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# Monitoring of GPS System Performance

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June 1985  
Final Report

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## PREFACE

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The work was performed by the Transportation System Center's Center for Navigation. This report represents one year of comprehensive study of the integrity and reliability of the GPS system for civil applications.

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## 1. INTRODUCTION

### 1.1 OBJECTIVES

The Global Positioning System (GPS), a worldwide satellite-based navigation system developed by the Department of Defense (DOD) is scheduled to become operational in late 1988. The system has the potential to become the primary radio navigation system for the United States. The Standard Positioning Service (SPS) portion of GPS, better known as coarse/acquisition (C/A) code, will be made available to all users worldwide and will provide 100-meter, 2 drms\* navigation accuracy.

The objective of this study is to assess the GPS system integrity and navigation performance using monitored satellite signals and processed navigation message data, as well as the analyzed data content of advisory and health messages. An additional objective is to discuss how the integrity of a basic GPS system may be improved using additional signal processing, aiding, and enhanced monitoring techniques.

Four major topics are addressed:

1. Methods by which the user alone\*\* can detect system problems with his own equipment.
2. Methods which enhance the monitoring of system performance by adding more data processing capability to the user's own onboard processor.
3. Methods by which the integrity of the system can be improved; i.e., through the addition of Very High Frequency Omnidirectional Range (VOR) or altitude measurement aiding, and integrity monitors for both general usage and fault detection.
4. Methods of maintaining navigation service when system problems occur.

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\*drms - distance root mean squared, a two-dimensional error term with a probability of 95.4 to 98.2 percent.

\*\*"alone" - no external measurement inputs; today's state-of-the-art receiver design without additional processing capabilities and operating solely in the baseline GPS mode.

receive the ground-control initiated correction data in the satellite message along with the navigation data. Detection of all other faults not identified in the message is left to the capability of the user receiver itself. Therefore, the amount of time needed to detect faults will vary greatly.

In some applications, specific performance requirements for civil service may be more restrictive than for the military. The integrity of the system during non-precision approach and landing may require that the user equipment issue a warning within 10 seconds when system performance is out-of-tolerance (i.e., the position error is greater than 100 meters 2 drms).

For example, the existing VOR system has a self-monitoring capability to check the quality of its transmitted signals and system accuracy by a monitor located at the site. Monitoring is done on a continuous basis. The VOR transmitter automatically shuts down within 10 seconds when the system performance degrades outside of specified limits. However, in such a case a shutdown affects only the local coverage area, whereas a GPS satellite shutdown will affect a large area. Providers of VOR systems claim these systems have a high integrity; consequently, they would expect similar performance from the GPS. Another aspect of VOR system integrity is the capability of the system to check position accuracy. The user may do this by flying over one of the stations or by parking at designated check points for real-time verification.

The update rate of data for a non-precision approach to an airport is another factor important to civilian users. The present requirement is approximately 2 to 4 seconds. This value has been derived to satisfy airport surveillance radar tracking requirements. These established requirements of the Federal Aviation Administration (FAA) are being applied to GPS system requirements without alteration, whenever GPS system compatibility for civil users is discussed.

This study considers the feasibility of a satellite-based navigation system with a fixed data rate for meeting civil navigation requirements as stated above. Future studies should verify whether the requirements identified above apply equally to the users of GPS.



Control and space segments support the constellation maintenance function and the user segment in terms of achieving user receiver navigation performance requirements.

Ultimately, user acceptance of GPS will be based on the integrity and reliability of the system. This means that acceptance of the future navigation system will involve reliability assurance that the system is providing sufficient information to enable position determination at the required accuracy performance and evaluation of the ability of the system to detect and indicate its malfunctions to ensure that the system is operating within specific performance limits. The degree of system effectiveness or value may be interpreted by the crosshatched area presented in Figure 1-1, in which both circles will completely overlap only under ideal conditions.

#### 1.4 SCOPE

This report addresses monitoring capability in resolving issues related to GPS integrity and reliability. Navigation data messages originated jointly by the control center and the satellite processor are the prime information-carrying sources available for monitoring GPS system navigation performance under the unaided user-own equipment concept. Receiver capability and response time to faulty operations are established using satellite signals and their message content at a fixed data rate. This report extends beyond the user receiver-alone concept to include enhancements to the user receiver, aided operation using measurements from other sources, and operation during outages or faulty data intervals.

## 2. GPS SYSTEM PERFORMANCE CONSIDERATIONS

### 2.1 GENERAL

The integrity and reliability performance of the GPS depends on the design of three elements - the space segment, the control segment, and the user segment - as shown in Figure 2-1. A high reliability of each segment, combined with rapid restoration time and redundancy in signal coverage (with good geometry to provide an acceptable GDOP) yields a highly reliable GPS system as a whole. Detection of satellite signals and faults is the basic means of ensuring integrity performance. Simplified integrity and reliability definitions given below, as defined by Braff and Shively (1983), are used as performance parameters for GPS system evaluation.

Integrity: Integrity is defined as the ability of a system to detect and indicate its malfunctions to ensure that the system is not used when it fails to operate within its specified performance limits.

The integrity of the GPS is dependent on all three elements; therefore, one segment used alone will not be sufficiently reliable. In this case, additional processing in the user's equipment, or additional measurement inputs from other sources or through supplementary ground-based monitors will greatly enhance integrity.

Reliability: The probability that over a specified period of time, at any given location, the navigation system is providing sufficient information to enable position determination at the required accuracy. Reliability decreases as the specified period increases.

The reliability of the space segment requires an adequate number of redundant satellites to maintain the required performance level. For example, 80 satellites including replacements will be required to maintain 95 percent system reliability up to the year 2000. This will be discussed in greater detail in Section 3.6.

Other performance parameters are often used:

Availability: The probability that at any point in time, at any required location, the navigation system is providing sufficient information to enable position determination at the required accuracy. Availability is closely

related to reliability. For most systems, availability reaches a constant steady-state value early in its life cycle. Hence, availability is independent of time.

The GPS specification\* states that the GPS shall be considered available as long as users have at least 4 operating satellites in view with geometry providing a PDOP of 6.0 and meeting the user equivalent range error (UERE) requirements. For user populations considered to be uniformly distributed over the earth and time, the GPS will be available at least 95 percent of the time.

Availability is 100 percent for receivers employing an 8-degree mask angle or less within the Continental U.S. (CONUS). This estimate is based on a 4-satellite availability criterion as a minimum at all times. A TSC study indicates that any one satellite failure will result in an outage of up to 36 minutes (PDOP greater than 6), somewhere within the CONUS. An attempt to coast during these outages by using a stable clock ( $10^{-10}$ ) or baro-altimeter is not always effective.

Results of the study indicate that poor geometries, which greatly amplify the effect of ranging errors, would also result in exaggerated sensitivity of the user position estimate to errors in the extrapolated receiver clock bias. Similarly, in certain circumstances altimeter aiding cannot compensate for poor geometry, regardless of the accuracy of altimeter itself.

System Value: System value is defined as a Figure of Merit that serves as a quantitative evaluation of performance. The system value is 1 for a geometric dilution of precision (GDOP) ranging from 1 to 6.

Constellation Value (Kruh): The constellation value is a measure of the quality of the 24-hour coverage for a PDOP threshold of 6, or less than 99.9 percent of the time over the entire world at a  $5^\circ$  mask angle.

The planned satellite constellation, 18/6/2 + 3, will result in a 99.7 percent average system value for global 24-hour coverage at a  $7.5^\circ$  mask angle. In this notation, the first term indicates the number of satellites, the second term indicates the number of orbits, the third term indicates the relative phase of satellites in adjacent orbits, and the last term indicates the number of active spares.

\*SS-GPS-300B, March 3, 1980.

Five unmanned monitoring stations - Hawaii, Colorado Springs, Ascension, Diego Garcia, and Kwajalein - provide 95 percent global coverage for each satellite. Similarly, three unmanned upload stations - Ascension, Diego Garcia and Kwajalein - provide an 89 percent continuous data uplink to each satellite.

A high reliability is maintained in all phases of system design. For example, satellite components have quadruple redundancies in clocks (two Cesium, two Rubidium); triple redundancies in RF components; and double redundancies in processors and in data links for monitoring-stations, upload-stations and the master control station. Automatic self-checking features are included in the system designs. GPS system operational characteristics are shown in Table 2-1.

Satellites are uploaded asynchronously\* at scheduled 8-hour intervals. The unscheduled upload interval is 45 minutes. For emergency situations or on command, this interval may be reduced to less than 10 minutes. Degradation of ephemeris and clock correction parameter values with time are significant factors in system performance. A summary of a performance estimate for up to 14 days is shown in Table 2-2.

The position accuracy depends on the user-satellite geometry as well as the pseudorange accuracy. This dependence may result in a range position error even if the pseudorange measurement error is good. This is the Position Dilution of Precision (PDOP) effect. The Figure of Merit defined previously considers the PDOP effect and defines the acceptance value for PDOP to be 6 or less. Significance of the PDOP values becomes more pronounced once the critical satellite fails and the three-dimensional solution using only three satellites is required. Any aiding by using a baro-altimeter or stable clock will greatly depend on the satellite geometry relative to the user.

## 2.2 SPACE SEGMENT INTEGRITY

Space system reliability directly affects measurements. Two critical areas of consideration are: (1) satellite onboard equipment reliability and (2) reliability of a particular constellation of interest. Whenever a satellite processor cannot locate the requisite valid control or data elements in its

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\*Staggered upload, as opposed to synchronous uploading, in which all satellites are uploaded at the same time.

TABLE 2-2. GPS CHARACTERISTIC DATA SUMMARY

<u>CLOCK</u>	CHANGED TO NEW DATA -	EVERY 1 HR DURING FIRST DAY OF UPLOAD EVERY 4 HR UP TO 14 DAYS FOR THE FOLLOWING DAYS $10^{-12} = 1.2 \text{ METERS/HR}$ , IN 15 MIN = 0.3 M (UERE)	
<u>EPHEMERIS</u>	CHANGED TO NEW DATA -	EVERY 1 HR DURING FIRST DAY OF UPLOAD EVERY 4 HR UP TO 14 DAYS FOR THE FOLLOWING DAYS	
SCHEDULED REGULAR UPLOADS UNSCHEDULED UPLOADS FLAG SYSTEM EMERGENCY UPLOADS		EVERY 8 HR 45 MIN 20 TO 25 MIN 10 MIN	
<u>ALMANAC</u>		<u>ALMANAC EPHEMERIS (UERE)</u>	<u>AGE OF DATA, ALMANAC (AODA)</u>
		900 M 1,200 3,600	AFTER 24 HR UPLOAD 1 WEEK 3 WEEK
<u>RANGE ACCURACY</u>		40 % IMPROVEMENT IF ALL SATELLITES UPLOADED SIMULTANEOUSLY 6 M (UERE 1 SIGMA) P-CODE 15 M (SEP) P-CODE 27-28 M (SEP) AFTER LOSS OF 4 MS, CLOCK $2 \times 10^{-13}$ 16 M (SEP) AFTER LOSS OF 2 MS, CLOCK $2 \times 10^{-13}$ 22-23 M (SEP) AFTER LOSS OF 4 MS, CESIUM CLOCK 120 M (SEP) AFTER LOSS OF MCS, RUBIDIUM CLOCK 120 M (SEP) AFTER LOSS OF MCS, CESIUM CLOCK	

TABLE 2-3. GPS NAVIGATION INTEGRITY: SPACE SEGMENT

- SATELLITE INITIATED FAULT ACTIONS

NAVIGATION DATA ERROR - DEFAULT DATA IN 6 SEC

CODE FAILURE (SA/AS) - NONSTANDARD CODE IN 6 SEC

- SATELLITE CLOCK FAILURES

POWER SUPPLY (MOST COMMON) - LINK SHUTDOWN IMMEDIATE

ATOMIC LOOP LOSES LOCK (BELOW  $10^{-10}$ ) AND INITIATES SEARCH MODE:  
USER EQUIPMENT WILL NOT TRACK; RESPONSE IMMEDIATE

## 2.4 USER SEGMENT INTEGRITY

This section describes user receiver performance in reception of satellite signals. The receiver's navigation processor converts the pseudorange measurements of each satellite in use into a user position estimate.

System elements which affect user segment integrity are: user receiver noise; constellation used; satellite signal quality; information message data received; and built-in self-test and user protection in the satellite.

The signal and noise-processing components (including signal path errors) are described and analyzed in technical publications covering both military and civilian usage of GPS. Performance values of various applications are summarized in Table 2-5. Satellite signal detection and processing may vary a great deal with the mode of operation. Therefore, bias error components in the pseudorange measurement may or may not be cancelled, as shown in Table 2-6. Using a system error breakdown, error components are identified and their contribution level are shown by segments: user, space, and control, and by modes of operation.

User segment integrity is demonstrated by the satellite-originated defaulting action and the information received by the way of health messages and parity bits. Table 2-7 provides a summary of user protection functions for Selective Availability;\* for implementation of advisories on signal and navigation data health status; for failure in parity; or in case of failures in navigation data. More detailed discussion on the health messages and on the user equipment Figure of Merit criterion appear in Section 3.1 and in the updated ICD-GPS-201.\*\*

## 2.5 MEASUREMENT RELIABILITY

### 2.5.1 Link Considerations

The signals from a satellite to a user experience delays while passing through the ionosphere and the troposphere. Ionospheric delay may be measured directly by using two frequencies, although this method is not available for

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\*Capability to deny full system accuracy to unauthorized users.

\*\*Space Segment (ONS, NDS) TT&C Control Segment Interfaces Document.

TABLE 2-6. GPS PSEUDORANGE ERROR COMPONENTS IN METERS

		ABSOLUTE (SAMSO)		DIFFERENTIAL (TSC)	
		P-CODE	C/A CODE	C/A CODE	
				0 KM	250 KM
SPACE	CLOCK STABILITY SPACE PERTURBATION	3.5	3.5	0.9	0.9
CONTROL	EPHEMERIS PREDICTION (DETERMINATION: 1.5 M) (PRESENTATION: 0.4 M/3 HRS)	4.3	4.3		
USER	IONOSPHERIC DELAY TEMPORAL DECORRELATION SPATIAL DECORRELATION	2.3	5.0 - 10.0		2.5
	TROPOSPHERIC DELAY	2.0	2.0		
	SELECTIVE AVAILABILITY			0.3	0.3
	RECEIVER NOISE	1.5	7.5	2.6	2.6
	MULTIPATH	1.2	1.2	1.2	1.2
	IONOSPHERIC BIAS				1.5
	MECHANIZATION			1.0	1.0
	MEASUREMENT UNCERTAINTY			0.4	0.4
	OTHERS	0.5	0.5		
UERE* (1 SIGMA)		6.6	10.8 - 13.9	3.2	4.3

\* UERE (1 SIGMA) AT 500 KM = 6.5 M



civilian use. The proposed method would use an atmospheric correction model, receiving correction information via the satellite data message. Neither of the above methods is fully accurate, the latter one being 50 to 75 percent effective at best.

An alternative method is to add to the baseline GPS a differential mode, which completely eliminates bias errors including ionospheric delay. Differential operation employs a fixed surveyed-in reference station which determines the pseudorange offset to each satellite caused by atmospheric delays, uncompensated satellite data errors, ephemeris errors, and Selective Availability; it then broadcasts these offsets to nearby users. The differential mode requires an additional data link to transmit error correction messages to users in real time. Considering both approaches, the user must be aware of the various error sources described below, which may not be obvious during system trade-offs.

#### 2.5.2 Local Time Dependence on Ionospheric Delay Measurement

When a constellation of satellites is being tracked, satellites of opposing horizons in combination with the other satellites in view provide the best horizontal position estimate. If the measurements are made at an inopportune time, such as during high ion increases in the atmosphere caused by the ionization process, the measurement differences in the ionospheric delay from opposing horizons may be significant.

Figure 2-2 represents a scaled actual measurement of the total ionosphere taken in Washington D.C. during March 1958, in which a maximum delay at a zenith of 11 meters was measured. A local time is a parameter. For example, a GPS user error in pseudorange, when measured at 9:00 AM local time, may have a 1:4 ratio in ionization delay difference when compared with two opposing east-west horizons.

#### 2.5.3 Observation Angle Dependence

The relative angle dependence for two observers of the same satellite is a characteristic condition in user-monitor station applications. To illustrate how the error enters into a measurement, the following operating conditions have

been assumed: (1) The nominal altitude of the ionosphere is 350 km above the surface of the earth, and (2) the ionospheric delay at zenith is equivalent to 10 meters and projects to 3.14 times this value when the satellite is observed at the horizon.

If two observers from two different locations observe the same satellite with some small relative difference in elevation angles (for example, looking along the baseline), the ionospheric delays or measured path will vary as shown in Figure 2-3. Separations between observers and the satellite elevation angles are used as parameters.

#### 2.5.4 Spatial and Temporal Dependence

The spatial and temporal errors related to the ionosphere have been adequately covered in published technical literature. Estimated changes of ionospheric delay, which also include Selective Availability, are shown in Figure 2-4.

#### 2.5.5 Time Synchronization

The following time control exists between a satellite and the control station:

1. Each satellite operates internally on one of two Cesium or Rubidium clocks.
2. All time-related data in the telemetry words (TLM)\* and handover words (HOW)\*\* are in satellite time.
3. All other data in the navigation message are referenced to GPS time.
4. Timing of the transmission of the navigation data in the messages to users is executed by the satellite on satellite time.

GPS time is established by the control center and is referenced to a Universal Control Time (UTC) zero point, established as midnight of January 6,

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\*Each TLM is 30 bits long, and occurs every 6 seconds in the data frame and in the first word in each subframe. See Appendix A.

\*\*The HOW is 30 bits long, and is the second word in each subframe immediately following the TLM word. See Appendix A.

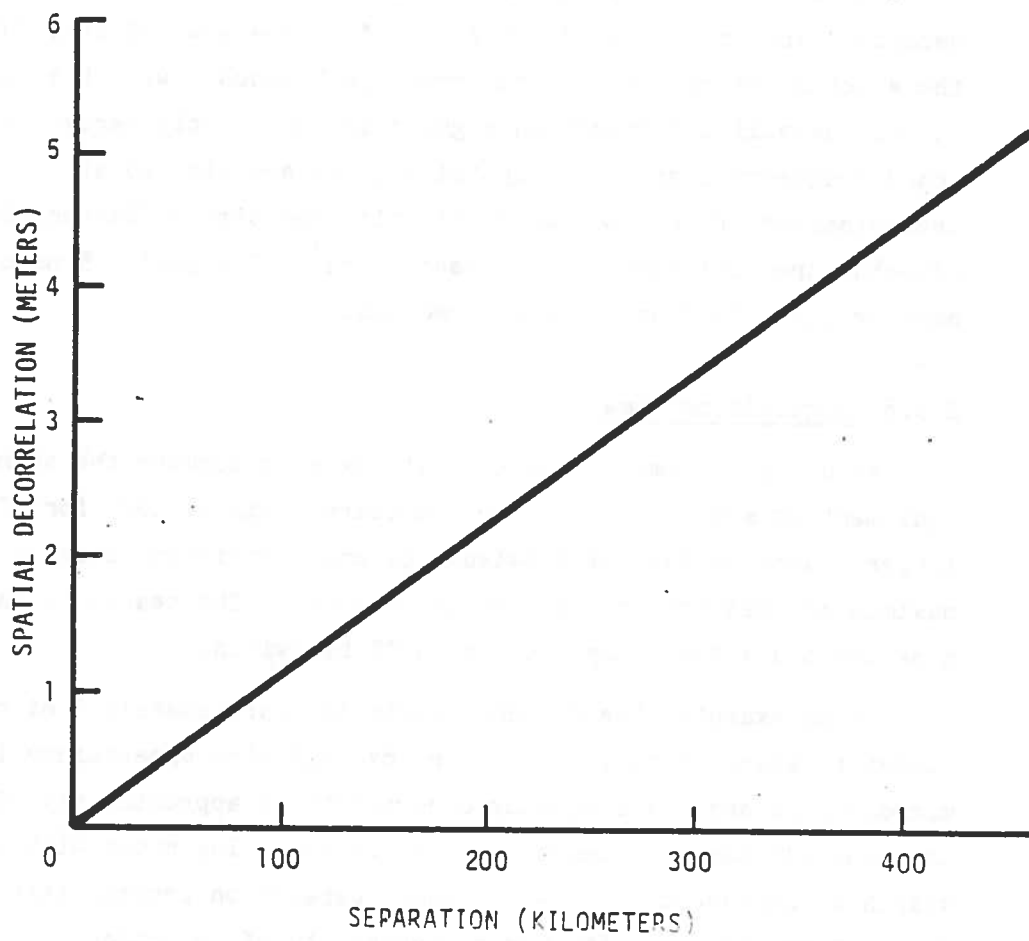


FIGURE 2-4. IONOSPHERIC SPATIAL DECORRELATION WITH SEPARATION

$$S_R = C/22N_0 \text{ with an IF bandwidth of } 22 S_R.$$

$$\text{For } \frac{C}{N_0} = 33 \text{ db (signal-to-noise spectral density),}$$

$$S_R = 90 \text{ or } 45 \text{ chips/second}$$

Doppler uncertainty =  $\pm 1$  kHz

Thus, the code search would take 22 seconds. In the event of a  $\pm 4$  kHz doppler uncertainty, it would take approximately 90 seconds to acquire a single satellite and six minutes to acquire a constellation of four by searching the sky from a cold start. Multipath would cause some signal delay, but a strong signal and a reasonable prediction of doppler uncertainty would be helpful.

An a priori knowledge of almanac is helpful. This almanac need be accurate only to a few kilometers. The almanac may be used for a week or more, and may be retained in the nonvolatile memory of a set from one usage to the next. The almanac can also be transferred from an active GPS receiver via a data link, cassette, or manual keyboard.

### 3. METHODS OF MONITORING GPS PERFORMANCE

GPS central control has two mission functions: the navigation performance function and the constellation maintenance function. GPS navigation performance is accomplished by monitoring the signals received from the satellites. Using these signals, accurate ranging measurements are performed on all satellites in view and information obtained on the atomic clocks onboard the satellites. All data are sent to the control center for processing to generate clock and ephemeris corrections. These corrections are sent to the ground antenna and uploaded to each satellite.

In the constellation maintenance function, the satellite state of health is monitored. This is done by making contact with the satellite, which in turn sends the information to the monitoring sites to be relayed back to the control center. At the control center, technical personnel review the data and, if required, send commands up to the satellite.

Because time delays in transferring the data via satellite (due to the lack of a direct communication link between a control segment and the users) would not meet the ten-second integrity requirement, civil users have been forced to look for alternative monitoring solutions.

Integrity and reliability performance are interrelated and depend on the performance of all three system segments: space, control, and user. Meeting the ten-second warning requirement as soon as the system is out-of-tolerance (error greater than 100 meters 2 drms) is a primary concern for the user. Can the user equipment alone meet this requirement, or are there alternative solutions (such as additional processing, aided navigation, or supplementary monitoring sites) to enhance system operation?

To find a satisfactory answer, it is assumed that the future role of GPS is as a supplementary, rather than sole, radionavigation system. The VOR radionavigation aid will remain the primary navigation system for the foreseeable future; thus, aircraft will be required to carry VOR receivers within controlled airspace. Other radionavigation aids will also be used, in various states of deployment. These aids are the MLS/ILS, LORAN-C, differential operation of GPS, and the aircraft baro-altimeter.

Messages sent by the satellite originate in two areas. One part of the satellite signal originates in the satellite itself, but the navigation message is uploaded from the ground. The actual message content is discussed in Section 3.1.1 and in Appendix A.

The GPS receiver performs two basic measurements, pseudorange and its rate of change. Both measurements are performed in the receiver using code and carrier loops respectively. The quality of the received signal is monitored by regular parity checks, using a built-in-test (BIT) at three levels: receiver, microprocessor and interface (see Table 3-1), and by the decoded advisories in the message data content.

Receiver performance is assured by good engineering designs incorporating self-testing and self-monitoring features, and by a method in which a balance between parallel channels is maintained.

Routine signal checks are summarized as follows:

- The parity check is capable of detecting up to three-bit errors in the received message by checking six-bit Hamming parity bits.
- Signal presence is checked by matched filter output and by signal tracking.
- Frequency is checked by a carrier loop.
- Pseudorange is checked by a code loop.
- Synchronization is checked by a HOW word and Z-count.
- Consistency of data is checked by comparison of old and new navigation solutions and Figure of Merit values.
- Overall quality of the signal and data are checked using telemetry and health bit advisories, including almanac\* data.

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\*A reduced-precision subset of clock and ephemeris parameters repeated at 25-page intervals (12.5 minutes) and uploaded at least once every six days. The expected accuracy of almanac data is 1000 to 2500 meters, 1 sigma. Each almanac datum is accompanied by its 8-bit health status.

The most essential fault monitoring data are contained in the decoded health messages and advisories. Timeliness or availability of this data is important to the system integrity; thus, the navigation message is discussed in considerable detail.

### 3.1.1 Navigation Message Data Format

Satellite signals are received at a data rate of 50 bits per second. A simplified data format is shown in Table 3-2. The 1500 bit-long frame is subdivided into five subframes each having ten 30-bit words. The ten 30-bit words for each subframe represent 6 seconds; five subframes represent 30 seconds for each frame. Thus, 25 frames in each almanac represent 12.5 minutes.

The five data subframes contain four major parts of data:

Subframe 1 - Clock Corrections

Subframe 2-3 - Ephemeris Data

Subframe 4 - Ionospheric Data (6 versions)

Subframe 5 - Almanac Data (2 versions)

A detailed description of data formats is shown in Appendix A. The most significant data are in Subframes 1 through 3. These data repeat with the latest updates in every subframe or frame, as specified by design. Subframes 4 and 5 are subcommutated 25 times each and recycled once every 12.5 minutes.

The most critical data in the format are contained in TLM and HOW words. TLM and HOW provide data updates, synchronization and status evaluation with every subframe, or within 6-second intervals.

The message data format is derived jointly by the satellite processor and ground control. Both TLM and HOW words (including parity bits) are generated by the satellite, and the eight remaining data words in each subframe - words 3 to 10 - are generated, along with their parity, by the control center.

### 3.1.2 Fault Detection Criteria

The navigation message provides a continuous flow of information: a 14-bit telemetry message; critical timing; signal and data health status in 6- and 8-bit health messages, and other signal advisories as required:

TLM Message - The TLM message contained in 14 bits provides information on the signal upload status, diagnostics and monitoring status. It is used by monitoring stations to verify the updated data and satellite uploads, and by users to establish an FOM\* number. The message format can be seen in Appendix A.\*\*

HOW Used with Configuration - The HOW word repeats every 6 seconds with the most recent data. The Z-count is contained in a 19-bit message truncated to 17 bits. This time-carrying information indicates the beginning of the next subframe. Two bits of the 19-bit truncated message provide synchronization. Both bits also provide the data update status if used with Subframe 4 of the given configuration message of the 25th page. This configuration is repeated at the slower rate of once every 12.5 minutes (the data page repetition rate). Table 3-3 summarizes information contained in the matrix of HOW and configuration combinations.

Six-Bit Health - The 6-bit health message consists of two parts: The most significant bit (MSB), which provides summary information on the good/bad status of the navigation data; and a 5-bit code on signal components. The 6-bit health information is included in the first subframe and renewed every 30 seconds. In addition, subframe 5 of the 25th frame repeats the 6-bit health status on all satellites in a single frame. Therefore, the 6-bit health status of a particular satellite is received every 30 seconds and every 12.5 minutes on all satellites at once.

Eight-Bit Health - The 8-bit health message is transmitted in the same data frame as the almanac data. The first three bits indicate the health of the navigation data, and the five bits of the message are the same as in the 6-bit health of a particular satellite. Almanac data with the corresponding health data are repeated sequentially for all satellites once every 12.5 minutes.

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\*Figure of Merit - indicates to what degree the Estimated Position Error agrees with the true position.

\*\*Further details of this information are contained in ICD-GPS-201.



User Range Accuracy (URA) - URA predictions are a statistical indicator of pseudorange accuracy. The URA exists in Subframe 1 and corresponds to the maximum value anticipated under a uniform level of Selective Availability during each frame interval. URA is interpreted as a "no better than" ranging accuracy. The value of URA\* includes errors in satellite position related to orbit plane and trajectory estimates, timing, and perturbations, as well as Selective Availability. The predicted accuracy of the curve fit (not of the absolute position) will not degrade below 0.4 meters for up to three hours without regular one-hour updates. Accuracy values are given in four bits. For example, N = ...1, ...3, 5 ...15, with corresponding accuracies better than 2.8 ... 5.7 ... 11.3 meters...,\*\* respectively.

Age of Data, Clock (AODC) - The AODC indicates the GPS time-of-week by means of an 8-bit issue number for which corresponding parameters were estimated. These data are contained in Subframe 1 and thus are repeated with every frame. New data on correction parameters are changed at 1-hour intervals in the satellite following the first day of upload and every four hours from then on, up to 14 days if no new upload is available.

Age of Data, Ephemeris (AODE) - The AODE is given an 8-bit issue number and represents the time difference between the ephemeris data reference time ( $t_{oe}$ ) and the time of the last measurement update ( $t_L$ ) used to estimate representation parameters. The  $AODE = (t_{oe} - t_L)$  modulo  $2^{14}$  is transmitted at the beginning of the ephemeris data in Subframe 2 and at the end of the ephemeris data in Subframe 3. Whenever the AODE of both subframes differ, the ephemeris data of that entire frame is not used for satellite position estimation.

---

\*One sigma error predicted in the pseudorange measurement based on measurements at the monitoring stations and evaluated at the control center for four-hour and six-hour curve fit intervals by a Kalman filter.

\*\*IRN-200 NC-002, paragraph 20.3.3.3.1.3.

corrected data, TEL messages, health bits, URA, and the 18th and 19th bit advisories of the truncated Z-count, in conjunction with an appropriate configuration as given in the 25th page of Subframe 4 (see Appendix A).

### 3.1.3 Fault Detection Summary: The User Receiver Alone

The GPS user may detect system faults by receiving warning advisories from the decoded health messages, or by inferring failure status from either an unrecognizable code or the receipt of no signal at all. The withholding of the signal is initiated by ground control or by a built-in protection in the satellite. Satellite-provided protection was discussed previously in Section 2.2 and control segment protection was discussed in Section 2.3.

Table 3-4 summarizes receiver fault identification capability to warn the user of detected system malfunctions. The most essential information is provided in a single bit (Z-count bit 19) in the overall navigation message status report (good/bad) within 6 seconds. The status of a particular data message or signal is provided within 30 seconds (6-bit health). The status of all satellites is provided every 12.5 minutes. The 6-bit health (25th page of Subframe 4) is provided instantaneously, and the 8-bit health (Subframe 5) is provided sequentially for each satellite.

Slowly varying drifts can be detected only over longer time intervals, and only when observed from a fixed location such as a monitoring station. Any change or correction in data, control station-originated command, or new upload would take up to 45 minutes. However, under emergency conditions, this interval could be shortened to less than 10 minutes.

Longer receiver warning intervals are associated with the reception of advisory and health messages. They are also associated with the decoding process, which requires time to receive a full data word or subframe. Faults for which there is an immediate warning in the receiver are a loss of carrier loop lock in case of a satellite clock failure, or satellite defaulting action due to the generation of unrecognizable code, or a parity failure.

The user receiver alone can detect:

- Message data updates and upload status - detected within 6 seconds.

- Advisory or anomolous results to be expected in measurement - detected within 6 seconds.
- Advisory on the expected pseudorange error - detected within 30 seconds (with a warning that data sent is worse than the data shown, provided in six seconds).
- Satellite signal and health status of satellites being tracked -detected within 30 seconds; including all other satellites together -detected within 12.5 minutes.
- Age of data - detected within 24 to 30 seconds.

The user receiver alone cannot detect:

- Satellite clock drifts
- User clock accuracy
- Satellite orbit position errors

### 3.2 THE USER RECEIVER WITH ENHANCEMENTS

Monitoring capability may be improved by additional processing in the navigation processor. Two areas of improvement may be identified. These are:

- Two-state error propagation
- Multiple solution processing

The enhancements above are proposed because more than one component of the user's solution should be used to improve the integrity of detection; i.e., error bias may not be always present in any particular component of the user's solution. Improvements in the position accuracy may be realized if more than one component of the solution is monitored from additional sources.

#### 3.2.1 Covariance Method in Two-State Error Propagation

The method proposed in this study uses a Kalman filter for determining the true values of the states.

TABLE 3-5. KALMAN FILTER IMPLEMENTATION

KALMAN

1. GAIN  $k_n = P_n(-) H_n^T (H_n P_n(-) H_n^T + R)^{-1}$
2. STATE  $\hat{x}_n(+) = \hat{x}_n(-) + k_n \left[ z - H_n \hat{x}_n(-) \right]$
3. COVARIANCE  $P_n(+) = P_n(-) - k_n H_n P_n(-)$

DIFFERENCE USED IN TWO COVARIANCE STATES

1.  $P_n(-) - P_n(+) = k_n H_n P_n(-)$

2. PROPAGATE AHEAD

$$\hat{x}_{n+1}(-) = \Phi \text{ END } \hat{x}_n(+)$$

$$P_{\text{END}}(-) = \Phi \text{ END } P_n(+) \Phi^T + Q$$

3. USE OF EPE AND FOM FOR COMPARISON

A two-state propagation of errors suggests that the a priori knowledge of the wayside coordinates or the landing point may greatly increase the accuracy of the track. The following parameters are involved:

1. Covariance and measurement error, which affect the gain matrix,  $k_n$ .
2. The state update, which depends on the gain matrix and residue in the measurement  $k_n(Z - H_n\hat{x}_n(-))$
3. Propagation, which is affected by the system dynamics or transition matrix, and state (source) noise  $Q$ .

All basic parameters affecting system performance are accounted for in this approach.

Using two-state error propagation, a pilot in the cockpit on his final approach may verify whether his cross-track error at the landing point is within safe limits.

### 3.2.2 Multiple Processing Method

A stable filter method (Bowen)\* is proposed for identifying a faulty satellite and is based on the available redundancy in satellite coverage at the time the fault occurs. This approach requires six or more satellites to identify the faulty satellite, although five satellites could provide information that one of the satellites had failed (but the satellite at fault could not be identified).

Of the total measurements, one individual measurement in the navigation processor is ignored consecutively, thus creating multiple filters. Differentiating these filters two at a time, results in matrix which reflects individual measurement contributions to the error estimate. If the Kalman filter is used for the navigation solution, state estimates can be used in forming the differences.

Large errors are easier to detect than small drifts. It is possible that an error can be detected and a faulty satellite identified with fewer than six satellites, as stated above. However, higher error thresholds may be required.

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\*Bowen, A.F., "Detection of GPS Satellite Clock Errors," ATM84 (4476-03)-8, Aerospace Corporation, March 13, 1984.

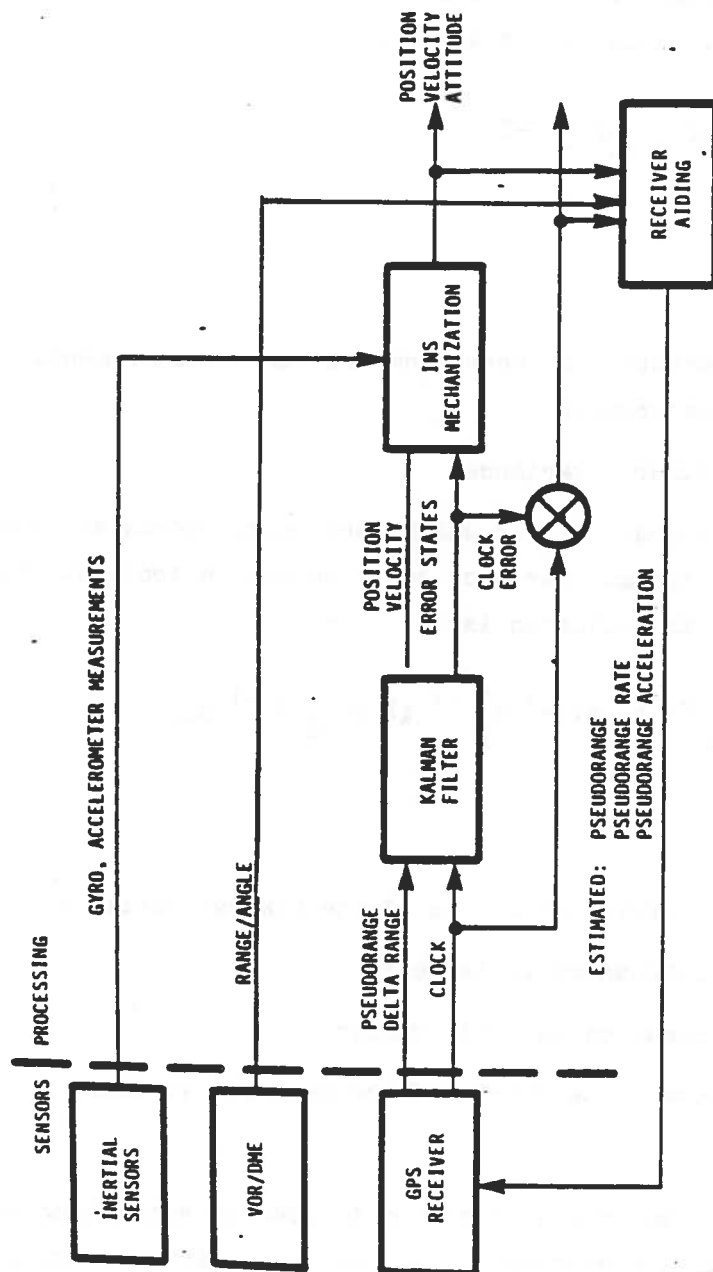


FIGURE 3-1. AIDED GPS MODEL

### 3.3.2 Aircraft Baro-Altimeter Aiding

The aircraft baro-altimeter may improve the reliability of the system in certain situations, depending on the geometry. Two solution methods will be demonstrated:

1. Solutions for two-dimensional and three-dimensional position estimates using an a priori altitude measurement with known variance.
2. Solution for a two-dimensional position estimate with three satellites and a perfect altitude measurement.

3.3.2.1 Two-Dimensional and Three-Dimensional Solution with Minimum Variance Estimate - The three-dimensional position solution can be readily obtained with four satellites under favorable geometry conditions. Using a conventional approach without altimeter inputs yields:

$$Hu = \Delta R$$

$$u = H^{-1} \Delta R$$

where

$H$  = predominantly a geometry matrix (directional cosines)

$$H = \begin{bmatrix} e_1^T & 1 \\ e_2^T & 1 \\ e_3^T & 1 \\ e_4^T & 1 \end{bmatrix}$$

$e_i^T$  = geometry matrix (directional cosines)

$u$  = error in position vector

$\Delta R$  = error in pseudorange.

Thus,

$$E(uu^T) = H^{-1} \text{Cov} \Delta R (H^{-1})^T$$

where

$E(u) = 0$  expectation in  $u$  (mean)

$E(uu^T)$  = covariance

By adding an a priori measurement  $P_0$  to the state estimate and assuming that both have gaussian noise distributions, the original estimate now becomes either the minimum variance Baye's estimate or the optimum Baye's estimate:

$$\hat{U} = (P_0^{-1} + H^T M^{-1} H)^{-1} H^T M^{-1} \Delta R$$

If there is little or no a priori information on baro-altimeter readings,  $P_0^{-1}$  is small and the matrix reduces to its original form. For an uncorrelated case in measurements,  $M$  is the diagonal matrix equal to  $\sigma_R^2 I$ .

Here, the variance of the a priori measurements was assumed to be  $\sigma_Z^2$ . Assuming only altitude dimension method by Brooks (1982), covariance  $P_0$  can be written as follows:

$$\begin{bmatrix} \infty & 0 & 0 & 0 \\ 0 & \infty & 0 & 0 \\ 0 & 0 & \sigma_Z^2 & 0 \\ 0 & 0 & 0 & \infty \end{bmatrix}$$

where  $\sigma_Z^2$  = known variance derived from previous measurements.

Thus, it can be demonstrated that a properly weighted a priori measurement may improve the three-dimensional navigation solution and is dependent on  $P_0$ ,  $M$ , and  $H$ . TSC at the present time is studying a technique to determine whether a priori altitude measurements can be obtained by establishing a bias error between an altimeter reading and the measured values at a level flight.

3.3.2.2 Two-Dimensional Solution with Three Satellites - The two-dimensional navigation solution uses perfect altitude with three satellites. The solution involves only a two-dimensional analysis with an unknown error in altitude variations:

$$\frac{(\hat{x} - x_i)\delta x + (\hat{y} - y_i)\delta y}{\hat{R} - ct} + ct = \Delta R_i$$



For a two-dimensional solution, only three satellites are required to estimate x, y and t, and z is assumed to be independently obtained from an altimeter input. A sample analysis presented for both the two-dimensional and three-dimensional cases is shown in Figure 3-2. This figure demonstrates that geometry will play a significant part in determining whether altimeter aidings during outages can provide an acceptable solution when a critical satellite fails.

### 3.3.3 VHF Omnidirectional Range Aiding

Very high frequency omnidirectional range radionavigation (VOR) aiding is discussed in Appendix B. VOR is used to structure domestic airspace for the continental U.S. (CONUS), with some guidance provided by the 842 Instrument Landing Systems (ILS) during approach and landing. The ILS service will be overtaken by 1250 planned Microwave Landing Systems (MLS) by the year 2000, with service sites distributed as shown in Figure 3-3.\*

The Area Radar Navigation (RNAV) will permit established navigation routes to preserve the system accuracy of VOR. The RNAV concept has been constrained to routes on which a flight check of the VOR transmitters has determined performance to be satisfactory. Therefore, VOR in its various forms will continue to serve as a radionavigation aid with 950 stations located throughout the CONUS, as shown in Figure 3-4. Of these 950 VOR stations, 112 will be test sites for the calibration and verification of aircraft VOR equipment, as shown in Figure 3-5.

Assuming that GPS will provide service as a supplementary system, aircraft in the controlled airspace would be required to carry a VOR receiver onboard along with GPS receiver. Therefore, both measurements from each receiver may be combined and used for monitoring purposes.

Performance may be improved by use of VOR measurements during the loss of a critical satellite. Providing an overdetermined solution may also improve performance by minimizing error variance in the position estimate.

The quality of the performance improvement will depend on the separation distance between a user receiver and the VOR transmitter. Two measurements are

\*DOT/FAA National Airspace System Plan, April 1984.

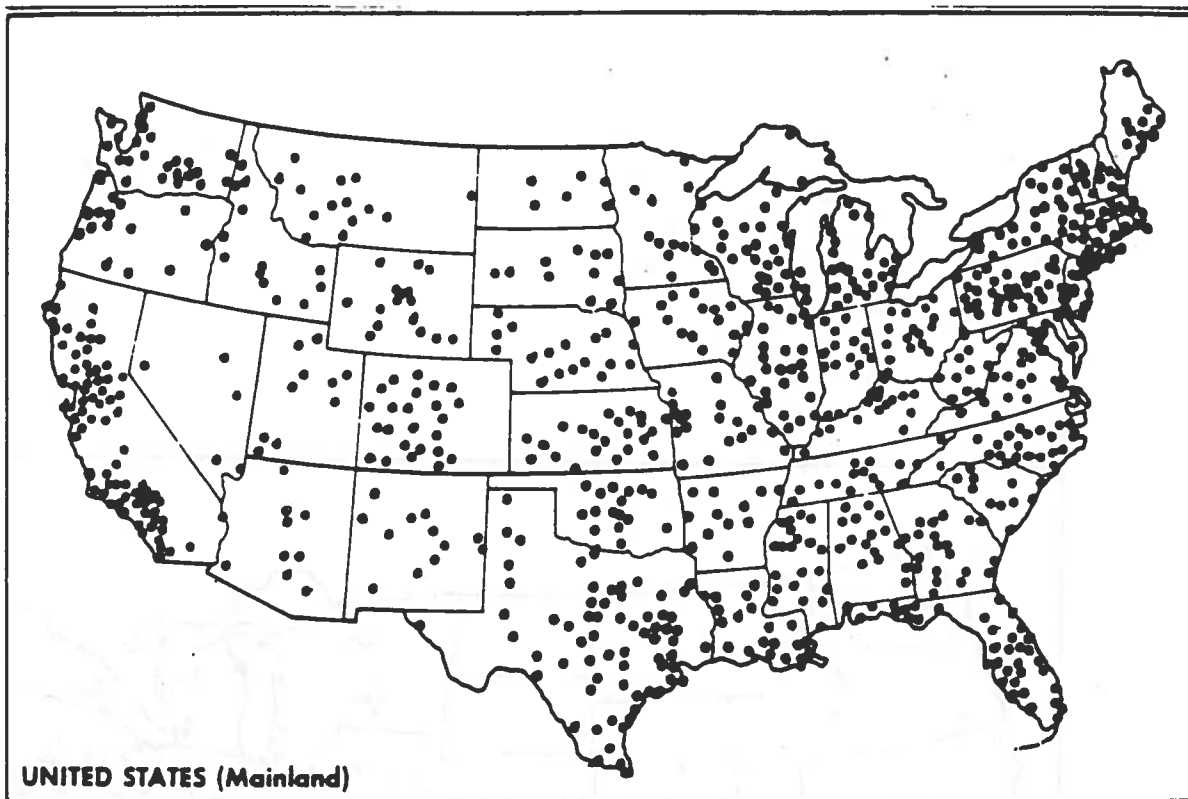


FIGURE 3-3. MICROWAVE LANDING SYSTEM SITES (1250 SITES BY YEAR 2000)

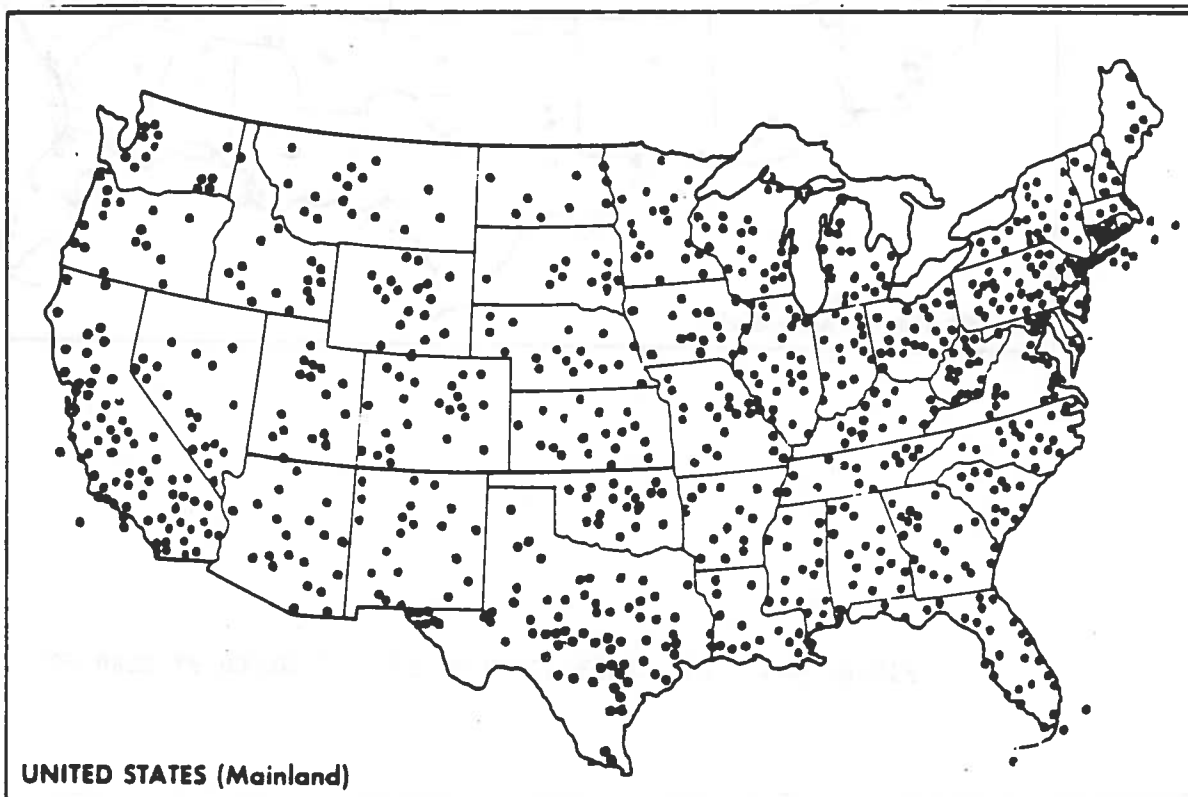


FIGURE 3-4. VHF OMNIDIRECTIONAL RANGE SITES (950 SITES BY YEAR 2000)

to the covariance with minimization potential similar to that discussed in Section 3.3.2.1. Then, using the variance minimization technique (which is possible when additional measurements are available), the resulting minimum error covariance matrix becomes:

$$P = [P_0^{-1} + H^T M^{-1} H]^{-1}$$

where

$$P_0 = \begin{bmatrix} \sigma_x^2 & 0 & 0 & 0 \\ 0 & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \infty & 0 \\ 0 & 0 & 0 & \infty \end{bmatrix}$$

and  $\sigma_x^2, \sigma_y^2$  are derived from measurements.

#### 3.3.4 Differential GPS Aiding

Alternative techniques may be used to implement differential GPS, as shown in Figure 3-6. All differential modes may be divided into two categories. In the first category, the processor receives satellite signals-in-space independently at both a user and a reference site. The estimated error at a fixed response site is then uplinked to the user over a suitable data link. In the second category of differential mode, satellite-originated signals are processed coherently when received over two paths directly as well as over the reference site. In the latter technique, the signal at the reference site is "reflected," and a time tag is added to the correction message and the result closely resembles relative navigation.

The monitoring of differential GPS system performance is recognized as one form of aided navigation. Performance integrity improvements are expected in the following areas, and will depend on the particular differential technique used:

- Bias error cancellation
- Reduction in GDOP
- Redundant measurements

- Additional health information
- Three-dimensional solution, with three satellites in relative-mode solution.

In the GPS solution, there are three ways in which the pseudorange measurements are used for position estimation:

- Intersection of four spheres (four pseudoranges, with one to each satellite).
- Intersection of three hyperboloids (three differences in times-of-arrival).
- Intersection of three ellipsoids (three satellites, with each satellite signal received by the user over two independent paths).

Figure 3-7 illustrates the signal reception geometry over two paths in synchronism, forming an ellipsoid to achieve complete common error cancellation and reduction in GDOP.

3.3.4.1 Common Error Cancellation - Common error cancellation in the differential mode is achieved in various ways, depending on whether the spatial or temporal correlation characteristic processing method is used.

A simplified analysis is presented to illustrate how the integrity of the system can be improved through the use of a relative differential mode. Using both the basic navigation equations derived earlier and the method of analysis developed by Mazur, Wong, and Mamen (1982), measurement and position errors are related by geometry of the user and constellation in use. The relationship is:

$$e_p = H^{-1} e_u$$

where

$e_p$  = position error in selected coordinates

$e_u$  = pseudorange measurement error

$H$  = directional cosine matrix referenced from the user to the satellites,  
as shown in Figure 3-8.

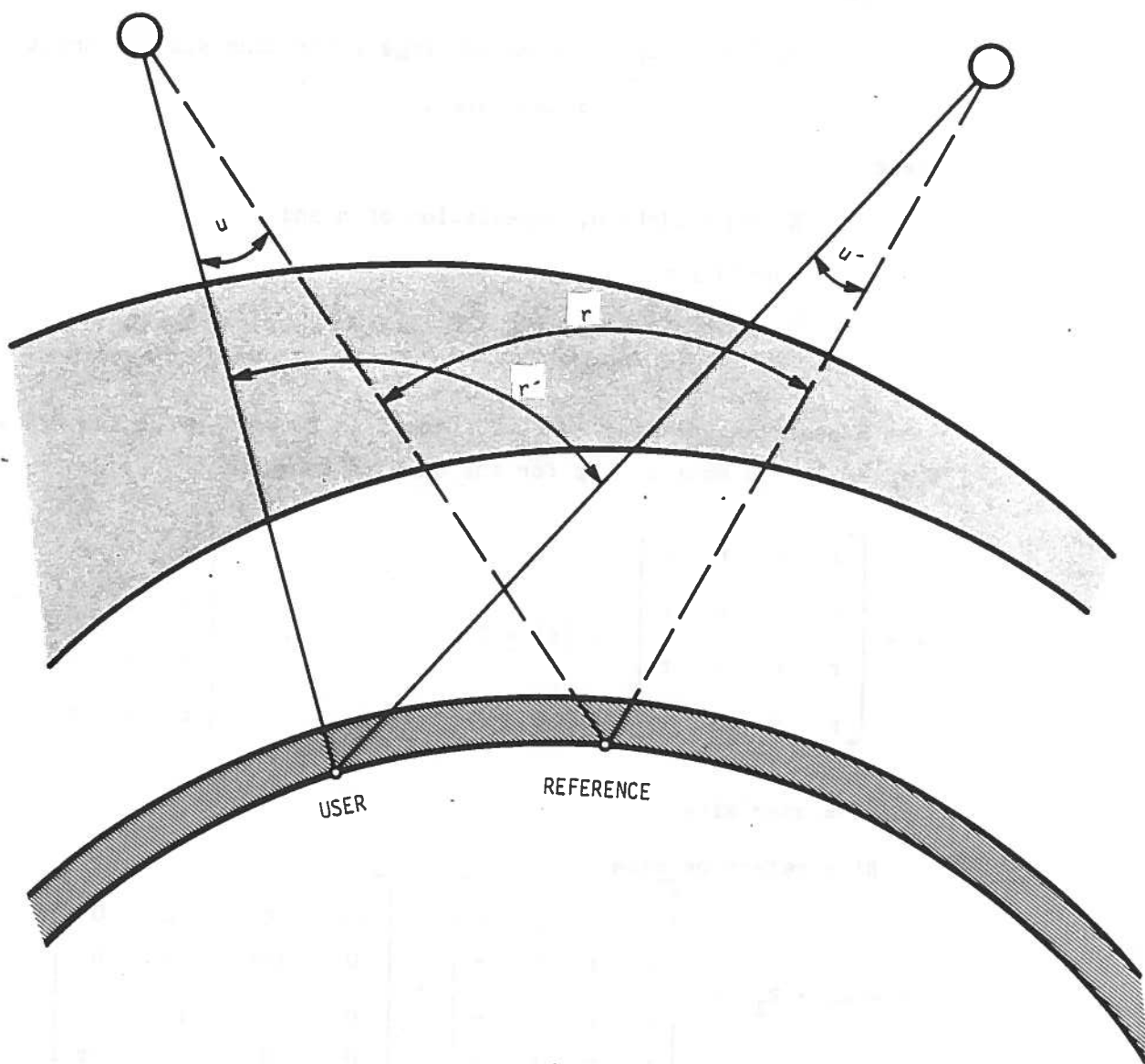


FIGURE 3-8. DIFFERENTIAL GPS GEOMETRY

For each user, the four-space vector error is defined as:

$$\underline{e}_G \triangleq \underline{x} - \hat{\underline{x}} = G^{-1} \hat{\underline{e}}_u$$

$$\underline{e}'_G \triangleq \underline{x}' - \hat{\underline{x}}' = G'^{-1} \hat{\underline{e}}'_u$$

$$\underline{\varepsilon}_G \triangleq \underline{e}_G - \underline{e}'_G = G^{-1} \underline{e}_u - G'^{-1} \underline{e}'_u$$

A working equation may be derived:

$$\text{Cov}(\underline{\varepsilon}) = E(\underline{\varepsilon} \underline{\varepsilon}^T) = \text{Cov}(H^{-1}(\underline{e}_u - \underline{e}'_u))$$

$$= H^{-1} \text{Cov}(\underline{\varepsilon}) (H^{-1})^T$$

$$= H^{-1} \text{Cov}(\underline{e}_u) (H^{-1})^T - H^{-1} \text{Cov}(\underline{e}'_u) (H^{-1})^T$$

where

$$\text{Cov}(\underline{e}_u) = \text{Cov}(\underline{n}) + \text{Cov}(\underline{c}) = \sigma_n^2 \underline{I} + \sigma_c^2 \underline{R}.$$

Therefore

$$\text{Cov}(\underline{\varepsilon}) = H^{-1} \text{Cov}(\underline{e}_u) (H^{-1})^T + H'^{-1} \text{Cov}(\underline{e}'_u) (H'^{-1})^T$$

$$- H^{-1} E(\underline{e}_u \underline{e}'_u^T) (H'^{-1})^T - H'^{-1} E(\underline{e}'_u \underline{e}_u^T) (H^{-1})^T.$$

Assuming  $r - r' \rightarrow 0$

$$\sigma_G^2 \approx 2 \left[ \sigma_n^2 + \underbrace{(1-r)\sigma_c^2 - (1-r')\sigma_c^2}_{\rightarrow 0} \right] \text{PDOP}^2 + 2\sigma_c^2 \underbrace{(r-r')}_{\rightarrow 0}$$

$$\sigma_G^2 \approx 2\sigma_n^2 \text{PDOP}^2$$

provided that both receivers have the same noise characteristics.

**3.3.4.2 Reduction in GDOP** - When a relative range between the user and the ground reference is made available to the user (usually through a time tag or ephemeris/range added to the error messages), the error in the position estimate may be reduced through the reduction of the relative GDOP. An analysis presented in published technical literature (Lee, 1983) claims that a three-dimensional solution with three satellites is possible.

According to this claim, the pseudorange measurement as derived from the slant range measurement is:

$$\rho_i = \text{true range} + \text{user clock error} + \text{UERE}$$

$$\rho_i = R_i + b_{u_i} + e_{u_i}$$

The reference position may or may not be known. The noise error is divided into two components (similar to Mazur, et al.):

$$e_{u_i} = \text{common noise} + \text{white noise.}$$

The same satellite signal at the user position and the reference position will result in two independent noise values (identified by subscripts u and r, respectively), as shown in Figure 3-9, in which:

$$\rho_u = R_u + b_u + n_u + c_u$$

$$\rho_r = R_r + b_r + n_r + c_r$$

$$\rho_u - \rho_i = \Delta\rho_i = \Delta R + \Delta b + N_i$$

= the difference in time of arrival, also equal to  
the difference in range + the difference in the user's  
clock + white noise

The time-of-arrival of the signal will differ because of the errors in both  
user clocks and the differences in range to the satellite:

$$T_u(t) = t + \frac{b}{c_u}$$

$$T(t) = t + \frac{b_r}{c}$$

$$T_r(t-\Delta t) = t - \Delta t + \frac{b}{c_u}$$

$$\Delta b = \left[ T_u(t) - T_r(t-\Delta t) \right] c - \Delta t \cdot c = \Delta T c - \Delta D$$

Using the original expression, the pseudorange difference becomes

$$\Delta\rho_i = \Delta R_i + \Delta T c - \Delta D + N_i$$

If the variables in the known and the estimated times-of-arrival are  
separated, the expression becomes:

$$\Delta\rho_i - \Delta T \cdot c = \Delta R_i - \Delta D + N_i.$$



where

$$e = \delta_x, \delta_y, \delta_z$$

$$N = N_1, N_2, N_3 \text{ (noise component)}$$

$$e = H_R^{-1} N .$$

Assuming that noise components are independent of each other and have the same variance, then

$$\sigma_G^2 = E[ee^T] = \sigma_N^2 (H_R^T H_R)^{-1} = \sigma_N^2 (\text{PDOP}_R)^2$$

This simplified expression will not hold if the noise variance is different from the UERE noise. In such cases, cross terms in the matrix should be considered in the position error estimate. Although the  $\text{GDOP}_R$  diagonal terms in the geometry matrix are reduced, the  $\text{GDOP}_R$  cannot be applied to the position error estimate in a straight way. This was done previously by assuming that all pseudorange noise components were the same and that noises were gaussian. Unless the same can be assumed here (that the noise contribution by the reference site had the same magnitude and gaussian distribution), the covariance solution would not be as simple.

### 3.4 GPS ENHANCED WITH INTEGRITY MONITORS

GPS operations are supported by five monitoring stations spread throughout the world. These monitoring stations are under complete control of the control center, to which they communicate exclusively. There is no direct communication link between the monitoring stations and GPS users. The only communication link possible for the users is through the control station in the form of satellite-transmitted messages. Supplementary monitoring sites using integrity monitors to establish such a direct link with the GPS users have been proposed.

#### 3.4.1 Local Area GPS Integrity Monitors: Introduction

The local area field monitoring concept is based on the enhanced differential mode design. Any differential site can easily be converted to perform GPS monitoring functions, although somewhat more complex hardware and software are necessary. A more sophisticated receiver to enable the detection of small changes in clock and signal power at extended time intervals will be required. A suitable ground-air data link would be an integral part of the differential operation with the capability of advising users within at least a 500 km effective range. Users could be addressed directly (discretely) or omnidirectionally (all call), with the addressing method determined by the differential mode implemented at the site. The pseudolite and some forms of relative reference modes would be broadcast with a direct address, but the baseline differential mode would broadcast omnidirectionally. The most practical location for the monitors could be at the Air Traffic Control (ATC) facilities throughout the CONUS.\* These facilities will be described in some detail.

The FAA's NAS plan,\* by the year 2000, proposes to consolidate all air traffic services in terminal and en route areas into 21 ACFs in the CONUS, as shown in Figure 3-10. These centers will combine both the 111 uniformly-distributed Air Route Surveillance Radars (ARSR) and the 213 Terminal Area Surveillance Radars (ASR). The ARSR site distribution is shown in Figure 3-11. The ACFs will be automated to various degrees.

Of all the terminal sites, 188 are either the terminal control radars TRACON or TRACAB, depending on the installation in which the radar is located. Some adjustments have been proposed for completion by the year 2000. From a set of 94 en route and 208 terminal radars, 197 sites will become Mode S transponders, compatible with distribution, as shown in Figure 3-12.

The NAS plan also establishes communication networks between ACFs and radar centers. A microwave communication link network is shown in Figure 3-13; a computer data link network is shown in Figure 3-14; the National Data Interchange Network (NADIN) is shown in Figure 3-15, and a CONUS-wide radio link is shown in Figure 3-16.

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\*DOT/FAA National Airspace System Plan, April 1984.

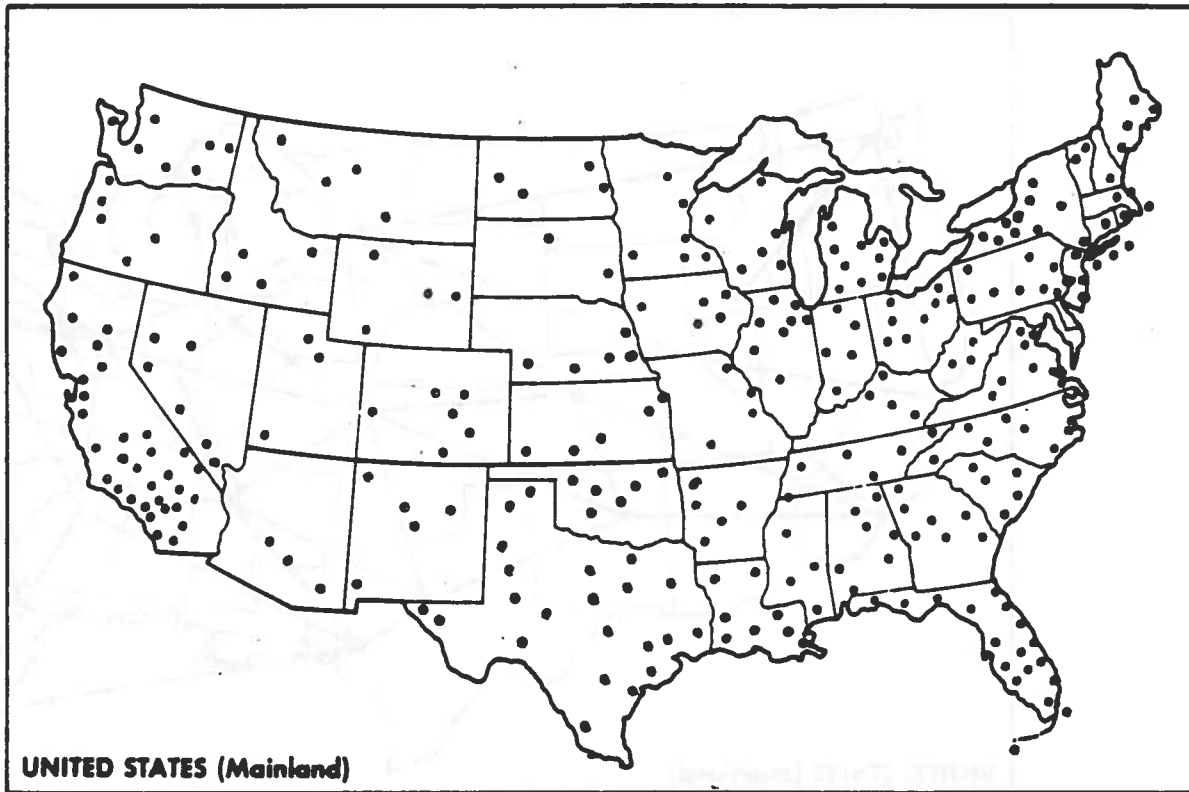


FIGURE 3-12. SURVEILLANCE RADAR SITE DISTRIBUTION BY YEAR 2000  
(INCLUDING 197 MODE S SITES)

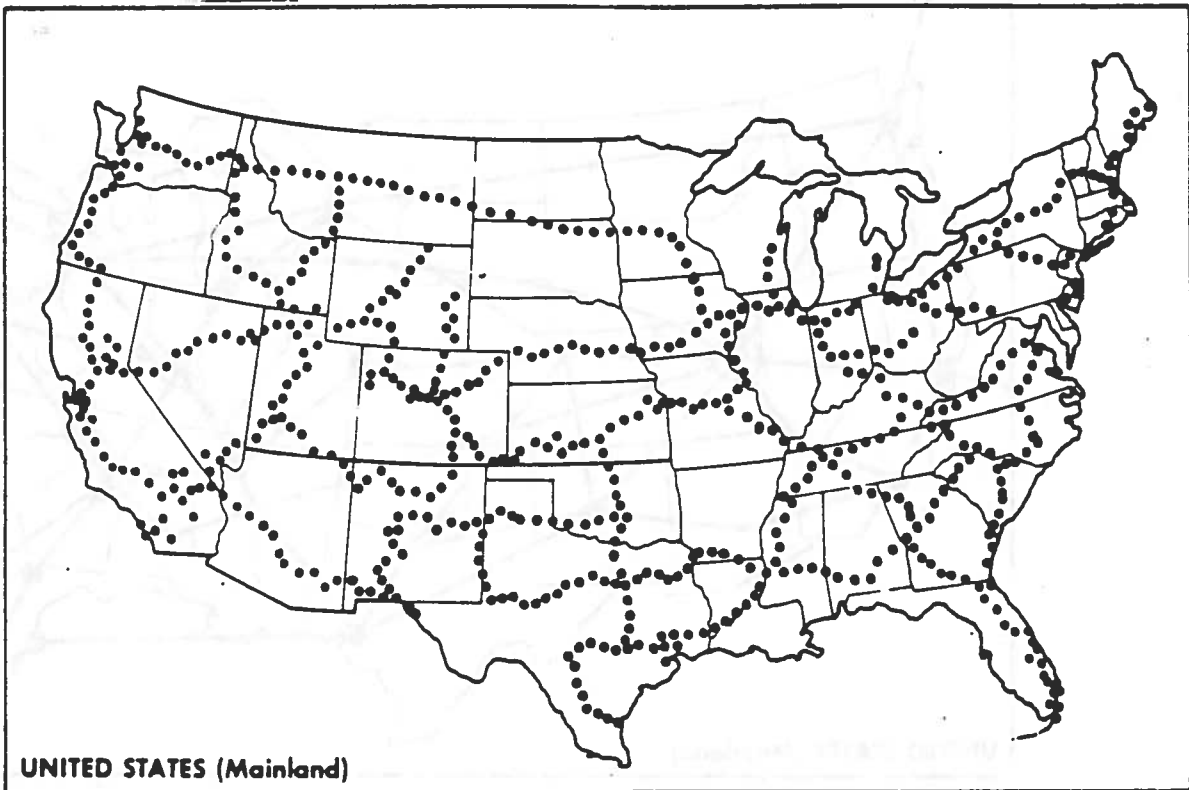


FIGURE 3-13. NAS MICROWAVE LINK

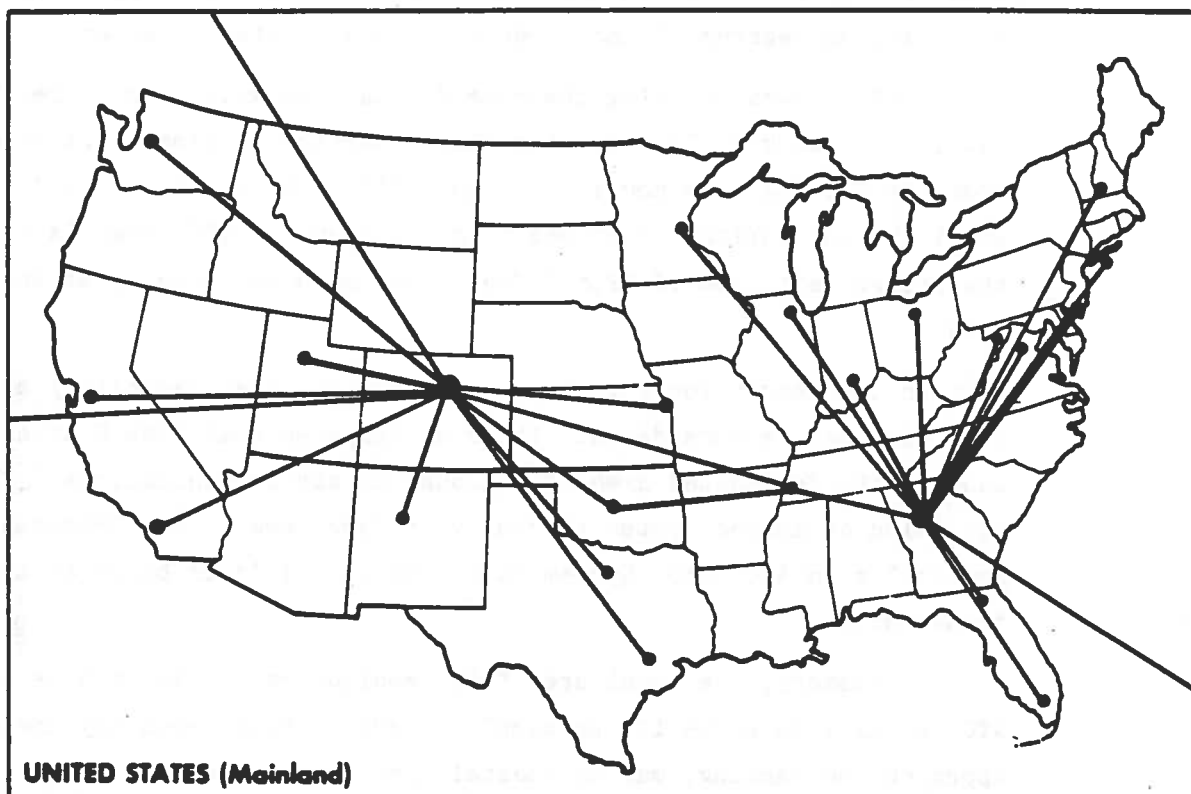


FIGURE 3-16. NAS RADIO LINK

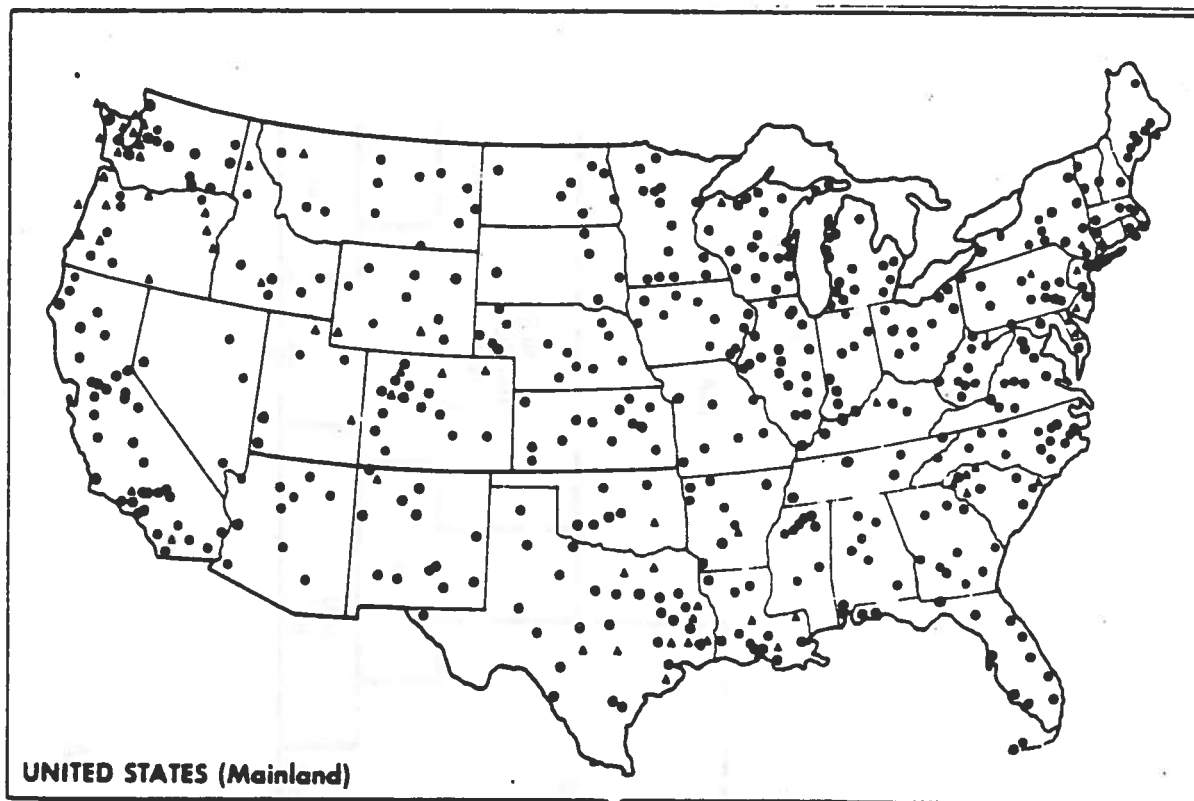


FIGURE 3-17. COMMERCIAL SERVICE AIRPORTS (650 TOTAL)

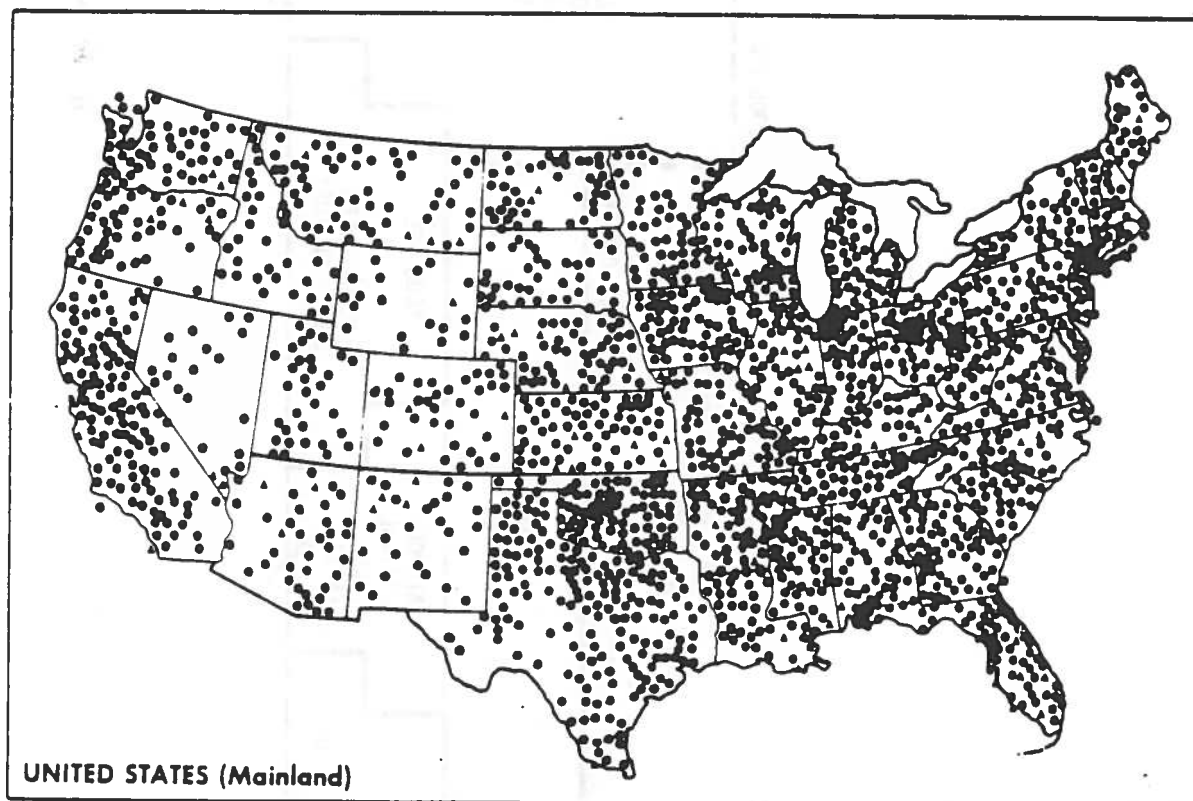


FIGURE 3-18. GENERAL AVIATION AIRPORTS (2866 TOTAL)

### 3.4.2 NAS-Integrated GPS Integrity Monitors

In this concept, the GPS integrity functions would be fully integrated with ACF operations, and could become an integral part of the NAS system. By the year 2000, service coverage would be from 6000 feet above mean sea level and would have a range characteristic to en route and terminal surveillance radars. Service area capability would include the CONUS, coastal zones, harbor and harbor approaches and offshore zones up to the altitudes compatible with ACF-provided control. Error messages and health advisories could be sent on in a manner similar to Mode S\* - UF-11, ALL CALL or UF-24, COM-C (ELM) data formats with an update intervals of four seconds in the terminal areas and 10 seconds in en route coverages. Handover between contiguous zones of various ACFs would be accommodated in accordance with Mode S boundary crossing protocol.

A NADIN or similar data link would connect all monitoring sites and would have a direct data link with ACF facilities and the GPS control center processor in Colorado Springs, CO. Figure 3-20 illustrates a typical site distribution of integrity monitor sites.

### 3.4.3 Low Orbit Satellite-Enhanced GPS Integrity Monitors

The GPS service coverage area may be improved to a zero altitude level anywhere in the CONUS and coastal zones. This is accomplished through the use of a low orbit satellite data link to communicate ground-derived uploaded corrections and advisories to all GPS users. It is proposed that sparsely located monitoring stations will upload error messages and health information using a GPS-coded upload channel at 1783.74 MHz or Mode S UF-24, Comm-C (ELM). Signal messages would return on the L1 (1575.42 MHz) frequency band, or on Mode S DF-24 (Comm-D) (ELM), separated between satellites by unique codes taken from the GPS Gold codes. These codes may be encrypted, if required.

The low orbit space segment may provide some tradeoffs or design options, depending on the ultimate service and coverage area requirements. For example, a constellation of five satellites in five planes in 1600 nm orbits would

\*Orlando, V.A. and P.R. Drouilhet, "Mode S Beacon System: Functional Description." DOT-FAA-PM-8318 (Project Report ATC-42, Revision C), July 15, 1983.

provide two-satellite visibility at all times. This design was conceived for extended TRANSIT and therefore may still have some supporting function with GPS. Similarly, six satellites in six planes in a 1200 nm orbit would provide three-satellite coverage over the CONUS and two-satellite coverage from 0° to 70° latitudes. The geometrical layout of the proposed system in the GPS environment is shown in Figure 3-21.

#### 3.4.4 GPS Enhanced with Integrity Data Link

A local area field monitor may be enhanced by a direct integrity data link with the GPS satellites. A few integrity sites will collect and evaluate the health status of the satellite messages from a widely-distributed integrity monitoring network over the CONUS. The centralized network modes selected for direct communication with the satellite are not intended to replace the functions of the control segment; the codes are to be used to improve integrity in a limited manner and to extend the average satellite visibility for the three globally-distributed uploaded stations from 89 percent to 100 percent over the CONUS. The health data control would consist of a warning flag for the navigation message frame, or the initiation of a default action by shutting down the satellite. This could be done by commanding a transmission of an unrecognizable C/A code in a case of gross error before the central processor either verifies and corrects the error, or reinstates the navigation message transmission.

It is estimated that it would take from 5 to 10 minutes for any command or unscheduled upload issued by the control center in an emergency to reach the user. An additional delay may be incurred due to visibility conditions. The nodal centers will enhance GPS visibility and integrity using independent commands, as well as commands approved or even requested by the control center. A ground communication network such as NADIN will link all nodes and integrity monitors and join with the ground control at Colorado Springs, CO. A change in the health status protocol, based exclusively on Block II satellites, may allow the use of bit-18 and bit-19 of the Z-count to convey ground-derived integrity monitor messages. GPS service frequencies are used for uplink at a frequency of 1783.74 MHz and for downlink at 2227.50 MHz. A proposed system layout is shown in Figure 3-22.

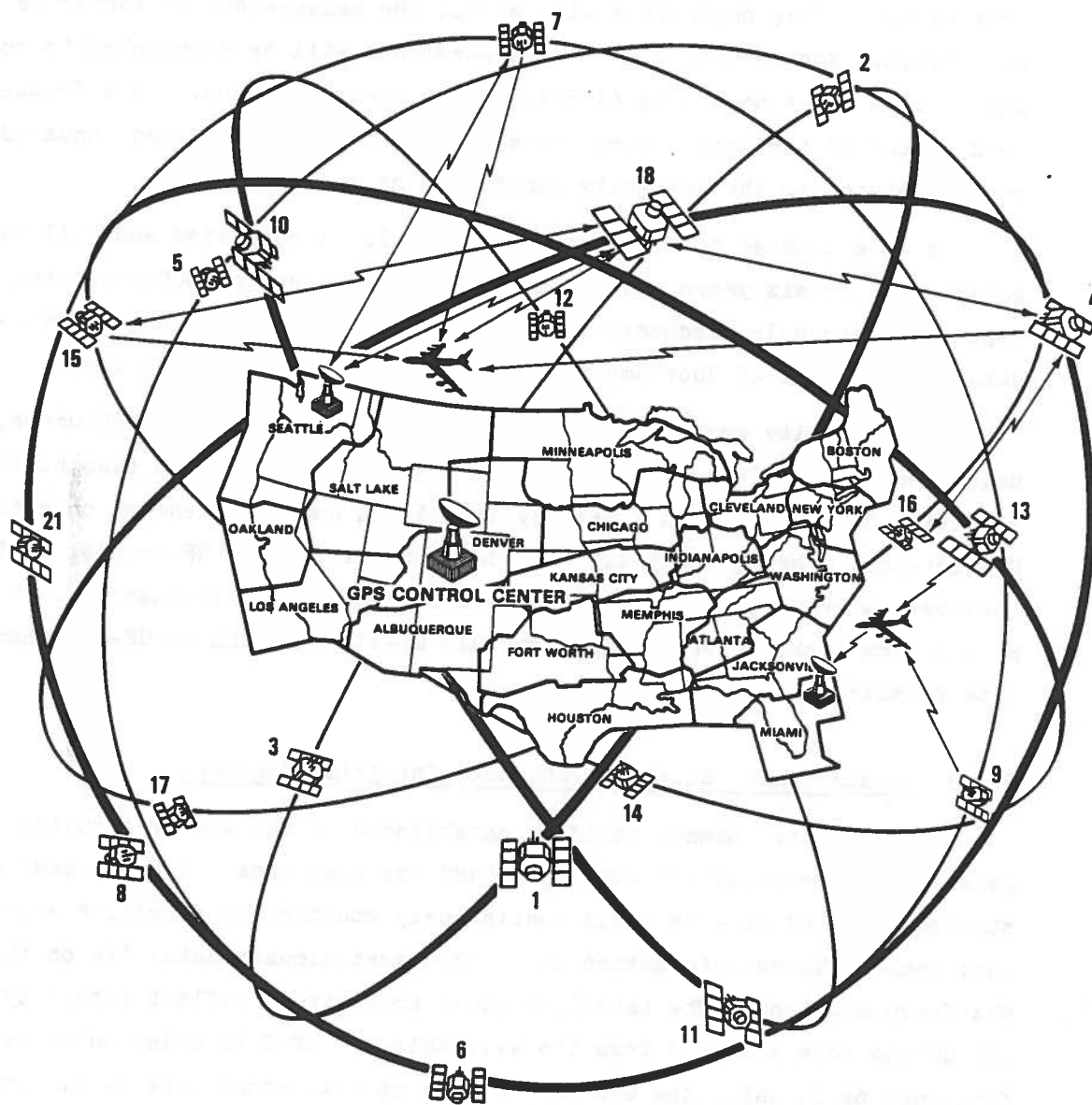


FIGURE 3-22. GPS ENHANCED WITH INTEGRITY DATA LINK SYSTEM LAYOUT



TABLE 3-6. GPS 21-SATELLITE REDEPLOYMENT

NEW SATELLITE PLACED IN ORBIT	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43
NUMBER AFTER OCTOBER 1986	0	1.5	4.0	6.0	7.5	8.5	10.0	11.5	14.0	15.5	17.5	19.0	20.0	21.5	23.0	24.5	26.0	27.0	28.5	30.0	30.5	32.0	33.0	34.0	35.5	36.5	37.5	41.0	48.0	54.0	59.0

45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
64.0	69.0	71.0	76.0	79.0	82.0	84.0	87.0	89.0	92.0	94.0	97.0	99.0	101.0	103.0	106	108	110	112	114	116	118	121	123	125	128	130	133	136	139	142	146	149	152	155	158

December  
1999

SATELLITES AVAILABLE AS OF OCTOBER 1986 - 12  
 SIZE SET OF SATELLITES PROVIDED (PROXIMATE) - 28  
 ADDITIONAL SATELLITES REQUIRED FOR  
 REPLACEMENT - 50  
 TOTAL NUMBER REQUIRED AS OF OCTOBER 1999 - 80

SATELLITE LIFE EXPECTED - 7.5 YEARS  
 90% MISSION DURATION - 6.2 YEARS

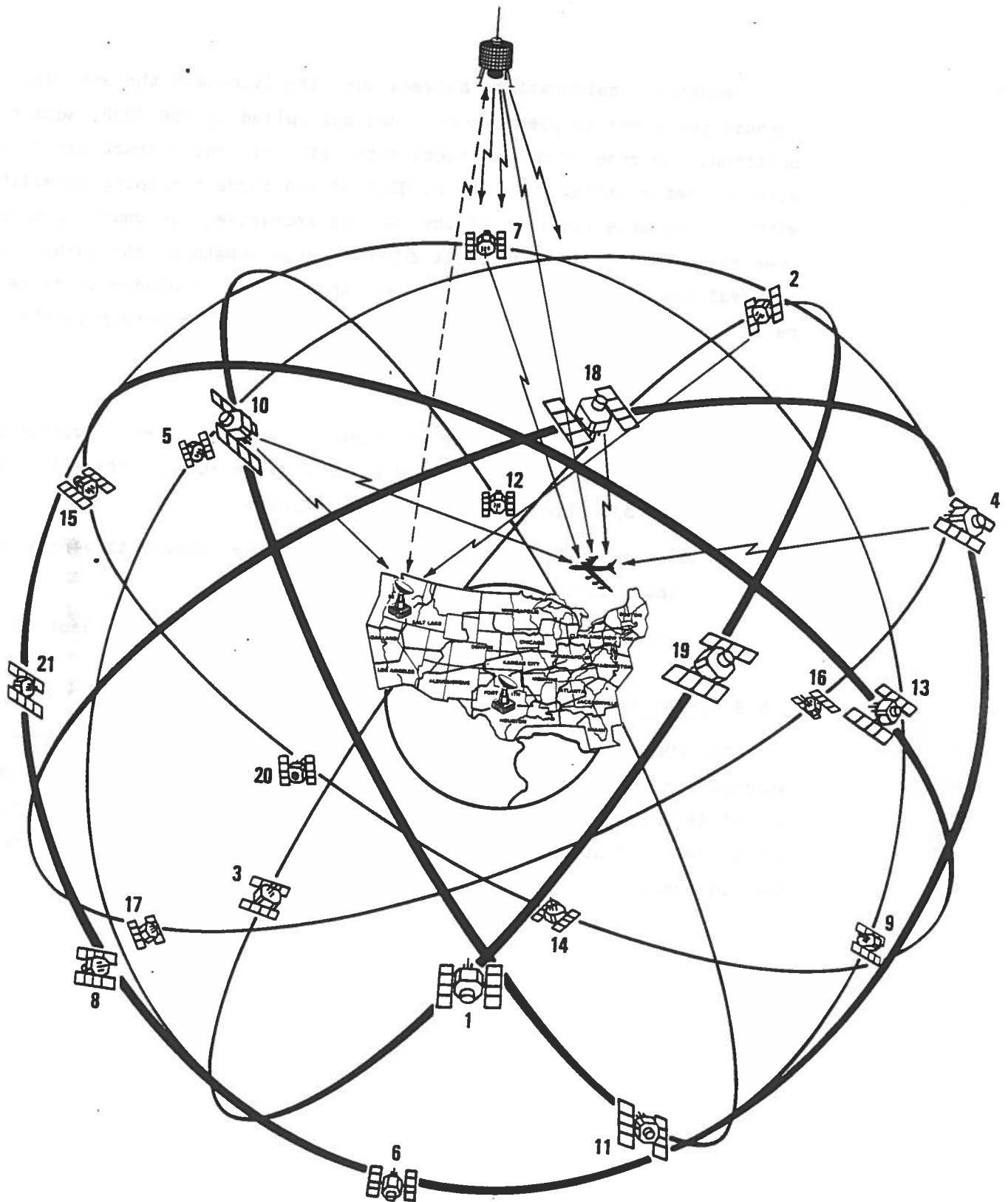


FIGURE 3-23. GEOSTATIONARY SATELLITE-ENHANCED INTEGRITY MONITOR SITE DISTRIBUTION

## 4. CONCLUSIONS AND RECOMMENDATIONS

### 4.1 CONCLUSIONS

Several conclusions may be drawn from the analysis of GPS compatibility with civil user requirements and the assessment of GPS navigation and integrity performance capabilities, limitations, and growth potential based on the monitoring capability of user equipment presented here. The GPS C/A code signals-in-space may provide a reliable worldwide navigation grid for air, marine and land users. Both navigation and integrity performances of this service can be continuously verified by user equipment with only a few exceptions. These exceptions are: user clock accuracy; slow drifts of satellite clock, power, and satellite position error; differences between a derived prediction of orbit and the satellite true position (error  $\pm 1.5$  meters, 1 sigma); and portion of the atmospheric delay of the signal path not accounted for by the use of an atmospheric model for the C/A code single-frequency users.

Performance assessments are based on the reliability of the C/A code signals, built-in self-protection in the design, and advisories sent in the navigation message content.

When user requirements are compared with GPS performance - for example, with the FAA requirements, particularly the ten-second integrity requirement listed in Table 4-1 - the crucial factor is the performance capability of the user equipment. It should be noted here that these FAA requirements refer specifically to warning the user when the system performance is out-of-tolerance (i.e., the error is greater than 100 meters 2 drms), not to the failure of any system function, element, or satellite, as is often misunderstood. Even a satellite failure does not constitute a system failure, contrary to the VOR concept, in which a local area is dependent solely on a single system. GPS has a certain degradation capability.

A GPS user is protected by system design, but protection is not in real time. Health bits, advisories and accuracy predictions are based on information collected by the monitoring stations, processed by central control, and ultimately require minutes to reach the user. Only certain information can reach the user in ten seconds: the protection initiated either by the satellite

(contained in TEL and HOW words) or by the satellite default capability/ contiguous subframe synchronization; the warning on bad data to follow (special indication if worse than URE); and the generation of unrecognizable codes. A realistic improvement over the present system can be achieved by the use of integrity monitors, and by the establishment of a direct link with the satellite. Advising users via the NAS system or low-orbit commercial satellites is another improvement alternative.

In response to air systems, specifically in regard to the FAA integrity requirements, the GPS may have the capability of warning users within ten seconds when navigation performance is out or suspected to be out-of-tolerance. However, 30 seconds or even 12.5 minutes may be necessary to fully meet the integrity requirement by identifying the cause of failure. Slowly degrading performance changes can be extrapolated and are not critical. The corrective steps to bring the system back to full operation or to warn users in advance can be achieved in a safe time interval by available TEL messages at six-second intervals.

#### 4.2 RECOMMENDATIONS

Future studies should address in depth the feasibility of fully integrating GPS functions into the NAS system. This would include:

- Integration of a differential GPS function.
- Use of Mode S for distributing GPS error messages and advisories in a controlled airspace.
- Establishment of a CONUS-wide coordinated coverage network, including a data link with the control center.
- Collocation of integrity monitors at NAS facilities.
- Development of a GPS function compatible with VOR, MLS, ILS and/or LORAN-C measurement inputs.

Additional studies should address system enhancements and tradeoffs (still within the NAS system concept) to improve GPS integrity and visibility. The areas of study should include:

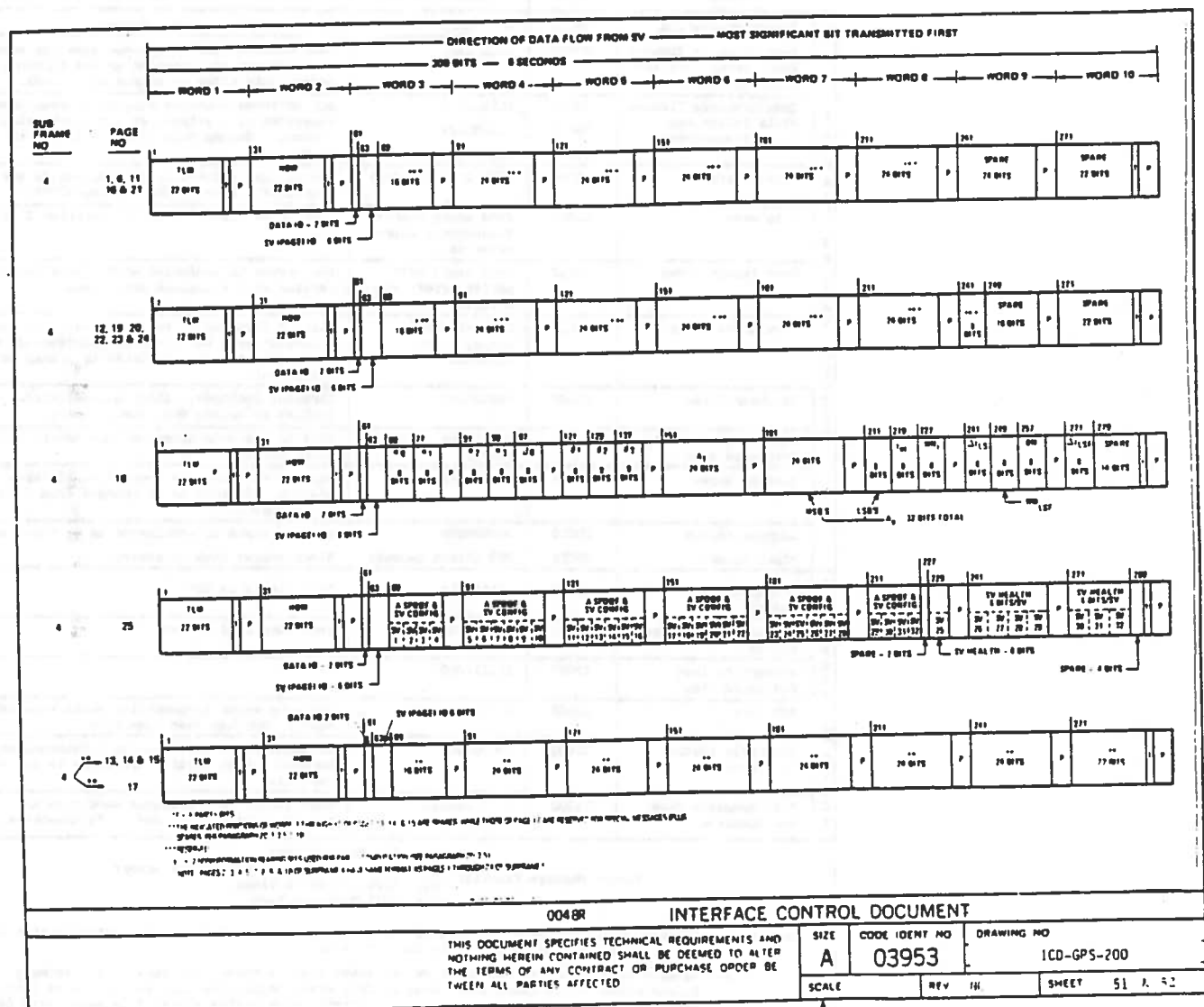
APPENDIX A  
GPS NAVIGATION DATA MESSAGE FORMAT

A unique C/A code is assigned to each GPS satellite. The C/A code satellite signals are transmitted on the L1 1575.42 MHz frequency. The signal is encoded at a chip rate of 1.023 MHz using biphase PSK modulation. Then, the 50 bps data is modulated in the PN (Gold) Codes.

The message structure utilizes a basic format of a 1500-bit frame made up of five subframes. Each subframe is 300 bits in length. The data sets transmitted by the satellite in Subframes 1, 2 and 3 have a transmission period of one hour (data content may differ during each hour to reflect updated clock and ephemeris information) during the first day of upload, and four hours during the second through fourteenth days. Subframes 4 and 5 are subcommutated and are referred to as Pages 1 to 25 of each subframe. The data content of each subframe is presented as follows:

- A-2 Data Formats of Subframes 1, 2, 3 and 5 (2 versions)
- A-3 Data Format of Subframe 4 (6 versions)
- A-4 Telemetry Message Content
- A-5 8-Bit Navigation Data Health Indication
- A-6 Brief Summary of 25-page Data
- A-7 6-Bit/8-Bit Health Common Code for Signal Components.

## DATA FORMATS, CONTINUED



## 8-BIT HEALTH INDICATIONS

### 8-BIT NAV Data Health Indications

BIT POSITION IN PAGE			INDICATION
137	138	139	
0	0	0	ALL DATA OK
0	0	1	PARITY FAILURE - some or all parity bad
0	1	0	TLM/HOW FORMAT PROBLEM - any departure from standard format (e.g., preamble misplaced and/or incorrect, etc.), except for incorrect Z-count, as reported in HOW
0	1	1	Z-COUNT IN HOW BAD - any problem with Z-count value not reflecting actual code phase
1	0	0	SUBFRAMES 1, 2, 3 - one or more elements in words three through ten of one or more subframes are bad.
1	0	1	SUBFRAMES 4, 5 - one or more elements in words three through ten of one or more subframes are bad.
1	1	0	ALL UPLOADED DATA BAD - one or more elements in words three through ten of any one (or more) subframes are bad.
1	1	1	ALL DATA BAD - TLM word and/or HOW and one or more elements in any one (or more) subframes are bad.

# 6-BIT/8-BIT HEALTH COMMON CODE FOR SIGNAL COMPONENTS

0	0	0	0	0	— ALL SIGNALS OK
0	0	0	0	1	— ALL SIGNALS WEAK (i.e., 3 to 6 dB below specified power level due to reduced power output, excess phase noise, SV attitude, etc.)
0	0	0	1	0	— ALL SIGNALS DEAD
0	0	0	1	1	— ALL SIGNALS HAVE NO DATA MODULATION
0	0	1	0	0	— L <sub>1</sub> P SIGNAL WEAK
0	0	1	0	1	— L <sub>1</sub> P SIGNAL DEAD
0	0	1	1	0	— L <sub>1</sub> P SIGNAL HAS NO DATA MODULATION
0	0	1	1	1	— L <sub>2</sub> P SIGNAL WEAK
0	1	0	0	0	— L <sub>2</sub> P SIGNAL DEAD
0	1	0	0	1	— L <sub>2</sub> P SIGNAL HAS NO DATA MODULATION
0	1	0	1	0	— L <sub>1</sub> C SIGNAL WEAK
0	1	0	1	1	— L <sub>1</sub> C SIGNAL DEAD
0	1	1	0	0	— L <sub>1</sub> C SIGNAL HAS NO DATA MODULATION
0	1	1	0	1	— L <sub>2</sub> C SIGNAL WEAK
0	1	1	1	0	— L <sub>2</sub> C SIGNAL DEAD
0	1	1	1	1	— L <sub>2</sub> C SIGNAL HAS NO DATA MODULATION
1	0	0	0	0	— P SIGNAL WEAK
1	0	0	0	1	— P SIGNAL DEAD
1	0	0	1	0	— P SIGNAL HAS NO DATA MODULATION
1	0	0	1	1	— C SIGNAL WEAK
1	0	1	0	0	— C SIGNAL DEAD
1	0	1	0	1	— C SIGNAL HAS NO DATA MODULATION
1	0	1	1	0	— L <sub>1</sub> SIGNAL WEAK
1	0	1	1	1	— L <sub>1</sub> SIGNAL DEAD
1	1	0	0	0	— L <sub>1</sub> SIGNAL HAS NO DATA MODULATION
1	1	0	0	1	— L <sub>2</sub> SIGNAL WEAK
1	1	0	1	0	— L <sub>2</sub> SIGNAL DEAD
1	1	0	1	1	— L <sub>2</sub> SIGNAL HAS NO DATA MODULATION
1	1	1	0	0	— SV <u>IS</u> TEMPORARILY OUT ~ do not use this SV during current pass
1	1	1	0	1	— SV <u>WILL BE</u> TEMPORARILY OUT ~ do not use this SV during period for which almanac is valid
1	1	1	1	0	— SPARE
1	1	1	1	1	— SPARE

SIZE  
A

CODE IDENT NO  
03953

DRAWING NO  
ME08-00002-400

SCALE

REV G

SHEET 51 OF



APPENDIX B  
RADIONAVIGATION AIDS

B.1 AIRBORNE VHF OMNIRANGE (VOR) SYSTEM DESCRIPTION

The VOR/VHF omnirange system operates in the 108.0 to 117.95 MHz frequency band. The ground station transmits a radio frequency (RF) signal with two 30-Hz modulated signals.

The relative phase of the two 30-Hz signals defines the radial lines in space with respect to the ground station (see Figures B-1 and B-2). The VOR ground station antenna is normally aligned so that the 0 deg radial of the antenna agrees with magnetic north, as shown in Figure B-1.

Each VOR transmits a three-letter identity code. The code provides azimuth information for aircraft with an error not greater than  $\pm 3$  percent and with a probability of 95 percent. For the coarse alignment the accuracy is within  $\pm 1$  degree.

Signals from stations located in rough terrain or surrounded by obstructions can degrade to lower performance levels. For example, certain propeller rpm settings for light aircraft can produce VOR course deviation indicator fluctuations up to  $\pm 6$  degrees. Helicopter motor speeds may also cause VOR course disturbances. When VOR is combined with Tactical Air Navigation (TACAN), the resulting system is called collocated VOR and TACAN, or VORTAC. VORTAC has capability in frequency channel arrangements and provides VOR azimuth, TACAN azimuth and distance.

VOR/VORTAC and TACAN aids are classed according to their operational uses. There are three classes: T (Terminal), L (Low Altitude), and H (High Altitude). The normal service range for the T, L, and H class aids are shown in Table B-1.

B.2 DME - DISTANCE MEASURING AID DESCRIPTION

Operating on the line-of-sight principle, DME uses frequencies in the UHF spectrum between 692 and 1213 MHz. Air-to-ground interrogations occur at a fixed frequency band from 1025 to 1150 MHz. Replies, displaced by 63 MHz and delayed by 50 microseconds, are generated by the ground station in the 962 to 1213 MHz frequency band.

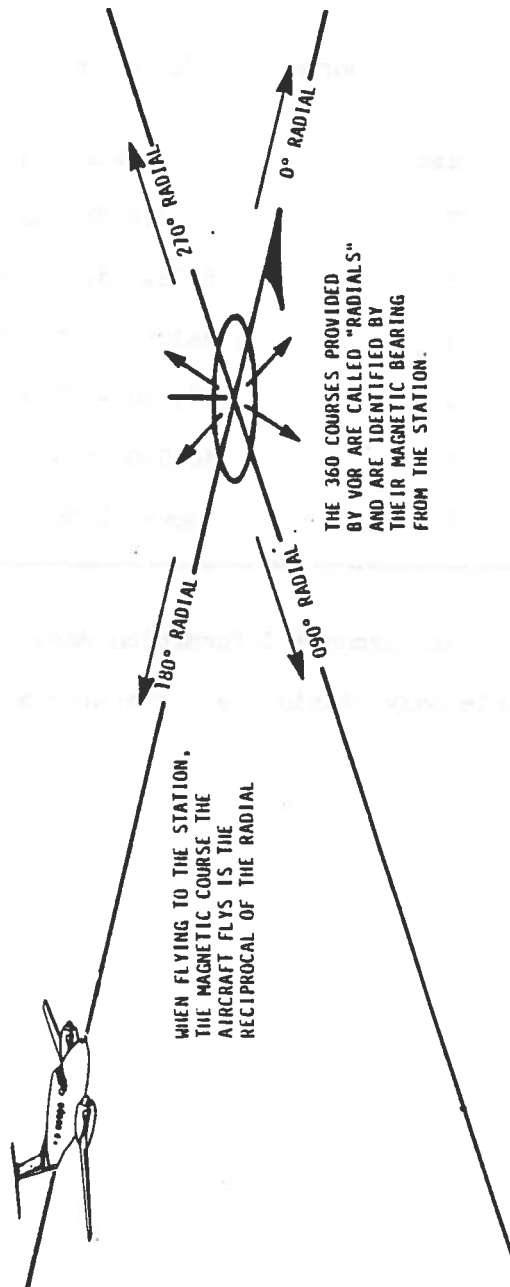


FIGURE B-2. SIGNAL PHASE ANGLE RELATIONSHIP

Slant range is measured by a pair of pulses, each lasting 3.5 microseconds and spaced by 12.5 microseconds, with a variable repetition rate ranging between 5 pulse-pairs/sec up to 150 pulse-pairs/sec. The same spacing is maintained for the downlink and uplink, but at different frequencies. DME operational range is 200 nm at line-of-sight altitude with an accuracy of better than 1500 feet (0.25 nm) or 2 percent of distance, whichever is greater.

### B.3 PERFORMANCE REQUIREMENTS OF NAVAIDS

#### B.3.1 Terminal Area Requirements

The present requirements for domestic navigation using VOR/DME/TACAN combinations are as follows:

Azimuth Navigation: 4.5 deg ( $2\sigma$ ) with 95 percent confidence

Range Navigation: 3000 ft ( $2\sigma$ ) or 3 percent, whichever is greater

#### B.3.2 CONUS En Route Separation Requirements

For distances greater than 5 nm from VOR, CONUS en route separation requirements are:

> 5 nm from VOR  $\pm$  4.5 degrees from center line

$\leq$  51 nm from VOR 8 nm width

For < FL 180 8 to 51 nm from VOR. Beyond 51 nm, the increase is 4.5°

#### B.3.3 En Route Navigation: Oceanic En Route

The oceanic routes have no VOR/DME services but have adopted a 60 nm separation standard with the standard deviation for lateral track errors being 6.3 nm (1 sigma) or 12.6 nm (2 sigma). According to Federal Aviation Regulations (FAR), the specified current separation requirement is:

2000 Feet > FL 290

1000 Feet < FL 290

where FL is the flight level.

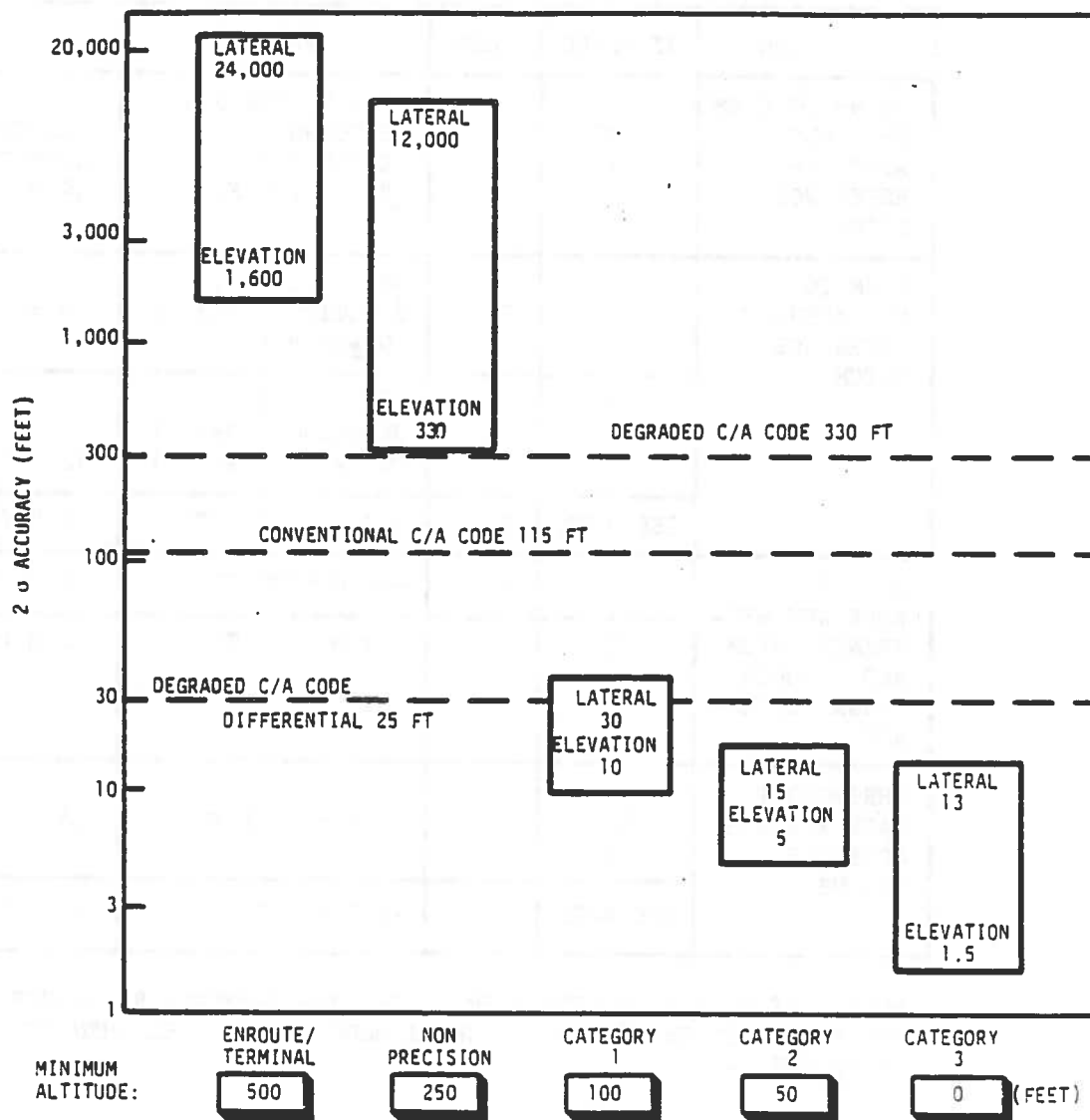
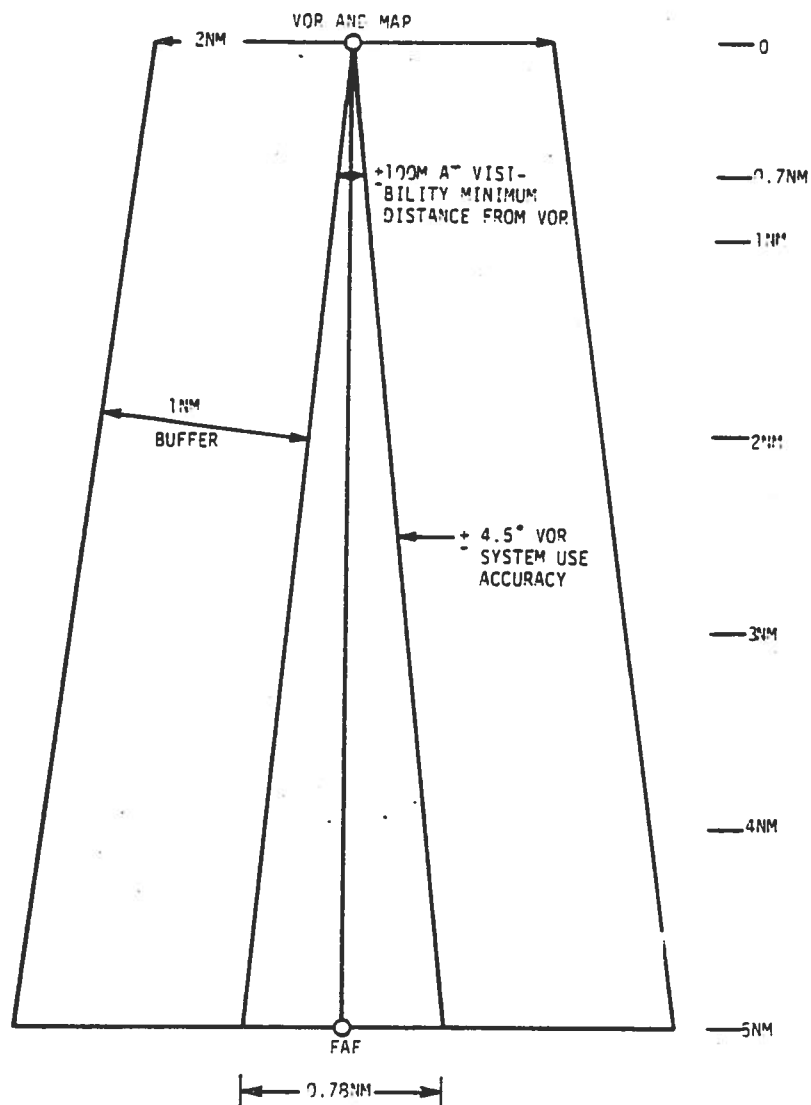


FIGURE B-3. FAA NAVIGATION STANDARDS AND GPS CAPABILITIES



MLS DISTANCE		PFE	OMN
5 NM	IA FA	$\pm 100M$ $\pm 12M$	$\pm 68M$ $\pm 12M$
20 → 5 NM	IA	$\pm 250M \rightarrow \pm 85M$	$\pm 68M \rightarrow \pm 34M$

FIGURE B-4. NON-PRECISION APPROACH OBSTACLE CLEARANCE AREA FOR CURRENT VOR WITH MAP AT VOR FACILITY

## APPENDIX C

### MINIMUM VARIANCE ESTIMATION\*

The minimum variance estimate includes a weighting matrix containing known measurement variances. The procedure for determining a minimum variance estimate when one or more of the coordinates of the position are directly measured can be developed through the following set of linear equations:

$$a_{11}x_1 + a_{12}x_2 \dots a_{1L}x_L = b_{11}y_1 + b_{12}y_1 \dots b_{1M}y_1M$$

$$a_{N1}x_1 + a_{N2}x_2 \dots a_{NL}x_L = b_{11}y_1 + \dots LNM y_M.$$

In the matrix form,

$$Ax = By$$

where the x vector contains the unknowns to be estimated, and the y vector contains the measurements.

The covariance matrix of the measurements is known and defined as:

$$K_y = [yy^T]$$

The following equations allow an estimation of the unknown vector X, in the least-square sense.

CASE I - As many equations as unknowns ( $N = L$ )

The covariance matrix here describes only the unmeasured variables.

$$x = A^{-1} By$$

$$K_x = (A^{-1} B) K_y (A^{-1} B)^T$$

\*Material in this appendix was obtained, in part, from Sinsky and Lew, "Minimum Variance Estimation," Bendix Corporation, BCN-TN-81-024 (May 1981).

When comparing the methods just discussed, it should be noted that if there is little or no a priori information:  $P_0^{-1}$  is very small and the above equation will reduce to  $\hat{x} = (A^T M^{-1} A)^{-1} A^T M^{-1} B y$ . For uncorrelated measurements, this equation reduces further to  $\hat{x} = (A^T A)^{-1} A^T B y$ .

The minimum variance solution is significant because it results in the smallest mean-squared error between the estimate and the truth. The trace is the mean-squared error.

$$\frac{1}{K} \sum_{K=1}^K \left\| \hat{x}_K - x \right\|^2 \rightarrow \sigma_{x_1}^2 + \sigma_{y_2}^2 \dots \sigma_{x_L}^2$$

$$K = \infty$$

To illustrate the utility of the technique described, a set of two equations with four variables may be solved (see Sinsky and Lew). The following equation may be considered:

$$A x_A - B y_B = C x$$

where

$x_A$  = the unknown variable vector.

$x_B$  = the known variable vector.

$$C_{11}x_1 + C_{12}x_2 + C_{13}x_3 + C_{14}x_4 = 0$$

$$C_{21}x_1 + C_{22}x_2 + C_{23}x_3 + C_{24}x_4 = 0$$

Case 3:

$x_2, x_3$ , and  $x_4$  are measured

$x_1$  and  $x_2$  are estimated

$$0x_1 + x_2 = x_2' + 0x_3 + 0x_4$$

$$C_{11}x_1 + C_{12}x_2 = 0x_2' - C_{13}x_3 - C_{14}x_4$$

$$C_{21}x_1 + C_{22}x_2 = 0x_2' - C_{23}x_3 - C_{24}x_4$$

$$\hat{x}_1 = 0.4921x_2' - 0.2571x_3 - 0.7680x_4$$

$$\hat{x}_2 = 0.6069x_2' - 0.3538x_3 - 0.9040x_4$$

$$K_x^\Lambda = \begin{bmatrix} 0.1999 & 0.2461 \\ 0.2461 & 0.3035 \end{bmatrix}$$

Case 4:

$x_1, x_2, x_3$  and  $x_4$  are measured

$x_1$  and  $x_2$  are estimated

$$x_1 + 0x_2 = x_1' + 0x_2' + 0x_3 + 0x_4$$

$$0x_1 + x_2 = 0x_1' + x_2' + 0x_3 + 0x_4$$

$$C_{11}x_1 + C_{12}x_2 = 0x_1' + 0x_2' - C_{13}x_3 - C_{14}x_4$$

$$C_{21}x_1 + C_{22}x_2 = 0x_1' + 0x_2' - C_{23}x_3 - C_{24}x_4$$

$$\hat{x}_1 = 0.3332x_1' + 0.3281x_2' - 0.1714x_3 - 0.5121x_4$$

$$\hat{x}_2 = 0.4102x_1' + 0.4051x_2' - 0.2483x_3 - 0.5890x_4$$

$$K_x^\Lambda = \begin{bmatrix} 0.1333 & 0.1641 \\ 0.1641 & 0.2025 \end{bmatrix}$$



## GLOSSARY

ACF	- Air Control Facilities
AODC	- Age of Data, Clock
AODE	- Age of Data, Ephemeris
ARSR	- Air Route Surveillance Radars
ASR	- Airport Surveillance Radar
ATC	- Air Traffic Control
C/A	- Coarse/Acquisition Code
DME	- Distance Measuring Equipment
DOD	- Department of Defense
DOT	- Department of Transportation
EPE	- Estimated Position Error
FAR	- Federal Aviation Regulation
FL	- Flight Level
FOM	- Figure of Merit
GPS	- Global Positioning System
HDOP	- Horizontal Dilution of Precision
HOW	- Handover Word
ILS	- Instrument Landing System
MLS	- Microwave Landing System
MODE S	- Discrete Addressable Secondary Radar System with Data Link
MSB	- Most Significant Bit
NADIN	- National Data Interchange Network
NAS	- National Airspace System

## BIBLIOGRAPHY

- Blythe, P.D., "Reliability of Navigation Systems, Final Report," Federal Aviation Administration, Contract DTFA 01-80-C-10030 (October 1983).
- Braff, Roland, C.A. Shively and M.J. Zeltser, "Radionavigation System Integrity and Reliability," Proceedings of the IEEE, Vol. II, No. 10, October 1983.
- Brooks, Richard A. et al., "GPS Error Budgets, Accuracy, and Applications Considerations for Test and Training Rates," Federal Electronic Corporation, WSWC TR 82-2 (December 1982).
- Civil Aviation Concerns for GPS Ad Hoc Technical Committee, presentation sponsored by the Joint Program Office (March 9, 1984).
- Federal Aviation Administration, "Microwave Landing System Ground Equipment Precision Distance Measuring Equipment (DME/P)," Specification # FAA-E-2721/3a (February 4, 1983).
- Gelb, Arthur, Applied Optimal Estimation (Cambridge, MA: M.I.T. Press, 1979).
- Jorgensen, Paul S., "Navigation Guidance and Control of Spacecraft Using the NAVSTAR Global Positioning System," presented at the National Telesystems Conference (San Francisco CA: November 14, 1983).
- Kalafus, Rudolph M. et al., "Simulation and Analysis Program," U.S. Department of Transportation, Research and Special Programs Administration, DOT-TSC-RSPA-83-11 (December 1983), 6-10.
- Lee, Ja Sung, "GPS Reference Relative (REFREL) Navigation Technique," presented at the National Telesystems Conference (San Francisco CA: November 14, 1983).
- Mazur, B.A., E. Wong and R. Mamen, "Preliminary Study of an Aeromarine Navigation System Making Use of Satellites: Appendix C, Details of System Error Analysis," ESTEC Contract No. 4631/81/F/RD(SC), (February 1982).
- Nakamura, Major Russell, "System Control," Proceedings of the Global Positioning System (GPS) Symposium (Arlington VA, April 21-22, 1983), 62-80.
- NAVSTAR GPS Space Segment/Navigation User Interferences, ICD-GPS-200 (January 2, 1983).
- Radio Technical Commission for Aeronautics, "Airborne VOR Receiving Equipment," SC-153 (Washington DC: November 28, 1983).
- Research and Special Programs Administration, U.S. Department of Transportation, "Federal Radionavigation Plan: Vol. 2, Requirements," DOT-TSC-RSPA-81-12-II (March 1982).