(Q,w,t) \tag{2.1}$$

where C is cost, Q is a vector of output, w a vector of factor price and t is time. This function represents long-run total cost by definition since it includes no fixed costs.

A short run cost function could be distinguished from a long run function because the short run function would include fixed cost - for example

$$C^S = G(Q,w,t) + D \qquad (2.2)$$

where C^S denotes the short run cost function and D represents fixed cost.

The long run total cost function is also the sum of the input prices times the optimal or profit maximizing factor input quantities for various levels of output

$$C = G(Q,w,t) = \sum_{i=1}^{n} w_i X_i \quad . \qquad (2.3)$$

The short run function is the same save for the inclusion of fixed cost

$$C^S = G(Q,w,t) + D \sum_{i=1}^{h-k} w_o X_i + D \qquad (2.4)$$

where there are k fixed factor inputs.

Given short run and long run total cost we can define <u>average total cost</u>, <u>average variable cost</u>, <u>average fixed cost</u> and <u>marginal cost</u>.

Average total cost is total cost divided by the level of output.

$$\bar{C} = C/Q = G(Q,w,t)/Q \quad . \qquad (2.5)$$

This is long run average total cost.

In the short run we have average cost as

$$\bar{C}^S = \bar{C}^S/Q = \left[G(Q,w,t) + D\right]/Q. \qquad (2.6)$$

Average fixed cost is

$$\bar{D} = D/Q \ , \qquad (2.7)$$

and marginal cost is

$$\partial C/\partial Q = \partial G(Q,w,t)/ \ Q \ . \qquad (2.8)$$

The marginal cost of the ith output for multiple product firms is

$$\partial C/\partial Q_i = \partial G(Q,w,t) \ Q_i \ . \qquad (2.9)$$

2.2 JOINT AND COMMON COSTS

Joint costs are the subject of considerable confusion in transportation economics. Joint costs are associated with the production of two or more products from a single production process. The confusion arises in part because this concept has been linked to the classification of the production process as separable. If the production process is separable, then all products are joint, and no individual production functions exist. If the production process is separable there exist true joint costs because only then will there exist joint products.*

Common cost is often used in a way that suggests that it is synonymous with joint cost.** Common costs should actually represent

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*The concept of jointness is further complicated in the application of the translog cost function because it is usually referred to as the joint cost function. This is not a prejudgment of the question of jointness in the production process, but a reference to the fact that the cost function and the associated factor share equations are estimated - e.g., simultaneously.

**For example Schuster says, "There is a fine distinction between joint and common costs. Joint costs arise when two or more products are always produced in fixed proportions by the same production process while common costs arise when two or more products may be produced by the same production process in varying proportions. See Pegrum, op. cit., pp. 155-157, and Miller, op. cit., pp. 161-163."

the costs of two or more outputs that, because of accounting practice, are not kept distinct. Common costs may exist regardless of whether there exist joint products and separable production functions. Common costs may be associated with joint products but this remains an accounting problem, not a question about the production technology.

Transportation test often use the term joint cost to mean costs associated with producing joint products produced only in fixed proportion. It is common to reference Alfred Marshall, considered by many to be the founder of modern microeconomics. Most of Marshall's examples of joint products were of products that could at least in the short run, be produced only in fixed proportions. For example,

> "§4. We may now pass to consider the case of *joint* products: i.e. of things which cannot easily be produced separately; but are joined in a common origin, and may therefore be said to have a *joint* supply, such as beef and hides, or wheat and straw. This case corresponds to that of things which have a joint demand, and it may be discussed almost in the same words; by merely substituting "demand" for supply and vice versa. As there is joint supply of things which have a common origin. The single supply of the common origin is split up into so many derived supplies of the things that proceed from it.

The examples of joint products was so heavily weighted toward those that could be produced only in fixed proportions, that an association of joint cost and fixed proportions was firmly rooted in the literature and minds of many. Garver and Hansen, authors of Principles of Economics (1928), said

> "JOINT SUPPLY AND JOINT COST. When two goods are the inseparable results of one productive provess, it is impossible to ascertain the cost of either as far as that process is concerned. Thus cotton fiber and cotton seed, which are the joint products of the cultivation of the cotton plant, have no separable cost of growing.

Marshall notes in Ch. VII § I that

> It often happens that a thing made in one branch of business is used as a raw material in another, and then the question of the relative profitableness of the two branches can be accurately ascertained only by an elaborate system of book-keeping by double entry; though in practice it is more common to rely on rough estimates made by an almost instinctive guess.

Marshall continues to discuss joint products using ocean freight as an example

> Another difficult case is that of the shipowner who has to apportion the expenses of his ship between heavy goods and goods that are bulky but not heavy.... .....in many ways the general principle can be applied that the relative proportions of the joint products of a business should be so modified that the marginal expenses of production of either product should be equal to its marginal demand price.[1]

In a footnote, Marshall notes that

> Of course this does not apply to railway rates. For a railway company having little elasticity as to its methods of working, and often not much competition from outside, has no inducement to endeavor to adjust the charges which it makes for different kinds of traffic to their cost to itself. In fact though it may ascertain the prime cost in each case easily enough, it cannot determine accurately what are the relative costs of fast and slow traffic, of short and long distance traffic, of light and heavy traffic; nor again or extra traffic when its lines and its trains are crowded and when they are nearly empty.

The example of ocean freight is clearly one of <u>variable</u> proportions-joint products. Marshall's definition and ours are compatible except that our definition attaches a more technical meaning to jointness. Secondly, Marshall uses "separable" in a different sense than we do; he says costs are not separable or allocable to each product. The methodology we use <u>does</u> allow the allocation or cost to each product (by taking the derivative of the fitted cost function). This is one of the principle advantages of the translog methodology.

Although Marshall frequently used examples of fixed proportions joint products, it is clear from a careful reading (also see Mathematical Appendix Notes XVII, XVIII, XIX, XX and XXI of the 8th (variorum) Edition) that he understood that fixed-proportions joint products are special cases of the variable proportions joint products. For example see footnote 3 p. 388; "We must illustrate by a simple example in which it assumed that the relative amounts of the two joint products are unalterable," (emphasis supplied) and in Mathematical Appendix Note XIV;

> Let factors of production of a commodity by a_1, a_2, etc. and let their supply equations be $y=\phi_1(X)$, $y=\phi_2(X)$, etc. Let the number of units of them required for the production of x units of A by m_1x, m_2x...respectively; where m_1, m_2,... are generally not constants but functions of x. [emphasis supplied]

and again in § b. p. 390 Marshall continues;

> There are very few cases of joint products that cost of production of both of which together is exactly the same as that of one of them along.... It is only when one of two things produced by the same process is valueless, unsaleable, and yet does not involve any expense for its removal, that there is no inducement to attempt to alter its amount;... For when it is possible to modify the proportions of these products, we can ascertain what part the whole expense of the process of production would be saved, slightly to diminish the amount of one of the joint products without affecting the amounts of the others.

In the fixed proportions case, joint products may be correctly treated as a single product. When the output proportions are variable , (many seemingly fixed proportions products such as beef and hides have long-run variable proportions through selective breeding) the marginal cost may be determined (as Marshall suggested in § b above) by varying the output of one joint product and observing the direction of change of total cost.

In motor carriers there are numerous examples of Marshallian joint cost and whether these are joint products in a technical sense is an empirical question. The principal example of Marshallian joint products are the head-haul and back-haul. If there were data available on empty mileage by commodity and costs by category, this would not be an insurmountable problem. The head-haul and back-haul might seem to be a fixed-proportions case. On reflection they are variable-proportions because head-haul, and back-haul are produced in the fixed proportions only for two points and a single commodity and fixed commodity demands. The use of a translog joint cost function allows us to allocate cost for joint products (or nonseparable products) if output data - data on ton miles back-haul and ton miles head-haul by commodity - were available.

There are other examples of Marshallian jointness in motor carriers; for example Taff notes:

> Two other examples of joint costs in the motor-carrier industry include the vehicle time consumed in running to and from pickup and delivery stops, and the so-called "contact time" on multiple shipment stops in collection and delivery service. The latter includes the time consumed at the shipper's or consignee's place of business, in starting and stopping the truck, locating the receiver of the freight, receiving instructions as to the location of freight, and other miscellaneous delays.

Taff also classifies peak-offpeak use of terminal facilities across the yearly or diurnal cycle as joint products:

> Still another example of joint costs is found in motor carrier operation. Vehicles on terminal facilities purchased to meet transportation demands at the peak hours of the day or during peak seasons of the year are available to serve transportation demands coming at off-peak hours or off-peak seasons......
> To sum up the subject of joint costs, there is no justification from a cost-of-service standpoint for distributing any more of these joint costs to any one unit of output resulting from the same operation than to any other unit of output resulting from the same operation. Furthermore, joint costs do not lend themselves to a distribution on the basis of directly assignable expenses, for no directly assignable expenses are present in a strictly joint-cost operation.

The assumptions made in Taff's statements are: 1) The products are actually joint in a technical sense; 2) There is no method of allocating joint costs. The first assumption in testable and the second is true only if an output variable cannot be defined and measured for each category.

2.3 THE THEORETICAL PROPERTIES OF THE JOINT COST FUNCTION

This section specified the properties of the cost function which we sill estimate in a somewhat more formal way than in Section 1.0.

2.3.1 The Transformation Function

The transformation function is defined by the implicit function

$$F(Q,X,t) = 0 \tag{2.10}$$

where

$Q = [Q_1,...,Q_m]$ a vector of output

$X = [X_1,...,X_n]$ a vector of inputs

t = time, a variable included to take into consideration shifts in the frontier over time due to technological change.

If the transformation is strictly convex in inputs, there exists a unique dual, joint cost function

$$G(Q,x,t) \tag{2.11}$$

where

$w = [w_1,...,w_n]$.

Shephard's lemma defines the dual cost function as

$$G(Q,w,t) = \min \sum_{j=1}^{n} w_j X_j \quad . \tag{2.12}$$

This dual cost function must equal the cost minimizing vector of factor inputs and the vector of cost minimizing inputs must equal the partials of the cost function with respect to factor prices

$$\partial G(Q,w,t)/\partial w = X^* \qquad (2.13)$$

where $\partial G(Q,w,t)/\partial x = [\partial G(Q,w,t)/\partial w_1,\ldots,\partial G(Q,w,t)/\partial w_m]$ and

$$X^* = [X_1^*,\ldots,X_2^*] \quad .$$

The assumption of cost minimization is testable, through the imposition of degree-one homogeneity in factor prices on the cost function, using a likelihood ratio test.

2.3.2 Returns to Scale

The assumption of constant returns to scale may be treated as a testable hypothesis; if

$$F(kQ,kX,t) = F(Q,X,t) = 0. \qquad (2.14)$$

it follows that

$$G(kQ,w,t) = \min \sum_{j=1}^{n} w_j kX_j \qquad (2.15)$$

$$= k \min \sum_{j=1}^{n} w_j X_j$$

$$= kG(Q,w,t)$$

and conversely 8) implies 7). The application of this definition to the joint cost function is discussed below.

Economies of scale show how the firm's underlying technology reacts when all inputs are increased equally. For cost functions, economies of scale relate changes in total cost to increase in output brought about by given percentage increases in inputs.

A firm experiences economies of scale when a given percentage increase in inputs causes a greater percentage increase in output; for example, a 10 percent increase in inputs might cause a 15 percent increase in output. On the cost side economies of scale show up as less than proportional changes in cost as output increases; if output increases 10 percent a cost increase of 7 percent indicate scale economies.

For multiple outputs, a variation in definition is necessary. In the case of a small output the parameter of interest is the log derivative of the cost function with respect to output or the elasticity of cost as output is increased

$$e = \frac{\partial \ln C}{\partial \ln Q} = \frac{\partial C}{\partial Q}\frac{Q}{C} \qquad (2.16)$$

In the multi-output case we are interested in the behavior of cost as all output is increased and so the parameter of interest is

$$e = \sum_{i=1}^{n} \partial \ln C/\partial \ln Q_i \quad . \qquad (2.17)$$

For example, the translog joint cost function becomes

$$\ell = \alpha_1 + X_2 + \delta_{11}\ln Q_1 + \delta_{12}\ln Q_2 + \rho_{11}\ln w_1 + \rho_{12}\ln w_2 . \qquad (2.17a)$$

The translog joint cost function may also be used to compute values for cost, scale economies and other values of interest by using the actual levels of output of a firm. For example, to estimate scale economies for the ith firm we can substitute values of output and inputs for that firm into the fitted cost function to estimate the individual scale economies it faces. If the underlying production technology is not homothetic (seperable) we cannot uniformly characterize scale economies over the entire range of possible outputs.

2.3.3 Output Measurement

A question that often arises in studies of motor carrier proudction technology is the correct unit of measurement for output ton-miles. This question may be posed statistically as one of whether or not there exist distinct and separate indices of input and output.

Let the transformation function be

$$F(Q,X,t) = F[J(Q),X,t] = 0 \qquad (2.18)$$

where J(Q) is a scalar function of Q. This function is an implicit function

$$F[J(Q),X,t] = J(Q,t) + k(X,t) = 0. \qquad (2.19)$$

If there exists an output index, J(Q), the existence of an input index k(K,t) is also implied. If both indices exist, the transformation function F(Q,X) is <u>input-output separable</u>. If separability exists, as neo-classical theory assumes, then all outputs are produced <u>jointly</u> and F(Q,X,t) is input-output separable and a single output measure <u>is</u> appropriate.

2.3.4 Separability and Jointness

The key theorem in duality theory shows that if the cost function is <u>weakly</u> input-output separable, the transformation function must be separable in inputs and outputs. <u>Weak</u> separability implies only that outputs are separable, <u>not</u> that the <u>cost function</u> is separable in factor prices. Separability requires that factor price-output interaction coefficients all be zero,* ($\rho_{ij} = 0$).

---

*Denny and Fuss argue that the conditions imposed by the use of translog functions is more restrictive than one might suspect <u>a priori</u>. In particular, they have shown that the linear separability conditions for the interpretation of the translog as an <u>exact</u> function require either a Cobb-Douglas function of translog aggregates on a translog function of Cobb-Douglas aggregates. Recall that our interpretation is that the translog is a differential or numerical approximation.

2.3.5 Estimation and Shephard's Lemma

The joint cost function may be estimated using the first order conditions implied by profit maximization. Let the cost functions be

$$G(Q,w,t) \tag{2.20}$$

Shephard's lemma is

$$\partial G(Q,w,t)/\partial w_j = X_j^* \quad j=1,\ldots,n. \tag{2.21}$$

Under the maintained hypotheses of constant returns, competitive product markets and cost minimization, an equation for each of the m outputs may be derived. If these two hypotheses are not fulfilled, the m equations have no useful meaning. The factor share equations and the joint cost function may be estimated simultaneously. This allows a number of a cross-equation coefficient restrictions to be applied that result in more efficient parameter estimation because all available information is used to estimate them.

The translog cost function is

$$\begin{aligned} \ln C = {} & \alpha_o + \sum_{i=1}^{m} \alpha_i \ln Q_i + \sum_{j=1}^{n} \beta_j \ln w_i \\ & + 1/2 \sum_{i=1}^{h} \sum_{j=1}^{m} \delta_{ij} \ln Q_i \ln Q_j + 1/2 \sum_{i=1}^{n} \sum_{j=1}^{m} \gamma_{ij} \\ & \ln w_i \ln w_j + \sum_{i=1}^{m} \sum_{j=1}^{n} \rho_{ij} \ln Q_i \ln w_j + \sum_{j=1}^{m} \gamma_{it} \ln w_j \cdot t \\ & + \sum_{i=1}^{m} \delta_{it} \ln Q_i \cdot t + 1/2 \gamma_{tt} \cdot t \end{aligned} \tag{2.22}$$

where

α_o = cost equation intercept,

α_1, β_j = first order parameter, m+n,

$\delta_{ij}, \gamma_{ij}, \rho_{ij}$ = second order parameters,

$\delta_{it}, \gamma_{it}, \gamma_{tt}$ = technological change parameters.

If Shephard's lemma holds and the firm minimizes cost, the cost function will be homogeneous of degree one in factor prices. This implies the following coefficient restrictions:

$$\sum_{j=1}^{n} \beta_j = 1 \tag{2.23}$$

$$\sum_{j=1}^{n} \gamma_{ij} = 0$$

$$\sum_{j=1}^{n} \rho_{ij} = 0 \ .$$

2.3.6 <u>Factor Cost Shares</u>

Shephard's lemma says that

$$\frac{\partial \ln C}{\partial \ln w_j} = (\partial C/\partial w_j) w_j / C \tag{2.24}$$

and these derivatives must equal a vector of cost minimizing input quantities factor in equilibrium

$$M_j = w_j X_j / C \quad j=1,\ldots,n \tag{2.25}$$

where M_j is the jth factor share in total cost.

The translog cost share relationships may be written

$$M_j = C_w^j \tag{2.26}$$

where

$$C_w^j = \partial \ln C / \partial \ln w_j \ .$$

Shephard's limma allows us to substitute $\partial C/\partial w_j$ for X_j^* where X_j^* represents the optimal (cost minimizing) employment of X_j. This substitution leads directly to the factor cost share equation.

The cost function must be linearly homogenous in factor prices if costs are minimized. The hypothesis of constant returns to scale can be tested by imposing constant returns and defining an appropriate likelihood ratio test. Constant returns require that

$$\sum_{i=1}^{m} \alpha_i = 1 \qquad (2.27)$$

$$\sum_{i=1}^{m} \delta_{ij} = 0$$

$$\sum_{i=1}^{m} \rho_{ij} = 0.$$

The cost function is regarded as a second order approximation to <u>any</u> cost function. Therefore, the parameters of the estimated function may be interpreted as the partial derivatives of the underlying function evaluated at the point of approximation.

2.3.7 Concavity

A function f(X) is concave if

$$f\Big((1-k)X + kX'\Big) \geq (1-k)f(X) + kf(X') - o < k < 1 \qquad (2.28)$$

and any X and X'. This means if a function is concave, that at any place along its linear approximation between X and X' the <u>actual</u> function always exceeds or equals the value of the approximation.

A function is <u>strictly concave</u> if this relationship holds with strict inequality. This means that the actual function is always greater than the approximation.

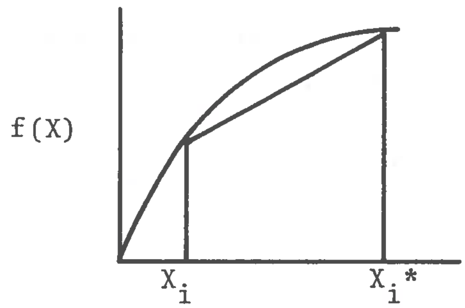

FIGURE 9. CONCAVITY

Any differentiable function, such as f(X), is strictly concave if for any X and X'

$$f(\overset{*}{X}) - f(X) < \sum_{i=1}^{n} f_i(\overset{*}{X}_i - X_i) \qquad (2.29)$$

where the f_i are the n partial derivatives of the function evaluated at a point X_i.

Strict convacity requires a negative definite Hessian. This can be shown by expanding f(X) around X using a Taylor's series expansion

$$f(X+V) = f(X) + \sum_{i}^{n} f_i V_i + 1/2\ v'Hv \qquad (2.30)$$

where $v = (\overset{*}{X}_i - X_i)$ is a vector and H a matrix of partial derivatives; since

$$f(\overset{*}{X}) - f(X) < \sum_{i=1}^{n} f_i(\overset{*}{X}_i - X_i) \qquad (2.31)$$

then

$$f(\overset{*}{X}) - f(X) - \sum_{i=1}^{n} f_i(\overset{*}{X} - X_i) < 0 \qquad (2.32)$$

and substituting

$$v = (\overset{*}{X}_i - X_i) \qquad (2.33)$$

we obtain the left side of 2). Since it must be negative and therefore

$$f(\overset{*}{X}) - f(X) - \sum_{i=1}^{n} f_i(\overset{*}{X}_i - X_i) = 1/2\ v'Hv \qquad (2.34)$$

then 1/2 v'Hv < 0 since

$$f(\overset{*}{X}) - f(X) - \sum_{i=1}^{n} f_i(\overset{*}{X}_i - X_i) < 0. \qquad (2.35)$$

Concavity of the Joint Cost Function

The joint cost function must be concave in factor prices and this requires that the quadratic form be negative (semi.) definite. The requirement may be stated mathematically as

$$v'Hv \leq 0 \qquad (2.36)$$

where

$$v' = \left[\partial G(Q,w,t)/\partial w_i, \ldots, \partial G(Q,w,t)/\partial w_j\right]$$

and the ijth element of H is

$$H_{ij} = \left(\partial^2 G(Q,w,t)/\partial w_i\, \partial w_j\right) . \qquad (2.37)$$

Stated another way this means

$$\sum_{i=1}^{n} \sum_{j=1}^{n} H_{ij} V_i V_j \leq 0 \text{ iff } V_i, V_j > 0. \qquad (2.37a)$$

If the concavity condition is met, the matrix H must have non-positive eigenvalues.*

---

*Given any square matrix H an eigenvector or characteristics vector is some nonzero vector such that

$$Hz = \lambda z$$
$$\text{or } H(1-\lambda)z = 0$$

where λ_i is an eignevalue or characteristic root of the matrix H. The eigenvalues of a negative semidefinite matrix are negative or zero valued. Furthermore, it is also true that the trace of a square matrix denoted by Tr(H) is the scalar

(footnote continued on next page)

2.3.8 The Elasticity of Substitution and Allen-Uzawa Partial Elasticities of Substitution

In a two factor world the elasticity of substitution, σ, measures the "ease" with which one input may be substituted for the other and ranges between zero and plus infinity. The elasticity of substitution may be seen geometrically as the slope of the isoquant for different input combinations. The isoquant slope represents the ratio of marginal products.

$$MRTS = -\ (dX_i/dX_j) = f_i/f_j \qquad (2.38)$$

where

$$Q = f(X_i, X_j) \qquad i,j=K,L$$

and

$$f_i = \partial Q/\partial X_i .$$

The elasticity of substitution in a two factor model is

$$\sigma = \frac{d(X_j/X_i)}{X_j/X_i} \Big/ \frac{d(f_i/f_j)}{f_i/f_j} \qquad i,j=K,L \qquad (2.39)$$

The elasticity of substitution may be defined using the partial derivatives of the production function:

$$\sigma = \frac{f_i f_j (X_i f_i \;\; X_j f_j)}{X_i X_j (f_{ii} f_j^2 - 2f_{ij} f_i f_j + f_{jj} f_i^2)} \qquad i,j=K,L \qquad (2.40)$$

---

*(Cont'd from previous page)

$$Tr(H) \quad \sum_{1=1}^{n} \quad H_{ij} .$$

The trace of a matrix is equal to the sum of its eigenvalues and the determinant of a matrix must equal the product of its eigenvalues; for the concavity requirement to be fulfilled we must have $Tr(H) \leq 0$ and $\det |H| \leq 0$.

The negative sign insures that σ is always positive or zero since the term in parentheses in the denominator is always negative. The elasticity of substitution is inversely proportional to the change in the slope of the isoquant as input ratios change. The limiting values of σ, zero and plus infinity, occur when the isoquants form right angles (when σ is zero) or straight lines (when σ is infinite); it is a unit-free number unrelated to the units in which inputs and outputs are measured.

The Cobb-Douglas production function has an elasticity of substitution that is constant and equal to one. This is a special case of the constant elasticity (CES) function for which the elasticity of substitution is constant but not necessarily unity.

Partial Elasticities of Substitution

The discussion on the elasticity of substitution assumes that there are only two inputs -- what happens if there are multiple inputs? The concept carries through as the partial elasticity of substitution or Allen-Uzawa Partial Elasticity of Substitution.

Assume a single output and n inputs $X_j \quad j=1,\ldots,n$

$$Q = f(X_1,\ldots,X_n).$$

Let

$$k_i = \frac{X_i f_i}{Z}$$

where

$$Z = \sum_{j=1}^{n} X_i f_i$$

for functions that are homogenous of degree one the summation of $\Sigma K_i=1$. Let the bordered Hessian of the production function be

$$H = \begin{bmatrix} f_1 & f_2 & \cdots\cdots & f_n & f_1 \\ f_{11} & f_{12} & \cdots\cdots & f_{1n} & f_2 \\ f_{21} & f_{22} & \cdots\cdots & f_{2n} & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & f_n \\ \cdot & \cdot & & \cdot & \\ f_{n1} & f_{n2} & \cdots\cdots & f_{nn} & 0 \end{bmatrix} \tag{2.41}$$

Let H_{ij} represent the cofactor of H. The partial elasticity of substitution between factor inputs X_i and K_j is

$$\sigma_{ij} = \sum_{j=1}^{n} X_j f_j / X_i X_j \ H_{ij} H^{-1} \tag{2.42}$$

Partial elasticities of substitution are symmetric, so that

$$\sigma_{ij} = \sigma_{ji} \tag{2.43}$$

For two inputs the partial elasticity of substitution reduces to the two factor elasticity of substitution previously defined. It can also be shown that

$$\sigma_{ii} < 0 \tag{2.44}$$

and

$$\sum_{j=1}^{n} k_j \sigma_{ij} > 0 \tag{2.44a}$$

since $k_j > 0$ some of the partial elasticities may be negative, but the elasticities and weights must have a positive sum. We infer that the positive elasticities outweigh the negative ones.

The inputs X_i and X_j may be said to be competitive (substitutable) or complements depending on whether the partial elasticity is positive or negative

PARTIAL ELASTICITY OF SUBSTITUTION	INPUT CLASSIFICATION
$\sigma_{ij} > 0$	Competitive
$\sigma_{ij} < 0$	Complements

This definition of the partial elasticity of substitution will be used in the empirical section of this paper.

In terms of the parameters of the joint cost function, the partial elasticity of substitution is defined

$$\sigma_{ij} = \frac{G\left(\partial^2 G/\partial w_1 \ \partial w_j\right)}{(\partial G/\ w_j)(\partial G/\ w_i)} \quad . \tag{2.45}$$

Because of the correspondence between parameter values and partial derivatives this becomes

$$\sigma_{ij} = 1 + \frac{ij}{\beta_i \beta_j} \quad , \tag{2.45a}$$

with variance,

$$v(\sigma_{ij}) = \frac{v(ij)}{(\beta_i \beta_j)^2} \quad . \tag{2.45b}$$

2.4 OTHER PROPERTIES OF THE COST FUNCTION

2.4.1 <u>Isocost Functions</u>

A goemetric interpretation of the cost function that offers some insight into the behavior of the multiproduct firm is the isocost curve examined in output space.

Assume that there are two outputs Q_1 and Q_2, two factor inputs with prices w_1 and w_2

$$C = g(Q_1, Q_2, w_1, w_2) \qquad (2.46)$$

The isocost function represents a line of constant cost. It is found by taking the total derivative of the cost function

$$dC = \frac{\partial g(\cdot)}{\partial Q_1}\, dQ_1 + \frac{\partial g(\cdot)}{\partial Q_2}\, dQ_2 + \frac{\partial g(\cdot)}{\partial w_1}\, dw_1 + \frac{\partial g(\cdot)}{\partial w_2} dw_2 = 0. \qquad (2.47)$$

There are no changes in cost along an isocost and also no changes in factor prices so the isocost reduces to

$$dC \Big|_{\substack{dC = 0 \\ dw_i = 0 \\ i = 1,2}} = \frac{\partial g(\cdot)}{\partial Q_1}\, dQ_1 + \frac{\partial g(\cdot)}{\partial Q_2}\, dQ_2 \, . \qquad (2.48)$$

The isocost function has the following slope in output space (Q_1, Q_2)

$$dQ_1/dQ_2 = \frac{\partial g(\cdot)/\partial Q_2}{\partial g(\cdot)/\partial Q_1} \, . \qquad (2.49)$$

When the isocost curve is shown in output space there are three possible configurations it may assume. It may be concave (curved away from the origin), convex (curved toward the origin) or a straight line.

A concave isocost function implies that specialization causes relative costs to increase; that is, more of the two outputs can be produced from the same cost by the selection of some combination of the two.

The convex isocost implies that specialization is associated with decreasing relative cost and is therefore advantageous to the firm.

No curvature - linearity - indicates no advantage or disadvantage to specialization.

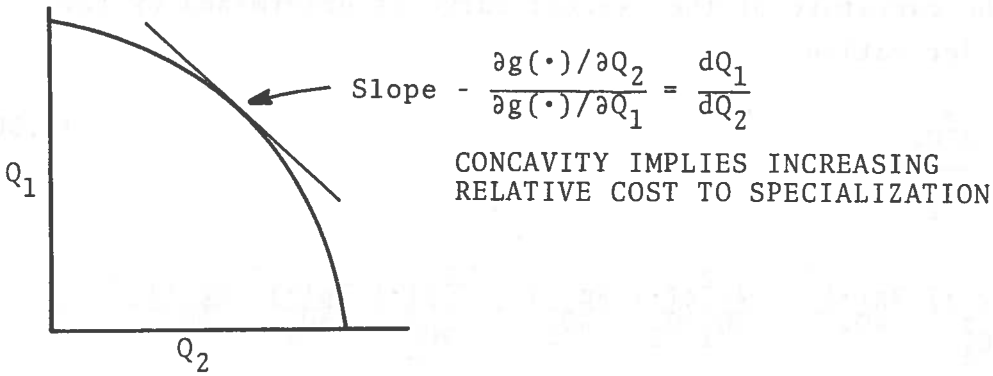

FIGURE 10. CONCAVE ISOCOST

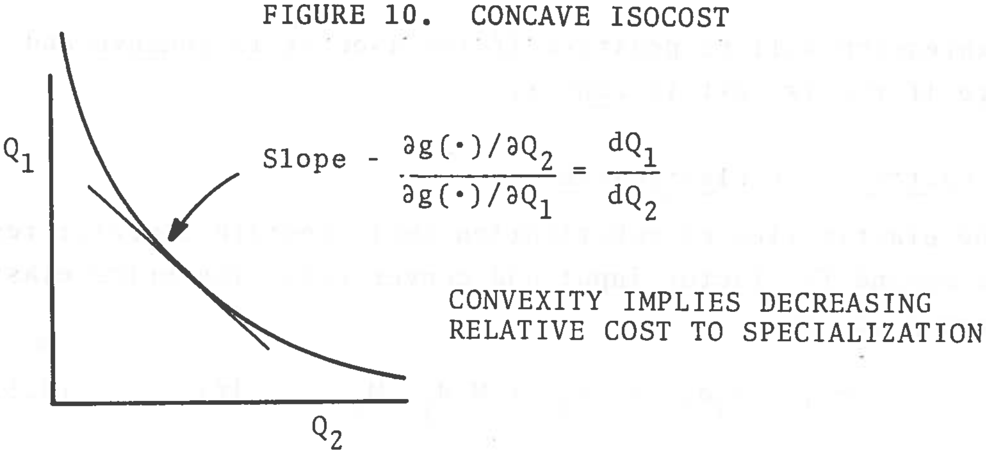

FIGURE 11. CONVEX ISOCOST

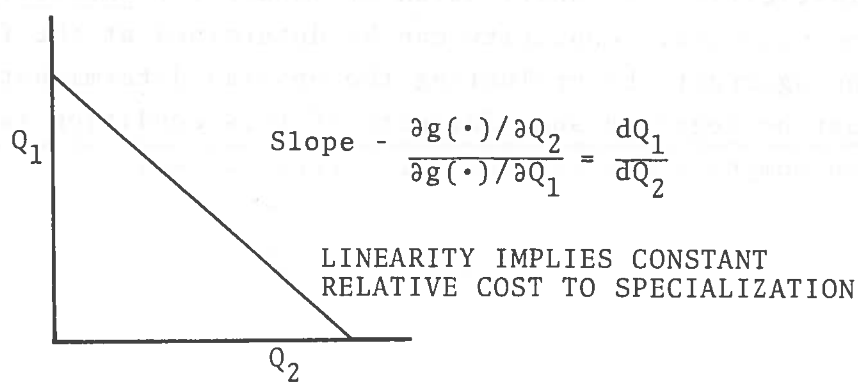

FIGURE 12. LINEAR ISOCOST

The curvature of the isocost curve is determined by the second derivative.

$$\frac{d^2Q_1}{dQ_2^2} = \tag{2.50}$$

$$- \frac{\partial^2 g(\cdot)}{\partial Q_1^2} \frac{\partial g(\cdot)}{\partial Q_2}^2 - \frac{2\partial^2 g(\cdot)}{\partial Q_1 \partial Q_2} \frac{\partial g(\cdot)}{\partial Q_2} + \frac{2^2 g(\cdot)}{\partial Q_2^2} \frac{2g(\cdot)}{\partial Q_2}^2 \frac{\partial g(\cdot)}{\partial Q_1}^3 .$$

This expression will be positive if the isocost is concave and negative if the isocost is convex.

2.4.2 Factor Price Elasticities

The elasticities of substitution imply certain elasticities of derived demand for factor input and conversely. The price elasticities are given by

$$\eta_{ij} = \hat{M}_i \sigma_{ij} = (\gamma_{ij} + \hat{M}_i \hat{M}_j)/\hat{M}_i \qquad i \neq j \tag{2.51}$$

and

$$\eta_{ii} = \hat{M}_i \sigma_{ii} = \left(\gamma_{ii} + \hat{M}_i \ \hat{M}_{ij}\right)/\hat{M}_i \tag{2.52}$$

where the γ coefficients are the coefficients of factor prices in the cost share equations and own and cross-price elasticities in the joint cost function. If the cost function is concave in factor prices, the elasticities of substitution of inputs for themselves the σ_{ii} must be negative. Concavity can be determined at the firm level or in the aggregate by evaluating the Hessian determinant; the Hessian must be negative semi-definite if this condition is satisfied. See Humphrey and Moroney for further detail.

3. PROBLEMS AND PROCEDURES OF ESTIMATION

3.1 HYPOTHESIS TESTS WITH MAXIMUM LIKELIHOOD ESTIMATION

Simple Hypotheses

The most common way of testing the statistical significance of hypotheses in econometrics is with a Student's 't' test. However, this procedure is appropriate only when the test involves the significance of a single values - such as regression coefficients or significant differences between the means of a distribution. A more general formulation requires us to assume some parameter space Ω, in which the parameter μ (the mean) lies. This space may consist of a set defined as the real number line

$$\Omega = \{\mu; -\infty < \mu < \infty\} .$$

The null hypothesis is a test of $\mu = \mu_o$ where μ is a point on the real number line. This situation is usually handled with a 't' test, and represents a simple hypothesis because it formulates the test around a single point in the parameter space Ω.

3.2 COMPOSITE HYPOTHESIS

When the null hypothesis involves a region in parameter space, a subset of the entire parameter space (universe), the hypothesis is a composite one. An example of a composite hypothesis is one that has two (or more) unknown parameters. Let Ω represent a two dimensional parameter space

$$\Omega = \left\{(\mu,\sigma^2); -\infty < \mu < \infty, 0 < \sigma < \infty\right\} .$$

If we formulate an hypothesis about the parameter μ and omit any hypothesis regarding σ^2 (the variance of the distribution) the relevant parameter space is a subset of the entire parameter space ω where

$$\omega = \left\{(\mu,\sigma^2); \mu_o = \mu, 0 < \sigma^2 < \infty\right\} .$$

This formulation of the null hypothesis means that ω is a straight line in Ω; a formulation that postulated some small value for σ^2 would represent a point in Ω. The former is a composite hypothesis, and the latter a simple one. The 't' test is appropriate for tests involving simple hypothesis, while composite hypothesis require something more general such as Scheffe's F test or the likelihood ratio test.

3.3 THE LIKELIHOOD RATIO

The likelihood ratio test for a composite hypothesis (such as the one represented by ω) is

$$L(\Omega) = (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2}\sum_{i=1}^{n}(X_i-\mu_o)^2/\sigma^2\right)$$

where the X_i elements of the vector X.

Taking the natural log of this expression

$$-\ n/2 \log 2\pi\sigma^2 + \frac{1}{2}\sum_{i=1}^{n}(X_i-\mu_o)^2/\sigma^2$$

in order to derive the maximum likelihood estimator we take the logarithmic derivative of the log likelihood ratio

$$-n/2\sigma^2 + \frac{1}{2}\sum_{i=1}^{n}(X_i-\mu_o)^2/\sigma^4 \quad .$$

Solving the derivative for σ^2 we have

$$\sigma^2 = \sum_{i=1}^{n}(X_i-\mu_o)^2/n \quad .$$

This solution is substituted back into the log likelihood function

$$L(\hat{\omega}) = e^{-n/2}\left(2\pi/n\sum_{i=1}^{n}(X_i-\mu_o)^2\right)^{n/2}$$

The caret over ω denotes the fact that the likelihood ratio is at a maximum when μ and σ^2 are restricted to be in ω; if this constraint is not imposed μ_o is replaced by $\bar{X}$ - the sample mean - and this defined the unrestricted likelihood. The ratio of these likelihoods is the likelihood ratio

$$L = L(\hat{\omega})/L(\Omega)$$

$$= \left(\sum_{i=1}^{n} (X_i - \bar{X})^2 \Big/ \sum_{i=1}^{n} (X_i - \mu_o)^2 \right)^{n/2}$$

where

$$0 < \Omega < 1 \qquad L = 1 \text{ iff } \mu_o = \bar{X}$$

The test statistic L is distributed as 't' with n-1 degrees of freedom.

The example shows that maximum likelihood estimation may lead to familiar tests of significance. In the case of linear regression, maximum likelihood leads to solution formulas identical to those that arise in the least squares procedure. Least squares esitmation does not impose any a priori assumptions about the distribution of estimated parameters. It is the tabulated theoretical distributions necessary in tests of hypotheses that require these assumptions. On the other hand, maximum likelihood estimation forces the distribution of single equation maximum likelihood estimators to be normal.

3.4 THE LIKELIHOOD RATIO IN REGRESSION ANALYSIS

In a linear regression the most common test is that of distinguishability of coefficients from zero.

Let

$$\Omega = \{\beta;\ -\infty < \beta < \infty;\ \sigma^2 > 0\} \qquad (3.1)$$

and

$$\omega = \{\beta^*;\ \beta^* = 0,\ \sigma^2 > 0\} \qquad (3.2)$$

where

$$\beta' = \left[\beta_o \beta_1 \ \dots \ \beta_k\right]$$

and

$$\beta^* = (\beta_{\dot{o}} \beta_1 \ \dots \ \beta^*_k) \ k \leq k = 1 \ .$$

The unrestricted likelihood function is

$$L(\Omega) = (1/2\pi\sigma)^n \ \exp\left(- \ 1/2 \ \sigma^2 (Y-X\beta)^1 (Y-X\beta)\right) dY;$$

and for the restricted regression,

$$L(\omega) = (1/2\pi\sigma)^n \ \exp\left(- \ 1/2 \ \sigma^2 (Y-X\overset{*}{\beta})^1 (Y-X\overset{*}{\beta})\right) dY$$

where

$$Y' = \left[Y_1 \ \dots \ Y_n\right] X = \begin{matrix} 1 & X_{11} & \dots & X_{1k} \\ . & & . & . \\ . & & . & . \\ . & & & . \\ 1 & X_{n1} & \dots & X_{nk} \end{matrix}$$

The likelihood ratio to test the null hypothesis of $\overset{*}{\beta} = 0$ is

$$\mathit{L} = L(\hat{\Omega})/L(\hat{\omega}) \qquad \mathit{L}=1 \text{ iff } \overset{*}{\beta}=0$$

$$= (S_\Omega/S_\omega)^{n/2}$$

where S is the determinant of the variance-covariance matrix of the unrestricted (S_Ω) and restricted (S_ω) regressions respectively, each divided by their respective variance matrix determinants;* $-2 \ln L$ is asymptotically distributed as χ^2 with degrees of freedom equal to the number of restrictions. Of course, the level of significance in a series of nested tests such as those for separability, etc. are not independent at each level. Therefore the appropriate method to take this into account is to select an overall level of significance for the n set of tests and treat each individual test as having a level of significance equal to 1/n times that overall level. Scheiffes F and the liklihood ratio are two possible methods of dealing with this test situation; a third is the Bonferroni 't' test. The latter was not used because Christensen (1973) has shown that the power of the test is quite low in cases in which the independent variables in the equation(s) are highly correlated - as they are in this case. We have chosen to ignore this problem in the interests of simplicity.

3.5 ESTIMATION BY FULL INFORMATION MAXIMUM LIKELIHOOD

Full information maximum likelihood (FIML) is a simultaneous method of estimation. This means that FIML estimates a system of equations - two or more equations - as though it were a single

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*If the null hypothesis is true, S_Ω and S_ω are distributed as χ^2 using an argument based on Cochran's theorem (see Wilks pg. 406), and we can deduce that the likelihood ratio has the distribution

$$L = \left(1 + (k-k^*)/(n-k)\ \tilde{S}\right)^{-1/2n}$$

where $$\tilde{S} = (n-k)(S_\omega - S_\Omega)/(k-\overset{*}{k})\ S_\Omega \quad .$$

$\tilde{S}$ is the Snedecor distribution (Wilks p. 186) which is a ratio of Gamma distributions; tables of the Snedecor distribution are available and Pearson's Tables of the Incomplete Beta Function may also be used given correspondence found in Wilks (pg. 187). Fortunately a simple transformation makes it unnecessary to use these relatively arcane distributions because $-2 \ln L$ is asymptotically distributed as χ^2 with degrees of freedom equal to the number of restrictions imposed.

equation. The FIML method uses an estimate of the system covariance matrix of distributions Σ. The Σ matrix includes any error associated with the specification of equations, so that any errors in specifying one equation is propagated throughout the system. This is a well known defect of using system estimation methods. We conclude that the advantages outweigh the disadvantages in this application. In particular, a major advantage is that FIML is not sensitive to which equation of the factor share set is deleted. It is also less sensitive to problems of collinearity than least squares methods and lastly, FIML is more efficient (has a smaller variance of the estimates) than least squares estimation.

The FIML method is now well known and widely used and details on its use are available in almost every intermediate and advanced econometric textbook. This presentation follows Johnston and Theil. It is included here only as a matter of convenience to the reader.*

Consider the following equation system written in implicit form

$$BY_t + \Gamma X_t = u_t$$

where Y is an n x g matrix of observations on the endogenous variables (variables determined within the system of equations) and B is a g x 1 vector of endogenous coefficients; Γ is a k x 1 vector of exogenous or predetermined variable coefficients, u_t is the vector of error terms or disturbances.

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*Because FIML is a system estimation method rather than a single equation method, the coefficient of determination R^2 is not displayed in the output of the estimation process, because R^2 has no valid meaning in a simultaneous equation system, and may have a negative value for correctly specified models. The lack of validity of R^2 may be explained by examining the equation for R^2; $R^2 = 1 - e'e/y'y$. We know that $0 \leq R^2 \leq 1$ will be satisfied only when $e'e \leq y'y$. In simultaneous equation systems $e'e \geq y'y$ is possible leading to negative R^2, so this statistic has no usefulness in simultaneous equation systems. See Christ, p. 319.

We assume that the expected value of the vector of error terms is zero

$$E(u_t) = 0$$

and the covariance matrix is

$$E(u_t u_t') = \Sigma$$

and

$$\det \Sigma > 0.$$

We assume that the error terms are distributed as standard normal with mean zero and covariance matrix Σ

$$u_t \sim N(0, \Sigma)$$

where

$$f(u_t) = 2\pi^{-G/2} \det\Sigma^{-1/2} \exp - 1/2 u_t' \Sigma^{-1} u_t .$$

It is assumed that the vectors are not serially correlated. The likekihood function is

$$\sum_{t=1}^{n} f(u_t) .$$

This makes the likelihood function for $Y = (y_1, \ldots, y_n)$

$$P(Y) = 2\pi^{-nG/2} (\det B)^n (\det \Sigma)^{-n/2}$$

$$\exp \left[- 1/2 \sum_{t=1}^{n} (BY_t + \Gamma X_t)' \Sigma^{-1} (By_t + \Gamma X_t) \right].$$

3.5.1 The Use of Pooled Cross-sectional and Time Series Data

The data represent a "panel" of observations on all trucking firms that existed continuously throughout the ten year period 1965-1974. There were a total of 252 such firms in each of the ten years for a total number of observations of 2520.

Due to the large data set, and across-equation coefficient restrictions, FIML is used as the estimation technique and we expect that any violations of assumptions would not lead to serious problems. Nevertheless, we should be aware of the consequences of violations of the assumptions and of possible alternative specifications, regression equations.

There are several specification options available to combine time series and cross section data: (1) We can estimate each year separately; (2) We can pool the time series and cross-section data by grouping all years for the same firm together; (3) We can pool all firms within a given year together; (4) We can use dummy variables to create separate intercepts for each year or each firm or both.

Choice (1) does not utilize all of the available information to estimate parameters, but because of the problems inherent in pooling, this would be the choice of some. The disadvantage of not pooling is that all of the econometric procedures--such as parameter restrictions--would have to be repeated within each year; the advantage is that this is a good way to get initial parameter estimates and test the estimating algorithm. In addition, it is of interest to observe how coefficient estimates behave over time.

The second method requires grouping all data for a single firm in a single year together. In the context of the regression model.

$$Y = X\beta + e$$

this specification means that

$$Y' = \left[Y_{1T}, \ldots, Y_{1T}, Y_2, \ldots, Y_{2T}, \ldots Y_{NT}, \ldots, Y_{NT}\right]$$

$$X = \begin{bmatrix} X^1_{11} & X^2_{11} & \cdots & X^K_{11} \\ X^1_{12} & X^2_{12} & \cdots & X^K_{12} \\ X^1_{1T} & X^2_{1T} & \cdots & X^K_{1T} \\ \cdot & \cdot & & \cdot \\ X^1_{NT} & X^2_{NT} & \cdots & X^K_{NT} \end{bmatrix} \qquad \begin{matrix} i=1,\ldots,K \text{ Regressors} \\ j=1,\ldots,N \text{ Firms} \\ k=1,\ldots,T \text{ Years} \end{matrix}$$

where X^i_{jk} refers to the ith regressor for the jth firm in the kth year

$$\beta' = [\beta_1, \ldots, \beta_k]$$

and

$$e' = e_{11}, \ldots, e_{1T} e_{21}, \ldots e_{2T} \ldots e_{N1}, \ldots, e_{NT} .$$

It is common to assume that the observations for individual firms are independent of one another but that the error variance is non-scalar due to some relationship between the error term and the size of an explanatory variable - <u>heteroscedasticity</u>. In the time domain, error terms are likely to be <u>autocorrelated</u>. The presence of heteroscedasticity means that

$$E(e'_{jk} e_{jk}) = \sigma^2_j$$

while independence within a time period means

$$E(e'_{jk} e_{ik}) = 0 \qquad (j \neq i).$$

Autocorrelation means that

$$e_{jk} = \rho_{jk-1} + U_{jk} .$$

3.5.2 Autocorrelation and FIML

One of the assumptions underlying the FIML estimates is that the error terms are uncorrelated; this assumption is likely to be unfulfilled. What then? This section deals with the problem of autocorrelation in simultaneous equation systems. Autocorrelation in single equation causes an underestimation of the absolute value of the regression coefficient (the estimated regression line is flatter than the true relationship, in a univariate linear relationship). In simultaneous equations systems there is a tradeoff between simultaneity and autocorrelation so that we often must to choose to deal only with one problem, and some believe that simultaneity may be less important than autocorrelation.

Consider FIML the application to a system with a first order autoregressive process. Rewrite the implicit model

$$BY_T + \Gamma X_t = u_t$$

let

$$X_t' = [Y_t \; Z_t] \text{ and } A = [B\Gamma]$$

then we can rewrite the system as

$$AX_t = u_t \, .$$

The autogregressive process is

$$u_t = Ru_{t-1} + \varepsilon t$$

where R is a matrix of autoregressive parameters the autoregressive error term ε_t are joint normal - $\varepsilon_t \sim N(O,\Sigma)$.

The implicit system may now be written

$$AX' - RAX_1' = E'$$

where $X' = (X_1,\ldots,X_n)$, $X_{-1}' = (X_{1-1},\ldots,X_{n-1})$ (e.g., all independent variables lagged one period) and $E' = (\varepsilon_1,\ldots,\varepsilon_n)$.

The probability function is

$$P(\varepsilon_t) = (2\pi)^{-n/2}(\det\Sigma)^{-1/2} \exp(-1/2\varepsilon_t'\Sigma^{-1}\varepsilon_t).$$

Taking logs of the likelihood function we have

$$C = n\log|\det B| - n/2 \log(\det\Sigma) - 1/2\mathrm{tr}\Big[(XA'=X_1A'R')\Sigma^{-1}$$
$$(AX' - RAX_1')\Big]$$

$$= C + n\log|\det B| - n/2 \log(\det\Sigma) - 1/2\mathrm{tr}\{\Sigma^{-1} AX'XA'$$
$$- 2RAX_1'XA' + RAX_1'X_1A'R'\}$$

where C is a constant.

We may also specify a second order autogressive process following the same procedure. The presence of a nonzero R matrix can be tested using a likelihood ratio against the version without the autoregressive specification.

Goldfeld and Quandt (GQ) performed a number of Monte Carlo experiments on simultaneous equation systems with autocorrelated disturbances and some models with collinearity as well. They tested ordinary least squares (OLS), two stage least squares (TLS) and four different versions of FIML, labelled FIML1 through FIML4 and defined as follows:

1) FIML1 - equation by equation application of FIML where equations are specified with a first order autoregressive process,
2) FIML2 - simultaneous estimation of equations without the correlation for autocorrelation,
3) FIML3 - simultaneous estimation of parameters <u>and</u> autoregressive process, and
4) FIML4 - simultaneous estimation with autoregressive process <u>assumed</u> to be know <u>a priori</u>.

The mean square error (MSE) for each case was then computed. On a coefficient-by-coefficient basis the ranking is

1. FIML3
2. FIML1
3. FIML4
4. FIML2
5. TLS
6. OLS

Their result indicates that, in the model tested, it was more important to account for autocorrelation than simultaneity - FIML1 ranks above FIML2 and FIML4. The FIML algorithm produced worse estimates of the equation intercept than coefficients and the rankings over a wide variety of cases, sample draws and assumptions about collinearity including the intercepts were (mean sum of ranks in parentheses)

1. FIML4 (20.7)
2. FIML3 (23.5)
3. FIML1 (30.3)
4. FIML2 (32.5)
5. TLS (36.0)
6. OLS (46.0)

Omitting the MSE of intercepts the ranking was unchanged but produced larger differences in mean rankings.

The GQ book has additional useful discussion of the properties of various estimators. Of course FIML4 is not a realistic specification because we never know the autoregressive process *a priori*, and FIML1 is not realistic because we would never apply FIML equation by equation. Lastly, note that these conclusions apply only for the specific models and assumptions used in the Monte Carlo experiments. The only way to be sure that these results apply is to do our own Monte Carlo experiments with the model we are estimating, but we can cautiously infer that FIML3 would provide efficient estimates and formulate null hypothesis based on this inference.

3.5.3 Heteroscedasticity

Regression analysis assumes that the variance of regression error terms is constant. The general linear model - the form in which the example translog production and cost functions were specified - is

$$Y = X\beta + e$$

where Y is an n x 1 vector of observations on the dependent variable, X is an n x k matrix of observations on the k regressors, β is a k x 1 vector of regression coefficients and e is an n x 1 error vector about which we assume that $E(e'e)=\sigma_e^2 I$, where I is a k x k identify matrix. Under this assumption we have homoscedasticity and the covariance matrix is scalar.

If $E(e'e) \neq \sigma_e^2$ we have heteroscedasticy, one of a family of demons that regularly attack econometricians, the others being multicollinearity, autocorrelation, specification error and underidentification. Heteroscedasticity biases the estimate of the coveriance matrix, so that the estimator is not BLUE because it does not have the smallest variance among linear estimators.

So, heteroscedasticity biases the estimate of the OLS error covariance; what operational significance does this have? First, the estimate of the slope remains unbiased. Secondly, tests of significance--such as t and F tests--are unvaried in the presence of heteroscedasticity because these tests assume that both the coefficients and their variances are unbiased. Thirdly, the direction of bias depends on the relationship between the error terms and explanatory variables. If these is a positive relationship it causes an underestimate of the error variance and a negative relationship an overestimate of the error variance.

3.5.4 Heteroscedasticity and Autocorrelation

There is no reason to believe that econometric problems occur singly. If we have both autocorrelation and heteroscedasticity not only is the covariance matrix nonscalar but the off-diagonal elements are nonzero. The effects of autocorrelation and heteroscedasticity are complementary for cases in which the size of explanatory variables and error terms are positively related. Both problems cause understatement of the true error variance and usually produce incorrectly narrow confidence intervals or hypothesis rejections.

Epps and Epps found that the Durbin-Watson statistic is not sensitive to the presence of heteroscedasticity in that neither the size or power of the test is adversely affected. Koerts and Abrahamse showed that the modified VonNeumann ratio is superior to the Durbin-Watson statistic under some circumstances, since it rejects the null hypothesis correctly more often than the Durbin-Watson. The two statistics are closely related; the Durbin-Watson is (n-k-1)/n times the modified VonNeumann ratio; see Theil for additional details.

Autocorrelation does cause serious problems in the application of some frequently applied tests for heteroscedesticy such as the Goldfeld-Quandt and the Glejser tests. This was especially true with negative autocorrelation. The two tests for heteroscedasticity became valid again where autocorrelation was tested first and, if present, correlated using the Cochrane-Orcutt procedure.

Application of Coefficient Restrictions

In estimating the translog joint cost function, the logical order is to begin by estimating an unrestricted function - whatever we conceive the most general case to be. Secondly, we impose cost minimization by applying homogeniety of degree one in factor prices. Thirdly, we impose, homogeneity in output. Fourth, we combine factor price homogeneity and output homogeneity. Fifth, we can test for a Cobb-Douglas with constant returns to scale. Stages five and six are the appropriate place to test functional forms other than the Cobb-Douglas - the multifactor CES for example. If at any stage we are unable to show that a restriction is justified, we should accept the **unrestricted specification.**

Summarizing the restriction tests run by run:

1) General unrestricted translog;

2) Homogeneity in factor prices (cost minimization);

3) Production homogeneity of degree k(or 1);

4) Factor price and production homogeneity;

5) Cobb-Douglas technology with possible non-constant returns;

6) Cobb-Douglas technology with constant returns;

7) Other functions forms with and without Constant Returns.

Two levels of coefficient restrictions are involved in the estimation process: One, across equation coefficient restrictions that require certain coefficients to be equal in different equations; Two, the various theoretical restrictions described above the validity of which are tested with likelihood ratio tests.*

*The tests described here assume that each succeeding level is independent of the previous level. Some would argue that this is not the case. If each test is not independent of the previous level, a Bonferroni t test is called for in which the level of significance for the entire set of tests is chosen *a priori* and each step has a level of significance that is a fraction of the overall level.

COEFFICIENT RESTRICTIONS IMPLIED BY VARIOUS THEORETICAL PROPERTIES

COEFFICIENT SYMBOL	ASSOCIATED VARIABLE
α_i $i=1,\ldots,m$	OUTPUT Q_i
β_j $j=1,\ldots,n$	FACTOR PRICE w_i
δ_{ij} $i=1,\ldots,n$ $j=1,\ldots,n$	OUTPUT INTERACTION Q_iQ_j
γ_{ij} $i=1,\ldots,n$ $j=1,\ldots,n$	FACTOR PRICE INTERACTION w_iw_j
ρ_{ij} $i=1,\ldots,m$ $j=1,\ldots,n$	OUTPUT-FACTOR INTERACTION Q_iw_j

THEORETICAL PROPERTY	ASSOCIATED RESTRICTIONS
FACTOR PRICE HOMOGENEITY (COST MINIMIZATION)	1. $\sum_{j=1}^{n} \rho_{ij} = 0$
	2. $\sum_{j=1}^{m} \gamma_{ij} = 0$ 3. $\sum_{j=1}^{m} \beta_j = 1$
DEGREE OF CONSTANT	1. $\sum_{i=1}^{m} \alpha_i = k \text{ or } 1$ 2. $\sum_{j=1}^{n} \delta_{ij} = 0$ 3. $\sum_{j=1}^{m} \rho_{ij} = 0$

4. A REVIEW OF SELECTED EMPIRICAL STUDIES OF MOTOR CARRIERS

This section reviews a select few studies of motor carrier cost. Many studies were not included because there was insufficient difference between them and those studies included, or because they are outdated. A full reference is found in the bibliography.

o Klem

This study used the most complete data avaiable prior to the present one. Klem's data came from the ICC 1974 Form A annual reports for Class I and II Instruction 27 Carriers.

Klem estimated a long run total cost curve using firm-level cross section data. His method estimates the scale economies as the ratio of incremental to average cost; if this ratio is less than one there exist scale economies.

The sample consisted of data from 510 Class I general freight carriers that receive more than 75% of their revenue from interstate general freight carriage over a three year period and are required by the Commission to provide additional detailed operating data on a form known as "Instruction 27". These carriers are referred to as Instruction 27 carriers.

When the set of carriers is restricted to those that receive at least 99% of their revenues from intercity general freight, 359 carriers remain in the sample; in addition, firms were also classified according to whether they carry principally truck load (TL) or less than truck load (LTL) traffic, and by ICC region.

Klem used number of shipments as an output measure. The independent variables included length of haul, shipment size, number of pick-ups and deliveries, and a dummy variable representing the geographical location of the firm. The shipment size coefficient was not reliably distinguishable from one, indicating no scale economies. Klem argues that any possible errors (such as measurement errors) would bias the estimates upward, so that the true values should be closer to unity then the estimated values.

The sample data was divided into TL and LTL classes and used to estimate separate cost functions by category; there were 141 LTL and 91 TL specialists. The TL cost function showed that their cost structure was similar to that of the aggregate sample; average cost was constant with respect to scale increases. The LTL regression showed that LTL specialists enjoy modest, but statistically significant, scale economies; for the LTL group incremental cost was found to be 94% of average cost.

Klem also classified firms by geographic area and included a corresponding dummy variable for this purpose. The appropriate covariance test showed no significant difference between geographical regions despite the fact that some firms had revenues that were twice the sample average. The larger LTL group showed smaller but statistically significant scale economies; Klem argues that this could be interpreted to mean that some firms in this group are larger than the optimum size.

o Roberts

This older study of motor carrier cost used a sample of 114 Class I, general commodities, carriers operating principally in the ICC Central Territory, which includes Indiana, Illinois, Ohio and the lower peninsula of Michigan.

Roberts used average cost per vehicle as the output measure, which -- it is argued -- has less sensitivity to varying traffic characteristics than other measures. For example, average cost per vehicle mile is significantly influenced by the percentage of total traffic that passes through a terminal. The accounting system has a terminal cost category that represents an average of 15% of total operating expenses; these were excluded from Robert's cost computation.

Another problem is the choice of a scale measure - Roberts chose total assets, because an accounting analysis led him to conclude that motor carriers did not have scale economies, but that there was "... a devious relationship between efficiency and financial health which renders motor carrier rate structures highly suspect and suggests serious maladjustments in rate-cost relationships."

o Emery

Emery's study of scale economies is similar to Robert's in many respects; using an accounting approach to the scale economies question, he concludes that:

> "Although many writers in the field of transportation discuss the possibility of the existence of scale economies in the various modes of transportation, few adequately define the term. Many avoid this defintional problem by speaking of diminishing returns to scale or of economies of large size. Even the terms used often are not properly defined and described. Much of this ambiguity is due to the fact that economists often are not in agreement as to the validity of this concept. The concept of economies of scale, as commonly interpreted by economists, implies not only the fact that there may exist certain economies resulting entirely from the size of operations for any given firm. Herein lies the trouble. Many feel that there is no limit, while others feel that there may be but if there is we will never be able to define the precise limit."

He contends that may writers confuse the concepts of limits on firm size with limits on plant size, and that, in addition to other objections listed above both of these are static and short-run concepts. In regard to short-run firm operations, he argues that for each fixed plant size there exists some efficient level of operations. Most common scale economies discussion begins with the idea of increasing a fixed plant size; if there exists some absolutely efficient plant size, it may never be known because we can only observe the effects of relative size. Most discussions of scale economies focus on relative size, although this distinction is often subsumed. It may be possible to infer from the data that some firm size is efficient, but Emery contends that this has little to do with "absolute" scale economies. He argues that it is insufficient to show that large firms are more efficient than small ones, because there may exist unobservable factors - other than firm size - which contribute to this apparent relative efficiency.

TABLE 2. SUMMARY OF SELECTED MOTOR CARRIER STUDIES

AUTHOR	DATE	TYPE OF STUDY	DATA BASE	TESTS OF SIGNIFICANCE	METHOD	FUNCTIONAL FORM	ESTIMATION METHOD	OUTPUT MEASURE	CONCLUSION ON SCALE ECONOMICS	INDEPENDENT VARIABLES	METHOD OF SCALE MEASUREMENT
Chisholm	1961	Cost	British milk processing firms milk collection vehicles	t, F	Single Equation Econometric	Cobb-Douglas	OLS	Gallons per vehicle mile	None	Gallons per vehicle mile, gallons per form number of vehicles as a percent of expected number	Coefficient of number of vehicles as a as a percent of excepted
Emery	1965	Cost	1964 Cl. I&II Gen. Frt. M.C. in Middle Atlantic Region	NA	Accounting	Linear *	NA	Ave. cost per ton mile	Yes	NA	Cost per ton mile
Roberts, Merril J.	1955	Cost	1954 Cl. I Gen. Frt. carriers in ICC Central Territory	NA	Accounting	Linear	NA	Ave. cost per vehicle	None with reservations	NA	Comparability of cost by size class
Warner	1962	Cost	1955-60 cross section time series for 72 Cl I&II firms	t, F	Single Equation, Econo-metric	Cobb-Douglas	OLS, with correct-ion for errors in variables	Shipments	Yes, small economics of scale	Shipments, at per shipment wt.	Coef. of shipments
Ladenson and Stoga	1974	Cost	1972 cross section of Cl I firms observations 116	t, F	Single Equation Econo-metric	Cobb-Douglas	OLS assum-ing profit maximum error	Ton-miles	No, in general yes for small firms	Ton-miles, labor capital and log of capital	Coefficient of log K is equal sum of labor and capital coef minus 1
Klem	1977	Cost	1974 cross section Cl I&II, I-27 Gen Frt carriers 518 observa-tions	t, F	Single Equation Econo-metric	Cobb-Douglas	OLS	Shipments	None or for most small for some LTL specialists	Shipments ave. haul, ave. wt. of shipment	Ratio of marginal to ave cost (output coef)
Lawrence	1973	Cost	1973 Cl I&II Gen Frt Carriers 236 Observations	t, F	Single Equation Econo-metric	Cobb-Douglas	OLS	Tons	Yes	Tons, distance ave. load 90 LTL, ave.wt. LTL shipments plus dummy for large cities in one variation	Coefficient of tons
Ayala Oramas	1975	Cost	1967-1972 data on Gen Frt and special commodity carriers	t, F	Single Equation Econo-metric	Cobb-Douglas	OLS	Revenue Ton-miles	None or smal.	Factor series and output	Coef. of ** output
Spady and Friedlænder	1978	Cost	1974 cross section 125 firms	Liklihood ratio, t, F	Simultanous Equation Econ-metric	Translog Joint cost function	FIML**	Hedonically adjusted ton miles	None	Tonmiles maintenance line haul terminal traffic Inf and Safety Adm & General, Deprecia-tion Taxes & Liscenses	Output coefficient

*Accounting methodology implicitly assumes that cost relationships are linear
**Full Information Maximum Liklihood

Emery's study used 1960 operating data for 233 Class I & II firms, the Middle Atlantic Territory. Averages were used to represent the various cost categories. Using total revenue as a measure of firm size, 7 groups were identified; the larger carrier groups had longer average hauls and higher load factors than smaller firms but there was no statistically significant relationship between these characteristics and the carriers' net profits. The operating ratios of carrier groups decreased slightly with carrier size, and larger carriers earned slightly greater profits per dollar of sales revenue. Capacity utilization was evaluated using average leased and owned vehicles on hand, divided by total vehicle miles during a year, giving the average mileage per vehicle; larger carriers appeared to be more efficient using this criterion. Part, but not all, of this increased vehicle utilization was concluded to be due to the longer average haul associated with larger carriers.

An examination of various motor carrier cost categories as a percent of total revenue, produced three conclusions:

1. Adminstrative and general expense ranged from - 11.4% for the smallest group to 5.8% for the largest;
2. Total fixed expenditure declined from 27.5% for the smallest group to 19.6% for the largest;
3. Transportation expenses, maintenance, and depreciation declined from the smaller to the larger carriers.

No attempt was made to determine if there were statistically significant differences in these ratios and this is a major flaw in the methodology. The wage bill declined as carrier size increased, but since larger carriers use more "leased" drivers, this expense category seems to be of equal importance for all carrier groups after adjustment for leasing. Emery concludes by saying:

> Analyzing the evidence presented, representing the cost study carrier operating statistics submitted to the Interstate Commerce Commission, there appear to be considerable scale economies among these carriers. This empirical evidence in addition to the commonly accepted advantages of large-scale operations (i.e., organizational advantages, specialized labor, better qualified managerial talent,

quantity discounts on particular purchases, and access to lower financing costs) presents a rather strong case for the existence of relative scale economies within this industry. Assuming that a carrier has not reached its optimum size to serve its present market, or that it contemplates increasing its market size, then the advantages from larger size appear to be significant.

o Warner

Warner used a combined cross-section of time series data set approach to estimate a Cobb-Douglas cost function as a measure of output Warner uses shipments. The coefficient of log shipments is the scale parameter and very mild economies of scale were found. This study is very similar to those by Lawrence and Klem even though it is much earlier than both of those studies.

o Lawrence

This study uses a group of Class I and II general freight carriers to examine the question of economics of scale. Lawrence has been a principal spokesman for the industry in arguing that there do exist scale economics in regulated motor carriers, particularly LTL specialists, and that consequently deregulation would lead to increased concentration. A principal focus of the study is the contention that previous studies failed to include firms that were sufficiently large to be representative of the larger firms in the industry. He also argues that there is a problem of heteroscedasticity because of error correlation with form size, although apparently no tests of hypothesis were performed. The response to the perceived problem of heteroscedasticity was to partition the sample and estimate separate cost equations. This situation is the appropriate one for use of tests of aggregation although apparently this was not done. In summary this study is very much in the same genre as the studies of Ladenson and Stoga, Klem and Warner. It is distinguished from those by a possibly inappropriate partitioning of the sample.

o Ladenson and Stoga

In a study of scale using the production function approach economies, Ladenson and Stoga estimate a Cobb-Douglas production function with cross-section data. The production function was specified in two different ways; either as output per unit of labor, or output per unit of capital. The authors advance several arguments for specifying the equations in this manner, even though the estimated parameters are theoretically equivalent in both.

The difference between the usual Cobb-Douglas specification and theirs is that they include among the independent variables the log of the variable by which output (i.e., output per unit of labor or capital) is divided. This makes the coefficient of the dividing variable equal to the coefficient of capital (or labor) plus the coefficient of labor (or capital) minus one, and this is provides a check on returns to scale estimates.

These functional forms impose a unitary value on the scale parameter that is equal to the sum of the coefficients of labor and capital. Neither specification allows a test of the hypothesis that returns to scale vary with firm size, but this may be done using size class dummy variables. Specifications with dummy variables do not permit direct tests of the hypothesis of equal factor input coefficients across size class. However, the authors do specify the estimating equation so such a test may be derived. Because of extreme collinearity among the variables, regression specifications in which output was expressed in per-unit labor terms were impossible to estimate.

Firms were initially divided into ten classes by sorting on the number of employees per firm. The size classes range from firms with less than 50 employees to those with more than 2,000 employees. No significant differences in the firm size coefficients appeared until firms were aggregated into two classes: those which have less than 50 employees and more than 50 employees; other taxonomies made no significant difference.

The authors also tried alternate definitions of labor and capital to test which was more "acceptable", than others and they experimented with various output measures. They concluded that, while ton miles is not the ideal measure of output, it is a "good" approximation of the "true" measure of output; they also concluded that firms with as few as 50 employees enjoyed increasing returns to scale. The estimated scale parameter was constant over a wide range of firm sizes, 50 to 2000 employees and more, and consequentially the authors conclude that firms have not achieved their optimum size, and that the optimum size is larger than other studies have suggested.

o Ayala-Oramas

Ayala-Oramas identifies two classes of scale economies: 1) Returns to traffic density, or "economies of utilization"; and 2) Returns to increased firm size--the usual scale definition. He recognizes that single-equation models have various limitations and makes some suggestions for improving single-equation cost models, but still uses a single-equation for estimation.

A separate set of equations is used to examine economies of utilization, and economies of scale in motor carriers; additional equations relating load factors to route characteristics and short-run output to long-run output are included. He finds no statistical evidence of economies <u>after</u> difference in capacity utilization are taken into account. He concludes that observed between-firm differences in scale economies and capacity utilization are due to the firm's scale of operation and the characteristics of the firm's service network.

Ayala-Oramas concludes that large firms may be able to get routes with more desirable network characteristics than small firms and that this explains the high load factor commonly associated with the larger firms. He recognizes there may exist difficult-to-quantify cost barriers that result from uneven economies of utilization across networks, and argues that this tends to increase concentration and differentiated growth over time:

> The apparent discrepancy between previous cost studies indicating constant return to scale and studies indicating minor but statistically significant increasing returns, can be resolved once we realize that there are economies

> of utilization and that such economies may be associated with the scale of operations, though there may not be economies or diseconomies of scale in the production of transportation capacity.

This could explain the significant scale economy variable for LTL carriers found in Klem's study.

He feels that since rates are set through rate bureaus under regulation, regulated rates are typically higher than unregulated rates. If the service differential opposed by regulated carriers is worth the rate differential, higher rates could be justifiable. However, there is no evidence showing that service differentials do justify higher rates since, where alternatives exist, shippers often choose private, exempt and illegal motor carriers over regulated carriers.

Ayala-Oramas classifies inter-city motor carrier output into three categories: pickup and delivery, line haul, and terminal operations. Pickup and delivery and line haul output have the same units of measurement, but pickup and delivery ordinarily uses less powerful trucks and line haul operations. The line haul rolling stock are composed of individual production units and that represents a small capacity with respect to market demand. He argues that there are scale economies to increased vehicle size which in turn decline as vehicle size increases. He also argues that there is some <u>ex ante</u> substitution possible between labor and capital (the classes of rolling stock and terminal facilities) but that <u>ex post</u>, the capital-labor ratio is changed primarily by varying capacity utilization.

Transportation output cannot be stored, is multi-dimensional, and has associated with it a joint product--the backhaul. Because of these characteristics, he argues that the long run, line-haul cost function for motor carriers should be linear and output separable. Line-haul costs amount to between 30 and 45% of total trucking expenses (up to 80% in specialized cases), and they can be divided equally between fixed expenses and running costs. Ayala-Oramas notes that

> Instead of trying to trace variations in total cost, the practical analysts have attempted to determine directly the increase in costs that would occur if output of one product were expended by one unit, and production of other items remains constant, or if one extra ton is moved over the same mileage, all else remaining constant. Unfortunately, this method is not valid with the multi-products, made in more or less fixed proportions with forward trips; it is only approximate in the data of output dimension given the high aggregation time of the available data.

Quality in transportation output is the quantity of "intrinsic characteristics" embodied in motor carrier output(s) which affect production cost. For regulated transportation industries, quality is not a predetermined variable; instead, the price-capacity decision determines quality. This problem is intensified because cost minimization of a given output is not dual to profit maximization, with respect to quality, given prices and quantities.

Ayala-Oramas distinguishes three market equilibrium models that lead to three separate cost specifications:

1) Cost is a function of output quality and output price
2) Output is a function of output price, quality and an exogenous demand shift variable;
3) Quality is a function of output price, input price, and an exogenous demand shift variable.

In the first case, the cost equation is under identified and ordinary least squares estimation of cost parameters is biased and inconsistent. In the second case, the cost function is identified, but the parameters of the production function may not be recovered from the cost function parameter estimates. In the third case, identification of production parameters from cost parameters would also be difficult.

o Johnson

Johnson combines cross-section and time series data to estimate farm cost functions, and uses analysis of covariance (ANCOVA) to avoid the "regression fallacy" or errors in variables.

He also argues that statistical cost curves may be under-identified because of Friedman's objection (discussed below).

The ANCOVA approach attempts to account for the variation in random variables using observable independent variables. This method seeks to find cost-output relationships corrected for differences in time and location. The pooling time series and cross-section data uses cost data over firms and time allows the introduction of dummy variables for firms and time which sum to zero and provide identification of the equation. For example, if there exists N firms and T years, then it will be necessary to estimate N + T + 2 parameters from N · T observations.

If the underlying assumptions are met, the cost function estimates have the following properties:

1) Persistent, time-related deviations from true cost for a given firm are removed from the estimated cost equation.
2) Deviations from true aggregate cost common to all firms specific to a particular year, are estimated; this prevents changes in the output price levels over time from biasing the coefficient estimates. Likewise, year to year changes in factor prices will not affect the coefficient estimates. Differences in price level are captured by the firm dummies, and differences in factor price changes are captured by the yearly dummies.

Short-run cost functions have traditionally been estimated using time series data, while long-run functions are usually estimated using cross-section data. The regression fallacy may occur because the output variable has observation error associated with it and Johnson argues that this problem may be avoided by combining cross-section and time series data.

However, pooling fails to satisfy Friedman's objection. Friedman argues that cost functions are under-identified because we assume profit maximization. But if firms face identical factor prices--as we would expect under competition--and all firms have identical production functions, then cost function estimates either have no variability, or are constant by the assumption of equal factor and output prices; if this were not true, all firms would not have identical cost functions. This objection can be answered by assuming that the equilibrium conditions have stochastic error terms.

Johnson estimates farm cost function, but the methodology has potential application in motor carrier research because of the suggested combination of time series and cross-section data. This study is outdated now, but deserves inclusion because it addresses the problems of pooling cross-section and time series.

o Spady-Friedlaender

Prof. Ann F. Friedlaender was a 1976 recipient of a three-year University Research Contract from DOT; the product of that contract has been a large number of scholarly papers. This research has produced econometric models of motor carrier and rail freight operations linked to a regional economic model; this framework will then be used to conduct policy experiments under various assumptions.

The most important contribution in the motor carrier field--(this also applied to rail)--was that of adjusting output for its inherent characteristics. Friedlaender and Richard Spady developed this idea into a full scale empirical procedure and established that its use produces better (as judged by a likelihood ratio test) estimates of a motor carrier cost function.

Initially, Spady and Friedlaender assumed that motor carrier cost functions were quality separable. This means that the effect of quality variations on output and cost, is independent of the relative prices of factor inputs so that the factor inputs combine to produce units of output called effective ton-miles of specific commodities. The effective ton-miles can vary depending on the combinations of ton-miles in "natural" units (unadjusted for quality). The quality separable assumption implies that the cost minimizing bundle of factor inputs does not depend on the composition of effective output; that is, factor prices do not affect the combinations of natural ton-miles, average hauls and average loads that can be produced for an equal outlay and equal values of other characteristics. The assumption of quality separability is prudent from a practical standpoint. To assume otherwise would require treating each characteristic as a distinct output and testing quality separability.*

The only drawback to the Spady-Friedlaender work is that they had to use TRINC's data, and consequently were unable to specify more than a single output; the hedonic specification may however substitute cost function used five characteristics in the hedonic output equation and four inputs.

These characteristics are average shipment size, average haul, percent LTL, insurance cost per ton-mile and average load. They are labor, fuel, capital and purchased transportation. They rejected separability (homotheticity) in both hedonic and non-hedonic specifications, and also rejected the hypothesis of constant returns to scale in both versions. The hedonic specifications proved to be uniformly superior. There were large economies of scale in the nonhedonic specifications but mild <u>diseconomies</u> of scale in the hedonic specification. The indication of diseconomies of scale was accepted, with the caveat that returns to scale cannot be uniformly characterized for nonseparable technologies.

They also computed the Allen-Uzawa elasticities of substitution for both hedonic and non-hedonic specifications and found that all factor inputs were substitutes (elasticities positive) and that the hedonic specification produced significantly lower elasticities than nonhedonic specifications; the same relationship held between the values of factor price elasticities.

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*Spady tested the assumption of quality separability with his own version of FIML and found that it could not always be justified.

In summary, Spady and Friedlaender estimated a long-run cost functions for the general freight motor carrier industry using a translog, hedonic specification. Their research is too varied to characterize simply, but they made a significant contribution to motor carrier research because of the translog joint cost specification. They show that the Cobb-Douglas or CES functions (cost or production functions) are inappropriate. The only significant criticism of the Spady-Friedlaender work is that they specify only a single output. Overall, this work is excellent and economists owe them much for the thorough theoretical basis they have given transportation cost function work.

Critique of Empirical Estimates in Motor Carriers

Estimates of motor carrier cost and production technology have, in the past, generally been characterized by the use of single equation, ordinary least squares methods. We can argue that the cost functions are identified because they are reduced form equations derived from the simultaneous solution of the necessary conditions for a profit maximizing firm. These studies would have benefited from a more explicit mathematical derivation of the cost functions for a regulated firm, so that efficient estimation, hypotheses about possible regulatory effects and the underlying technology could have been formulated.

The production function estimation could easily incorporate the dummy variable procedure used by Klem and Johnson, and use a time series of cross-section data for estimation purposes. Examples of theoretical work from which these suggestions arise are Hall, DeVany, Ayala-Oramas, and Anderson.

A CRITIQUE OF ICC COSTING METHODOLOGY

One of the more serious difficulties with ICC methodology is the failure to account for capacity utilization of plant and equipment. It is based on the historical operating characteristics of firms that reflect a capital equipment utilization that itself resulted from ICC policies. In addition, ICC costs do not permit the separation of cost changes that result from changes in factor prices; there is no way to sort cost changes by causality.

Also, the methodology does not account for the value to the firm which arises from regularity of service. Seasonal users of trucking services may require large investments in motor carrier equipment than shippers facing more uniform demand.

Costs from ICC Form A are average costs, and while some regional adjustments are possible, if there exist joint or common costs, average costs can't be determined with certainty because this requires an (arbitrary) allocation of the joint costs. Therefore, the average costs of particular movements can't be determined _except_ on an arbitrary basis. This also holds true for the allocation of capital costs; these are not separated by the accounting practices so they can be non-arbitrarily included in average costs.

Joint and capital cost allocation, are among the most troublesome cost categories in trucking. To allocate capital costs or joint costs, a general inventory and evaluation method is necessary. Even if this were feasible--and it is not because of the large number of firms in the industry--the results would be sensitive to the choice of the accounting method.

Average cost data distorts the actual costs of several different types of service. For example, actual costs are usually _higher_ than average for small loads and average capacity utilization, for irregular service, and for service with seasonal or daily peaks. Actual costs are lower than average for multiple shipments and truckload movements, regular traffic, and off-peak movements. Because of the failure to account for capacity utilization, acutal costs are distorted in ways that cannot always be predicted _a priori_.

The ICC cost studies have often attempted to examine the question of increasing returns to scales. Since increasing returns imply decreasing long-run average costs, the usual method is to examine the scale economies questioned by measuring the percent variable cost. The purpose of percent variable cost, (defined equivalently either as incremental cost divided by total cost, or ratio of marginal cost to average cost, or the elasticity of total cost with respect to a change in output) to measure the returns to scale.

While total cost functions may be curvilinear, the ICC assumes that cost functions are linear, and this implies that the long-run cost function is a straight line with origin at zero. In the short run, with fixed costs, the function may have a positive intercept. (The intercept must be nonnegative because it would otherwise imply negative fixed cost-a subsidy.) For linear functions marginal cost is constant, independent of the output level, equal to the slope of the function; average cost declines continuously with the output level. In the short run, percent variable cost will be equal, greater or less than one hundred, depending on whether the intercept is positive or zero. In either case, percent variable cost is variable cost and is not a function of the level of output.

Because of these problems it is important to determine what level of output is assumed for the cost under discussion. Therefore, if average cost is to be measured or if cost is to be measured for some "average" level of output, it is important to specify this level of output.

The Commission's cost methodology aggregated annual report data from both large and small firms, to determine the percent variable cost. By aggregating over firm size, considerable distortion is introduced, and it is not possible to discuss the accurate measurement of percent variable cost without specifying both average firm size and traffic density.

As Griliches notes, if an econometric (or accounting) cost relationship is a good representation of true cost it must have four characteristics:

1) It must be defined correctly.
2) The variables must be measured in relevant units.
3) It must include all of the important variables.

4) The excluded variables must be distributed randomly about zero and uncorrelated with levels of included variables.

No study has been carried out by the ICC Cost Section to determine whether these criteria are satisfied.

There is also the problem of untested aggregation over outputs; for example, pickup and delivery, terminal operations and line haul operations. What we would like to do is estimate the long-run relationship between output and cost. For this purpose, cross-section data is preferred to time series because it is less affected by short-run and transitory phenomena than time series.

If there exists a transitory and permanent part to cost, there will be a bias in the elasticity estimates unless the transitory cost portion is extremely small. If it is not small the estimated elasticity of output with respect to cost will be biased downward.

In summary, the ICC methodology has seven major faults: (1) It does not account for between-firm differences in capacity utilization of plant and equipment; (2) It does not allow the separation of changes in cost which result from changes in factor prices; (3) It does not allow us to compute the value of regularity of service to the firm. (4) It focuses principally on average costs. (5) It does not deal with cost changes as network density increases, or with the allocation of capital costs; (6) The assumption of linearity imposes a straight line total cost function on the data without any examination of whether or not this assumption is correct; (7) It assumes separability. Variable cost would likely be underestimated using standard ICC cost methodology.

5. THE DATA

Regulated motor carrier data has always posed a problem to researchers because of its quality and availability. The ICC annual report data is published in summarized, aggregate form (as Part 7 of The National Transportation Statistics) without many of the details desirable for economic analysis. This data is often unavailable for several years after it is collected. The other major source is TRINC's, which also publishes firm-level data but is also aggregated excessively for many purposes. Many published studies use TRINC's data simply because it is widely available and easy to access. The American Trucking Associations publish Financial and Operating Statistics (F&OS) for Class I regulated carriers on a quarterly basis, but these data are also aggregated excessively.

The ICC has prepared annual reports for Class I & II carrier data in machine readable form since 1965. These data have not always been easily accessible principally because: (1) There is an enormous quantity of data; (2) The data processing characteristics of the data are not consistent from year to year; and (3) The computer tapes were not readily accessible physically because of the expense of collecting and organizing them. Further, the data are not systematically audited for accuracy so that in reading the tapes it is impossible to discriminate between sources of error. For example, it is common to update tapes by adding the updated data at the end of the tape rather than with the records being updated.

The data consist of a panel of data for 252 Class I, general freight motor carrier firms that existed continuously during the ten year period from 1965 to 1974, making 2520 observations per variable. The study firms are also Instruction 27 (I-27) carriers, which means that they derive 75 percent or more of their annual revenues from intercity freight operations. These firms are required to report additional statistics on their operating characteristics (such as number of truckload (TL) and less than truckload (LTL) shipments) that are extremely useful in economic analysis.

Because of the data problems, no previous study used time series of cross-section (panel) data, although Klem used several subsets of the 1974 Class I and II data; Friedlaender and Spady used TRINC's data as did Ayala-Oramas.

Another obstacle to the use of the data is the change in account definition from year to year. Between 1965 and 1973 there were only minor changes in Motor Carrier Form A, but in 1974 Form A was changed substantially. This required a major reconciliation of the accounts to achieve comparability of data for 1965-1973 with that for 1974.

The factor price and output variables used to specify the model were constructed from approximately 50 ICC accounts or line items on the annual reports. The inputs are represented by factor prices and the outputs are constructed from operating statistics.

SPECIFICATION OF THE COST FUNCTION

The translog joint cost function allows the specification of functions with multiple inputs and outputs. Because the translog functions requires numerous terms for each possible combination, this can be burdensome as well as advantageous as it requires large numbers of coefficients. For example, Table 3 shows the various numbers of coefficients that would result by specifying different quantities of inputs and outputs. The model has four outputs and nine inputs, leading to 105 coefficients in nine total equations (assuming symmetry).

The output variables are:

1. Q1 = number of truckload (TL) shipments--TL output.
2. Q2 = number of less than truckload (LTL) shipments--LTL output.

3. Q3 = tons per day of pickup and delivery output.

4. Q4 = tons of freight--terminal platform output.

The factor price variables are:

1. W1 = salaried, clerical and other labor wage
2. W2 = linehaul wage
3. W3 = pickup and delivery and terminal platform wage
4. W4 = other inputs not elsewhere classifed price
5. W5 = purchased transportation price
6. W6 = owner-operator compensation (including tractor rental)
7. W7 = materials--tires, oil, lubricant price
8. W8 = fuel price
9. W9 = capital services price.

The output variables were taken from Schedule 9003, while the input prices were constructed in two ways. Labor categories use wage rates constructed from expense and quantity data (person hours worked) from Schedule 9002. The labor categories were aggregated using a Divisia index of inputs that were divided into the total factor expenditure $w_i x_i$ to estimate the wage rate.

Non-labor input variables present a more difficult problem because there is no information available on quantity. The exception to this is fuel. Total fuel expenditures are available, and the number of gallons of fuel used can be estimated by tallying the federal fuel tax dollars and dividing through by the tax per gallon. (Fuel tax expense - tax rate x gallons, therefore, gallons = fuel tax expense/federal tax rate.)

Factor prices are not available for the other category, purchased transportation, owner-operators, tires, oil and lubricants (materials) and capital services price. Leaving capital price until last, we will discuss an alternative method to estimate factor prices when quantities are unobservable.

Diewart has shown that either price or quantity indices with highly desirable, or superlative to use his terminolgy,

TABLE 3. NUMBER OF TRANSLOG COEFFICIENTS RESULTING FROM VARIOUS COMBINATIONS INPUTS AND OUTPUTS*

Total Number of Coefficients = (m + 1)(m/2)(n+1)(n/2) +mn +m +n + 1

Total Number of Equations = Total Cost function + (n-1) factor share equations

OUTPUTS m	INPUTS n	TOTAL COEFFICIENTS	SYSTEM EQUATIONS
1	4	21	4
1	6	36	6
1	8	55	8
2	4	28	4
2	6	38	6
2	8	48	8
4	4	45	4
4	6	66	6
4	8	91	8
4	9	105	9
4	12	153	12

* Assuming Symmetry which means that coefficient $\theta_{ij} = \theta_{ji}$. Since the coefficients are interpreted as partical derivatives, the ij th must equal the ji th by Young's Theorem (discussed in Section 1.0). These numbers include all coefficients - the number actually estimated can be reduced using Young's Theorem and imposing cost minimization as a maintained hypothesis and further reduced if constant returns can be imposed.

properties may be estimated using a translog "aggregation" function. (For a detailed description see Diewert, or Spady and Friedlaender). We discuss aggregation by both the use of Divisia indices and estimated factor prices; the two approaches are theoretically equivalent.

The Divisia Index

The naive approach to aggregation is to add cost categories together and divide by an aggregate sum of quantities to get an "average" factor price. This is inappropriate because of the average factor does not accurately measure the true factor price or quantity. What does "accurate" mean? For a cost function, accurate means one that gives identical measures of total factor expenditure regardless of whether a factor price index or a factor quantity index is constructed. For example, let

$$C_1 = g(h(Q),w) \qquad (5.1)$$

and

$$C_2 = g(Q,h(w)) \qquad (5.2)$$

then the index is "accurate" if and only if $C_1 = C_2$.

The appropriate index has the following properties:

1) If factor prices double, the index doubles (and likewise for quantities).
2) The index number between any two periods is independent of the choice of the base period.
3) The index must be independent of the units of measurement of prices and quantities.

One class of functions with these desirable properties are homothetic aggregation functions. (Homothetic functions are monotonic transformations of homogenous functions.)

Suppose we need a factor price index, how is one constructed? The necessary elements are:

1) total expenditures for a factor.
2) quantities of each of the subgroups to be aggregated over time or (for cross section data) across firms or both.

To get an index of the unobservable factor prices, we must define a quantity index and divide factor cost ($w_i X_i$) by that index

$$\tilde{w}_i = \frac{w_i X_i}{\tilde{X}_i} \qquad (5.3)$$

where actual X_i is unobservable.

The dependent variable in the factor share equations is the share of that factor input in total expenses; $w_i X_i / C = M_i$. The capital share equation was omitted because one equation must be omitted to avoid a singular error matrix. The estimated equations are shown in Table 10. The variables are in logarithmic form. Each equation has the mean of each variable factor subtracted from it. This is called mean-centering and defines the point of approximation.

The coefficient restrictions are applied implicitly by "omitting" a variable and subtracting the omitted variable from all other variables in the equation, which is equivalent to dividing out of the logs.

The econometric factor price estimation has as the dependent variable the percentage that a "micro" factor represents of total expenditure in a micro-group. The specification for this procedure is*

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*An often neglected component of production technology overtime is technological change. Omission of technological change parameters is a misspecification and leads to biases in parameter estimates. Many production studies assume that technological change is Hicks neutral and proceed to estimate technological change as a constant exponential function of time. This means that (see Gollop and Hnylicza) we can write,

$$X_j^* = X_j \, e^{\mu_j t}, \qquad (A)$$

where X_j^* = jth input in <u>efficiency</u> units

X_j = jth input in <u>natural</u> units

μ_j = rate of technological augmentation or change for jth input

t = time

(footnote continued.)

$$\frac{w_i X_i}{wX} = \log \tilde{W}_i - \lambda_i T + \log g\,(Q_i) + e$$

where

$w_i X_i$ = factor input bill for factor estimated

$wX = \sum_{i=1}^{m} w_i X_i$ all expenditure for a class of input input to be aggregated

$\log \tilde{w}$ log of estimated factor price

λ_i = rate of price diminution--the negative of the rate of factor input technological change.

T time

$g(Q_i)$ = a translog--specified set of proxies for quantity.

The factor price estimate required is obtained by exponentiating the intercept and factor.

That is $we\text{-}\lambda_i \tau = \exp(\log\tilde{w}_i - \lambda_i \log T)$

Factor prices estimated in this manner are assumed to be constant across firms as are the rates of price diminution. Individual factor prices could be estimated for each firm and year by specifying dummy variables. This was not done because of the large number of dummy variables that would have to be estimated. (It would require 2520 dummy variables to get a separate factor price for each year and firm.) The estimation of the factor price with the rate of price diminution gives variation across time but not across firms. See Spady and Friedlaender (1977) or Diewert (1976) for further discussion of this procedure. The price diminution parameter--is discussed by Hnylicza. The procedure suggested in Hnylicza is more complicated. It requires appending

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*(Cont'd)
Technological change reduces the effective cost of the factor inputs, so we may treat it as a cost reduction,

$$w_j^* = w_j e^{-\lambda_j t} \qquad (B)$$

The rate of factor augmentation must equal the rate of effective input cost reduction, the observed value of a factor share

$$w_j X_j = w_j^* X_j^* \qquad (C)$$

Since

$$-\mu_j = \lambda_j .$$

TABLE 4. FACTOR SHARES

Share	Symbol
1. Salaried Clerical, other	M1
2. Line haul	M2
3. Pickup and delivery, terminal and platform	M3
4. Other inputs, NEC	M4
5. Purchased transportation	M5
6. Owner operator	M6
7. Materials, tires, oil and lubricants	M7
8. Fuel	M8
9. Capital	M9

a 'share' of technological change equation to the model set. The procedure used here estimates factor price diminution outside the main model by estimating each factor price in order to get the rates of price diminution. Even though the estimated factor prices were not used in every case, the price diminution factors were used with the actual price when these could be computed.

Table 6 lists the correspondences between the theoretical functions specified in Section 1 and 2 and the estimated functions. The theorectical sections use greek letters while the econometric specification use English mnemonic equivalents (where feasible). The independent variables themselves use their own means as the points of approximation, although this is not explicitly in the econometric specification.

Appendix I describes the data from which the variables in the models were formulated.

TABLE 5. DEFINITION OF OUTPUT AND FACTOR PRICE VARIABLES

Output	Symbol	Source
• TL Shipments	Q1	Schedule 9003
• LTL Shipments	Q2	Schedule 9003
• Ton per hour pick up and delivery	Q3	Schedule 9003
• Tons, terminal-platform	Q4	Schedule 9003

Factor Price	Symbol	Source
• Labor, salaried-clerical, other	W1	Constructed by Divisia Index
• Labor, linehaul	W2	Constructed by Divisia Index
• Labor, pickup and delivery plus terminal-platform	W3	Constructed by Divisia Index
• Other inputs not elsewhere classified (NEC)	W4	Estimated
• Purchased transportation	W5	Estimated
• owner-operator	W6	Estimated
• materials, tires, oil and lubricants	W7	Estimated
• Fuel	W8	Observed directly
• Capital	W9	Observed directly

TABLE 6. COEFFICIENT SYMBOLS AND VARIABLES

Coefficient Symbol	Variable*
α_0	intercept
α_1	Q_1
α_2	Q_2
α_3	Q_3
α_4	Q_4
δ_{11}	$1/2\ Q_1^2$
δ_{12}	$1/2\ Q_1\ Q_2$
δ_{22}	$1/2\ Q_2^2$
δ_{31}	$1/2\ Q_3\ Q_1$
δ_{32}	$1/2\ Q_3\ Q_2$
δ_{42}	$1/2\ Q_4\ Q_2$
δ_{43}	$1/2\ Q_4\ Q_3$
δ_{44}	$1/2\ Q_4^2$
β_1	W_1
β_2	W_2
β_3	W_3
β_4	W_4
β_5	W_5
β_6	W_6
β_7	W_7
β_8	W_8
β_9	W_9
γ_{11}	$1/2\ W_1^2$
.	
.	
.	
γ_{99}	$1/2\ W_9^2$
ρ_{11}	$1/2\ Q_1\ W_1$
ρ_{49}	$1/2\ Q_4\ W_9$

*All variables are in log form conditioned on own arthmetic means

TABLE 7. COMPUTATIONAL METHOD FOR OMITTED COEFFICIENT VALUES ASSUMING MINIMIZATION

COST MINIMIZATION IMPLIES

1. $\Sigma\beta_j = 1$

2. $\Sigma\gamma_j = 0$

3. $\Sigma\rho_j = 0$

CONSTRAINT IMPLIED BY OMITTING VARIABLE j AND ALL COMBINATIONS THEREOF

THEREFORE

$$\beta_j = -\sum_{i=1}^{m-1}\beta_i \qquad \gamma \neq j$$

γ_{11}	*	*	*	*	*	*	*	$-\Sigma\gamma_{1j}$ †
γ_{21}	γ_{22}	*	*	*	*	*	*	$-\Sigma\gamma_{2j}$
γ_{31}	γ_{32}	γ_{33}	*	*	*	*	*	$-\Sigma\gamma_{3j}$
γ_{41}	γ_{42}	γ_{43}	γ_{44}	*	*	*	*	$-\Sigma\gamma_{4j}$
γ_{51}	γ_{52}	γ_{53}	γ_{54}	γ_{55}	*	*	*	$-\Sigma\gamma_{5j}$
γ_{61}	γ_{62}	γ_{63}	γ_{64}	γ_{65}	γ_{66}	*	*	$-\Sigma\gamma_{6j}$
γ_{71}	γ_{72}	γ_{73}	γ_{74}	γ_{75}	γ_{76}	γ_{77}	*	$-\Sigma\gamma_{7j}$
γ_{81}	γ_{82}	γ_{83}	γ_{84}	γ_{85}	γ_{86}	γ_{87}	γ_{88}	$-\Sigma\gamma_{8j}$
$-\Sigma\gamma_{j1}$	$-\Sigma\gamma_{j2}$	$-\Sigma\gamma_{j3}$	$-\Sigma\gamma_{j4}$	$\Sigma\gamma_{j5}$	$-\Sigma\gamma_{j6}$	$-\Sigma\gamma_{j7}$	$-\Sigma\gamma_{j8}$	$\Sigma_j^m(\Sigma_i^m\gamma_{ij}) = 0$

†Stars indicate variables omitted due to Young's Theorem (symmetry) so that $\sigma_{32} = \sigma_{23}$.

TABLE 8. CROSS PRODUCT COEFFICIENT RESTRICTIONS TO SATISFY THE CONSTRAINT $\Sigma\rho_{ij} = 0$

ρ_{11}	ρ_{12}	ρ_{13}	ρ_{14}	ρ_{15}	ρ_{16}	ρ_{17}	ρ_{18}	$-\sum_{j-1}\rho_{1j}$	$= 0$
ρ_{21}	ρ_{22}	ρ_{23}	ρ_{23}	ρ_{25}	ρ_{26}	ρ_{27}	ρ_{28}	$-\Sigma\rho_{2j}$	$= 0$
ρ_{31}	ρ_{32}	ρ_{33}	ρ_{34}	ρ_{35}	ρ_{36}	ρ_{37}	ρ_{38}	$-\Sigma\rho_{3j}$	$= 0$
ρ_{41}	ρ_{42}	ρ_{43}	ρ_{44}	ρ_{45}	ρ_{46}	ρ_{47}	ρ_{48}	$-\Sigma\rho_{4j}$	$= 0$

---

since $\gamma_{ij} = \gamma_{ji}$, $\Sigma\gamma_{ij} = \Sigma\gamma_{ji}$
so the final implied coefficient value in the lower right hand corner is the sum of the others previously calculated sums so as to satisfy the constraint $\Sigma\gamma_{ij} = 0$

TABLE 9. COST MODEL ARGUMENTS

Econometric Specifications	Theoretical Corresponsence	Explanation
A1 to A4	α_1 to α_4	Output Coefficien
B1 to B4	β_1 to β_4	Factor Price Coefficients
D11 to D44	δ_{ij}	Output inter-action Coefficient
G11 to G99	α_{ij}	Factor Price Interaction Coefficient
R11 to R49	ρ_{ij}	Output Factor Price Interaction terms
M1 to M9	M_i	Factor Shares
LG1 to LG4	$\log Q_i$	Log of outputs
LW1 to LW9	$\log w_i$	Log of factor prices
LW1.W1 to LW9.W9	$\log w_i \log w_j$	Factor Price Interactions
LW1.Q1 to LG4.Q4	$\log Q_i \log Q_j$	Output interactior
Q1.W1 to Q4.W9	$\log Q_i \log w_I$	Output factor price interactions

6. RESULTS AND CONCLUSIONS

Several results are presented in this section: (1) The estimates of factor prices and technological change rates; (2) Joint estimates of the translog cost functions; and (3) Inferences about the underlying technology implied by the estimated joint cost function.

Since this presentation is lengthy we offer a preview of those results:

(1) Class I motor carrier of general freight show no evidence of system economies of scale;
(2) We accept the hypothesis of cost minimization;
(3) Separability (homotheticity) <u>cannot</u> be accepted so that the production or cost functions such as the Cobb-Douglas and CES are <u>inappropriate</u> descriptions of technology in this industry.

<u>Estimates of Factor Prices</u>

Some factor prices are not observable, (as discussed in Section 2.0) and, must therefore be estimated econometrically. It is also necessary to estimate all of the factor prices to obtain the rates of price diminution (the negative of the technological change rate). Therefore, <u>all</u> factor prices were estimated, even though not all estimates were actually employed in the model.

The factor price estimation assumes that there is some "micro" production function that processes "micro" inputs, converting them into the "macro" (or actual inputs) that the firm uses. This function may be treated as homothetic and separable without loss of generality with respect to the overall technology of the joint cost function.

Factor price estimation involves estimating an equation in which the dependent variable is the percentage of cost in a micro category and the independent variables are suitable proxies for quantities of the variable. The estimated factor price is the intercept of the equation, shown below.

The factor price estimating equation is

$$h_i = \ln w_1 + \phi(X) + e_1 \qquad (6.1)$$

where h is

$$h_i = w_1 X_1 / \sum_{i=1}^{n-1} w_i X_i \qquad (6.2)$$

w_1 is the estimated factor price

$\phi(X)$ is a vector of quantity proxies

and e_1 is an error term.

The quantity proxies in ϕ are specified in translog form. The equations were estimated simultaneously using FIML, and the results are presented in Table 10.

The estimates of factor prices themselves were actually used only in cases for which no prices could be observed, but the rate of technological change was used in all cases. Therefore, estimated factor prices were used for inputs: 4. Other expenditures not elsewhere classified (NEC); 5. Purchased Transportation; 6. Owner-Operators; 7. (materials) Tires, Oil, Lubricants; and 9. Capital

The price of capital was omitted from the estimation process, to impose cost minimization implicity after cost minimization was accepted in a likelihood ratio test.

Estimated Cost Function

The results presented in Table 11 were chosen from those estimates with various parameter restrictions. The test for cost minimization leads us to conclude that firms do minimize cost and therefore the joint cost function is estimated with cost minimization as a maintained hypothesis. Cost minimization is imposed by deleting one factor price from the estimated equation; this is equivalent to normalizing or dividing both sides of the equation by that variable. Since these equations are log-linear, "dividing" through means substracting the log of the omitted variable from the remaining variables in the equation (including the dependent variable). The coefficients of omitted variables are computed by

TABLE 10. FACTOR PRICE AND PRICE DIMINUTION ESTIMATES

	$\log W_i$ †	λ_i
1. Salaried, Clerical, Other Labor per ton mile	.23125	-.00196††
2. Linehaul Labor per ton mile	.28332	.0057
3. Pickup & Delivery Terminal Platform per ton	.48434	.0062
4. Other Expenditures NEC per ton mile	.3964	.0002
5. Purchased Transportation per ton mile	.17810	.0007
6. Owner Operator per ton mile	.3942	.0004
7. Tire, oil, lube (materials) per mile	.1378	.0002
8. Fuel per mile	.19015	.0003
9. Capital per ton mile	.27351	.0008
System likelihood	63334.45	

†All of the estimated factor prices and rates of price diminution were reliably distinguishable from zero at above the .0001 confidence level.

††The negative value of the coefficient indicates a net decrease in productivity over time.

TABLE 11. JOINT ESTIMATES OF COST AND FACTOR SHARE EQUATIONS

Log likelihood function = 2670 No. OBS = 200

9 equations 105 coefficients

	Coefficient Value	Standard Error		Coefficient Value	Standard Error
α_0	16.2402	.0049	γ_{31}	-.0006	.0011
α_1	.1085	.0043	γ_{32}	.0715	.0011
α_2	.2483	.0036	γ_{33}	.3966	.0018
α_3	.1243	.0016	γ_{41}	-.1646	.0024
α_4	.5699	.0041	γ_{42}	-.0144	.0025
δ_{11}	.1018	.0020	γ_{43}	.3360	.0045
δ_{12}	.2009	.0040	γ_{44}	.0036	.0043
δ_{22}	.0703	.0259	γ_{51}	-.0007	.0025
δ_{31}	-.1982	.0009	γ_{52}	-.1076	.0027
δ_{32}	.0291	.0082	γ_{53}	.2619	.0047
δ_{33}	.0049	.0196	γ_{54}	.3827	.0044
δ_{41}	.4382	.0006	γ_{55}	.0174	.0095
δ_{42}	-.2716	.0066	γ_{61}	.0004	.0011
δ_{43}	.0140	.0009	γ_{62}	-.0252	.0012
δ_{44}	.4480	.0021	γ_{63}	.5405	.0018
β_1	.1006	.0105	γ_{64}	.0020	.0017
β_2	.0582	.0007	γ_{65}	-.0204	.0034
β_3	.3801	.0105	γ_{66}	.0142	.0006
β_4	.0702	.0008	γ_{71}	.0080	.0007
β_5	.1743	.0045	γ_{72}	.0001	.0006
β_6	.0056	.0098	γ_{73}	.0003	.0008
β_7	.01420	.0112	γ_{74}	-.0061	.0010
β_8	.0244	.0036	γ_{75}	.0015	.0024
β_9	.1700	--	γ_{76}	.0107	.0015
γ_{11}	.2053	.0027	γ_{77}	1.628	.0008
γ_{21}	.3074	.0007	γ_{81}	.0008	.0027
γ_{22}	.0826	.0007	γ_{82}	1.4742	.0029
			γ_{83}	.0023	.0051

TABLE 11 (Cont'd)

	Coefficient Value	Standard Error		Coefficient Value	Standard Error
γ_{84}	.1653	.0049	ρ_{32}	.0230	.0124
γ_{85}	.0314	.0106	ρ_{33}	.0679	.0145
γ_{86}	.0423	.0008	ρ_{34}	.0668	.0085
γ_{87}	.5975	.0006	ρ_{35}	3.8966-E05	.0023
γ_{88}	.1014	.0404	ρ_{36}	.0114	.0040
γ_{91}	- .5600	--	ρ_{37}	.0002	.0007
γ_{92}	- 1.6500	--	ρ_{38}	- .0304	.0120
γ_{93}	- 1.4655	--	ρ_{39}	.1460	--
γ_{94}	- .3729	--	ρ_{41}	- .0278	.0175
γ_{95}	- .5662	--	ρ_{42}	- .0191	.0241
γ_{96}	- .5645	--	ρ_{43}	.0296	.0374
γ_{97}	- .4492	--	ρ_{44}	.0198	.0298
γ_{98}	- 2.0856	--	ρ_{45}	- .0192	.0094
γ_{99}	6.7681	--	ρ_{46}	- .0008	.0156
ρ_{11}	.0011	.0141	ρ_{47}	- .0006	.0012
ρ_{12}	- .0252	.0201	ρ_{48}	.0946	.0403
ρ_{13}	- .0197	.0349	ρ_{49}	.0377	--
ρ_{14}	- .0247	.0241			
ρ_{15}	.0095	.0075			
ρ_{16}	.0013	.0013			
ρ_{17}	- .0006	.0011			
ρ_{18}	- .0246	.0320			
ρ_{19}	.0829	--			
ρ_{21}	.0003	.0039			
ρ_{22}	- .0098	.0091			
ρ_{23}	- .0097	.0142			
ρ_{24}	.0078	.0070			
ρ_{25}	.0090	.0020			
ρ_{26}	.0070	.0034			
ρ_{27}	.0004	.0009			
ρ_{28}	- .0651	.0096			
ρ_{29}	.0601	--			
ρ_{31}	- .0075	.0046			

subtraction as explained in Table 7.

The computation of the coefficients of the omitted cross factor price terms is illustrated in Table 8. These computations are based on the requirement for linear homogeneity in factor prices; this means that the cross and square terms γ must sum to zero $\Sigma\gamma_{ij} = 0$. We can rearrange this requirement to apply to groups of coefficients - the groups being the rows or columns of the symmetric coefficient matrix. The estimated coefficients form a lower triangular matrix and the symmetrical off-diagonal elements are equal. The missing coefficients are calculated using the requirement that the sum of each row or column must equal the negative of the missing coefficient value. The final missing coefficient is the value that sets the sum of the coefficients to zero.

Coefficient Interpretations

The intercept α_o, represents the value of total cost evaluated out the means of all variables while the α_i terms are the coefficients of the output variables (or marginal cost evaluated at the mean). All of these must be positive if the marginal cost is of an output is positive. Marginal cost is evaluated at the mean of each variable. They are equal to the log partial of the cost function multiplied by average cost--since all variables other than the output coefficient vanish at the mean. Also, the factor price coefficients equal the estimated factor shares when the remaining terms drop out.

The output-square and output cross-product terms show how cost varies with output. If positive the cost function has the traditional 'U' shape with respect to an output, while if negative, they have an inverted 'U' shape. These output-square terms were all positive indicating a 'U' shaped cost curve.

The output-factor price cross terms are reliably distinguishable from zero (as determined by a likelihood ratio test) and hence the production technology is non-separable (non-homothetic). Therefore factor intensitities will vary with the level of output. Relating this to the isoquant map this means the isoquants are not evenly and uniformly spaced throughout the isoquant map, reflecting the fact that different levels of output require different factor intensities. The shape of the isoquants also differs throughout the production surface. (See Table 12.)

Summary

- The industry does not enjoy significant economies of scale.
- The structure of technology is non-separable.
- Firms minimize cost.

Further Research

The findings presented here did not make use of the hedonic regression technique or output adjustment equations, developed by Friedlaender and Spady. It would be a significant addition to this work to use this technique.

The characteristics of the networks of individual firms should somehow be included in the hedonic cost functions. No data representing network characteristics is presently available but a proxy, such as the number of states in which operations are carried out, as might serve a proxy for network extensiveness. This can be accomplished in future research, into the certificates of networks authorized by the carriers. The ICC does have a centralized record of the routes a carrier is authorized to serve so that this is not a simple matter of using available information but requires a laborious search of numerous operating certificates.

The estimates presented here are not divided by regions; Friedlaender and Spady found that doing so makes a significant difference in the estimates. Work has begun to classify the sample carriers by TRINC's regions and the functions will be re-estimated. There is some practical difficulty with identifying carriers as operating in a given region, because the larger carriers may have their operations scattered over several regions and have a corporate headquarters in yet another region. Interregional carriers, such as transcontinental carriers can be classified separately but this does not eliminate the problem. The fact that the regional dummy variables lead to significantly different parameter estimates indicates that some firms have different technologies. Some classifying scheme based on degree of specialization in a particular technology might be more appropriate.

It was the philosophy of these estimates that pickup and delivery and terminal platform activity were separate outputs. It would be useful to test this view of the motor carrier firm by treating terminal/platform and pickup and delivery activities as variables in an hedonic regression.

No tests of alternative firm objectives were made, nor were tests of the possible existence of firm-level regulatory effects tested.

TABLE 12. LIKELIHOOD RATIO TEST FOR SCALE ECONOMIES

Log Likelihood

Unrestricted	Restricted	Result
2670.15	2672.39	accept

AND

LIKELIHOOD RATIO TEST FOR COST MINIMIZATION

Log Likelihood

Unrestricted	Restricted	Result
2076.15	2670.15	accept

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APPENDIX I

ICC ACCOUNT RECONCILIATION

1. Arrow Transp. Co., Inc.
2. Auclair Transp. Inc.
3. Beacon Fast Freight Co. Inc. (NY Corp.)
4. Blue Line Express Inc.
5. Transportation Co. (A Corp.)
6. Coles Express
7. Fox & Ginn Inc.
8. Hemingway Transport
9. Intercity Transp. Co.
10. Lombard Bros. Inc.
11. ND Transportation Company
12. Old Colony Transp. Co., Inc.
13. St. Johnsbury Trucking Company Inc.
14. Sanborn's Motor Express Inc.
15. Schuster Express Inc.
16. Shawmut Transportation Co. Inc.
17. Valleries Transportation Service Inc.
18. P. Wajer & Sons Express, Co., Inc.
19. H. P. Welch Co.
20. A A A Trucking Corp.
21. A P A Transport Corp.
22. Adley Express Co.
23. Allegheny Freight Lines Inc.
24. American Freightways Co. Inc.
25. Arrow Carrier Corp.

26. Associated Transport Inc.

27. B & F Motor Express Inc.

28. Bair Transport Inc.

29. Berman's Motor Express Inc.

30. Boss Linco Lines Inc.

31. Branch Motor Express Co.

32. George W. Brown Inc.

33. Burgmeyer Bros. Inc.

34. P. Callahan, Inc.

35. Canny Trucking Co. Inc.

36. Charlton Bros Transp. Co. Inc.

37. W. T. Cowan, Inc.

38. Dornson Transfer & Storage Co.

39. Dorns Transportation Inc. (NY Corp.)

40. Eastern Freight Ways Inc.

41. Eazor Express, Inc.

42. Elliott Bros. Trucking Co. Inc.

43. Feuer Transportation Inc.

44. Follmer Trucking Co.

45. Fowler & Williams Inc.

46. Hall's Motor Transit Co.

47. Herman Forewarding Co.

48. Transport or Delaware Inc.

49. Inland Express Inc.

50. Inland Transportation Company

51. Miller S Motor Freight Inc.

52. Modern Transfer Co. Inc.
53. Moon Carrier
54. Motor Freight Express Inc.
55. Mushroom Transportation Co. Inc.
56. Nelson Freightways Inc.
57. New Penn Motor Express Inc.
58. Oneida Motor Freight Inc.
59. Penn Yan Express Inc.
60. Pinter Bros. Inc.
61. Preston Trucking Co., Inc.
62. Snyder Bros. Motor Freight Inc.
63. South Bend Freight Line Inc.
64. Spector Freight System Inc.
65. Surburban Motor Freight Inc.
66. Tobler Transfer Inc.
67. Trans American Freight Line Inc.
68. Transport Motor Express Inc. (Del. Corp.)
69. Transportation Service Inc.
70. Tucker Freight Lines Inc.
71. U. S. Truck Co. Inc.
72. United Trucking Service Inc.
73. Western Transportation Co.
74. Western Trucking Co.
75. White Star Trucking Inc.
76. Wilson Freight Co.

77. Wolverine Express Inc.
78. Akers Motor Lines Inc.
79. Barnes Freight Line Inc.
80. Brown Transport Corp.
81. Carolina Freight Carriers Corp.
82. Central Motor Lines Inc.
83. Central Truck Lines Inc.
84. Colonial Motor Freight Line Inc.
85. ET & WNC Transportation Inc.
86. Ecklar-Moore Express Inc.
87. Estes Express Lines
88. Erickson Motor Express Corp.
89. Florida Alabama Transp. Co.
90. Georgia Highway Express Co.
91. Gordons Transports Inc.
92. Howard & Hall Co. Inc.
93. Burris Express Inc.
94. Hennis Freight Lines Inc.
95. Houff Transfer Inc.
96. M. R. & R. Trucking Co.
97. Southeasten Freight Lines Inc. (Ala Corp.)
98. McLean Trucking Company
99. Mercury Motor Express Inc.
100. Old Dominion Freight Line
101. Osborn Transportation Company

102. Overnite Transp. Co.

103. Pilot Freight Carriers Inc.

104. Riverside OR Lines Inc. (Fla. Corp.)

105. Southeastern Freight Lines Inc.

106. Standard Trucking Co.

107. Tennessee-Carolina Transp. Inc. (Tenn Corp.)

108. Terminal Transport Co. Inc.

109. Carolina Freight Lines Inc.

110. Wilson Trucking Corp.

111. Admiral Merchants Motor Freight Inc.

112. Advance United Expressways

113. All-American Inc.

114. Badger Freightways Inc.

115. Barber Transportation Co.

116. Brigos Transportation Co. (Minn Corp.)

117. Mercury Motor Freight Inc.

118. Chippewa Motor Freight Inc.

119. Clairmont Transfer Co.

120. Fore Way Express Inc.

121. Gateway Transp Co. Inc.

122. Glendenning Motorways Inc.

123. Gross Common Carrier Inc.

124. Hart Motor Express Inc.

125. Midwest Motor Express Inc.

126. Wisconsin Truck Lines Inc.

127. Motor Transport Co.

128. Murphy Motor Freight GHT Lines Inc. (Minn Corp.)

129. Werner Continental Inc.

130. Witte Transportation Co.

131. Byers Transportation Co. Inc.

132. Campbell Sixty-Six Express Inc.

133. Capital Truck Lines Inc.

134. Chicago-Kansas City Freight Line Inc.

135. The Chief Freight Lines Co.

136. Churchill Truck Lines Inc.

137. Crouch Freight Systems Inc.

138. Crouse Cartage Co.

139. Darling Transfer Inc.

140. Frisco Transportation Co.

141. Graves Truck Line Inc.

142. H & W Motor Express Company

143. Manley Transfer Co. Inc.

144. Mid-American Lines Inc.

145. Middlewest Freightways Inc.

146. Riss International Corp.

147. Alamo Express Inc.

148. S-Best Freight System Inc.

149. Brown Express Inc.

150. Central Freight Lines Inc.

151. Curry Motor Freight Lines Inc.

152. Curry Motor Freight Lines Inc.
153. Herder Truck Lines Inc.
154. Jones Truck Lines Inc.
155. Mercer Motor Freight Inc. (Del Corp.)
156. Mercury Motors Inc.
157. Lines Inc. (Okla Corp.)
158. Mistletoe Express Service
159. Red Arrow Freight Lines Inc.
160. Red Ball Motor Freight Inc.
161. Carolina Motor Freight Line Inc.
162. Stern Motor Transport Inc.
163. Sourtwestern Transp. Co.
164. Strickland Transp Co. Inc.
165. T I M E-DC Inc.
166. Texas-Oklahoma Express Inc.
167. Pacific Motor Transport Co.
168. Garrett Freightlines Inc.
169. IML Freight Inc.
170. Illinois-California Express
171. Milne Truck LInes Inc.
172. Navajo Freight Lines Inc.
173. Ringby Truck Lines Inc.
174. Rio Grande Motor Way Inc.
175. Salt Creek Freightways
176. Cherokee Freight Lines (Calif Corp.)

177. Colonial Motor Freight Lines Inc.

178. Consolidated Freightways Corp. of Del.

179. Delta Lines Inc.

180. Di Salvo Trucking Co.

181. Elsen Freight LInes, Inc.

182. Pacific Intermountain Express Co.

183. Smith Transportation Co.

184. Sterling Transit Co. Inc.

185. Thunderbird Freight Lines Inc.

186. Transcon Lines

187. Western Gillette Inc.

188. Red Star Express Lines

189. Reisch Trucking & Transportation Co. Inc.

190. Service Transp. Co.

191. Smith & Solomon Trucking Co.

192. Smith's Transfer Corp.

193. Tose Inc.

194. Ward Trucking Corp.

195. Wooleyhan Transport Co.

196. A & H Truck Line Co.

197. Advance Transportation Co.

198. American Transit Lines Inc.

199. Anderson Motor Service Inc.

200. Be-Mac Transport Co. Inc.

201. Loudon Motor Freight Inc.

202. Blue Arrow-Douglas Inc.

203. Brady Motorfrate Inc.

204. Freight Lines Incorporated

205. Central Transport Inc.

206. Centralia Cartage Co.

207. Checker Express Co.

208. Cleveland, Columbus & Cincinnati Highway Inc.

209. Commercial Motor Freight Inc.

210. Commercial Motor Freight Inc. of Ind.

211. Consolidated

212. Cook Motor Lines Inc.

213. Cooper-Jarrett Inc.

214. Dohrn Transfer Co.

215. Duff Truck Line Inc.

216. Eastern Express Inc.

217. Ellis Trucking Co. Inc.

218. Express Freight Lines Inc.

219. General Expressways Inc.

220. Hajek Trucking Co. Inc.

221. Herriott Trucking Co. Inc.

222. Hollard Motor Express Inc.

223. R C & D Motor Freight Inc.

224. Inter-City Trucking Service Inc.

225. Interstate Motor Freight System

226. Jones Transfer Co.

227. Kain's Motor Service Corp.

228. Knox Motor Service Inc.

229. Liberty Trucking Co.

230. Long Transportation Co.

231. Lovelace Truck

232. W L Mead Inc.

233. Mohawk Motor Inc.

234. Morrison Motor Freight Inc.

235. Motor Express Inc.

236. Motor Freight Corp. (Indiana Corp.)

237. National Transit Corp.

238. Nighthawk Freight Service Inc.

239. The O-K Trucking Co.

240. Ogden & Moffett Co.

241. Parker Motor Freight Inc.

242. Pic-Walsh Freight Co.

243. Putnam Transfer & Storage Co.

244. Carrier Unknown

245. The Reinhardt Transfer Co.

246. Renner's Express Inc.

247. George Rimes Trucking Co.

248. Roadway Express Inc.

249. Rooks Transfer Lines Inc.

250. Scherer Freight Lines Inc.

251. Short Freight Lines Inc.

252. Earl C Smith Inc.

APPENDIX II

DATA RECONCILIATION

Class I Motor Carriers of Properties Form A
Keyed to Pre-1974 Accounts

Title and Schedule Number	Line	Acct. No.	Pre-1974
(1200) Carrier Operating Property			
Land	1	1211	1201
Structures	2	1213	1210
Revenue Equipment	3	1221	1220
Service Cars and Equipment	4	1223	1230
Shop and Garage Equipment	5	1233	1240
Furniture and Office Equipment	6	1235	1250
Miscellaneous Equipment	7	1237	1260
Improvements to Leasehold Property	8	1241	1270
Undistributed Property	9	1243	1280
Unfinished Construction	10	1245	1290
Total	11	sum 1-10	
Carrier Operating Property Leased to Others	12	1251	1300
Grand Total	13	sum 1-12	

Title and Schedule Number	Line	Acct. No.	Pre-1974	
Carrier Operating Leased to Others				
(1260) <u>Carrier Operating Property Leased to Others (Total)</u>		1260	1400	
Property Used in Other than Carrier Operations		1261		
(1220) <u>Revenue Equipment Owned</u>		1220	1220	1220
Total Trucks	9			
Truck Tractors	10			
Total Semi-Trailers	19			
Total Full Trailors	28			
Other Revenue Equipment	32			
Total	33			
(3000) <u>Operating Revenues</u>				
Freight Revenue -common Carriers	1	3100	3100	Same
Freight Revenue -contract Carriers	2	3200	3110	Same
Freight Revenue -cartage	3	3300	3120	Same
Interlining	4	3400	3130	Same
Other Revenue	5	3900	3900	Same
Total	6			
(4000) <u>Operating Expenses</u>				
Equipment Maintenance		4110	4110	4100
Office and Other Expenses		4246 4296 4346 4396 4516 4526	4120	4120

Title and Schedule Number	Line	Acct. No.
Office and Other Expenses (Continued)		4536
		4546
		4556
		4596
		4616
		4666
		4676
		4696
		4716
		4726
		4736
		4766
		4776
		4786
		5126
		5526
LH Repairs and Servicing -		4241
		4341
		4521
		4531
		4541
PD Repairs and Servicing	PD sum	4242
		4342
		4522
		4532
		4542
LH Tires and Tubes		4151
PD Tires and Tubes		4552
Transportation Supervision		4111
		4112
		4121
		4122
		4131
		4132
		4211
		4212
		4311
		4312
Office and Other Expense		4291

Title and Schedule Number	Line	Acct. No.
Other Transporta-		4292
tion Expense		4391
		4392
		4591
		4592
		4611
		4612
		4661
		4662
		4671
		4672
		4691
		4692
		5121
		5122
Line haul		5521
		5522
Labor, Drivers		4221
& Helpers		4251
		4321
		4351
Pick up and delivery, Labor,		4222
		4252
		4322
		4352
Employees Wel-		4401
are Expense		4402
Line haul, Fuel,		4511
Pick up and delivery, Fuel,		4512
Line haul, Oil etc.		4521
Pick up and delivery, Oil etc.		4522
Equip. Rents W.		5411
Driver		
Equip. Rents		5421
W.O. Driver		5437
Other Purchased		5441
Transportation		5451
		5461
		5471
Equip. Rental	Pick up and delivery	5412
W. Driver		
Equip. Rental	Pick up and delivery	5422
W.O. Driver		5432
Other Purchased T.		5472
Pick up and delivery		5472
		5482

Title and Schedule Number	Line	Acct. No.
(4000) <u>Terminal</u>		
<u>Expense</u>		sum 4113
Supervisory		4114
Salaries		4115
		4123
		4124
		4125
		4133
		4134
		4135
Salaries & Fees		4213
Billing &		4313
Collecting		4653
Other Office		4214
Employees		4215
		4314
		4315
Office and Other		4244
Expenses		
Other Terminal		
Expense		4245
		4515
		4525
		4535
		4545
		4555
		4594
		4595
		4613
		4614
		4615
		4663
		4664
		4665
		4673
		4674
		4675
		4693
		4694
		4695
		4715
		4725
		4735
		4765
		4775

Title and Schedule Number	Line	Acct. No.	
Other Terminal Expense (Continued)		4785 5123 5124 5125 5523 5524 5525	
Total Expense		4000	
(9002) Officers, Directors Employees Service and Compensation - Carriers of Freight			
A. Classification of Employees and Their Compensation		All	
(9003) Operating Statistics Intercity, Carriers of Freight		All Including	I27 Statistics
9005) Trucks and Tractors in Intercity Revenue Service		All	
9006) Percentage Distribution of Intercity Traffic - Common Carriers of Property		All	
100) Comparative Statement of Financial Position			
Total Current assets	13 sum	100 1020 1030 1110 1120	

Title and Schedule Number	Line	Acct. No.
Total Current Assets (Continued)		1130
		1140
		1160
Total Tangible Property	21	1300
	22	1300A
	24	1340
	25	1340A
	26	
	27	
Investment Securities & Advances	36	
Deferred Charges	40	
Total Assets	**41**	

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