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RAIL-HIGHWAY CROSSING RESOURCE ALLOCATION MODEL

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16. Abstract

This report describes a methodology developed at the Transportation Systems Center for the Federal Railroad Administration and the Federal Highway Administration to aid in determining the most effective allocation of funds to improve safety at rail-highway crossings. One way to improve safety is to install active motorist warning devices at the crossings. However, since available funds are insufficient to provide an active warning device at all of the 216,000 public rail-highway crossings in the United States, a method has been developed for selectively allocating these funds in a optimal way.

The resource allocation model employs an accident prediction formula which was determined statistically from the extensive data base of the DOT-AAR National Rail-Highway Crossing Inventory and the FRA accident files. The predicted accident rates from this formula, combined with the warning system effectiveness and cost parameters, provide a funding priority ranking of crossing/warning device combination options which are based upon benefit/cost ratios. By selecting from this list, decisions can be made which should increase the accident reduction benefits for any given funding level.

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PREFACE

This report presents the results of a study performed at the Transportation Systems Center to develop a mathematical model which would optimize the allocation of money for rail-highway crossing safety improvements.

The study was sponsored by the U.S. Department of Transportation, Federal Railroad Administration's Office of Safety, and the Federal Highway Administration's Office of Research. This study supports a program which was outlined in the 1973 Highway Safety Act on safety improvement at rail-highway crossings in the United States.

This report is part of a TSC rail-highway program under the management of Robert Coulombre. Considerable technical advice was contributed by John Hitz.

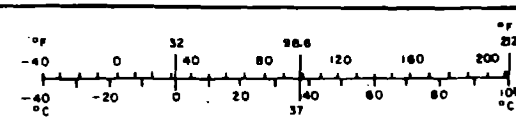
METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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SYMBOLS AND ABBREVIATIONS

AAR	Association of American Railroads
ACC	accidents
B	benefit
B'	cumulative benefits
B/C ratio	benefit/cost ratio
C	cost
C'	cumulative costs
CMAX	funding level
DOT	Department of Transportation
E	effectiveness
FHWA	Federal Highway Administration
FRA	Federal Railroad Administration
H (or H.I.)	Hazard Index
K	constant
LTS	lights
TSC	Transportation Systems Center
X	crossing

SUMMARY

This report describes a methodology, developed by the Transportation Systems Center (TSC) for the Federal Railroad Administration (FRA) and the Federal Highway Administration (FHWA), to aid in determining the most effective allocation of funds to improve safety at rail-highway crossings. The Federal Aid Highway Acts of 1973 and 1976, and the Surface Transportation Assistance Act of 1978, provide funds which amount to authorizations of over $1 billion. However, there are some 216,000 public rail-highway crossings in the United States and this amount is insufficient to provide active warning devices at all of them. Therefore, a method of determining optimum allocation is necessary.

The TSC resource allocation model was designed to provide information to assist in making such allocation decisions. The methodology employs a rail-highway crossing accident prediction formula which was statistically determined from the extensive data base of the DOT-AAR National Rail-Highway Crossing Inventory and the FRA accident files. The resource allocation model, combining the predicted accident rates with warning system effectiveness and cost parameters, provides a funding priority ranking of allocation options. From this prioritized list, it can be determined which of the 216,000 crossings should be considered for improvements, and which type of warning system should be installed at each crossing to maximize the total benefit for any given funding level.

The TSC resource allocation model can be applied to all of the crossings in the DOT-AAR Crossing Inventory or to any subset of crossings such as a state, a region, or a railroad. On the national level, it can determine the maximum benefit possible for any given budget. It can be used to determine the effect of different rail-highway crossing safety situations which could be used in setting national policy. On the state and local

levels, it can be used to prioritize crossing options by their benefit/cost ratio.

It is not intended that the algorithm dictate the final decisions, but provide aid to state and local officials and railroad management for making decisions. Local conditions, and the judgement of state and local officials, play a major role in this evaluating process, as well as in the final decision.

Benefit versus cost curves for the DOT-AAR Crossing Inventory have been developed by using the model for a wide range of warning device parameters. This demonstrates the sensitivity of benefits to the uncertainties in the effectiveness and cost assumptions. In this analysis, the model was applied to crossings in one state and the warning device options were ranked by benefit per dollar. Additional experience in using this methodology will lead to further refinement of the model, as well as its adapation for different situations.

1. INTRODUCTION

This report presents the results of a study which was designed to develop a methodology for allocating funds for warning device improvements at public rail-highway crossings. The ultimate goal of the study is to improve rail-highway crossing safety.

The Federal Aid Highway Acts of 1973 and 1976, and the Surface Transportation Assistance Act of 1978, provide funds for public rail-highway crossing safety projects. These statutes have authorized over $1 billion with the stipulation that at least half of the funds must be used to install new warning systems at rail-highway crossings or to upgrade existing systems. A state-by-state apportionment of the $550 million which is authorized by the 1973 and 1976 Acts is listed in an Association of American Railroads (AAR) brochure. (Ref. 1)

To assist in the systematic planning and evaluation of programs for improved crossing safety, a comprehensive inventory of the characteristics of all rail-highway crossings in the nation was carried out through a joint DOT, state, and AAR program. (Ref. 2) Obviously, there are not sufficient funds available to install an automatic warning system at each of the 216,000 public rail-highway crossings in the United States.[1] In fact, a DOT report to Congress recommended that active warning systems be installed or improved at the 30,000 crossings having the highest accident rate. (Ref. 3)

The TSC analysis was designed to determine which crossings should have a specific warning system installed to achieve the greatest benefit. Figure 1-1 illustrates the basic outline of the TSC resource allocation model. Inventory information and the accident histories of the crossings were used to develop an

[1]The present analysis was done for the inventory of 1975, when there were approximately 219,000 public crossings.

accident prediction formula which determines the expected number of accidents at each of the 216,000 public crossings. (Ref. 2 and Ref. 4)

An initial reaction may be that crossings should be considered in the order of their accident prediction rates with the crossing having the highest accident rate treated first, the crossing with the second highest accident second, and so forth. However, if the established criteria are the maximum benefit for a given total cost, this procedure will not suffice, due to the different warning system options which are available for different crossings and their differing costs and benefits. For example, installing a flashing light at the crossing with the tenth highest accident rate might yield a higher benefit/dollar ratio, rather than installing an automatic gate at the most hazardous crossing.

Consequently, a priority ranking was produced based upon the benefit per dollar for each available option, determined by combining the calculated accident predictions with the warning system cost plus a factor of merit. A factor of merit represents the effectiveness of various warning system options. From this list of funding options, recommendations can be made on which crossings should be selected and which type of warning system should be installed to achieve the maximum benefit for a given budget level.

The resource allocation model can be used on the national level, the state level, regionally, or for a railroad. On the national level, it can provide estimates of the maximum benefit which is possible for given budget. It may be used to evaluate the sensitivity of benefits to the changes in equipment cost, effectiveness, or installation strategy. On the state level, it could be used to prioritize crossing options by their benefit/cost ratios. The algorithm will not dictate the final decisions but aid state and local officials, as well as railroad personnel, in making informed decisions. Local conditions, and the judgment of state and local officials, still play a major part in making the final decision. For example, state warrants must take precedence over the resource allocations model's decision. In addition, as experience using this methodology develops, the model should evolve to a point where it could meet other objectives.

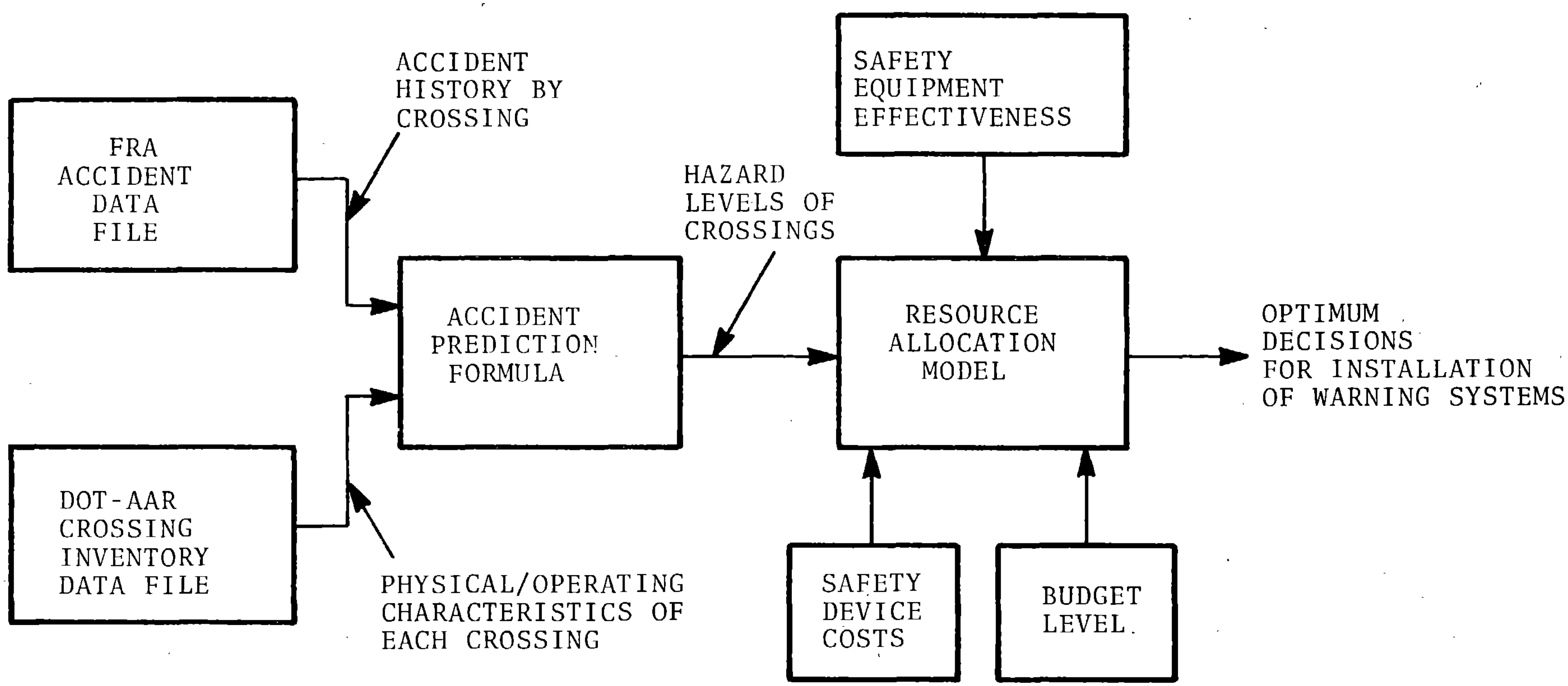

FIGURE 1-1. RAIL-HIGHWAY CROSSING RESOURCE ALLOCATION METHODOLOGY

2. RESOURCE ALLOCATION MODEL

The key elements of the rail-highway crossing resource allocation model and their inter-relationships are shown in Figure 1-1. The accident prediction formula can be any formula which computes the expected number of accidents per year for each crossing to which the resource allocation model is to be applied. There are numerous accident prediction and hazard formulas in use today. Several of these are cited in Appendix A.

The resource allocation model for this study employed the DOT accident prediction formula which was recently developed from the DOT-AAR National Railroad-Highway Crossing Inventory data base and previous rail-highway crossing accident data. A brief description of the DOT accident prediction formula is contained in Appendix B.

The effectiveness of different warning systems has been determined in a California Public Utilities Commission study. (Ref. 5) "Effectiveness" is defined as a number between 0 and 1 which represents the factor by which accidents are reduced when a specific warning system is installed at a crossing having an existing and identifiable warning system. An existing warning system might be the absence of an active warning system. It must be stressed that effectiveness is a relative measure which involves both existing and proposed warning systems. If automatic gates have an effectiveness of 0.9 when installed at a crossing which has an existing passive warning device, the accident rate at the crossing will be reduced by 90 percent. Of course, when automatic gates are installed at a crossing with flashing lights, they would have a different, lower effectiveness. A device which completely eliminates accidents, such as a grade separation, has an effectiveness value of 1; it is 100 percent effective. The California study appears to provide the best available effectiveness data because accident rates were compared before and after a system was installed at a crossing. A study has recently been completed at TSC, which used the FRA accident files to calculate new effectiveness values. (Ref. 6)

All crossings in the DOT-AAR inventory are assigned to one of eight warning device classes. The eight classes and the number of public crossings in each class are shown in Table 2-1. These figures represent inventory data as of June 1980.

TABLE 2-1. INVENTORY WARNING DEVICE CLASSES, 1980

WARNING DEVICE	CLASS OF DEVICE		NUMBER OF CROSSINGS
No signs or signals	1		14,419
Other signs	2		1,034
Stop signs	3		3,515
Crossbucks	4		137,141
Special warning devices	5		7,473
Highway signals, wigwags, bells	6		3,116
Flashing lights	7		34,420
Gates	8		15,005
		TOTAL	216,123

Three categories of crossings were established for the present analysis in order to simplify and realistically reflect the accuracy of the data. Inventory classes 1 through 4 were grouped together and called "passive" warning systems, meaning that they are not train-activated devices. Inventory classes 5, 6, and 7 were grouped together and called "flashing lights," since public crossings which are equipped with flashing lights predominate in this category. Inventory class 8 remained as a separate warning device category.

Table 2-2 is a matrix showing the effectiveness and cost symbols used in this analysis for the three warning system groupings. It was assumed that gates were the most effective warning system possible and no attempt would be made to upgrade them. They were not included as an existing warning system. For the proposed warn-

TABLE 2-2. EFFECTIVENESS-COST MATRIX

	PROPOSED WARNING SYSTEM			
	FLASHING LIGHTS		AUTOMATIC GATES	
EXISTING WARNING SYSTEM	EQUIPMENT EFFECTIVENESS	EQUIPMENT COST	EQUIPMENT EFFECTIVENESS	EQUIPMENT COST
Passive	E_1	C_1	E_2	C_2
Flashing Lights	—	—	E_3	C_3

ing system, two possible options were considered, flashing lights and automatic gates with flashing lights. For flashing lights, no distinction was made as to whether cantilevered flashing lights were used. While this may seem to be a significant factor, effectiveness measures and accurate cost determinations are not known for cantilevered flashing lights. A study is currently underway to determine accurate costs for warning systems and identify the major cost components, including cantilevered flashing lights. (Ref. 7)

For any given crossing and/or proposed warning system, a pair of parameters (E_j, C_j), as shown in Table 2-2, must be provided for the resource allocation algorithm. The first parameter (E_j) is the effectiveness of installing a proposed warning system at a crossing which now has an existing warning system. The second parameter (C_j) is the corresponding cost of the proposed warning system. Table 2-2 shows the six warning system parameters (E_1, C_1, E_2, C_2, E_3, C_3) that are needed to use the resource allocation algorithm.

There are only two independent effectiveness parameters, because E_3 is functionally related to E_1 and E_2. This is due to the fact that if flashing lights are installed at a passive crossing with the ensuing effectiveness E_1, the flashing lights are removed and gates are installed with the ensuing effectiveness E_3, then the same accident prediction rate should result for this crossing, as if gates had directly been installed at this passive crossing with the ensuing effectiveness E_2. This means: $(1-E_1)(1-E_3) = 1-E_2$ or, $E_3 = 1 - [(1-E_2)/(1-E_1)]$.

In the algorithm for the resource allocation model, all crossings which did not have existing gates were candidates for improved warning systems. Consider a given crossing (i) which may have a passive device or flashing lights. H_i is the crossing's predicted number of accidents per year. The benefit achieved in installing warning system j with effectiveness E_j equals H_iE_j accidents prevented per year, where $j = 1, 2$ or 3. The cost of obtaining this benefit is C_j. The algorithm systematically computes the incremental benefits and costs of all such improvement options which could be implemented for all crossings under consideration. The individual benefit/cost ratios which are associated with these improvements are selected by the algorithm in an efficient manner to produce the maximum benefit which could be obtained for a predetermined total cost. This total cost is the sum of an integral number of equipment costs C_1, C_2, and C_3. The total, maximum benefit is the sum of the individual benefits of the form H_iE_j.

A flow diagram describing the logic of the resource allocation algorithm is shown in Figure 2-1. The input to this program consisted of the set of crossings for which the model was to apply, the accidents predicted per year for these crossings, the six warning parameters (E_1, E_2, E_3, C_1, C_2, C_3), and the funding level (CMAX) which determined where the calculation was to stop.

The algorithm described in Figure 2-1 proceeded according to the following steps in computing optimal resource allocations.

Step 1: A reasonable assumption is made for the algorithm that $E_2 > E_1$ and $C_2 > C_1$. This is an algebraic statement of the idea that gates are more effective additions at passive crossings than flashing lights. Also, gates cost more. However, the effectiveness/cost ratio for flashing lights (E_1/C_1) could be greater than or less than that for gates (E_2/C_2). If $E_1/C_1 > E_2/C_2$, the algorithm computes incremental benefit/cost ratios for all possible improvements at each crossing according to the procedure outlined in step 2A. The step 2A procedure was based on the assumption that flashing lights have a greater effectiveness/cost ratio than gates. If the opposite is true — that

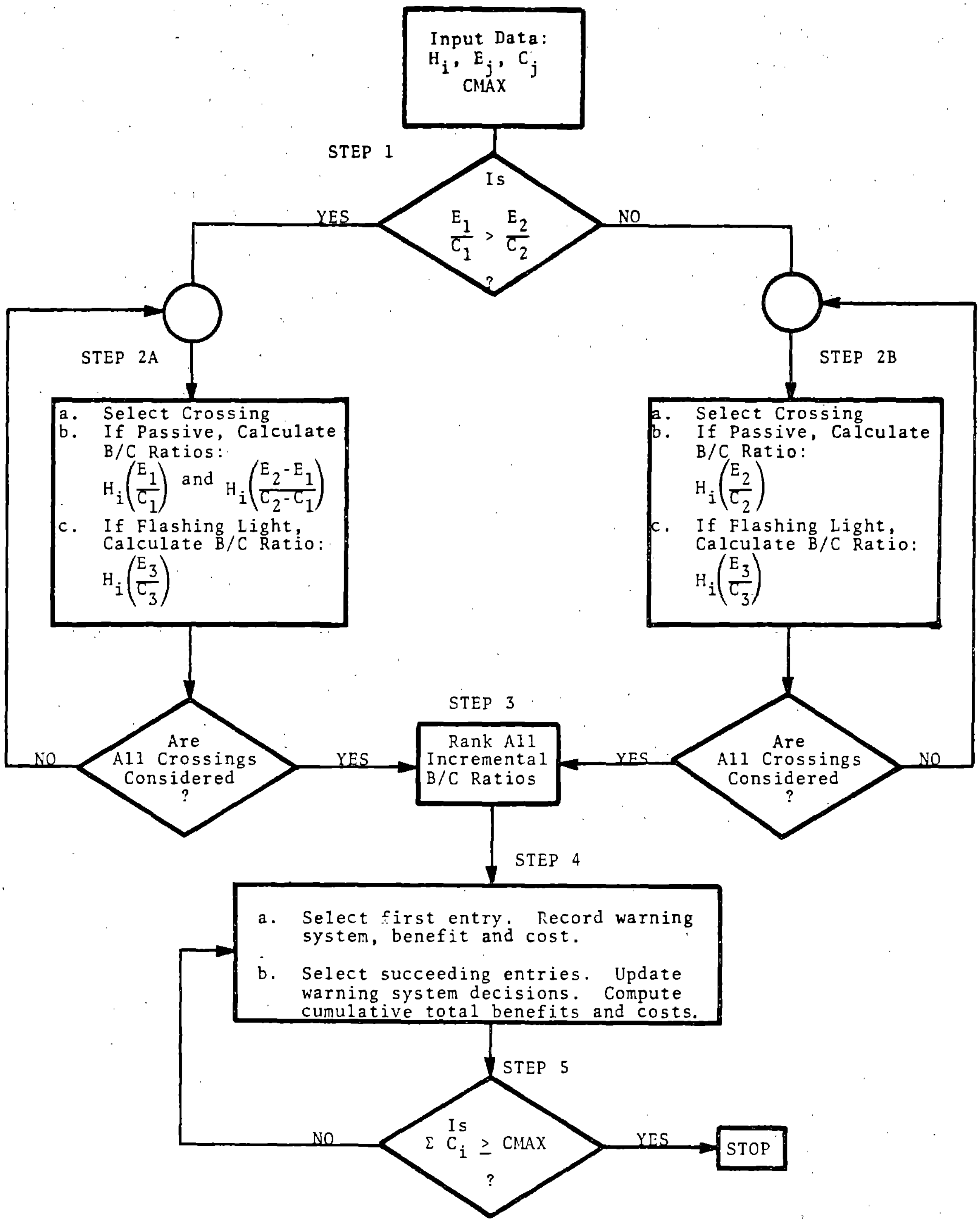

FIGURE 2-1. RESOURCE ALLOCATION ALGORITHM

gates have an effectiveness/cost ratio equal to or greater than flashing lights ($E_1/C_1 \leq E_2/C_2$) — then step 2B was followed for computing the improvement benefit/cost ratios.

Step 2A: In step 2A, two incremental benefit/cost ratios were calculated for each passive crossing, H_iE_1/C_1 and $H_i\ [E_2-E_1)/(C_2-C_1)]$, where H_i is the number of accidents predicted per year for the crossing. These two ratios correspond to the two actions available for passive crossings, either to install flashing lights or a revised decision to install gates. For each crossing equipped with flashing lights, the algorithm computed H_iE_3/C_3, corresponding to an upgrading to gates. The incremental benefit/cost ratio was represented in units of accidents prevented per year per dollar.

Step 2B: The algorithm computed the incremental benefit/cost ratio H_iE_2/C_2 for passive crossings and the ratio H_iE_3/C_3 for crossings with flashing lights. These benefit/cost ratios are associated with installing only gates at crossings. For the step 2B case, these actions were always optimal to the alternative of installing flashing lights, since the benefit/cost ratio and the absolute cost of gates are greater than for flashing lights.

Step 3: Regardless of whether step 2A or 2B was followed, all of the incremental benefit/cost ratios which were calculated by the algorithm were ranked with the largest first. The list of benefit/cost ratios corresponded to the sequence of optimal decisions which were made, starting with the top of the list.

Step 4: This consisted of a series of repeated steps, where the algorithm progressed down the list of ranked benefit/cost ratios. This process was equivalent to making the optimum decision of achieving the maximum benefit for each additional increment in cost incurred. If the benefit/cost ratio at any given step on the list was calculated as H_iE_1/C_1, a decision was made to install flashing lights at this passive crossing, with an incremental benefit of H_iE_1 and an incremental cost of C_1.

If the benefit/cost ratio was $H_i[(E_2-E_1)/(C_2-C_1)]$, a previous decision to install flashing lights was changed to installation of gates. The incremental benefit of changing the previous decision was $H_i(E_2-E_1)$. Similarly, the incremental cost was C_2-C_1. If the benefit/cost ratio was H_iE_3/C_3, then a decision was made to install gates at a crossing which had flashing lights. The incremental benefit was H_iE_3 at an incremental cost of C_3. The total benefit at each step is the sum of the incremental benefits and the total cost is the sum of the incremental costs.

Since monies are allocated step-by-step in the order specified by the algorithm, the algorithm also determined the particular warning systems which were to be installed at particular crossings. The total cumulative cost and benefit was determined at each step. Since the crossings which were affected were known, the predicted accident rate, location, and all of the information in the inventory for those crossings was also known. Thus, the output of the program could include any of this information and any computations based on this information. Several types of output are shown in Section 3.

Step 5: The cumulative total cost at each step, proceeding down the list of benefit/cost ratios, was compared with the total funding limit specified as input to the algorithm. When cost equaled or exceeded this limit, the program ended. Otherwise, the sequential procedure described in step 4 continued.

To illustrate the algorithm, an example follows which considers the three crossings described in Table 2-3. The predicted accidents per year and current warning device information for the crossings, together with the warning device cost and effectiveness parameters presented in Table 2-4, constitute the input data for the algorithm. The algorithm proceeds through the proper steps. (See Figure 2-1)

TABLE 2-3. SAMPLE CROSSINGS FOR THE ALGORITHM

CROSSING	CURRENT WARNING DEVICE	PREDICTED ACCIDENTS PER YEAR H_i
X_1	Passive	$H_1 = 0.3$
X_2	Flashing Lights	$H_2 = 0.2$
X_3	Flashing Lights	$H_3 = 0.1$

TABLE 2-4. EFFECTIVENESS-COST INPUT DATA

	PROPOSED WARNING SYSTEM			
	FLASHING LIGHTS		AUTOMATIC GATES	
EXISTING WARNING SYSTEM	EQUIPMENT EFFECTIVENESS	EQUIPMENT COST	EQUIPMENT EFFECTIVENESS	EQUIPMENT COST
Passive	$E_1 = 0.7$	$C_1 = \$25,000$	$E_2 = 0.9$	$C_2=\$45,000$
Flashing	—	—	$E_3 = 0.667$	$C_3=\$35,000$

Step 1: The effectiveness/cost ratio for flashing lights, E_1/C_1, is greater than that for gates, E_2/C_2, so the algorithm followed step 2A. (See Figure 2-1) This implies that the most effective first action which can be taken at a passive crossing is the installation of flashing lights.

Step 2A: The crossings were selected in the order as they appear in Table 2-3, until all were considered. For each crossing selected, the appropriate incremental benefit/cost ratios were calculated, corresponding to all the possible warning device improvements which could be made, as shown in Table 2-5.

Step 3: The incremental benefit/cost ratios, as calculated in step 2A, were ranked in descending order, beginning with the largest. The warning device improvement action at each crossing,

TABLE 2-5. STEP 2: CALCULATION OF INCREMENTAL BENEFIT/COST RATIOS

CROSSING	CURRENT WARNING DEVICE	INSTALL FLASHING LIGHTS AT PASSIVE CROSSING: $B/C = H_i\left(\frac{E_1}{C_1}\right)$	REVISE DECISION FROM INSTALLING FLASHING LIGHTS TO GATES AT PASSIVE CROSSING: $B/C = H_i\left(\frac{E_2-E_1}{C_2-C_1}\right)$	INSTALL GATES AT FLASHING LIGHT CROSSING: $B/C = H_i\left(\frac{E_3}{C_3}\right)$
X_1	Passive	$0.3\left(\frac{0.7}{25,000}\right)$ $= 0.0084 \times 10^{-3}$	$0.3\left(\frac{0.9 - 0.7}{45,000-25,000}\right)$ $= 0.003 \times 10^{-3}$	—
X_2	Flashing Lights	—	—	$0.2\left(\frac{0.667}{35,000}\right)$ $= 0.0038 \times 10^{-3}$
X_3	Flashing Lights	—	—	$0.1\left(\frac{0.667}{35,000}\right)$ $= 0.0019 \times 10^{-3}$

represented by the ratios and corresponding cumulative benefits and costs, is tabulated in Table 2-6.

Step 4: From the ranked list in Table 2-6, the first action selected by the algorithm was designated by the first ranked incremental benefit/cost ratio: installation of flashing lights at crossing X_1 with a cost of $25,000. The next action selected by the algorithm corresponded to the next ranked incremental benefit/cost ratio, installation of gates at crossing X_2, resulting in a cumulative cost of $60,000 for the first two projects.

The algorithm proceeded in this manner until the cumulative total cost of all improvement actions equaled the available funding (CMAX). It should be noted that the third action selected by the algorithm did not involve an additional crossing, but revised an earlier decision to install gates at crossing X_1 rather than flashing lights. This type of revision was typical of the algorithm for normal applications, as additional funding was made available. For the above example, if a total of $115,000 were available for improvements (CMAX = $115,000), the algorithm would proceed through the fourth item on the list involving crossing X_3. The overall improvement actions for $115,000 would result in the installation of gates at all three crossings.

Appendix C presents mathematical verification that this algorithm is optimum.

TABLE 2-6. STEP 3: RANKING OF INCREMENTAL BENEFIT/COST RATIOS

RANK	INCREMENTAL BENEFIT/COST RATIO	WARNING DEVICE IMPROVEMENT ACTION	E_jH_i BENEFIT, ACCIDENTS REDUCED PER YEAR	ΣE_jH_i CUMULATIVE BENEFITS, ACCIDENT REDUCED PER YEAR	CUMULATIVE COSTS (DOLLARS)
1	0.0084×10^{-3}	Install Flashing Lights at Crossing X_1	0.21	0.21	25,000
2	0.0038×10^{-3}	Install Gates at Crossing X_2	0.13	0.34	60,000
3	0.0030×10^{-3}	Install Gates at Crossing X_1	0.06	0.40	80,000
4	0.0019×10^{-3}	Install Gates at Crossing X_3	0.07	0.47	115,000

3. APPLICATIONS

General

The resource allocation model was applied to several sample situations to demonstrate its usefulness. This section describes the use of the method to determine the sensitivity of program benefits and costs at a national level regarding assumptions which concern warning device effectiveness and cost, and warning device installation policy.

The section on state application describes the use of the method in determining warning device installation decisions for a hypothetical state. For each application, different combinations of effectiveness and cost input data were used, as identified by the run numbers in Table 3-1. The runs encompassed parameter values which were representative of the range found in practice.

TABLE 3-1. EFFECTIVENESS-COST PARAMETERS

RUN NUMBER	E_1	E_2	E_3	C_1	C_2	C_3
1	0.7	0.9	0.667	$25,000	$45,000	$35,000
2	0.7	0.9	0.667	25,000	35,000	35,000
3	0.7	0.9	0.667	25,000	45,000	25,000
4	0.7	0.9	0.667	15,000	45,000	35,000
5	0.6	0.8	0.500	25,000	45,000	35,000
6	0.5	0.7	0.400	25,000	45,000	35,000
7	0.7	0.0	0.000	25,000	----	----
8	0.0	0.9	0.667	----	45,000	35,000

Reviewing the results of the method, it is important to stress two points. First, the method determines the maximum benefit which can be obtained for a given expenditure of funds with given parameter values. The method considers all crossings and all possible warning system options, and recommends which crossings should receive which warning systems in order to produce the optimum benefit. Second, the optimum decision produced by the model, recommending which equipment is to be considered for which crossing, is optimum for a given funding level only. If an increased funding level is desired at a later date, this will not result in the installation of more equipment at additional crossings. Previous decisions specifying flashing lights for certain crossings may be changed to the installation of gates at these crossings. Similar reasoning shows that if the funding level is decreased at a later date, the previous decisions may no longer be optimal.

Sensitivity to Equipment Cost and Effectiveness

Eight different combinations of effectiveness and cost values, represented by run numbers 1 through 8 (Table 3-1), were used as input to the resource allocation model in order to perform sensitivity analyses of these parameters. A tabulation of the results for run number 1 is shown in Table 3-2. Results for the remaining runs are contained in Appendix D. The first column in Table 3-2 shows the cumulative amount to be spent for warning system improvements in dollars. The second column gives the number of accidents prevented per year. The third, fourth, and fifth columns show the number of crossings where warning systems will be installed for possible transitions from existing systems to proposed systems.

The effectiveness values in runs 1 through 4 were the same as those contained in the California Public Utilities Commission study. A recent study of FRA's accident data file produced effectiveness values which closely agreed with the California study. The cost data were estimated costs which were thought to exist at the time that the computations were made. Since then, the installation costs were found to be reasonably accurate. (Ref. 7) There was still some uncertainty concerning actual effectiveness and cost. Therefore, the results of runs 1 through 8 were thought to define the range of uncertainty in the resulting

TABLE 3-2. RESOURCE ALLOCATION RESULTS FOR RUN NUMBER 1

COST (DOLLARS)	BENEFIT (ACC PREVENTED/YR)[1]	PASSIVE TO FLASHING LTS[2]	PASSIVE TO GATES	FLASHING LTS[2] TO GATES
40,015,000	471	702	10	629
80,015,000	769	1,505	25	1,179
120,005,000	1,020	2,346	42	1,699
160,000,000	1,241	3,190	73	2,199
200,005,000	1,440	4,032	110	2,693
240,005,000	1,624	4,859	156	3,186
280,000,000	1,795	5,758	200	3,630
320,000,000	1,955	6,632	265	4,065
360,000,000	2,106	7,425	340	4,545
400,015,000	2,250	8,298	409	4,976
440,010,000	2,386	9,160	486	5,404
480,015,000	2,517	10,033	575	5,809
520,015,000	2,642	10,899	660	6,224
560,020,000	2,762	11,711	772	6,643
600,020,000	2,878	12,604	863	7,031
640,005,000	2,989	13,470	978	7,407
680,010,000	3,096	14,242	1,113	7,825
720,000,000	3,200	15,088	1,261	8,173
760,000,000	3,300	15,872	1,411	8,563
800,030,000	3,398	16,639	1,561	8,966
840,015,000	3,492	17,475	1,709	9,321
880,010,000	3,584	18,315	1,876	9,649
920,025,000	3,673	19,071	2,059	10,017
960,015,000	3,759	19,793	2,244	10,406
1,000,005,000	3,843	20,563	2,435	10,753

[1] Accidents prevented per year

[2] Flashing lights

benefits. The most likely set of values were those from run number 1.

Figure 3-1 shows the benefit/cost curves which resulted from runs 1 through 8. These curves show the sensitivity of benefits to warning system effectiveness and cost. As expected, if effectiveness is reduced with fixed equipment costs, the benefit is reduced. Also, if equipment cost is reduced with fixed effectiveness, the benefit is increased.

These results can be further extended by analysis and simple computation. For given values of effectiveness (E_1, E_2, E_3) and equipment cost (C_1, C_2, C_3), the benefit/cost equation is $B = f(C)$. In this equation, B is the total benefit and C is the total cost. Considering the case where equipment effectiveness is the same but costs were $C_1' = kC_1$, $C_2' = kC_2$, $C_3' = kC_3$, these new costs were changed by a single constant k over the previous costs. Therefore, the new incremental benefit/cost ratios were multiplied by this single constant and the order of the decisions was not changed.

Further, the incremental benefit was a product of the effectiveness and predicted number of accidents per year, neither of which is changed. This means that the cumulative benefits B' for the new parameters will be the same as B for the same number of decisions made by the algorithm. In addition, for any given number of decisions with an associated value of total cost C, the new cost C' satisfies the relationship $C' = kC$. Substituting this equation in the benefit/cost equation resulted in the following:

$$B' = f(C'/k).$$

The key result was that the same function (f) could be used to obtain a new curve, B' versus C'. Any point (B,C) on the original curve could be mapped onto a point (B', C') of the new curve, where $B' = B$ and $C' = kC$.

The interpretation of this reasoning was that if the cost of all three equipment options was changed by the same fractional amount, the new benefit/cost curve is easily obtained. This

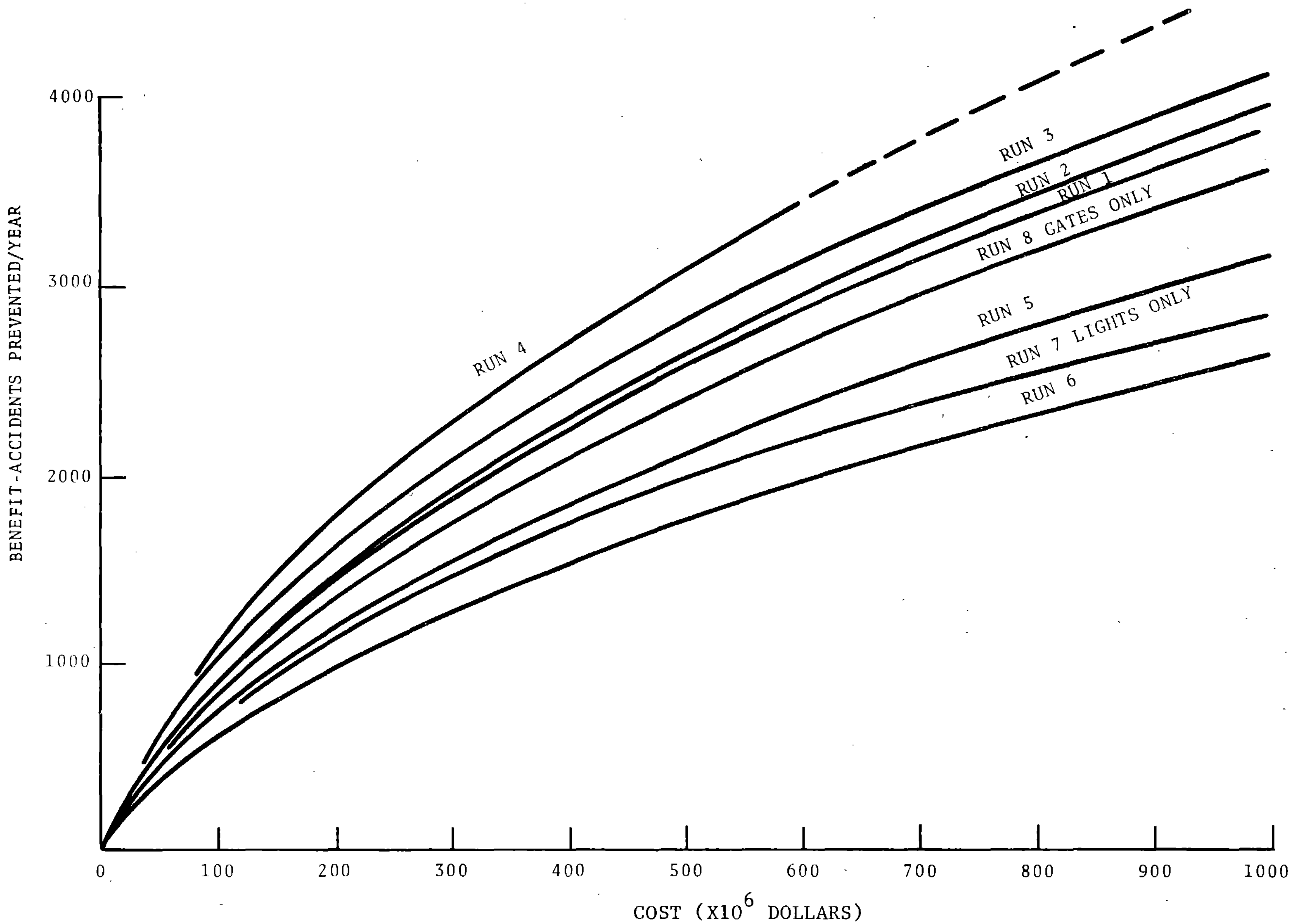

FIGURE 3-1. MAXIMUM BENEFIT VERSUS COST FOR EFFECTIVENESS-COST PARAMETERS GIVEN IN TABLE 3-1

change in equipment cost, when expressed as a percent change, would be (k-1) x 100.

The families of curves, obtained for each of the eight curves in Figure 3-1, were plotted and the results for run number 1 are shown in Figure 3-2. Graphs showing similar curves obtained from the other runs are contained in Appendix E. The equipment cost changes shown are -20 percent, -40 percent, -50 percent, +20 percent, and +40 percent. The three parameters associated with each curve are C_1, C_2, and C_3. For the curves shown in Figure 3-2, the effectiveness values are specified and held fixed.

With a given funding level, given effectiveness values, and a given percent change in initial equipment cost, the increase or decrease in benefits can be determined. By considering the change in initial equipment cost, expressed as a percentage, as constant, and by holding effectiveness constant, the percent change in benefit became a function of the total funding level. Tables E-1 through E-5, contained in Appendix E, show this relationship for the eight runs.

The calculations for run number 1 for a 20 percent reduction in equipment cost can be demonstrated. As shown in Figure 3-2, the benefit for COST = 100 for C_1 = 25, C_2 = 45, and C_3 = 35 is 871. The benefit for COST = 100 for C_1 = 20, C_2 = 36, and C_3 = 28 is 1009. This is a benefit increase of 15.8 percent. For the same two curves for COST = 200, the benefits are 1439 and 1675, producing a benefit increase of 16.4 percent. For COST = 400, the benefits are 2249 and 2571, resulting in a benefit increase of 14.3 percent. In a similar way, increases of 13.9 percent and 13.2 percent are calculated for COST = 600 and COST = 800, respectively.

Tables E-1 through E-5 show that the sensitivity of benefits to the changes in funding level is quite large, but relatively independent of equipment effectiveness or initial equipment cost. The benefit percent change for a given percent change in initial equipment cost will not be significantly affected by any lack of

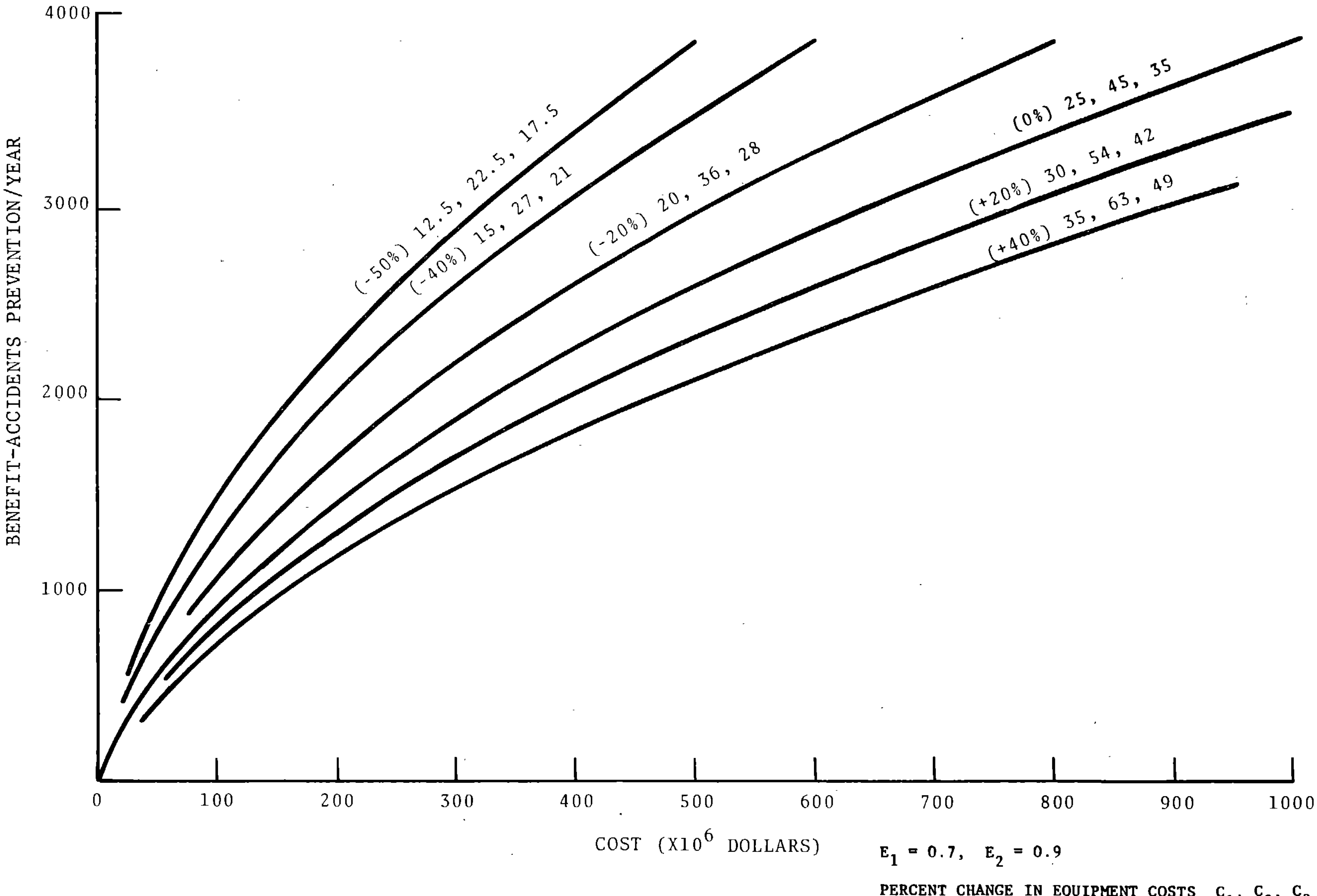

FIGURE 3-2. MAXIMUM BENEFIT VERSUS COST FOR DIFFERENT CHANGES IN EQUIPMENT COST APPLIED TO RUN NUMBER 1

precision in the estimates of equipment effectiveness or initial equipment cost. The average percent change in benefits for the eight different combinations of cost and effectiveness values, listed in the "average" columns in Tables E-1 through E-3, was plotted as a function of funding level. This produces five curves, one for each percentage change in equipment cost.

The results illustrated in Figure 3-3 could be used to measure the effect of an investment in research and new technology, aimed at reducing the cost of rail-highway crossing warning systems. For example, for a national rail-highway crossing program of $500 million, a research program producing a 40 percent reduction in equipment costs provides a 35 percent increase in benefits. Figure 3-3 also shows the resultant effect when costs increase because of inflation or other factors. For a $500 million program, a 20 percent increase in cost would result in about a 10 percent decrease in benefits.

Sensitivity to Equipment Installation Policy

By inserting the artificial effectiveness of zero for flashing lights, run number 8, the algorithm produced a result representing a "gates only" policy. The idea of using only gates for active warning devices has been considered by safety engineers. (Ref. 8) Figure 3-1 shows that the "gates only" policy results in reduced benefits, as compared with the optimum benefits of run number 1. For these parameter values, the reduction was relatively small, about 7 percent.

By inserting the artificial effectiveness of zero for "gates," run number 7, the result represents a "lights only" policy. The idea of only using flashing lights for active warning devices also has been considered by safety engineers. (Ref. 8) Figure 3-1 shows the reduced benefit produced by the "lights only" policy. In this case, the reduction was considerably greater than for the "gates only" policy.

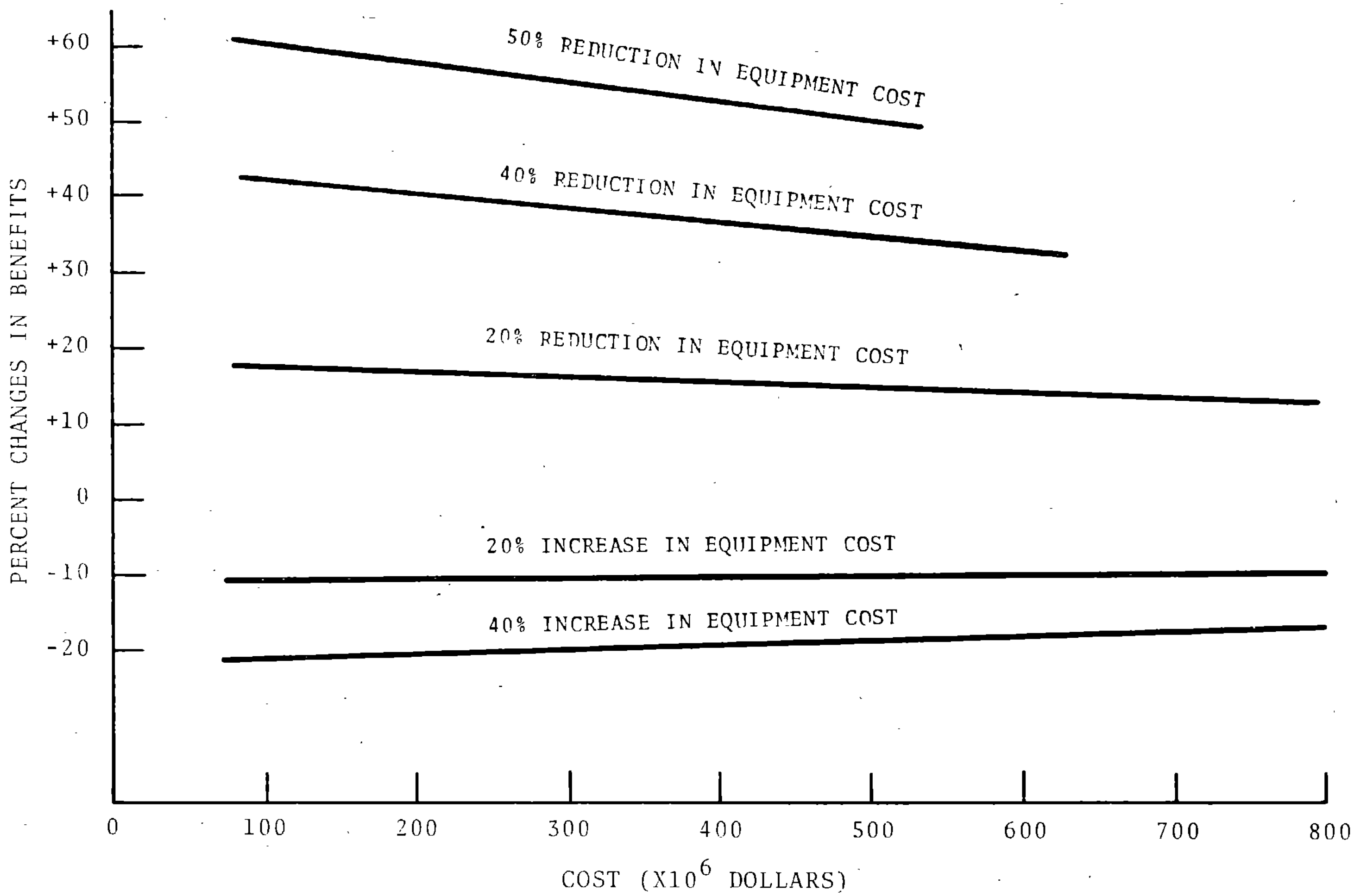

FIGURE 3-3. CHANGE IN BENEFIT VERSUS COST FOR DIFFERENT PERCENT CHANGES IN EQUIPMENT COST (SMOOTHED REPRESENTATIONS OF DATA IN APPENDIX E)

State Application

The resource allocation model can be used for smaller crossing groups, other than the full DOT-AAR Crossing Inventory, such as states, railroads, or regions.

Table 3-3 shows the results obtained using the resource allocation model for a hypothetical state, given a rail-highway crossing budget of $5 million and employing the parameters used for run number 1. (Table 3-1) The affected crossings were ranked by the benefit/cost ratio of the final decision at each crossing and suggested a preferred order in which these crossings should be further investigated.

The accident prediction is listed for convenience and to illustrate how the accident prediction ranking compares with the resource allocation model ranking. The first five crossings are also in ranked order by the number of accidents predicted. Overall, accident predictions are closely correlated with the benefit/cost ratios for this application.

The indicated benefit of approximately 21 accidents prevented per year is much lower than the nationwide benefits. This was a logical conclusion because each state's crossings are a subset of the DOT-AAR Crossing Inventory and given a specific funding level, the crossings to be improved would contain fewer high-risk crossings.

TABLE 3-3. OUTPUT OF RESOURCE ALLOCATION MODEL FOR A HYPOTHETICAL STATE SHOWING CROSSING DECISIONS RANKED BY BENEFIT/COST RATIO

PREDICTED ACCIDENTS PER YEAR	CUMULATIVE COST (DOLLARS)	CUMULATIVE BENEFIT	BENEFIT/COST PER CROSSING	PRESENT WARNING DEVICE CLASS	RECOMMENDED WARNING SYSTEM
1.7307	35,000	1.1595	0.000033129640	5	Gate
0.9783	70,000	1.8150	0.000018727105	6	Gate
0.9563	105,000	2.4557	0.000018305602	5	Gate
0.7256	140,000	2.9418	0.000013890540	7	Gate
0.5991	175,000	3.3433	0.000011468685	5	Gate
0.3370	220,000	3.6465	0.000006739992	4	Gate
0.3351	265,000	3.9482	0.000006702672	4	Gate
0.3345	310,000	4.2493	0.000006690232	4	Gate
0.3465	345,000	4.4814	0.000006632120	7	Gate
0.3458	380,000	4.7131	0.000006620212	7	Gate
0.3380	415,000	4.9396	0.000006470186	7	Gate
0.3380	450,000	5.1660	0.000006470186	7	Gate
0.3380	485,000	5.3924	0.000006470186	7	Gate
0.3205	530,000	5.6807	0.000006409088	4	Gate
0.3236	565,000	5.8976	0.000006193947	5	Gate
0.3176	600,000	6.1104	0.000006079641	7	Gate
0.3176	635,000	6.3233	0.000006079641	7	Gate
0.3030	680,000	6.5959	0.000006060768	1	Gate
0.3165	715,000	6.8079	0.000006058209	7	Gate
0.2992	760,000	7.0771	0.000005983640	4	Gate
0.2955	805,000	7.3431	0.000005909000	4	Gate
0.2908	850,000	7.6048	0.000005816944	4	Gate
0.2813	895,000	7.8580	0.000005625368	4	Gate
0.2880	930,000	8.0509	0.000005512874	7	Gate
0.2743	975,000	8.2977	0.000005486040	4	Gate
0.2855	1,010,000	8.4891	0.000005465248	7	Gate
0.2851	1,045,000	8.6801	0.000005458104	7	Gate
0.1897	1,070,000	8.8129	0.000005311880	4	Light
0.2734	1,105,000	8.9961	0.000005234254	7	Gate
0.2712	1,140,000	9.1779	0.000005191390	5	Gate
0.2651	1,175,000	9.3555	0.000005074703	7	Gate
0.2509	1,220,000	9.5813	0.000005018296	4	Gate
0.2588	1,255,000	9.7547	0.000004953252	7	Gate
0.1738	1,280,000	9.8764	0.000004866030	4	Light
0.2494	1,315,000	10.0434	0.000004774650	7	Gate
0.2365	1,360,000	10.2563	0.000004729688	1	Gate
0.2365	1,405,000	10.4691	0.000004729688	1	Gate
0.2386	1,440,000	10.6290	0.000004567470	7	Gate
0.2318	1,475,000	10.7842	0.000004436495	6	Gate
0.1580	1,500,000	10.8948	0.000004423664	4	Light
0.2153	1,545,000	11.0886	0.000004306728	4	Gate
0.2152	1,590,000	11.2823	0.000004304240	4	Gate
0.1499	1,615,000	11.3873	0.000004197256	4	Light
0.1499	1,640,000	11.4923	0.000004197256	4	Light
0.1482	1,665,000	11.5961	0.000004148491	4	Light
0.1474	1,690,000	11.6993	0.000004127592	4	Light
0.2061	1,735,000	11.8848	0.000004122616	4	Gate
0.1442	1,760,000	11.9857	0.000004037029	4	Light
0.2105	1,795,000	12.1268	0.000004029280	5	Gate
0.2059	1,830,000	12.2647	0.000003941170	7	Gate
0.2045	1,865,000	12.4017	0.000003914975	7	Gate
0.1387	1,890,000	12.4988	0.000003883768	4	Light

TABLE 3-3. OUTPUT OF RESOURCE ALLOCATION MODEL FOR A HYPOTHETICAL STATE SHOWING CROSSING DECISIONS RANKED BY BENEFIT/COST RATIO (CONT.)

PREDICTED ACCIDENTS PER YEAR	CUMULATIVE COST (DOLLARS)	CUMULATIVE BENEFIT	BENEFIT/COST PER CROSSING	PRESENT WARNING DEVICE CLASS	RECOMMENDED WARNING SYSTEM
0.1977	1,925,000	12.6313	0.000003783999	7	Gate
0.1919	1,960,000	12.7600	0.000003674456	7	Gate
0.1905	1,995,000	12.8876	0.000003645879	7	Gate
0.1903	2,030,000	13.0151	0.000003643498	7	Gate
0.1293	2,055,000	13.1055	0.000003619045	4	Light
0.1815	2,090,000	13.2272	0.000003474421	7	Gate
0.1799	2,125,000	13.3477	0.000003443463	7	Gate
0.1798	2,160,000	13.4682	0.000003441082	7	Gate
0.1794	2,195,000	13.5883	0.000003433938	5	Gate
0.1745	2,230,000	13.7053	0.000003341064	5	Gate
0.1734	2,265,000	13.8215	0.000003319631	5	Gate
0.1725	2,300,000	13.9370	0.000003302962	7	Gate
0.1153	2,325,000	14.0178	0.000003228926	4	Light
0.1657	2,360,000	14.1287	0.000003171987	5	Gate
0.1128	2,385,000	14.2077	0.000003159262	4	Light
0.1626	2,420,000	14.3167	0.000003112452	7	Gate
0.1618	2,455,000	14.4252	0.000003098165	5	Gate
0.1105	2,480,000	14.5026	0.000003093082	4	Light
0.1596	2,515,000	14.6095	0.000003055300	7	Gate
0.1082	2,540,000	14.6853	0.000003030384	4	Light
0.1079	2,565,000	14.7608	0.000003019934	4	Light
0.1554	2,600,000	14.8649	0.000002974333	5	Gate
0.1548	2,635,000	14.9686	0.000002962426	7	Gate
0.1536	2,670,000	15.0714	0.000002940994	7	Gate
0.1536	2,705,000	15.1743	0.000002940994	7	Gate
0.1521	2,740,000	15.2762	0.000002912417	5	Gate
0.1478	2,775,000	15.3752	0.000002829069	7	Gate
0.1475	2,810,000	15.4741	0.000002824307	7	Gate
0.1006	2,835,000	15.5445	0.000002817909	4	Light
0.1006	2,860,000	15.6149	0.000002817909	4	Light
0.1472	2,895,000	15.7136	0.000002817162	6	Gate
0.1462	2,930,000	15.8115	0.000002798112	6	Gate
0.1460	2,965,000	15.9094	0.000002795730	7	Gate
0.1454	3,000,000	16.0068	0.000002783823	7	Gate
0.0993	3,025,000	16.0763	0.000002779594	4	Light
0.1446	3,060,000	16.1732	0.000002767154	7	Gate
0.1446	3,095,000	16.2702	0.000002767154	7	Gate
0.1444	3,130,000	16.3669	0.000002764773	7	Gate
0.1433	3,165,000	16.4630	0.000002743340	7	Gate
0.0974	3,190,000	16.5311	0.000002727346	1	Light
0.1403	3,225,000	16.6252	0.000002686187	7	Gate
0.0933	3,250,000	16.6905	0.000002612400	4	Light
0.1357	3,285,000	16.7814	0.000002598077	7	Gate
0.1353	3,320,000	16.8721	0.000002590932	5	Gate
0.1345	3,355,000	16.9622	0.000002574263	5	Gate
0.1340	3,390,000	17.0520	0.000002564737	7	Gate
0.1329	3,425,000	17.1411	0.000002543305	7	Gate
0.1325	3,460,000	17.2299	0.000002536160	7	Gate
0.0882	3,485,000	17.2916	0.000002469589	4	Light
0.1289	3,520,000	17.3779	0.000002467101	7	Gate
0.0866	3,545,000	17.4385	0.000002424307	4	Light
0.1265	3,580,000	17.5232	0.000002421855	5	Gate

TABLE 3-3. OUTPUT OF RESOURCE ALLOCATION MODEL FOR A HYPOTHETICAL STATE SHOWING CROSSING DECISIONS RANKED BY BENEFIT/COST RATIO (CONT.)

PREDICTED ACCIDENTS PER YEAR	CUMULATIVE COST (DOLLARS)	CUMULATIVE BENEFIT	BENEFIT/COST PER CROSSING	PRESENT WARNING DEVICE CLASS	RECOMMENDED WARNING SYSTEM
0.1263	3,615,000	17.6078	0.000002417092	7	Gate
0.1260	3,650,000	17.6923	0.000002412329	5	Gate
0.1259	3,685,000	17.7766	0.000002409948	6	Gate
0.1258	3,720,000	17.8609	0.000002407567	5	Gate
0.1255	3,755,000	17.9449	0.000002402803	5	Gate
0.1254	3,790,000	18.0289	0.000002400422	7	Gate
0.1248	3,825,000	18.1125	0.000002388516	7	Gate
0.1246	3,860,000	18.1960	0.000002386134	7	Gate
0.0852	3,885,000	18.2557	0.000002385992	4	Light
0.1238	3,920,000	18.3387	0.000002369464	7	Gate
0.1225	3,955,000	18.4208	0.000002345650	7	Gate
0.1213	3,990,000	18.5020	0.000002321838	5	Gate
0.1210	4,025,000	18.5831	0.000002317074	5	Gate
0.0799	4,050,000	18.6390	0.000002236214	4	Light
0.1159	4,085,000	18.7166	0.000002219438	5	Gate
0.0784	4,110,000	18.7715	0.000002194416	1	Light
0.0773	4,135,000	18.8256	0.000002163067	4	Light
0.0770	4,160,000	18.8794	0.000002156101	4	Light
0.0770	4,185,000	18.9333	0.000002156101	4	Light
0.1126	4,220,000	19.0087	0.000002155142	5	Gate
0.1126	4,255,000	19.0841	0.000002155142	7	Gate
0.1120	4,290,000	19.1591	0.000002143234	7	Gate
0.1107	4,325,000	19.2332	0.000002119420	5	Gate
0.0756	4,350,000	19.2862	0.000002117786	4	Light
0.1106	4,385,000	19.3604	0.000002117039	7	Gate
0.0754	4,410,000	19.4131	0.000002110819	4	Light
0.1098	4,445,000	19.4868	0.000002102751	5	Gate
0.1098	4,480,000	19.5604	0.000002102751	7	Gate
0.0743	4,505,000	19.6125	0.000002079470	4	Light
0.1086	4,540,000	19.6852	0.000002078937	5	Gate
0.0739	4,565,000	19.7369	0.000002069021	4	Light
0.0739	4,590,000	19.7887	0.000002069021	4	Light
0.1079	4,625,000	19.8610	0.000002064649	7	Gate
0.1079	4,660,000	19.9332	0.000002064649	7	Gate
0.0728	4,685,000	19.9841	0.000002037672	4	Light
0.1057	4,720,000	20.0550	0.000002024166	7	Gate
0.0720	4,745,000	20.1054	0.000002016773	4	Light
0.1050	4,780,000	20.1757	0.000002009877	7	Gate
0.0718	4,805,000	20.2259	0.000002009806	4	Light
0.1047	4,840,000	20.2961	0.000002005115	5	Gate
0.0715	4,865,000	20.3462	0.000002002840	4	Light
0.0713	4,890,000	20.3961	0.000001995874	4	Light
0.1042	4,925,000	20.4659	0.000001995589	5	Gate
0.1039	4,960,000	20.5355	0.000001988446	7	Gate
0.0705	4,985,000	20.5848	0.000001874974	4	Light
0.0703	5,010,000	20.6340	0.000001968008	4	Light

4. CONCLUSIONS

The resource allocation model was designed to help determine the optimum allocation of funds to improve rail-highway crossing safety. The criterion for optimum allocation is the maximum benefit, measured in terms of the number of accidents prevented per year. The model nominates crossings for further investigation on the basis of the crossing's annual predicted number of accidents and on the effectiveness and cost of the available warning system options.

The maximum benefit can be determined for any given funding level and for a wide range of effectiveness-cost parameters for warning systems. When applied to national crossings, assuming warning system parameters and a funding level of $500 million, the estimated maximum benefit was 2,575 accidents prevented per year. The model recommends the particular crossings to be upgraded and the type of warning systems to be installed at these crossings. According to the model, there was no other set of crossings from the DOT-AAR Crossing Inventory and no other selection of warning systems which would produce a greater benefit than 2,575 accidents prevented per year for this funding level.

By properly specifying warning device parameters, the model can be made to restrict the new warning device options to automatic gates only. This "gates only" policy produces a benefit which is slightly less (≃7 percent) than that obtained by allowing both flashing lights and gates. A similar result was observed in previous analysis. (Ref. 9) A benefit/cost curve was also obtained for a "lights only" policy. In this case, the benefit was diminished by about 25 percent.

The optimum rail-highway crossing decisions recommended by the model change were a function of assumed funding levels. An optimum set of decisions were specified for a particular funding level. If at a later date, more or less money is made available, some of the optimum decisions for a new funding level might be different than those which were determined for the original funding level. This highlights the importance of a careful determination of available funding levels before a program is specified.

Any accident prediction formula which calculates the expected number of accidents per year for the rail-highway crossings can be used with this model. In this study, the DOT accident prediction formula was used. The equipment effectiveness and cost figures used in this analysis were considered to be close to national averages. These data could vary due to local conditions. For this reason and the inherent uncertainties concerning the data, the sensitivity of benefit/cost results to these equipment parameters were determined. The cost variation was from -50 percent to +40 percent of the assumed national averages. At a funding level of $500 million, this resulted in a benefit variation which ranged from +50 percent to -20 percent. If the effectiveness for lights and gates, relative to passive warning devices, were reduced from 0.7 and 0.9 to 0.5 and 0.7, respectively, the benefit for a $500 million funding level was reduced by 40 percent. The sensitivity of benefits to the effectiveness and cost inaccuracies proved constant, regardless of the absolute values.

APPENDIX A
COMMON RAIL-HIGHWAY CROSSING HAZARD MODELS

Many accident prediction formulas have been proposed over the years. These procedures fall into one of two categories, absolute or relative. Procedures which produce a "stand-alone" accident prediction figure are considered to be absolute. Procedures resulting in a hazard index number which has value only when compared to another number derived from the same source are considered to be relative. Absolute accident predictions are produced by accident prediction formulas such as Coleman-Stewart, Peabody-Dimmick, and the DOT Accident Prediction Formula. Other commonly used procedures, hazard index models, produce a relative hazard index. The best known of these is the New Hampshire Formula. The most widely used formulas are described on the following pages.

1. The Coleman-Stewart Model[1]:

$$\text{LOG } H = C_0 + C_1 \text{ LOG } C + C_2 \text{ LOG } T + C_3 (\text{LOG } T)^2$$

where 1/2 is used for C or T, if C = 0 or T = 0, respectively.

Category	C_0	C_1	C_2	C_3
Single-track, Urban				
Automatic gates	-2.17	0.16	0.96	-0.35
Flashing lights	-2.85	0.37	0.16	-0.42
Crossbucks	-2.38	0.26	0.78	-0.18
Single-track, Rural				
Automatic gates	-1.42	0.08	-0.15	-0.25
Flashing lights	-3.56	0.62	0.92	-0.38
Crossbucks	-2.77	0.40	0.89	-0.29
Multiple-track, Urban				
Automatic gates	-2.58	0.23	1.30	-0.42
Flashing lights	-2.50	0.36	0.68	-0.09
Crossbucks	-2.49	0.32	0.63	-0.02
Multiple-track, Rural				
Automatic gates	-1.63	0.22	-0.17	0.05
Flashing lights	-2.75	0.38	1.02	-0.36
Crossbucks	-2.39	0.46	-0.50	0.53

C = vehicle movements per day

T = train movements per day

H = the average number of accidents per crossing-year

[1] J. Coleman and G.R. Stewart, "Investigations of Railroad-Highway Grade Crossing Accident Data," Transportation Research Record, No. 611, 1976.

2. Peabody-Dimmick[1]:

$$A_5 = 1.28 \frac{V^{0.170} T^{0.151}}{P_f^{0.171}} + K$$

3. Mississippi Formula[2]:

$$H.I. = \frac{\frac{SDR}{8} + A_5}{2}$$

4. New Hampshire Formula[3]:

$$H.I. = VTP_f$$

5. The Ohio Method[4]:

$$H.I. = A_f + B_f + G_f + L_f + N_f + SDR$$

6. The Wisconsin Method[5]:

$$H.I. = \frac{\left(T \frac{V}{20} + \frac{P_1}{50}\right)}{5} + SDR + A_e$$

---

1. L.E. Peabody and T.B. Dimmick, "Accident Hazards at Grade Crossings," Public Roads, Vol. 22, No. 6 pp. 123-130, (Washington, D.C.: GPO, August 1941).
2. John Dearinger, "Cross Section and Pavement Surface," (Automotive Safety Foundation, 1970).
3. National Transportation Safety Board, "Special Study of Rail-Rapid Transit," (Washington, D.C.: GPO, June 6, 1971).
4. State of Ohio Department of Highways, "Ohio Railroad Grade Crossing Priority Report," (State of Ohio, 1959).
5. California Public Utilities Commission, "The Effectiveness of Automotive Protection in Reducing Accident Frequency and Severity at Public Grade Crossings in California," (State of California, June 30, 1974).

7. Contra Costa County Method[1]:

$$H.I. = TZ\left(1 - 2.718^{\frac{-Vt}{1400Z}}\right)$$

8. The Oregon Method[2]:

$$H.I. = \left(V_1T_1P_f + 1.4\ V_2T_2P_f\ \frac{A_e}{A_5}\right)$$

9. North Dakota Rating System[3]:

$$H.I. = \left(N_f+L_f\right) + \left(P_f+D_f+G_f+X_f\right) + \left(VT_f\right) + SDR$$

10. Idaho Formula[4]:

$$H.I. = V_f \times T_f \left(CB_f + SDR + N_f + Y_f\right)$$

11. Utah Formula[5]:

$$H.I. = \frac{T}{1000}\left[\left(\frac{P}{10} + \frac{F}{20} + \frac{S}{30}\right) + SDR + N_f + X_f + R_f\right] + 2A_e$$
$$+ \frac{P_1}{100{,}000}\left(\frac{P}{10} + \frac{F}{20} + \frac{S}{30}\right) - P_f$$

---

1. Cynthia K. Danner, "Critique of Report #50, "Factors Influencing Safety at Highway-Rail Grade Crossings," California Public Utilities Commission, Case No. 8767, Exhibits 112 and 113, (Southern Pacific Transportation Company, 1969).
2. "Relative Hazards at Railroad Grade Crossings on State Federal-Aid-Highway Systems," (Oregon State Highway Department, April 1956).
3. Donald G. Newman, "An Economic Analysis of Railway Grade Crossings on the California State Highway System," Report EEP-16, (Stanford University, June 1965).
4. E.L. and Ireson Grant, Engineering Economy, Fourth Edition, (New York: Ronald Press Co., 1964).
5. Federal Highway Administration, "Highway Progress," (Washington, D.C.: U.S. Department of Transportation, 1971).

12. City of Detroit Formula[1]:

$$H.I. = \frac{T}{1000} \left[\frac{P}{10} + \frac{F}{20} + \frac{S}{30} \ SDR + N_f + X_f + R_f \right] \left(100\% - \%P_f\right) + 2A_e$$

Key:

A_5 = expected number of accidents in 5 years
A_e = accident experience
A_f = accident probability factor
B_f = train speed factor
CB_f = type and speed of train factor
D_f = alignment of track and highway factor
F = number of freight trains in 24 hours
G_f = approach gradient factor
H.I. = hazard index
K = parameter specified in graphical form
L_f = angle of crossing factor
N_f = number of tracks factor
P = number of passenger trains in 24 hours
P_1 = number of pedestrians in 24 hours
P_f = protection factor
R_f = road approach factor
S = number of switch trains in 24 hours
SDR = Sight Distance Rating
t = time crossing blocked
T = average 24-hour train volume
T_1 = average daylight train volume
T_2 = average train volume during dark hours
T_f = train volume factor
V = average 24-hour traffic volume
V_1 = average daylight traffic volume
V_2 = average traffic volume during dark hours
V_f = traffic volume factor
VT_f = exposure factor
X_f = condition of crossing factor
Y_f = severity factor
Z = number of traffic lanes

1. Jack E. Lersch and Associates, "Alignment," (Automotive Safety Foundation, 1971).

APPENDIX B

DOT ACCIDENT PREDICTION FORMULA

The resource allocation model for this study used the DOT accident prediction formula to obtain input on predicted crossing accidents per year. The DOT formula uses information which is contained in the DOT-AAR Rail-Highway Crossing Inventory. (Ref. 2) All of the factors in the inventory were subjected to thorough statistical analysis. Only those factors which were selected for their significance in predicting rail-highway crossing accidents are shown in the formula on page 37. This is the only available formula which has been developed from DOT-AAR Crossing Inventory information.

Determination of the constants used in this formula is discussed in full detail by Peter Mengert in Rail-Highway Crossing Hazard Prediction Research Results. (Ref. 11) This formula was developed using 1975 accident data and was normalized so that the sum of the accident predictions for all 219,162 rail-highway crossings equaled 11,354, the number of rail-highway crossing accidents that occurred in 1975. (Ref. 4) When used with inventory data for 1975, the results which involve the number of accidents per year, refer to 1975 accidents. This normalizing factor does not affect the model's ability to rank crossings in a relative sense. It does affect the absolute quantity of accidents reduced per year, computed by the resource allocation model. However, the decisions in the algorithm, as to which crossings should receive which warning system, are not affected by the normalizing factor. Thus, an accident prediction procedure, producing "expected accidents per time period" which are in error by a multiplication constant, will still produce correct optimal decisions.

The accident prediction formula used in the present analysis consists of three equations.

Warning Device Classes 1, 2, 3, and 4

$$H = 0.389 \ EXP \ (2X_1)$$

where

$$X_1 = 0.74982 \ HVOL_1 + 0.19474 \ LOG_{10})DT+1) + 0.17491 \ MAIN \ TRACKS + 0.17780 \ HWY \ PAVED + 0.045405 \ POP - 0.13139 \ FC,$$

$$HVOL_1 = -0.13711 + 0.38069h - 0.66800h^2 - 0.19171h^3,$$

$$h = -3.0264 + 1.1580 \ LOG_{10}(T+1) + 0.48654 \ LOG_{10}(C+1) - 0.22122 \ [LOG_{10}(T+1)]^2.$$

Warning Device Classes 5, 6, and 7

$$H = 1.084 \ EXP \ (2X_2)$$

where

$$X_2 = 1.0422 \ HVOL_2 + 0.13737 \ MAIN \ TRACKS - 0.097584 \ [LOG_{10}(T+1)]^2 + 0.018064 \ LANES - 0.036259 \ LOG_{10}(DT+1) + 0.018944 \ POP,$$

$$HVOL_2 = 2.8395 + 0.75477 \ LOG_{10}(T+1) + 0.083292 \ [LOG_{10}(C+1)]^2.$$

Warning Device Class 8

$$H = 0.820 \ EXP \ (2X_3)$$

where

$$X_3 = -0.83656 + 0.74849 \ HVOL_3 + 0.19139 \ MAIN \ TRACKS + 0.093829 \ LANES,$$

$$HVOL_3 = -1.9674 + 0.18621 \ LOG_{10}(T+1) \ LOG_{10}(C+1).$$

Key:

H	= expected number of accidents per year
T	= number of trains per day
C	= number of highway vehicles per day
DT	= number of day thru-trains per day
MAIN TRACKS	= number of main tracks
HWY PAVED	= 1 if highway paved, 0 if not paved
POP	= population - tens digit of the functional classification of road crossing[1]
FC	= units digit of functional classification of road over crossing[1]
LANES	= number of traffic lanes
EXP(x)	= natural base e (2.71828), raised to the power (x)

[1]Functional classification, as well as the other parameters in the model are contained in the DOT-AAR Crossing Inventory. (ref. 2 and Ref. 12)

APPENDIX C
MATHEMATICAL PROPERTIES OF THE RESOURCE ALLOCATION MODEL

The mathematics involved in the resource allocation algorithm consist basically of calculating all incremental benefit/ cost ratios; sorting them; and making decisions sequentially from the ranked listing, starting with the highest. For each decision, the incremental benefit and incremental cost are recorded, and the total benefit and total cost are obtained. Thus, the mathematics would seem simple, amounting to a counting type of procedure. However, there are non-trivial mathematical aspects which should be considered.

Optimality of Algorithm

It must be proven that the algorithm is optimum. For a given cost, the total benefit must be maximum. The term "strategy" means a specific assignment of warning systems to be installed at a specific set of crossings. The strategy produced by the algorithm is called the optimum strategy. The optimum strategy, compared with any other strategy, is presented in Figure C-1. The total benefit and total cost for any strategy is the sum of a set of benefit and cost increments, where there is a pair of increments for each crossing and warning system combination. The incremental benefit/cost ratios for any other strategy are ranked and compared to the set of increments for the optimum and competing strategies. Repetitious increments which are produced by the same crossing warning system combination are deleted from these lists. The total benefit and total cost for these identical increments will be the same and is denoted as point P on the benefit/cost curve shown in Figure C-1. Point P could be the origin. It is necessary to be concerned only with the results when the other increments are added.

A comparison of the benefit and cost curves for the two strategies is illustrated in Figure C-1, where $\{B_{1i}\}$ and $\{C_{1i}\}$ are the sets of benefit and cost increments for the algorithm which remain

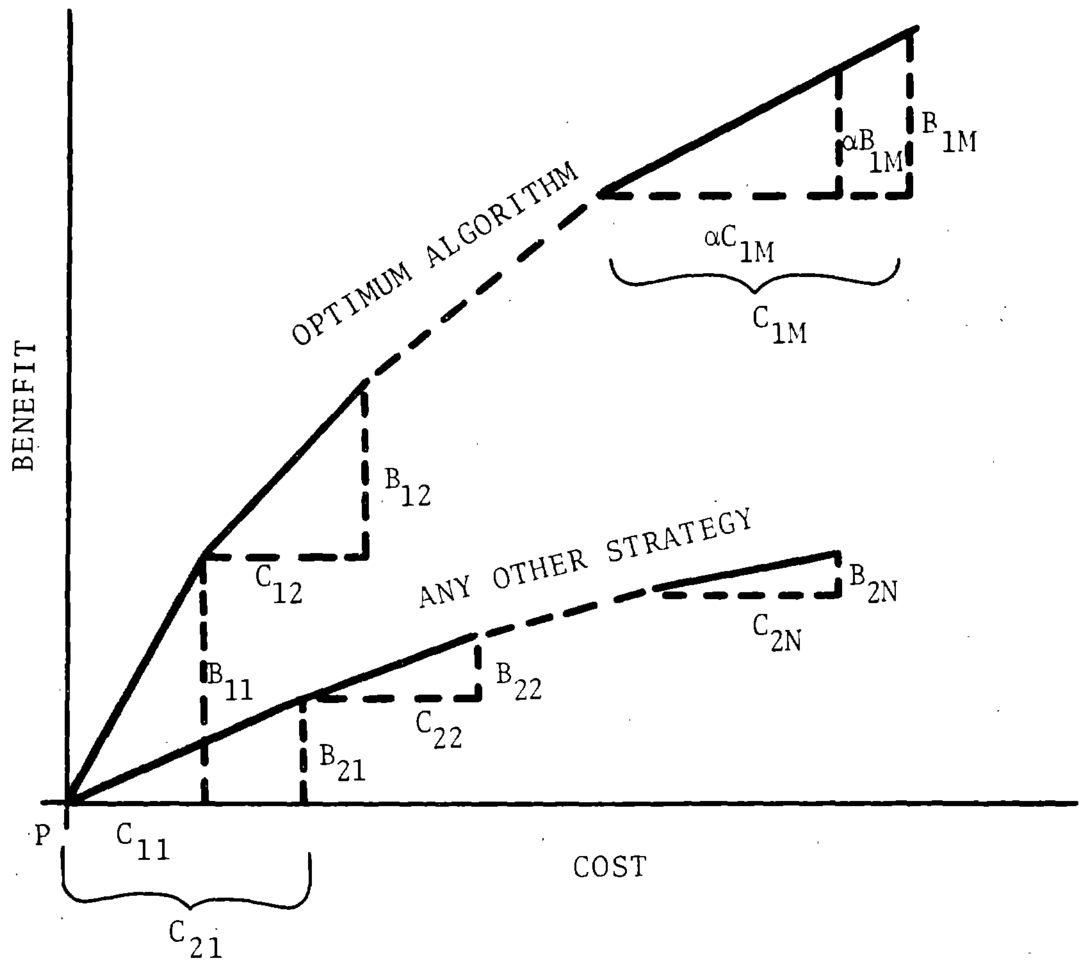

FIGURE C-1. COMPARISON OF BENEFIT OF OPTIMUM ALGORITHM WITH ANY OTHER STRATEGY after the above deletion, where $i = 1, 2, \ldots, M$. The benefit and cost increments $\{B_{2j}\}$ and $\{C_{2j}\}$ remain for the other strategy, where $j = 1, 2, \ldots, N$.

Due to the discrete nature of the problem, the total cost of the other strategy may not equal the total cost of the optimum strategy. Consider the case where the N segment for the other strategy terminates within or at the right end of the M segment for the algorithm. Thus:

$$C_{11} + C_{12} + \cdots + \alpha C_{1M} = C_{21} + C_{22} + \cdots + C_{2N}$$

$$0 < \alpha \leq 1 \qquad \text{(Equation 1)}$$

This suggests the need to be more precise in the meaning of "optimum" when referring to the algorithm. Optimum is defined so that the total benefit, $\sum_{j=1}^{N} B_{2j}$, is no greater than the line joining the end points of the M segment of the algorithm. It is not meaningful to consider a case where the total cost of the other strategy falls outside the M segment of the algorithm.

Since the incremental benefit/cost ratios $\{B_{1i}/C_{1i}\}$ comprise those of the algorithm, they cannot be less than the numbers $\{B_{2j}/C_{2j}\}$ which comprise the competing strategy. If this were not true, there would be a contradiction; if any member of the second set were greater than a member of the first set, this member of the second set would be a member of the first set. In such a case, it would not be greater than any member of the first set. Assuming that each set is in rank order, the following inequalities hold:

$$\frac{B_{11}}{C_{11}} \geq \frac{B_{12}}{C_{12}} \geq \cdots \geq \frac{B_{1M}}{C_{1M}} \geq \frac{B_{21}}{C_{21}} \geq \frac{B_{22}}{C_{22}} \geq \cdots \geq \frac{B_{2N}}{C_{2N}} \qquad \text{(Equation 2)}$$

The inequalities in equation 2 can be used to produce:

$$B_{21} + B_{22} + \cdots + B_{2N} = \frac{B_{21}}{C_{21}} C_{21} + \frac{B_{22}}{C_{22}} C_{22} + \cdots + \frac{B_{2N}}{C_{2N}} C_{2N}$$

$$\leq \frac{B_{1M}}{C_{1M}} \left(C_{21} + C_{22} + \cdots + C_{2N}\right)$$

Substituting equation 1 produces the inequality:

$$B_{21} + B_{22} + \cdots + B_{2N} \leq \frac{B_{1M}}{C_{1M}} \left(C_{11} + C_{12} + \cdots + \alpha C_{1M}\right) \quad \text{(Equation 3)}$$

The inequality in equation 2 again produces:

$$\frac{B_{1M}}{C_{1M}} \left(C_{11} + C_{12} + \cdots + \alpha C_{1M}\right) \leq \frac{B_{11}}{C_{11}} C_{11} + \frac{B_{12}}{C_{12}} C_{12} + \cdots + \alpha \frac{B_{1M}}{C_{1M}} C_{1M}$$

Substituting this in equation 3, and simplifying, produces the inequality:

$$B_{21} + B_{22} + \cdots + B_{2N} \leq B_{11} + B_{12} + \cdots + \alpha B_{1M} \quad \text{(Equation 4)}$$

Equation 4 proves that the benefit for any other strategy cannot be higher than that for the optimum strategy. This means that the benefit for the other strategy can be no higher than the interpolated benefit specified by the straight line segments of the algorithm curve. Note that the equality in equation 4 could hold if, for example, $M = N = 1$ and $B_{11}/C_{11} = B_{21}/C_{21}$ but $C_{11} \geq C_{21}$.

Other Properties

According to the algorithm, a decision to install gates at a passive crossing will only be made after a previous decision is made to install flashing lights at the crossing. Once gates have been chosen for a passive crossing, the decision will not be reversed if the algorithm proceeds and more money is allocated. If gates are chosen for a particular crossing currently equipped with flashing lights, this decision will not be reversed. For example, with the symbols E_1 and C_1 denoting the effectiveness and cost of installing flashing lights at a passive crossing and E_2 and C_2 denoting the effectiveness and cost of installing gates at a passive crossing, if $E_1/C_1 > E_2/C_2$, it follows that:

$$E_2 C_1 < E_1 C_2$$

$$\left(E_2 C_1\right) - \left(E_1 C_1\right) < \left(E_1 C_2\right) - \left(E_1 C_1\right)$$

$$\left(E_2 - E_1\right) C_1 < E_1 \left(C_2 - C_1\right)$$

$$\frac{E_2 - E_1}{C_2 - C_1} < \frac{E_1}{C_1}$$

If $E_1/C_1 \leq E_2/C_2$, only gates will be installed and the decisions will not be reversed.

The algorithm demonstrates that if $E_1/C_1 > E_2/C_2$, decisions may be changed concerning the device to be installed at a certain crossing as the level of expenditure increases. For example, at a given passive crossing with a given level of expenditure, a decision could be made that flashing lights be installed. If a greater expenditure is considered, the optimum decision may be the installation of automatic gates at the crossing.

According to the optimal strategy, a warning system will not be installed at a crossing having a lower predicted-accidents-per-year rate than another crossing having the same warning system and the same upgrade cost unless that warning system is first installed at the crossing with the higher annual predicted accident rate. For a passive crossing, flashing lights will not be installed unless all passive crossings having higher rates of predicted accidents per year are equipped with lights. If two passive crossings are considered for possible upgrading to lights, the crossing having the higher rate of predicted accidents per year will produce a greater benefit for the same cost. The same reasoning applies when two crossings with flashing lights are considered for gate installations.

If one of two passive crossings is to be equipped with flashing lights and the other with gates, gates will be installed at the crossing with the higher number of predicted accidents per year. To illustrate this point, H_1 and H_2 are the two hazard indices with $H_1 > H_2$ and from this inequality it follows that:

$$H_1 \; (E_2 - E_1) > H_2 \; (E_2 - E_1)$$

assuming that $E_2 > E_1$.

It then follows that:

$$H_1E_2 + H_2E_1 > H_1E_1 + H_2E_2$$

The left side of the inequality ($H_1E_2 + H_2E_1$) is the benefit when gates are installed at a crossing with a higher number of predicted accidents per year and lights are installed at the other crossing. The right side of the inequality ($H_1E_1 + H_2E_2$) is the benefit for the reverse installation. The cost ($C_1 + C_2$) is the same in both cases.

This reasoning indicates that if the crossings were either all passive or all equipped with flashing lights, the algorithm would dictate that warning systems would be installed at crossings selected consecutively from the list, being ranked by the number of predicted accidents per year, beginning with the crossing with the highest annual number of predicted accidents. However, with a mix of flashing lights and passive crossings, crossings are not always consecutively selected. Some crossings give greater benefit/cost ratios than crossings which have a higher rate of predicted accidents per year. This is due to different effectiveness/cost ratios for different systems.

Using the parameters of run number 1, the ratio for flashing lights to gates is 1.91×10^{-5}, and the ratio for passive to flashing lights is 2.8×10^{-5}. The benefit/cost ratio for passive to flashing lights is $2.8 \times 10^{-5}H$. H is the predicted number of accidents per year. For a crossing equipped with flashing lights and having a higher annual predicted accident rate, the benefit/cost ratio is 1.91×10^{-5} $(H+\Delta)$, where $(H+\Delta)$ is the predicted number of accidents per year. The passive crossing with fewer predicted accidents will be chosen over the flashing light crossing if:

$$2.8 \times 10^{-5} H > 1.91 \times 10^{-5} (H+\Delta)$$

This inequality is satisfied when $\Delta/H < 0.47$. For example, a passive crossing, where $H = 0.5$, will have the same benefit/cost ratio as a flashing light crossing, where $H = (0.5 \times 0.47) + 0.5 = 0.735$.

The accident prediction function is a set of positive numbers which denote the predicted number of accidents per year. This function is a monotonic function of the result of the regression analysis, often called a "hazard function." The value of the hazard function is called the hazard index and can theoretically be a number from $-\infty$ to $+\infty$. In ranking crossings by relative hazard, the original regression output, or any monotonic function of that output would give the same ranking.

The question arises as to whether all monotonic functions of a regression equation would produce the same configuration of equipment assignment from the resource allocation model. This question is reduced to whether the ranking of incremental benefit/cost ratios would be the same for all monotonic functions of a regression equation. The result that equipment assignment, or ranking of incremental benefit/cost ratios, can be different may be demonstrated in the example which follows.

In the case of two passive crossings X_1 and X_2 with the parameters shown in the following table:

	FLASHING LIGHTS	GATES	H*
X_1	0.7, $25,000	0.9, $45,000	0.3
X_2	0.7, $25,000	0.9, $45,000	0.2

The hazard indices are 0.3 and 0.2. The warning system parameters are the same as those for run number 1: $E_1 = 0.7$, $C_1 = 25{,}000$; $E_2 = 0.9$, $C_2 = 45{,}000$. The incremental benefit/cost ratios are:

	FLASHING LIGHTS	GATES
X_1	0.0084×10^{-3}	0.003×10^{-3}
X_2	0.0056×10^{-3}	0.002×10^{-3}

The algorithm produces the following ranking:

INCREMENTAL B/C RATIOS	COST	BENEFIT	DECISION[1]
0.0084×10^{-3}	$25,000	0.21	FL on X_1
0.0056×10^{-3}	50,000	0.35	FL on X_1, X_2
0.003×10^{-3}	70,000	0.41	G on X_1, FL on X_2
0.002×10^{-3}	90,000	0.45	G on X_1, X_2

With the same parameters, but a new hazard function, the parameters are as follows:

	FLASHING LIGHTS	GATES	H*
X_1	0.7, $25,000	0.9, $45,000	0.3
X_2	0.7, $25,000	0.9, $45,000	0.1

*H may be either a relative or absolute measure.

[1]G = gates
FL = flashing lights

The incremental benefit/cost ratios are:

	FLASHING LIGHTS	GATES
X_1	0.0084×10^{-3}	0.003×10^{-3}
X_2	0.0028×10^{-3}	0.001×10^{-3}

The algorithm produces the following ranking:

INCREMENTAL B/C RATIOS	COST	BENEFITS	DECISION[1]
0.0084	$25,000	0.21	FL on X_1
0.0030	45,000	0.27	G on X_1
0.0028	70,000	0.34	G on X_1, FL on X_2
0.0010	90,000	0.36	G on X_1, G on X_2

The change is the second decision; gates instead of flashing lights were installed at crossing X_1.

The benefit/cost curves for both of these cases are shown in Figure C-2.

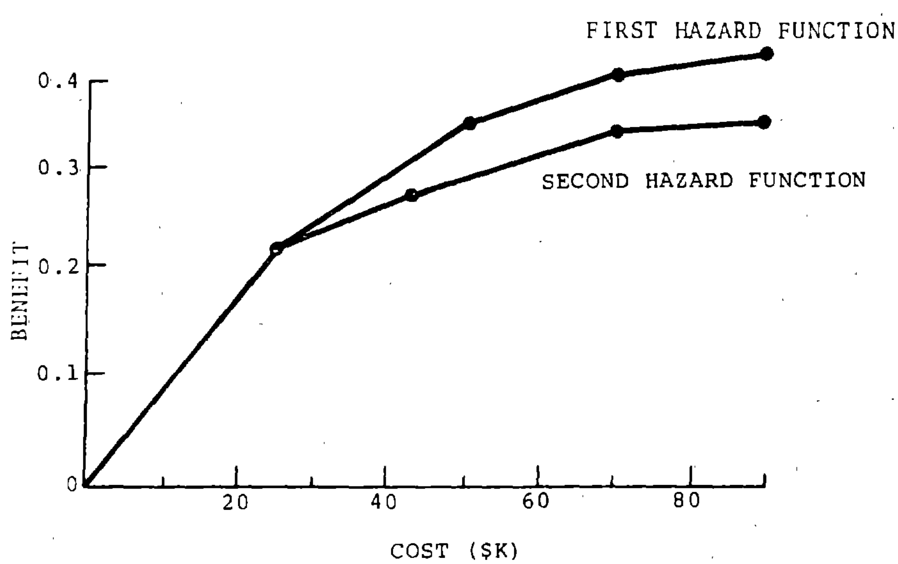

FIGURE C-2. BENEFIT/COST CURVES FOR TWO HYPOTHETICAL MONOTONIC HAZARD FUNCTIONS

[1] G = gates
FL = flashing lights

Where two hazard functions differ only by a constant multiplier, the ranking of the incremental benefit/cost ratios would be the same. Therefore, the rail-highway crossing warning device decisions made by the resource allocation algorithm are the same. The value of the benefit, when using one hazard function, would differ by this constant multiplier from the other hazard function.

Method of Calculating Additional Values

The results obtained in this report could be used to estimate the benefit for any incremental change in either effectiveness or cost. To demonstrate this, if B denotes the benefit and C the cost, a functional relationship exists: $B = f(E_1, E_2, C_1, C_3, C)$. C_3 denotes the cost of installing gates at a crossing which is equipped with flashing lights. Figure 3-1 provides some values of this function. With B_o the benefit at some known point $(E_{10}, E_{20}, C_{10}, C_{20}, C_{30}, C_o)$, B_1 is the estimated benefit at some other point $(E_{11}, E_{21}, C_{11}, C_{21}, C_{31}, C_1)$, using the linear terms of the Taylor's series expansion:[1]

$$B_1 \approx B_o + \left.\frac{\partial f}{\partial E_1}\right|_o \left(E_{11} - E_{10}\right) + \left.\frac{\partial f}{\partial E_2}\right|_o E_{21} - E_{20} + \left.\frac{\partial f}{\partial C_1}\right|_o \left(C_{11} - C_{10}\right)$$

$$+ \left.\frac{\partial f}{\partial C_2}\right|_o \left(C_{21} - C_{20}\right) + \left.\frac{\partial f}{\partial C_3}\right|_o \left(C_{31} - C_{30}\right) + \left.\frac{\partial f}{\partial C}\right|_o \left(C_1 - C_o\right)$$

The subscripts after the partial derivatives signify that the partial derivatives should be evaluated at $E_{10}, E_{20}, C_{10}, C_{20}, C_{30}, C_o$. These partial derivatives can be approximated by a ratio of increments:

[1] G.B. Thomas, Jr. and R.L. Finney, Calculus and Analytic Geometry, Fifth edition, (Reading, Ma.: Addison-Wesley Publishing Company, 1979), p. 810.

$$B_1 \approx B_o + \frac{\Delta f}{\Delta E_1}\left(E_{11}-E_{10}\right) + \frac{\Delta f}{\Delta E_2}\left(E_{21}-E_{20}\right) + \frac{\Delta f}{\Delta C_1}\left(C_{11}-C_{10}\right)$$

$$+ \frac{\Delta f}{\Delta C_2}\left(C_{21}-C_{20}\right) + \frac{\Delta f}{\Delta C_3}\left(C_{31}-C_{30}\right) + \frac{\Delta f}{\Delta C}\left(C_1-C_o\right)$$

As an example, in run number 1, C_o is equal to 400 and B_o 2239, as seen in Figure 3-1.[1] The problem is estimating B_1 for the same cost, $C_1 = C_o = 400$, and the same effectiveness values: $E_{11} = E_{10} = 0.7$ and $E_{21} = E_{20} = 0.9$ but $C_{11} = 20$, $C_{21} = 36$, and $C_{31} = 28$. It follows that:

$$B_1 \approx 2239 + \frac{2706-2239}{15-25}(20-25) + \frac{2301-2239}{35-45}(36-45)$$

$$+ \frac{2488-2239}{25-35}(28-35)$$

$$\approx 2703 \text{ accidents prevented/year.}$$

A benefit value of 2301 was obtained from run 2, a benefit value of 2488 was obtained from run 3, and a benefit value of 2706 was obtained from run number 4. The true benefit value is 2581, as shown in Figure 3-2, so these figures are reasonable. Accuracy is improved if the values at which B_1 is evaluated are closer to the values at which B_o is evaluated and if there are finer approximations of the partial derivatives. This method is the conventional multi-dimensional linear interpolation.

[1] See page 20 of this report.

APPENDIX D
RESOURCE ALLOCATION RESULTS

Resource allocation results for different combinations of rail-highway crossing warning equipment effectiveness and costs are shown in Tables D-1 through D-7 and correspond to runs 2 through 8. The parameter values which were used are listed below:

RUN NO.	E_1	E_2	E_3	C_1	C_2	C_3
2	0.7	0.9	0.667	$25,000	$35,000	$35,000
3	0.7	0.9	0.667	25,000	45,000	25,000
4	0.7	0.9	0.667	15,000	45,000	35,000
5	0.6	0.8	0.5	25,000	45,000	35,000
6	0.5	0.7	0.4	25,000	45,000	35,000
7	0.7	----	----	25,000	----	----
8	----	0.9	0.667	----	45,000	35,000

The symbols E_1, E_2, E_3, C_1, C_2, and C_3 used in this table are defined by the following matrix:

	PROPOSED WARNING SYSTEM			
	FLASHING LIGHTS		AUTOMATIC GATES	
EXISTING WARNING SYSTEM	EQUIPMENT EFFECTIVENESS	EQUIPMENT COST	EQUIPMENT EFFECTIVENESS	EQUIPMENT COST
Passive	E_1	C_1	E_2	C_2
Flashing Lights	-	-	E_3	C_3

The symbols E_1 and C_1 denote the effectiveness and cost of installing flashing lights at a passive crossing. The symbols E_2 and C_2 denote the effectiveness and cost of installing gates at a passive crossing. The symbols E_3 and C_3 denote the effectiveness and cost of installing gates at a crossing which is equipped with flashing lights.

In Tables D-1 through D-7, the first column shows the cumulative amount to be spent for warning system improvements in actual dollars. The second column lists the number of accidents prevented per year. The third, fourth, and fifth columns show the number of crossings where warning systems are to be installed for the three possible transitions from the existing system to the proposed system.

TABLE D-1. RESOURCE ALLOCATION RESULTS FOR RUN NUMBER 2

COST (DOLLARS)	BENEFIT (ACC PREVENTED/YR)[1]	PASSIVE TO FLASHING LTS[2]	PASSIVE TO GATES	FLASHING LTS[2] TO GATES
40,010,000	475	507	178	603
80,015,000	778	1018	436	1,123
120,025,000	1033	1511	733	1,617
160,000,000	1259	1990	1,060	2,090
200,005,000	1464	2431	1,442	2,536
240,010,000	1653	2844	1,826	3,000
280,015,000	1828	3257	2,244	3,430
320,010,000	1994	3720	2,657	3,829
360,020,000	2150	4101	3,099	4,258
400,005,000	2299	4509	3,536	4,672
440,000,000	2441	4909	3,994	5,071
480,000,000	2576	5326	4,451	5,459
520,020,000	2706	5755	4,907	5,840
560,020,000	2831	6130	5,405	6,217
600,010,000	2951	6415	5,958	6,603
640,000,000	3067	6784	6,480	6,960
680,010,000	3180	7235	6,956	7,305
720,000,000	3288	7506	7,530	7,680
760,005,000	3393	7821	8,088	8,040
800,015,000	3495	8209	8,633	8,361
840,015,000	3594	8535	9,156	8,748
880,030,000	3690	8842	9,747	9,081
920,005,000	3784	9062	10,398	9,415
960,005,000	3875	9430	10,965	9,728
1,000,030,000	3963	9673	11,586	10,077

[1] Accidents prevented per year

[2] Flashing lights

TABLE D-2. RESOURCE ALLOCATION RESULTS FOR RUN NUMBER 3

COST (DOLLARS)	BENEFIT (ACC PREVENTED/YR)[1]	PASSIVE TO FLASHING LTS[2]	PASSIVE TO GATES	FLASHING LTS[2] TO GATES
40,020,000	548	440	6	1,150
80,015,000	882	1,060	17	2,110
120,005,000	1158	1,772	29	2,976
160,010,000	1399	2,533	53	3,772
200,010,000	1615	3,308	78	4,552
240,000,000	1812	4,119	115	5,274
280,005,000	1994	4,923	159	5,991
320,010,000	2164	5,800	203	6,635
360,015,000	2324	6,653	267	7,267
400,000,000	2475	7,485	345	7,894
440,005,000	2618	8,381	414	8,474
480,000,000	2754	9,234	495	9,075
520,015,000	2884	10,158	587	9,586
560,020,000	3008	11,026	681	10,149
600,005,000	3127	11,896	794	10,675
640,000,000	3242	12,791	885	11,216
680,020,000	3352	13,658	1016	11,714
720,015,000	3459	14,477	1162	12,232
760,000,000	3561	15,364	1305	12,687
800,020,000	3661	16,166	1461	13,205
840,005,000	3757	17,009	1609	13,695
880,000,000	3850	17,856	1770	14,158
920,010,000	3941	18,649	1958	14,627
960,020,000	4028	19,438	2141	15,109
1,000,000,000	4114	20,166	2360	15,586

[1] Accidents prevented per year

[2] Flashing lights

TABLE D-3. RESOURCE ALLOCATION RESULTS FOR RUN NUMBER 4

COST (DOLLARS)	BENEFIT (ACC PREVENTED/YR)[1]	PASSIVE TO FLASHING LTS[2]	PASSIVE TO GATES	FLASHING LTS[2] TO GATES
40,005,000	583	1,887	1	333
80,005,000	952	3,738	3	680
120,010,000	1257	5,573	5	1034
160,010,000	1523	7,343	6	1417
200,030,000	1763	9,102	8	1804
240,005,000	1982	10,878	15	2176
280,005,000	2184	12,607	18	2574
320,025,000	2373	14,330	25	2970
360,010,000	2550	16,034	29	3377
400,010,000	2717	17,735	32	3787
440,010,000	2874	19,424	46	4188
480,025,000	3024	21,084	63	4598
520,010,000	3167	22,673	75	5044
560,000,000	3302	24,354	93	5443
600,010,000	3432	25,965	113	5870

[1] Accidents prevented per year

[2] Flashing lights

TABLE D-4. RESOURCE ALLOCATION RESULTS FOR RUN NUMBER 5

COST (DOLLARS)	BENEFIT (ACC PREVENTED/YR) [1]	PASSIVE TO FLASHING LTS [2]	PASSIVE TO GATES	FLASHING LTS [2] TO GATES
40,020,000	377	846	25	507
80,025,000	619	1,766	63	944
120,015,000	822	2,655	114	1386
160,015,000	1002	3,546	175	1814
200,020,000	1165	4,409	261	2230
240,020,000	1315	5,247	367	2638
280,010,000	1456	6,147	450	3031
320,025,000	1587	6,987	570	3420
360,005,000	1712	7,830	693	3802
400,020,000	1830	8,657	828	4181
440,020,000	1943	9,507	953	4556
480,030,000	2050	10,309	1108	4927
520,005,000	2154	11,060	1296	5291
560,000,000	2253	11,889	1466	5623
600,000,000	2349	12,683	1640	5975
640,010,000	2442	13,411	1840	6341
680,000,000	2531	14,133	2053	6694
720,010,000	2617	14,905	2255	7026
760,000,000	2701	15,598	2481	7383
800,000,000	2782	16,373	2699	7692
840,005,000	2861	17,162	2900	8013
880,015,000	2938	17,848	3163	8328
920,005,000	3012	18,482	3421	8686
960,000,000	3085	19,131	3677	9036
1,000,020,000	3155	19,828	3945	9337

[1] Accidents prevented per year

[2] Flashing lights

TABLE D-5. RESOURCE ALLOCATION RESULTS FOR RUN NUMBER 6

COST (DOLLARS)	BENEFIT (ACC PREVENTED/YR)[1]	PASSIVE TO FLASHING LTS[2]	PASSIVE TO GATES	FLASHING LTS[2] TO GATES
40,015,000	310	841	58	468
80,015,000	509	1,724	139	876
120,015,000	677	2,579	255	1259
160,020,000	826	3,368	393	1661
200,020,000	962	4,147	538	2061
240,005,000	1087	4,949	708	2412
280,015,000	1203	5,748	867	2780
320,000,000	1313	6,469	1075	3140
360,010,000	1417	7,173	1300	3491
400,015,000	1516	7,870	1528	3843
440,010,000	1610	8,625	1743	4170
480,005,000	1700	9,281	1992	4524
520,020,000	1787	9,950	2249	4859
560,005,000	1871	10,606	2511	5196
600,005,000	1951	11,312	2772	5499
640,020,000	2029	11,928	3060	5832
680,000,000	2104	12,560	3343	6159
720,030,000	2177	13,207	3625	6478
760,000,000	2248	13,760	3954	6802
800,025,000	2316	14,412	4244	7107
840,010,000	2383	15,030	4559	7403
880,000,000	2448	15,663	4858	7709
920,020,000	2511	16,182	5207	8033
960,020,000	2572	16,693	5588	8321
1,000,000,000	2632	17,192	5969	8617

[1] Accidents prevented per year

[2] Flashing lights

TABLE D-6. RESOURCE ALLOCATION RESULTS FOR RUN NUMBER 7

COST (DOLLARS)	BENEFIT (ACC PREVENTED/YR)[1]	PASSIVE TO FLASHING LTS[2]	PASSIVE TO GATES	FLASHING LTS[2] TO GATES
40,000,000	364	1,600	0	0
80,000,000	598	3,200	0	0
120,000,000	793	4,800	0	0
160,000,000	962	6,400	0	0
200,000,000	1115	8,000	0	0
240,000,000	1254	9,600	0	0
280,000,000	1383	11,200	0	0
320,000,000	1503	12,800	0	0
360,000,000	1616	14,400	0	0
400,000,000	1722	16,000	0	0
440,000,000	1823	17,600	0	0
480,000,000	1918	19,200	0	0
520,000,000	2009	20,800	0	0
560,000,000	2095	22,400	0	0
600,000,000	2178	24,000	0	0
640,000,000	2257	25,600	0	0
680,000,000	2332	27,200	0	0
720,000,000	2404	28,800	0	0
760,000,000	2474	30,400	0	0
800,000,000	2541	32,000	0	0
840,000,000	2606	33,600	0	0
880,000,000	2668	35,200	0	0
920,000,000	2729	36,800	0	0
960,000,000	2787	38,400	0	0

[1] Accidents prevented per year

[2] Flashing lights

TABLE D-7. RESOURCE ALLOCATION RESULTS FOR RUN NUMBER 8

COST (DOLLARS)	BENEFIT (ACC PREVENTED/YR) [1]	PASSIVE TO FLASHING LTS [2]	PASSIVE TO GATES	FLASHING LTS [2] TO GATES
40,000,000	439	0	262	806
80,005,000	714	0	633	1,472
120,000,000	944	0	1,045	2,085
160,030,000	1148	0	1,513	2,627
200,030,000	1333	0	1,985	3,163
240,005,000	1503	0	2,486	3,661
280,020,000	1663	0	2,998	4,146
320,000,000	1812	0	3,503	4,639
360,005,000	1954	0	4,035	5,098
400,015,000	2089	0	4,578	5,543
440,000,000	2217	0	5,108	6,004
480,010,000	2339	0	5,679	6,413
520,000,000	2457	0	6,255	6,815
560,010,000	2571	0	6,840	7,206
600,005,000	2680	0	7,399	7,630
640,000,000	2786	0	8,007	7,991
680,040,000	2888	0	8,616	8,352
720,030,000	2987	0	9,192	8,754
760,020,000	3083	0	9,803	9,111
800,015,000	3176	0	10,432	9,445
840,015,000	3267	0	11,065	9,774
880,040,000	3355	0	11,676	10,132
920,020,000	3441	0	12,301	10,471
960,015,000	3524	0	12,922	10,815
1,000,015,000	3606	0	13,562	11,135

[1] Accidents prevented per year

[2] Flashing lights

APPENDIX E
SENSITIVITY OF BENEFITS TO COST

Figure E-1 through Figure E-7 and Tables E-1 through E-5 show the sensitivity of benefits to the changes in warning device costs. Each graph shows a family of benefit/cost curves for given effectiveness values and for equipment costs which are: 20 percent less, 40 percent less, 50 percent less, 20 percent greater, and 40 percent greater than the nominal equipment costs [C_1 = \$25,000, C_2 = \$45,000, and C_3 = \$35,000].

The information shown in the tables was obtained from information contained in Figure E-1 through Figure E-7, assuming the percent change in equipment cost and effectiveness is constant. The precent change in benefits was determined as a function of the total cost. The numbers listed under each run number represent these functional values. The last column of each table contains the average of the functional values.

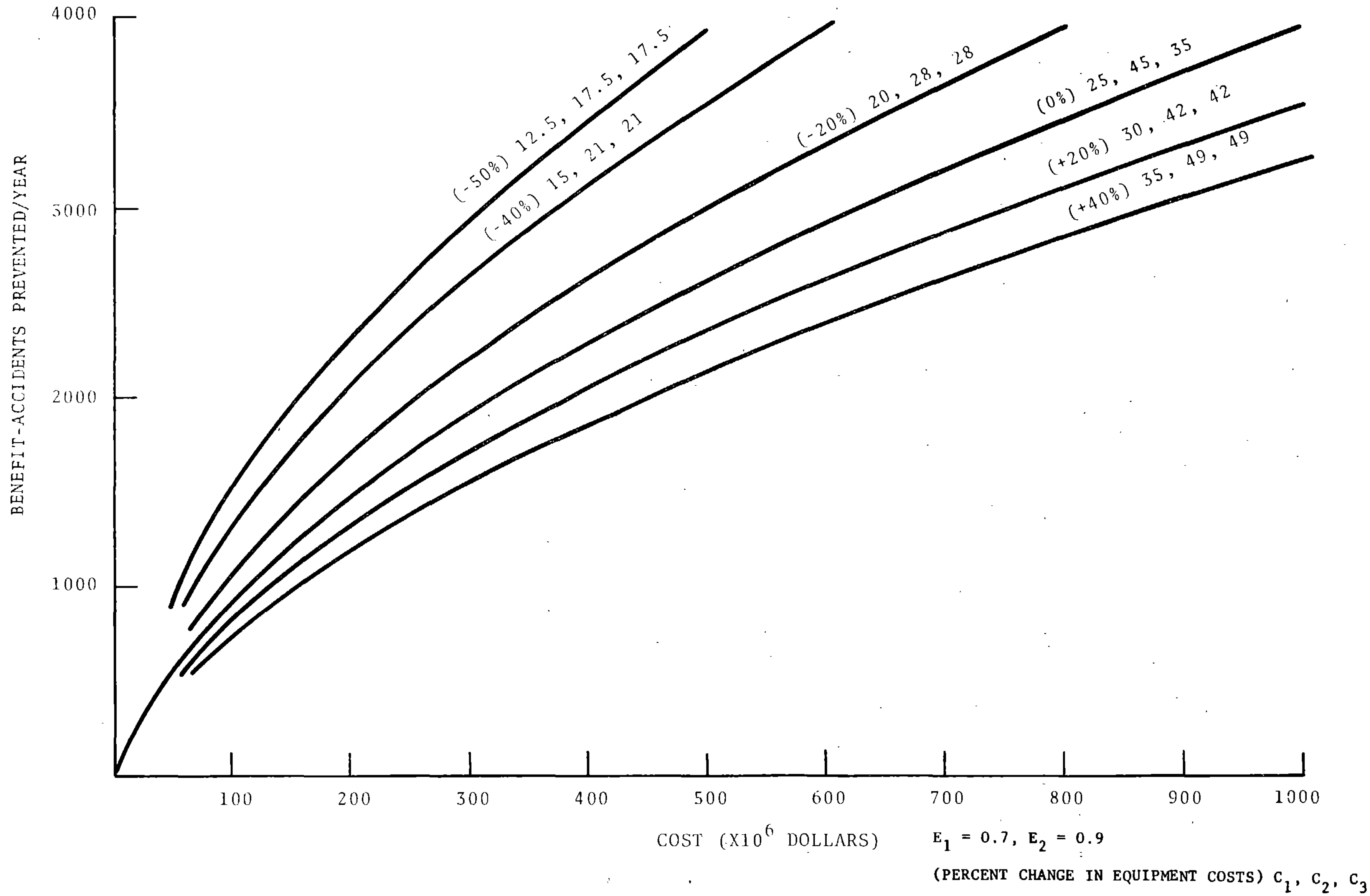

FIGURE E-1. MAXIMUM BENEFIT VERSUS COST FOR DIFFERENT CHANGES IN EQUIPMENT COST - APPLIED TO RUN NUMBER 2

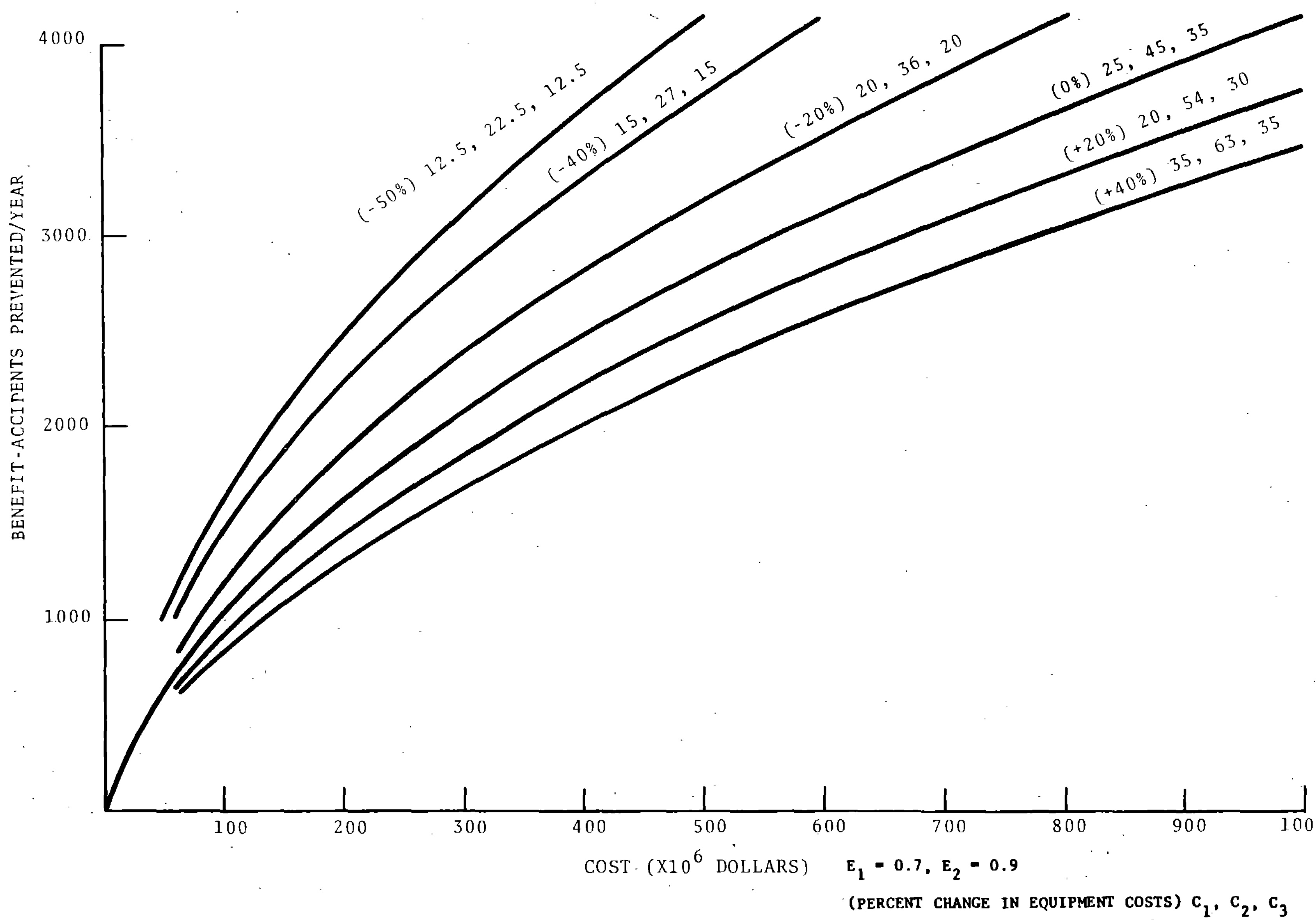

FIGURE E-2. MAXIMUM BENEFIT VERSUS COST FOR DIFFERENT CHANGES IN EQUIPMENT COST - APPLIED TO RUN NUMBER 3

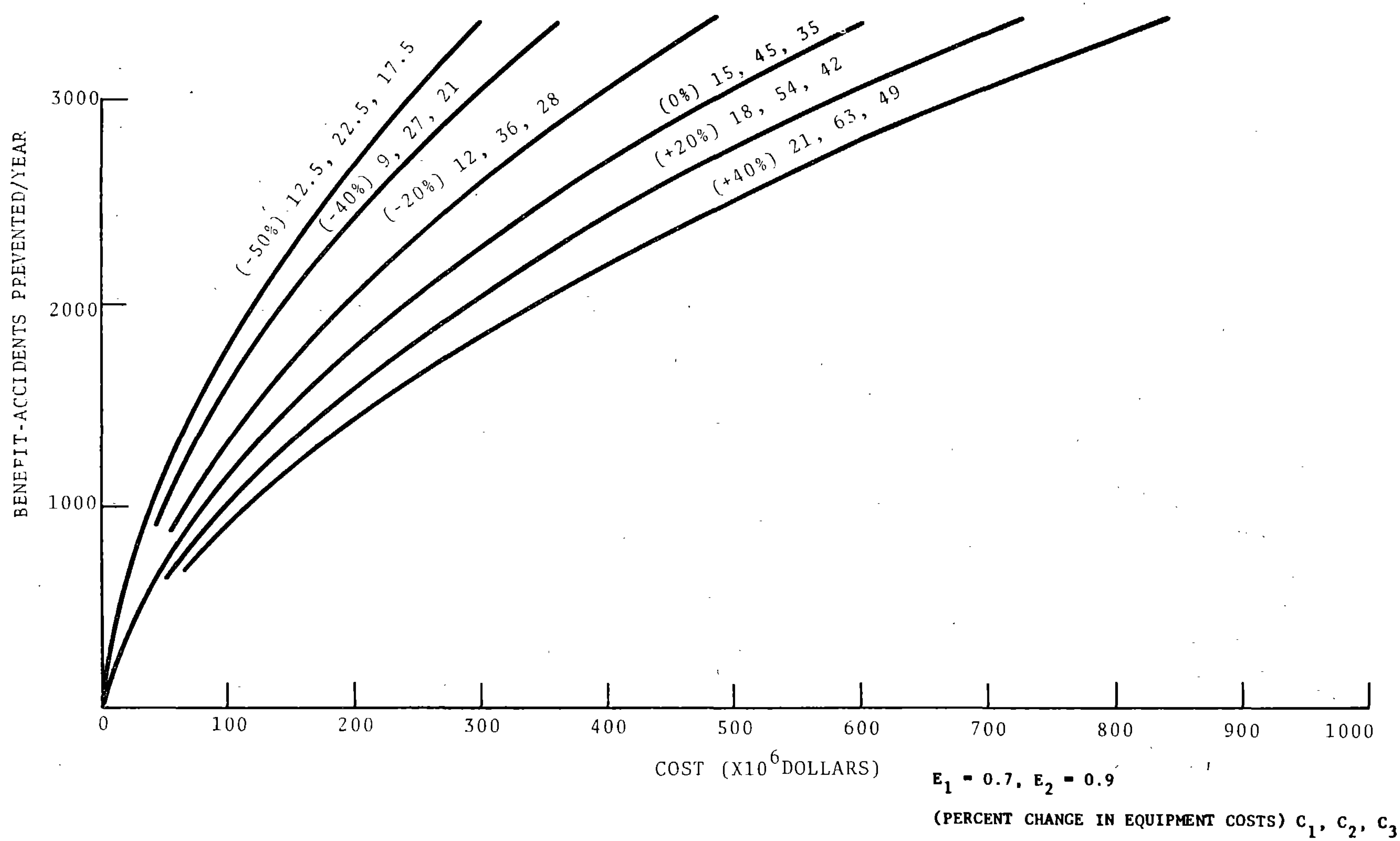

FIGURE E-3. MAXIMUM BENEFIT VERSUS COST FOR DIFFERENT CHANGES IN EQUIPMENT COST - APPLIED TO RUN NUMBER 4

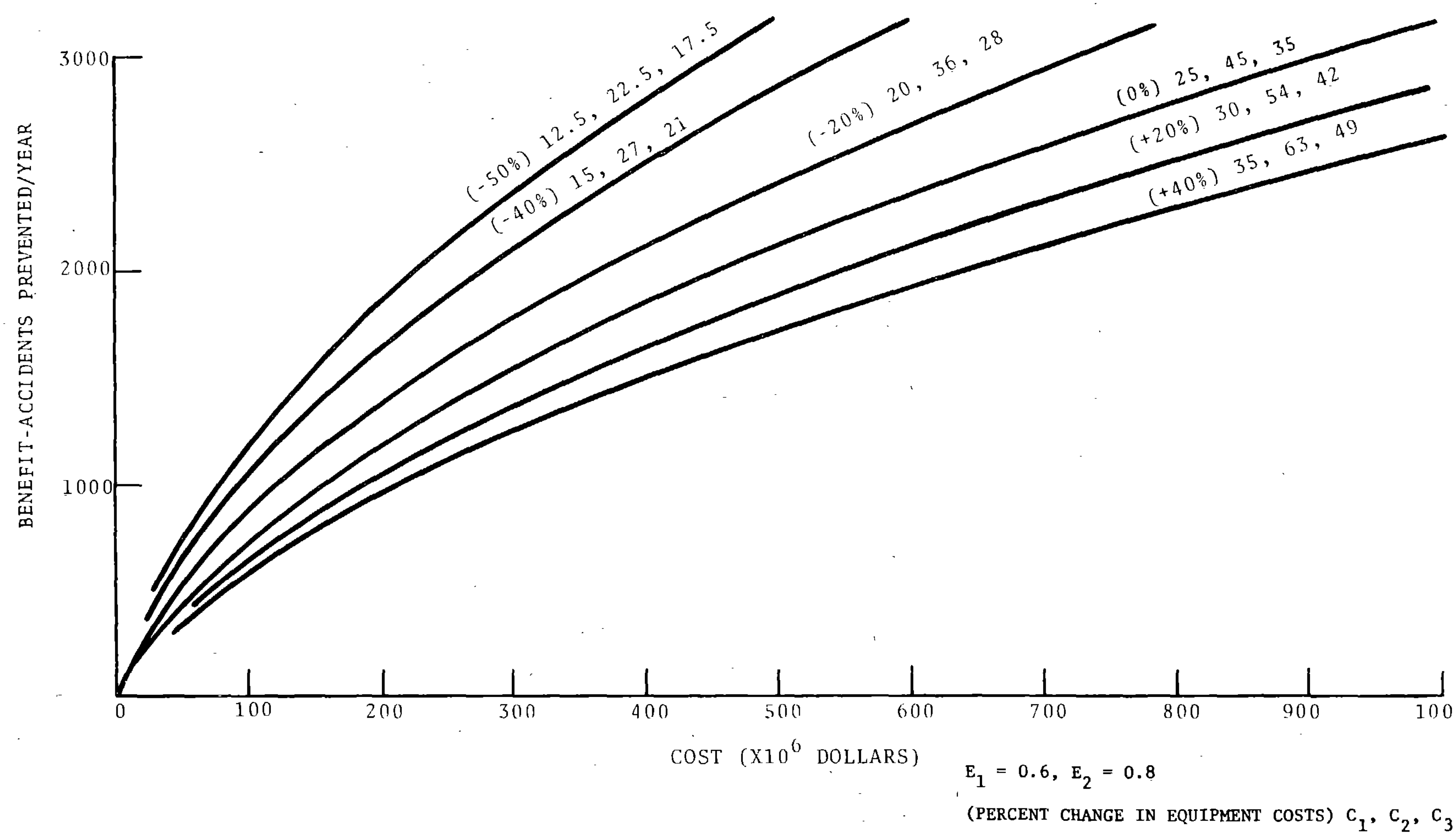

FIGURE E-4. MAXIMUM BENEFIT VERSUS COST FOR DIFFERENT CHANGES IN EQUIPMENT COST — APPLIED TO RUN NUMBER 5

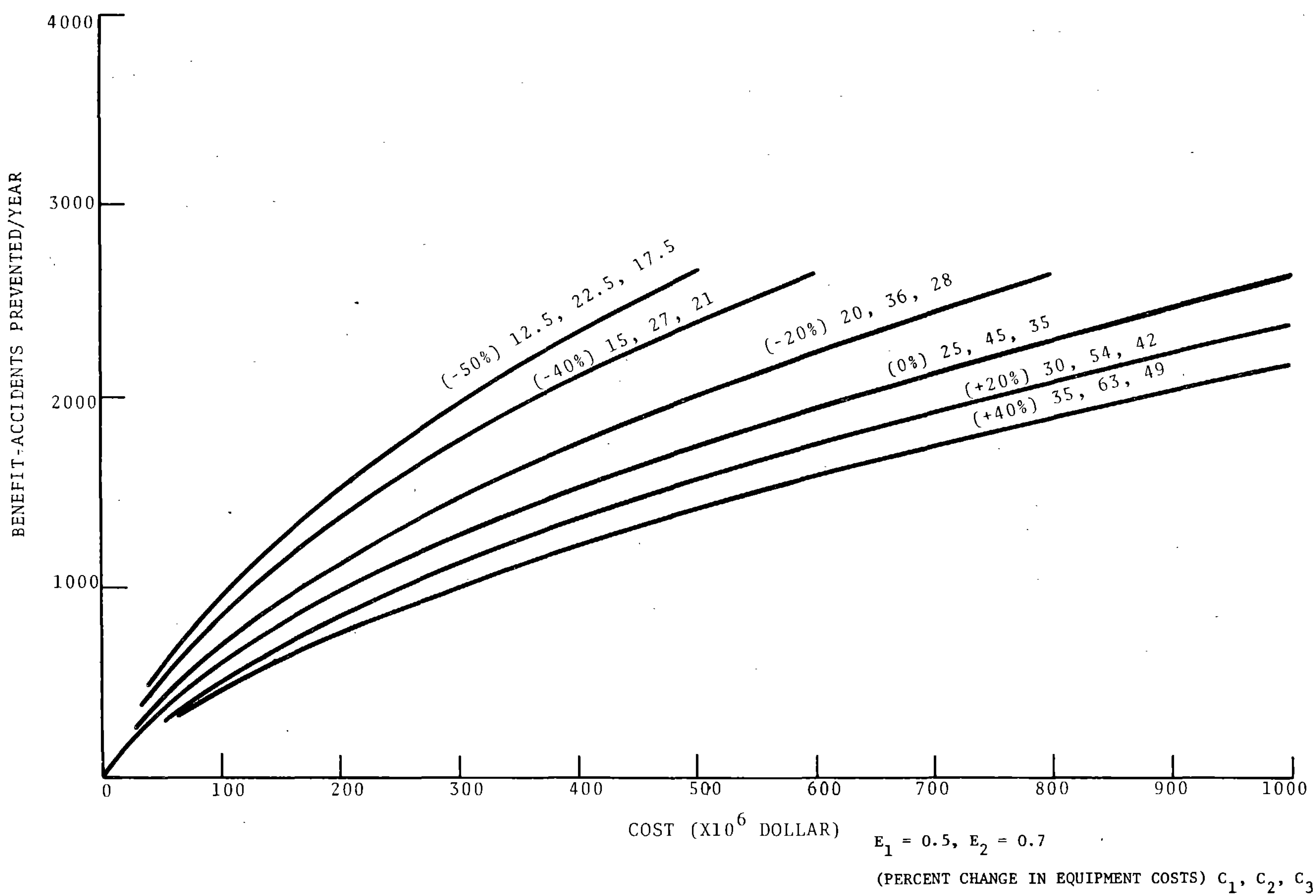

FIGURE E-5. MAXIMUM BENEFIT VERSUS COST FOR DIFFERENT CHANGES IN EQUIPMENT COST - APPLIED TO RUN NUMBER 6

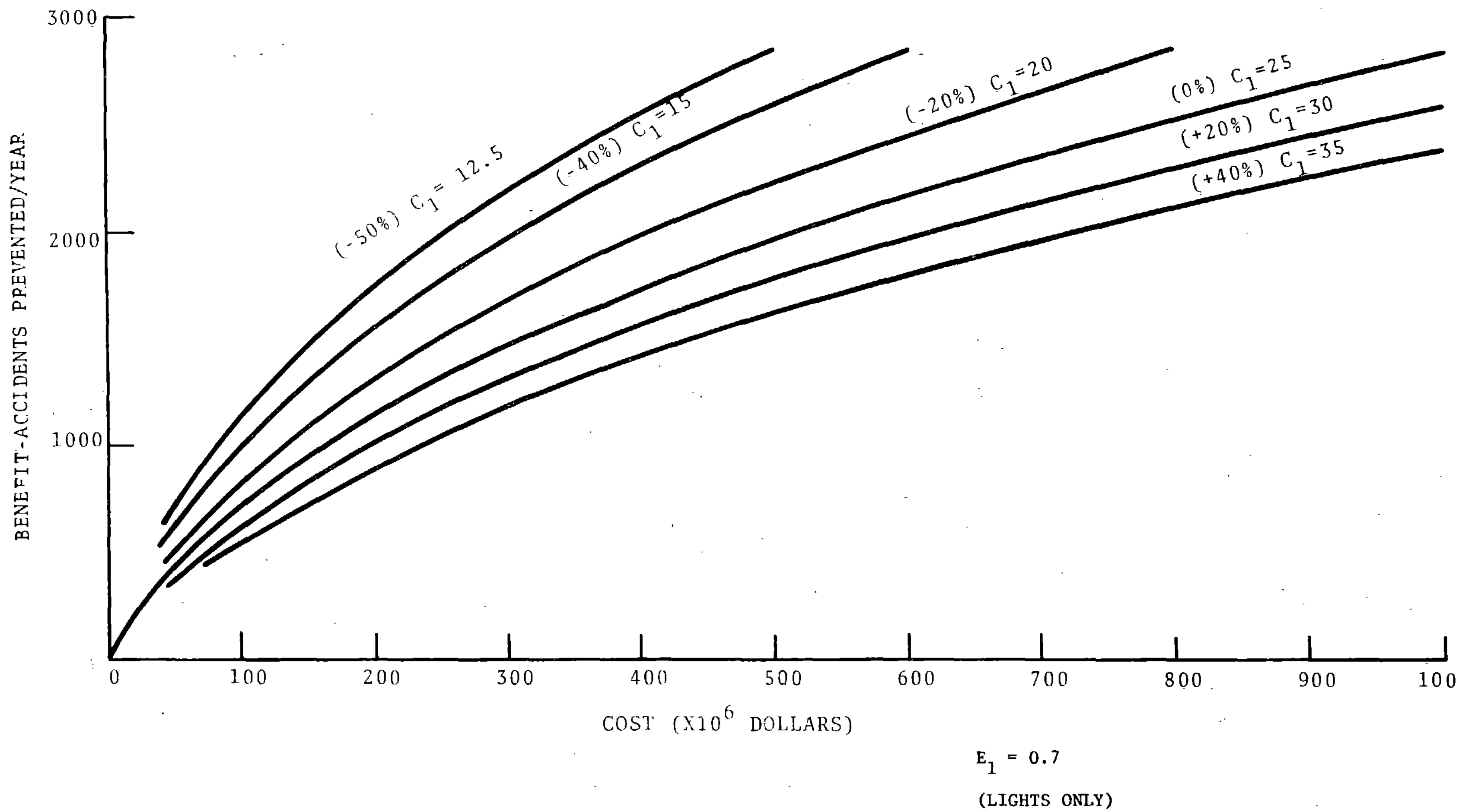

FIGURE E-6. MAXIMUM BENEFIT VERSUS COST FOR DIFFERENT CHANGES IN EQUIPMENT COST — APPLIED TO RUN NUMBER 7

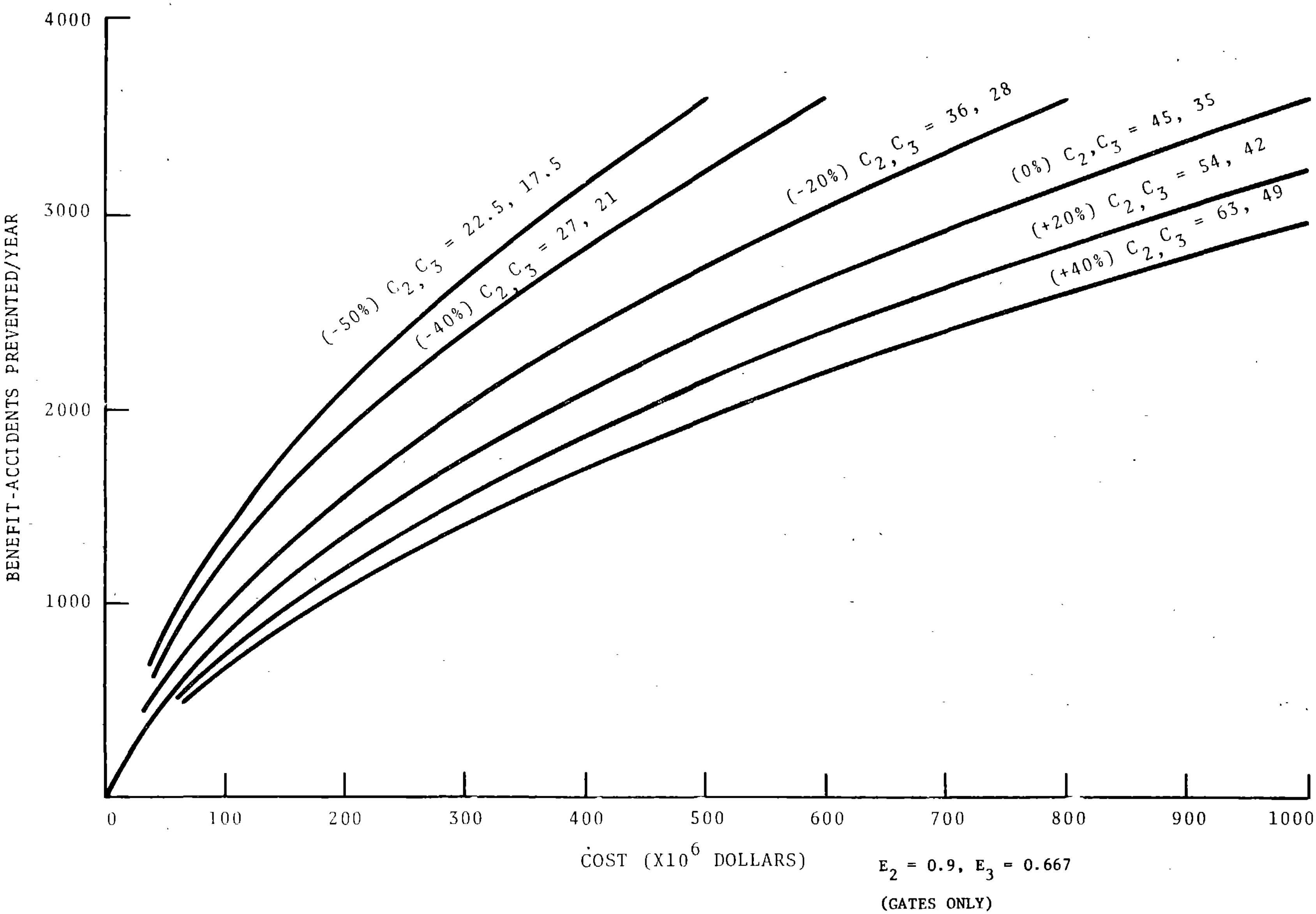

FIGURE E-7. MAXIMUM BENEFIT VERSUS COST FOR DIFFERENT CHANGES IN EQUIPMENT COST - APPLIED TO RUN NUMBER 8

TABLE E-1. PERCENT INCREASE IN BENEFITS FOR A 20 PERCENT REDUCTION IN EQUIPMENT COST

TOTAL COST (DOLLARS-MILLIONS)	RUN NUMBER								AVERAGE
	1	2	3	4	5	6	7	8	
100	15.8	16.4	15.3	16.1	16.2	15.8	15.0	17.9	16.1
200	16.4	16.2	15.8	14.8	16.6	16.2	15.6	16.3	16.0
400	14.3	15.6	14.1	12.9	14.6	14.8	14.1	14.9	14.4
600	13.9	14.2	12.9	—	13.8	14.6	12.9	14.0	13.8
800	13.2	13.4	13.1	—	13.6	14.6	11.7	13.7	13.3

TABLE E-2. PERCENT INCREASE IN BENEFITS FOR A 40 PERCENT REDUCTION IN EQUIPMENT COST

TOTAL COST (DOLLARS-MILLIONS)	RUN NUMBER								AVERAGE
	1	2	3	4	5	6	7	8	
100	41.4	41.8	41.1	40.6	43.6	43.2	40.7	43.3	42.0
200	39.7	39.6	37.8	35.9	41.2	40.3	38.3	40.0	39.1
400	35.0	37.2	34.3	——	35.6	37.3	33.9	36.4	35.7
600	33.8	34.8	32.5	——	34.4	35.4	31.4	34.9	33.9

TABLE E-3. PERCENT INCREASE IN BENEFITS FOR A 50 PERCENT REDUCTION IN EQUIPMENT COST

TOTAL COST (DOLLARS-MILLIONS)	RUN NUMBER								AVERAGE
	1	2	3	4	5	6	7	8	
100	58.6	61.6	59.5	58.9	62.1	62.1	57.5	60.4	60.1
200	56.0	56.2	52.9	51.4	57.8	58.4	54.4	57.2	55.5
400	50.1	55.2	47.4	___	51.5	53.3	47.3	51.9	51.0
500	49.3	51.2	46.8	___	51.3	52.1	45.4	50.6	49.5

TABLE E-4. PERCENT DECREASE IN BENEFITS FOR A 20 PERCENT INCREASE IN EQUIPMENT COST.

TOTAL COST (DOLLARS-MILLIONS)	RUN NUMBER								AVERAGE
	1	2	3	4	5	6	7	8	
100	10.3	11.0	10.4	11.1	12.0	11.6	11.5	13.4	11.4
200	10.3	10.6	11.2	10.9	11.2	12.3	11.1	11.2	11.1
400	10.5	10.4	10.3	10.3	10.8	11.1	9.7	10.4	10.4
600	10.0	10.8	9.8	9.5	10.6	10.2	9.1	10.5	10.1
800	9.9	10.7	9.5	—	10.5	10.2	9.5	10.2	10.1

TABLE E-5. PERCENT DECREASE IN BENEFITS FOR A 40 PERCENT INCREASE IN EQUIPMENT COST

TOTAL COST (DOLLARS-MILLIONS)	RUN NUMBER								AVERAGE
	1	2	3	4	5	6	7	8	
100	22.1	21.9	19.6	20.0	19.7	21.1	20.4	21.5	20.8
200	19.4	19.1	18.0	19.7	19.8	22.1	19.4	19.9	19.7
400	19.0	18.9	18.1	18.9	19.0	20.1	18.1	18.5	18.8
600	18.2	18.5	17.3	17.4	18.5	18.5	17.1	18.1	18.0
800	17.6	18.6	17.0	—	18.1	17.8	16.6	17.6	17.6

GLOSSARY

Accident Prediction Formula - A hazard function whose values represent predicted accidents per year at a crossing.

Active Warning Device - A warning device activated by an approaching train; e.g., gates, flashing lights, highway signals, wigways and bells.

Benefit/Cost Curves - Curves which show a plot of benefit versus cost. Benefit is specified in accidents prevented per year. Cost is in dollars and is equivalent to program budget.

Benefit/Cost Ratio - Ratio of benefit in accidents prevented per year to cost of warning systems in dollars.

Effectiveness - Accident reduction factor for a warning device relative to some presently installed warning device. It is a number between zero and one with zero meaning no effectiveness and one meaning total effectiveness.

Flashing Lights - An active warning device consisting of flashing red lights that are either cantilevered or mast mounted.

Gates - Automatic gates and flashing lights.

Gates Only Policy - Refers to a policy where only automatic gates will be installed at a crossing in the future.

Hazard Function - Any function which gives a numerical value of the likelihood of an auto-train collision at a crossing.

Hazard Index - A value of the hazard function. This need not be the predicted number of accidents per year.

Lights Only Policy - Refers to a policy where only flashing lights will be installed at a crossing in the future.

Optimum - The best or most favorable point.

Optimum Strategy - A strategy which maximizes the benefit in accidents prevented per year.

Passive Warning Device - Warning device not activated by an approaching train.

Relative Hazard - A hazard index which has value only when compared to another number derived from the same hazard function.

Strategy - Decisions for upgrading safety for a set of crossings.

Warning Device - A device which warns highway traffic that a railroad crosses the highway.

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