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A Parametric Cost Study of AC-DC Wayside Power Systems

Transportation Systems Center

**Prepared For
Alexander Kusko, Inc., Needham Heights, Mass.**

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16. Abstract The wayside power system provides all the power requirements of an electric vehicle operating on a fixed guideway. For a given set of specifications there are numerous wayside power supply configurations which will be satisfactory from a technical standpoint. The purpose here is to determine among a set of technically feasible designs, the one which is most cost effective. The primary cost tradeoff used in this study is between power rails and substations. Included is a presentation of the major technical and cost characteristics of each and a means of parameterizing these quantities, a procedure for optimizing costs, identification of the principal characteristics of a cost effective solution, and a comparison of ac and dc wayside power systems. For purposes of illustration, numerical values and costs for the Tracked Levitated Research Vehicle and the wayside power rail at the High Speed Train Test Center at Pueblo, Colorado, are used.			
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PREFACE

The work described in this report was performed in the Power and Propulsion Branch at the Transportation Systems Center under the sponsorship of the Advanced System Division of the Office of Research and Demonstrations, Federal Railroad Administration.

The objective of this work was to identify and evaluate cost effective designs for ac and dc wayside power systems. The work was performed by Alexander Kusko, Inc. under contract DOT-TSC-203, Task Directive DOT-TSC-203-101, and under contract DOT-TSC-965, Task Directive DOT-TSC-965-10.

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DEFINITIONS

Basic Power Rail - A power rail design with a minimum power handling capacity of one vehicle between two substations.

Low Impedance (Resistance) Power Rail - A modified basic power rail with reduced per-mile impedance (resistance).

Power Rail Ampacity - The load capacity of the power rail expressed either in amperes or in terms of the S/H ratio.

Substation Spacing (S) - The distance between two substations.

Headway Spacing (H) - The distance between two vehicles traveling in the same direction.

S/H Ratio - The ratio of the substation spacing (S) to the vehicle headway (H). The ratio defines the vehicle density in a single direction of travel between two substations.

Banked Secondary System - A wayside power distribution system in which the power rail sections between two substations are fed from both ends.

Supply-Regulated Substation - A substation design which regulates the output bus voltage against supply voltage changes by means of a transformer with a conventional load tap changer (ac and dc substations).

Completely Regulated Substation - Substation designs which regulate the output bus voltage against load and supply voltage changes by means of a transformer with a static or vacuum-switch assisted load tap changer (ac substations) or by means of a thyristor controlled rectifier (dc substations).

Substation Spacing Voltage Limited - The condition in which the maximum substation spacing is determined by the voltage drop at the vehicle(s).

Substation Spacing Power-Rail Ampacity Limited - The condition in which the maximum substation spacing is determined by the power-rail ampacity.

Substation Rating Impedance Limited - The condition in which the minimum substation rating is determined by the substation internal voltage drop due to its impedance.

Substation Rating Overload Limited - The minimum substation rating determined by its overload capability.

Substation Rating Thermally Limited - The minimum substation rating determined by its RMS load capability.

1.0 Introduction

The purpose for this study is to achieve cost-effective designs for ac and dc wayside power systems for electrically propelled vehicles. In this report we have addressed the following:

- Technical features of present and future ac and dc wayside power systems.
- Procedure for optimizing the costs of wayside systems.
- Comparison of ac with dc wayside power systems.
- Factors that reduce the costs of wayside power systems.

As a specific system for numerical values and actual costs, we have used the Tracked Levitated Research Vehicle (TLRV), and the wayside power rail installed at the High Speed Train Test Center (HSTTC) at Pueblo, Colorado. Our procedures, design factors and conclusions, however, can be applied to wayside electric power systems in general.

1.1 AC and DC Wayside Power Systems

DC wayside systems are generally used in rapid-transit systems; voltages in the 570-to-1000 V range are most common in North America. These

voltages, although relatively low compared to electric-utility distribution voltages, are tolerated because of short vehicle-to-substation distances. For longer distances and lower traffic density, such as electrified sections of conventional or commuter railroads, up to 3 kV dc, or ac systems are used. In general, economics prescribes higher voltages as the distances increase between vehicles and substations.

Single-phase ac systems operating at 11-to-50 kV are common in conventional electrified railroads in North America. Where the transformer on board the vehicle can be tolerated, the higher voltage ac system is desirable, as it permits the use of smaller catenary conductors and longer substation spacing. The transformer cost is offset by savings in catenary and substation costs. Three-phase ac systems have found use for levitated vehicles requiring high power at medium voltage, where the additional weight of a transformer and single-phase propulsion equipment cannot be tolerated. System voltage is limited to on-board utilization levels. For example, the propulsion plant of the TLRV is supplied with 8.25-kV 3-phase power directly from the collector. PRT vehicles, such as those at Morgantown, using industrial-type rectifier-dc-motor drive systems, operate from 3-phase power rails at voltages up to 600 V.

The question addressed in this report is whether to use ac or dc wayside power, and at what voltage level.

1.2 Approach

In this report, the wayside power-system costs include only power rails and substations; vehicle associated costs are not included.

Fundamental to any economic optimization study is the recognition of the various cost trade-offs available. The principal cost trade-off here is between the per-mile power-rail costs and the per-mile substation costs. For example, by incurring additional per-mile power-rail costs to lower power-rail impedance, substations may be spaced further apart, thus reducing per-mile substation costs. Per-mile substation costs can also be optimized by selecting the substation rating and by varying the manner of regulating the output bus voltage. Close regulation permits greater substation spacing and a possible savings in per-mile costs.

The factors in this study, which influence power-rail and substation costs, fall into three categories:

- Performance requirements - input power and voltage drop at the vehicle.
- Design parameters - voltage, power factor, and headway spacing of the vehicle.
- Design variables - spacing, rating, and regulation of the substation, power-rail impedance.

In the study, the performance requirements are fixed and must be met by any viable design for a specific vehicle. The design parameters span candidates for a viable design; with the performance requirements, they complete the description of the interface between the vehicle and the wayside power system. The design variables are the factors which can be manipulated to achieve a cost-effective design, subject to the design parameters and performance requirements. Design variables in this report are given numerical values, but the concept which is represented by each numerical value is equally as important.

The formulation of the cost optimization problem is based on the policy of maximizing substation spacing subject to the performance and design factors and given power-rail conditions. It will be seen in Art. 1.6 that the character of a cost-effective design is determined by the constraints on the design variables. Solutions lying on constraint boundaries are common in many optimization problems where constraints are present.

1.3 Performance Requirements and Design Parameters

The performance requirements of the TLRV are used for both ac and dc vehicles in this study:

- vehicle input power (cruise) - 11.25 MW (constant power, cruise)
- voltage range at the vehicle - +0, - 25%

The following design parameters are used:

- ac vehicle voltage - 3-phase, 8.25 kV and 4.16 kV.
- ac vehicle power factor - 0.75 lagging, 0.9 lagging, 1.0, and 0.9 leading.
- dc vehicle voltage - 1.5 kV, 3 kV, 6 kV, and 12 kV.
- vehicle headway - 10-to-100 mi.

For the purpose of evaluating substation per-mile costs, a 100-mi stage length is used throughout.

1.4 Design Variables

To achieve cost-effective designs, the design variables are varied within the bounds of the following equipment configurations:

1.4.1 Wayside System

A banked-secondary dual-rail system is assumed. That is, the power rail is broken only at the feed points adjacent to the substations and each section is fed from both ends as shown in Fig. 4.1. The dual-rail provides power for simultaneous travel in two directions. Our limitation to a banked-secondary dual-rail system does not detract from the generality of the analysis. Alternative designs can be handled by introducing additional design parameters.

1.4.2 Power Rail

The basic power rail for this study is the 3-phase 8.25-kV TLRV power rail to be installed at HSTTC; by definition it has the capacity to supply one 15-MVA 0.75-PF vehicle between two substations. This rail design, modified for two conductors is also used for the dc systems in this study. To obtain "low-impedance" (ac systems) and "low-resistance" (dc systems) designs we assume that parallel booster cables or additional rail conductor cross section, respectively, will be added to the basic power rail. The use of parallel cables causes step changes in the power-rail impedance variable. For uniformity in approach, we use only discrete changes in conductor cross section for the dc power rails.

1.4.3 Substations

Substations with two types of output voltage regulation are considered: (1) completely regulated if the output voltage holds constant for all loads and input voltages and (2), supply regulated if the output voltage is compensated only for slow changes in the input voltage, not for the voltage drop in the transformer.

1.4.4 Rectifiers

Wayside-power rectifier sets are not available commercially for over 3 kV dc. Technically, the rectifiers can be built, but higher voltage dc traction service awaits development of dc circuit breakers, or other

acceptable protection schemes. Thyristor rectifiers with gate control will be considered as an alternative method of achieving protection above 3 kV dc. With thyristor rectifiers complete regulation of the substation will be assumed.

1.4.5 Substation Spacing

The substation-spacing design variables does not translate directly into equipment. Its range of admissible values, however, depends on the remaining design variables, the design parameters, and the performance requirements. Since substation spacing defines the number of substations on a given route, this variable could also be used to introduce the costs for supplying substations from the utility systems. However, those costs are unique to a specific route and utility system, and were omitted in this study.

1.5 Specific Results - TLRV Wayside Power System

Over the range of design parameters, we have found that the least-cost and most technically feasible wayside system for the TLRV is the 3-phase 8.25-kV system for which the vehicle was designed. Higher voltages may be more cost effective, but we have assumed that 8.25 kV is the limit for direct on-board operation of the propulsion plant.

DC wayside power systems above 6 kV are cost competitive with the 8.25-kV ac system. However, above 3 kV dc conventional circuit breakers are not

available; thyristor rectifier sets are specified as an alternative means of obtaining protection. In view of the marginal cost benefit of a dc system over an ac system, and the lack of experience with wayside thyristor rectifier sets, we feel that the risks of dc systems for this application are not warranted.

The total per-mile costs (1973 prices) for cost-effective ac and dc wayside power systems are given in Figs. 1.1 and 1.2. Each point on the curves represents a cost effective design for the performance requirements and design parameters, reflecting the following decisions:

- The basic power rail for single vehicle loads between substations is used. Reduction of impedance to increase substation spacing and reduce substation per-mile costs, or increase of ampacity for multiple vehicle loads between substations, are not cost effective. Modifications to the basic power rail increase direct per-mile costs and are difficult to offset in the substations, particularly when the power rail represents the majority of total system costs. (Power rail costs are noted in Figs. 1.1 and 1.2).
- Completely regulating the output voltage of an ac substation when the vehicle power factor is 0.9 lagging or better yields little cost advantage. The completely regulated substation design uses a vacuum-switch assisted load tap changer (LTC), and remains to be tested in commercial wayside equipment. At a vehicle power

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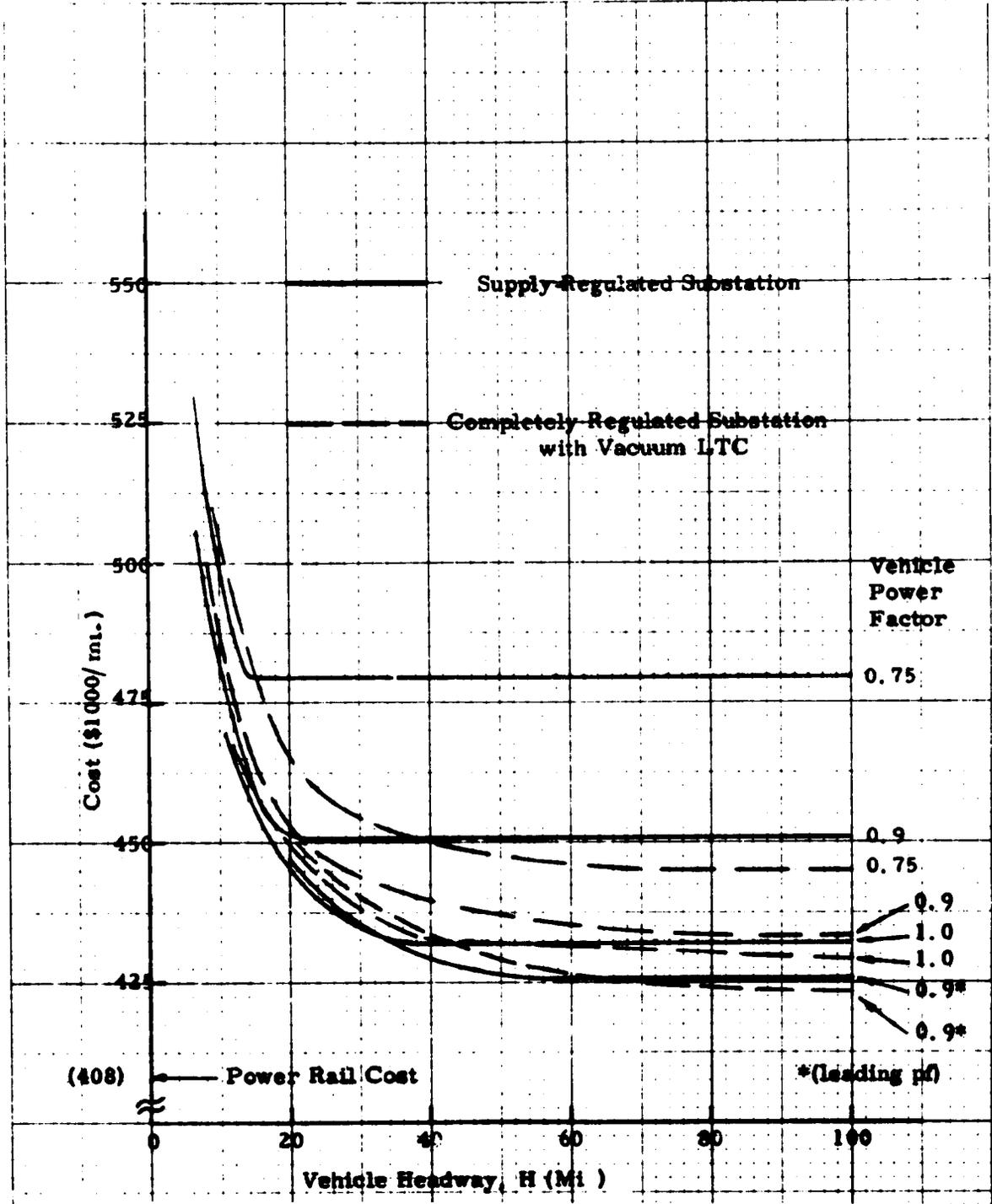


Fig. 1.1 Total Wayside Power-System Per-Mile Costs with Supply Regulated Substations and Completely Regulated Substations with Vacuum LTC, 8.25-kV ac System, Basic Power-Rail Design. Based on 1973 Costs

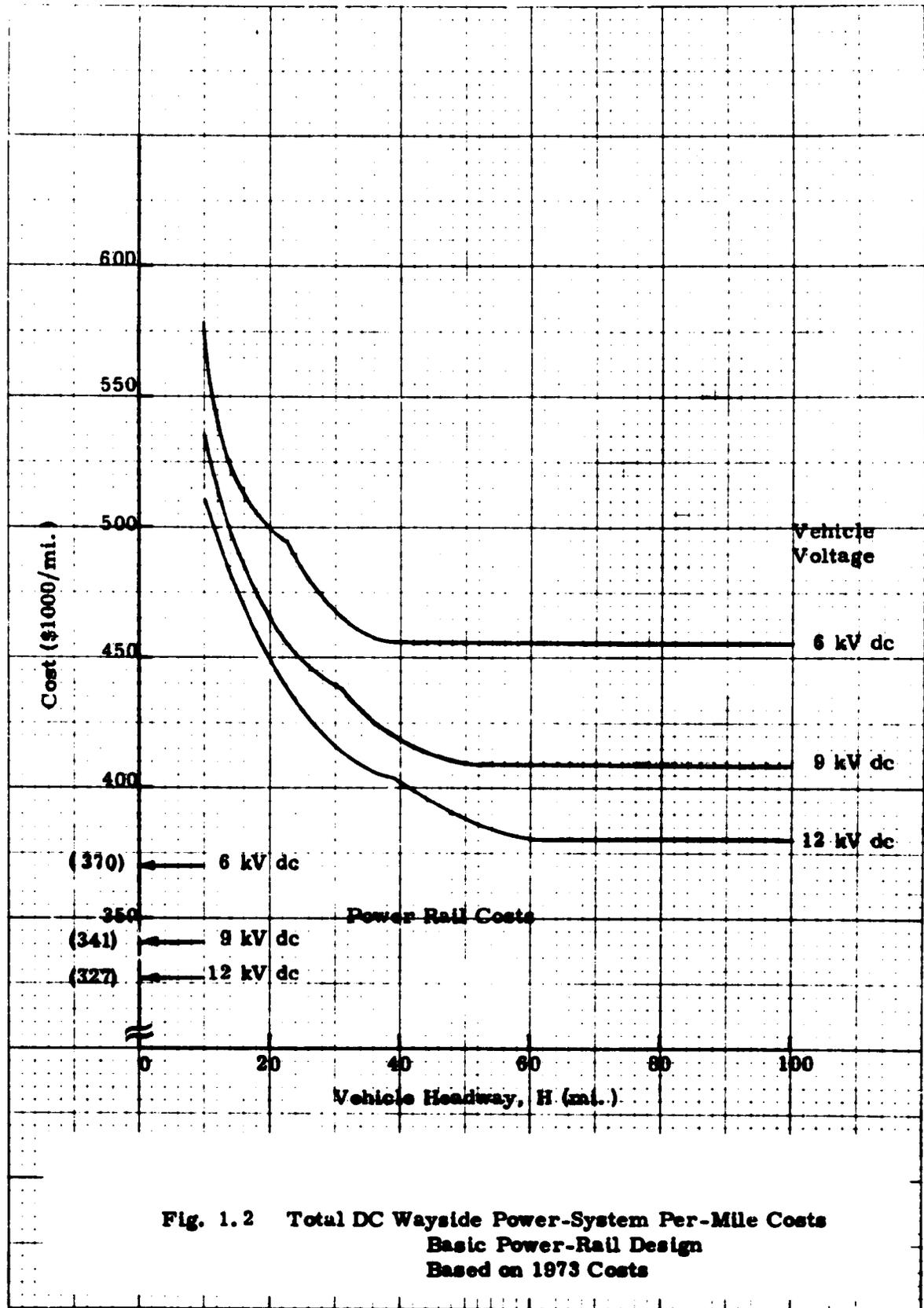


Fig. 1.2 Total DC Wayside Power-System Per-Mile Costs
Basic Power-Rail Design
Based on 1973 Costs

factor of 0.75 lagging, the cost savings may warrant the use of a completely regulated substation. The supply-regulated substation, which corrects only for primary voltage changes, uses a conventional load tap changer of current practice. Complete regulation of the dc substation is an inherent feature of the thyristor rectifier.

1.6 Character of Cost Effective Designs

To achieve a cost-effective design, equipment must be exploited to its maximum capacity or to other limits.

1.6.1 Design Constraints

Constraints on the variables introduced in Art. 1.2 define the basic character of a cost-effective design. Our policy for achieving cost effectiveness is to space substations at the maximum distance allowable by the constraints and to rate them as necessary.

Substation Spacing Limit. Substation spacing is said to be voltage limited when the voltage drop at the vehicle determines spacing. Maximum substation spacing is then a function of the power-rail impedance, the vehicle current, the vehicle power factor, the number of vehicles between substations, and, for supply-regulated substations, the substation impedance.

Substation spacing is said to be power-rail ampacity limited when the vehicle headway and the power-rail ampacity determine the spacing. For a given power-rail ampacity the vehicle headway can be reduced, with the substation spacing fixed at the voltage limit, until the vehicle density causes the power rail to reach its ampacity. For further reduction of headway, the substation spacing must be reduced as well to prevent the power rail loading from exceeding the ampacity.

Power-rail designs are identified as basic or as low-impedance designs. The basic design sets a reference level for power-rail impedance before modifications and the power-rail ampacity. A basic power rail will by design have a minimum power-rail ampacity of 1 pu (one vehicle on one power-rail section between two substations). The power-rail ampacity for a low-impedance design will depend on the method of reducing the power-rail impedance.

Substation Rating Limits. Substation rating is based on either a thermal limit (rms rating) or on an impedance limit (voltage drop). In dc substations, impedance limits may be superseded by the short-term overload capability of the rectifiers. For vehicle headways in the order of substation spacing or less, substation loading is fairly uniform and thermal limits determine the rating. Substation rating is then said to be thermally limited. At greater headways, the

ratio of the peak-to-rms loading increases. Either an impedance limit is imposed on the rating to bound the voltage drop in the substation, or a short-term overload limit is imposed to protect equipment. Substation rating is then said to be impedance or overload limited, respectively.

1.6.2 Typical Per-Mile Cost Characteristics

The influence of the various factors on per-mile costs is shown in Fig. 1.3. The 6-kV dc wayside power system using a basic power-rail design is selected for the example. The power-rail cost of \$370,000/mi (1973 costs) is assumed constant as a function of vehicle headway. The following comments generally are also applicable to ac systems. Where differences between ac and dc systems occur, they are noted. The cost curve in Fig. 1.3 has three regions defined by headway, and substation rating or substation spacing limits:

- Headway from 22.6 to 37 Miles. Substation Rating Thermally Limited, Substation Spacing Voltage Limited. Substation spacing is constant at 22.6 mi, the maximum allowable by the voltage drop at the vehicle. As vehicle headway decreases from 37 mi, the traffic density between substations increases and substation rating, based on rms loading, increases accordingly. Since substation spacing is fixed, substation per-mile costs rise only as rating rises.
- Headway from 37 to 100 Miles. Substation Rating Impedance or Overload Limited, Substation Spacing Voltage Limited. Substation

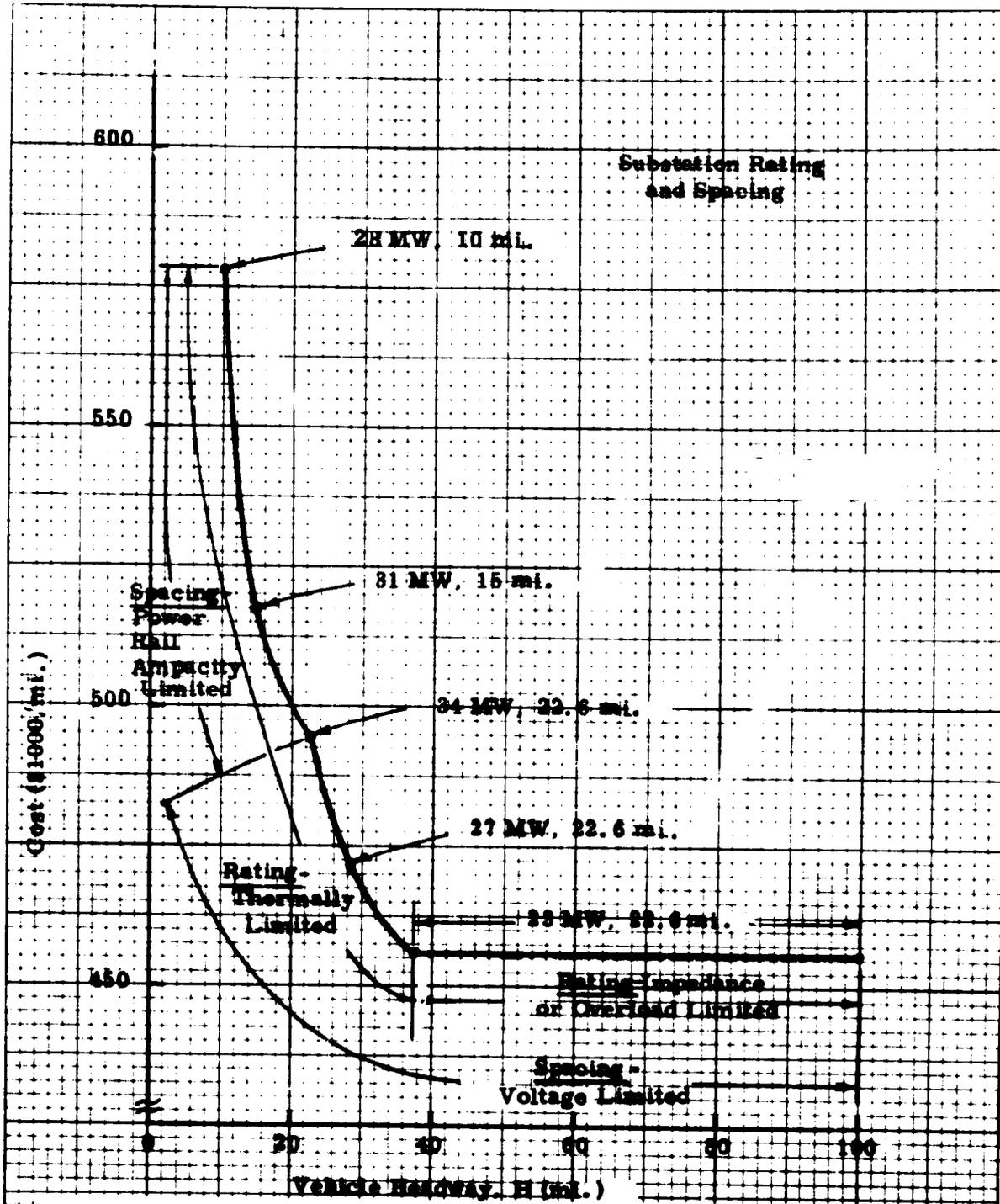


Fig. 1.3 Typical Wayside Power System Per-Mile Costs
6-kV dc System
Basic Power-Rail Design
Based on 1973 Costs

spacing is constant at 22.6 mi. Substation rating of 23 MW does not vary even though the substation rms load increases with decreasing headway. The peak substation load, however, is constant; since the example is a dc system, the substation rating is limited by the overload capability of the rectifiers. For an ac system, the limit would correspond to a maximum voltage-drop requirement in the substation. In either case, the characteristic of a constant per-mile cost for this headway range is the same.

Headway from 10 to 22.6 Miles. Substation Rating Thermally Limited, Substation Spacing Power-Rail Ampacity Limited.

The basic power-rail ampacity is the load of one vehicle between substations. As the headway is reduced from 22.6 mi, the substation spacing must be reduced in step for maximum power-rail utilization and for least per-mile substation cost. Substation rating is gradually reduced with reduced vehicle headway because the power-rail losses decrease. Substation per-mile costs, however, increase rapidly, because the rating decreases slower than the spacing decreases. In most of the ac wayside power systems investigated, the minimum headways reached with substation spacing set by the voltage limit covered the range of vehicle headways under consideration. The headways in which the substation spacing must be reduced to keep the power-rail at its ampacity are not shown on the cost curves.

1.7 General Design Guides

The following guides for the design of wayside power systems both for the TLRV and for vehicles of similar electrical characteristics are drawn from this study:

- Power-Rail Impedance. Reducing power-rail impedance in order to increase substation spacing is not generally cost-effective for the TLRV. The incremental per-mile cost of reducing power-rail impedance is greater than the incremental per-mile saving resulting from increased substation spacing. The cost benefit, if any, with low-impedance power rails will occur at vehicle headways greater than the maximum substation spacing set by voltage limit with the basic power rail.
- Power-Rail Ampacity. Power-rail ampacity in the cost study is treated as a constraint from a particular power-rail design, rather than a variable. The modifications to reduce power-rail impedance have varying effects on power-rail ampacity. For the ac power rails, the use of parallel cables does not necessarily result in a net increase in ampacity. With cable sizes giving the desired reduction in impedance, the current sharing between the power-rail proper and the parallel cables is such that the cables are the limiting factor in total

ampacity. AC power-rail ampacity may be improved by increasing rail-conductor cross section; but this has little effect on the impedance. For the dc power rail, resistance may be lowered by adding rail-conductor cross section, which also increases the ampacity.

Substation Regulation. The greater substation spacing (voltage limit) obtainable with completely regulated substations is often insufficient to offset the additional costs over the supply regulated substations. Special equipment will be required in completely regulated substations as the number and frequency of step changes exceeds the economical level for conventional load tap changers. Substation designs calling for static or vacuum-switch assisted load tap changers, or thyristor rectifiers, rely on equipment which is undeveloped or untested in wayside power systems. Where these devices appear to be advantageous from a cost standpoint, the cost benefit should be weighed against potential development problems.

In general, the wayside power system should be designed for the highest possible voltage, and the least costly construction method. The substations should then be placed to meet the requirements for voltage drop at the vehicle and ampacity of the power rail.

2.0 Problem Formulation

The method used in this report to arrive at a cost-effective wayside power-system design is simply to consider several power-rail and substation alternatives, evaluate the cost-optimal configuration of each combination, and select the one with the minimum per-mile cost. Determination of the most cost-effective design among all the candidates is therefore a trivial mathematical problem.

The essential economic problem lies in finding the cost-optimal configuration for each design alternative. The solution form, which is known a priori, is the maximum substation spacing consistent with system performance requirements and design parameters. Since the power-rail per-mile cost is fixed as a function of design this policy seeks to minimize the per-mile substation cost and therefore produces a cost-optimal configuration.

Much of this report is concerned with solving this problem. The number of factors which influence the solution are numerous, but the procedures are straightforward.

2.1 Study Variables and Parameters

In approaching the problem outlined above, the choice of variables and parameters is important in obtaining the most general conclusion, despite the

adoption of a baseline system to obtain numerical results.

The factors considered in this study, which directly influence design and cost, are classified into three groups:

- Design Variables Substation spacing, substation regulation, substation rating, and power-rail impedance.
- Design Parameters. Vehicle voltage, vehicle power factor, and vehicle headway spacing.
- Performance Requirements. Vehicle power and voltage drop at the vehicle.

Determination of cost-optimal system configuration involves optimizing simultaneously over the four design variables; this is difficult. The character of three of the design variables permits a simplification of the optimization procedure.

Substation regulation and power-rail impedance are quantities which relate to specific equipment configurations. For all practical purposes they assume only a finite number of discrete values assigned by the substation and power-rail designs considered. Each combination of designs therefore defines a new problem of reduced dimension. Substation rating is not an independent variable and can be expressed as a function of the substation spacing and

vehicle headway. Substation spacing remains as the one free variable. The four dimensional problem is now reduced to a finite number of one-dimensional problem.

The design of a wayside power system will depend strongly on the density and distribution of vehicles between substations. A simple parameter which combines all these conditions in a useful form is the ratio S/H of the substation spacing (S) to the vehicle headway (H). In terms of this ratio, generalized data on power-rail ampacity, substation loading, substation rating, and substation spacing can be generated. A normalizing effect introduced by the maximum substation spacing criteria (voltage-drop limit) combined with the generalizing effect of the ratio S/H , yields a compact presentation of design information.

2.2 Procedures

The several steps used in this report to arrive at the cost of candidate wayside power-system designs are enumerated below. The procedure for establishing per-mile substation costs is somewhat tedious, because several parameters are involved. In the following, vehicle power requirements refer collectively to the vehicle voltage, input power, power factor and allowable voltage drop at the vehicle.

1. Determine candidate power-rail designs and per-mile costs as a function of power-rail impedance and vehicle power requirements.
2. Determine substation design and cost as a function of rating and regulation.
3. Determine the maximum substation spacing as a function of the ratio S/H , power-rail impedance, substation regulation, and vehicle power requirements.
4. Determine substation rating as a function of the ratio S/H and regulation.
5. Combine the results of 2, 3, and 4 above to obtain substation per-mile costs as a function of vehicle headway, power-rail impedance, substation regulation, and vehicle power requirements.
6. Combine the results of 1 and 5 above to obtain total wayside power-system per-mile costs as a function of vehicle headway, power-rail impedance, substation regulation, and vehicle power requirements.

A flow chart of the above steps is shown in Fig. 2.1.

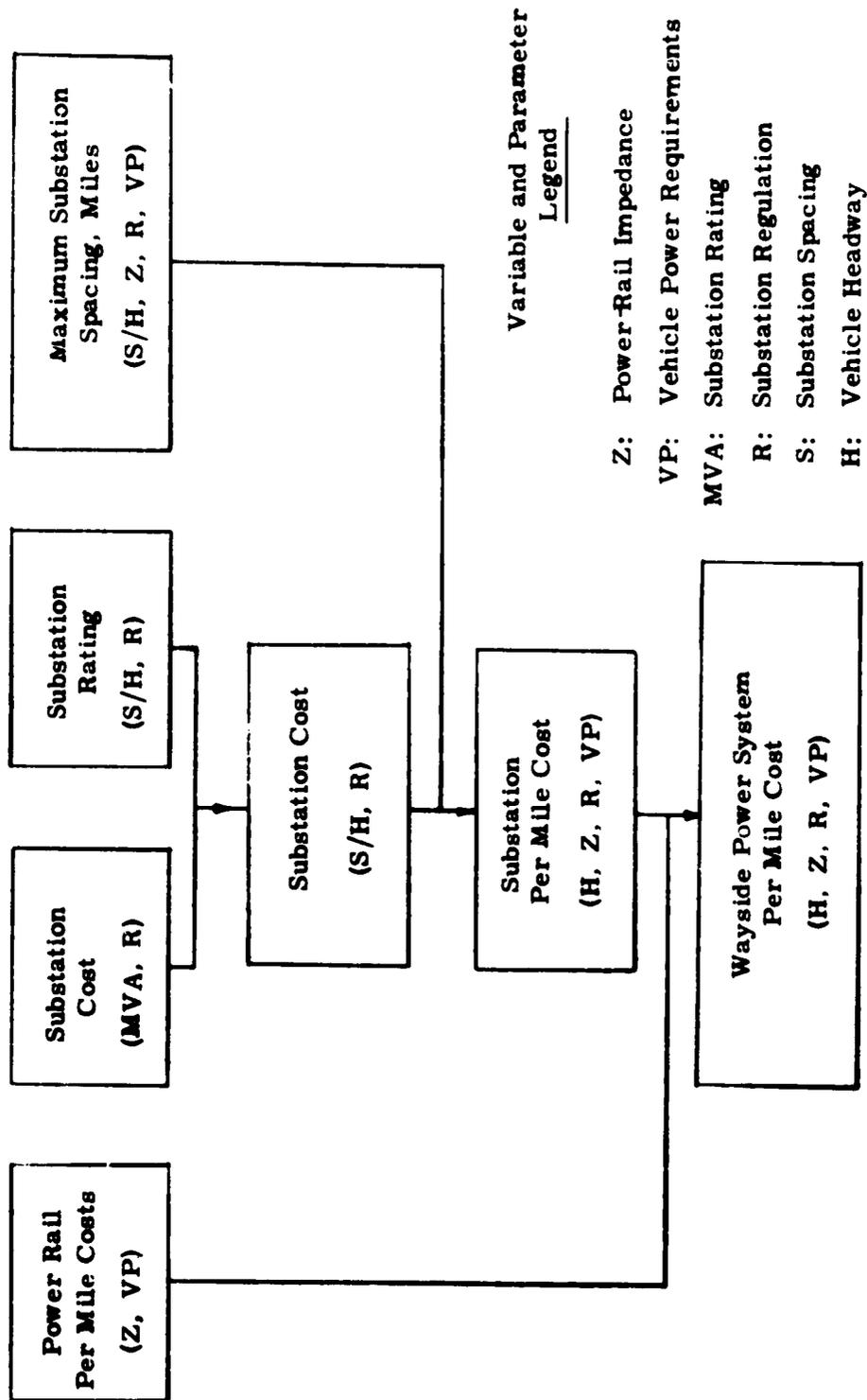


Fig. 2.1 Flow Chart for the Determination of Wayside Power System Per-Mile Costs.

From the results of 6 above, the lowest system per-mile cost can be selected for given headway and vehicle power requirements. The selection specifies power-rail impedance and substation regulation and thereby determines the most cost-effective power-rail and substation designs.

The number of design candidates to be evaluated can be reduced by judicious choice of the power-rail and substation alternatives. For example, the cost trend of a particular modification to a basic power-rail design can usually be established with one sample. If the modification proves to be cost effective, then additional samples can be generated to locate the best configuration. If not, the modification can be abandoned.

3.0 AC Power-Rail Design and Cost

The basic ac power rail presented here is based upon an existing 3-phase 8.25-kV design to be installed at the High Speed Test Center in Pueblo, Colorado. The power capacity of the rail is one 15-MVA 0.75-PF lagging vehicle between two substations.

The characteristics of the 4.16-kV power rail are not developed here; they are similar to that for the 8.25-kV design.

3.1 Basic Power Rail

The basic ac power rail consists of three sets of 3 x 1/8 in. copper bars each welded in a rigid sheet steel structure. Fig. 3.1 illustrates the rail configuration and support. These copper and steel rails are mounted on post insulators in a radial configuration with the rubbing copper edges pointing inward. The post insulators are bolted to the inner wall of a C-shaped supporting structure which is close to ground level. The supports are expected to be placed at 12-ft intervals alongside the roadway. The power-rail ampacity is 1100 A.

A power-collector arm will reach out from the side of the vehicle, extend through the opening in the "C" of the power-rail structure, and connect to a bullet-shaped plug which carries the brushes and slides along the center-line of the three power rails.

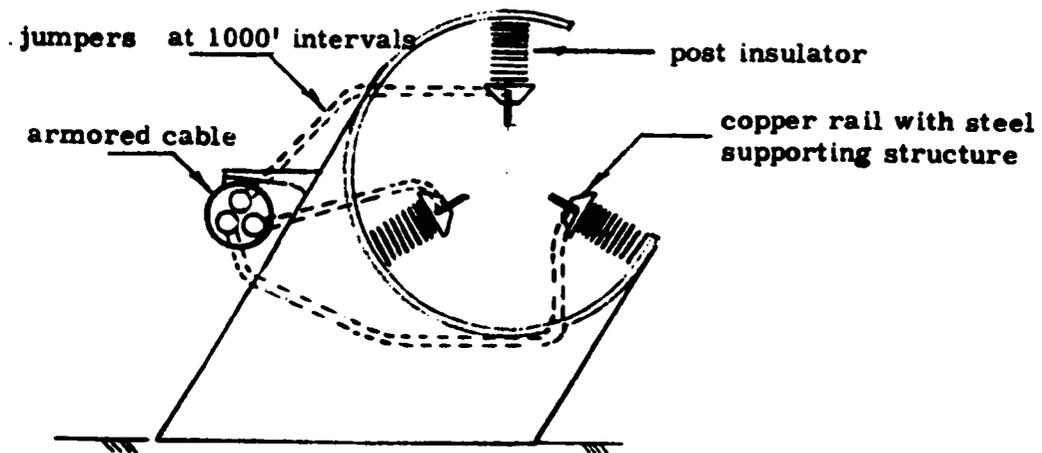


Fig. 3.1 AC Power Rail with One Parallel Cable

Table 3.1 shows a cost breakdown of the basic ac power-rail system. The costs are 1973 estimates reflecting large quantities of rail produced by automated techniques. For two rail directions, the installed cost is expected to be \$408,000/ mi. The rail impedance is $0.125 + j 0.397\Omega$ / mi.

The design of the basic ac power rail will not be adjusted for the lower current of those vehicles with high power factors. Reductions in the conductor cross section are limited by mechanical considerations. Consequently the vehicle load capacity of the basic ac power-rail will be somewhat greater for vehicles with high power factors.

Most of the voltage drop in the ac power rail is from its inductive reactance rather than its resistance; the ratio of reactance to resistance is more than 3:1. The reactance is primarily a function of the power-rail geometry and spacing, which are constrained by mechanical and electrical requirements. It is not possible to further reduce the spacing as a means of lowering the power-rail reactance. The only alternative is to lower the per-mile impedance with parallel cables.

Table 3.1
Costs for AC Power Rail*
 (Basic Design)

<u>Item (per vehicle direction)</u>	<u>Per-Mile Cost</u>
Power Rail (3 rails each 3 x 1/8 in cross section)	\$64,000
Support Structure and Insulation	33,000
Protective Cover	12,000
Hardware for Insulator Bracket	24,000
Protective Cover Hardware	1,000
Installation Labor	70,000
	\$204,000
Subtotal	\$204,000
x2 rail directions	\$408,000

*Based on 1973 prices

3.2 Low-Impedance Power Rail

To determine the feasibility and economy of lowering the impedance of the power rail, two alternative rail designs are considered. Each design uses 1000-kcmil Al-conductor armored cables. These would be installed on brackets mounted to the outer wall of the C-shaped rail supporting structure and connected to the copper power rails at 1000-ft intervals as shown in Fig. 3.1. The alternatives are either one or two of these cables in parallel, and are denoted low-impedance designs.

The use of parallel or "booster" circuits, while simple in concept, poses some problems. In comparison with the basic ac power rail, the impedance, and in particular the inductive reactance, of the cables is low because of the close conductor spacing. Consequently, a large portion of the load current will be carried by the cables rather than the power rail. Adequate ampacity can be provided by selecting cables of sufficient size. However, as the cable size increases, the cable impedance decreases (mostly the resistive component) and the cable carries an even larger portion of the load current.

The characteristics of the two parallel-circuit designs is shown in Table 3.2. The ratio of the impedance Z_t of the cables and the power rail to the impedance Z_c of the cables alone, gives the fraction of the total current carried by the cables. The load currents tabulated are for a single vehicle

Table 3.2

Electrical Characteristics of Parallel Cable Systems

Cable	Total Z_t (Ω /mi) ^t	$\frac{Z_t^{(1)}}{Z_c}$	Ampacity ⁽²⁾ (A)
1000 kcmil	0.0718 + j 0.119	0.68	430
2 x 1000 kcmil	0.0460 + j 0.0688	0.81	860

Vehicle			Cable Current (A)	
P.F	MVA	Current(A) ⁽³⁾	1000 kcmil ⁽⁴⁾	2 x 1000 kcmil ⁽⁴⁾
0.75	15	1050	714	851
0.9	12.5	875	595	708
1.0	11.25	788	536	638
0.9 (leading)	12.5	875	595	708

(1) 1000 kcmil: $Z_c = 0.125 + j 0.161 \Omega/\text{mi}$

2 x 1000 kcmil: $Z_c = 0.0625 + j 0.0805 \Omega/\text{mi}$

(2) 40°C ambient, 90°C insulation rise

(3) $\text{MVA} / (\sqrt{3} \times 8.25 \text{ kV}) = 1 \text{ pu vehicle current}$

(4) $\frac{Z_t}{Z_c} \times 1 \text{ pu vehicle current}$

Note: Power rail ampacity = 1100A.

Table 3.3

Costs for AC Power Rail Systems with Parallel Cables

(Low Impedance Design)

<u>Item (per vehicle direction)</u>	<u>Per-Mile Cost*</u>
2 x 1000 kcmil Cable	\$99,000
Hardware	13,500
Labor	10,500
Basic Power Rail	<u>204,000</u>
Total	\$327,000
x2 rail directions	\$654,000

*Based on 1973 Prices

at rated voltage. From Table 3.2, it is evident that two 1000 kcmil Al cables are required in order not to exceed the ampacity of the cables.

The cost of the power rail with two 1000-kcmil Al cables is given in Table 3.3, including a breakdown of items. The cables are supported by brackets every 12 ft and cost \$11.80 each per cable. Splices and rail connections will occur at about the same frequency (every 1000 ft), and could be made up in the same junction box. Materials for splices and rail connections are estimated at \$1.00 /ft.

3.3 Power-Rail Ampacity

Power-rail load capacity is usually defined in terms of a current carrying limit or ampacity. For the purposes of this report, it is desirable to translate ampacity into the maximum number of vehicles which may occupy a rail section between two substations and their relative positions. These conditions are described precisely by the ratio S/H of the substation spacing (S) to vehicle headway (H). The maximum value of S/H for a power rail is defined as its ampacity.

The conversion from ampacity to a maximum value of S/H is found from the electrical loading in the power rails. For a single power-rail section fed at both ends, the maximum and minimum currents into the power rail from a single substation are shown in Fig 3.2. The currents are given

as a function of the ratio S/H . The current is expressed in per-unit vehicle current where 1 pu is the current drawn by a vehicle at its rated voltage.

The curves in Fig. 3.2 are derived from substation loading patterns presented in Chapter 4. The maximum substation spacing is assumed for a worst-case voltage drop of 25% at the vehicle (voltage limit). It is the maximum substation spacing for a given voltage-drop requirement that provides the normalization for the per-unit representation of the power-rail current. The curves are independent of the vehicle and power-rail characteristics and vary only with the value of S/H .

When the ampacity of the power rail is known in per-unit vehicle current, the ampacity of the power rail in S/H can be found from Fig. 3.2. Since the power rail carries maximum current for long durations, the maximum load curve is used.

Maximum loading of the power rail occurs when a vehicle is close to a substation, since almost all the current to the vehicle is supplied by that substation. As the vehicle approaches the point of minimum voltage, the current to the vehicle is divided between the substations at both ends of the rail section. The situation is obvious for the case of one vehicle on a rail section, $S/H \leq 1$. These conditions also hold when two

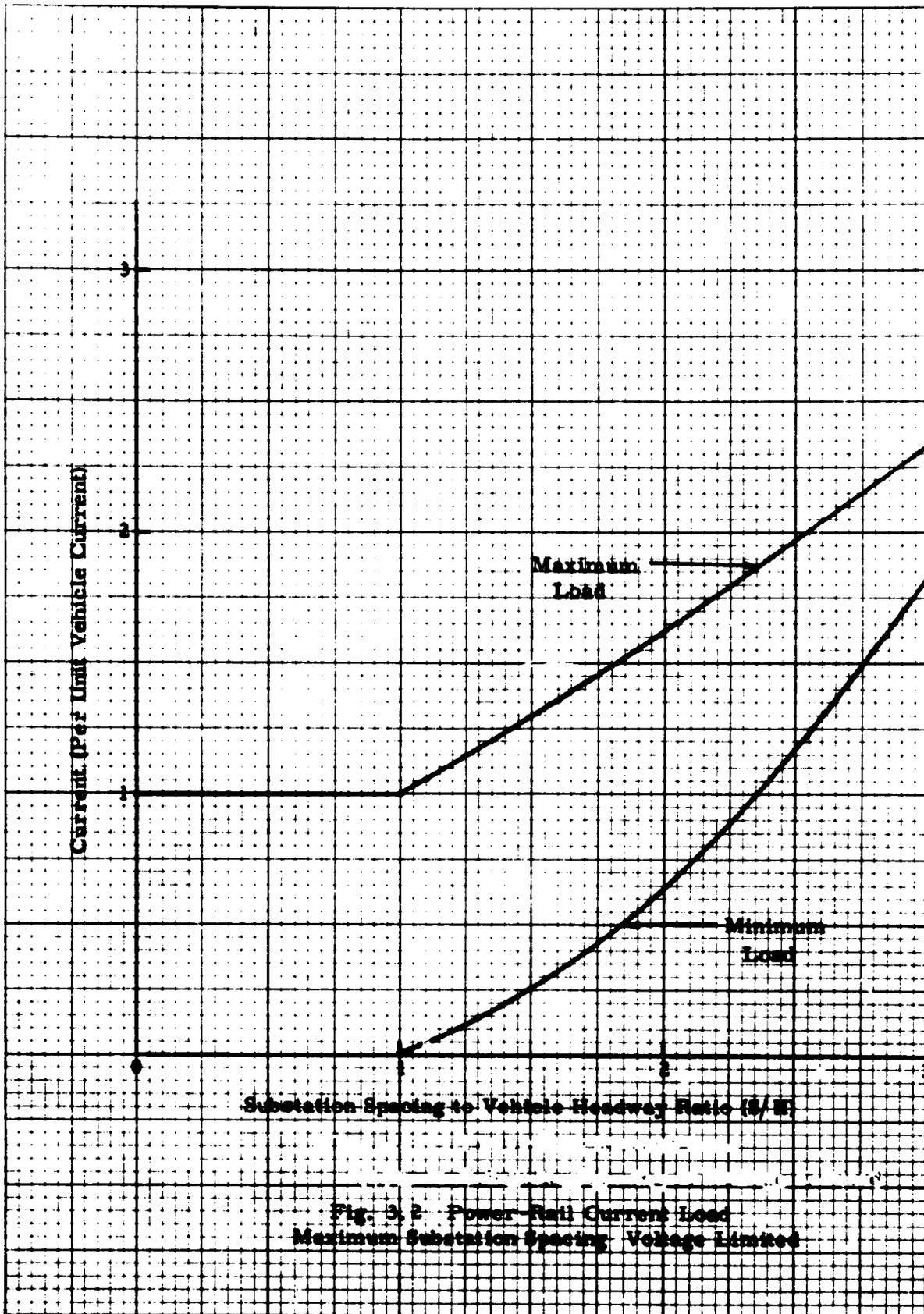


Fig. 3, 2 Power-Rail Current Load
Maximum Substation Spacing Voltage Limited

or more vehicles occupy a rail section, $S/H > 1$. The current to the vehicle at the minimum-voltage point, however, does not necessarily divide equally between the two substations.

From Table 3.2 and Fig. 3.2, it is evident that for traffic conditions where S/H is somewhat greater than unity (more than one vehicle between substations), the ampacity of the power rails and the parallel cables will be exceeded. The actual limit of S/H depends upon vehicle power factor, since the magnitude of the current changes with this parameter. For the basic rail design, dividing the ampacity of the power rail (1100 A) by the vehicle currents shown in Table 3.2 gives the load capacity of the power rail in per-unit vehicle current. For the low-impedance rail design, the ampacity of the parallel cables sets the limit. Dividing the ampacity of the cables (860 A) by the cable loads shown in Table 3.2 gives the load capacity of the low-impedance power rail in per-unit vehicle current. The maximum values of S/H corresponding to the ampacities of the power rail or the cables can then be read directly from Fig. 3.2. These values are given in Table 3.4.

Table 3.4 shows that the use of parallel cables in this particular application does not result in improved power-rail load capacity. Higher S/H ratios obtained with parallel cables, does not imply that traffic density can be increased. Higher S/H ratios imply more vehicles between substations; since the maximum substation spacing also increases due to the lower

Table 3.4
Power-Rail Ampacity

<u>Vehicle (PF)</u>	<u>Maximum S/H</u>	
	<u>Basic Power Rail</u>	<u>Low-Impedance Power Rail*</u>
0.75	1.05	1.01
0.9	1.45	1.35
1.0	1.65	1.60
0.9 (leading)	1.45	1.35

*2 x 1000 kcmil parallel cables

impedance of the power rail, the actual vehicle headways may be smaller.

3.4 Power-Rail Utilization

The power-rail ampacities found in Art. 3.3 provide the maximum utilization of the power rail when substations are spaced at the voltage-drop limit. If the design specification for the wayside power system calls for small vehicle headways, power-rail ampacity may require that substations be spaced closer together than necessary just to meet the voltage-drop limit at the vehicle. Substation spacing is then said to be power-rail ampacity limited.

When substation spacing is not at the maximum allowable by the voltage-drop limit, the curves in Fig. 3.2 are no longer valid for the determination of the power-rail ampacity in S/H . If new curves are calculated, the currents will change primarily because of the reduced transmission losses in the power rail.

To bound the difference in the value of S/H when substation spacing is voltage-drop limited and power-rail ampacity limited, consider as the limiting case, an almost lossless power rail. For $S/H \leq 1$, the maximum current from a substation into the power rail will still be 1.0 pu vehicle current, although the duration of the maximum value will be short.

For $S/H = 2$, the maximum current will be 1.5 pu. Thus between 1.0 and 1.5 pu current, the difference found in the power-rail ampacity in terms of the S/H value in Fig. 3.2 will increase from about 0-to-10% in an almost linear fashion. Actual differences will be much smaller; the curves in Fig. 3.2 can be used with little error when substation spacing is power-rail ampacity limited.

In conclusion, the ampacities given in Table 3.4 provide good estimates for maximum utilization of the power-rail when substation spacing is power-rail ampacity limited.

4.0 AC Substation Design and Cost

The cost estimates and designs for substations are based on a banked-secondary system. The substations consist of two independent half-substations, each feeding the power rails in one direction of travel. Substation designs are considered with complete regulation of the output bus, and with regulation of only the daily variation of the ac supply voltage to the substation. Design and cost data are given for 8.25-kV substations; the results are generally applicable to 4.16-kV substations.

4.1 Substation Voltage Regulation

Substations which provide regulation of the output bus voltage against load and supply voltage changes are termed completely regulated. Since load changes occur frequently, long life is the principal consideration in the selection of a load tap changer (LTC). The anticipated number of tap changes per unit time exceeds the economical duty of a conventional mechanical LTC. Static tap changers or vacuum-switch assisted tap changers must be considered.

The vacuum-switch assisted LTC consists of a heavy-duty (long-life) mechanical load tap changer with a vacuum switch to interrupt the circuit when each tap change is made. The current interruption takes place in a vacuum instead of under oil. Contact life is therefore increased; maintenance is essentially confined to replacing the vacuum switches.

Vacuum-switch assisted LTC's have been used in industrial applications where long life is a primary consideration. Lifetime is almost a linear function of the current which must be interrupted: 750,000 operations at 500 A can be conservatively expected. Transformer primary voltages up to 230 kV can be used. For a 15-MVA transformer with a primary voltage of 69 kV, 1,725,000 operations at rated load current can be expected, and at 110 kV, 2,750,000 operations. Even under heavy traffic conditions of 30 vehicles/h, 16 h/day, 365 days, and 10 operations per vehicle transit, a lifetime of over one year for the vacuum switches is possible. Sufficient data to estimate the lifetime of mechanical components is not yet available.

Static tap changers using thyristor switches are not commercially available, but appear to be technologically feasible. Our inquiries to potential suppliers, however, yielded only negative responses. High cost and long development time were universally cited. This may be the result of little demand for such a product. Cost estimates were obtained.

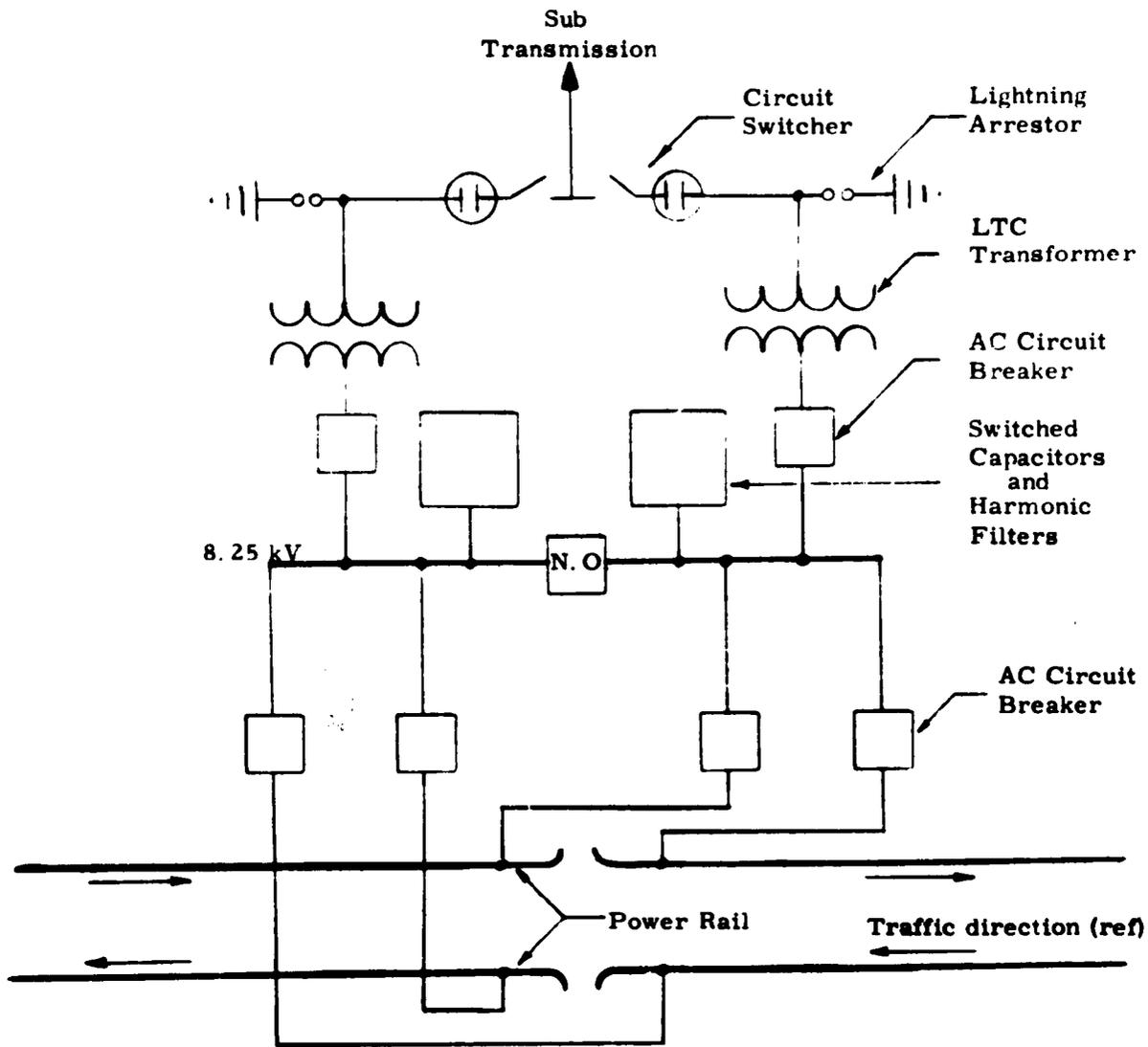
Static tap changers are solid-state devices and should meet ratings similar to those established for traction rectifiers. *

The application of special LTC equipment can be avoided if the voltage drop in the transformers can be tolerated. Maximum voltage drop can be controlled by selecting transformers of adequate size. Such substations should include in the transformer a conventional LTC to compensate only for daily variations in the primary supply voltage, and are termed supply-regulated.

*Traction Ratings - "Silicon Rectifier Units for Transportation Power Supplies", NEMA Standards Publication No. RI 9-1968, reaffirmed 1973.

Heavy Traction: 100% rated load continuously followed by 300% rated load for 1 min superimposed on 150% rated load for 2 h.

Extra-heavy Traction: 100% rated load continuously followed by 300% rated load for five periods of 1 min and 450% rated load for 15 sec superimposed on 150% rated load for 2 h.



**Fig. 4.1 Banked Secondary AC Substation
Half-Substation Feeding Power
Rail In One Direction of Travel**

Table 4.1

Cost Estimate for AC Substations
(15 MVA per half-substation)

<u>Item</u>	<u>Unit Cost*</u>	<u>No. of Units</u>	<u>Total Cost</u>
LTC Transformer	\$106,000	2	\$212,000
Increment for Static Tap Changer	200,000	2	400,000
Increment for Vacuum Tap Changer	22,000	2	44,000
69-kV Circuit Breaker	15,000	2	30,000
15-kV Circuit Breaker	20,000	7	140,000
Harmonic Filter	30,000	2	60,000
Capacitor Bank	10,000	2	20,000
Ground Resistor	7,000	2	14,000
Rail Connections (per cable)	3,000	12	36,000

Totals - Completely Regulated Substation (a) with Static LTC - \$1,140,000

(b) with Vacuum LTC - \$695,000

Total - Supply-Regulated Substation- \$640,000

***Based on 1973 prices.**

****includes 25% for installation.**

4.2 Substation Components and Cost

A one-line diagram of an ac substation is shown in Fig. 4.1. Note that each half-substation only feeds traffic in one direction of travel. In the event of a failure the normally open tie breaker can be closed for combined operation at reduced power. Equipment cost estimates based on 1973 prices for an ac substation with a rating of 30 MVA (15 MVA per half-substation) are given in Table 4.1, and discussed in the following:

4.2.1 LTC Transformer

The transformers are rated 55° C temperature rise, 7.5% impedance, self-cooled, load tap changing units. Basic list price about a nominal rating of 15 MVA, including the standard LTC equipment adder costs, is \$58,000 + \$3200 x MVA, or \$106,000 at 15 MVA.

4.2.2 Increment for Static LTC

Cost estimates for a static LTC rated at 15 MVA varied from \$150,000-to-\$250,000. An average of \$200,000 is used.

4.2.3 Increment for Vacuum LTC

The premium for a vacuum-assisted LTC is small, since it represents a minor variation to standard LTC equipment. The estimated incremental cost is \$2000. Replacement vacuum interrupters cost about \$200 each,

three per transformer. The present worth of replacements over a 40-y period including labor and incidentals is estimated at \$20,000. The lifetime of mechanical components is not known but replacement costs are not expected to be significant and are not included.

4.2.4 Harmonic Filter

Filter size will depend on a number of factors, such as the type of vehicle power conditioners and the ac supply system impedance, which are not specified. For completeness, 2 MVAR of filtering at \$15,000/MVAR is included. Costs may be significantly higher if communication interference is a problem.

4.2.5 Capacitor Bank

Power factor at the substation can fall below 0.7 when the vehicle power factor is 0.75. For completeness, 2 MVAR of power factor correction capacitors at \$5000/MVAR is included to correct the secondary power factor to at least 0.8. Correction will not be required when the vehicle power factor is greater than 0.9.

4.2.6 Switchgear

AC circuit breakers are rated 15 kV 1200 A maximum continuous, with an interrupting capability of 31,000 A. The circuit switchers, which supply the transformer have a continuous current rating of 600 A, an

interrupting rating of 4000 A, and will sustain 14,000 A for 1s.

4.3 Substation Rating and Cost

Substation costs at higher MVA ratings are calculated from the figures in Table 4.1 for 30 MVA assuming a 15% constant cost with the remainder proportional to the MVA rating. On this basis, the costs are:

- (1) completely regulated substation with static LTC, $\$171,000 + \$32,300 \times \text{MVA}$
- (2) completely regulated substation with vacuum LTC, $\$104,000 + \$19,700 \times \text{MVA}$; and
- (3) supply regulated substation, $\$96,000 + \$18,200 \times \text{MVA}$.

Note that these figures are for total substation MVA - the sum of two half-substations for travel in both directions.

5.0 Substation Spacing

For economic reasons, the largest substation spacing which meets system performance requirements should be used. When the maximum spacing is primarily limited by the vehicle voltage requirements, substation spacing is said to be voltage limited. Vehicle headway requirements may also be a consideration. If this leads to multiple vehicles between two substations, the ampacity of the power rail may be exceeded. Substations spaced at less than the distanced permitted by voltage requirements would then be necessary. Substation spacing is then said to be power-rail ampacity limited.

The maximum spacing for the basic and low-impedance power-rail designs and for completely and supply-regulated substations are presented in this chapter. A maximum voltage drop of 25% at the vehicle is assumed. The results given are for 8.25-kV substations; the approach is similar for 4.16-kV substations.

5.1 Vehicle Voltage Profiles

To determine the maximum substation spacing (voltage limit), the positions of the vehicles for minimum voltage at one of the vehicles must be known.

The minimum - voltage positions depend wholly on the spacing and number of vehicles on the section between two substations. General solutions to the problem are readily formulated when these two variables are expressed in terms of a single parameter, - the ratio S/H of the substation spacing S to the vehicle headway H .

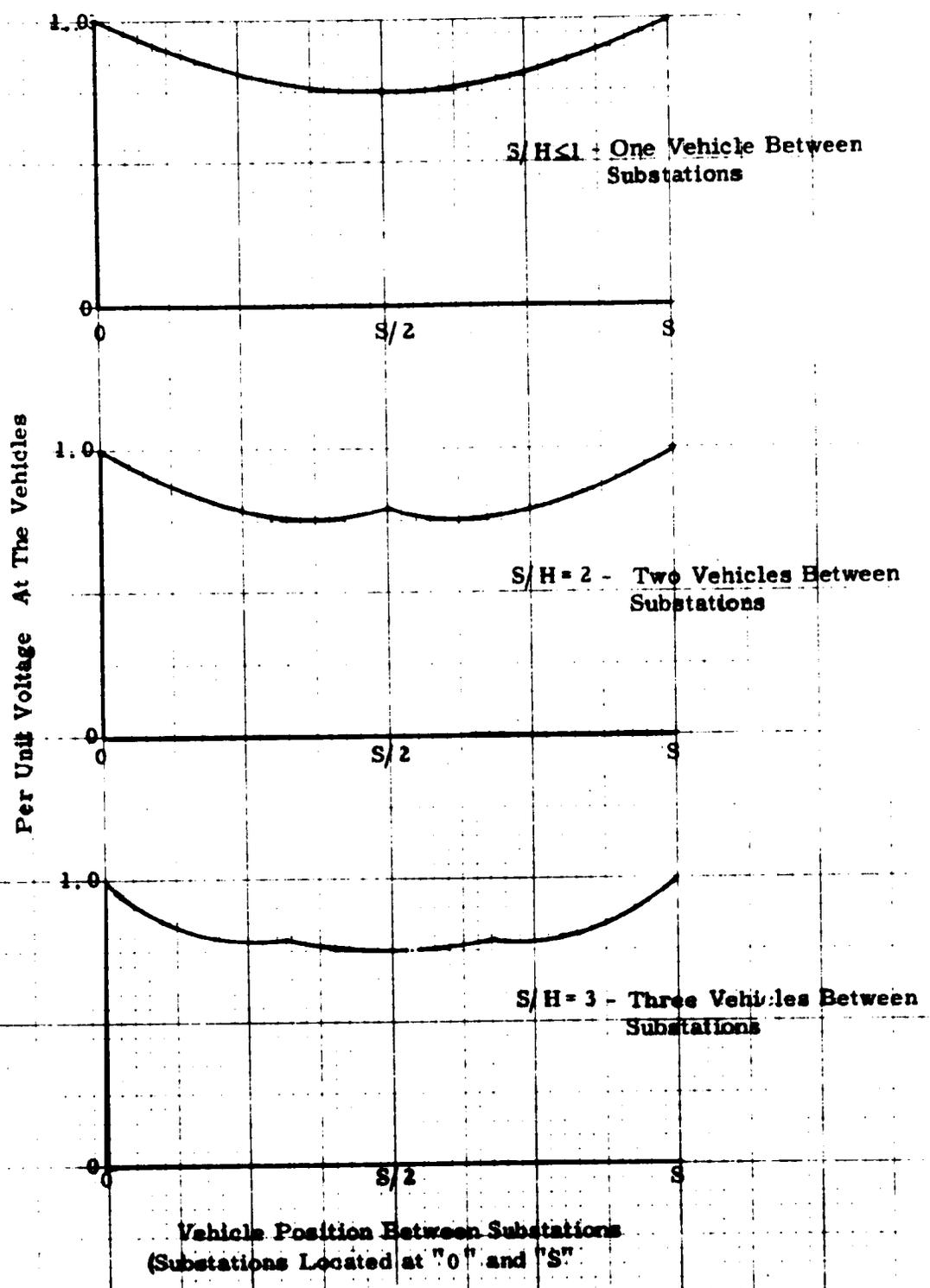
The simplest cases to consider are those for which S/H is an integer since the number of vehicles per section is the same as S/H . Cases for which $S/H < 1$ are, of course, all identical to the case $S/H = 1$, since there can be only one vehicle between substations.

When S/H is an odd integer, it is evident that the minimum voltage occurs at a vehicle halfway between the substations, i. e. , when the vehicles are distributed symmetrically between the substations. When S/H is an even integer, however, the minimum voltage at a vehicle corresponds to a non-symmetrical distribution.

Figure 5.1 illustrates the vehicle voltage profiles for $S/H \leq 1$, and $S/H = 2$, and 3. In each case it has been assumed that the substation spacing has been adjusted so that the minimum vehicle voltage is 0.75 of the rated substation voltage. It has also been assumed that the substation voltage is completely regulated. The voltage profiles are still representative when the substations are supply regulated, and can be used with little error for such substation designs. In the special case when the vehicle power factor is leading, resonance conditions may alter the shape of the profiles somewhat.

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F.3. 5.1 Vehicle Voltage Profiles

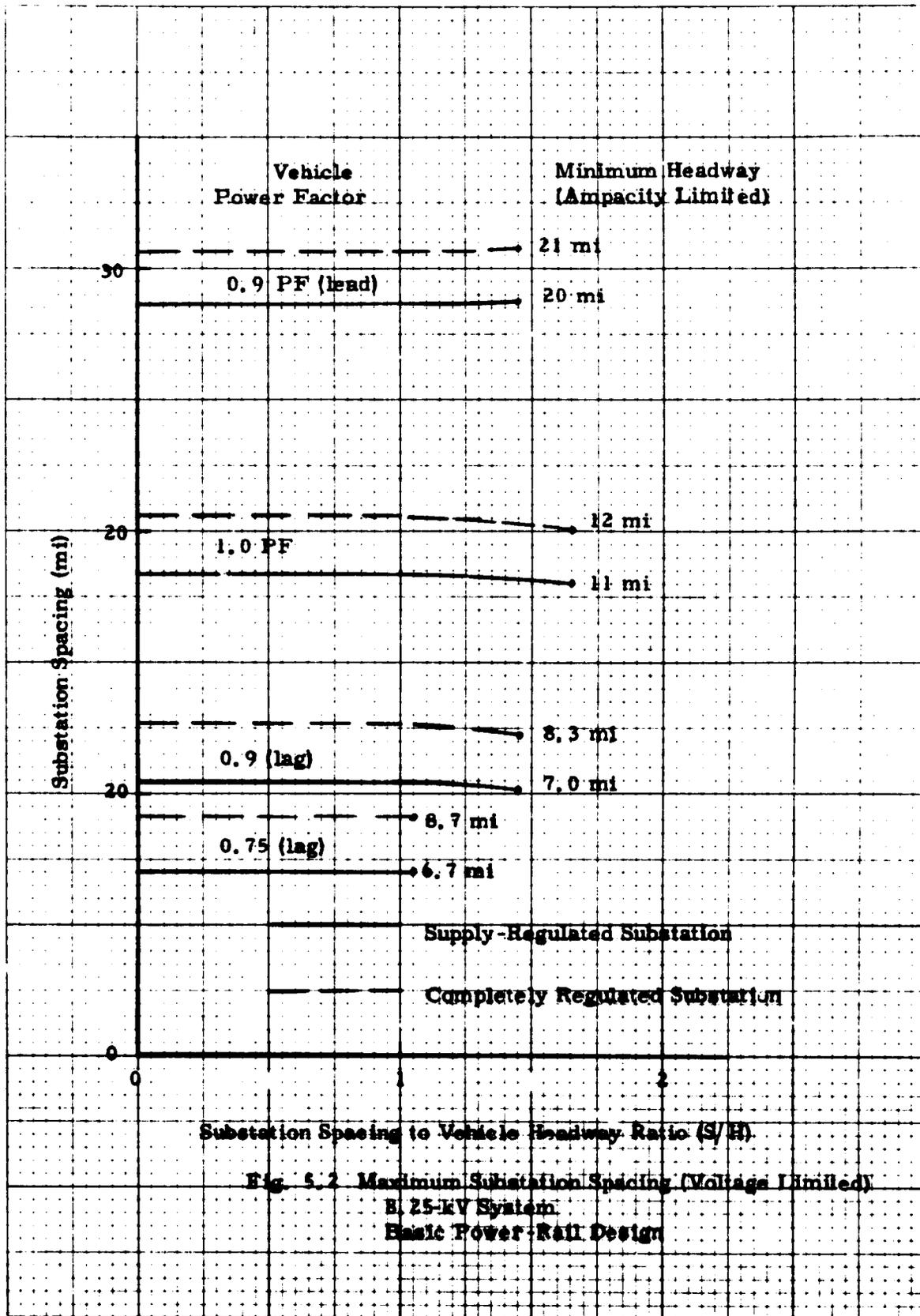
S = Substation Spacing
H = Vehicle Headway

5.2 Calculation of Maximum Substation Spacing (Voltage Limited)

The problem of calculating the maximum substation spacing is in essence no more than evaluating the voltage drop in the power rail between a vehicle and a substation. Since the minimum-voltage positions for a given number of vehicles between two substations is always the same when the ratio S/H is specified, it is simply a problem of solving a network with impedances that bear linear dependence on vehicle position and substation spacing. The vehicles, however, are considered as constant-power loads and are, therefore, nonlinear circuit elements. Except in the simplest of cases ($S/H \leq 1$), closed form solutions are analytically unattractive and iterative methods must be used. Algorithms for solving these problems are given in Appendix A.

5.3 Maximum Substation Spacing (Voltage Limited) - Completely Regulated Substation

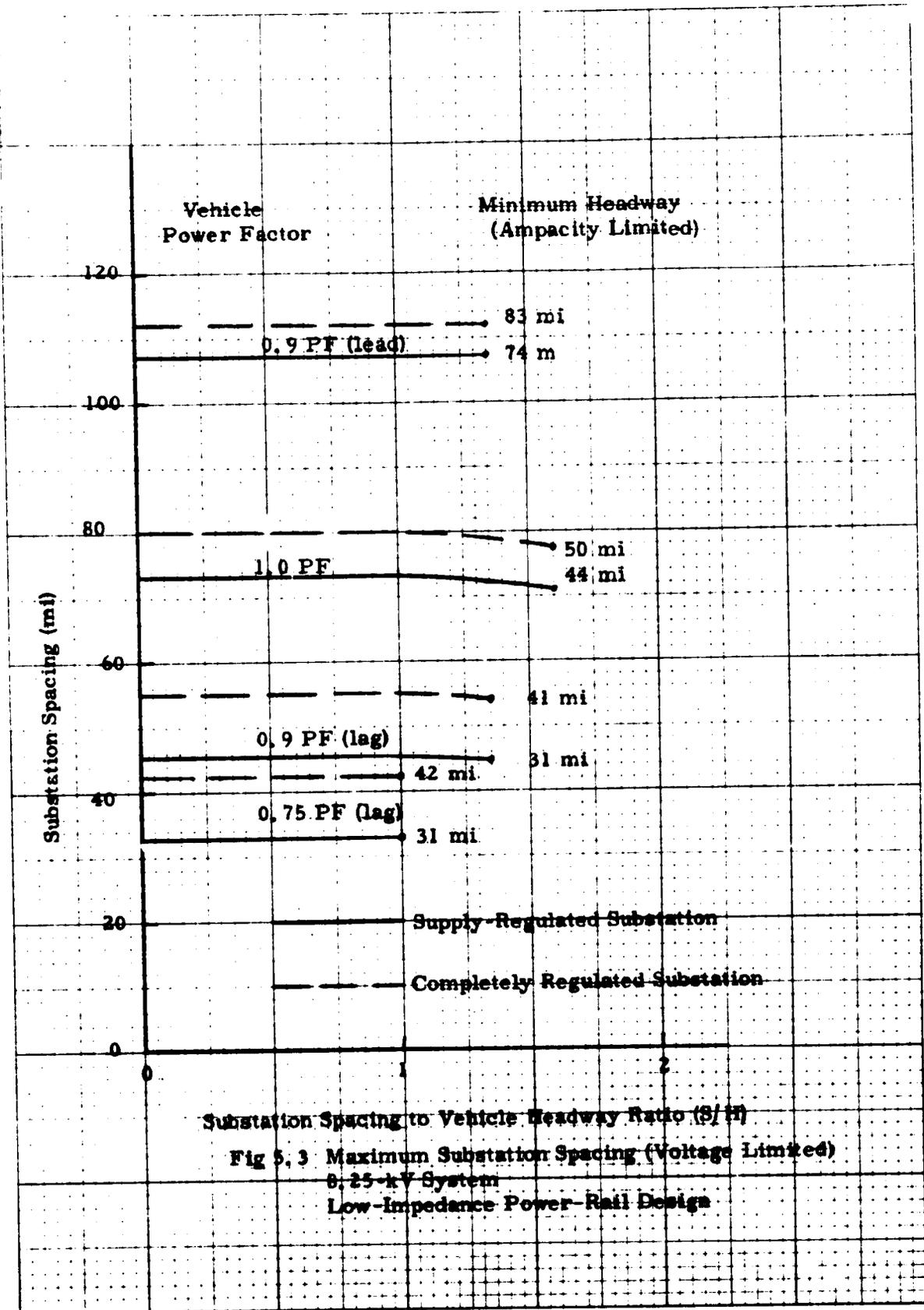
The maximum substation spacings corresponding to a voltage drop of 25% at a vehicle are given in Figs. 5.2 and 5.3 for the two power-rail designs developed in Chapter 3. The maximum substation spacing, expressed as a function of the substation spacing to vehicle headway ratio S/H , is almost constant. Substation spacing increases as the vehicle power factor rises, since the voltage drop in the power rail decreases as the vehicle reactive current is reduced.



Substation Spacing to Vehicle Headway Ratio (S/H)

Fig. 5.2 Maximum Substation Spacing (Voltage Limited)
B. 25-kV System
Basic Power-Rail Design

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Substation Spacing to Vehicle Headway Ratio (S/H)

Fig 5.3 Maximum Substation Spacing (Voltage Limited)
8.25-kV System
Low-Impedance Power-Rail Design

The right hand cut-off point of the curves indicates the limit for multiple vehicle loads, i. e. , the maximum values of S/ H and the minimum permissible headway. For values of S/ H greater than the indicated limits, the current in the power rail or in the parallel booster cables will exceed the ampacity of these conductors (Table 3. 4).

5. 4 Maximum Substation Spacing (Voltage Limited) - Supply Regulated Substation

Maximum substation spacings with supply-regulated substations are somewhat smaller than for the completely regulated substations because of the additional voltage drop in the transformer. The reactance of a 15-MVA 8. 25-kV 7. 5% impedance transformer is $0. 34 \Omega$, or about the reactance of one mile of the basic power rail. As conditions permit larger substation spacings, this added impedance between the substation and vehicle will have proportionally less effect.

Since the ohmic impedance of a transformer depends on the rating, the rating must be known prior to calculating the substation spacing. The maximum substation spacings for completely and supply-regulated substations do not differ appreciably and good estimates on substation size can be obtained from the former and applied to the latter. Substation rating curves will be presented in Chapter 6. Supply-regulated substations are rated at 0. 70 pu

of the vehicle load for $S/H \leq 0.5$; 1.0 pu for $S/H = 0.8$; 1.25 pu for $S/H = 1$; and 2.45 pu for $S/H = 2$.

5.5 Substation Spacing (Power-Rail Ampacity Limited)

To operate with vehicle headways smaller than the minimums tabulated in Figs. 5.2 and 5.3, substation spacing must be reduced in order to remain within the power-rail ampacity. As noted as Art 3.4, good estimates of the S/H ratio for the maximum utilization of the power-rail at reduced headway are S/H ratios given by the endpoints of the curves. The power-rail ampacity-limited substation spacing is determined from the S/H ratio and the vehicle headway.

6.0 AC Substation Rating

Substation loading is determined by vehicle traffic patterns. Since a half-substation only feeds the power rail for one direction of travel, the substation loading pattern is completely specified by the ratio S/H . Substation ratings are found to be either thermally limited or impedance limited. The results are applicable to both the 4.16kV and 8.25kV substations.

6.1 Substation Load Patterns

Substation loading patterns for several values of S/H are given in Figs. 6.1 and 6.2. The indicated loads are for vehicles on both sides of the substation in one direction of travel. The power-rail feed arrangement is shown in Fig. 4.1. The curves are plotted as a function of per-unit time, where one per-unit time is the time for a vehicle to travel the distance between two substations. At time zero a vehicle is adjacent to the substation. The periodicity of the load pattern is H/S per-unit time. Substation loading is expressed in per-unit vehicle load where one per-unit load is the MVA requirement of a single vehicle.

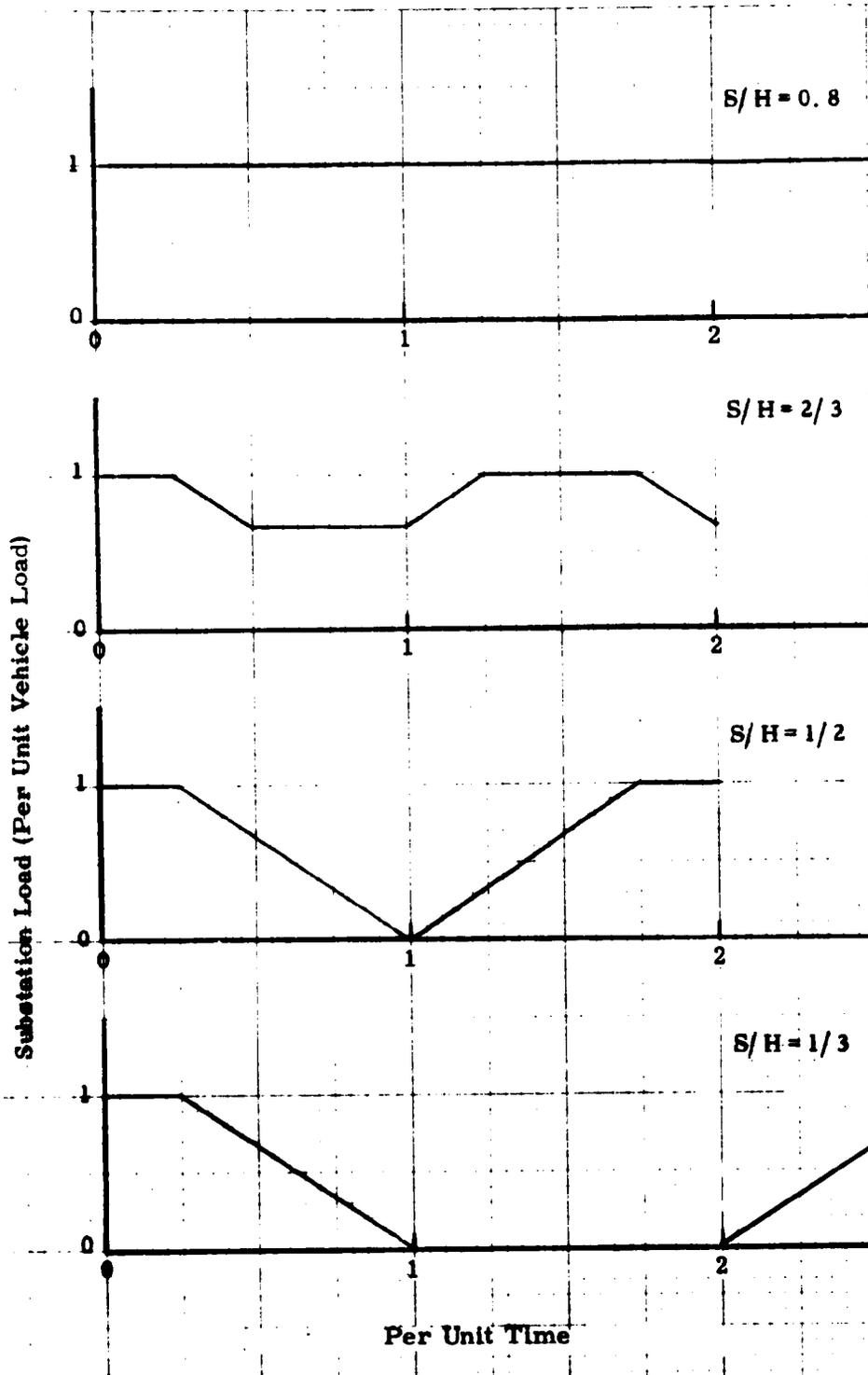


Fig. 6.1 Substation Loading - One Direction of Travel, for $S/H = 0.8, 2/3, 1/2,$ and $1/3.$

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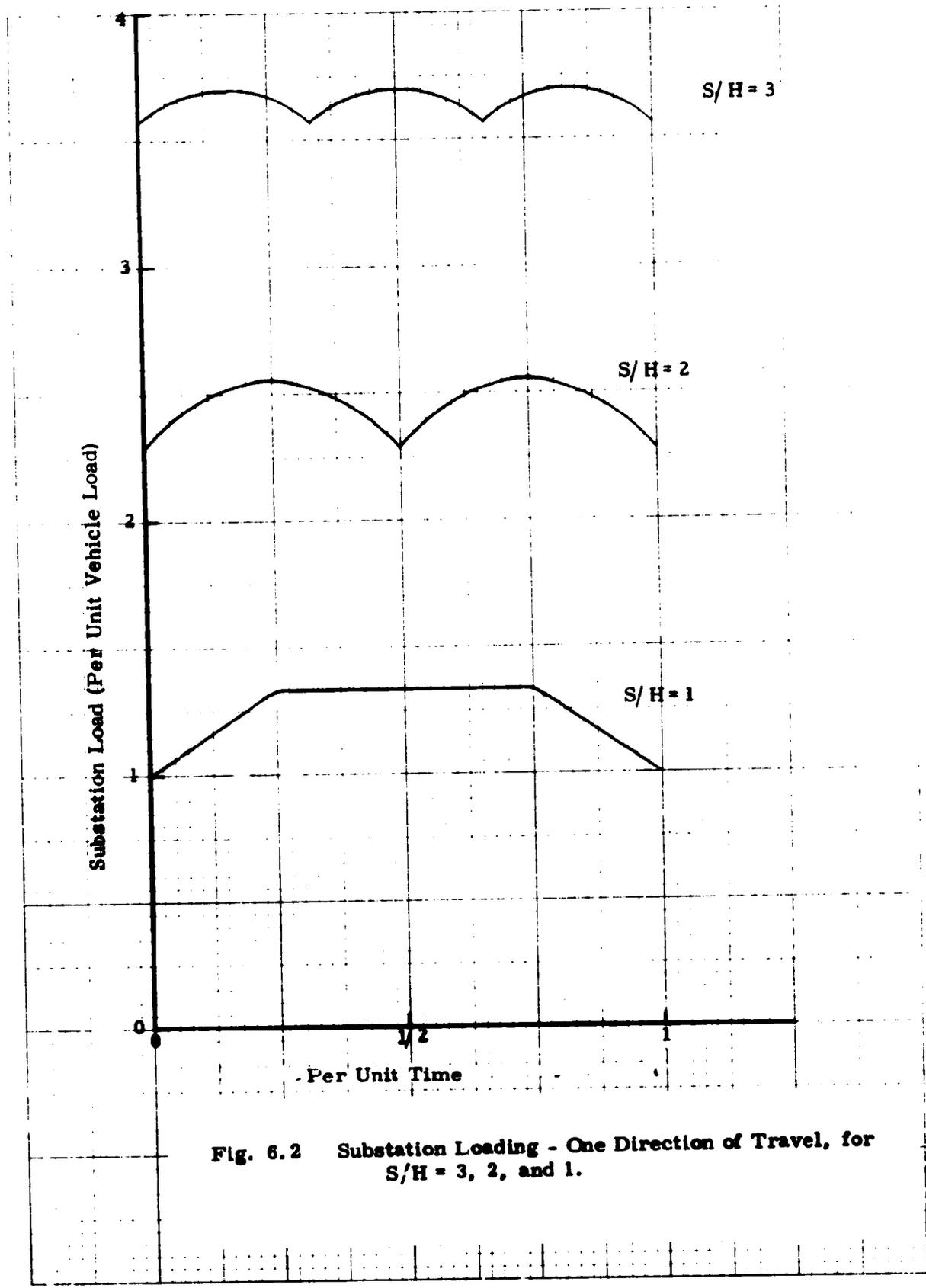


Fig. 6.2 Substation Loading - One Direction of Travel, for $S/H = 3, 2, \text{ and } 1.$

The loading curves presented in Figs. 6.1 and 6.2 assume the maximum substation spacing for a vehicle voltage drop of 25%. Setting the substation spacing for 25% voltage drop allows the normalization for the per unit representation of substation loading. The load patterns are independent of vehicle and power-rail characteristics and depend only with the value of S/H .

6.2 Substation Peak and RMS Loads

The rms and peak values of substation loading are summarized in Fig. 6.3 as a function of S/H . It is evident that for $S/H \leq 0.8$, the peak and rms loads are almost identical, i.e., substation loading is fairly uniform as vehicle headways are in the order of substation spacings or less. The results of Fig. 6.3 will be used to determine the appropriate MVA rating of substations when substation spacing is voltage limited.

6.3 Transformer Voltage Drop

Depending on the voltage drop of the substation transformer, substation ratings may be based either on rms loading (thermal limits) or on impedance limits.

Since transformer impedance is nearly all reactance, the voltage drop in the transformer is due essentially to reactive (quadrature) current. If the transformers are rated for rms loading, then the voltage drop in the transformer is given from Appendix B, by

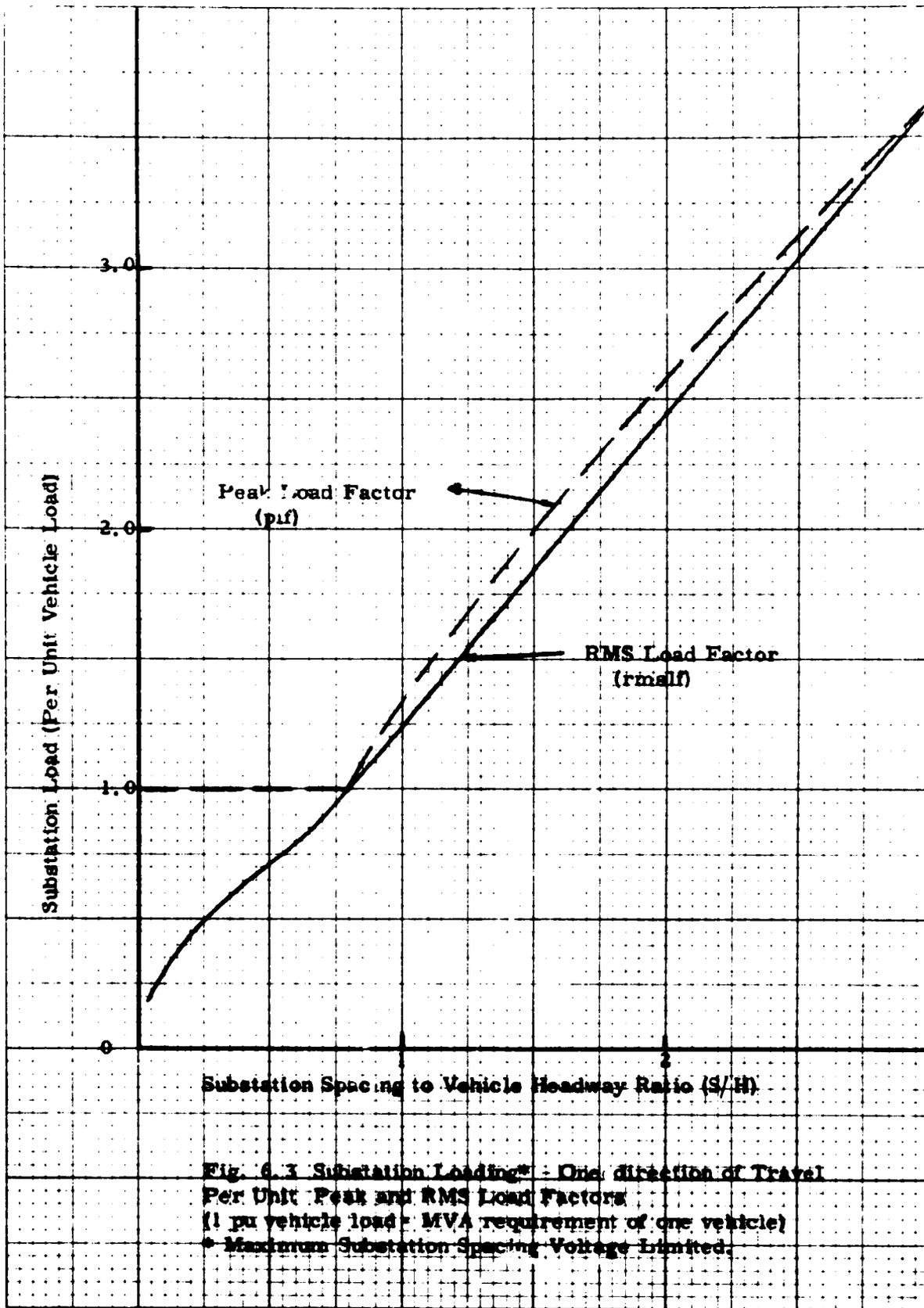


Fig. 6.3 Substation Loading* - One direction of Travel
Per Unit Peak and RMS Load Factors
(1 pu vehicle load = MVA requirement of one vehicle)
* Maximum Substation Spacing Voltage Limited.

$$\% \text{ voltage drop} = \% \text{ impedance} \frac{\text{p}f \times \text{RF}}{\text{rms}f}$$

where

$\%$ impedance = impedance of transformer on its on base in $\%$,

$\text{p}f$ = peak load factor in pu vehicle load (Fig. 6.3)

$\text{rms}f$ = rms load factor in pu vehicle load (Fig. 6.3)

RF = reactive factor of transformer current.

The substation will be power factor corrected to at least 0.8 PF lagging, so that $\text{RF} \leq 0.6$. Transformer impedances are 7.5%. Then,

$$\% \text{ voltage drop} \leq \frac{\text{p}f}{\text{rms}f} \times 4.5\%$$

For $S/H \geq 0.8$, $\text{p}f/\text{rms}f \leq 1.06$ and the worst case transformer voltage drop is in the order of 4.8%. For $S/H < 0.8$, $\text{p}f/\text{rms}f$ ratios can be determined from Fig 6.3. For $S/H = 0.5, 0.25, 0.125$, the worst case transformer voltage drops are 6.4%, 9.0%, and 12.9%, respectively.

It is quite evident that transformer voltage drop only becomes a serious problem when, simultaneously, vehicle headways are much greater than substation spacing ($S/H \ll 1$), and the transformers are rms load rated. The voltage drop can be limited to 6.4% or less if the transformers are rated at 0.70 pu for $S/H < 0.5$.

It should be noted that the above voltage-drop figures are those due to the frequent vehicle load variations seen by the substations. They do not include daily variations in the supply-system voltage.

6.4 Ratings - Completely Regulated Substation

The substation output-bus voltage in this case is held constant for all loads; the effects of transformer impedance are cancelled. Only when the voltage drop in the transformer and the excursion in the supply voltage exceed the tap range must the transformer rating be limited by impedance.

The rating of the substations can be based on the rms load except in the case of very small S/H ratios. Assuming supply variations do not exceed 7%, then the standard load tap range of 20% will be adequate in the worst case for S/H as small as 0.125, corresponding to a transformer rating of 0.35 pu of vehicle load (Fig 6.3).

6.5 Ratings - Supply-Regulated Substations

The rating of transformers for supply-regulated substations will be based on rms loading for $S/H > 0.5$. For $S/H < 0.5$, the transformer is impedance limited and a minimum rating of 0.70 pu of vehicle load is used. This will limit the voltage drop under load to 6.4%. The substation spacing calculations (Art 5.3) for supply-regulated substations are based on these assumptions.

Comparing Figs. 5.1 and 6.1 for $S/H \leq 0.8$, it is evident that maximum substation loading does not coincide with a vehicle in the minimum voltage position (halfway between substations). Hence the maximum transformer voltage drop of 6.4% will have a smaller effect on substation spacing than might be anticipated.

Supply-regulated substations are provided with load tap changers (LTC), which are slow responding units, designed only to compensate for daily changes in the supply voltage. These LTCs can also be used to compensate transformers rated for rms loading. Then for $S/H > 0.8$, for which the rms and peak loads are about the same, the voltage drop fluctuation in the transformer due to frequent load changes would be small. For $S/H < 0.8$, the rms load may be much smaller than the peak load; the LTC would be biased to offset the voltage drop of the rms load. Since peak loading for $S/H < 0.8$ occurs in the vicinity of a substation, (Fig 6.1), a suitable choice would be to set the LTC to obtain the nominal bus voltage at 1 pu vehicle load. When the vehicle is outside the section, the overvoltage would be no more than the maximum voltage drop of the transformer, or 6.4% in the worst case when $S/H < 0.5$.

Using the slow response LTC to compensate transformers rated for rms loading would permit substation spacings almost equivalent to those obtainable with completely regulated substations. This compensation was not used in calculating the maximum substation spacings given in Art. 5.3.

6.6 Rating - Substation Spacing Power-Rail Ampacity Limited

When the headway specification is such that the maximum substation spacing is power-rail ampacity limited, rather than voltage limited, the loading curves in Figs. 6.1, 6.2, and 6.3 are no longer valid. However, reasonable approximations can be determined from those curves when the substation spacing must be reduced from the voltage limited value.

The vehicles are assumed to be constant-power devices. For a given value of S/H , the substation rating changes only by the reduction in transmission losses due to the shorter distances between vehicles and substations. Assuming lossless power transmission, the rms substation load in per-unit vehicle load is approximately equal to the value of S/H . By referring to Fig 6.3, the transmission loss with the substation spacing voltage limited can be calculated. An estimate to the transmission loss at reduced substation spacing can be obtained by decreasing the above determined loss in proportion to the reduction in substation spacing. A new rms substation load can then be calculated.

7.0 AC Wayside Power-System Costs

The total per-mile cost of the wayside power system is the sum of the power-rail per-mile cost and the substation per-mile cost. Results are expressed as a function of vehicle headway with vehicle power factor, substation voltage drop, and power-rail impedance as parameters. Cost figures are for dual-rail system (two directions of travel).

7.1 Power-Rail Per-Mile Costs

Power-rail per-mile costs are presented in Tables 3.1 and 3.2: (1) \$408,000/mi for the basic power rail, and (2) \$654,000/mi for the low-impedance power rail (with 2 x 1000-kcmil cables in parallel). Power-rail costs are assumed constant as a function of vehicle headway.

7.2 Substation Per-Mile Costs

Substation per-mile costs, as a function of vehicle headway, with vehicle power factor, substation regulation, and power-rail impedance as parameters, can be determined from the results of Chapters 4, 5, and 6. For every admissible value of S/H , and combination of parameters, (1) substation spacing and vehicle headway values (S, H) can be determined from Figs. 5.2 and 5.3, and (2) the substation rating can be determined from Fig. 6.3. Substation cost for each rating can then be calculated from the results of Art. 4.4. Per-mile costs are averaged out over a number of

substations. The results in the following section are based on an integral number of substations with a total stage length in the order of 100 mi.

7.3 Total Per-Mile Costs

Total wayside power-system per-mile costs as a function of vehicle headway with vehicle power factor, substation regulation, and power-rail impedance as parameters are shown in Figs. 7.1, 7.2, and 7.3. Note that since power-rail costs are constant with respect to vehicle headway, these curves also indicate the relationship between substation per-mile costs and vehicle headway.

Figs. 7.1 and 7.2 show the per-mile costs of the various substation designs (supply-regulated, completely regulated with static LTC, completely regulated with vacuum LTC), and the basic power-rail design. Per-mile costs of substations and the low-impedance power rail are shown in Fig. 7.3.

Comparing Fig. 7.3 with Figs. 7.1 and 7.2, we see that there is no economic advantage in lowering the power-rail impedance in order to increase the maximum substation spacing. The additional cost in the power rail overshadows any reduction in substation per-mile costs resulting from increased substation spacing. Moreover, the ampacity of the low-impedance power-rail design does not permit operation with several

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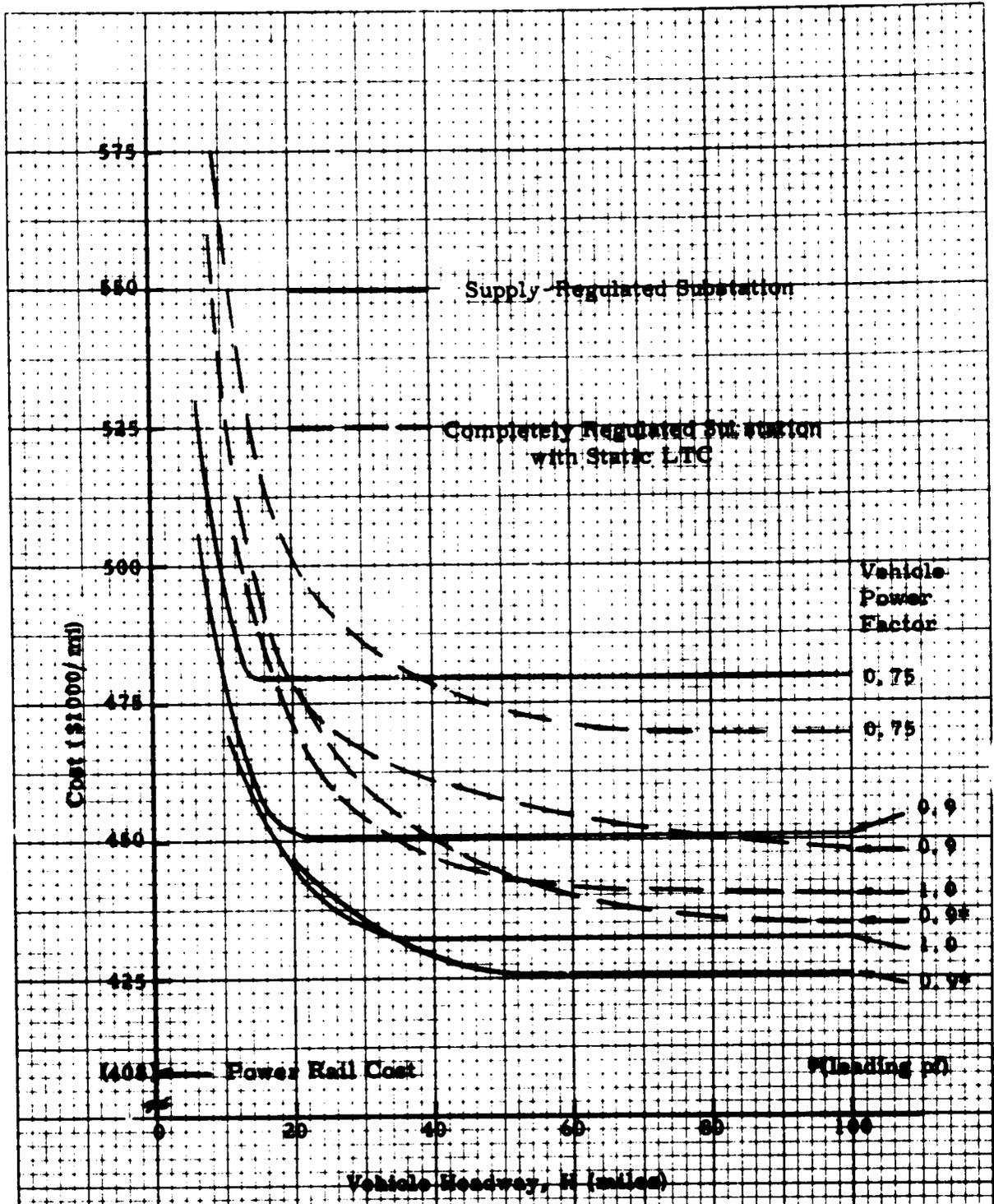


Fig. 7.1 Total Wayside Power-System Per-Mile Costs with Supply Regulated Substations and Completely Regulated Substations with Static LTC, 8.25-kV ac System, Basic Power-Rail Design. Based on 1973 Costs

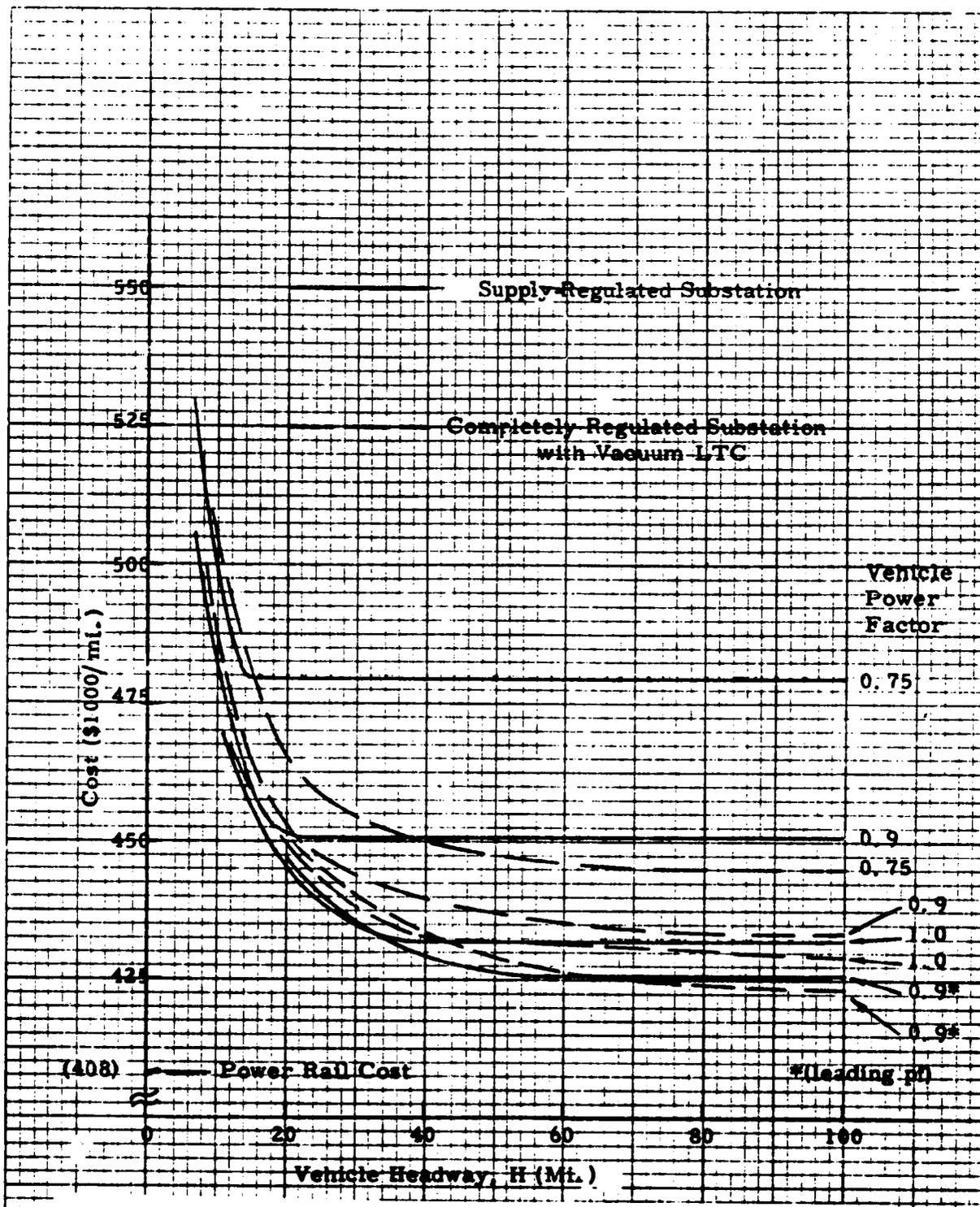


Fig. 7.2 Total Wayside Power-System Per-Mile Costs with Supply Regulated Substations and Completely Regulated Substations with Vacuum LTC, 8.25-kV ac System, Basic Power-Rail Design. Based on 1973 Costs

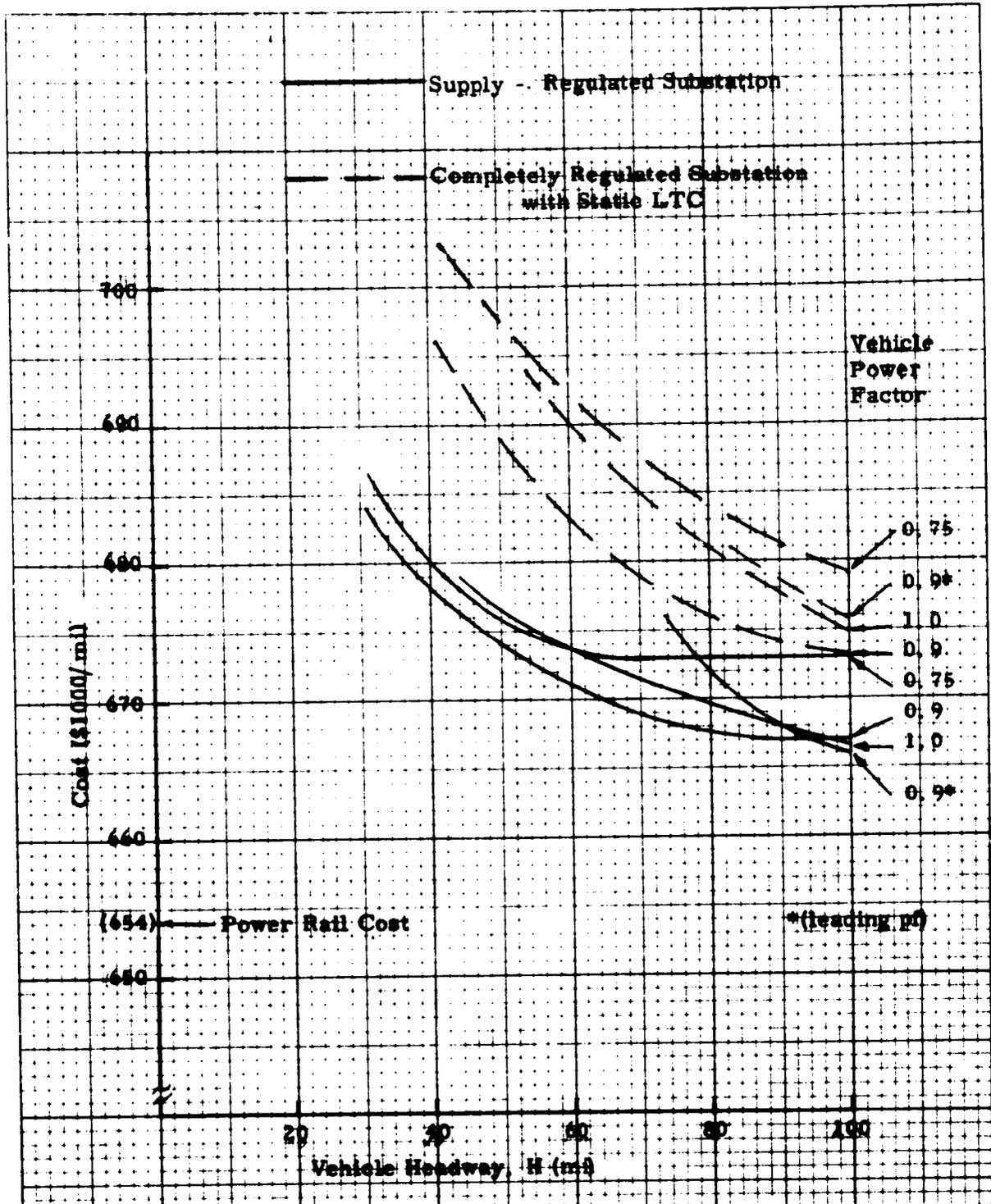


Fig. 7.3 Total Wayside Power System Per-Mile Costs with Supply Regulated Substations and Completely Regulated Substations with Static LTC, 8.25-kV ac System, Low-Impedance Power-Rail Design. Based on 1973 Costs

vehicles between substation. Increasing substation spacing therefore also increases the minimum vehicle headway that can be attained with substations spaced at the voltage limit. The left-hand cut-off points of the curves in Figs. 7.1, 7.2, and 7.3 indicate the minimum headways corresponding to the ampacity limits of the power rail. Because of the obvious economic disadvantage of a wayside power system utilizing a low-impedance power-rail design, the curves in Fig. 7.3 have not been extended to smaller headways. The extension of these curves would correspond to the case where maximum substation spacing is power-rail ampacity limited.

The economic advantage of the supply-regulated substation design under certain conditions is also evident from Figs. 7.1 and 7.2. The cost curves reach a minimum value and remain constant with increasing headway when the substation transformers reach their impedance limits. The impedance limits for completely regulated substations are higher than for supply-regulated substations, and costs for the former decrease over a much wider load, i. e., headway, range. Where the curves (for the same vehicle power factor) intersect, the tradeoff between accommodating voltage drop with transformer rating (supply-regulated substation) or with load tap changers (completely regulated substation) occurs. As the vehicle power factor increases, the voltage drop in the substation transformers decreases and the cost intersection point occurs at greater headways.

From Figs. 7.1 and 7.2, it is apparent that the supply-regulated substation is economically advantageous compared to the completely regulated substation with a static LTC at headways less than 20-to-40 mi, depending on vehicle power factor. Even at greater headways, the cost savings in the latter design does not exceed \$10,000/ mi. In consideration of the greater complexity of the static LTC, its benefit may be marginal.

The supply - regulated substation is economically competitive with the completely regulated substation with a vacuum LTC at headways less than 15-to-45 mi, depending on vehicle power factor. Cost benefit of the latter design ranges from \$2500-to-\$35,000/ mi, depending on headway and vehicle power factor. The maximum benefit is experienced with large headways and low vehicle power factor. It is noted however, that the longevity of the mechanical components in a vacuum LTC has not been established (insufficient application data) and must be carefully weighted against the cost benefits that may exist.

In conclusion, the use of completely regulated substations compared to supply-regulated substations does not result in a decisive economic benefit. Where an economic advantage does exist, it is usually small and must be carefully weighed against the use of undeveloped or untested load tap changers.

7.4 Lower Voltage AC Wayside Power-System Costs

The calculation of the per-mile costs for lower-voltage systems is similar to that presented for the 8.25-kV system. No new design concepts or procedures are required. Lower-voltage systems for the TLRV are not cost competitive because of the increase in power-rail costs for the heavier current. Total costs for one lower-voltage system are presented for comparison.

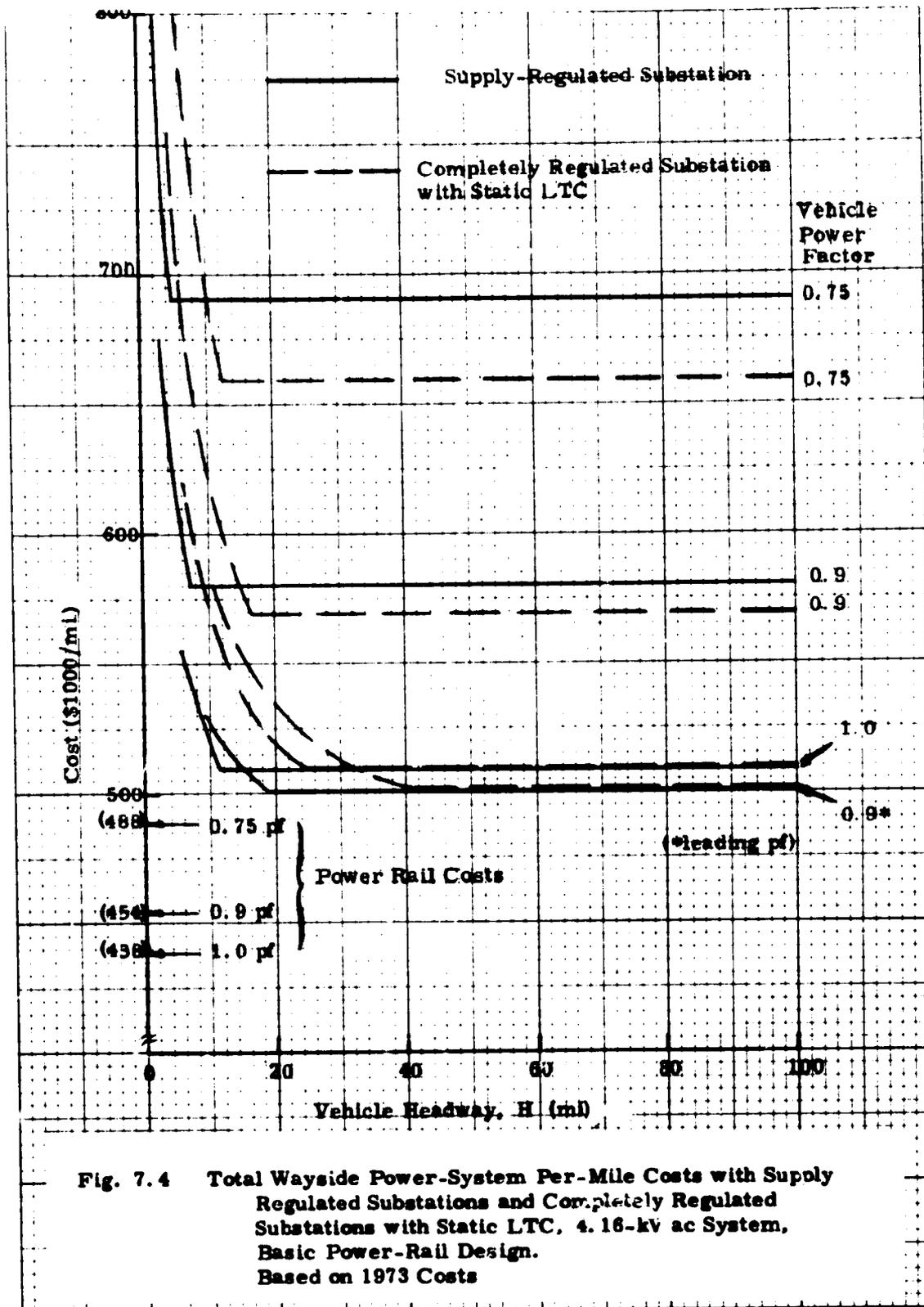
The total wayside power-system per-mile costs for a 4.16-kV ac system are shown in Figs. 7.4 and 7.5. These figures are the analogs to Figs. 7.1 and 7.2 for the 8.25-kV ac system. The comments in Art. 7.3 are applicable.

Only basic power-rail designs are considered. To lower the power-rail impedance, four 1000-kcmil parallel booster cables would be required in order to meet the ampacity of these conductors. (See Art. 3.2 for the 8.25-kV analog). The cost is prohibitive, almost equal to the per-mile cost of the basic power rail. (See Table 3.3)

Adequate ampacity of the rail conductors is provided by increasing the copper cross section. For each vehicle configuration, the power-rail ampacity is adjusted for a maximum of one vehicle between substations. Power-rail costs are assumed to vary only with the rail size.

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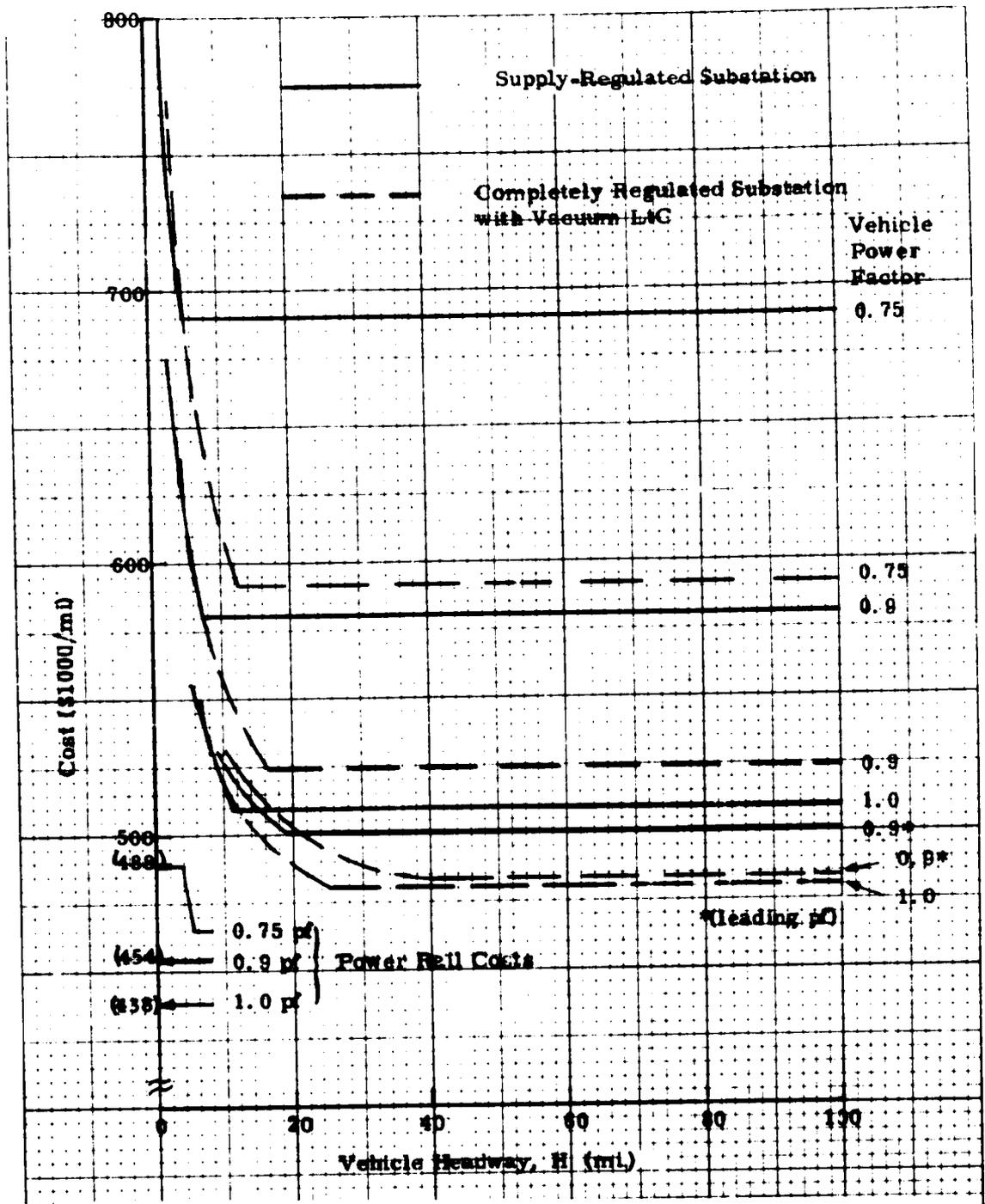


Fig. 7.5 Total Wayside Power-System Per-Mile Costs with Supply Regulated Substations and Completely Regulated Substations with Vacuum LTC, 4.16-kV ac System, Basic Power-Rail Design. Based on 1973 Costs

Substation costs are as given in Art. 4.3, and are assumed to vary with power rating only. Substation ratings are determined from the curves of Fig. 6.3.

8.0 DC Power-Rail Design and Cost

The TLRV ac power rail, modified for two rail conductors, is the basis for the design of the dc power rail. The rail conductor cross section is adjusted to obtain various ampacity and resistance characteristics.

Configurations for the 11.25-MW vehicles at voltages from 1.5 kV dc to 12 kV dc are given. Potential size and cost reductions of the overall power-rail assembly at the lower voltages are discussed.

8.1 Power-Rail Design

The rated current for the various vehicle configurations under consideration varies from 938 A to 7500 A dc. With multiple vehicles between substations even higher currents must be carried by the power rail.

To maximize the ampacity of the copper rail for a given conductor cross-sectional area, wide thin bars should be used. This provides a large ratio of surface area to volume for more efficient cooling. The sheet steel structure supporting the copper rail can accommodate rail bars up to 6 in. in width with minor modifications. Thicker rail bars can also be used without creating any difficult power collection problems. The use of larger copper bars does not necessarily imply an increase in the outline dimensions of the power-rail structure. It is assumed that larger bars would be fabricated in a form which would provide mechanical strength and stiffness and retain a favorable surface-area-to-volume ratio. Such designs would also

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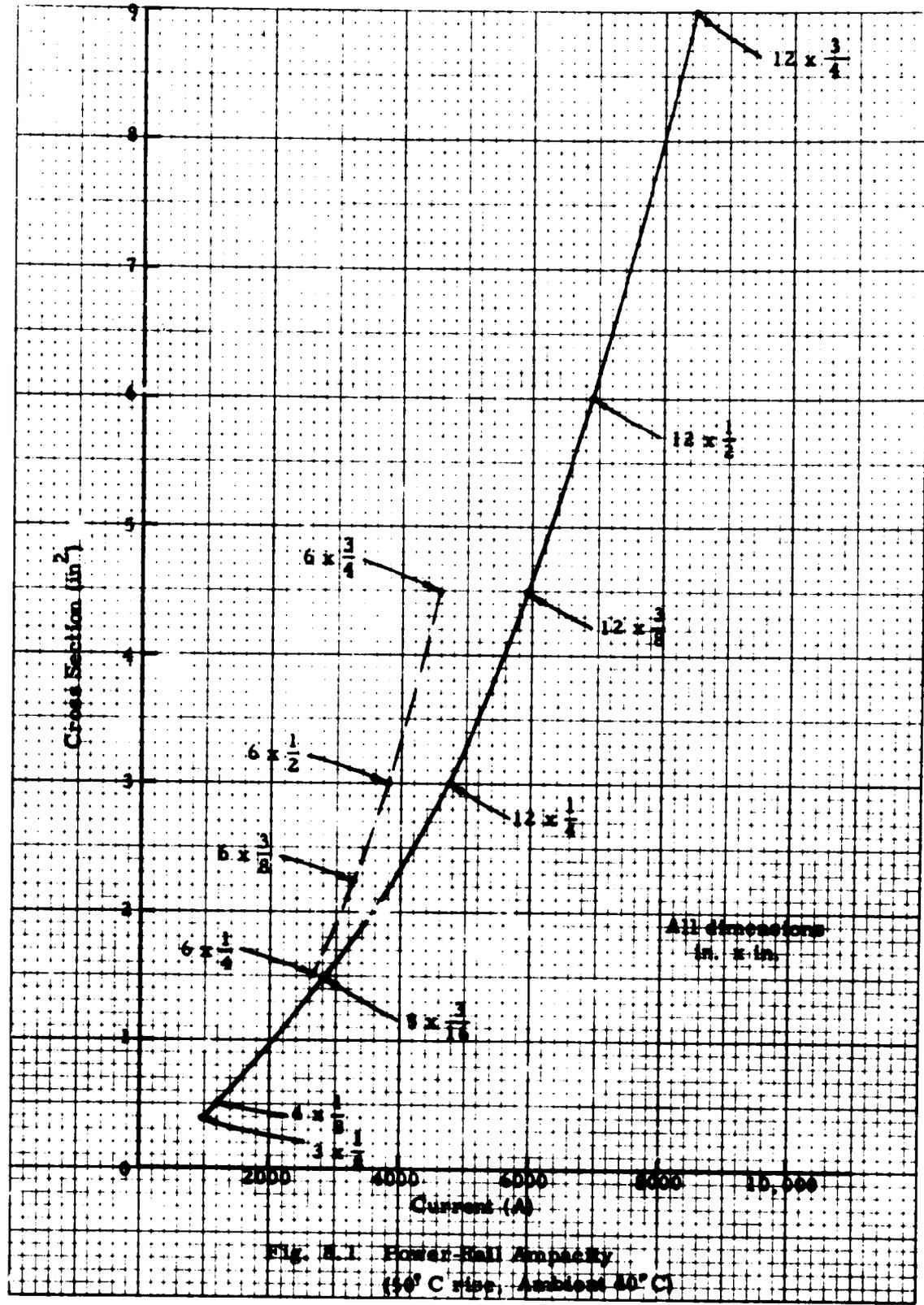


Fig. H-1 Power-Full Ampacity
(50° C rise, Ambient 30° C)

eliminate the need of the supporting sheet steel structure to connect the rails to the post insulators. See Fig. 3.1.

The ampacity of copper rail bars is shown in Fig. 8.1. A gradual transition from 3-in. width bars to 12-in. width bars is used as the current increases. The dimensions of the bars are indicated along the length of the curve. The ampacity of 6-in. width bars is also shown for comparison. Above 3000 A it is evident that significant savings in copper are available by using wider bars.

The use of parallel booster cables to reduce the resistance of the power rail is not economical compared to adding power-rail cross section. (Note, that for the ac system, parallel cables are the only means of reducing the power-rail reactance).

8.2 Power-Rail Component Costs

Table 8.1 shows a cost breakdown of the dc power rail. Most of the figures are derived from the costs given in Table 3.1 for the ac power rail. Cost is assumed to vary only with the power-rail copper cross section. Price for copper is taken at \$1.05/lb (1973 prices). A reduction in costs at lower operating voltages does not appear to be justified. Reduced insulation requirements at the lower voltages may permit some reduction in the overall size of the power-rail assembly. Potential cost savings, however,

Table 8.1

Costs for DC Power Rail

(Basic and Low Resistance Designs)

<u>Item (per vehicle direction)</u>	<u>Per-Mile Cost*</u>
Power Rail ² (2 rails each A in. in cross section)	\$32,000 + 41,000 A
Support and Insulators	27,000
Protective Cover	12,000
Insulator Bracket Hardware	16,000
Protective Cover Hardware	1,000
Installation Labor	60,000
	<hr/>
Subtotal	\$148,000 + 41,000 A
x 2 rail directions	\$296,000 + 82,000 A

*Based on 1973 prices.

may be more than offset by the increased structural strength required to withstand the higher short-circuit current forces in the lower-voltage systems.

8.3 Power-Rail Ampacity

Evaluation of the dc power-rail ampacity in terms of the ratio S/H of the substation spacing (S) to the vehicle headway (H) is based upon material presented in Art. 3.3

Two power-rail designs are presented. The basic design of smaller copper cross section will have by design an ampacity of $S/H = 1$, or at most one vehicle between the substations. A low-resistance design, with a greater copper cross section will have an ampacity of $S/H = 2$, or at most two vehicles between substations. Note that power-rail ampacity is not strictly a parameter in this study. Because of the strong dependence between ampacity and resistance in a dc power rail (unlike the ac power rail), it is possible to select ampacity and still achieve the desired resistance characteristics. Hence, "high ampacity" and "low resistance" are synonymous terms.

The maximum power-rail currents for traffic conditions defined by $S/H = 1$ and $S/H = 2$ are given in Table 8.2 for the various vehicle configurations under consideration. The maximum currents in per-unit vehicle current, as a function of S/H are found from Fig. 3.2. (Fig. 3.2 is

Table 8.2

DC Power Rail Characteristics

<u>Vehicle Voltage kV dc</u>	<u>Maximum Current* (A)</u>	<u>Power Rail Cross Section (in²)</u>	<u>Resistance Ω/mi</u>	<u>Cost** (\$/mi)</u>
Basic Design: (Ampacity Limit S/H=1)				
1.5	7500	7.0	0.0136	870,000
3	3750	2.2	0.0433	476,000
6	1875	0.90	0.106	370,000
9	1250	0.55	0.173	341,000
12	938	0.38	0.251	327,000
Low Resistance Design (Ampacity Limit S/H=2)				
1.5	12,250	17	0.00561	1,690,000
3	6113	4.7	0.0203	681,000
6	3056	1.7	0.0561	435,000
9	2038	1.0	0.0953	378,000
12	1529	0.70	0.136	350,000

*By design these values also define the "power rail ampacity."

** Based on 1973 prices.

applicable to ac and dc systems, provided the substations are at the maximum spacing for a vehicle voltage drop of 25%). For $S/H = 1$ the load currents are 1 pu vehicle current, and for $S/H = 2$, 1.63 pu vehicle current.

The dc power rail characteristics are summarized in Table 8.2. The required power-rail cross section to carry the currents is taken from Fig. 8.1, and the per-mile costs calculated from Table 8.1. The resistance is calculated from $\zeta = 0.75 \times 10^{-6} \Omega - \text{in.}$

9.0 DC Substation Design and Cost

The design of dc substations hinges upon the availability of dc circuit breakers. Below 3 kV dc, circuit breakers are available and conventional traction (diode) rectifier substation designs are used. Above 3 kV dc, we assume that protection is provided by gate control of thyristor rectifiers as in industrial practice, backed up by the ac breaker. Although thyristors have not been used in wayside power systems, they have been used on electric locomotives.

Substations with and without a regulated dc bus are considered only below 3 kV dc. Above 3 kV dc the thyristor rectifier is inherently capable of providing regulation by phase control.

A banked-secondary distribution system is assumed for all voltage levels; each substation consisting of two independent half-substations. Below 3 kV dc, a half-substation only feeds the power rails in one direction of travel. Because dc circuit breakers are available, rail sections can be easily isolated under fault conditions. Above 3 kV dc, each half-substation feeds the power rails for both directions of travel in one rail section only.

9.1 Substation Voltage Regulation

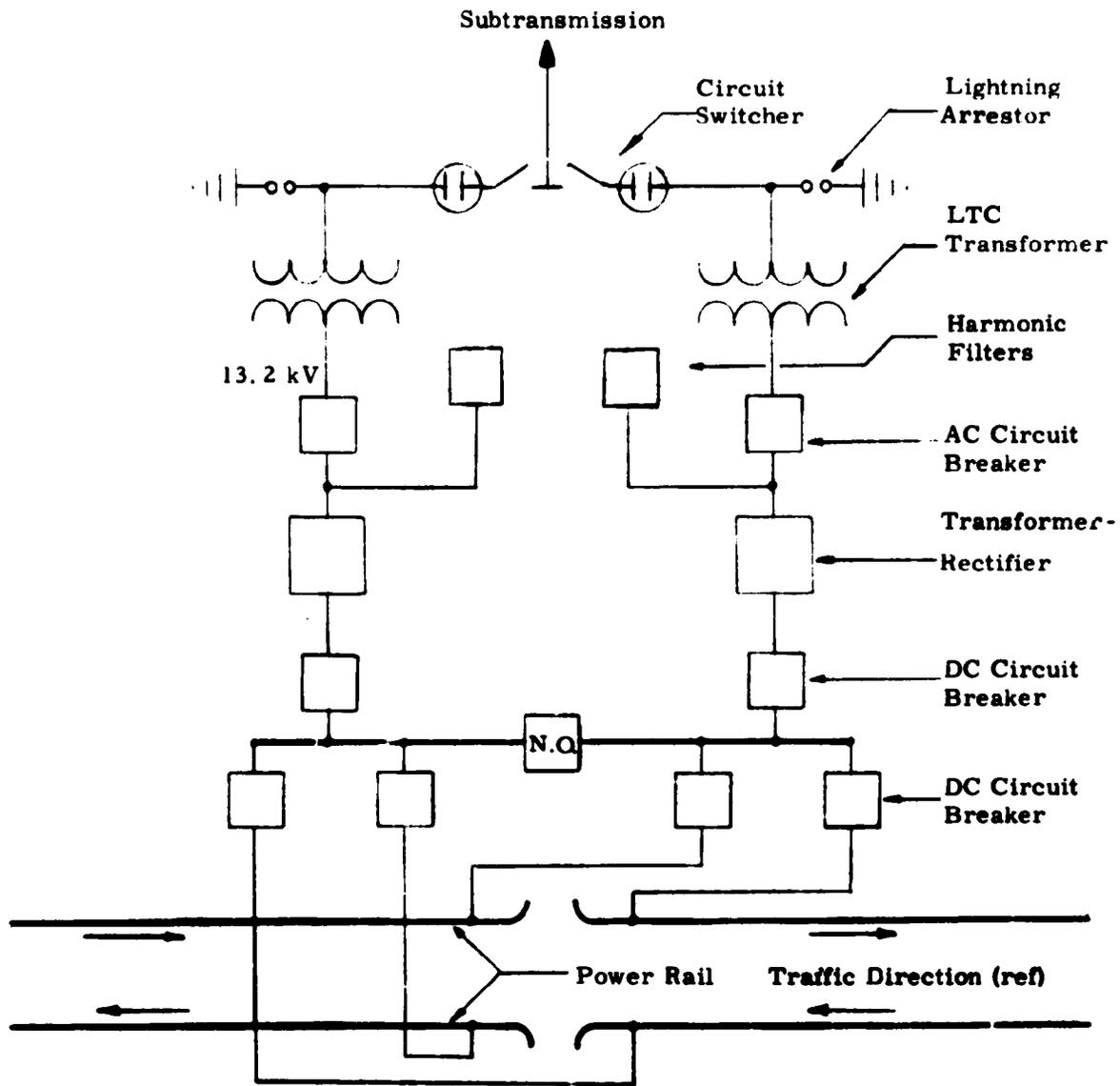
Substation designs providing regulation of the dc bus voltage against frequent

load changes are termed completely regulated substations. Below 3 kV dc, regulation will be accomplished with load tap changers (LTC). Like the ac system, static tap changers or vacuum-switch assisted tap changers are considered. (The characteristics of these components have been discussed in Art. 4.1). Above 3 kV dc, regulation is accomplished by phase control of the thyristor rectifiers.

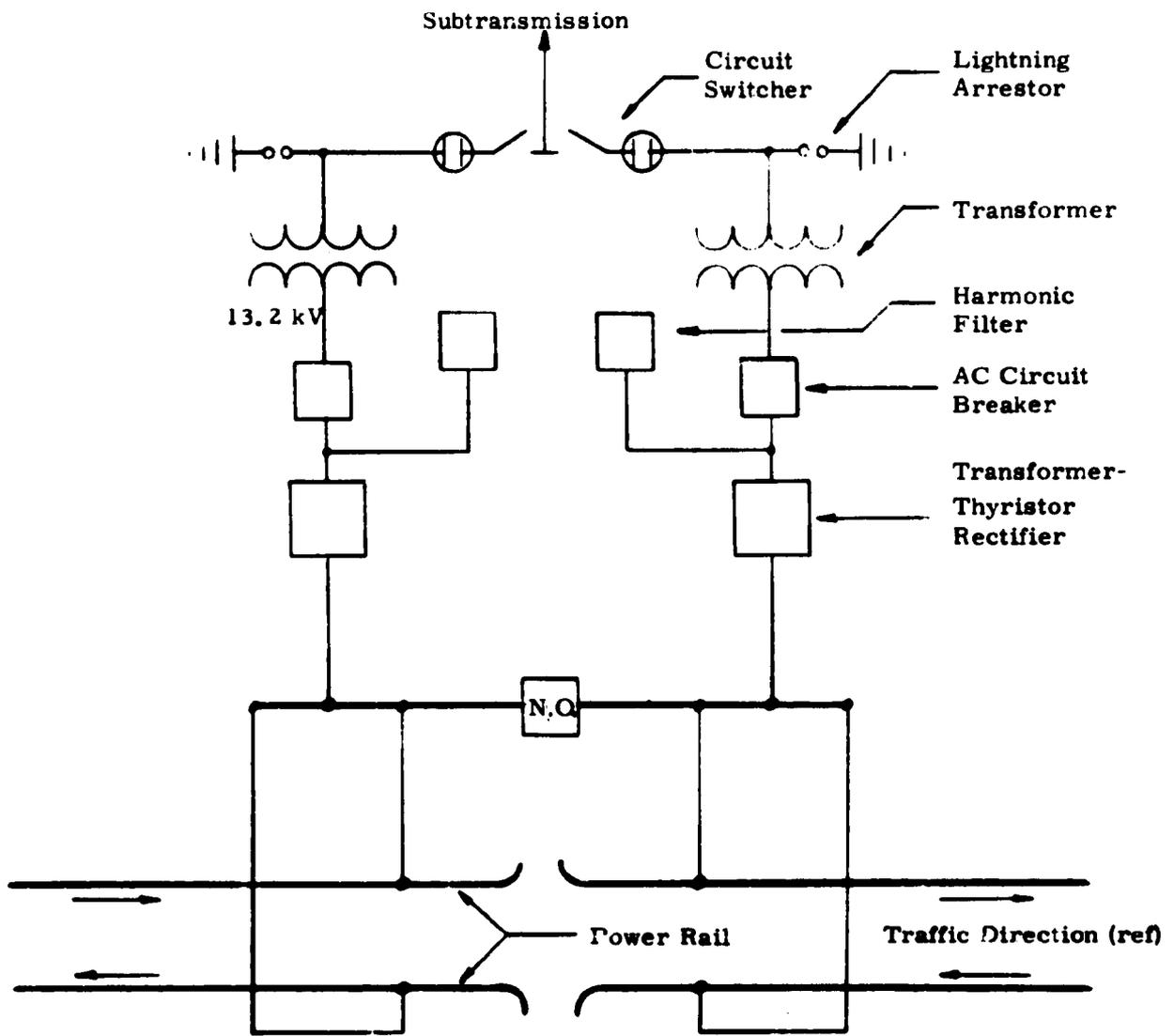
Complete regulation of the substation voltage can be avoided in substations below 3 kV dc if the voltage drop in the rectifier sets can be tolerated. The voltage characteristics of rectifiers will be presented in Chapter 9. Substations which do not regulate against the effects of frequent load changes will include a conventional slow-response tap changer to compensate for variations in the ac supply voltage during the day.

9.2 Substation Components and Costs

One-line diagrams for dc substations below and above 3 kV dc are shown in Figs. 9.1 and 9.2 respectively. Note in Fig. 9.1 that each half-substation only feeds traffic in one direction of travel. In the event of a fault, rail sections will be isolated by the dc switchgear. In Fig. 9.2, each half-substation feeds traffic in both directions of travel, but only in one section. The rail sections are inherently isolated; in the event of a fault, traffic in both directions is interrupted.



**Fig. 9.1 Banked-Secondary DC Substation
(Below 3 kV dc)
Half-Substation Feeding Power Rail
In One Direction of Travel**



**Fig. 9.2 Banked-Secondary DC Substation
(Above 3 kV dc)
Half-Substation Feeding Power Rail
In Two Directions of Travel**

The substation feed arrangement shown in Fig. 9.1 can also be used above 3 kV dc if each half-substation contains two independent thyristor rectifiers. Each rectifier feeds one of the power rails extending in each direction from the substation for a single direction of travel. This approach will not be pursued as the costs are substantially the same as the design shown in Fig. 9.2.

Equipment cost estimates for substations below and above 3 kV dc are given in Tables 9.1 and 9.2 respectively. A nominal rating of 20 MW (10 MW per half-substation) is assumed. The components and costs (1973) for the two types of substations are discussed in the following.

9.2.1 LTC Transformer

The transformers are rated 55°C temperature rise, 7.5% impedance self-cooled, under load tap changing. Basic list price for transformers over 10 MVA including the standard LTC equipment adder costs is \$58,000 + \$3200 x MVA. The power factor of diode rectifiers is about 0.95 and the rating of the transformer for a 10-MW rating per half-substation is $10/0.95 = 10.5$ MVA. The cost is \$92,000. Basic list price for transformers over 10 MVA without LTC equipment is \$19,000 + \$2750 x MVA. The average power factor for the thyristor rectifier is estimated at 0.8, and the rating of the transformer for a 10-MW rating per half-substation is $10/0.8 = 12.5$ MVA. The cost is \$53,000.

Table 9.1

Cost Estimate for DC Substation Below 3 kV dc

(10 MW per half-substation)

<u>Item</u>	<u>Unit Cost*</u>	<u>No. of Units</u>	<u>Total Cost*</u>
LTC Transformer	\$ 92,000	2	\$184,000
Increment for Vacuum LTC	22,000	2	44,000
Increment for Static LTC	140,000	2	280,000
Diode Rectifier	300,000	2	600,000
AC Switchgear	45,000	2	90,000
DC Switchgear	13,000	7	91,000
Harmonic Filter	15,000	2	30,000
Rail Connections (per cable)	3,000	8	24,000

Totals Completely Regulated Substation**

(a) with Static LTC - 1,620,000

(b) with Vacuum LTC - 1,330,000

Total - Supply-Regulated Substation - \$1,270,000**

*Based on 1973 prices

**includes 25% for installation.

9.2.2 Increment for Static LTC

The average cost estimate for a static LTC is \$13,000 per MVA. The static LTC should be rated for traction service.*

9.2.3 Increment for Vacuum LTC

Initial costs plus the replacement of the vacuum interrupters once per year is estimated at \$22,000. A discussion of the costs is given in Art. 4.2.

9.2.4 AC Switchgear

The cost of ac switchgear is the same for both substation designs. Ratings are discussed in Art. 4.1 for ac substations.

9.2.5 Rectifiers

The basic cost for a diode rectifier rated for traction service* is \$30,000 per MW, including the transformer for 12-pulse operation. Estimates for thyristor rectifiers, including controls and transformers for 12-pulse operation are \$42,000 per MW. The cost difference is not as great as might be expected. Although thyristors may cost 2-to-3 times as much as diodes, diode costs represents only about 10% of the total costs.

* (Traction ratings are given in Art 4.1)

Moreover, diodes must have a 5-cycle surge rating (clearing time of the dc circuit breakers) whereas the thyristors require only a 1-cycle surge rating.

9.2.6 DC Switchgear

Available only for substations below 3 kV dc. Cost per unit in the 1.5- to- 3 kV dc range is \$13,000.

9.2.7 Harmonic Filters

For traction applications, reactors on the dc side of the rectifiers are not required. Harmonic filters will probably not be required on the ac side unless the per-unit supply system impedance is relatively high viewed on the base of the substation rating. Costs for 1 MVAR of filtering are included for substations below 3 kV dc (diode rectifiers) and 2 MVAR of filtering for substations above 3 kV dc (thyristor rectifiers), at \$15,000/MVAR. Costs could be significantly higher if communication interference is a problem.

9.3 Substation Rating and Cost

Substation costs at different MW ratings are calculated from the figures in Tables 9.1 and 9.2, assuming a 15% fixed cost with the remainder proportional to the MW rating. On this basis the costs are given by:

1. Supply-regulated substation (below 3 kV dc), \$191,000 +
\$54,000 x MW
2. Completely regulated substation (below 3 kV dc) with vacuum
LTC, \$200,000 + \$57,000 x MW
3. Completely regulated substation (below 3 kV dc) with static
LTC, \$243,000 + \$69,000 x MW
4. Completely regulated substation (above 3 kV dc), \$210,000 +
\$60,000 x MW.

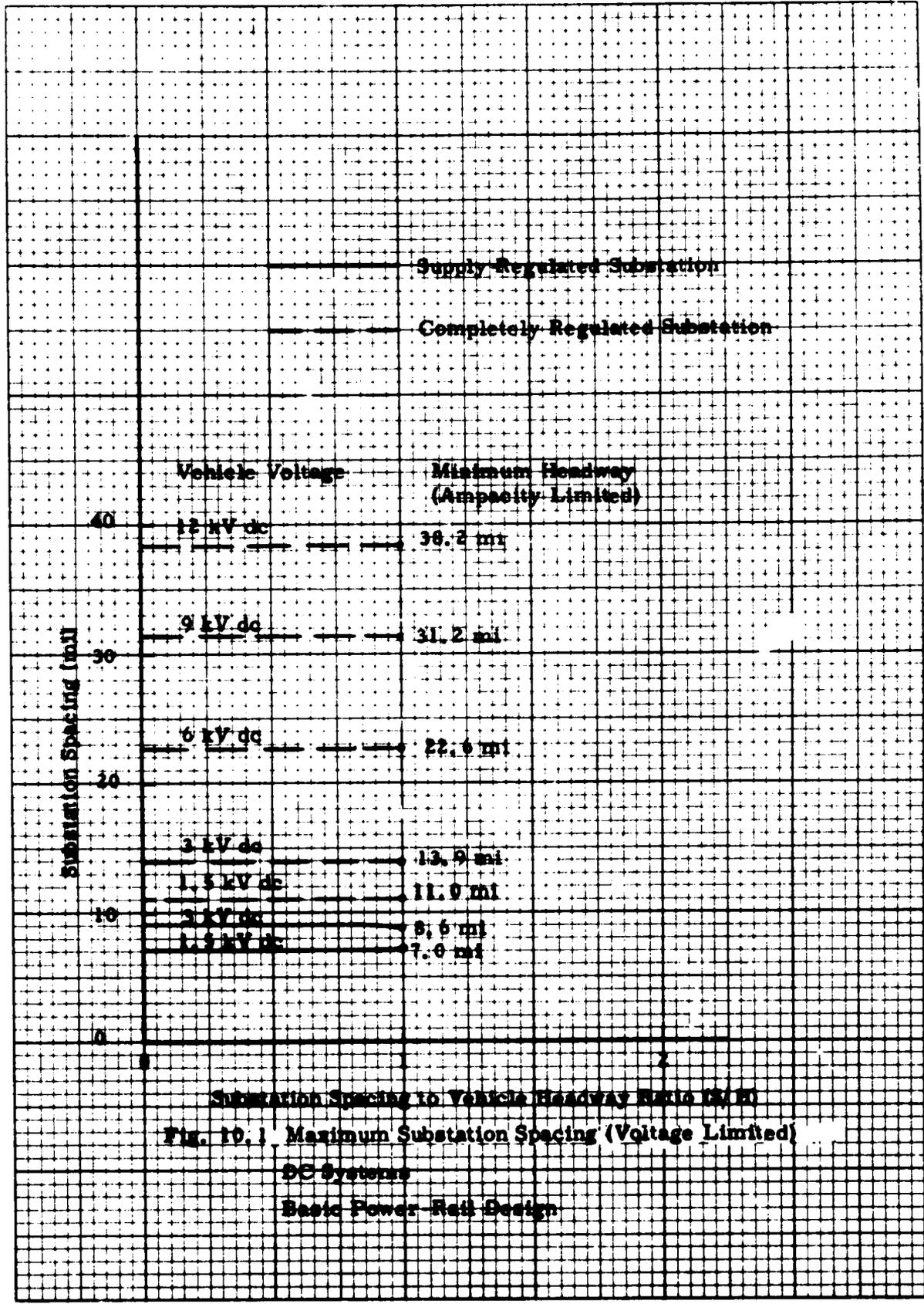
10.0 DC Substation Spacing

The calculation of the maximum substation spacing for dc wayside systems is based on the same considerations presented in Chapter 5 for ac systems. The treatment in Arts. 5.1 and 5.2 of vehicle voltage profiles and methods of calculation is completely general and applicable to both dc and ac systems.

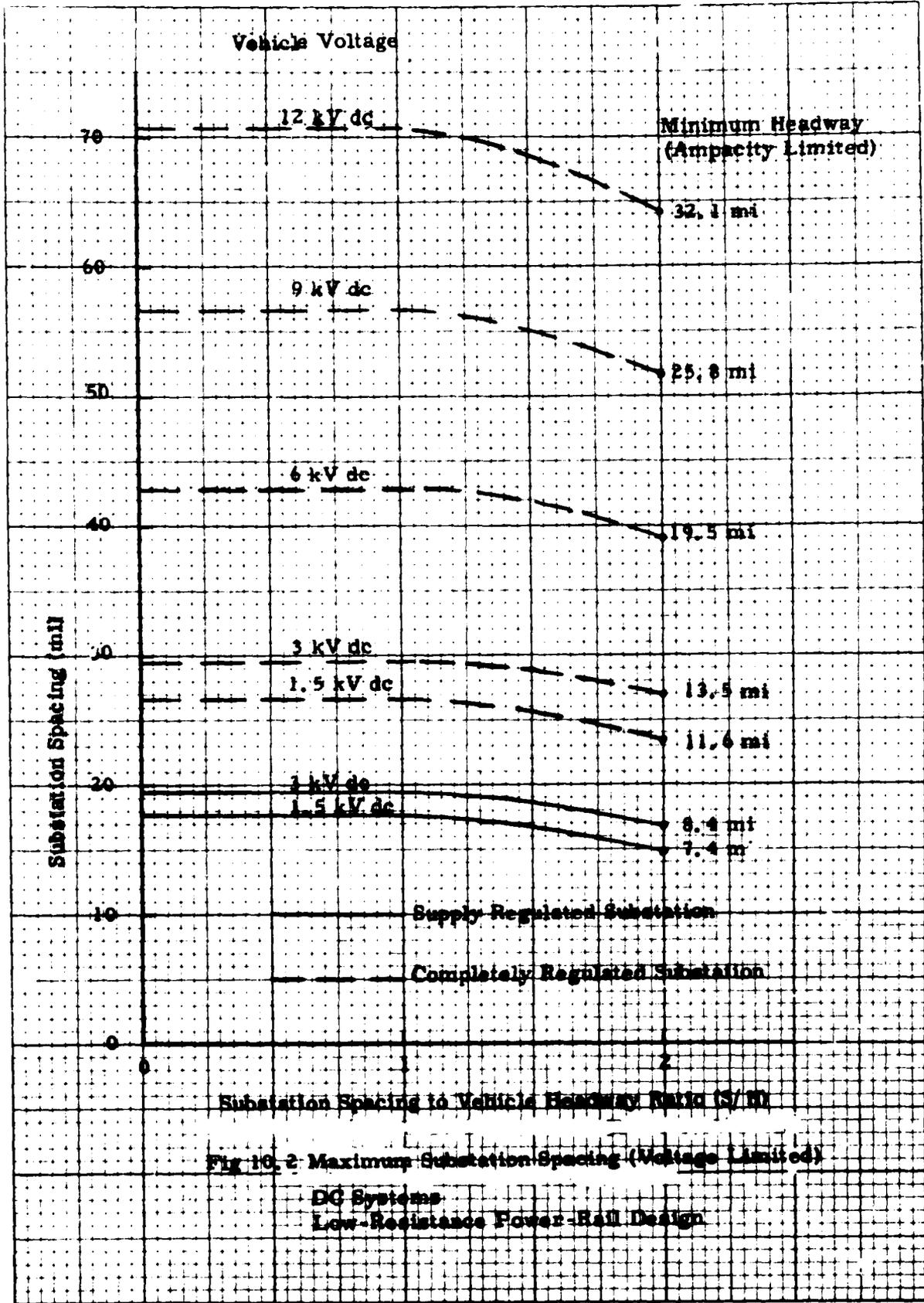
The maximum substation spacings for the basic and low-resistance power rail designs described in Chapter 8 are calculated. Both completely and supply-regulated substations are considered for dc voltages below 3 kV dc. Above 3 kV dc, only completely regulated substations are considered. Hence the change in the substation loading patterns due to the power-rail feed for substations above 3 kV dc (Fig. 9.2) will not affect substation voltage or spacing.

10.1 Maximum Substation Spacing (Voltage Limited) - Completely Regulated Substations

The maximum substation spacings for a voltage drop of 25% at the vehicle are given in Figs. 10.1 and 10.2 for the basic and low-resistance power rails.



Substation Spacing to Vehicle Headway Ratio (S/H)
Fig. 10.1 Maximum Substation Spacing (Voltage Limited)
 DC Systems
 Basic Power-Rail Design



The right hand cut-off point of the curves indicates the limit for multiple vehicle loads, with the maximum values of S/H and the minimum permissible headways. For greater values of S/H , the power-rail current exceeds its ampacity.

10.2 Maximum Substation Spacing (Voltage Limited) - Supply-Regulated Substations

The maximum spacings with supply-regulated substations is less than for completely regulated substations because of the voltage drop in the diode rectifier sets. To protect the rectifier under fault conditions, the impedance behind the rectifier must be high. A source impedance of 16% on the base of the rectifier transformer has been used, producing a voltage drop of about 8% at the rated rectifier load.

Since the voltage drop in the rectifier depends on its rating, the rating must be known in order to calculate the substation spacing. Substation rating curves were presented in Chapter 6 for ac half-substations feeding the power rails in one direction of traffic. They are also applicable for the dc sub-station designs below 3 kV dc. (Supply regulation above 3 kV dc is not considered).

The maximum spacings for 1.5 kV dc and 3 kV dc substations with the basic and low-resistance power-rail designs are given in Figs. 10.1 and 10.2 (solid-line curves). In comparison with the supply-regulated ac substations,

Figs. 5.2 and 5.3, the reduction in spacing from the completely regulated dc substations is greater because of the higher voltage drop of the dc substations.

10.3 Substation Spacing - (Power-Rail Ampacity Limited)

The procedure for determining the appropriate substation spacing for vehicle headways less than the minima tabulated in Figs. 10.1 and 10.2 is the same as described in Art. 5.5 for ac systems.

11.0 DC Substation Rating

Substation loading is determined by the vehicle traffic patterns. For substations below 3 kV dc, a half-substation only feeds the power rail in one direction of travel and the loading is completely specified by the ratio S/H . The results of Arts. 6.1 and 6.2 are therefore applicable. When maximum substation spacing is used as a design criteria and the same vehicle voltage drop of 25 % is specified, the results expressed in per-unit values are valid for both ac and dc systems. Above 3 kV dc, the power-rail feed arrangement is changed and new load patterns must be derived.

11.1 Substation Load Patterns

The load patterns for a half-substation above 3 kV dc feeding traffic in both directions of travel in a single rail section are easily determined from the load patterns of Fig 11.1 for a single direction of travel. Note that for $S/H \leq 1$, the shape of the load curves are identical to those in Fig. 6.1 for $S/H \leq 0.5$ since in each of these cases there is at most only one vehicle which loads the substation at a given time. The pattern for $S/H = 2$ was determined by direct calculation using the procedures presented in Appendix A.

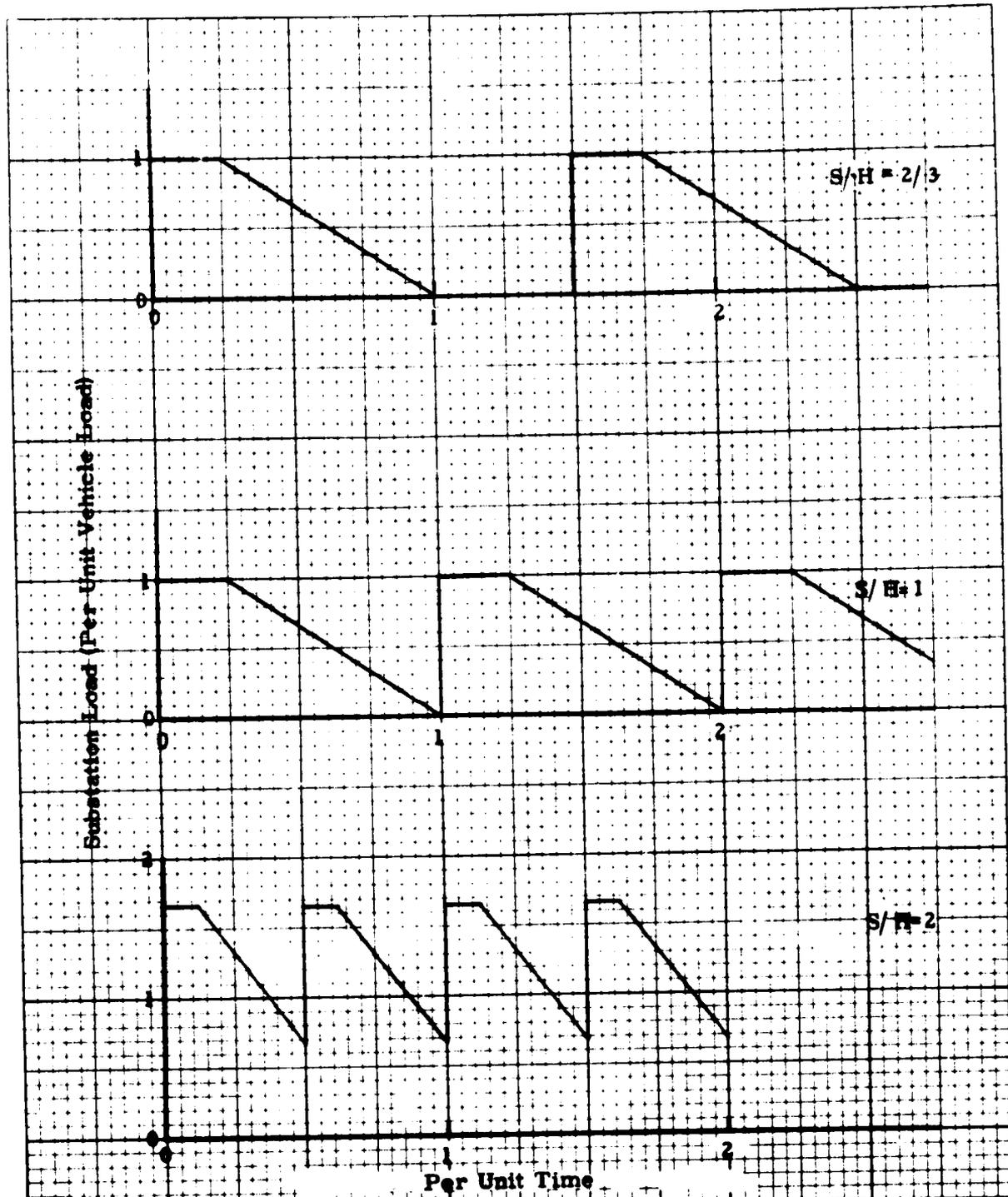


Fig. 11.1 Substation Loading* - One Direction of Travel
(For Substations Feeding One Rail Section)

S - Substation Spacing
H - Vehicle Headway

*Maximum Substation Spacing Voltage Limited

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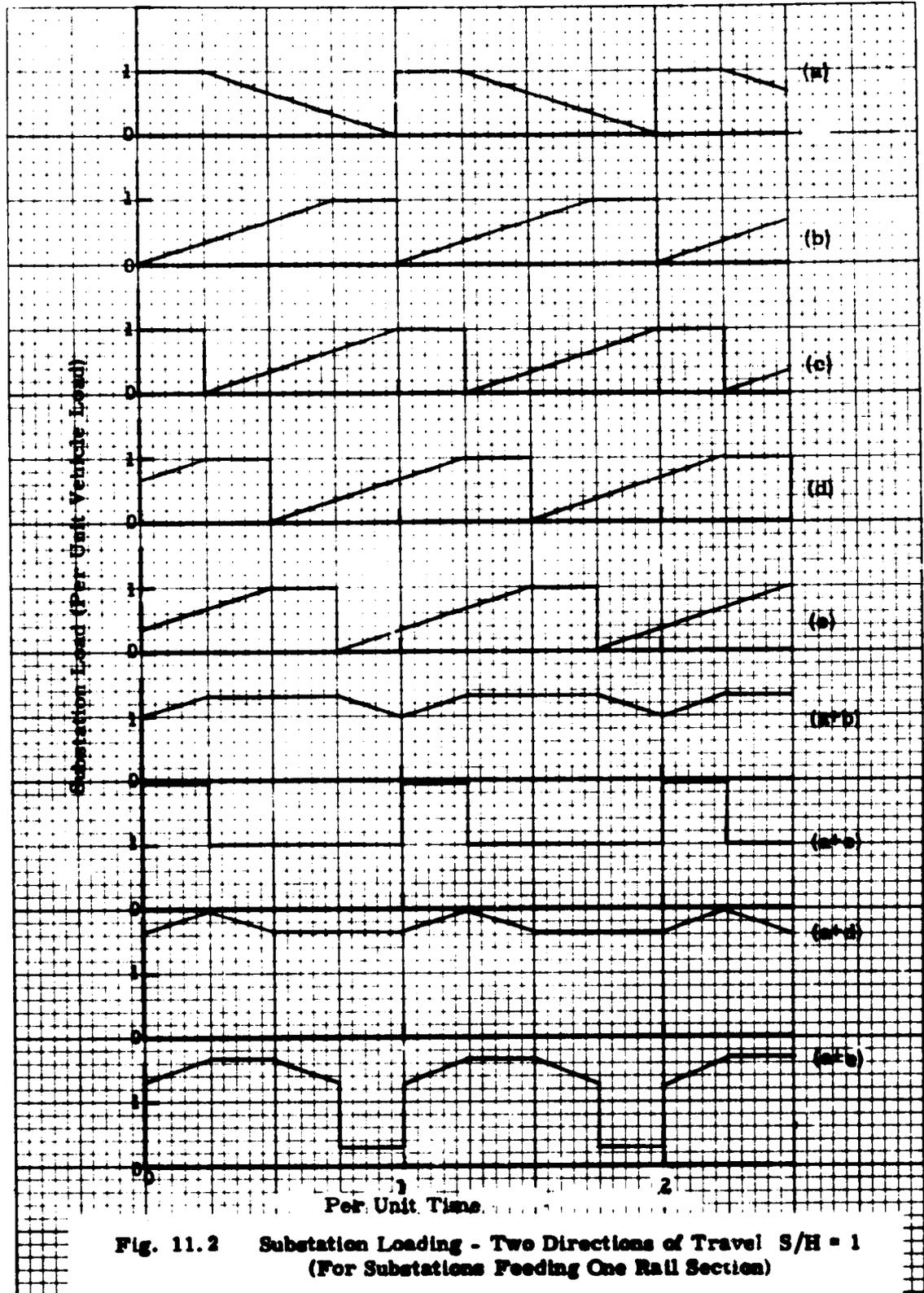


Fig. 11.2 Substation Loading - Two Directions of Travel $S/H = 1$
(For Substations Feeding One Rail Section)

The total substation load for two directions of travel is then obtained by adding the loads for each direction of travel. Since the phase relationship between vehicles in each direction of travel is not known, an infinite family of loading patterns is possible. Upper and lower bounds can be established, however, by judicious selection of the phasing. The process is illustrated in Fig. 11.2 for $S/H = 1$. Curve "a" represents the loading for one direction of travel and curves "b," "c," "d," "e" represent the loading in the opposite direction of travel with various degrees of phase shift between the vehicles. The total load patterns are represented by the curves "a + b," "a + c," "a + d," and "a + e."

The loading curves presented in Figs. 11.1 and 11.2 assume the maximum substation spacing for a vehicle voltage drop of 25%. As before, it is the maximum substation spacing for a given voltage drop that provides the normalization for the per-unit representation of substation loading.

The load patterns are independent of the vehicle and power rail characteristics and depend only on the value of S/H .

11.2 Peak and RMS Loads

The rms and peak loading curves for substations over 3 kV dc are given in Fig 11.3 as a function of S/H . These curves were derived from loading patterns similar to those shown in Fig. 11.2. From Fig 11.2 it is apparent

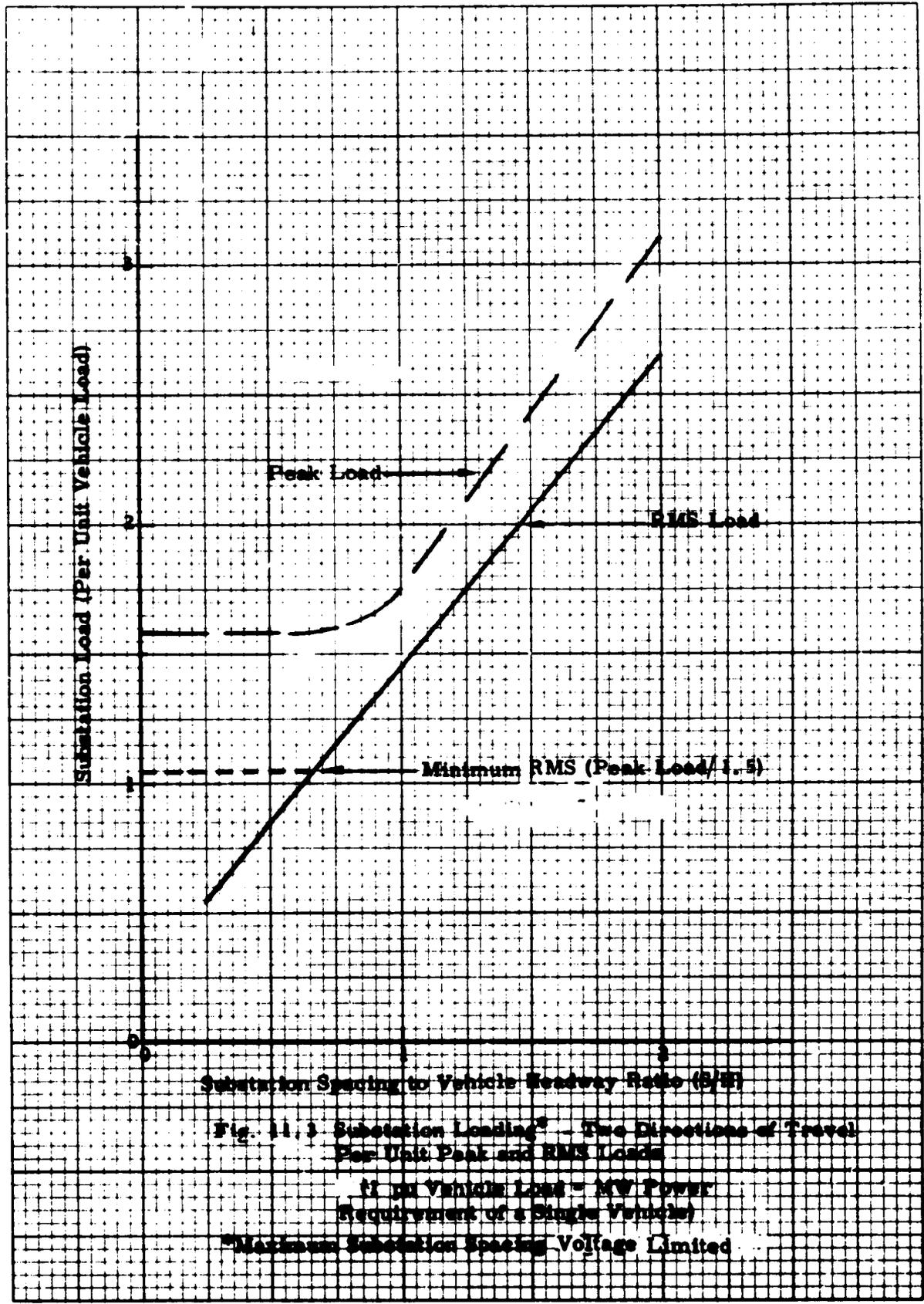


Fig. 11.3 Substation Loading* - Two Directions of Travel
Per Unit Peak and RMS Loads

(1) per Vehicle Load - MW Power Requirement of a Single Vehicle

*Maximum Substation Spacing Voltage Limited

that a range of peak and rms values can be obtained as a function of the traffic conditions for a given value of S/H. The curves in Fig 11.3 represent averages over a number of traffic conditions. Note from Fig. 11.2, that peak loads of 2 pu can be experienced for $S/H \leq 1$. The curves of Fig. 11.3 will be used to determine the MW rating of substations above 3 kV dc when substation spacing is voltage limited.

11.3 Rectifier Rating

DC substation ratings may be based on rms-thermal loading, voltage drop, or overload.

Rectifiers rated for traction service are designed to handle large overloads for short periods of time. (See Art. 4.1) Because peak loading is persistent and repetitive, a maximum value of 1.5 for the ratio of peak to rms-thermal loading is established in this study for rating of dc substations. This applies to both diode and thyristor-rectifier substations. When the rms-thermal rating of the substation is governed by the peak loading/ 1.5, the substation is said to be overload limited.

The voltage drop in a diode-rectifier set is due to commutation in the source impedance behind the rectifier bridge. For a full-wave 3-phase rectifier, the voltage drop is given by (Standard Handbook for Electrical Engineers, 1969, McGraw-Hill, Section 12)

$$\Delta V_d = \frac{3 X_c}{\pi} I_d$$

where

X_c = the source or commutation reactance (Ω).

I_d = direct current (A).

$\frac{3X_c}{\pi}$ = apparent resistance of the rectifier.

Assuming that X_c is 16% on the rectifier transformer base, then

$$X_c = \left[\frac{V_d}{1.35 \cos \phi} \right]^2 \times \frac{\cos \phi}{P} \times 0.16 \Omega$$

where

V_d = rectifier output voltage, kV dc

$\cos \phi$ = rectifier displacement power factor = 0.95

P = rectifier rating in MW

These expressions will be used to evaluate the effect of rectifier voltage drop on substation spacing. Typically, the percent voltage drop at the rated load current is about one-half X_c , or in this case, 8%. Note that these expressions also describe the inherent voltage drop in a thyristor rectifier.

11.4 Rating - Completely Regulated Substations

The dc-bus voltage is held constant so that the effects of rectifier voltage drop are cancelled. Assuming that the rectifier set is rms-thermal rated at no less than the peak load/ 1.5, then the inherent voltage drop due to commutation is limited to about $1.5 \times 8\% = 12\%$ at peak load. If the ac-supply regulation does not exceed a band of 8% a regulation capability of about 20% is required. Substations are therefore rms-thermal rated until the constraint of the overload limit is reached.

For substations above 3 kV dc, Fig. 11.3 must be used to determine the substation rating. A minimum rating of 1.05 pu of vehicle load is applied for $S/H \leq 0.65$ (overload limit). Note that this is based upon average rms and peak loads over the various traffic patterns possible. Peak loads of 2.0 pu are possible for $S/H \leq 1$ so that the actual peak-to-rms-thermal ratio would exceed 1.5. However, these peak loads generally occur when vehicles cross near a substation, so that the deviation from complete regulation will probably not produce below-tolerance voltages at the vehicles.

For substations below 3 kV dc, Fig. 6.3 can be used to determine the substation rating. Applying the same constraints as above, a minimum rating of 0.7 pu of vehicle load is found for $S/H \leq 0.5$ (overload limit).

11.5 Rating - Supply-Regulated Substations

Supply regulation is only considered for substations below 3 kV dc.

Therefore, Fig. 6.3 can be used to determine the substation rating. For $S/H > 0.5$ rms-thermal ratings are used, and for $S/H < 0.5$ a minimum rating of 0.7 pu of vehicle load is applied (overload limit).

For $S/H < 0.5$, the peak substation load is 1.0 pu of vehicle load. The worst-case voltage drop is therefore $(1.0/0.7) \times 8\% = 11.5\%$. However, as noted before (Art. 6.5), maximum substation loading does not coincide with a vehicle at the minimum voltage position on the track when $S/H < 0.8$. Thus, the effect of substation voltage drop on the maximum substation spacing (voltage limited) is not as much as might be anticipated by the worst case regulation.

Supply-regulated substations are also provided with load tap changers but these are slow-response units designed only to compensate for changes in the ac supply voltage over the day. The comments in Art. 6.5 concerning the use of this type of regulator to compensate for long-term load levels are also applicable here.

11.6 Rating - Substation Spacing Power Rail Ampacity Limited

The procedure for determining substation rating when vehicle headways are less than the minima tabulated in Figs. 10.1 and 10.2 is based on the same considerations discussed in Art. 6.6 for ac systems.

12.0 DC Wayside Power-System Costs

The total wayside power-system per-mile cost is the sum of the power rail and the substation per-mile costs. Results are expressed as a function of vehicle headway with vehicle voltage, substation voltage drop, and power-rail resistance, as parameters. Cost figures are for dual-rail systems (two directions of travel).

12.1 Power Rail Per-Mile Costs

Power-rail per-mile costs (1973) are presented in Table 8.3. They vary from \$1,690,000/mi at 1.5 kV dc to \$353,000/mi at 12 kV dc for the low-resistance design (maximum capacity of two vehicles per track between substations) and from \$870,000/mi at 1.5 kV dc to \$327,000/mi at 12 kV dc for the basic design (maximum capacity of one vehicle per track between substations). Power-rail costs are assumed constant as a function of vehicle headway.

12.2 Substation Per-Mile Costs

Substation per-mile costs, as a function of vehicle headway, with vehicle voltage, substation regulation, and power-rail resistance as parameters, can be determined from the results of Chapters 9, 10 and 11. For every admissible value of the ratio, S/H , and combination of parameters, (1) substation spacing and vehicle headway values (S, H) can be determined from Figs. 10.1 and 10.2, and (2) the substation rating can be determined from Fig. 6.3 (below 3 kV dc) and Fig. 11.3 (above 3 kV dc).

Substation costs for each rating can then be calculated from the results of Art. 9.3. Per-mile costs are averaged out over a number of substations. The following results assume an integral number of substations with a total stage length in the order of 100 mi.

12.3 Total Per-Mile Costs

Total wayside power-system per-mile costs, as a function of vehicle headway, with vehicle voltage, substation regulation, and power-rail resistance, are shown in Figs. 12.2 and 12.2. Since power-rail costs are constant as a function of headway, these curves also show the relationship between substation per-mile costs and vehicle headway.

Per-mile costs associated with the basic and low-resistance power-rail designs are shown in Figs. 12.1 and 12.2 respectively. Completely regulated substations below 3 kV dc use either static or vacuum LTC - the total costs are about the same. Completely regulated substations above 3 kV dc use thyristor rectifiers. Supply regulation is not considered above 3 kV dc.

From Figs. 12.1 and 12.2 it is evident that there is no economic advantage in reducing the power-rail resistance in order to increase substation spacing. The additional cost of the power rail is not offset by the reduction in per-mile substation costs.

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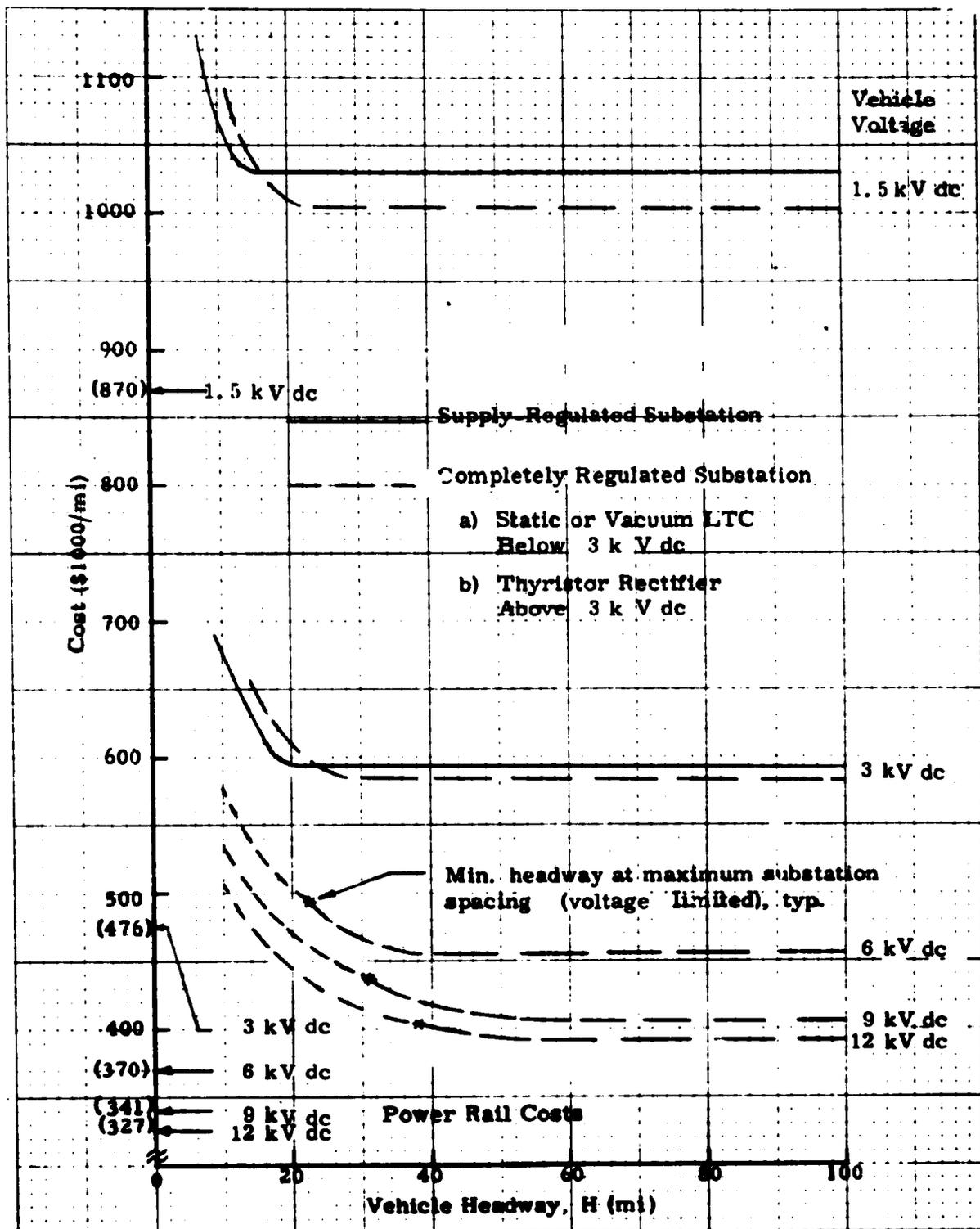


Fig. 12.1 Total DC Wayside Power-System Per-Mile Costs with Supply Regulated Substations and Completely Regulated Substations, Basic Power-Rail Design. Based on 1973 Costs

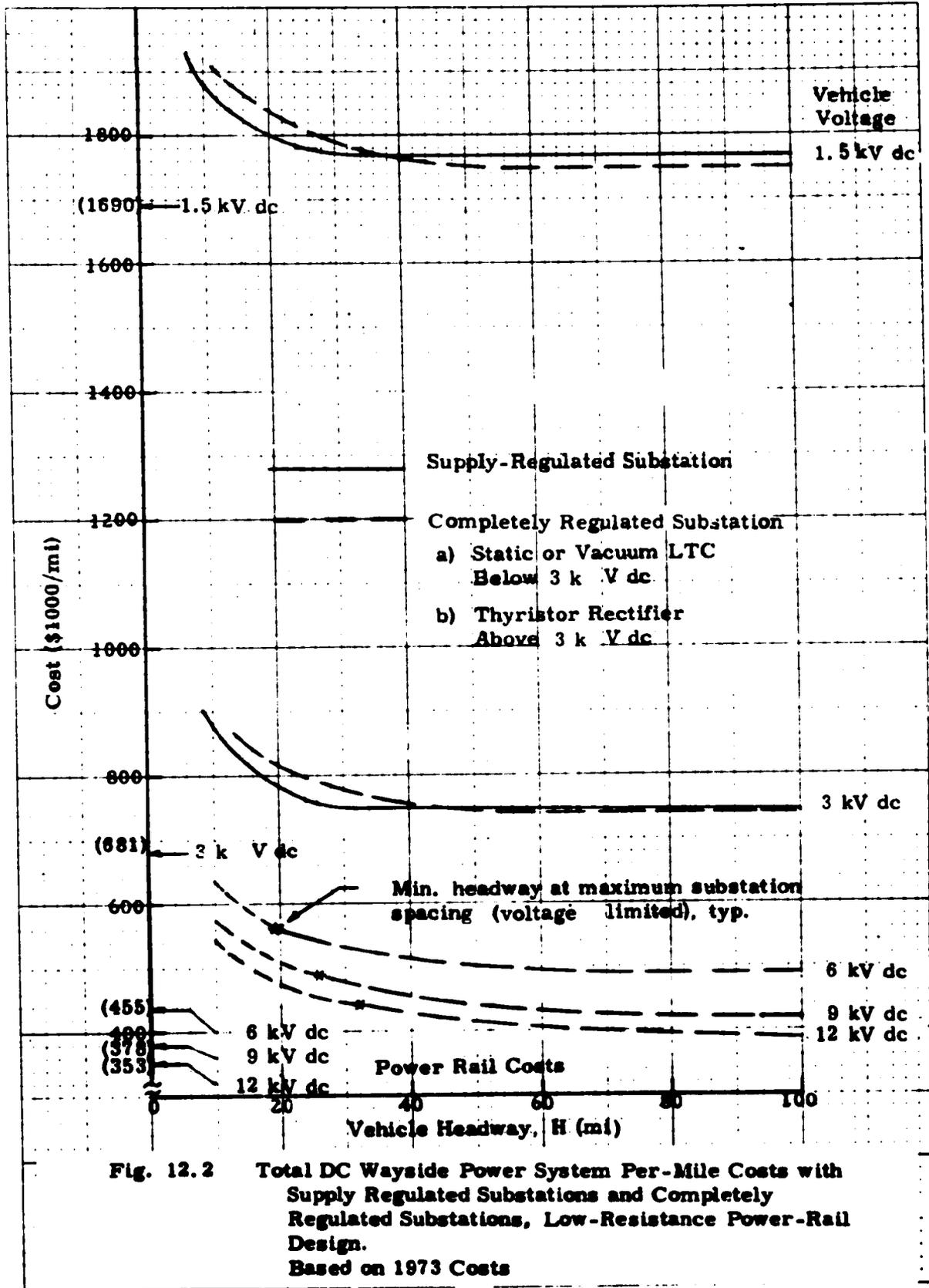


Fig. 12.2 Total DC Wayside Power System Per-Mile Costs with Supply Regulated Substations and Completely Regulated Substations, Low-Resistance Power-Rail Design. Based on 1973 Costs

One benefit of reducing the resistance of the dc power rail is an increase in power-rail ampacity. Although the ampacity of the low-resistance power rail is double that of the basic power rail, the maximum substation spacing set by vehicle voltage-drop requirements is also almost doubled. Consequently, the minimum vehicle headway in miles attainable with the maximum substation spacing is about the same, as noted in Figs. 12.1 and 12.2. For vehicle headways below the indicated minimums, substation spacing must be reduced to keep the load of the vehicles between substations within the ampacity of the power rail (power-rail ampacity limited).

The comparison of dc wayside power-system designs, as a function of vehicle voltage, is thoroughly dominated by the cost of the power rail. At low voltages, the rail conductor cross sections (and costs) to carry the load currents are large. The power requirements (11.25 MW) of the vehicles in this study make it impractical to consider voltages below 6 kV dc.

Appendix A

Calculation of Maximum Substation Spacings

Substation Loading Patterns and Vehicle Voltage Profiles

The problem of calculating the maximum substation spacing is essentially no more than calculating the voltage drop in the power rail between the vehicles and the substations. A general approach is assumed so that substation loading and vehicle voltages may also be calculated from the network equations.

The various cases to be solved are defined by the number of vehicles on a rail section between two substations (banked secondary distribution system) In terms of the notation adopted in this report, the cases are equivalently defined by the ratio S/H of the substation spacings (S) to the vehicle headway (H).

The solution for $S/H = 2$ is derived in detail. Solutions for $S/H \leq 1$ are special cases of this problem. For greater values of S/H , the general approach may be expanded. The presentation here assumes an ac distribution system. The results are easily reduced for dc systems as a special case.

A.1 Vehicle Voltage Profiles

The procedure of solving for the maximum substation spacing assumes that the positions of the vehicles between substations corresponding to the minimum permissible voltage at one of the vehicles is known. When $S/H =$ odd integer, this occurs when the vehicles are symmetrically distributed between substations.

When $S/H = \text{even integer}$, the distribution is non-symmetric. Both cases can be verified by direct calculation using the techniques presented here.

The notation for describing vehicle positions between substations is shown in Fig. A.1 for the case, $S/H = 2$. Distances between vehicles and between vehicles and substations are expressed as a fraction of S through the parameters f_1 , f_2 , and f_3 . Configuration (b) corresponds to a vehicle at the minimum voltage position, and is used to determine the maximum substation spacing, S_{max} . Configurations (a) and (c) are used to obtain additional information on substation loading and vehicle voltage profiles with $S = S_{\text{max}}$.

A.2 Network Equation

The electrical network to be solved corresponding to the vehicle distribution patterns shown in Fig. A.1 is given in Fig. A.2. Referring to Fig. A.2, the following conventions are adopted. All voltages and currents are per phase quantities, and are complex numbers unless otherwise designated by subscript as complex components: r for real, q for complex. For example, the voltage and current at vehicle 1 are given by $V_1 = V_{1r} + j V_{1q}$, and $I_1 = I_{1r} + j I_{1q}$.

The vehicles are constant power devices with a known total power, VA , and power factor angle, θ . Thus for the i -th vehicle

$$3 V_i I_i^* = VA (\cos \theta + j \sin \theta) \quad \text{A.1}$$

where $*$ denotes the complex conjugate.

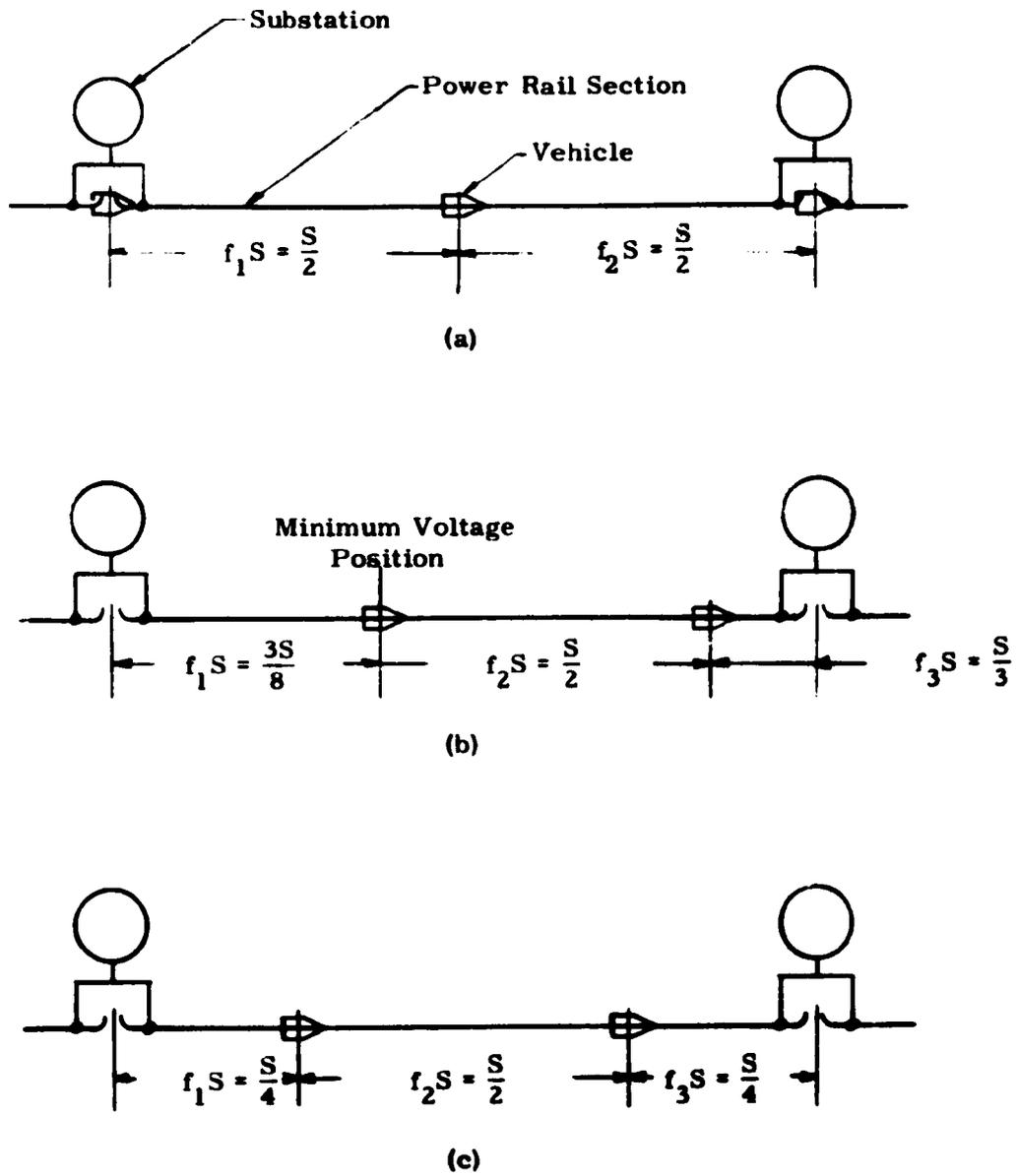
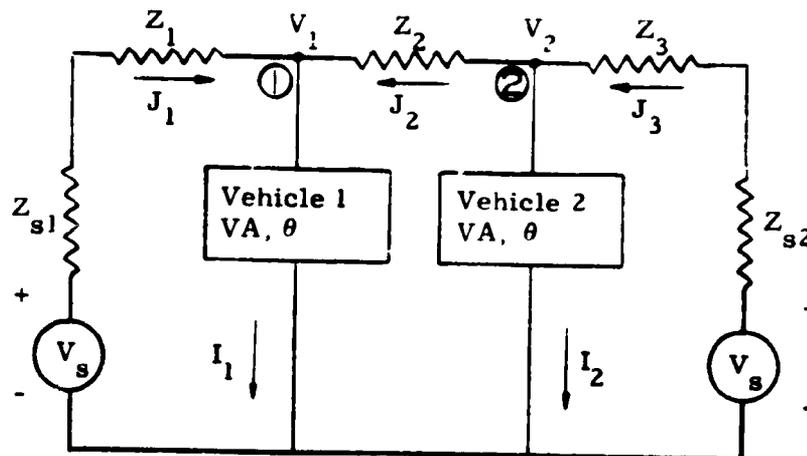


Fig. A.1 Vehicle Distribution Patterns

$S/H = 2$

S = Substation Spacing

H = Vehicle Headway



- V_i, I_i = Voltage and current at Vehicle i
 V_s = Substation voltage
 VA, θ = Vehicle power and power factor angle
 Z_{s1}, Z_{s2} = Substation impedances
 Z' = Per mile power rail impedance
 S = Substation spacing (miles)
 $Z_i = f_i S Z'$ = Impedance of $f_i S$ miles of power rail
 (Fig. A.1)

**Fig. A.2 Electrical Network for Two Vehicles
 Between Substations**

The substation voltages, V_s , are assumed to be in phase. This assumption holds whenever substations feed traffic in one direction only and the adjacent rail sections are not electrically isolated (See Fig.4. 1). The total load seen by each substation, by symmetry, is the same and the phase shifts in the substations are equal. Even when this condition is not strictly met, the error is small. The phase shift in the power rail is large in comparison to the subtransmission lines to the substations.

Node 1, corresponding to vehicle 1, is adopted as a reference point. Then, without loss of generality (and for computational convenience) the complex part of V_1 may be set to zero, i. e., $V_{1q} = 0$.

If the division of the currents at node 1 is known, then the remaining network currents are completely specified. Two parameters, a and b, are introduced to define the current division such that

$$J_1 = (1-a) I_{1r} + j(1-b) I_{1q} \quad \text{A. 2}$$

$$J_2 = a I_{1r} + j b I_{1q} \quad \text{A. 3}$$

where J_1 and J_2 are defined in Fig. A.2.

Then, the voltage at vehicle 2 can be calculated from

$$V_2 = J_2 Z_2 + V_1 \quad \text{A. 4}$$

and the current I_2 calculated from equation A. 1. The remaining network current J_3 can then be expressed as

$$J_3 = a I_{1r} + I_{2r} + j(b I_{1q} + I_{2q}) \quad \text{A. 5}$$

Taking the network loop equation

$$J_1 (Z_1 + Z_{s1}) - J_2 Z_2 - J_3 (Z_3 + Z_{s2}) = 0 \quad \text{A. 6}$$

substituting A. 2, A. 3, and A. 5, and equating the real and complex parts, an equation of the form

$$\underline{A} \begin{bmatrix} a \\ b \end{bmatrix} = \underline{B} \quad \text{A. 7}$$

is obtained where A and B are 2x2 and 2x1 arrays, respectively. The components of these arrays are functions of the vehicle currents and the network impedances.

The currents at vehicle 2, however, are a function of the parameters a and b (Eqs. A. 3 and A. 4). Because the vehicles are constant power devices, the dependence on these parameters is nonlinear as a matrix inversion is required to solve equation A.7. Solutions are therefore obtained by iteration.

The solution to equation A. 7 is also a function of the network impedances which depend on the substation spacing, S. The correct substation spacing is obtained when the equation

$$|V_s| = |V_1 + J_1 (Z_1 + Z_{s1})| \quad \text{A. 8}$$

is satisfied. Since the convention adopted assumes that the complex part of V_1 is defined, $V_{1q} \approx 0$, equation A. 8 may be readily solved for V_1 without ambiguity.

A. 3 Maximum Substation Spacing Algorithm

The solution to the network equations is found by a two level iterative procedure. The flow chart for the algorithm is given in Fig. A. 3. Iterative quantities are denoted by the superscripts, i and j .

Input constants, which define the specific problem to be solved are: The power rail per mile impedance, Z' ; the vehicle power, VA , and power factor angle, θ ; the magnitude of the substation voltage, $|V_s|$; the vehicle distribution parameters, f_1, f_2, f_3 ; and the minimum permissible voltage at a vehicle, V_1 . The parameters $f_1, f_2,$ and f_3 are adjusted so that vehicle 1 is at a minimum voltage point. The algorithm is initialized by estimating the substation spacing S^0 , and the parameters a^0 and b^0 .

Convergence to the solution is obtained by successive substitution of iterates. Assuming current values of a^i, b^i and S^j , iterates in a and b are defined by the solution to equation A. 7,

$$\begin{bmatrix} a^{i+1} \\ b^{i+1} \end{bmatrix} = \underline{A}^{-1} (a^i, b^i, S^j) \underline{B} (a^i, b^i, S^j) \quad \text{A. 9}$$

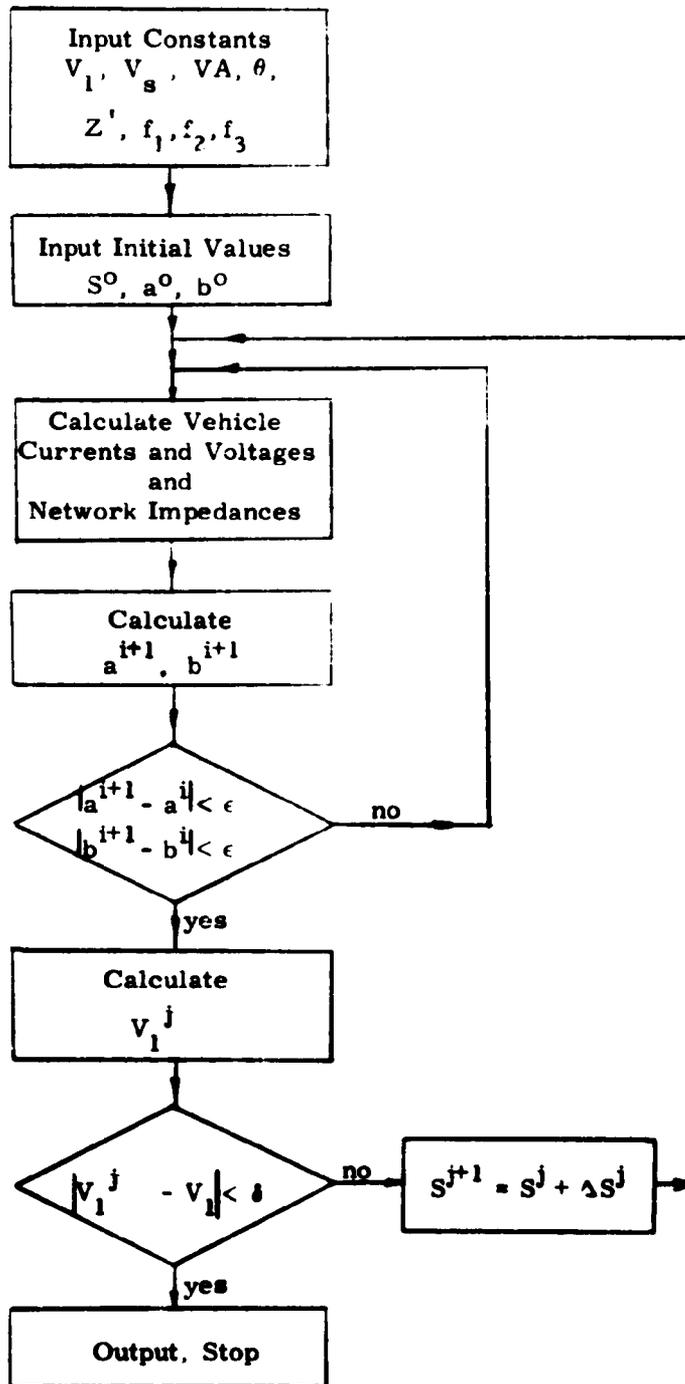


Fig. A. 3 Algorithm Flow Chart

Convergence is assumed when

$$|a^{i+1} - a^i| < \epsilon \quad \text{and} \quad |b^{i+1} - b^i| < \epsilon \quad \text{A.10}$$

for some suitable error bound ϵ .

Convergence in S is obtained when the solution to equation A. 8, V_1^j , satisfies

$$|V_1^j - V_1| < \delta \quad \text{A.11}$$

for some suitable error bound δ . Iterates in S are defined by

$$S^{j+1} - S^j = \Delta S^j = S^j \left(\frac{V_1 - V_1^j}{V_1} \right) \quad \text{A.12}$$

Note, by the convention adopted, the complex components of V_1 and V_1^j are zero so that S is real.

For each new estimate of S, convergence in a and b must be obtained. The convergence rate is at best linear but reasonable solution times can be obtained on even the smallest of programmable computers.

Solutions are denoted by $S = S_{\max}$.

A.4 Substation Loading Algorithm

The power delivered by each substation can be easily calculated by adding to the previous algorithm, expressions to evaluate $V_s J_1^*$ and $V_s J_3^*$ (*denotes the complex conjugate).

Substation loads at the maximum substation spacing can be found for other vehicle distribution patterns by interchanging the roles of S and V_1 . The maximum substation spacing S_{\max} is known and becomes an input constant. The initial value of the voltage at vehicle 1 is estimated, V_1^0 , to start the algorithm. Since equation A. 8 must still be satisfied, it may be solved for V_1 to obtain the iterate V_1^j . Convergence is obtained when the successive iterates differ by some error bound.

Vehicle voltage profiles are simply determined by evaluating V_1 and V_2 at the solution points for various values of the vehicle distribution parameters.

A.5 Computational Considerations

As with any iterative procedure, selection of initial values should be within the range of convergence of the algorithm. In most cases, setting $a^0 = b^0 = 0$ and almost any estimate at S^0 was sufficient.

Since $V_{1q} = 0$ has been assumed in defining the reference node 1, the reference current $I_{1q} = 0$ when vehicle 1 has a unity power factor. Equations A. 2 and A. 3 are then no longer valid. This difficulty may overcome by assuming a vehicle power factor arbitrarily close to unity (e. g. , 0. 99999) without significant loss of accuracy.

When the substation output voltage is regulated (constant voltage source) the substation impedance Z_s is effectively zero. Substation impedance must be included when the substation output voltage is unregulated. Moreover, since the problem structure in Art. A.2 only considers the vehicle load on a single section, the effective impedance for each section must be used.

In Fig. A.4 (a), a single substation with a total impedance Z_s is shown feeding into the ends of two rail sections. The power flow in each direction is indicated by VA_1 and VA_2 . The equivalent network is shown in Fig. A.4 (b), where Z_{s1} and Z_{s2} are the effective impedances associated with each rail section. The power flow quantities, which are not known, can be estimated with adequate accuracy from the solutions obtained with regulated substations. Note that the power flow changes when the vehicle distribution pattern is altered.

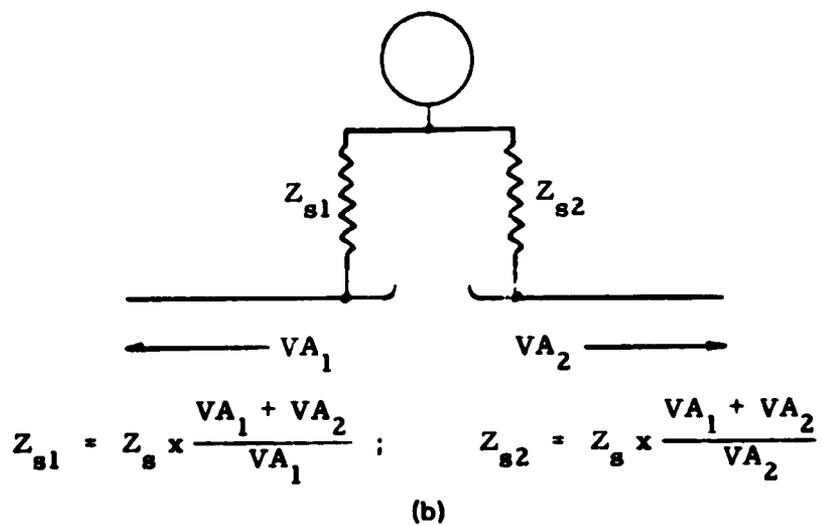
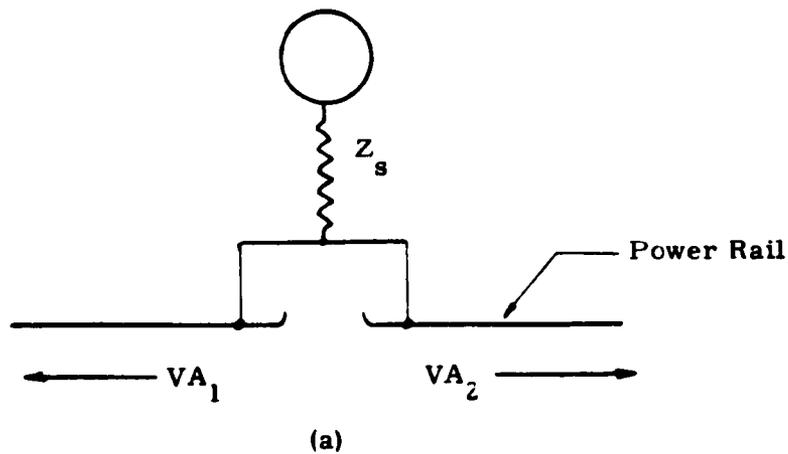


Fig. A.4 Substation Equivalent Impedance

Appendix B

Transformer Voltage Drop

Transformer impedances are nearly all reactive so that voltage drop is primarily due to the flow of quadrature current.

The quadrature current at a substation is given by

$$I_q = \frac{pff \times MVA \times RF}{\sqrt{3} \times 8.25 \text{ kV}}$$

where

pff = peak load factor in per unit vehicle load (Fig. 6.3)

MVA = power per vehicle at 8.25 kV

RF = reactive power factor of load

Assuming the transformer is rated for the rms load, the rated current of the transformer is:

$$I_b = \frac{rmsff \times MVA}{\sqrt{3} \times 8.25 \text{ kV}}$$

where

$rmsff$ = rms load factor in per unit vehicle load (Fig. 6.3)

Then the voltage drop in the transformer due to the quadrature current

I_q is:

$$\begin{aligned} \% \text{ voltage drop} &= \% \text{ impedance} \times \frac{I_b}{I_q} \\ &= \% \text{ impedance} \times \frac{pff \times RF}{rmsff} \end{aligned}$$

where

% impedance = impedance of the transformer on its own base.