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IMPROVEMENT OF METROLINER TELEPHONE CHANNEL CAPACITY AND MODELING OF TELEPHONE CHANNEL DEMANDS

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16. Abstract

The channel capacity of the present Metroliner telephone system is analyzed and methods are proposed to increase that capacity without increasing the overall bandwidth. To determine the number of channels required, calculations have been carried out using two available mathematical models: the Erlang Modal and the Waiting Model. Three criteria have been used: (1) the probability that no channel is available, (2) the mean waiting time and (3) the probability of having to wait at least t minutes.

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1.0 INTRODUCTION

Public telephone services have proved to be very popular to the passengers riding on the Metroliner between New York City and Washington D.C. According to an AT&T survey[1], 65% of the telephone users indicated that they had to wait to place their calls because all channels were busy. Of these persons, one out of ten had to wait 30 minutes or longer. The following studies are aimed at providing guidelines for improving the present service. Basic to this aim is an analysis to determine the actual telephone channel capacity of the present system. In addition, studies were made to improve that capacity subject to the following restrictions:

1. The communication link remains a ground-to-ground radio link between the Metroliner mobile station and the base station.
2. The overall radio frequency bandwidth remains the same.

Of the various ways to increase channel capacity we have considered these methods: They are 1) modify the train schedule, 2) reduce the number of buffer zones and 3) increase the number of carrier frequencies by reducing the bandwidth per channel.

Two theoretical models were applied in this study: the Erlang Model and the Waiting Model. A number of basic parameters: probability of waiting, mean waiting time and minimum waiting time were obtained. These parameters are used to relate the channel capacity to the demand. The models should apply to all kinds of demands.

The second section presents a brief description of the present telephone system. An analysis of the channel capacity of the present system is given in the third section. A number of methods to increase the channel capacity are suggested. Theoretical models are discussed and applied in section four.

2.0 DESCRIPTION OF METROLINER TELEPHONE SYSTEM

The Metroliner telephone system[2,3] consists of four major subsystems in addition to the regular telephone networks. They are:

1. The telephone unit on board the train and the markers installed on the track to distinguish radio zones.
2. Base stations.
3. Carrier systems between the base station and the central control terminal located at Philadelphia.
4. The central control terminal.

From the central terminal, the mobile telephone is connected to the regular telephone network through which the party at the stationary end is reached.

Figure 1 shows all the major systems of the Metroliner telephone system. The units marked 1, 2, and 4 are used exclusively for the Metroliner telephone system. There are six radio carrier pairs available. Each pair consists of one frequency for transmitting and one frequency for receiving. The following table shows the frequency allocations.

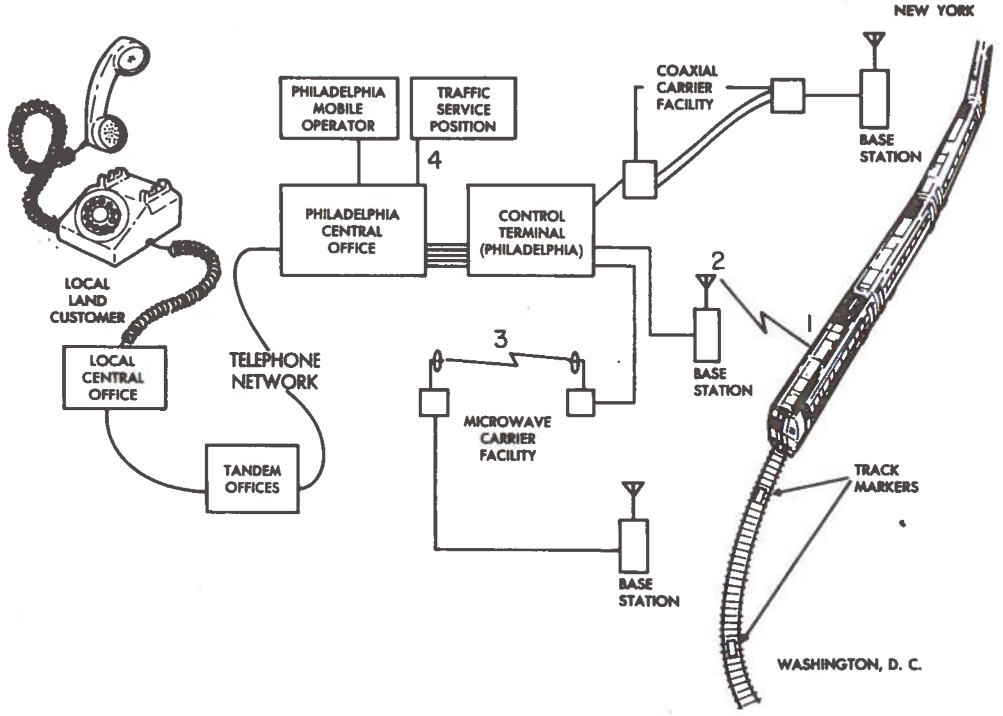

Figure 1. High-Speed Train Radio Telephone System

Mobile Station Transmitting Frequency	Base Station Transmitting Frequency
406.025 MHz	416.125 MHz
406.075	416.175
406.125	416.225
406.775	416.875
406.825	416.925
406.875	416.975

The frequency separation is 50 KHz.

The entire railroad track from Washington to New York is divided into 9 radio zones, approximately 25 miles in length. Each zone has one base station located near the center of the zone. Each base station can handle three or four pairs of radio carriers. Fig. 2 shows the present arrangement of the radio carriers distribution.

Figure 2. Present Radio Carrier Zonal Distribution

The functions of each subsystem will be discussed in the following.

2.1 TELEPHONE SYSTEM ON BOARD

This system consists of the following units:

1. power supply,
2. track-marker sensor,
3. channel control unit,
4. radio unit,
5. coin-control unit,

6. antenna,
7. telephone set,
8. track-markers.

2.1.1 Power Supply

The power supply provides the power to the entire train unit. The input could be either from the line or the internal battery system on board the train.

2.1.2 Track-marker Sensor

The track-marker consists of two mutually coupled coils which are connected to input and output respectively, of an amplifier in the channel control unit. When the train passes a track-marker on the track, the coupling of the input and output coils increases and causes the amplifier to oscillate at the marker's frequency.

2.1.3 Channel Control Unit

This unit consists of a high gain amplifier by which input and output are connected to input and output of track-marker sensor coils respectively, a bank of seven filters and logic circuitry. The output of the amplifier is fed to a bank of seven filters tuned to the seven resonant frequencies of the track-markers. The outputs of the filters are fed into the logic circuitry, which operates on a sequential two-out-of-seven basis to identify radio zones. The logic circuitry then switches the radio unit to the best available channel.

2.1.4 Radio Unit

This unit consists of six pairs of fixed tuned transmit-receive units. The logic circuit dictates which pair is to be used.

2.1.5 Coin-Control Unit

This is a standard unit used on all pay phones.

2.1.6 Antenna

One antenna is provided for each telephone unit to be operated in the 406-416 MHz frequency range.

2.1.7 Telephone Set

This is a touch-tone dialing, single-slot coin telephone. A dial tone is provided before depositing coin.

2.1.8 Track-Markers (installed on track)

The track-markers are mounted between rails in protective wooden enclosures. There are ten track-markers along the entire track. Each enclosure contains two markers, and each marker is resonant at a different frequency. From the combination of frequencies used by the two markers, logic circuitry on the train determines the zone boundary being passed. The sequence in which the frequencies are detected also tells the logic circuitry the direction in which the train is moving.

2.2 BASE STATION

The base station contains either three or four fixed tuned transmit-receive units depending on the location of the station in the radio zone. Each unit has two antennas which point in opposite directions along the track. The message is either transmitted to the mobile train phone or transmitted to the carrier facility for relaying the message to the central office.

2.3 RELAY LINK

The two available relay links are the microwave carrier facility and the coaxial carrier facility. These facilities are used to relay the telephone conversation between the base stations and the central control terminal.

2.4 CENTRAL CONTROL TERMINAL

2.4.1 Monitoring Unit

This unit is used to monitor the signal-to-noise ratio of all transmit-receive pairs at the base station, The unit determines which base station is to be used and when and where to switch base stations.

2.4.2 Terminal

This terminal handles a maximum of 12 simultaneous conversations and interfaces with the central office which is tied in with the telephone network.

3.0 IMPROVEMENT OF CHANNEL CAPACITY

3.1 GENERAL DISCUSSION

In this section the ways and means to increase the channel capacity of the present system are discussed. In addition, the possibility of modifying and improving the equipment is considered. The increase of the channel capacity is subject to the following restrictions:

1. The communication link remains as a ground-to-ground radio link between the Metroliner telephone and the base stations.
2. The number of radio zones remains the same as in the present system.
3. The overall radio frequency bandwidth remains the same.

There are three ways of increasing the channel capacity. The degree of improvement is different in each case as is the equipment change involved. These three ways are: 1) A slight change of train schedule. For example, the minimum time separation between successive train departures could be 15 minutes. This modification involves no change of equipment. On the other hand, the amount of increase of channel capacity is probably the least. 2) Increase the zone coverage of the radio channels by reducing the number of buffer zones from three to two. According to Bell Telephone Laboratories[4], a two-zone buffer will probably not introduce co-channel interference. Zonal distribution patterns with the reduced number of buffer zones will be discussed in Section 3.2. Additional equipment will be required when the number of buffer zones is reduced and the distribution of the radio carriers is rearranged for each base station. For example, an additional number of transmit-receive pairs will be needed at the base station, and the logic circuitry must be revised to take this into account. 3) The present radio channel separation of 50 kHz may be reduced. For example, the land mobile 2-way radio in the 450-470 MHz band has a 25 kHz frequency separation. Therefore, it would be reasonable to reduce the frequency separation from 50 kHz to 25 kHz for the Metroliner system and increase the number of carrier pairs from six to twelve. Furthermore, in England[5] the channel separation for 2-way radio has been reduced to 12.5 kHz in the VHF band (30 MHz to 300 MHz). The practice in England has proved to be successful for both AM and FM modulations. The 12.5 kHz frequency separation has also been proposed for the land mobile frequency band in UHF (300 MHz to 3 GHz). It is conceivable that a 12.5 kHz frequency separation could be applied to the radio carriers of the Metroliner telephone system if the telephone demand is high. In this way, with the same total bandwidth, the number of radio carriers can be increased two fold or four fold respectively. Of course, with the reduced channel separations, the equipment at the Metroliner car, base station and central control terminal will need some changes and some additions. For example, the transmit-receiver parts will require a narrower bandwidth, the logic circuitry will need to control more radio channels, the control terminal will need more equipment to handle more telephone channels, and other changes may be needed. Before a final decision can be made, consultation and discussion with the personnel of AT&T is necessary. In the following sections, we shall discuss the channel capacity improvements that result from a) an increase of the zone coverage by a radio carrier and b) an increase of the total number of radio carriers. The approach we are taking is to consider the channel capacity for both single and multiple trains on the track. For the multiple train case, the peak hours of the present train schedule are considered.

3.2 CHANNEL CAPACITY

It is possible to modify the present arrangement of telephone channels in the operating Metroliner communication system in various ways. Figure 3 shows the present pattern of six radio carriers pairs with eleven telephone channels. The radio carrier separation is 50 kHz. Buffer areas three zones wide separate regions where radio frequencies may be re-used. These buffers prevent interference between two calls on the same frequency.[2] It is possible to reduce the length of the buffer areas without increased interference. Modified patterns are shown in Figures 4 and 5 with two zone buffers. Pattern A is similar to the present pattern with the exception that there are fifteen zonal spaces unused as opposed to twenty-three unused spaces in the present pattern. Pattern B, a different possible pattern has only twelve unused spaces.

For a given pattern, the number of telephone channels available to passengers whose train is located in a specific zone depends on the direction in which the train is moving. In order to assure continuous service through at least one zone, it is not possible to initiate a call on a telephone channel that falls on the last zone being traversed.[2] However, calls can be initiated at all end-of-run zones in Landover and Newark, that is when the train is ending its journey.

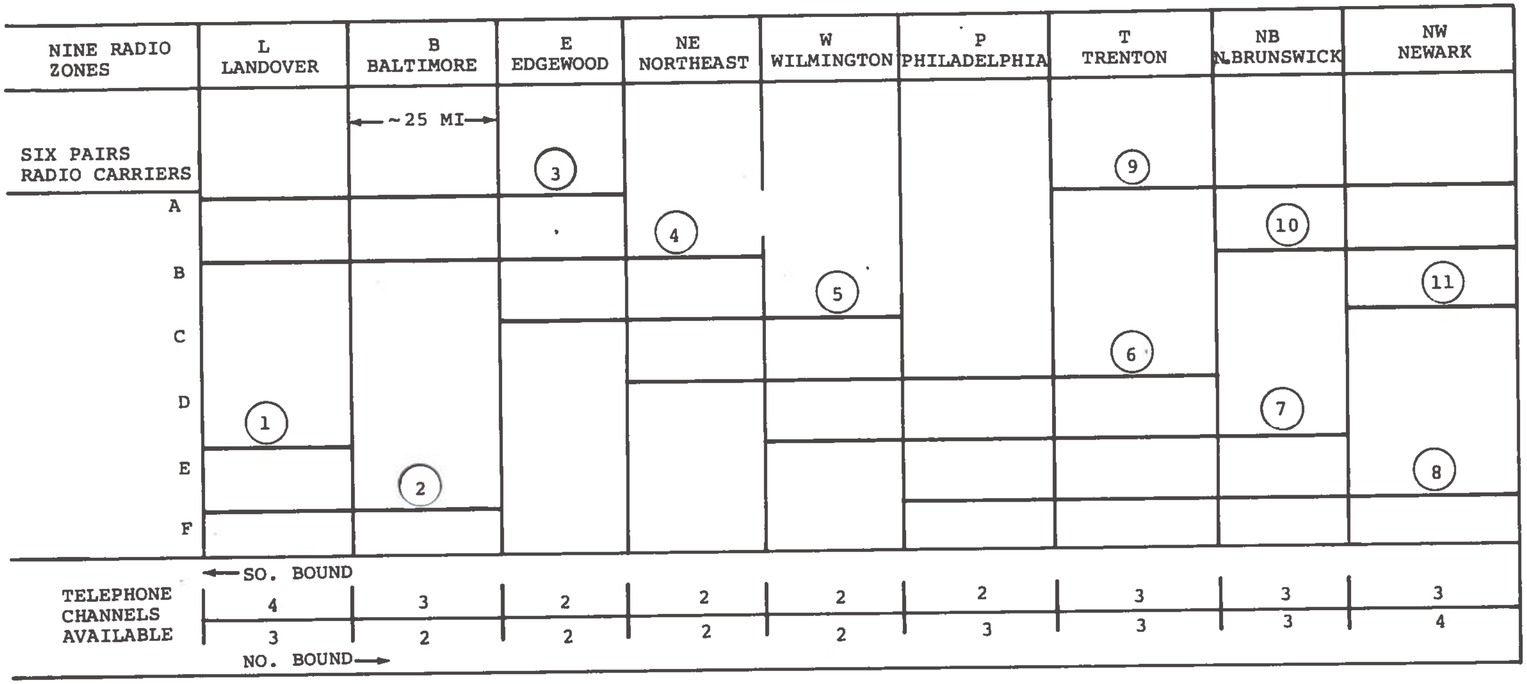

Figure 3. Present Distribution of Telephone Channels
Metroliner: New York-Washington D.C.

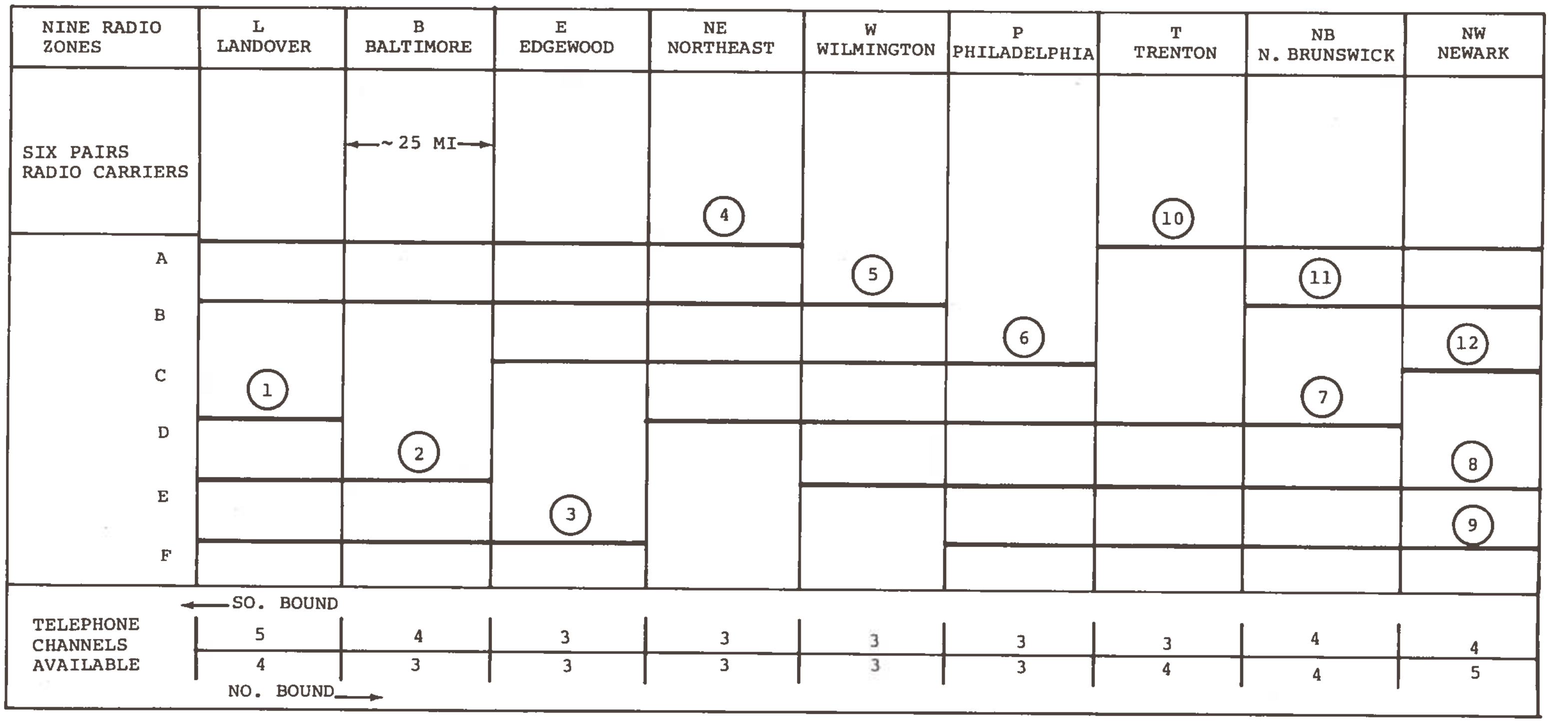

Figure 4. Possible Distribution of Telephone Channels Two-Zone Buffer, Pattern A.

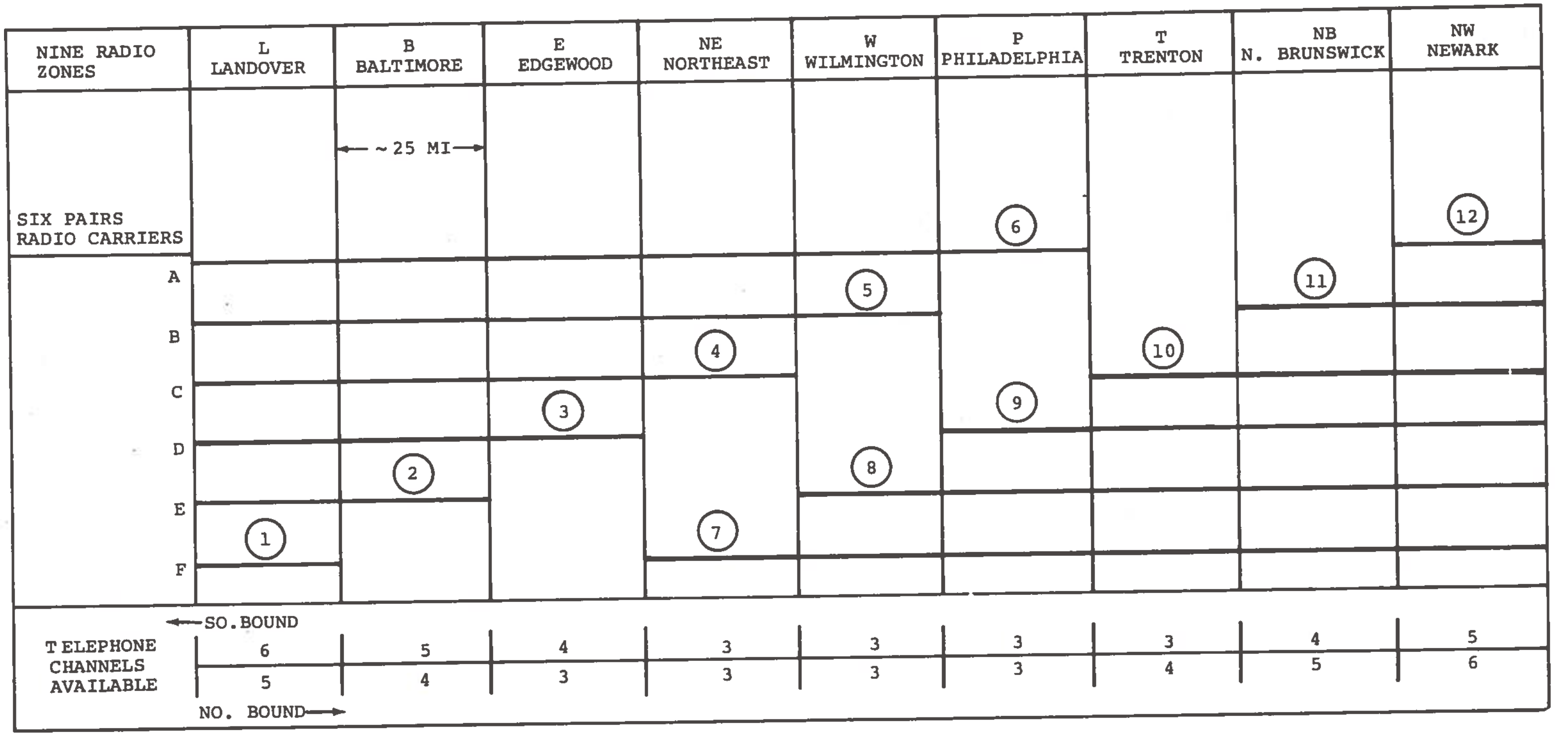

Figure 5. An Alternate Distribution of Telephone Channels, Two-Zone Buffer, Pattern B.

3.2.1 Channel Capacity For A Single Train On The Track

The simplest measures of channel capacity are the maximum possible number of calls and the average number of channels available to a single train on the track between New York and Washington. These quantities have been calculated for the morning executive according to the Metroliner schedule of November 14, 1971.[6] The calculations were made for three different buffer zone patterns and three different carrier frequency separations (50, 25, 12.5 kHz). The results, assuming constant speed of travel, are shown in Tables 1 and 2.

TABLE 1. MAXIMUM POSSIBLE NUMBER
OF CALLS DURING THE TRIP FOR A SINGLE TRAIN
ON THE NEW YORK - WASHINGTON TRACK.
THE TRIP DURATION IS ABOUT 2 HR. 45 MIN.
THE AVERAGE LENGTH OF THE TELEPHONE CONVERSATION
TIME IS 4 MIN.

	6 RADIO CARRIER PAIRS	12 RADIO CARRIER PAIRS	24 RADIO CARRIER PAIRS
PRESENT PATTERN	110	220	440
PATTERN A	146	292	548
PATTERN B	165	330	660

TABLE 2. THE AVERAGE NUMBER
OF TELEPHONE CHANNELS AVAILABLE TO
A SINGLE METROLINER TRAIN
ON THE NEW YORK - WASHINGTON TRACK.

	6 RADIO CARRIER PAIRS	12 RADIO CARRIER PAIRS	24 RADIO CARRIER PAIRS
PRESENT PATTERN	2.7	5.3	10.6
PATTERN A	3.6	7.1	14.2
PATTERN B	4.0	8.0	16.0

3.2.2 Channel Capacity of a Train Sharing Telephone Channels with Other Trains on the Tracks.

To obtain an over-all visual impression of the locations of the trains on the track, a space-time diagram showing the Metroliner Train trajectories utilizing the current train schedule was plotted (Fig. 6). The instantaneous train locations can be obtained by looking at a section of the diagram corresponding to a given time. The left hand column of the diagram shows the radio zones along the track, each zone being about 25 miles in length. With this diagram the number of trains on the tracks versus the time of the day was determined (Fig. 7). Both north-bound and south-bound trains are included. The figure shows the morning and evening rush-hour peaks and also a lesser peak in the early afternoon.

In order to determine the number of channels available to a train during the worst conditions, the time periods during the morning and evening peak-hours were chosen for consideration. The time during these periods was divided into half hour intervals, and at the midpoint of each interval the zonal location of each train on the track was determined. Then for a given time division the telephone channels available to each train with a given channel pattern was listed. When a specific telephone channel is shared by say n trains at a given time, it is supposed that only 1/n channels is numerically alloted to each train. The total number of channels at a division of time is thus calculated. Figures 8A, B and C show the variation of telephone channel availability during the successive half hour periods of a train trip. A typical channel distribution is shown by train #102. The distribution shows that the train has more channels available at the beginning and at the final phases of the trip. The number of channels is much reduced in the middle of the trip. One should therefore be aware that the distribution of available channels in time is non-uniform. The reasons for the non-uniformity are 1) there are more channels available at the two ends and 2) there are more trains sharing the channels during the middle of the trip (due to the train schedule). If the demand for the telephone is constant throughout the trip, the user may experience congestion (the probability of getting a free channel is low and the waiting time is longer) during the middle of the trip. One would like to know whether the demand for a typical train varies during a trip. If it does vary, how does it vary? Furthermore, the demand may vary from train to train (commercial and vacationing passengers differ in their demands). Hence one might like to know the maximum demand. Then the required channels could be provided accordingly.

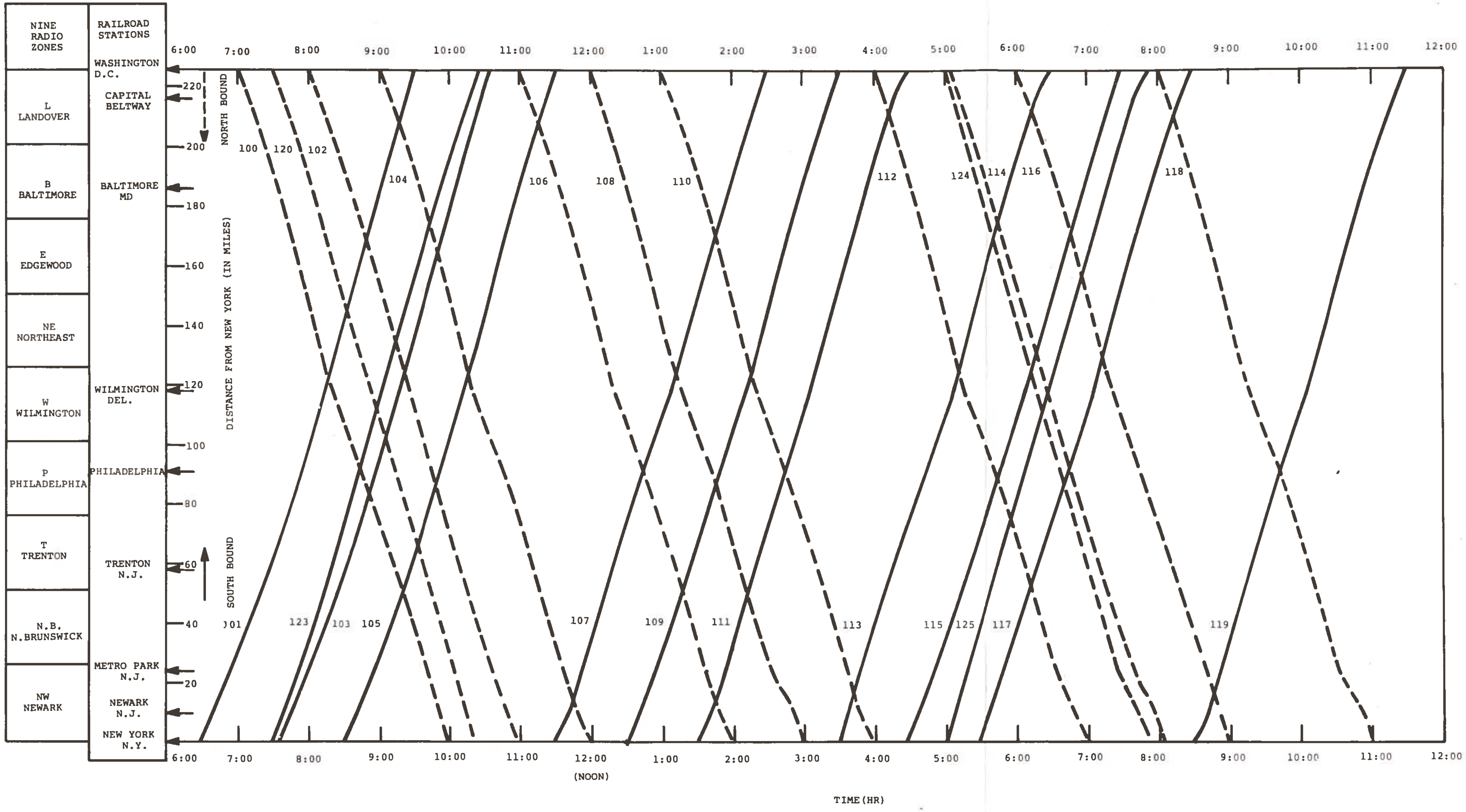

Figure 6. Space-Time Diagram showing Trajectories of Currently Scheduled Metroliner Trains. (Numbers on Trajectories Indicate the Train Number)

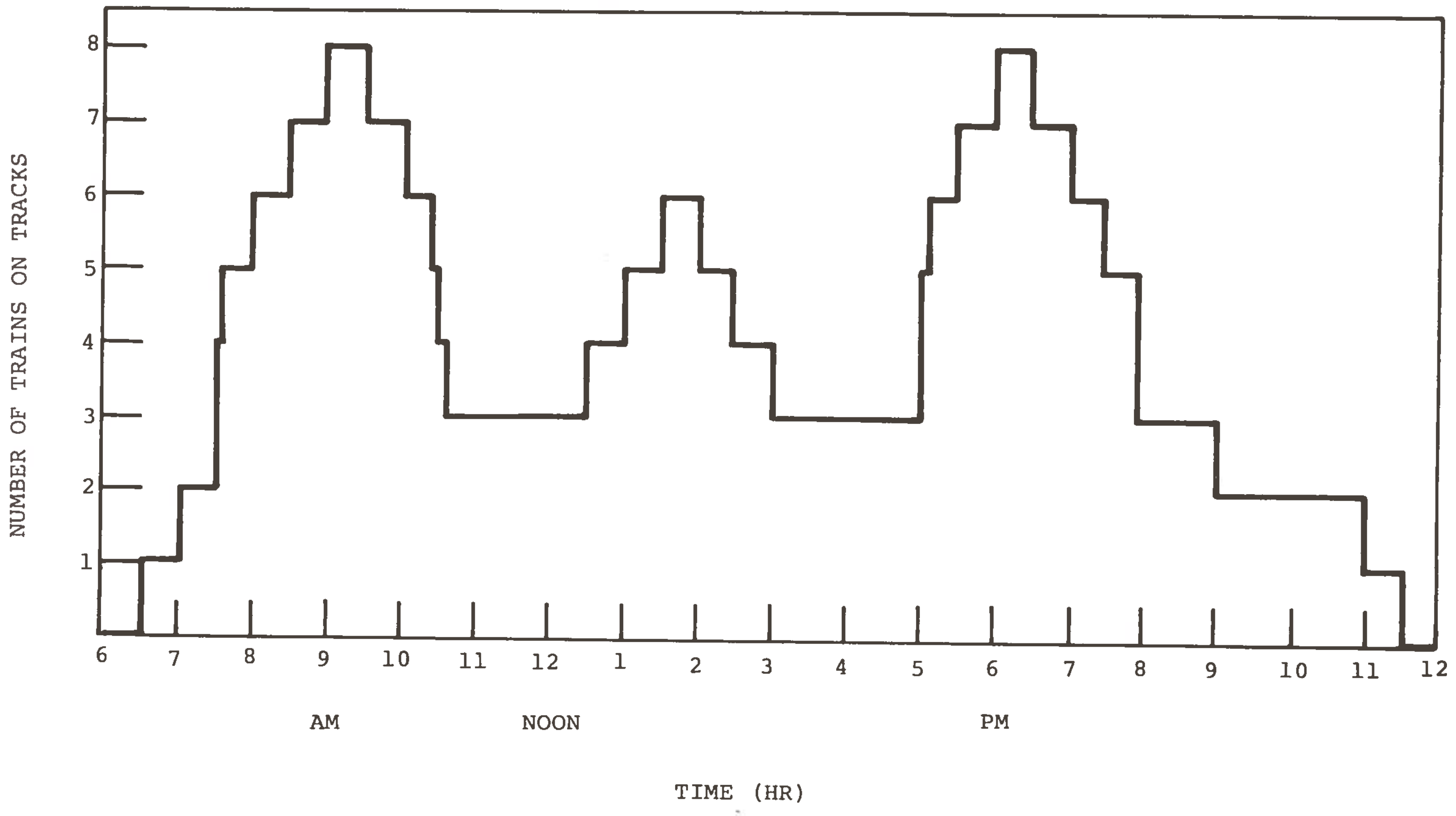

Figure 7. Number of Trains on Tracks Versus Time of the Day.

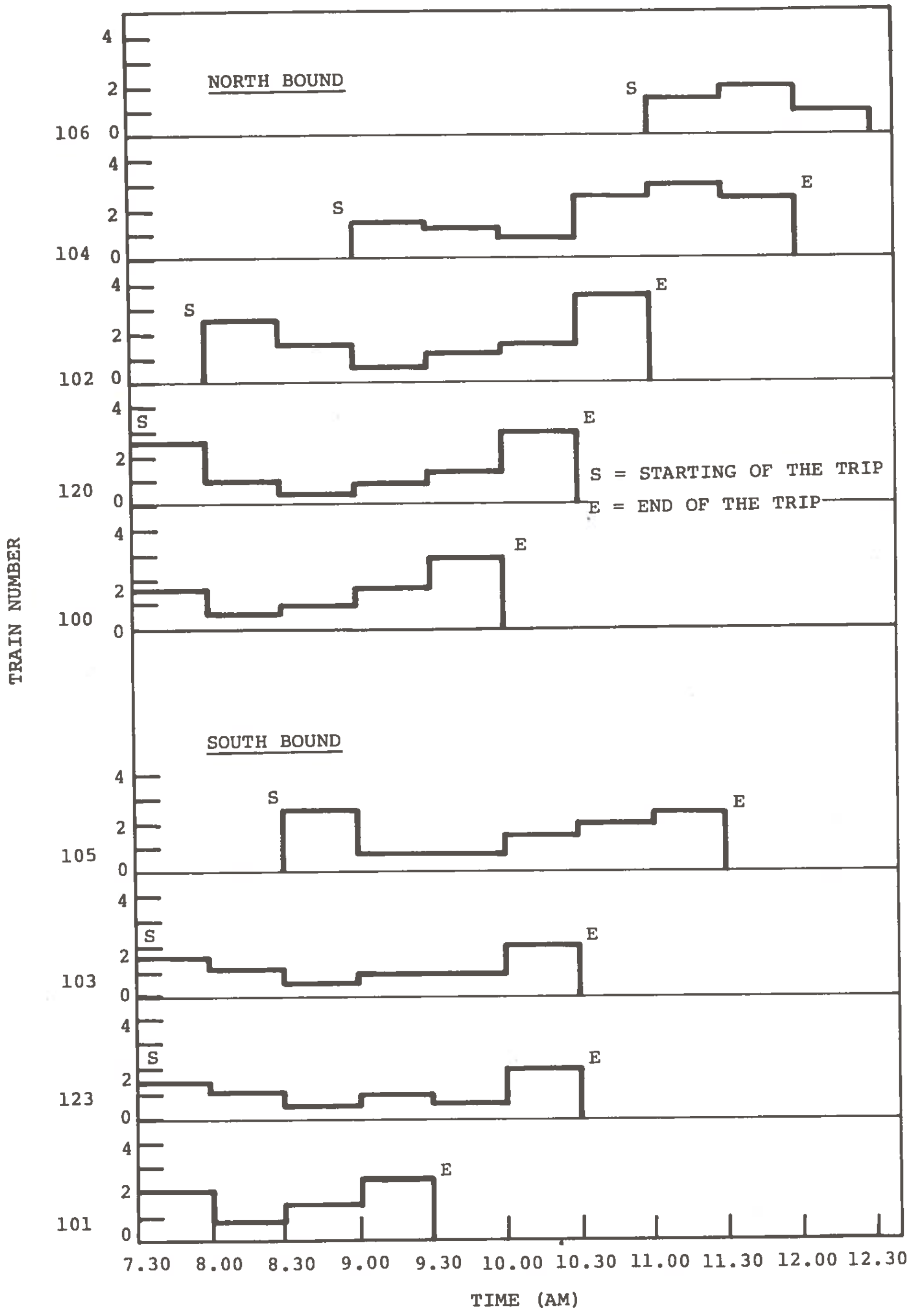

Figure 8A. Telephone Channel Time-Distribution of Morning Hours Metroliner Trains, Present Pattern.

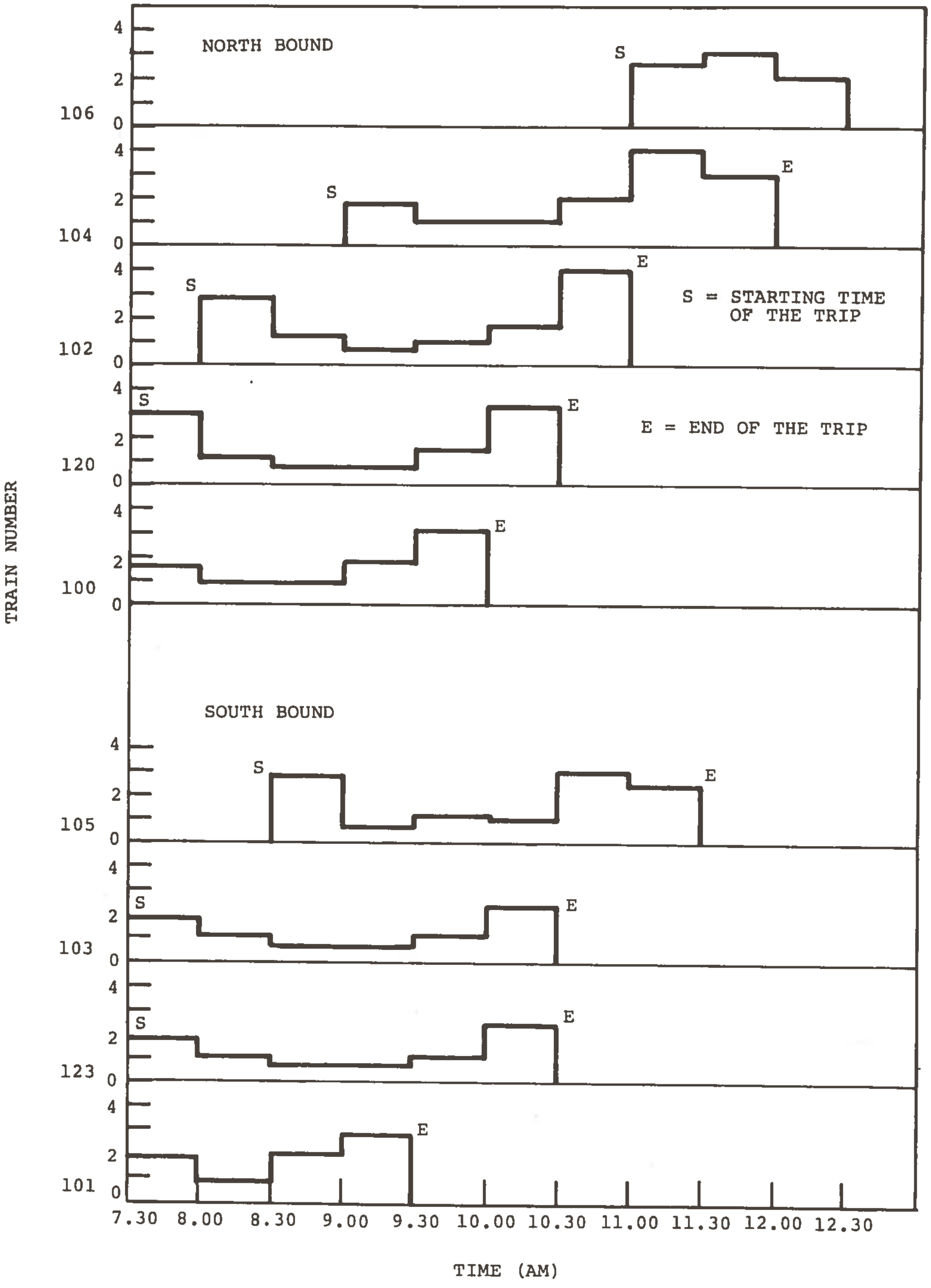

Figure 8B. Telephone Channel Time-Distribution of Morning Hours Metroliner Trains, Pattern A.

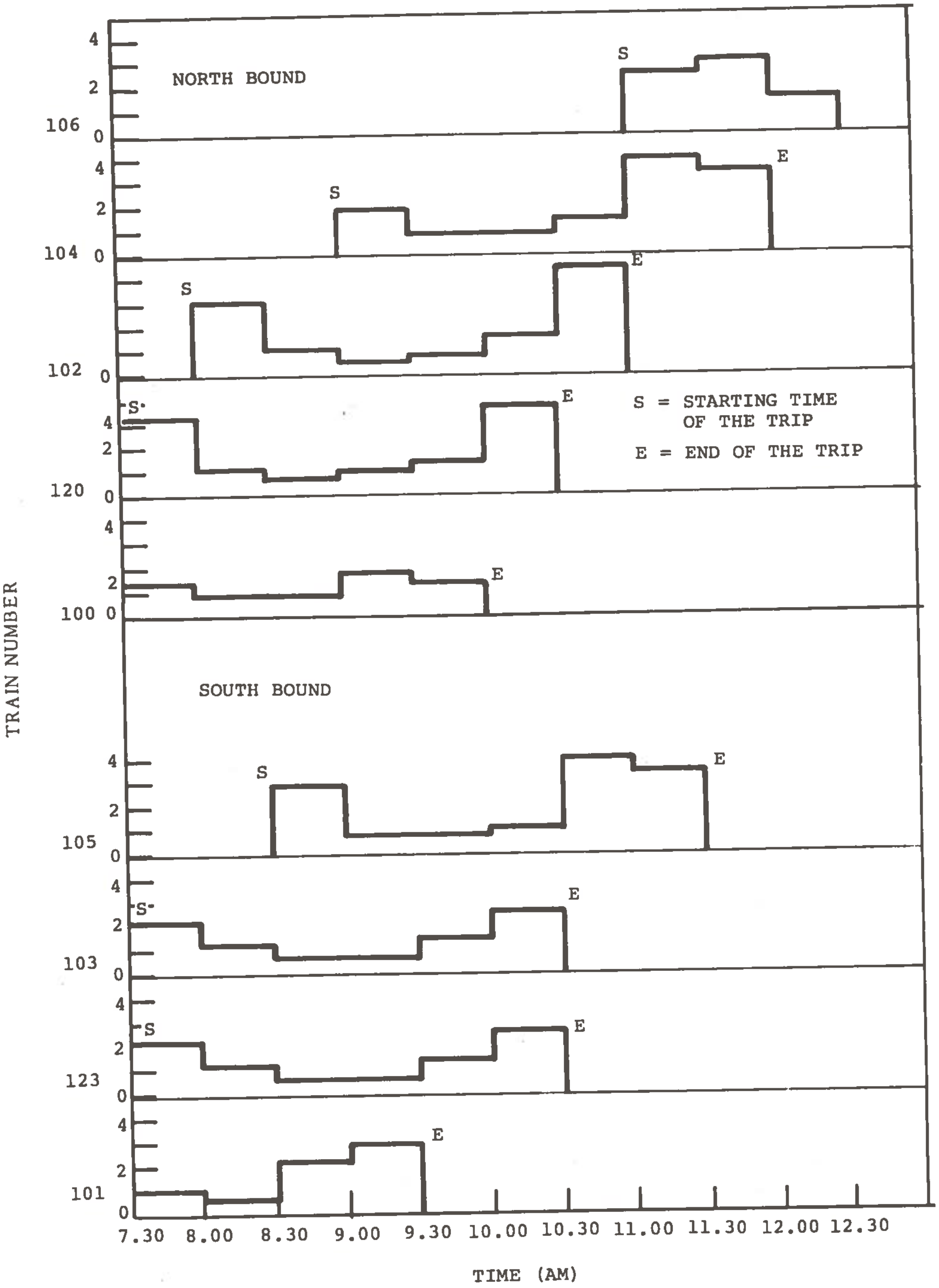

Figure 8C. Telephone Channel Time-Distribution of Morning Hours Metroliner Trains, Pattern B.

Answers to these questions require further data from AT&T and from train surveys. The presentations here show a number of ways to improve channel availability; the degree of improvement is about proportional to the investment required in equipment changes.

The following table shows the number of channels that can be used by all the morning trains for the time periods from 7:30 to 11:30 A.M. As can be seen, more of the available channels can be used with patterns A and B as compared with the present pattern.

TABLE 3. CHANNEL AVAILABILITY FOR MORNING TRAINS FOR SUCCESSIVE INTERVALS OF TIME.

	7:30	8:00	8:30	9:00	9:30	10:00	10:30	11:00	11:30	TOTAL NUMBER OF CHANNELS AVAILABLE
PRESENT PATTERN	9	7	8	9	9	9	8	7		11
PATTERN A	10	8	9	10	10	12	9	9		12
PATTERN B	10	8	9	10	10	12	10	10		12

Table 4 shows the average number of channels available to each train during the entire trip between New York and Washington. Table 5 shows the average for the trains considered. From these numbers it can be seen that some degree of improvement may be achieved by changing the zonal distribution patterns. One feature of Table 4 is worth special note. It is that on the average and for all three zonal patterns, the smallest number of channels is available to trains 123 and 103 in the morning and to trains 124 and 114 in the evening. The next lowest average is about 20% higher than this (Table 6). A look at the departure time (Figure 6) shows that the morning departure of trains 123 and 103 are separated by only five minutes, as are the afternoon departures of trains 124 and 114. Thus, each pair of trains occupy the same radio zone throughout almost all of the trip, which accounts for the reduced number of available channels. All other departures are separated by at least half an hour or more. In the following section it is shown that by modifying the train schedule the number of channels available to these trains can be increased.

3.2.3 Modification of the Train Schedule

One way to improve the lowest average channel availability, discussed in the preceding section, is to increase the separation of the trains in time. Since 15 minutes is about the

TABLE 4. AVERAGE NUMBER OF TELEPHONE CHANNELS AVAILABLE ON EACH TRAIN RUNNING BETWEEN NEW YORK CITY AND WASHINGTON D.C. (MORNING AND EVENING PEAK HOURS).

TRAIN NUMBER	PRESENT PATTERN	PATTERN A	PATTERN B
AM			
SOUTHBOUND			
123*	1.1	1.3	1.41
103*	1.1	1.3	1.41
105	1.69	1.87	2.12
NORTHBOUND			
100	1.75	1.80	2.19
120	1.49	1.73	1.78
102	1.77	1.9	1.97
104	1.97	2.1	2.11
PM			
SOUTHBOUND			
113	2.08	2.25	2.36
115	1.32	1.63	1.68
125	1.39	1.58	1.65
117	1.42	1.66	1.72
NORTHBOUND			
112	1.72	2.0	2.08
124*	1.13	1.34	1.4
114*	1.13	1.34	1.4
116	1.81	1.86	2.27

*TRAINS LEAVING 5 MINUTES BEFORE OR AFTER ANOTHER.

minimum time for a train to travel through a radio zone, two trains 15 minutes apart will always be in different zones, which means that this interval should probably be the minimum headway between trains. In the specific case of trains 123 and 103, we have recalculated the channel availability of the morning trains using 15 minutes intervals in two cases:

1. Both trains 123 and 103 departing at their regularly scheduled time (five minutes apart).
2. Train 103 delayed 10 minutes behind its regularly scheduled departure.

TABLE 5. AVERAGE NUMBER OF TELEPHONE CHANNELS AVAILABLE TO METROLINER TRAINS ON THE NEW YORK - WASHINGTON TRACK DURING THE MORNING AND EVENING PEAK HOURS (*).

	6 RADIO CARRIER PAIRS	12 RADIO CARRIER PAIRS	24 RADIO CARRIER PAIRS
PRESENT DISTRIBUTION	1.5	3.1	6.1
DISTRIBUTION A	1.7	3.4	6.8
DISTRIBUTION B	1.8	3.7	7.4

*AVERAGE OVER THE INDIVIDUAL TRAINS SHOWN IN TABLE 4.

TABLE 6. COMPARISON OF THE AVERAGE TELEPHONE CHANNEL CAPACITY FOR THE TWO LOWEST AVERAGES.

	PRESENT PATTERN	PATTERN A	PATTERN B
LOWEST AVERAGE CHANNEL CAPACITY	1.1	1.3	1.4
NEXT LOWEST AVERAGE CHANNEL CAPACITY	1.32	1.58	1.65
% DIFFERENCE	20%	22%	18%

Table 7 shows the average number of telephone channels available for these trains with the present schedule and with the modified schedule. For train #123 there is an increase of 26.2% in channels available. The lowest number for the pair of trains under consideration is raised from 1.22 to 1.40. The capacity of both trains is increased. No substantial changes occur in the other trains. To alter the channel availability for other trains where they are low, as for example #120, certain particular changes in the schedules must be made. Using a simple intuitive method in this example, it has been shown that a

TABLE 7. AVERAGE NUMBER OF TELEPHONE CHANNELS AVAILABLE TO MORNING TRAINS WITH REGULAR AND MODIFIED SCHEDULE*

		AVERAGE NUMBER OF TELEPHONE CHANNELS AVAILABLE	
		PRESENT SCHEDULE	MODIFIED SCHEDULE
	TRAIN NUMBER	123 AND 103 5 MINUTES APART	123 AND 103 15 MINUTES APART
SOUTHBOUND	123	1.22	1.54
	103	1.29	1.40
	105	1.67	1.56
	100	1.69	1.69
	120	1.26	1.25
NORTHBOUND			
	102	1.79	1.81
	104	1.92	1.90
	AVER.	1.55	1.59

* The above results were obtained by using 15 minute intervals instead of the 30 minute intervals for the results of Table 4. Therefore the numbers given in the first column are more accurate than those in the first column of Table 4.

change of 10 minutes may make the channel availability for a group of trains more uniform. Modification of the schedule of two trains five minutes apart could be effected by advancing one train schedule by five minutes and delaying the other train by five minutes. Then the two trains will be 15 minutes apart. To effect improvement for the trains as a whole, however, some type of mathematical optimization method must be employed in combination with a computer program.

4.0 SYSTEM REQUIREMENTS

4.1 INTRODUCTION

The requirements which should be placed on the Metroliner telephone system depend on what is considered satisfactory service. It is unrealistic to hope to provide immediate service for all possible users under all possible conditions. Instead a criterion for acceptable quality of service must be set in terms of, for example, percent of the time a telephone is available or the length of waiting times. The actual values of such criteria is a policy decision which will not be made here. Instead data are provided which will allow such decisions to be quickly converted into system requirements.

Aspects of the system which affect service are 1) the availability of telephones, 2) the availability of channels once an open booth has been obtained, and 3) the availability of operators once a channel has been obtained. The second factor, determined by the number of channels, is by far the most permanent. The other two can be improved relatively easily if they prove inadequate. Therefore, we shall focus here on the relation between the number of available channels and the quality of service.

A variety of mathematical models which describe demand for limited facilities have been studied in the theory of congestion, and the results have been extensively applied to telephone traffic problems during the past half-century. Any approach necessarily assumes a model which, with varying accuracy, describes real situations. The complex and unpredictable nature of human behavior precludes an exact description, even in a statistical sense. Nevertheless, increasingly refined models may be proposed, and the one eventually adopted is generally chosen because further accuracy does not justify increased mathematical complexity.

In the analysis of Metroliner telephone service, we shall employ two models: an Erlang model and a Waiting model. Both models are probabilistic and describe random fluctuations in demand for service. However, they assume that the system and the conditions which affect demand are unchanging so that a steady-state condition has been reached. The actual system and its use will differ from these models in certain aspects. For instance, the average demand might increase as a principal destination is approached. Furthermore, in the present system the number of channels available to a train may change significantly as the train progresses from zone to zone and as other trains enter and leave regions of channel sharing. Although the models will not fully incorporate these realities, it is felt that they will be an adequate guide for setting system requirements.

In the following section the theory of these models is briefly described giving assumptions and results. Detailed derivations can be found in References 7 and 8.

4.2 THEORY

4.2.1 An Erlang Model

Consider a system with N channels and assume that attempts to initiate calls are made at random intervals. If all channels are in use at a given moment, an attempt to initiate a call, i.e., to obtain a channel, will not be successful. If one or more channels are unused at a given moment, an attempt to initiate a call will be successful. The probability that k channels are in use is p_k, $0 \leq k \leq N$.

An Erlang model assumes that attempts to place a call are a Poisson process which is independent of the number of channels occupied. That is, for a small interval of time Δt, the probability of an attempt is $\lambda \cdot \Delta t$, where λ is a constant. We, therefore, can identify λ as the average number of attempts per unit time.

For a call in progress, it is assumed that the probability that the call will terminate during an interval Δt is $\mu \cdot \Delta t$, where μ is a constant. This means that the duration of calls or holding times will have an exponential distribution, with the number $1/\mu$ representing the average length of a conversation. While this assumption about calling habits is motivated partially by mathematical simplification, it has been observed to reasonably approximate both local and long-distance calls.

An Erlang model can describe systems not in equilibrium, but such a refinement is not warranted in the present application. Here we shall assume that the system is in equilibrium and that, consequently, the p_k are not functions of time. It can be shown that a necessary condition for equilibrium is $\lambda/\mu < N$. The model described here is unrealistic outside that range.

On the basis of the above assumptions, the probability that exactly k channels are in use can be derived and is given by

$$p_k = \frac{a^k/k!}{1+a+a^2/2!+....+a^N/N!} \qquad (1)$$

where

$a = \lambda/\mu.$

The probability that all channels are in use, P_n, is also the probability that an attempt to obtain a channel will meet with a busy signal. This probability is given by

$$p = \frac{a^N/N!}{1+a+a^2/2!+....+a^N/N!}$$

In many applications it is difficult to obtain data for a direct determination of λ, the average number of attempts per unit time. It is far easier to obtain the average number of successful attempts per unit time, which we label σ. The variables λ and σ are related by

$$\sigma = (1 - p)\lambda$$

or

$$b = (1 - p)a \qquad (2)$$

where

$$b = \sigma/\mu.$$

The variable a can be eliminated between Equations (1) and (2), and p can be computed as a function of b. The results are given in Figure 9. The region in which this equilibrium model is indicated to be invalid corresponds to $\lambda/\mu \geq N$.

4.2.2 A Waiting Model

For this model we consider a system with N channels and assume that customers arrive to demand service at random intervals. If one or more channels are unused, the customer immediately initiates his call. If all channels are in use, he waits his turn along with any other people who wish to use the telephone. People in this waiting group obtain open channels as they become available

We assume a Poisson influx of customers θ, given in customers per unit time, and calls of exponential holding time with average length $1/\mu$. A state of statistical equilibrium exists provided $\theta/\mu < N$. Then the probability of having to wait for an open channel can be shown to be

$$p = \frac{\frac{d^N}{N!}\cdot\frac{N}{N-d}}{1+d+d^2/2!+....+\frac{d^{N-1}}{(N-1)!}+\frac{d^N}{N!}\cdot\frac{N}{N-d}}$$

where $d = \theta/\mu$. This relationship is displayed numerically in Figure 10.

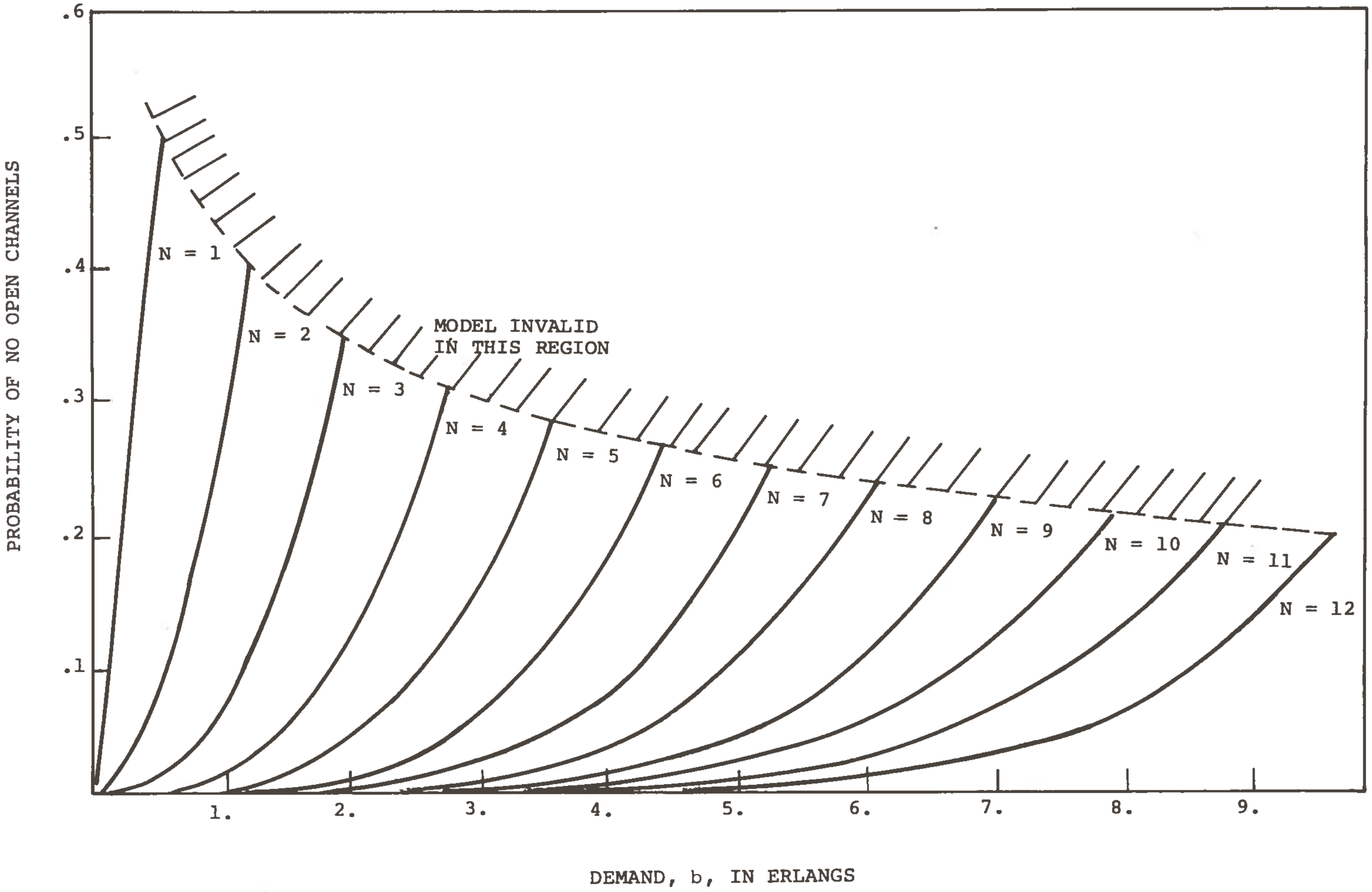

Figure 9. Erlang Model

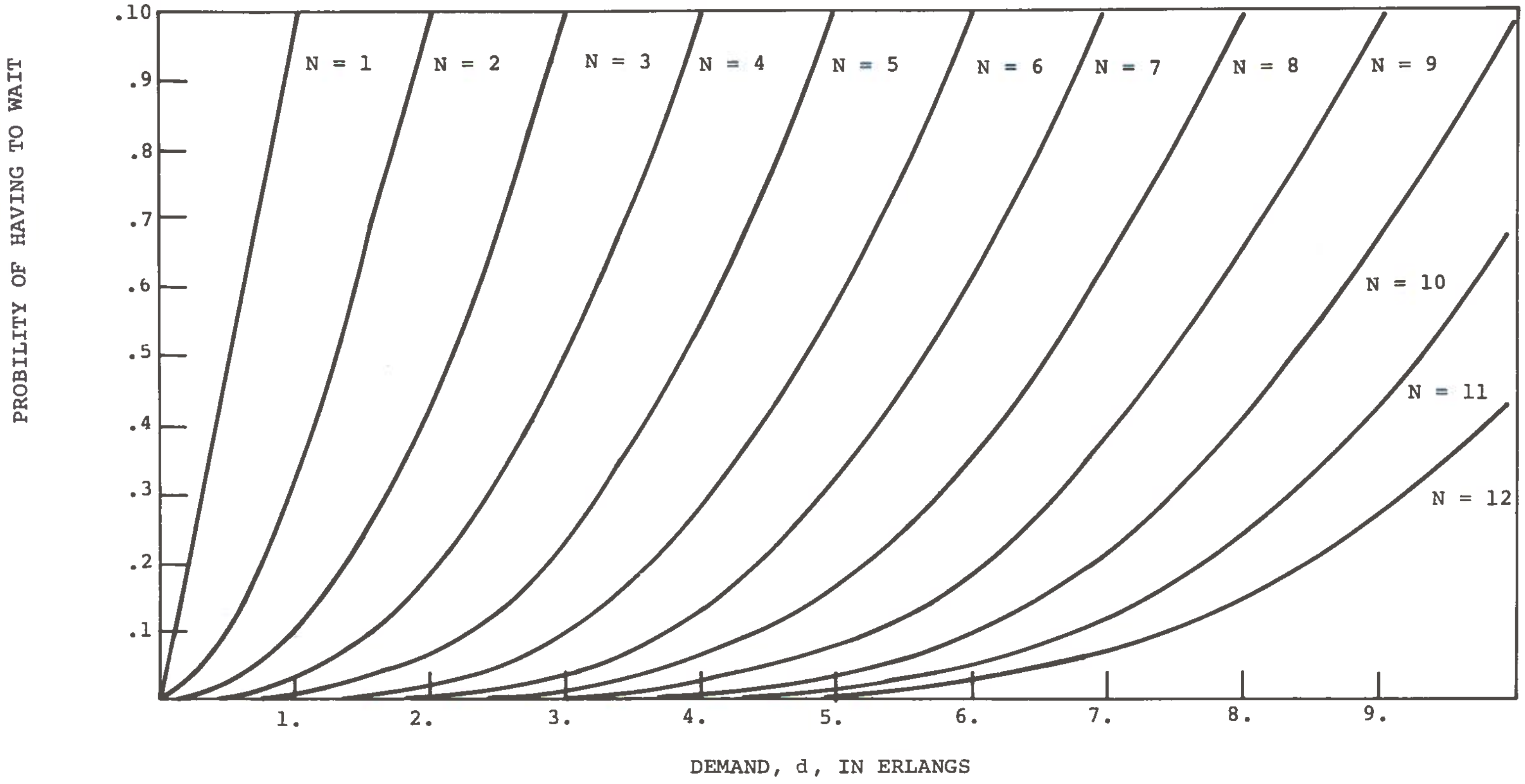

Figure 10. Waiting Model

The probability of having to wait more than a time t is

$$W(t) = p(d) \cdot e^{-\mu(N-d)t}$$

Some typical distributions of waiting time are given in Figures 11A and B.

The mean waiting time, including the case of zero waiting time, is

$$M = \frac{p(d)}{\mu(N-d)} .$$

Numerical results are given in Figure 12.

When the necessary condition for statistical equilibrium, $\theta/\mu < N$, is violated, waiting lines grow without limit.

4.3 DISCUSSION AND APPLICATIONS

4.3.1 Choice of Models

The choice of model to describe the Metroliner telephone service will depend upon customer behavior patterns in the use of the telephone. The Erlang model describes a situation in which customers attempt to use the telephone at random intervals. If unsuccessful in obtaining a channel, a customer does not stand ready to seize one as soon as it becomes free. Rather, he leaves for the moment, perhaps to try later. This description is most likely to apply to a low demand situation in which the channel congestion is only temporary. The customer expects a reasonable chance of success if he tries later and therefore does not stand and wait at a telephone. The intuitive conclusion that the Erlang model applies when demand is low and the channels seldom congested is confirmed by mathematical considerations. As mentioned in Section 4.2.1, the Erlang model can be in a stable state only when $\lambda/\mu < N$.

If competition for an open channel is heavy, customers will tend to line up to preserve their places. Although there may be waiting lines at several telephone booths on a given train, all are competing for the same channels and may be treated in the analysis as a common waiting group. This suggests the Waiting model described in Section 4.2.2. However, even when a customer waits at a telephone, it is not strictly true that he seizes a channel the moment it becomes free. Rather he must periodically lift the receiver to check for an open channel. Nevertheless, in heavy demand situations he can be expected to check frequently enough that little time lapses before he seizes a freed channel.

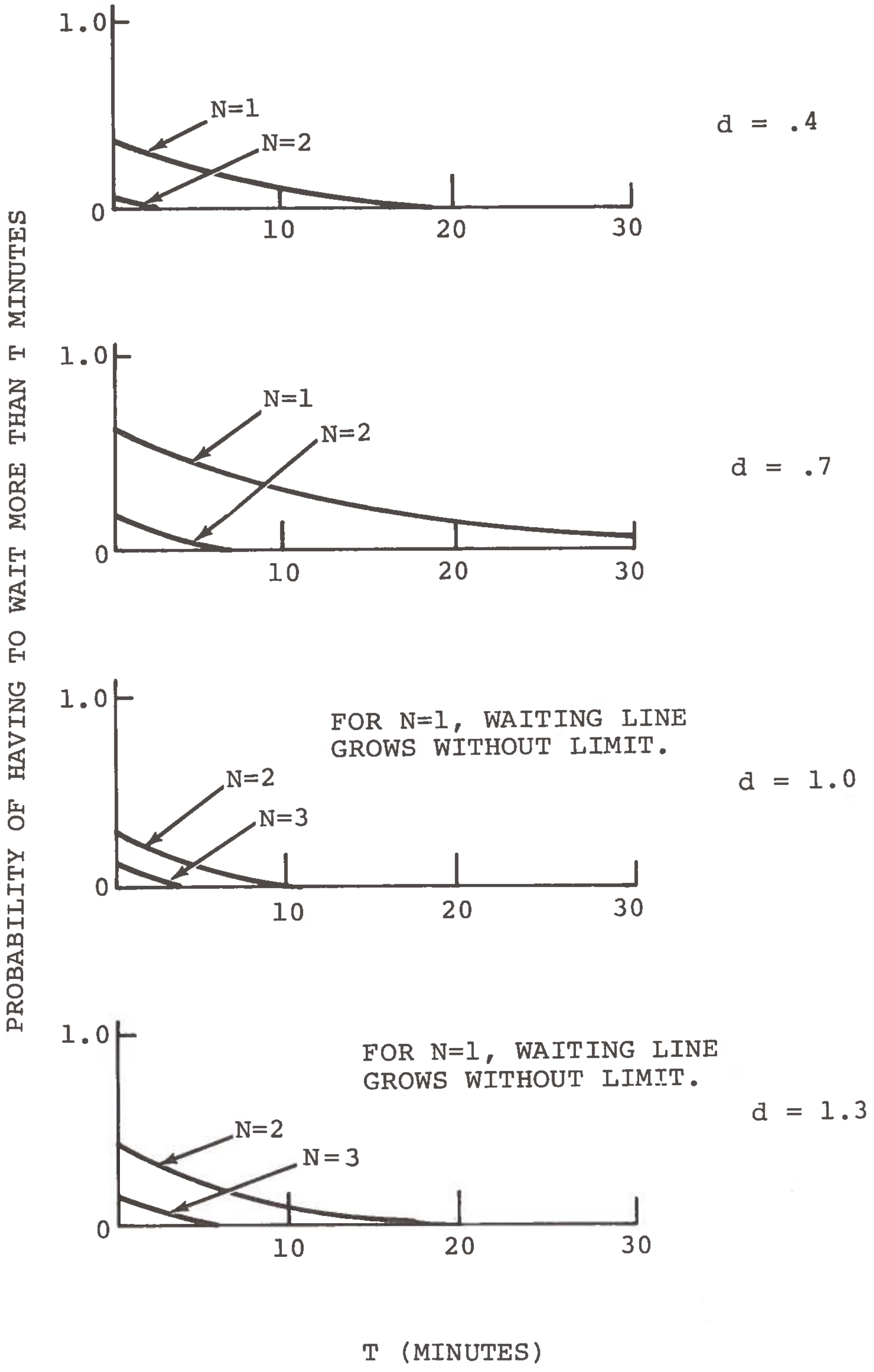

Figure 11A. Waiting more than t minutes

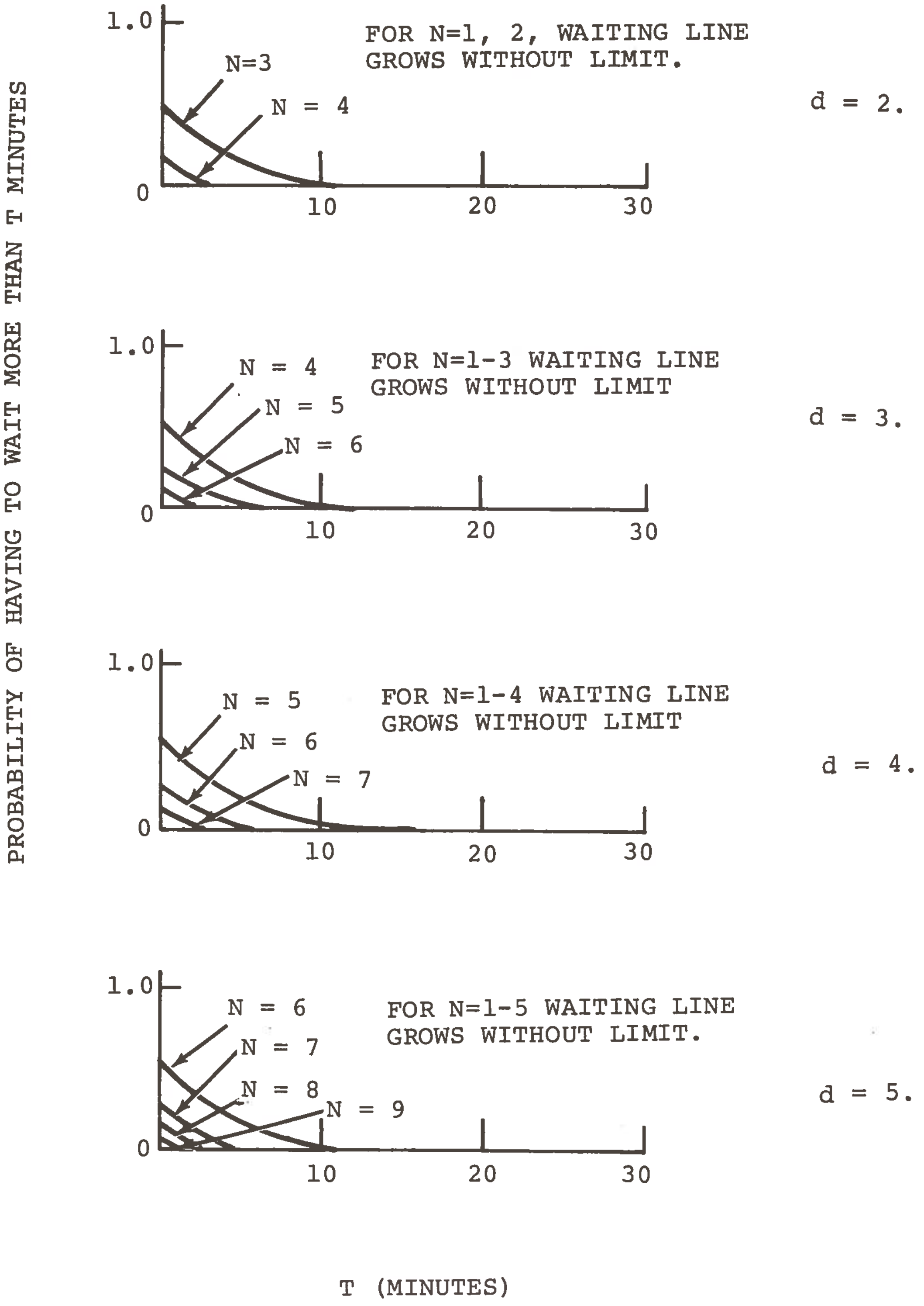

Figure 11B. Waiting more than t minutes

4.3.2 Applications

In Sections 4.2 and 4.3.1 two models, the Erlang model and Waiting model were described. In this section the models and the parameters are applied to determine the requirements and some of the critical parameters. For example, it will be seen that the mean waiting time is a very sensitive function of the demand as it approaches channel capacity.

The average demand is given by Mr. E. Price, AT&T New York[9], as 3.5 erlangs per 1000 passenger-hour. This average demand must be the lower limit because Reference 1 states that some people could not obtain a free channel to use the telephones and some could not complete the telephone conversation due to the lack of time or the lack of channels. It appears that the actual demand must be somewhat higher than 3.5 erlangs per 1000 passenger-hour. Furthermore, this demand is an average value; the actual demand on a train will generally fluctuate, and the manner in which the demand varies during a trip may determine the congestion and selection of the line. For example, one possibility is that the demand may be higher near the end of the trip. A more precise value for the average demand and its variation will be needed for future study which may lead to the optimal way of distributing the finite number of telephone channels.

According to Reference 1, 35% of the telephone users cannot complete their calls, so that the real average demand may be 35% higher than the value given by E. Price. Furthermore, the demand varies from train to train, so that the maximum demand may be a few times higher than the average. It is likely that the maximum demand is not more than three times greater than the average. Various demand estimates are given in the following table.

	AT&T Av. Demand	Actual Av. Demand	Peak Demand
Demand in Erlangs/train-hours	.7	1.1	3

Since the average number of passengers is $\approx$ 200, the AT&T average demand is one fifth of the demand given by E. Price of AT&T. The actual demand is based on the fact that approximately 35% of the potential telephone users have not been able to obtain a free channel.

Since there are three criteria present (the probability of no channel, the mean waiting time, and the probability of having to wait) for determining the number of channels, a further study will be needed to determine which is the best criterion from the user's point of view and what value of each should be chosen as a yardstick to determine the number of channels to be made available.

On the mean waiting time, one should note that when the demand nears the channel capacity, the rate of change of waiting time increases very rapidly. For example, given that there are two channels available and that the demand is 1.7 erlangs, then from Figure 12 the mean waiting time is 1 minute. If the demand increases by 10% to 1.87 erlangs, the mean waiting time will increase to 1.5 minutes which is an increase of 50%. What this really means is that when demand nears the channel capacity, the waiting time will increase sharply. The same thing applies to the probability that no channel is available. (Figs. 9 and 10).

If in the future passenger traffic increases, the demand will increase accordingly:

	50% Increase	100% Increase
Average demand	1.0 erlangs	1.4 erlangs
Actual demand	1.8	2.4
Peak demand	4.5	6

The average number of telephone channels required for 10% probability of waiting: (using Fig. 10)

	Present Load	50% Increase	100% Increase
No. of Average Telephone Channels	2.3	3.5	4
No. of Peak Telephone Channels	6	8	10

Using the mean waiting time criterion of 1 min: (Fig. 12).

	Present Load	50% Increase	100% Increase
No. of Average Telephone Channels	2.4	3.1	3.8
No. of Peak Telephone Channels	4.5	6	7.8

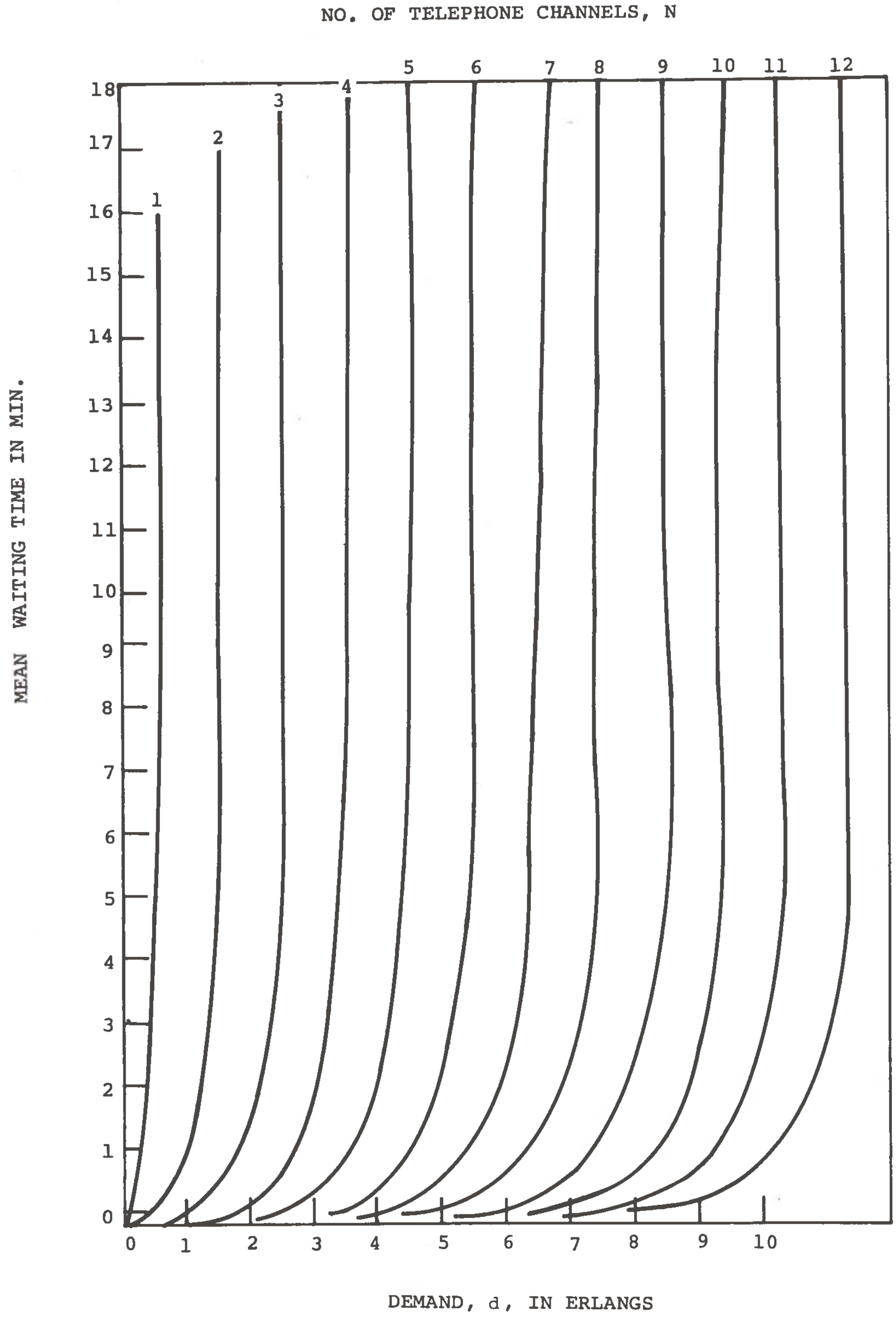

Figure 12. Mean Waiting Time

At the present time the actual distribution of demand is not known but if the demand is constant over the entire trip, then at times it will be greater than the number of available channels, and a waiting line will form. The customer who is waiting in line has a number of choices:

1. Give up on making the phone call.
2. Wait his turn.
3. Wait for his turn but give up because he has reached the destination.

To provide the right number of user's channels it is necessary to determine:

1. The demand on each individual train as well as the overall demand during the trip.
2. The optimum channel distribution along the way.

Recent data furnished by the AT&T Washington office (Table 8) show that the maximum number of phone calls is 319. The average length of a phone call is about 4 minutes. Total number of hours of phone calls per day is about 4 x 319/60 = 21.3 hr. Reference 1 states that 25% of the phone calls occur during the morning hours, i.e., 5.3 hr. From these figures the percentages of the available channels being used in the morning hours can be calculated. The following table shows the number of phone channels available to be used for every one-hour period starting from 7:30 AM to 11:30 PM. The total number of channel hours for the four periods is 33 hr for the present distribution 38.5 hr and 39.5 hr for distribution A and distribution B respectively.

	Total Hr.	% of Hr. Used
Present Pattern	33	16 %
Pattern A	38.5	13.8%
Pattern B	39.5	13.5%

It appears that the channels are not being used very efficiently. The reasons for this inefficiency are many. The most obvious ones are:

1. The demand may be out of phase with the channel availability.
2. Equipment failures may be excessive.

It is believed that by studying the above two problems one may find ways to improve the system so that optimum use of the presently available channels can be reached.

4.4 FUTURE STUDY

During the short period of this study information about the average demand and average channel capacity of 15 trains during the morning and afternoon peak hours has been obtained. Possible topics for future study are:

1. The time variation of demand for each train.
2. The optimum channel distribution for a given train schedule.

TABLE 8. METROLINER TELEPHONE CALLS, 1971

	Week Beginning																
	6/20	6/27	7/4*	8/22	8/29	9/5	9/12	9/19	9/26	10/3	10/10	10/17	10/24	10/31	11/7	11/14**	11/21
Sunday	83	59	31	65	36	40	88	81	68	80	92	105	120	108	114	88	84
Monday	146	112	35	98	107	90	106	76	155	71	172	159	206	218	200	218	272
Tuesday	171	136	64	108	121	120	129	125	159	154	176	140	239	223	214	265	196
Wednesday	154	146	129	139	146	148	148	146	134	171	159	27†	255	184	192	205	224
Thursday	186	158	125	110	128	152	158	118	198	193	162	245	319	205	196	249	183††
Friday	161	144	146	287	136	136	154	150	206	165	225	292	286	218	214	256	157††
Saturday	60	44	49	74	66	88	82	70	75	113	82	136	93	103	128	83	75
Business Week	788	696	499	742	638	646	695	615	852	754	894	863	1305	1048	1016	1193	1032

*Data not available because of strike

†Re 10/17 week - 27 apparently statistical error

††Re 11/21 - Thanksgiving week - Thursday and Friday traditionally light

**New Train Schedule effective 11/14/71

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