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HIGHWAY TRAFFIC KINEMATICS AND THE CHARACTERISTIC RELATION

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16. Abstract

A new relation describing the fundamental formula of road traffic is presented, where the approach is guided by an emphasis on the parameter determination aspects of the problem. The proposed relation includes the degrees of freedom sufficient to allow independent satisfaction of the inherent highway constraints. It is shown how the relation can be fitted to an empirically determined curve through appropriate choice of the parameters.

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PREFACE

The work described in this report was performed as part of a project at the Transportation Systems Center on the analysis and control of traffic flow. The project is one component of TARP — the Transportation Advanced Research Program.

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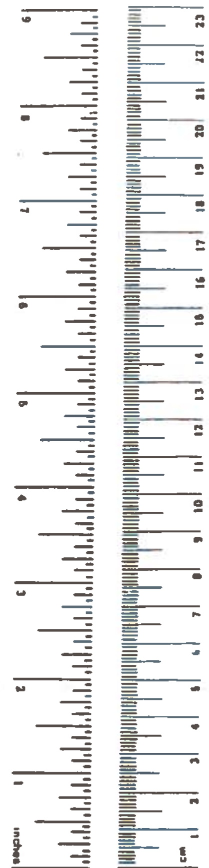
METRIC CONVERSION FACTORS

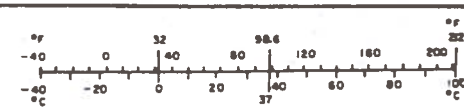
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	AREA			
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
	MASS (weight)			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
	VOLUME			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
	TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
	AREA			
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
	MASS (weight)			
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
	VOLUME			
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
	TEMPERATURE (exact)			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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1. INTRODUCTION

There is a continuing interest in the design of a simple and adequate formula describing the basic characteristics of traffic flow on freeways. The formula, complemented by the dimensional relation

flow = density x speed,

describes the relation that necessarily obtains between any chosen pair of the three basic quantities.

Many forms have been proposed in the literature of which mention will be made of a representative selection. The relations of Pipes (1953) and Gazis et al. (1959) were each derived from a car-following model. The latter, involving a logarithmic form of the speed-density relation, when combined with the exponential form proposed by Underwood (1961) yields the composite form of Edie (1961). As examples of results derived from fluid-analogy mechanical models we note the work of Greenberg (1959) and Drew (1965, 1968). The formulas derived by Haight (1958, 1960, 1963) were based on statistical formulations; the relations proposed by Greenshields (1934) and by Guerin and Palmer (1968) were based on direct empirical considerations.

Certain features of the relation have been discussed in a previous report (O'Mathuna and Haim, 1973) in which the approach is guided by an emphasis on the parameter determination aspects of the problem. There was also included an alternate mathematical description of the fundamental relation which avoided some of the more obvious drawbacks of previously derived forms. It was further indicated how the proposed formula could be generalized to admit further degrees of freedom, thereby allowing the added flexiblilty necessary for the identification and determination of the appropriate characteristic parameters. The present work is concerned with the further exploration of these features of the problem.

We note that among the features to be described by the formula are the following five terminal requirements:

1. A finite maximum mean-speed at zero density
2. A zero mean-speed at jam density
3. A zero flow at zero density
4. A zero flow at jam density
5. The vanishing of the speed/density derivative in the limit of zero density.

The physical necessity of the first four is clear: the fifth requirement, reflecting the fact that on a nearly empty road the mean-speed is unaffected by moderate changes in the density, has been fully discussed by Haight (1963).

A further, physically necessary requirement does not seem to have received its due attention in previous discussions: namely, the requirement that the dimensionless form of the formula fit the empirically determined curve in such a manner that the assignment of appropriate values to the basic parameters (free-speed and jam-density) implies realistic values for such derived quantities as capacity and capacity-speed. Also, the derived range of wave-speeds should fall within realistic limits.

The satisfaction of these requirements, which is a principal concern of the present work, can be most conveniently accomplished through the perspective of a nondimensional formulation. It will be shown how the logarithmic form of the speed-flow relation previously proposed can be extended in such a manner that the extra degrees of freedom are adequate for fitting the formula to an empirically determined curve, through the satisfaction of the parametric requirements.

2. DIMENSIONAL SCALING AND THE CHARACTERISTIC RATIO

It is customary to let q denote the flow, while u and k represent the corresponding mean-speed and density, respectively. The dimensional relation then reads

$$q = ku, \tag{2-1}$$

while the fundamental formula describes the relation subsisting between two of the three quantities.

The maximum value of the mean-speed, called the free-speed, will be denoted by u_0, and the maximum density or jam-density will be denoted by k_*. With these scale factors we define normalized dimensionless vairables λ and m by setting

$$k = k_*\lambda, \ 0 \leq \lambda \leq 1 \tag{2-2}$$

$$u = u_0 m, \ 0 \leq m \leq 1 \tag{2-3}$$

Then if we introduce the dimensionless flow-variable ϕ by writing

$$q = k_* u_0 \phi \tag{2-4}$$

the dimensional relation (2-1) becomes

$$\phi = \lambda m \tag{2-5}$$

and the fundamental formula will relate two of the three quantities ϕ, λ, and m. The relation between either of the other pairs will then follow from relation (2-5).

If we use the subscript c to denote the maximum values of the flow variables q and ϕ, namely, the values assumed at capacity flow, we see from (2-4) that

$$q_c = k_* u_0 \ \phi_c \tag{2-6}$$

This suggests defining a characteristic highway ratio R by setting

$$R = \frac{q_c}{k_* \, u_0} \tag{2-7}$$

and equation (2-6) when written as

$$\phi_c = R \tag{2-8}$$

may be read as a constraint on ϕ.

It is therefore appropriate to inspect the potential range of values for the characteristic ratio R. First, we note that

1. The free-speed for highways normally lies in the 60 to 70 mph range, with perhaps a rising tendency to a bias towards the lower half of the range.

2. The effective car length is generally taken in the 20 to 30 ft range so that the implied jam-density can vary from 175 to 260 cars per mile; a value of 200 cars per mile represents a typically acceptable figure for the jam-density, corresponding to an effective car length of 26.4 ft.

3. The accepted value for per lane capacity flow is around 2,000 cars per hour; although no figure in excess of 2,200 cph has been observed, the reputed tendency of the upper limit to rise is attributed to the constant improvement in driver performance - equally reputed!

Combining the above ranges appropriately, we obtain the following limits for the acceptable values of the characteristic ratio:

$$\frac{2000}{70\text{x}260} \leq R \leq \frac{2200}{60\text{x}175} \tag{2-9}$$

or, in decimal form:

$$.11 \leq R \leq .21$$

It is likely that most highways are characterized by values close to the center of this range; accordingly, we shall take the value

$$R = .16 \tag{2-10}$$

as typical; for a highway with a per-lane capacity of 2,000 cars per hour, and with a mean free-speed of 63.5 mph, this would correspond to a jam-density of approximately 200 cars per mile (the exact value would by 197).

Finally, we introduce a normalized measure of the flow (the standard flow/capacity ratio) by setting

$$\rho = \frac{\phi}{\phi_c} \tag{2-11}$$

so that, in terms of the normalized variables the dimensional relation (2-1), or its dimensionless form (2-5), reads

$$\rho = \frac{1}{\phi_c} \lambda m = \frac{1}{R} \lambda m \tag{2-12}$$

which will be used in the construction of the normalized curves.

3. STRUCTURE OF THE CHARACTERISTIC RELATION

The most reliable data describe speed and flow characteristics, and consequently, the most generally accepted characteristic curves are those derived from such data - typified by the speed-flow curves produced in the Highway Capacity Manual (1965). It is therefore appropriate to consider the fundamental formula as a speed-flow ralation in the form

$$\phi = \phi(m), \tag{3-1}$$

and investigate how the function ϕ should be constructed so as to meet the basic requirements and allow a fitting to the empirical curves of the Highway Capacity Manual.

In the previous report (O'Mathuna and Haim, 1973) we have shown that the basic logarithmic formula

$$\phi_0(m) = - \left[(1-m) \ln (1-m)\right] \tag{3-2}$$

meets the five terminal requirements and follows the general pattern expected of the characteristic curve. However, some extension of the formula is necessary to admit the degrees of freedom necessary for the satisfaction of the parametric requirements. It was indicated in the previous report how this extension could be effected; if the relation is taken in the form:

$$\begin{aligned} \phi(m) &= \phi_0(m)\ f\ (m) \\ &= - \left[(1-m) \ln (1-m)\right] f(m) \end{aligned} \tag{3-3}$$

then the five terminal conditions are satisfied and the general pattern of the characteristic curve is retained, if the multiplying function $f(m)$ satisfies the following conditions:

1. $f(m)$ remains bounded, is strictly positive and has non-positive derivative on the interval (0, 1) (3-4)

2. $f(m)$ satisfies the terminal conditions:

i. $f(0) = 1$

ii. $f(1) = a, \; o < a \leq 1$

iii. $|f'(1)| < \infty$ (3-5)

The present investigation focuses on choosing an appropriate form for the function $f(m)$ and on the determination of the constants appearing therein. The procedure will be to choose a form for $f(m)$ with the flexibility necessary to meet the parametric requirements; when appropriate values have thus been assigned to the constants appearing in the function f, a straightforward inspection can check that conditions (3-4) and (3-5) have been satisfied.

In order to gain some insight into the desired form for f with the appropriate number of degrees of freedom, we note that besides satisfying condition (2-8) it is also necessary that the implied value of the dimensionless velocity fit the value specified by the data. Thus, if u_c is the value of the mean-speed at the capacity point and we set

$$m_c = \frac{u_c}{u_0} \tag{3-6}$$

then in order to fit the curve at the capacity point it is necessary that

$$m = m_c \Longrightarrow \frac{d\phi}{dm} = 0 \tag{3-7}$$

$$\Longrightarrow \phi = \phi_c = R \tag{3-8}$$

The simultaneous satisfaction of these conditions can be achieved with two disposable constants. Having thus nailed the curve at the capacity point, it will be found that matching at one other intermediate point will lead to an adequare fit over the entire range. This further matching can be accomplished by means of one additional disposable constant. The three constants together with the implicit use of the parameter m_c in general imply four degrees of freedom for the function of f.

By taking the function f in the form:

$$f(m) = 1 - am - bme^{-\alpha(m-m_c)} \tag{3-9}$$

the constants a and b can be chosen to nail the capacity point, while the exponent α can be chosen to achieve a matching at a suitably chosen intermediate point. That conditions (3-4) and (3-5) are satisfied then follows from a straightforward verification.

The next section will give the computational details for fitting the formula to the characteristic curve of the Highway Capacity Manual.

4. THE FUNDAMENTAL FORMULA

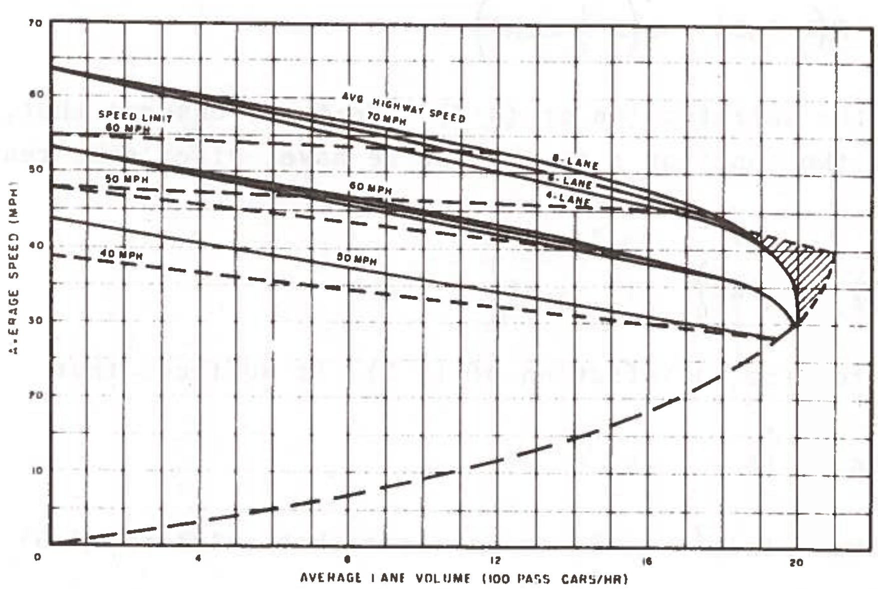
For the typical curves in the Highway Capacity Manual reproduced in Figure 1 we see that:

$$q_c = 2000 \text{ cph}, \; u_0 = 63.5 \text{ mph} \tag{4-1}$$

No value is specified for the jam-density or the effective car length. We shall take the typical value of .16 for the characteristic ratio which, as was noted following relation (2-9), is consistent with a jam-density of approxiamtely 200 cars per mile. Moreover, in the typical case illustrated in Figure 1, the capacity speed appears as exactly half the free-speed so that conditions (3-7) and (3-8) take the specific form:

$$m = m = \frac{1}{2} \Longrightarrow \frac{d\phi}{dm} = 0 \tag{4-2}$$

$$\Longrightarrow \phi = \phi_c = .16 \tag{4-3}$$



Source: Highway Capacity Manual, 1965.

FIGURE 1. TYPICAL RELATIONSHIPS BETWEEN VOLUME PER LANE AND AVERAGE SPEED IN ONE DIRECTION UNDER IDEAL UNINTERRUPTED FLOW CONDITIONS ON FREEWAYS AND EXPRESSWAYS

for the fixing of the capacity point.

Introducing the form (3-9) for f(m) — with m = 1/2 — into the characteristic relation (3-3) we obtain:

$$\phi(m) = -\left[(1-m)\ \ln\ (1-m)\right]\left[1 - am - bme^{-\alpha\left(m-\frac{1}{2}\right)}\right] \tag{4-4}$$

which yields the following form for the derivative:

$$\frac{d\phi}{dm} = \left[1 + \ln(1-m)\right]\left[1 - am - bme^{-\alpha\left(m-\frac{1}{2}\right)}\right] + \left[(1-m)\ \ln\ (1-m)\right]\left[a + be^{-\alpha\left(m-\frac{1}{2}\right)} - \alpha bme^{-\alpha\left(m-\frac{1}{2}\right)}\right] \tag{4-5}$$

For m = 1/2, the value of the derivative can be arranged to read:

$$\left.\frac{d\phi}{dm}\right|_{m=\frac{1}{2}} = 1 - \ln 2 - \frac{1}{2}a - \frac{1}{2}b\left(1 - \frac{1}{2}\alpha\ln 2\right) \tag{4-6}$$

so that taking

$$a = 2\left(1-\ln 2\right) - b\left(1-\frac{1}{2}\alpha\ln 2\right) \tag{4-7}$$

ensures the satisfaction of (4-2). Next, we observe that, for the value of the function ϕ at m = 1/2 we have, after some rearrangement,

$$\phi\left(\frac{1}{2}\right) = \frac{1}{2}\left(\ln 2\right)^2\left[1 - \frac{1}{4}\alpha b\right] \tag{4-8}$$

so that for the satisfaction of (4-3) it suffices that

$$\frac{1}{2}\left(\ln 2\right)^2\left[1 - \frac{1}{4}\alpha b\right] = .16. \tag{4-9}$$

Noting that $(\ln 2)^2 = .48$, it is clear that relation (4-9) simplifies to:

$$\alpha b = \frac{4}{3} \tag{4-10}$$

which, when used in relation (4-7), yields the relation between a and b,

$$a = 2 - \frac{4}{3}\ln 2 - b \tag{4-11}$$

It remains to effect a calibration at a suitably chosen intermediate point; from inspection of the curves in Figure 1, it would appear that a point with speed in the 40 to 50 mph range would be appropriate. On the curve for the six-lane highway we observe that:

$$u = 47.5 \text{ mph} \Longrightarrow q = 1525 \text{ cph} \tag{4-12}$$

which in terms of normalized variables (mean speed/free-speed and flow/capacity) reads:

$$m = \frac{3}{4} \Longrightarrow \rho = .7625 \tag{4-13}$$

and recalling relation (2-11) with $\phi_c = .16$, we see that in terms of m and ϕ the requirement (4-13) may be written:

$$m = \frac{3}{4} \Longrightarrow \phi = .122 \tag{4-14}$$

Setting $m = \frac{3}{4}$ in relation (4-4) we obtain:

$$\phi\left(\frac{3}{4}\right) = \frac{1}{2}\left(\ln 2\right)\left[1 - \frac{3}{4}a - \frac{3}{4}be^{-\alpha/4}\right] \tag{4-15}$$

Substituting for a from (4-7) and for b from (4-10) we obtain, after some rearrangement,

$$\phi\left(\frac{3}{4}\right) = \frac{1}{2}\left(\ln 2\right)\left[\ln 2 - \frac{1}{2} + \frac{1}{\alpha}\left(1 - e^{-\alpha/4}\right)\right] \tag{4-16}$$

A direct calculation shows that by taking:

$$\alpha = 4 \tag{4-17}$$

in (4-16) we obtain:

$$\phi\left(\frac{3}{4}\right) = .122 \tag{4-18}$$

for the satisfaction of requirement (4-14).

Inserting the value (4-17) for α into equations (4-10) and (4-11) yields the following values for a and b:

$$a = \frac{5}{3} - \frac{4}{3}\ln 2, \qquad b = \frac{1}{3} \tag{4-19}$$

and $\phi(m)$ takes the explicit form:

$$\phi(m) = -\left[(1-m)\ln(1-m)\right]\left[1 - \left(\frac{5}{3} - \frac{4}{3}\ln 2\right)m - \frac{1}{3}\,me^{-4\left(m-\frac{1}{2}\right)}\right] \tag{4-20}$$

for which a direct check will show that conditions (3-4) and (3-5) are satisfied. Written in terms of the normalized variable ρ (Definition (2-11)) relation (4-20) becomes

$$\rho(m) = -6.25\left[(1-m)\ln(1-m)\right]\left[1 - \left(\frac{5}{3} - \frac{4}{3}\ln 2\right)m - \frac{1}{3}e^{-4\left(m-\frac{1}{2}\right)}\right] \tag{4-21}$$

where we have used the fact that $\frac{1}{\phi_c} = \frac{1}{.16} = 6.25.$

The graph of relation (4-21) is illustrated in Figure 2. The corresponding points from the curves of the Highway Capacity Manual (Figure 1) are also shown indicating how well the fitting has been achieved.

Moreover from equations (2-5) and (4-20), we have

$$\lambda = -\frac{1}{m}\left[(1-m)\ln(1-m)\right]\left[1 - \left(\frac{5}{3} - \frac{4}{3}\ln 2\right)m - \frac{1}{3}e^{-4\left(m-\frac{1}{2}\right)}\right] \tag{4-22}$$

The relation giving m in terms of λ,

$$m = m(\lambda) \tag{4-23}$$

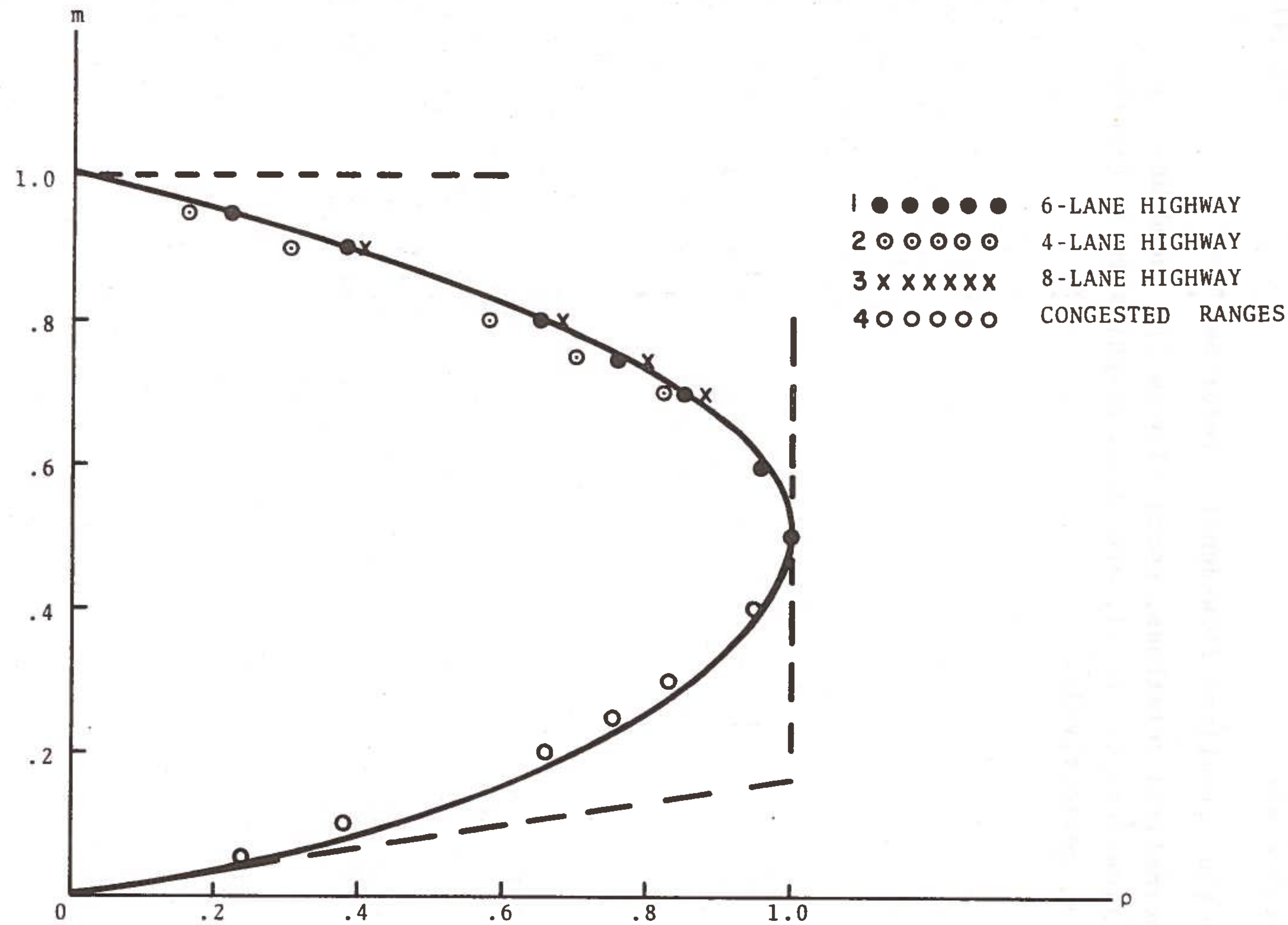


FIGURE 2. SPEED/FLOW (m-ρ) RELATION

implied by (4-22) can then be used with relation (2-12) to yield

$$\rho = 6.25 \lambda m(\lambda) \tag{4-24}$$

describing the normalized flow-density relation.

The normalized relations, speed-flow (m-ρ), speed-density (m-λ) and flow-density (ρ-λ), are shown graphically in Figures 2, 3, and 4, respectively.

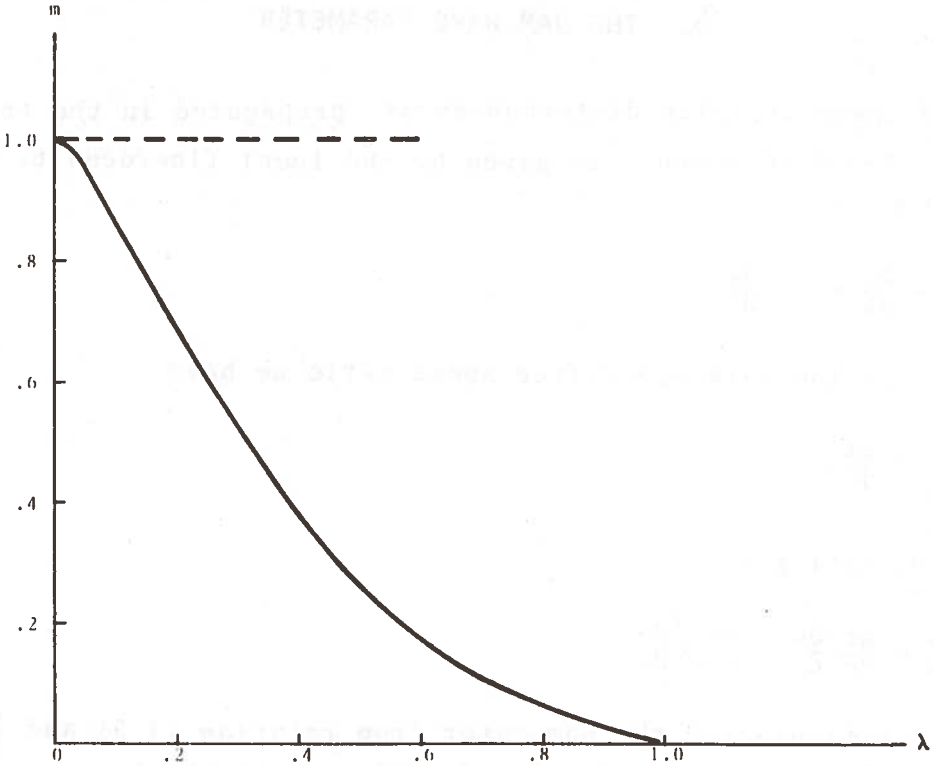


FIGURE 3. SPEED/DENSITY (m-λ) RELATION

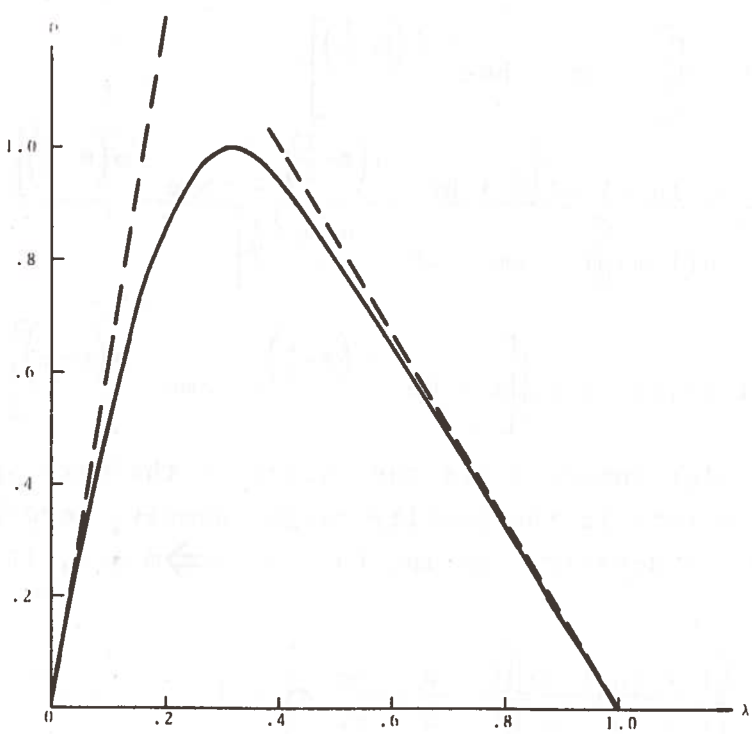


FIGURE 4. FLOW/DENSITY (ρ-λ) RELATION

5. THE JAM WAVE PARAMETER

The speed at which disturbances are propagated in the traffic stream -- the wave speed — is given by the local flow/density derivative, namely,

$$c = \frac{dq}{dk} = u_0 \frac{d\phi}{d\lambda} \tag{5-1}$$

so that for the wave-speed/free-speed ratio we have

$$\frac{c}{u_0} = \frac{d\phi}{d\lambda} \tag{5-2}$$

Moreover, noting that

$$\frac{d\phi}{d\lambda} = \frac{d\phi}{dm}\,\frac{dm}{d\lambda} = \frac{d\phi}{dm}\bigg/\frac{d\lambda}{dm} \tag{5-3}$$

we may substitute for the numerator from relation (4-5) and insert the derivative of (4-22) for the denominator to obtain

$$\frac{d\phi}{d\lambda} = \frac{\left[1 + \ln(1-m)\right]\left[1 - am - bme^{-\alpha\left(m-\frac{1}{2}\right)}\right] + \left[(1-m)\ \ln\ (1-m)\right]\left[a + be^{-\alpha\left(m-\frac{1}{2}\right)} - \alpha bme^{-\alpha\left(m-\frac{1}{2}\right)}\right]}{\frac{1}{m}\left[1 + \frac{1}{m}\ln(1-m)\right]\left[1 - am - bme^{-\alpha\left(m-\frac{1}{2}\right)}\right] + \frac{1}{m}\left[(1-m)\ \ln\ (1-m)\right]\left[a + be^{-\alpha\left(m-\frac{1}{2}\right)} - \alpha bme^{-\alpha\left(m-\frac{1}{2}\right)}\right]} \tag{5-4}$$

Of particular interest are the values of the wave-speed at the three pivotal points in the density range, namely, zero-density, capacity, and jam density. Noting that $\lambda \to 0 \Rightarrow m \to 1$, it is easily checked that

$$\lim_{\lambda \to 0} \frac{d\phi}{d\lambda} = \frac{\left[1 + \ln(1-m)\right]\left[1 - a - be^{-2}\right]}{\left[1 + \ln(1-m)\right]\left[1 - a - be^{-2}\right]} = 1 \tag{5-5}$$

reflecting the well-known fact that in this density range disturbances are propagated at the vehicle speed. For the capacity point we have

$$\left.\frac{d\phi}{d\lambda}\right|_{\lambda=\lambda_c} = 0$$

so that disturbances tend to remain stationary, though the stationarity is not very stable.

For the jam density region we note that $\lambda \to 1 \Longrightarrow m \to 0$, so that from relation (5-4) we have

$$\lim_{\lambda\to 1} \frac{d\phi}{d\lambda} = \frac{1}{\frac{1}{m}\left[-\frac{1}{2}m - \frac{1}{3}m^2 - \cdots\right] + \frac{1}{m}\left[-m - \frac{1}{2}m^2 - \cdots\right]\left[a + be^2\right]}$$

$$= \frac{1}{-\frac{1}{2} - \left(a+be^2\right)} = -\frac{1}{3.706} = -.27 \qquad (5\text{-}7)$$

where we have inserted the values (4-19) for a and b. The value given by (5-7) indicates that the proposed formula implies that the backward jam shock-wave speed is roughly one-quarter of the free speed. While this value appears reasonable, it should be tested against actual observations. In fact, this parameter, which should be relatively easy to measure, appears to have been ignored as a practical check on the theoretical formula.

Recalling that $\phi = \phi_c \rho$, we have

$$\frac{d\rho}{d\lambda} = \frac{1}{\phi_c}\frac{c}{u_0} = 6.25\,\frac{c}{u_0}$$

so that

$$\lim_{\lambda\to 0} \frac{d\rho}{d\lambda} = 6.25, \quad \lim_{\lambda\to 1} \frac{d\rho}{d\lambda} = -1.67$$

for the slopes of the limiting tangents to the curve (Figure 4).

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