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DYNAMIC URBAN GROWTH MODELS

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16. Abstract

This report describes the further development of the dynamic models of urban evolution derived from concepts of self-organization that have recently emerged in the physical sciences. The first section contains the new inter-urban model which describes the evolution of an interacting hierarchy of urban centers and the development of internal structure within each center. It is shown how mutual interactions between elements of the system lead to a self-organization of the system through successive instabilities of the collective structure. The second section introduces the techniques of Boolean algebra to describe the evolution of the internal structure of a city and how these techniques may be used by urban planners. The final section illustrates the importance of behavioral fluctuations in transportation mode choice showing how a system reorganizes itself when critical size thresholds are exceeded.

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PREFACE

The dynamic urban growth models discussed in this report build upon research begun in 1976 and reported on in the U.S. Department of Transportation Report No. DOT-TSC-RSPA-78-20.I, II October 1978. In this previous report we developed two models: the inter-urban model which describes the evolution of urban centers within a region, and the intra-urban model which describes the structural evolution within each center. This report presents a further extension of these models including an analysis of an urban system's dynamic, "collective" organization.

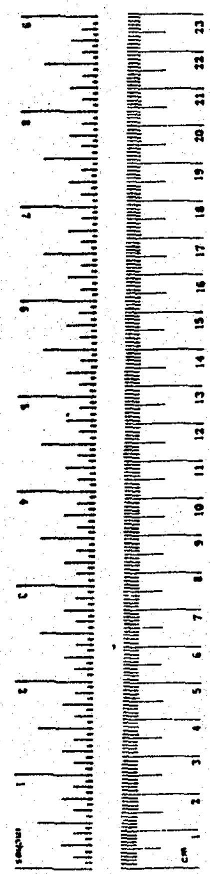
Section 3 of this report on dynamic models of competition between transportation modes has also been published in Environment and Planning International Journal of Urban and Regional Research, Volume II, 1979 co-authored by J.L. Deneubourg and A. de Palma from the University of Brussels and D. Kahn from the Transportation Systems Center.

The technical monitor, D. Kahn, of the contract under which this work was performed would like to take this opportunity to acknowledge the copy and production editing of Caron Tsapatsaris for this report.

METRIC CONVERSION FACTORS

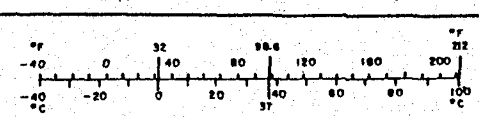
Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C



Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



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EXECUTIVE SUMMARY

This report describes the further development and exploration of the dynamic models of urban evolution for which the basic methodology was laid down in work performed by our group under a previous contract (TSC-1185 - Final Report).[1] These methods are derived from new concepts that have recently emerged in the physical sciences in connection with the discovery of "dissipative structures"[2,3]. These occur in physical systems having elements which interact in a non-linear manner, involving positive and negative feedback loops, and which are open to the exterior, exchanging matter and/or energy with the outside world, and in this way remaining far from thermodynamic equilibrium.

The evolution of such systems involves both deterministic stable periods, as well as bifurcation points in the vicinity of which instabilities occur, when "fluctuations," small local inhomogeneities, are amplified and carry the system to some new, qualitatively different state of organization. This process of "order by fluctuation" is of great generality for the evolution of complex systems, and applies to systems composed of basic units which are themselves already macroscopic objects containing mechanisms governing their interactions with the environment and with the other elements of the systems. Thus, given some basic "behaviour pattern" of the individual elements, their mutual interaction can lead to a self-organization of the system through successive instabilities of the collective structure.

In our previous report we developed two models, one describing the evolution of urban centers within a region, the inter-urban model, and the other describing the structural evolution of an urban center, the intra-urban model. In this report we describe the further extension of these models at the level of dynamic organization, and also in the direction to the "collective" aspects of the behaviour patterns" used to describe consumer choice, for example, in the urban system. As we shall see, the modelling of the dynamic evolution of the urban system entails the description of a collective organization which results from the mutual interaction of the behaviour patterns of the various populations, which may in their turn reflect a "collective organization" at a lower level as the individuals within the populations, interact. The situation is one of instabilities within instabilities and so on!

In the first section the inter-urban model is modified so as to give a much more realistic representation of the evolution of the urban centres of a region, where large centers sprawl outwards forming residential suburbs. The modified version of the model now corresponds to a picture of the evolution of a region wherein we not only have the formation of an interacting hierarchy of urban centers, but also one in which there is an internal structural evolution within each center. Only such a model can assess the real global effects of a modification, for example, in the transportation system within a particular urban center.

The second section is devoted to the development of a new method which offers the perspective of an enormous simplification and saving of time in the analysis of a urban evolution. It is a method based on the techniques of Boolean algegra, in which continuous variables are replaced by discrete ones, the yes/no, 0/1, of the binary system. Boolean algebra has been applied in the first instance, to the evolution of the internal structure of a city. By describing the "presence" or "absence" of a given population according to whether it is above or below a certain percentage of the local residents, the very large number of <u>distinct</u> stationary states which may characterize the continuous variable differential equations of intra-urban structure , is reduced remarkably to those which can be distinguished according to the threshold criteria chosen. Furthermore, the "dynamics" of such a problem consist in the assignment of probabilities for the passage from one stationary state to another, and this is simplified to a problem of time-delays. It is assumed that a "change" when it occurs, concerns only one variable at a time, and this choice is determined by the probability distributions of the time-delays of the various variables.

Despite these rather sweeping simplifications, the Boolean model can nevertheless suffice to answer many of the practical questions that decision makers and planners may pose - particularly questions concerning the qualitative evolution of the system. The possible structural repercussions of different strategies can be estimated therefore in a very simple economical manner.

The models that have been developed so far have attempted to describe the evolution of urban structures on the basis of some supposed basic "behaviour pattern" of the populations involved. Until now we have also treated the choices concerning transportation in a somewhat oversimplified manner. However, it is clear that different populations may exhibit different behaviour, not only as concerns their place of residence, shopping center or place of work, but also the mode of transport that they choose for their regular travel. This choice, however, of train, bus, subway or car will be vital importance in shaping the urban structure, and of course conversely, the character of the urban structure will affect the choices of transportation mode. The problem of the competition between different modes of transport, within a city for example, is a vital step in understanding the city's structural evolution.

Our study in the third section explores the behaviour of the demand for a particular mode of transport when the population requiring to go from A to B is offered a choice. In this case we assume that there are two modes of transport in competition with each other, and that the level of "satisfaction" for the average user of a particular mode has some functional dependence on the level of use - that is on the fraction that does in fact adopt that mode.

For some simple, fairly realistic functional dependencies of the user utility function on the level of use, we show that the system can possess more than one stationary state. Futhermore, we can have more than one stable stationary state, in the vicinity of which the collective reaction of users to any slight deviation from this state, is negative. That is to say that the response of the users is such as to damp any small perturbation of this ratio for the modal split fractioning of users. Nevertheless, it is still possible that these stable stationary states correspond to very different levels of global "satisfaction," but that some large scale re-organization is required in order to leave the less favourable stable state. This is clearly an important feature for planning decisions, and also for the implications of inter-modal competition for the evolution of urban structure. As the urban structure evolves, travel demand between various points in a city can exceed or fall below thresholds which can lead to a sudden, discontinuous, change in modal use, and hence in transport "costs." Any integrated model of the global effects of transport investment, for example on the urban structure and economy, must take such factors into account.

The integration of these three separate aspects that are the subject of this report will have to be left for the future when general equations may be written down which encompass all these different elements, and which reduce to the various simpler equations under well defined assumptions.

The equations for the inter-urban evolution are being tested by an application to the time evolution of the urban hierarchy of the Bastogne region of Belgium, and clearly the Boolean methods of the second section can be extended to this model as well. Similarly, the section devoted to intermodal competition is also relevent to an inter-urban model where apart from the various modes of passenger travel, there are also those available to the transport of merchandise, (air/sea/rail/road/pipeline). Also this third section raises the question of the relation between the models which we have described here (and in our previous report), and various other methods of urban and economic modelling (global utility functions; optimization techniques; Pareto maxima; entropy maximization etc.), The clarification of this relation should lead to a much more thorough understanding of the real significance and status of these various methods, which in turn lead to the establishment of a much more solid foundation to a "theoretical social dynamics," and hence to the problems of evolving urban systems.

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2. Nicolis, G., and I. Prigogine, Self-Organization in Non-Equilibrium Systems, Wiley, New York, 1977, 490 p.

3. Prigogine, I., P.M. Allen, and R. Herman, "The Laws of Nature and the Evolution of Complexity," in Goals for a Global Community, Eds. E. Laszlo and J. Bierman, Pergamon Press, New York, 1977, 83 p.

1. A DYNAMIC URBAN MODEL

1.1 INTRODUCTION

In our previous reports a dynamic version of central place theory was developed based on the mutual interaction of the spatial distributions of population and employment. This interaction constituted a positive feedback, which, when the effects of fluctuations are included explicitly, leads to a self-organization of the system into an urban hierarchy which reflects the the dual effects of historical chance and economic necessity. This introduces the possibility of describing qualitative changes in the spatial organization of a region, changes which ususlly mark the breakdown of previously successful extrapolations in the behaviour of the system.

Our model consists of two sets of equations, one for the population of each point i, x_i, and the other for the growth and decay of economic functions k, at each point i, $S_i^{(k)}$. In the equation for x_i, the population responds to the employment opportunities at the point i,

$$\frac{dx_i}{dt} = bx_i \left(N + \sum_k S_i^{(k)} - x_i\right) - mx_i \qquad (1)$$

where b and m are related to the birth and death rates respectively as well as to the mobility of the population. N represents the "natural" carrying capacity of each point of the system in the absence of economic exchange between different localities. $\sum_k S_i^{(k)}$ represents the employment potential at i resulting from the different economic functions situated there.

We have separate equations for the $S_i^{(k)}$ which grow according to the economic demand that is attracted to the point i,

$$\frac{dS_i^{(k)}}{dt} = \alpha\, S_i^{(k)} \left(\text{Demand for k at i} - S_i^{(k)}\right). \qquad (2)$$

The demand arriving at the point i for the function k is then related to the "attractivity" of the point i to each population x_j at j, relative to that of others offering the function k. We shall return to this point later.

The other important feature of our model was the random appearance of economic functions at different points in the system. The first model was as general as possible, and the probability of the launching of a particular function was taken as being uniform over the whole region. In any given experiment, however, a particular sequence occurs and this leads to a distribution of urban centers following the growth of some centers and the elimination of others according to the economic laws contained in equations (1) and (2).

Typically, we have a result such as is shown in Figure 1 after the stochastic launching of two economic functions. At each point we already have domestic functions, and the two new functions concern economic interaction between the points, of medium and long range respectively.

Although as we see from Figure 1 the equations (1) and (2) give rise to a reasonable form for the distribution of centers; there is an important mechanism missing from our description: the competition for space that will occur at a given point. That is to say that in equations (1) and (2) we have assumed that employment and residences can be stacked on top of one another without limit at a given locality. The number of jobs divided by the number of residents, a ratio known as the coefficient of employment, is equal to unity for each point separately. One of the improvements that we shall describe here is therefore the correction of this inadequacy in the simplest possible manner. We shall suppose that with a certain probability, as the effects of crowding become more intense, a certain fraction of the population decides to reside on the points neighbouring that of its place of employment. This is represented by adding two terms into equation (1), which express the idea that for every "route" out of an urban center a fraction of the population having

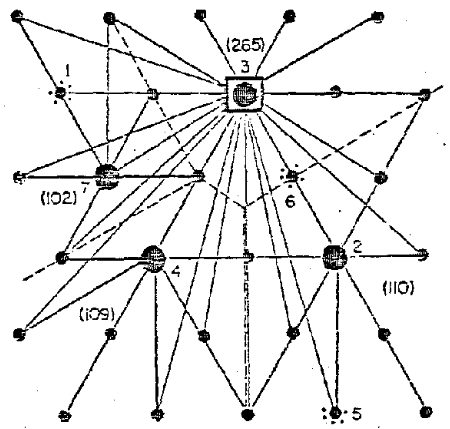


FIGURE 1. DISTRIBUTION OF URBAN CENTERS FOLLOWING THE RANDOM LAUNCHING OF TWO EXPORT FUNCTIONS OF MEDIUM AND LONG RANGE RESPECTIVELY

- Small Centers with only the lowest order function K = 1
- Centers with Functions 1 and 2 (pop. in brackets)
- Center with Functions 1, 2 and 3

employment in the center itself will choose to reside at some distance out, a fraction which depends on the "crowding" at the center. Of course, the people employed at any two points will both "decentralize" along the route between them, and the resident population of each will only change by the net difference of flows. This idea is expressed by the following equation:

$$\frac{dx_i}{dt} = bx_i(N + \sum_k S_i^{(k)} - x_i) - mx_i - \sum_j e^{-\beta d_{ij}} (x_i^2 - x_j^2) \quad (3)$$

which implies that employment at the point i can be filled by people residing at the point i, and to a lesser extent by residents at the neighbouring points. Each point is "losing" $e^{-\beta d_{ij}} x_i^2$ residents to its neighbours, and we see that this gives rise to a very simple representation of "residential sprawl" as the crowding in urban centers becomes severe.

As we shall see, the addition of such a simple term produces a much more complicated pattern of growth, as well as an internal dynamics of growing centers. This is because the coupling between the equations of employment and residence for each point implies that the pattern of employment will respond to the "decentralization," so we will have a complex process of mutual adjustments as the changes of residence and employment act on each other. Let us now describe the urbanization pattern of an initially rural area according to our modified scheme of equations (2) and (3).

1.2 URBANIZATION OF A REGION

The technique used in modelling the urbanization process is basically similar to that used in our earlier reports. However, instead of explicitly "launching" the economic functions, we have chosen population thresholds above which economic functions appear spontaneously at a point, and have fluctuated the values of the population variables by a small precentage around their values as dictated by the equations of evolution. In this way we allow for the necessary uncertainty in the exact value of the population at a point, supposing some 5 percent variation for small villages, and decreasing to 3 percent for more densely occupied points.

Because of this, when several points are approaching the necessary value to receive some new level economic activity, these fluctuations will result in them attaining the threshold in a random manner, giving rise to a "stochastic launching" of economic functions similar to our previous method. The economic activity, once launched, will either find a sufficient market and grow and prosper, or will be eliminated by its competitors. The advantage of this new method is that it retains at all times the possibility of the system adapting to new circumstances, functions appearing where it was impossible before, owing perhaps to changes in transport technology, or in the relative attractivity of the region to migrants.

Our equations are:

$$\frac{dx_i}{dt} = bx_i\ (N + \sum_k S_i^{(k)} - x_i) - mx_i - \sum_j \varepsilon^{-\beta d_{ij}}\ (x_i^2 - x_j^2) \quad (4)$$

$$\frac{dS_i^{(k)}}{dt} = \alpha\ S_i^{(k)}\ (\text{Demand} - S_i^{(k)}) \quad (5)$$

where the demand is,

$$\text{Demand} = \sum_j \frac{x_j \varepsilon^{(k)}}{(P_i^{(k)} + \phi^{(k)} d_{ij})^e} \cdot \frac{A_{ij}}{\sum_{i'} A_{i'j}} \quad (6)$$

where $\varepsilon^{(k)}$ is the demand per individual for (k) at unit price.

$P_i^{(k)}$ is the cost of production of k at the point i.

$\phi^{(k)}$ is the cost of transport for k per unit distance.

d_{ij} is the distance between i and j.

A_{ij} is the attractivity of the center i to population j for the function k.

$\sum_{i'} A_{i'j}$ is the sum over the total attractivity to which the x_j are subjected.

e is some power law.

The demand arriving at the point i for the activity k, falls therefore into two parts: the fall off in demand with price involving distance, and secondly, the fraction of x_j that in fact chooses i out of all the possible centers offering k.

The precise form that we have used for A_{ij} is,

$$A_{ij} = ((1.1 - (1 + \rho\ (x_i - x_{th}))^{-1}/(P_i^{(k)} + \phi\ d_{ij}))^I \qquad (7)$$

where ρ is a constant and x_{th} the threshold at which the function may appear at i. The form of this function corresponds to the idea that the attractivity grows initially with the intensity of the activity at the center (measured by the excess population), but then saturates at some upper level.

Let us now look at the evolution of the population distribution of a region which starts off initially as a purely rural area with no substantial economic interaction between local centers. As three export functions of successively greater range and market threshold appear in the system, the urbanization process will occur.

We have once again chosen a triangular lattice numbered as in Figure 2 and have, for this particular experiment taken the most general case where the natural carrying capacity of each point is the same. The values chosen for the various parameters are:

$b = .01 \quad m = .1 \quad \beta = 1.53 \quad e = 2 \quad P_i^{(k)} = 1 \quad d_{12} = 6$

$\varepsilon^{(1)} = .25$	$\phi^{(1)} = 1$	$x_{th}^{(1)} = 50$
$\varepsilon^{(2)} = .15$	$\phi^{(2)} = .15$	$x_{th}^{(2)} = 68$
$\varepsilon^{(3)} = .1$	$\phi^{(3)} = .1$	$x_{th}^{(3)} = 84$
$\varepsilon^{(4)} = .05$	$\phi^{(4)} = .05$	$x_{th}^{(4)} = 100$

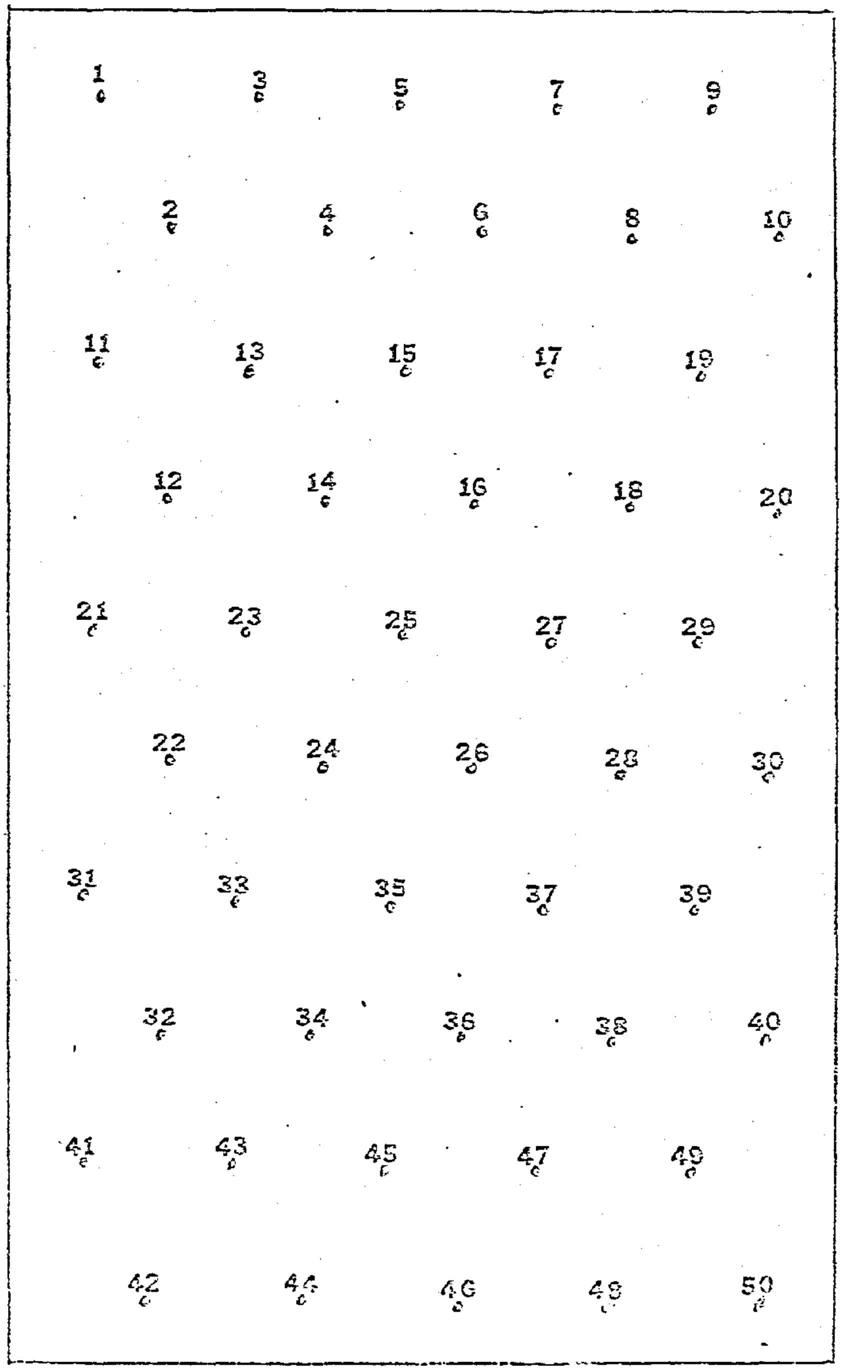


FIGURE 2. SIMULATION LATTICE

At time t = 0, the points have all approximately 66 units of population. They are however, subject to fluctuations of the order of 5 percent, and when a point exceeds 68 it receives the second function and begins to grow if there is a sufficient market.

At time t = 4, the situation is depicted in Figure 3, and we see that five points have received the function 2 and have grown to a population greater than 75. These are the "nuclei" of future cities, and already lay down the skeleton of the urban structure that will emerge.

In Figure 4 the situation at t = 12 is shown. The structure that was only embryonic at t = 4 has "solidified" and we see that five large centers are growing. The points 15 and 31 have already received all four functions considered in our simulation while points 10, 40 and 44 have three functions. In particular, the examination of the evolution around point 15 reveals how the crowding at this point results first in the build up of residential suburbs, with a coefficient of employment less than unity, and then how, later, a certain decentralization of economic functions occurs, as the short and medium range activities find sufficient market in the suburbs. This has important consequences for the evolution of the urban area as a whole, but during the interval t = 12 to t = 20 the central core density continues to grow, but attains a maximum at about this time.

Also of interest is the formation of a "twin-city" on the points 38 and 40 due simply to the particular sequence of events that the random fluctuations of our particular simulation has produced.

KEY TO FIGURES 3 TO 11 AND FIGURES 12 TO 14

- • Center having only function 1
- ● Center with functions 1 and 2
- ▲ Center with functions 1, 2 and 3
- ◮ Large center having functions 1, 2, 3, and 4

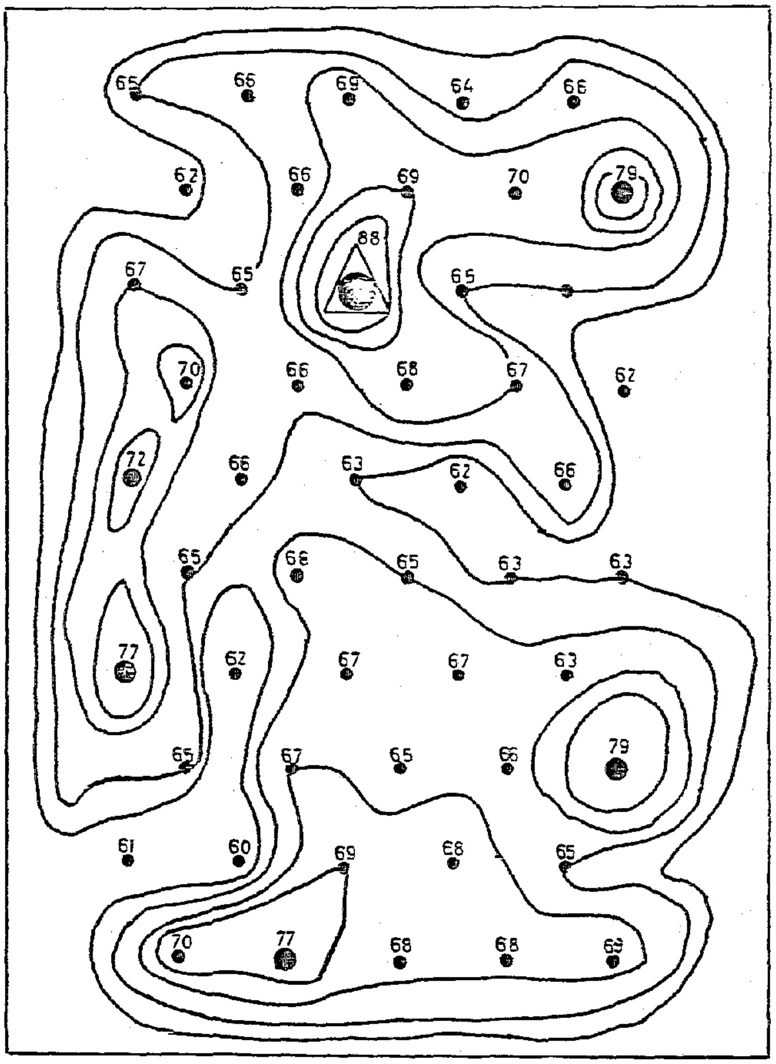


FIGURE 3. THE DISTRIBUTION OF POPULATION ON A RECTANGULAR PLAIN REPRESENTED BY FIFTY POINTS AT TIME t = 4 UNITS. (AT t = 0 ALL POINTS HAD 67 UNITS.)

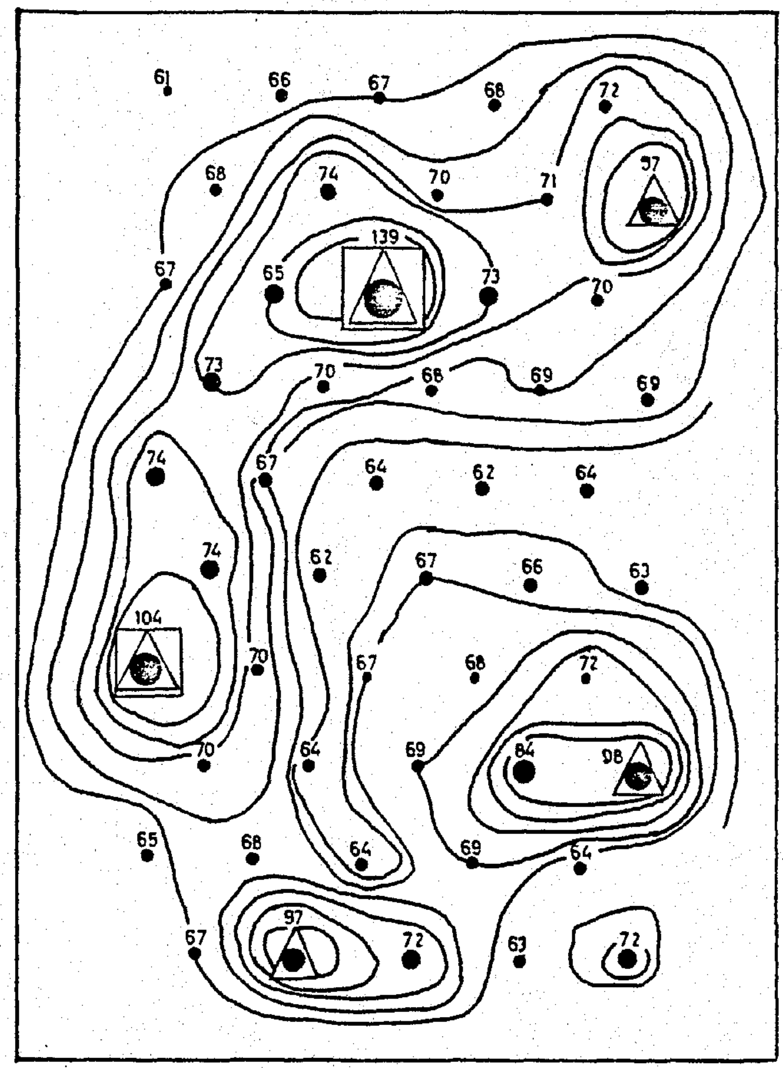


FIGURE 4. THE DISTRIBUTION OF POPULATION AT TIME t = 12 UNITS. THE STRUCTURE IS BEGINNING TO "SOLIDIFY" AROUND FIVE MAIN CENTERS.

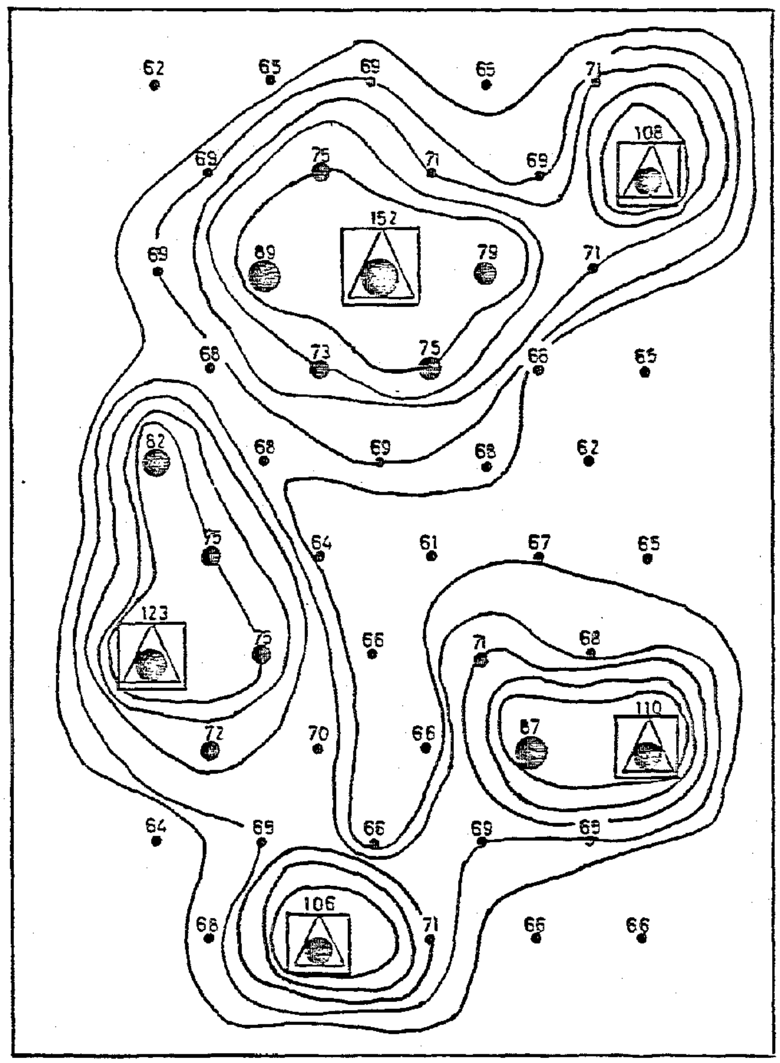


FIGURE 5. AT TIME $t = 20$, THE CENTRAL CORE DENSITY OF THE LARGEST CENTER IS GOING THROUGH A MAXIMUM (152). THERE IS MARKED "URBAN SPRAWL" AROUND THIS CENTER TOO.

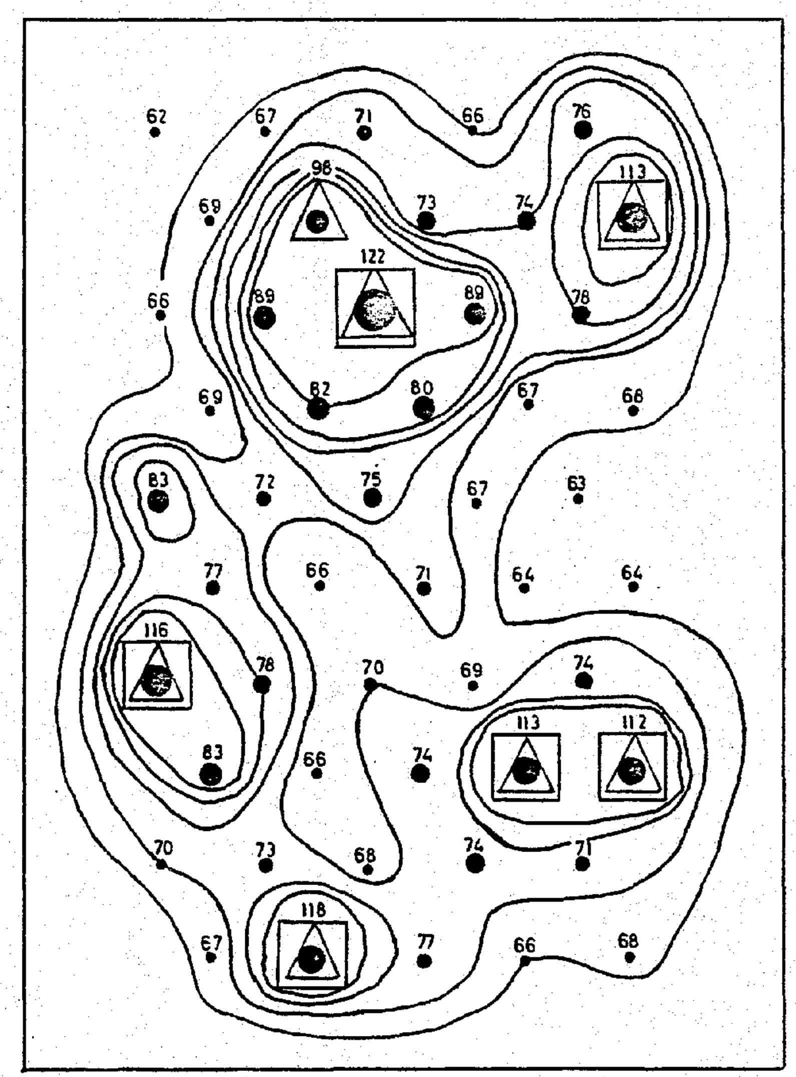


FIGURE 6. AT TIME t = 34 THE BASIC STRUCTURE IS ESSENTIALLY STABLE. TWO CENTERS HAVE UNDERGONE CENTRAL CORE DECAY.

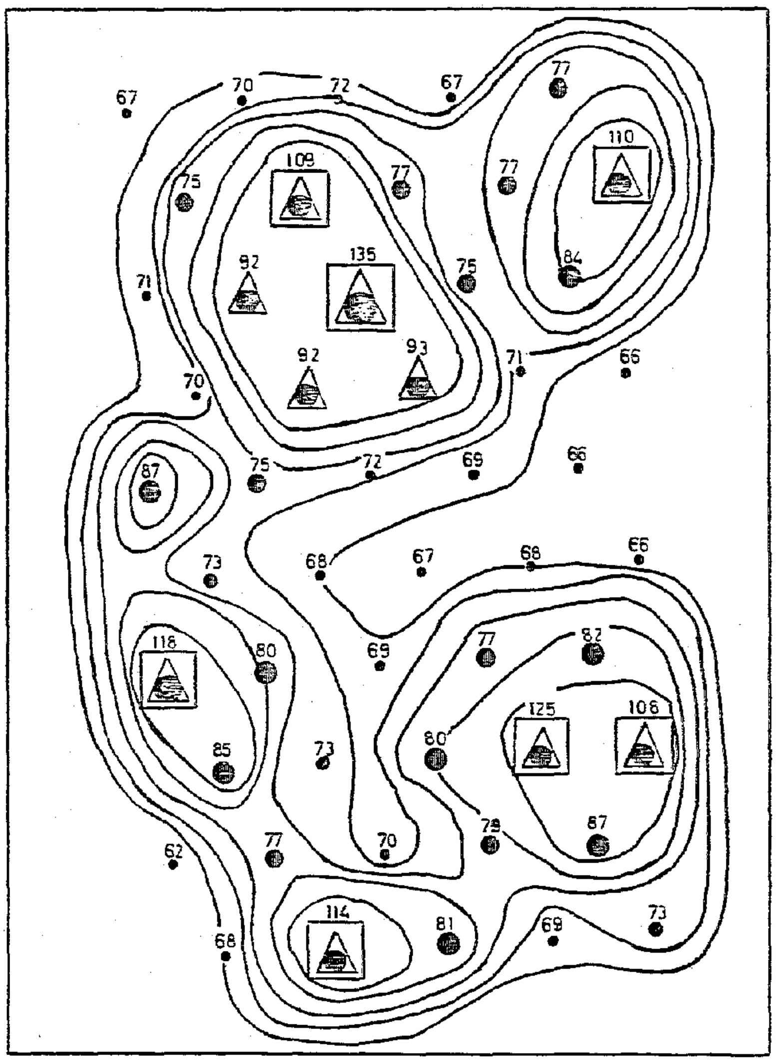


FIGURE 7. BETWEEN TIME 34 AND 46 THE BASIC PATTERN IS STABLE. NOTICE HOWEVER THE SHIFT IN CENTER OF THE "TWIN CITY" BETWEEN t = 20 AND t = 46.

At t = 20 we see that five central places have received the four functions present in our simulation and have deformed the population density contours in consequence, the residences and economic functions sprawling outwards to a distance depending on the size of the center (Figure 5).

Between t = 20 and t = 34 the structure remains more or less unmodified (Figure 6). The second center of the "twin-city" captures the fourth function and owing to its superior geographical situation begins to dominate its partner, which was by chance the first to appear. Another important feature is that the "oldest" and largest center on point 15 has, during this period, suffered a severe decline in its central core density. This results from the complex non-linear dynamics of our system, whereby the residences of the population that is employed at 15 spread outwards, and then attract local economic functions into the suburbs. These, however, once present, act as a source of local employment and in addition, act as a screen for the lower order functions diverting the clients of the central core which, consequently suffers a loss both of employment and of population.

Continuing the simulation from t = 34 to t = 46 shows that the structure remains basically unmodified, although as the growth analysis will show there now occurs a polarization of the growth in the system between the upper and lower halves of our lattice (Figure 7).

In Figures 8 to 11 we show the zones in which the growth is concentrated during the different periods of the evolution of our system. Initially, in the first period going from t = 0 to t = 10, we see that the growth is highly concentrated spatially in the five centers which are at the origin of the urban structure of the region. The areas of above average growth generally encompass only a single point, and this point has a very large growth rate. This can be referred to as the period of "central urbanization."

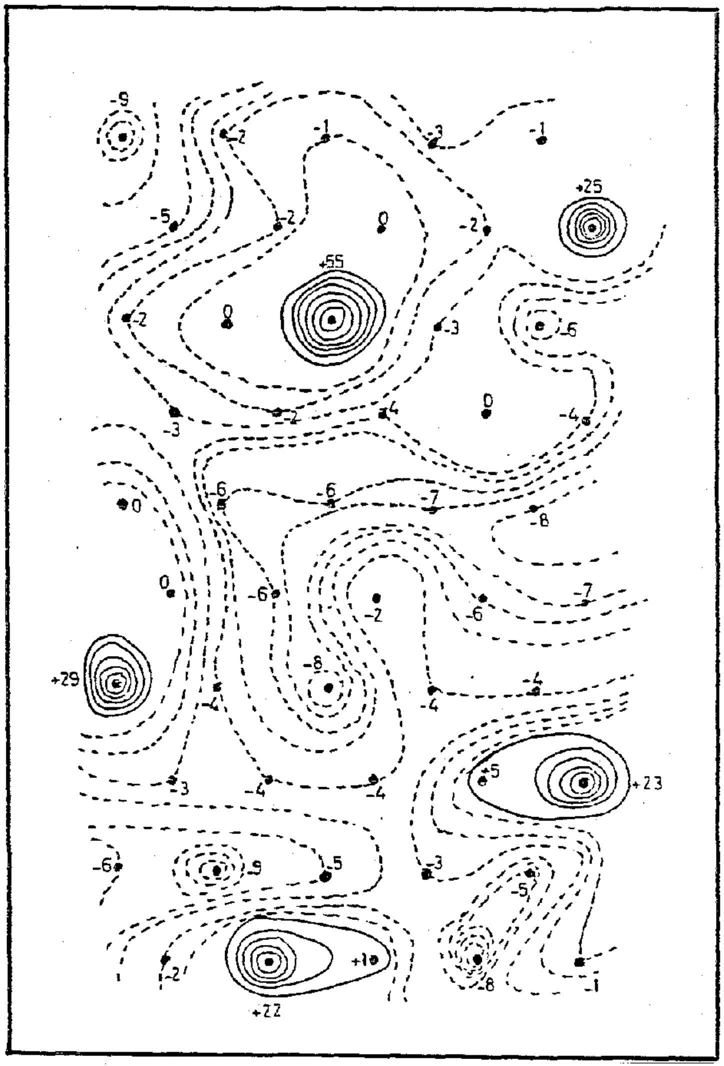
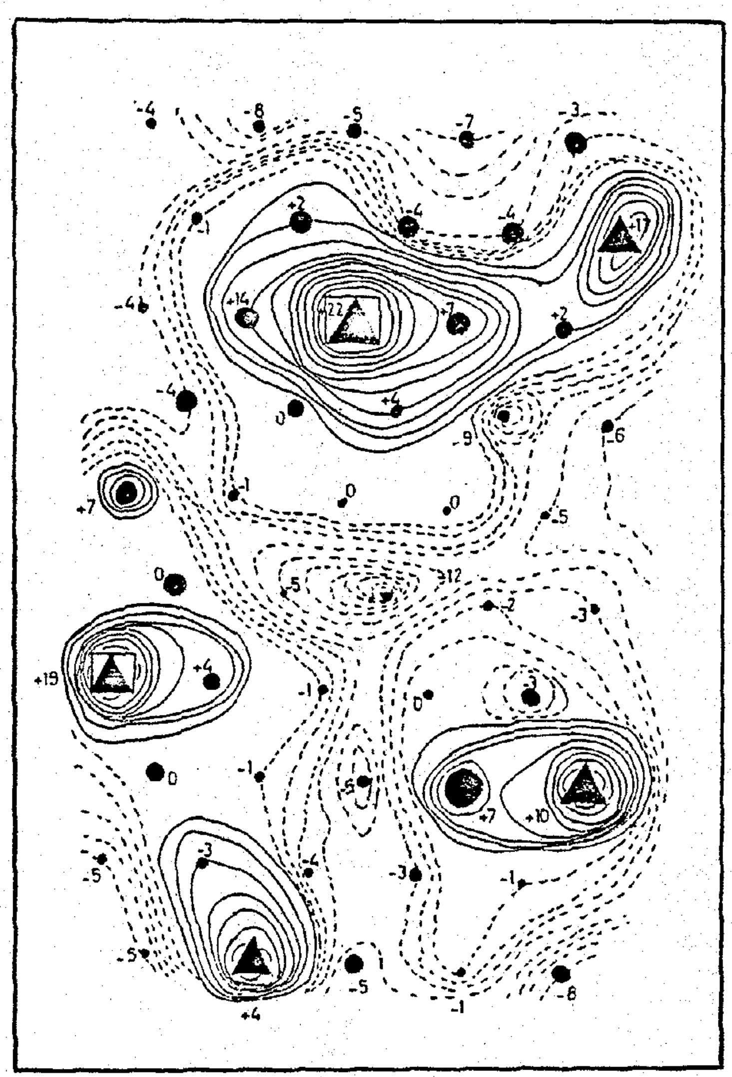


FIGURE 8. THIS SHOWS THE ABOVE OR BELOW AVERAGE GROWTH THAT HAS OCCURRED AT EACH POINT IN THE PARTICULAR PERIOD 0 → 10. THE ABOVE AVERAGE GROWTH IS VERY STRONGLY CONCENTRATED IN THE FIVE POINTS WHICH WILL BECOME THE DOMINANT URBAN CENTERS.



t = 10 → 20

FIGURE 9. THE ABOVE AVERAGE GROWTH IS NOW SPREAD OUT, CORRESPONDING TO THE FORMATION OF RESIDENTIAL SUBURBS. THE INTERURBAN SPACE IS SUFFERING CONTINUED DECLINE.

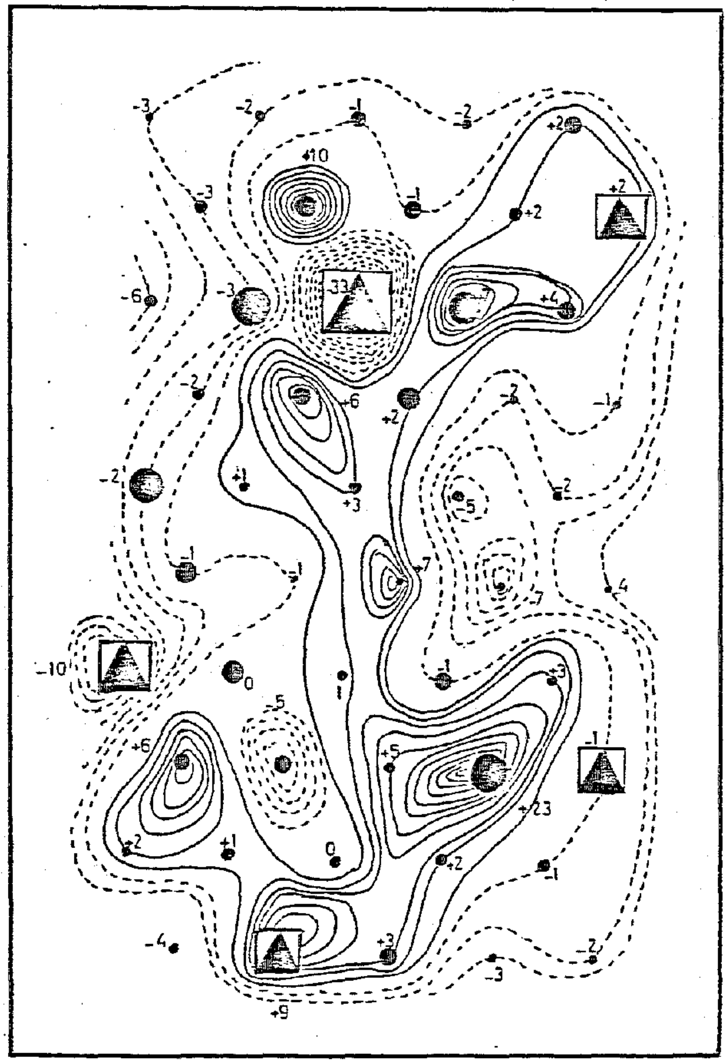


FIGURE 10. IN THE PERIOD 20-34 TWO LARGE URBAN CENTERS SUFFER A SEVERE DECLINE OF THEIR CORES, AND ABOVE AVERAGE GROWTH IS NOW ALMOST EXCLUSIVELY CONCENTRATED IN THE INTER-URBAN SPACE.

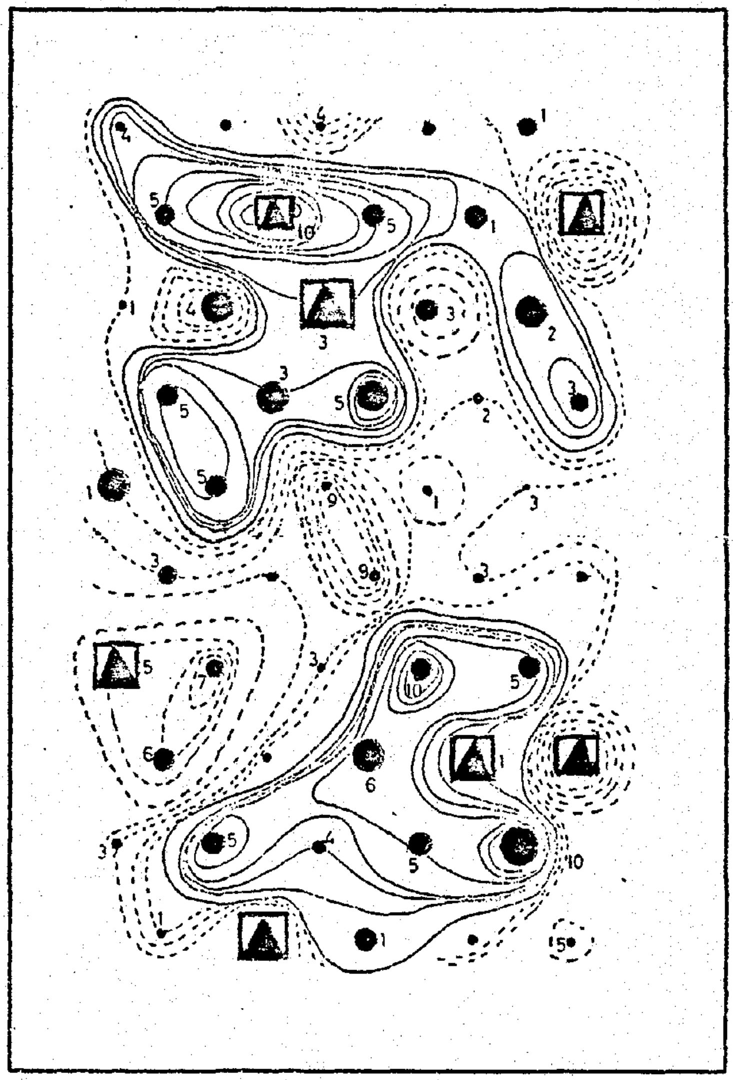


FIGURE 11. THE URBAN CENTERS COMPETE AMONG THEMSELVES AND THIS LEADS TO A POLARIZATION OF GROWTH.

In the next stage, Figure 9, which covers the period t = 10 up to t = 20 we see that, while the central cores are still growing strongly, the "growth plateaus" are much broader, showing the effects of suburban growth, that is of urban sprawl.

In the period t = 20 to t = 34, however, Figure 10 shows an entirely different pattern. Here, the central cores of three centers suffer a strong decline, and the remaining two grow very little. The zone of "above average growth" is nearly all concentrated in the inter-urban region, and marks a period of "counter urbanization."

In the final period of our simulation, between t = 34 and t = 46 (Figure 11) it is clear the inter-urban growth of the preceding interval marks the beginning of real competition between the upper and lower halves of the lattice, and although the growth remains essentially non-urban it shows the effect of the competitive growth of the different parts of the urbanized area.

1.3 ALTERNATIVE STRATEGIES FOR DECISION MAKERS

In this section we look at the effects on the global evolution of the system and of different decisions taken at time t = 34. This allows us to demonstrate the potential of our methods for the exploration of decisional alternatives, where either local or global changes can be imposed on the system, and where we begin to see the possibility of studying quantitatively the most basic issues of government: 1) Who should a decision favor and how much, and at the expense of whom? and 2) What hierarchy or decisional power will lead to which local strategies, and what will be the impact of the latter on the evolution of the whole?

In this section, in the very simple, somewhat artificial urbanization example we have presented earlier, we shall demonstrate the principle on which such fundamental questions can be explored. The importance of this section should therefore be judged, not on the details of the particular example used, but on the basic human difficulties of a "collective dimension" to individual acts, which is today perhaps the most important, almost wholly unanswered question facing our increasingly interdependent society.

Having explained the wider background of the discussion, let us turn to the example. Let us return to the simulation at t = 34 with an urbanization pattern as shown in Figure 6. We shall investigate the effects of three different decisions, and afterwards the question of a decisional strategy.

First, let us suppose that the population of the region as a whole is fixed over the next period, and explore its relative growth and decline from the time t = 34 to the time t = 50. First of all, if there is no intervention, no decision, and all the parameters are unchanged then the "growth and decline" zones are those shown in Figure 12. We note that the system undergoes a certain "polarization," and that in particular, the area across the center of our region, which has no urban development continues to decline, and in terms of percentage-change is most marked.

Now, let us examine the effects on the growth/decline patterns of the system, of some governmental "road building" program, or of some new technology, which has the effect of halving transportation costs (that is the values of $\phi^{(1)}$, $\phi^{(2)}$, $\phi^{(3)}$ and $\phi^{(4)}$ relevant up to t = 34, are now halved). This is in fact a strategy that has been proposed in various countries in order to help arrest the decline of different regions. In the case of our simulation, as has been found in reality for those countries, the improved transport efficiency has the effect of accelerating the decline of the rural areas between centers, and of favouring most the largest center (Figure 13).

The third strategy which we shall examine concerns the possibility of directly interfering in the urban structure by the placing of a specific investment at a particular point. This corresponds to the idea of a "New Town" or of the strategic development of a hitherto undeveloped center, in the hope of generating self-sustaining economic growth in the otherwise declining zone (Figure 14).

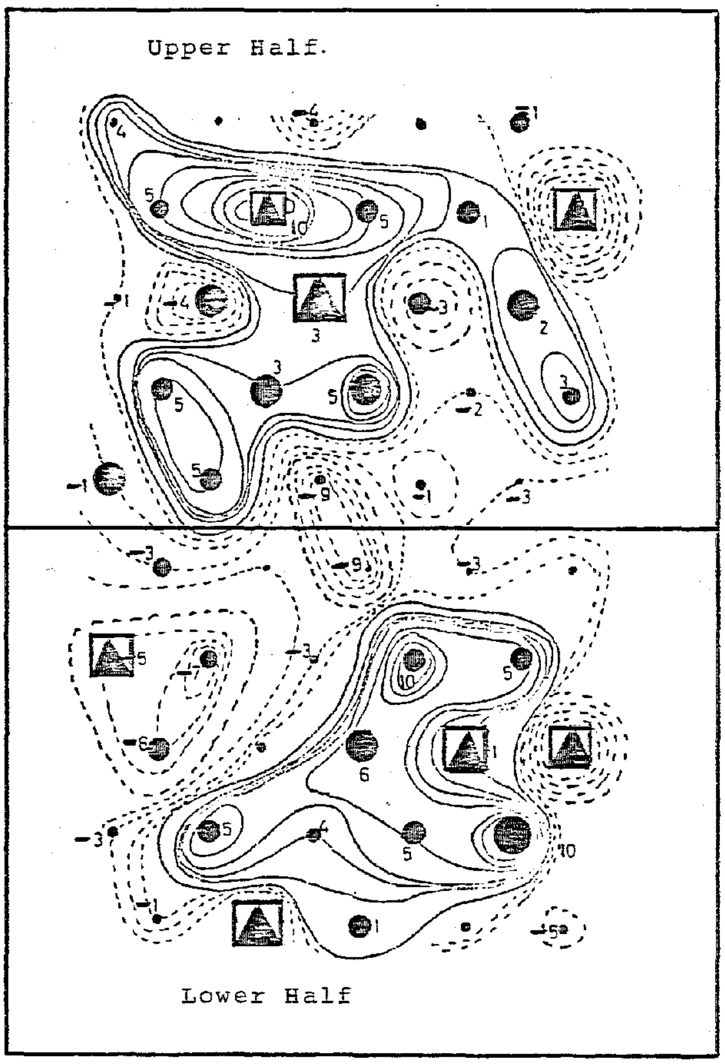


FIGURE 12. GROWTH PATTERN FOR THE PERIOD t = 34 → 50, IN THE ABSENCE OF ANY CHANGE IMPOSED AT t = 34. IT IS THE PATTERN AGAINST WHICH ALTERNATIVES MUST BE JUDGED.

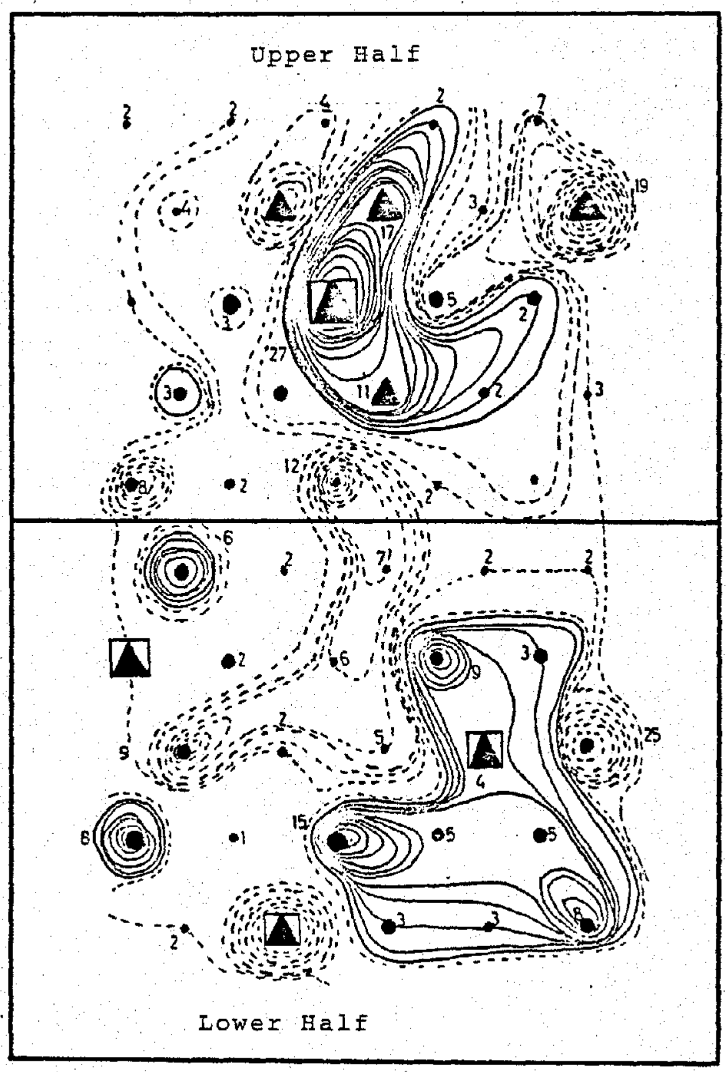


FIGURE 13. IF THE TRANSPORTATION COSTS ARE HALVED EVERYWHERE AT TIME t = 34 THE SYSTEM EVOLVES AS SHOWN HERE BETWEEN t = 34 AND t = 50.

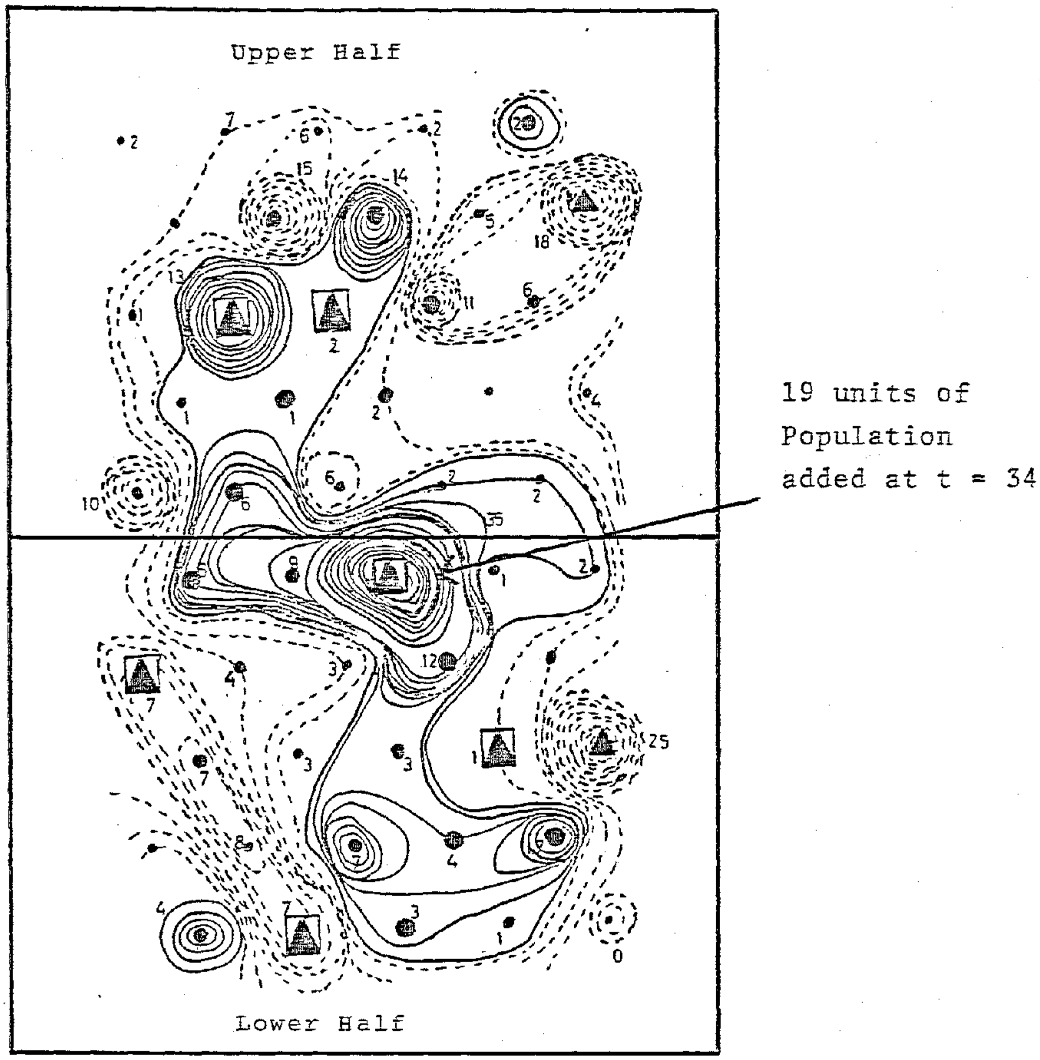


FIGURE 14. IF A POPULATION IS INJECTED AT THE POINT INDICATED AT TIME t = 34 THEN THE SYSTEM EVOLVES AS SHOWN HERE BETWEEN t = 34 AND t = 50.

The first important remark that must be made is that in all our simulations there are present small fluctuations of population and employment which test the stability of the basic structure, and could if this is not assured lead to the amplification of a particular fluctuation and the adoption of a new spatial pattern. However, we may see from the Figures 4 to 7 that the basic structure becomes stable to these small fluctuations by about $t = 16$. Thus, we know already that if we wish to modify the pattern, and in particular to move to a structure without the "declining rural hole" in the middle, then a perturbation of some larger size is required. In fact, in a series of computer simulations it was possible to ascertain that for almost certain, self-sustaining growth at the chosen point 26, it is necessary to invest 19 units at time $t = 34$. If less than this is inserted then the chances are that it will simply waste away since the basic structure is stable.

In Figure 14 we see the growth/decline pattern for a simulation where 19 units of population and employment were added to point 26. The investment flourishes, producing a remarkable increase in population and jobs at and around this point. Of course this is at the expense of other points which would otherwise have grown, but it can be shown that the final structure resulting at time $t = 50$ following the perturbation, is more efficient than otherwise, since there is less transportation required for the same total consumption as before, which means that mean haul distances are shorter and variations in the consumptions of goods between urban and rural populations is less marked.

However, before drawing any hasty conclusions about which strategy should be adopted let us briefly examine the manner in which the administrative division of a region may affect which decisions are adopted. Consider the case where our lattice is divided into two separately governed districts: the upper half, and the lower half.

Let us briefly discuss the consequences for each half of each of the three strategies above. First, if there is no change, (Figure 12) we see that growth occurs in both districts, but slightly more in the upper than in the lower. We have between t = 34 and t = 50

upper half + 11 lower half - 11.

Second, if we halve the transportation costs in the system (Figure 13) we find that although the greatest growth occurs in the largest center (point 15), this growth is in some way achieved at the expense of the district itself, since we find for the period t = 34 to t = 50,

upper - 6 lower + 6.

The third strategy consists in placing 19 units of population on point 26, which is in the lower half. Not surprisingly, when the investment pays off we find that the lower half gains greatly:

upper - 41 lower + 41.

We see from this, that in fact, it pays the lower half to invest the 19 itself, since rather than do nothing it gains:

41 - 19 + 11 = + 33.

Thus the "strategy" played by the lower half is to invest in a center on its frontier with the upper, which causes a growth at the expense of the upper half. This basic idea of strategy corresponds clearly to many problems such as the conflict of two political parties where effort must go into attracting supporters from the middle ground, and similarly for competing firms with different ranges of products.

The importance of these results is not however, in their detail. It is rather in the principle which is demonstrated that in a complex system of interdependent entities, the decisions made by individuals, or by collective entities representing certain localities have a real effect on the evolution of the system and of everyone in it. This is the "collective" aspect of individual actions which characterizes our society, and decisions should be made as far as possible in the knowledge of these collective effects, rather than finding that the "system" is sweeping the various actors in a quite unexpected and undesirable direction, as a result of their individual behaviour.

This is the basic aim of the methods that we have described here, since, by choosing the various parameters so that they correspond to a particular urban hierarchy, it is possible to simulate not only the long term repercussions of a given strategy for the immediate locality involved, but also the consequence of that strategy for the region in which it is embedded.

2. A BOOLEAN FORMALISM FOR INTRA-URBAN DYNAMICS

2.1 A SIMPLIFIED FORMALISM SUITABLE FOR MODELLING COMPLEX SYSTEMS

The description which we have developed in the previous reports, and in the above sections, based on differential equations, becomes rapidly cumbersome when applied to complex systems involving a very heterogeneous system with many interacting populations. Although very large computers can still simulate the evolution of such systems, for the purposes of reflection and understanding concerning the qualtitative evolution of the structure of the system, it may be sufficient and indeed convenient to have some simplified description. The "full" description given by the differential equations contains information on both the "qualitative" nature of the structure, and the details of the "quantitative" change in the variables involving a growth or decay of the densities at each point. In the pursuit of a particular aim such as, for example, a plan concerning the internal structure of a city, then as a first step in the comparison of different strategies the planner may require only knowledge of the "qualitative" repercussions on the structure, and this may in turn demand the establishment of certain "key values" (thresholds) for the variables.

Let us consider for example the intensity of white emigration out of a particular neighbourhood as a function of the percentage of black inhabitants of that neighbourhood. We may find a curve having three stages: for the first two critical values of the black population, perhaps there is only a small increase in white emigration, but for the third one there may be a large increase. In this case the planner may well choose to concentrate simply on this latter jump, and consider the two others as negligible, in which case it becomes possible to reduce the complicated functional relation between the two variables to a very simple logical binary function. This expresses the following approximate relationship: white emigration is negligible (= 0) as long as the percentage of black residents in the neighbourhood remains below (= 0) the third critical value; it becomes non-negligible (= 1) as soon as the third critical percentage is attained (= 1).

This small example shows us how, Boolean formalism, as has been developed for the theory of genetics,[1] could be introduced to the problem of urban evolution if the fundamental hypothesis, rather natural in genetics, were found to be acceptable in this case too. Indeed we should be able to classify urban populations into discrete sets, each characterized by a behaviour implying a series of dichotomic choices. This would mean that threshold could be meaningfully defined for these populations as a whole and that nothing essential is lost by neglecting the obviously continuous distribution of the individual behaviours.

The Boolean approach is basically an approximation of the dyanmics of a complex non-linear system, which is more properly described by differential equations. Because of this, it clearly cannot give as much information to the user. Thus, for example, it can only permit the discussion of the stationarity of a structure and not its stability since stability analysis necessarily implies non-binary "perturbations." For the description in terms of differential equations, "stability" refers both to the quantitative values of the variables as well as to the "organization" of these values in some macroscopic pattern. Thus, in the Boolean formalism the whole aspect of "growth" or "decline" is missing except when it is involved in triggering structural changes.

As a tool for the planner the formalism has to be taken with care since it presupposes a workable separation of the "input" or "control" variables, on which the planner is supposed to be able to act, from the "internal" variables which reflect the response of the system resulting from individuals of the various populations present, reacting according to the criteria specific to these populations. Planning strategies are meant to be compared by studying the effects of different sequences of changes in the "input" or "control" variables, perhaps involving a subtle fine-tuning of the timings of the events in a particular sequence.

The internal variables undergo some corresponding sequence of transitions and this may be compared with the various goals desired.

In order to extend the temporal range of the model, it is possible to take into account a coupling between the evolution of the internal variables and that of the control variables, and different possibilities can be envisaged and studied by supposing different scenarios.

In the model presented here, it should be pointed out that the descirption is not a spatial one; the evolution of each particular neighbourhood, (each with given values of the "control" variables) is independent: the interaction with the neighbouring zones is not taken into account. This is one direction in which further research will be directed since it is felt that this is a rather serious defect. Despite the various simplifications and perhaps over-simplifications involved in this model, the dynamical evolution it describes is essentially similar to that of the much more cumbersome differential equation formalism. It offers therefore some tool of reflection in weighing different strategies for planners and decision makers, if the decisional criteria of the various groups and populations which are in interaction in an urban area were successfully identified. The Boolean formalism can therefore also be thought of, not only as being useful in itself, but also as a preliminary qualitative study, which it is worth doing before moving to a more complete, quantitative/qualtitative model, should this be necessary.

2.2 A SIMPLE MODEL OF RESIDENTIAL LOCATION

In its present version,[2] the model introduced here is aimed at showing the possibilities of the method rather than simulating some aspects of reality: it is not based on field work; it is guided only by our intuition of individual behaviours. It is thus quite speculative. Nevertheless it draws its inspiration from the situations with which we have been confronted in the city we personally know best: the Agglomeration of Brussels, Belgium.

We will start by describing the various individual relationships that maintain the different socio-economic agents (these are the internal variables: four classes of population and the housing price). The behavioural equations that follow express each group's conditions of immigration into and emigration out of any neighbourhood. Some of the behavioural factors found in these equations describe the characteristics of the urban environment that we choose as relevant to the problem (these are the input variables: population density, quality of the neighbourhood and home-to-work travel time). After that we will build up the spontaneous sequences of immigration and emigration.

2.2.1 The Behavioural Equations

Let us imagine a hypothetical "city" having four classes of dwellers whose different behaviours can be explained, for instance, by cultural and socio-economic differences:

- high income residents (A)
- middle income resident (B)
- low income indigeneous residents (C)
- low income foreign minority group ($\mathcal{D}$)

Each class chooses its residential location as a function of a series of constraints and requirements which are characteristic and define each class with respect to the others. Among these are the relevant "physical" characteristics of the city: population density (D), quality of the neighbourhood (H) and home-to-work travel time (T). Four behavioural equations follow (the functions a, b, c, d), expressing each group's potential demand for residential locations. If the system, that is the neighbourhood, meets the requirements of one of several classes, the residential immigration can take place and this leads to the satisfaction of the potential demand (the memory variables

α, β, γ, δ). At this stage, we suppose that the housing supply is instantaneously adjusted to the demand, i.e., that one immediately builds all types of housing necessary to satisfy the demand. In reality, there can be a long time delay between the expression of a demand and the building of the corresponding housing type (it is often the case with social housing). The opposite can however also happen: the supply comes before the demand and the buildings stay empty. In any case, the method could be modified in order to describe situations where demand and supply are not necessarily in equilibrium.

The time delay that occurs between the moment where the potential demand is expressed and the moment where it is satisfied is a characteristic delay we will call "migration delay." One of the advantages of the Boolean formalism is that one may differentiate the migration kinetics, by giving different values to a definite group's migration delay for immigration and emigration, but we will not do so here.

To the four internal variables already cited (the four groups of people), we will add a fifth one of another nature: the housing price (P). The potential variation of the housing price depends on several conditions; their formulation leads to a fifth behavioural equation (function p). When these conditions are met in a neighbourhood, the housing price can rise (memory variable π).

Now what are the location constraints and requirements of each class of population and what are the conditions of variation of the housing price? Since we are in the process of writing urban planning fiction, let us imagine that the relationships between people are tainted by a peculiar tendency towards discrimination: the first three classes reject the foreigners belonging to the minority group $(.\bar{\delta}.)$. The foreigners stick together $(.\delta.)$ for cultural reasons but also because they are rejected by the other groups (δ is partially due to the $\bar{\delta}$ of the other groups). As to the three first classes, they are willing to mix, except when the population density is high: in that case, each of them rejects the one which is just below in the income scale

$(\overline{D.\beta}$ and $\overline{D.\gamma})$. Besides these general characteristics, each class of population has requirements reflecting a value system of its own.

Let us imagine for instance that <u>high income residents</u> (A) accord great importance to the quality of the environment $(.H.)$. On the other hand, their attitude towards indigenous low income people is less drastic than that towards middle income people they accept to mix with the former, even at high population density, providing residents of their own class have already migrated in the neighbourhood before them $(\alpha + \bar{\gamma})$. This bridge connecting the two ends of the economic scale can be interpreted as a tendency of some high class people to be interested in low income people. Their hypothetical behavioural equation is then the following:

$$a = \bar{\delta} .H. (\overline{D.\beta}) \cdot (\alpha + \bar{\gamma}).$$

<u>Middle income residents</u> (B) are relatively sensitive to the cost of urban living. The urban space is not entirely accessible to them for financial reasons: they cannot afford at the same time expensive housing and high home-to-work travel time or cost $(\overline{\pi \cdot T})$; but they can afford the three other possibilities $(\bar{\pi} \cdot T$, $\bar{\pi} \cdot \bar{T}$, $\pi \cdot \bar{T})$. Yet there is a rider to add here, revealing a wish to climb the rungs of the social ladder: they like to live in the vicinity of wealthy people even if it is costly $(+ \pi \cdot T \cdot \alpha)$. After simplification, their equation is the following:

$$b = \bar{\delta} \cdot (\overline{D.\gamma}) \cdot (\overline{\pi \cdot T} + \alpha).$$

<u>Indigenous low income residents</u> (C) are highly sensitive to the cost of urban living. Their possibilities of choosing a residential location are even more limited: they can afford to live in the city providing they have cheap housing and a low home-to-work travel time or cost $(\bar{\pi} \cdot \bar{T})$.

$$c = \bar{\delta} \cdot \bar{\pi} \cdot \bar{T}.$$

Foreign low income residents ($\mathcal{D}$) respond to the same economic constraints as the indigenous low income residents. They differ from them however, by the necessity or choice of living together.

$$d = \bar{\delta} \cdot \bar{\pi} \cdot \bar{T} .$$

To these four behavioural equations, let us add now the housing price ($\mathcal{P}$) equation. In a first trial, we considered the housing price as an input variable, but further on we have transformed it into an internal variable, because decision makers generally cannot control it in a free enterprise society. We have made it essentially sensitive to the nature of the residents: it goes down when low income people of any kind immigrate into the neighbourhood; when these are missing, the housing price goes up when wealthy people come in or when the neighbourhood is characterized by a good quality environment and low density.

$$p = \bar{\gamma} \cdot \bar{\delta} \cdot (\alpha + H.\bar{D}).$$

At this stage of modelling, each economic and social variable is very simply represented by one binary variable, the value of whose threshold in the system has not been determined. Field work would reveal a greater complexity in the behaviours. For instance, the critical percentage of the foreign residents in a neighbourhood, leading to some reaction on the part of the other social classes - i.e., the "social efficiency threshold" of the $\mathcal{D}$ variable - could well be different from one social class to another. One would then need to use several binary variables to express these nuances between classes or a delayed multi-level logical representation.[3] What is true for an internal variable is true too for an input variable: each social class may perceive differently the physical characteristics of the urban space.

The equations we have just set up are purely deterministic: they describe the behaviour of the average person of each social class. Chance - i.e., the existence of some irrationality in the behaviours or the local presence of other factors than the ones that have been introduced in the equations - this chance does not affect the individual behaviours (as it does in the models using differential equations). Nevertheless it does influence the final states by its intervention in the choice of the initial states.

At the end of this paper, we will moderate the deterministic nature of the individual behaviours by introducing a random variation around the mean of migration time delays. From given input and initial states, we will finally end up with an estimation of the probabilities linked to each stable state (see the results of the simulations in Figures 26 and 27 and in Reference 7).

2.2.2 The Decision Variables

The input variables express the influence of the outside world on the system. They immediately emerge from the behavioural equations of which they are parameters. They characterize the "physical" nature of the neighbourhood. For the urban planner, for instance, they are decision variables, the channel by which he can make the urban structure change.

Population density (D) is indeed often determined by local land use plans.

Neighbourhood quality (H) is also largely influenced by land use plans. One could consider it as a function of other input variables, such as the type of housing, the type of environment, ... that we will not define here.

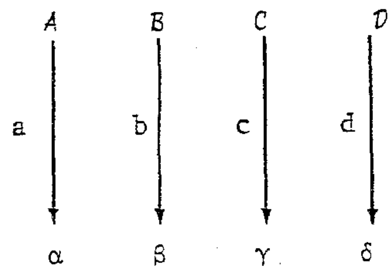
Home-to-work travel time (T) is largely determined by the location of jobs and transport networks; it can be interpreted also as the cost of transportation. It is typically the kind of variable that the urban planner can control.

In the model, the three binary variables associated with the three decision variables take the value 1 when they overstep an arbitrarily chosen threshold.

Figures 15a and b synthesize the functioning of this hypothetical model.

Internal variables

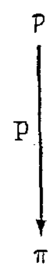
Residents:



migration conditions or potential demand for residential location

effective migration or satisfaction of the potential demand

Housing Price:



conditions of variation

effective variation

Input variables

D : population density

H : neighborhood quality

T : home-to-work travel time or cost

FIGURE 15a. VARIABLES OF THE MODEL

A_{ij}	π	α	β	γ	δ
π	0	0	-	-	-
α	+	+	+	0	0
β	0	-	0	0	0
γ	-	-	-	0	0
δ	-	-	-	-	+
D	-	-	-	0	0
H	+	+	0	0	0
T	0	0	-	-	-

A_{ij} : Effect of the variable in the row i on the variable in the column j.

+ : positive feedback
- : negative feedback
0 : no feedback

FIGURE 15b. FUNCTIONING OF THE MODEL

2.2.3 The Collective Structures, Projected into a Theoretical City

The state table (or flow table - Figure 16) describes all the possible situations for the system. The system - as we have defined it - is the neighbourhood. Consequently the table gives us a complete theoretical description of all types of neighbourhoods: each column represents a "physical" situation and each line a social content. Nevertheless the whole table does not represent a city because the neighbourhoods do not interact with each other (as they do in the models using differential equations). It would be an important improvement to the method to introduce interactions between first neighbours; the model would then give a global description of the city instead of a local one. In this table, almost all states except for a few, are unstable.

It is then important to define the concept of stability when it is applied to the city which is characterized by change. This will lead us to specify the nature of the urban problems which will be the most suitable for Boolean treatment, as well as to define the time scale in which we operate. The stability of the final state is such that neither input variables, not internal variables will be allowed to change value. This implies a certain permanence of the urban structures which will not be allowed to vary enough to make any variable pass from one logical state to another. This constrains the neighbourhoods to remain qualitatively unchanged but does not exclude quantitative modifications, such as people's and firms' migrations, on the condition that the qualitative balance be untouched. Then we see that the Boolean formalism suits urban development problems (qualitative changes) better, by definition, than urban growth problems (quantitative changes), because of its intrinsic sensitivity to the former. As to the time scale, it has to include both the characteristic time required to go from the initial state to the final state, as well as a time sufficient to show the stability of the final state. This implies that the input variables be unchanged, and consequently the "physical" structures of the city be relatively permanent.

internal states	D=0 H,T 0 0	0 1	1 0	1 1	D=1 H,T 0 0	0 1	input states 1 0	1 1
00000	00110	00100	11110	11100	00110	00100	01110	01100
00001	00001	00000	00001	00000	00001	00000	00001	00000
00010	00110	00100	00110	00100	00010	00000	00010	00000
00011	00001	00000	00001	00000	00001	00000	00001	00000
00100	00110	00100	11110	11100	00110	00100	00110	00100
00101	00001	00000	00001	00000	00001	00000	00001	00000
00110	00110	00100	00110	00100	00010	00000	00010	00000
00111	00001	00000	00001	00000	00001	00000	00001	00000
01000	10110	10100	11110	11100	10110	10100	11110	11100
01001	00001	00000	00001	00000	00001	00000	00001	00000
01010	00110	00100	01110	01100	00010	00000	01010	01000
01011	00001	00000	00001	00000	00001	00000	00001	00000
01100	10110	10100	11110	11100	10110	10100	10110	10100
01101	00001	00000	00001	00000	00001	00000	00001	00000
01110	00110	00100	01110	01100	00010	00000	00010	00000
01111	00001	00000	00001	00000	00001	00000	00001	00000
10000	00100	00000	11100	11000	00100	00000	01100	01000
10001	00000	00000	00000	00000	00000	00000	00000	00000
10010	00100	00000	00100	00000	00000	00000	00000	00000
10011	00000	00000	00000	00000	00000	00000	00000	00000
10100	00100	00000	11100	11000	00100	00000	00100	00000
10101	00000	00000	00000	00000	00000	00000	00000	00000
10110	00100	00000	00100	00000	00000	00000	00000	00000
10111	00000	00000	00000	00000	00000	00000	00000	00000
11000	10100	10100	11100	11100	10100	10100	11100	11100
11001	00000	00000	00000	00000	00000	00000	00000	00000
11010	00100	00100	01100	01100	00000	00000	01000	01000
11011	00000	00000	00000	00000	00000	00000	00000	00000
11100	10100	10100	11100	11100	10100	10100	10100	10100
11101	00000	00000	00000	00000	00000	00000	00000	00000
11110	00100	00100	01100	01100	00000	00000	00000	00000
11111	00000	00000	00000	00000	00000	00000	00000	00000

π, α, β, γ, δ

Input variable :

D : population density
H : neighbourhood quality
T : home-to-work travel time or cost

Internal variables :

π : housing price
α : high income residents
β : middle income residents
γ : indigenous low income residents
δ : foreign low income residents

Potential demands :

p : variation of the housing price
migration of :
a : high income residents
b : middle income residents
c : indigenous low income residents
d : foreign low income residents

Nature of the states

00000 : unstable
00000 : stable

FIGURE 16. STATE TABLE - VALUES TAKEN BY THE BEHAVIOURAL EQUATIONS (p, a, b, c, d) FOR EACH POSSIBLE INPUT AND INTERNAL STATE

Since these physical structures (means of transportation, firms' technical requirements, ...) are largely dependent on the technico-economic system, the time scale is then reduced to the short and middle term which, by definition, excludes the possibility of any modification of the technico-economic system. Nevertheless it is possible to lengthen the prediction term of this type of model by defining a temporal sequence of input states. This sequence has to be arbitrarily chosen since it cannot be predicted by the model.

One can give a spatial image to this state table by imagining a theoretical city which would show the main tendencies observed in reality. A real city may be cut up on the basis of criteria such as the ones we have defined as input variables (population density, neighbourhood quality and home-to-work travel time), since we observe that:

- the population density decreases exponentially to the periphery and increases again in the surrounding satellites; in the center, two cases are possible: the density reaches its maximum or decreases to form what is called a density crater;

- the main employment areas are the central business district (C.B.D.) and one of several industrial areas; generally arranged along an axis;

- the neighbourhood quality is generally mediocre in the C.B.D. and around industrial areas; good in the periphery, in the historical center and, in some cases, in that part of the C.B.D. that has been recently renewed.

In order to define the specific theoretical city we are presenting, we have had to make two choices. On the one hand, we have chosen its physical characteristics so as to make it look very much like the Agglomeration of Brussels, Belgium, so that we would finally be able to illustrate theoretical results with the real situation. Nevertheless the theoretical city is much larger than Brussels (since it includes statellites) and the variety of neighbourhood situations is wider in order to make it match all the cases contained in the state table. On the other hand, we have been forced to locate the social efficiency thresholds in an arbitrary manner, since we have not determined their values or their existence.

The resulting cutting is inevitably arbitrary with the following characteristics (Figure 17): the industrial areas are arranged along a S.W.-N.E. rapid transit axis, going through the C.B.D.; the factories located at the S.W. end of the axis have in fact closed down. The population density forms a central crater, diminishes to the periphery and increases again in three residential satellites located in the urban fringe. One of them only is well linked to the C.B.D. by a rapid means of transport. The neighbourhood quality is mediocre along the industrial axis and in the Northern satellite.

Let us build up now the input state map (Figure 18). If one takes n criteria to each of which is associated a binary variable (as it is the case in this study), each portion of the city will be characterized by one of the 2^n possible combinations of values of the n binary variables. In our case, each area of the theoretical city will be defined by three numbers, corresponding to the columns of the state table. If D represents the population density, H the neighourhood quality and T the home-to-work travel time, each input state will have the following meaning (s):

D H T

0 0 0 - depopulated industrial center
- secondary pole of industrial employment

0 0 1 - low quality suburb (residential area mixed with abandoned factories for instance)

0 1 0 - depopulated historical center, renewed center of C.B.D.
- pleasant peripheral residential area, well linked to the employment areas

0 1 1 - pleasant residential suburb, badly linked to the employment areas

1 0 0 - part of the first dense fringe, surrounding the C.B.D., where residences and factories are mixed

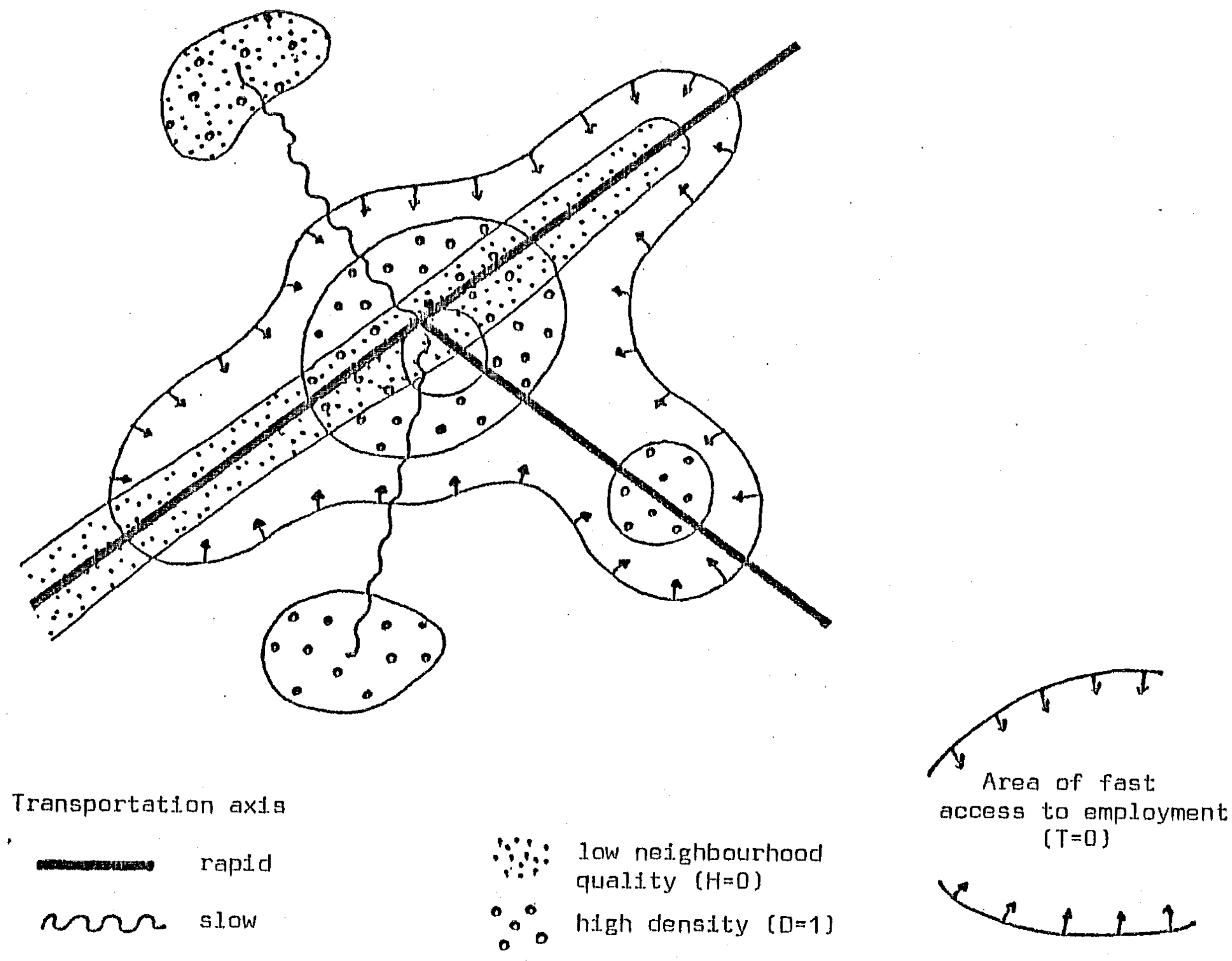


FIGURE 17. THEORETICAL CITY - SOCIAL EFFICIENCY THRESHOLDS OF THE INPUT VARIABLES

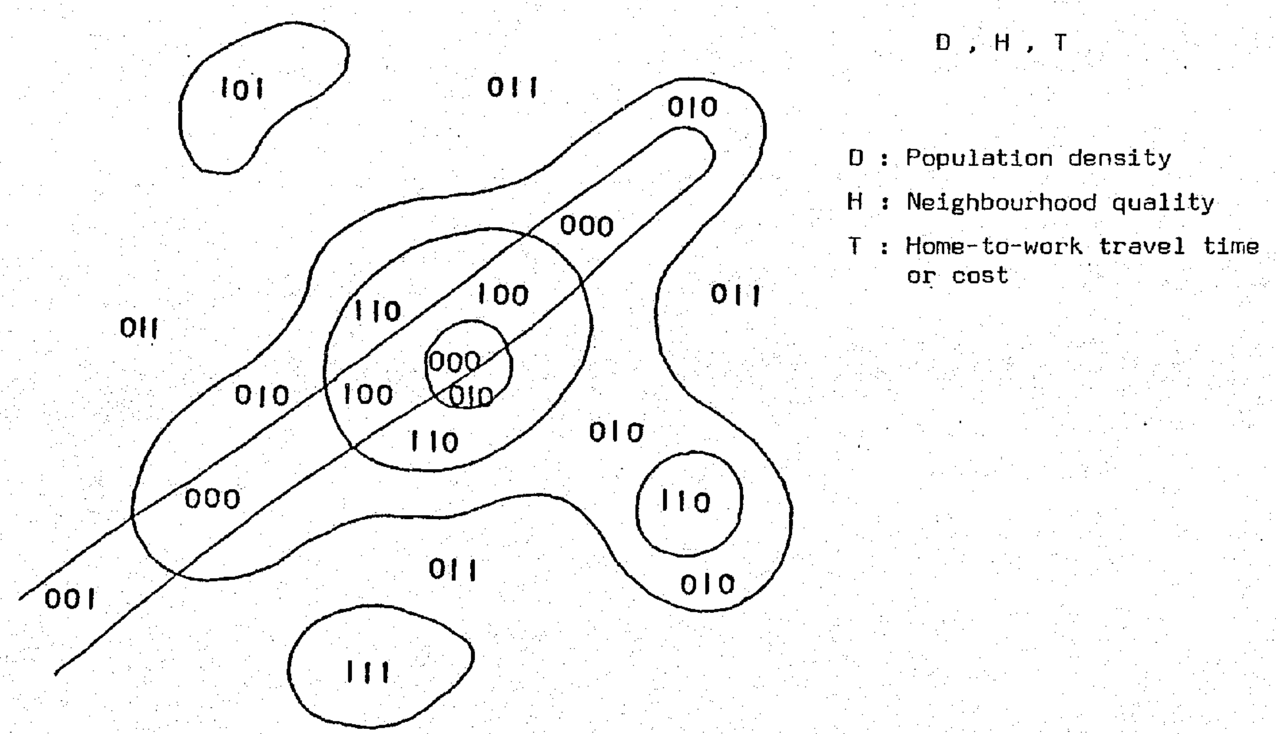


FIGURE 18. THEORETICAL CITY - INPUT STATES MAP

1 0 1 - industrial satellite, badly linked to the employment areas, showing a dominant dormitory function

1 1 0 - part of the first dense ring, surrounding the C.B.D., showing a high neighbourhood quality (renewed or historical populated area)

- pleasant residential satellite, with a dominant dormitory function, well linked to the employment areas by a rapid means of transport

1 1 1 - pleasant residential satellite, with a dominant dormitory function, badly linked to the employment areas.

Looking at the state table (Figure 16), we notice that a neighbourhood can reach different stable states even though it is in the same physical state (column). Indeed, the behavioural equations, while assuming an average mechanism of interaction at the individual scale, do not determine in an unequivocal manner the final state of the system at a collective scale, i.e., the distribution of people and housing price in the urban space. The factors influencing behaviour do not intervene solely; the choice of the initial state - that is the history of the system - may considerably modify the final state. The change of value of one internal variable, is in some cases enough to make a very different situation arise.

The following example illustrates the influence on the system of a change of value of the internal varaible δ. Let us take also this opportunity to explain in detail how the state table should be used.

<u>First case</u>

Input state (D,H,T): 000

Initial internal state (π, α, β, γ, δ): 00000

internal states		D,H,T: 000
t	00000 →	00110
$t + \Delta t_1$	→ 00100 →	00110
$t + \Delta t_2$	→ 00110 →	00110

Going back to the state table (Figure 16), let us choose the column corresponding to the input state (000). In this column are written the values taken by the behavioural equations (p, a, b, c, d) for each possible internal state (π, α, β, γ, δ) indicated in the very first column on the left. Let us pick up the chosen internal state (00000) at time t. We see that the corresponding values of the behavioural equations are 00110. This means that there exists a demand for location in the neighbourhood from the part of two groups: middle income residents and indigenous low income residents. At time $t + \Delta t$ the two groups might be present in the neighbourhood at a concentration higher than their threshold and the corresponding internal state might become 00110. However, the interesting point here is that the immigration delay of each group in the neighbourhood will probably be different. Two situations are possible:

- The middle income residents enter the neighbourhood more quickly and exceed their threshold at time $t + \Delta t_1$. The next internal state is then 00100.

Now let us go back to the first column on the left and select the new internal state 00100. What are the corresponding values of the behavioural equations? It is still the same: 00110. This means that the second group does not see any objections to the presence of the first one in the neighourhood. So they can come in and the new internal state at $t + \Delta t_2$ will then be 00110. Since the corresponding values of the behavioural equations are identical to the internal state, the system has reached a stable state. In order to have new spontaneous transitions, something will have to change in the input state.

- if the indigenous low income residents migrate first, the sequence will be the following:

internal states		D,H,T,: 000
t	00000 →	00110
$t + \Delta t_1$	00010 →	00110
$t + \Delta t_2$	00110 →	00110

The final state will be the same but we will see later on that it is not always the case.

The two possible sequences of spontaneous transitions are summarized the following way:

t		$t \div \Delta t_1$		$t \div \Delta t_2$	
$00\bar{0}\bar{0}0$	β	$001\bar{0}0$	γ	$\underline{00110}$	first case
	γ	$00\bar{0}10$	β	$\underline{00110}$	second case

The definition of the time delays will make the system choose one trajectory or the other.

Second case

Input state (D,H,T): 000
Initial internal state (π, α, β, γ, δ): 00001

In the same input state column as the previous case (000), let us choose 00001 as initial internal state. The corresponding values of the behavioural equations are:

internal states	D, H, T: 000
00001 →	$\underline{00001}$

We see that the potential demand for migration expressed by the two previous groups does not exist, not because of the "physical" conditions of the neighbourhood but because of the presence of foreign low income residents at a rate above their social efficiency threshold. Their presence is considered as repulsive. The system will then stay stable. So under the same "physical" conditions, the final structure of the system will be very different.

The system - as it has been defined - can reach seven different states (π, α, β, γ, δ) (Figure 16):

- at low housing price:
  three situations of one class dominance:

  00001 foreign low income residents
  00010 indigenous low income residents
  00100 middle income residents

two possibilities of mixing two groups:

  00110 middle income and indigenous low income residents
  01010 high income and indigenous low income residents

one possibility of partial integration:

  01110 mixing of the three indigenous classes

- at high housing price, there is only one possible stable state:

  11100 the mixing of the two highest income classes

The projection in space of the stable states (Figure 19), on the basis of the input states map (Figure 18), gives an image which is a bit confused of the spatial organization of the theoretical city. However, the maps showing the possible locatons of each social class at stable states (Figure 20 a,b,c,d) are somewhat clearer. We will comment on these later on.

Doing the same operation for the housing price, we get a map showing the stable spatial distribution of housing price (Figure 21), which is qualitatively consistent with the real situation of a city like Columbus, Ohio (Figure 22).[4]

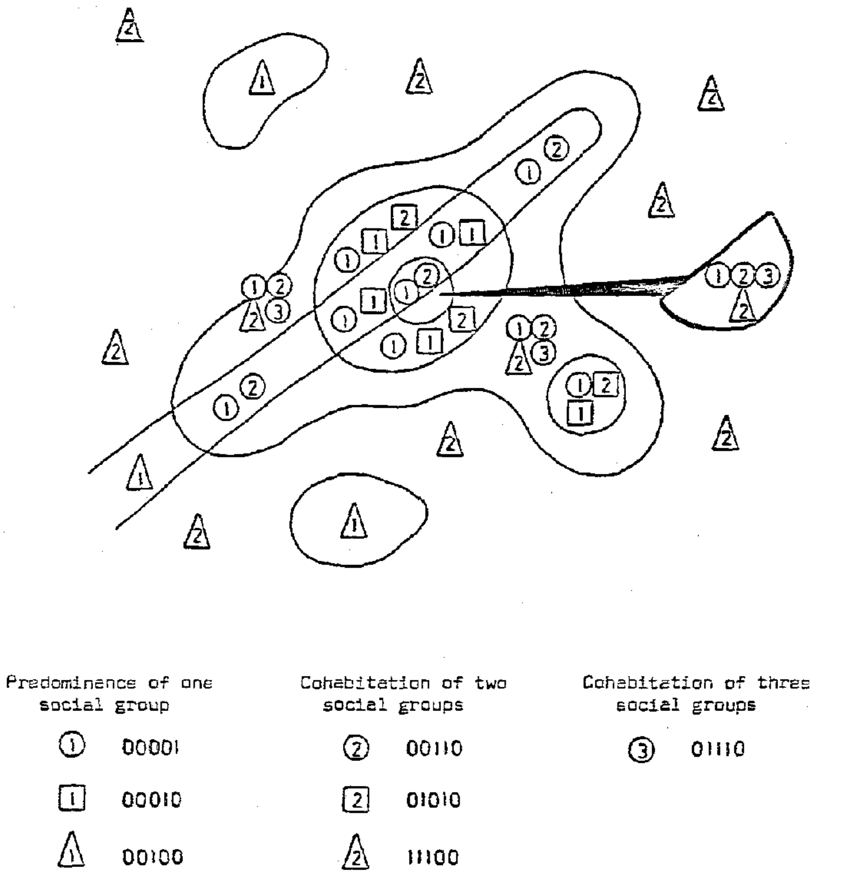


FIGURE 19. THEORETICAL CITY - STABLE STATES MAP

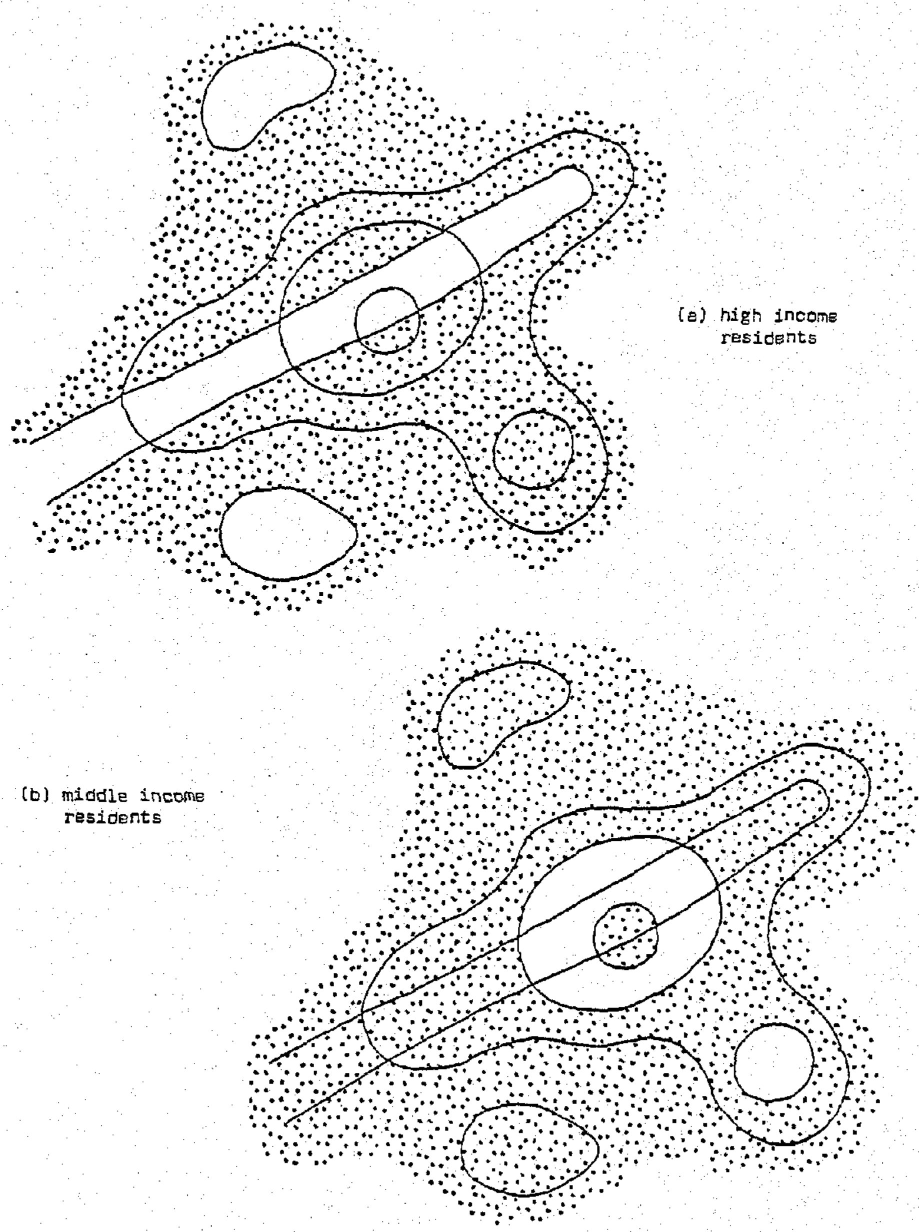


FIGURE 20a,b. THEORETICAL CITY - POSSIBLE LOCATIONS OF THE DIFFERENT SOCIAL GROUPS AT STABLE STATE: (a) HIGH INCOME RESIDENTS, (b) MIDDLE INCOME RESIDENTS

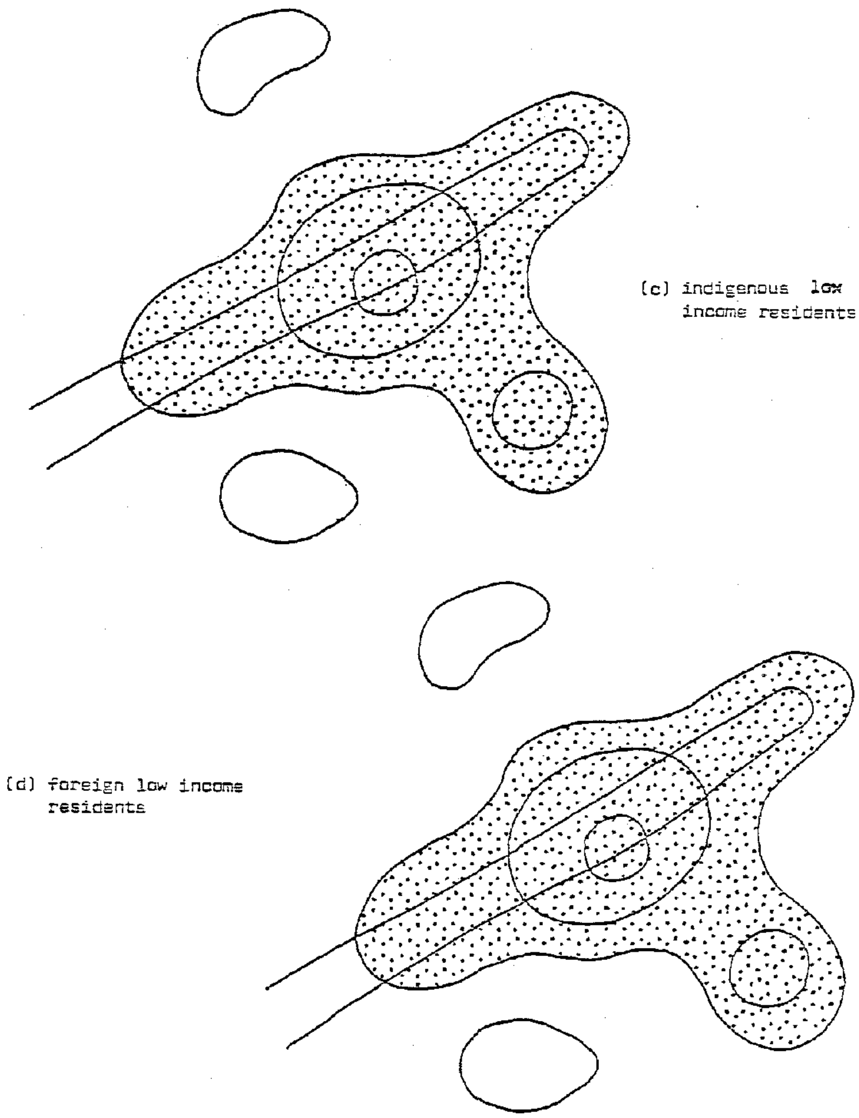


FIGURE 20c,d. THEORETICAL CITY - POSSIBLE LOCATIONS OF THE DIFFERENT SOCIAL GROUPS AT STABLE STATE: (c) INDIGENOUS LOW INCOME RESIDENTS, (d) FOREIGN LOW INCOME RESIDENTS

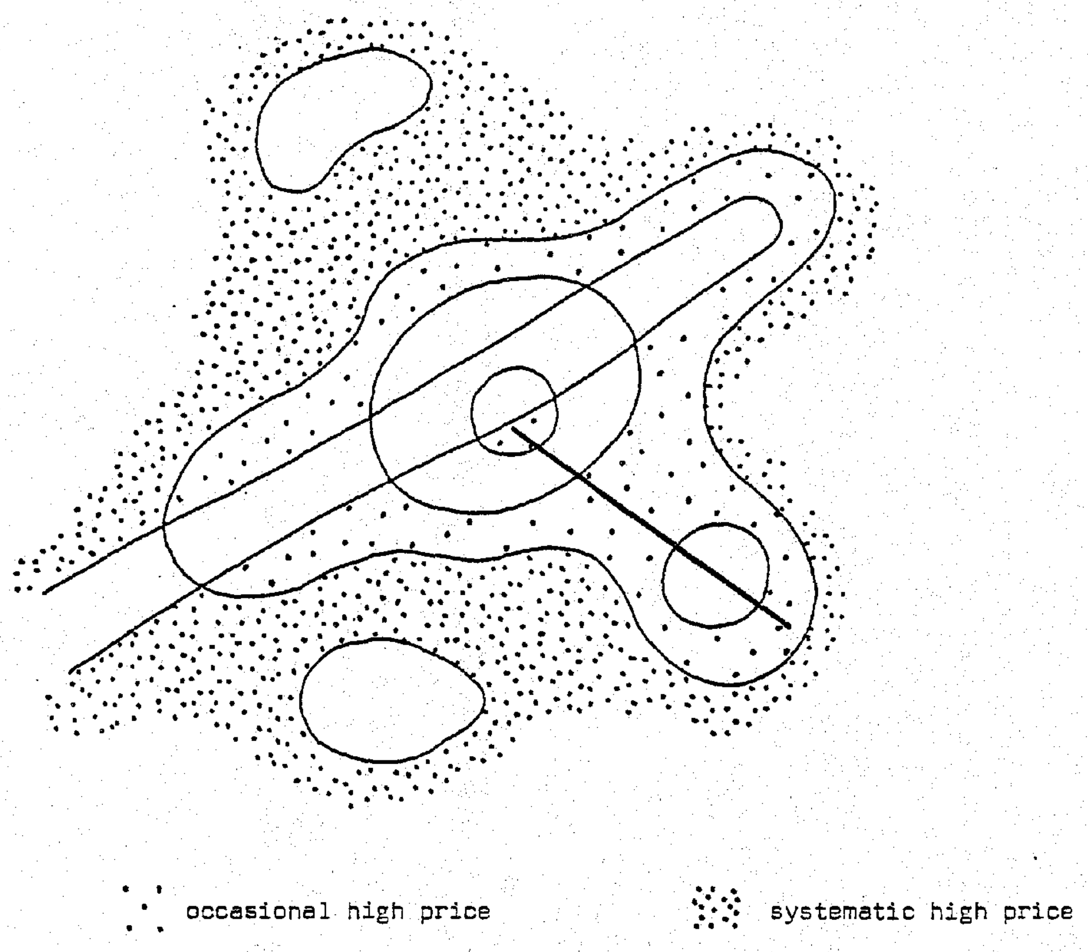


FIGURE 21. THEORETICAL CITY - HOUSING PRICE AT STABLE STATE

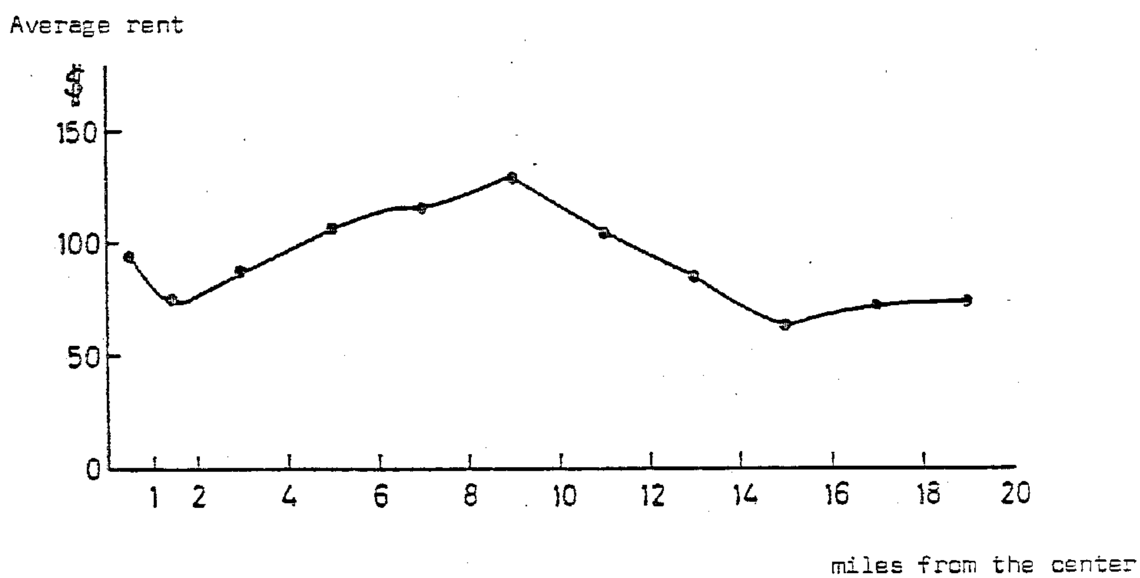


FIGURE 22. COLUMBUS, OHIO - AVERAGE RENT AS A FUNCTION OF THE DISTANCE TO THE CITY CENTER - THIS PROFILE CORRESPONDS TO THE AXIS DRAWN ON FIGURE 21.

2.2.4 The Comparison of the Theoretical City with a Real City

Before "comparing" the theoretical results with the latest census of the Brussels' population, it is necessary to describe the "physical" characteristics of the Agglomeration (Figure 23).[5] Divided into two parts by a S.W.-N.E. canalized river, hugged by a railroad line, the Agglomeration of Brussels is crossed today by an industrial axis passing through its center (this center is symbolized on all maps by the pentagon that used to form the walls of the ancient city). Small factories and often damaged old houses are intimately mixed along this axis. The ancient city, depopulated today, has developed a classical central business district on the eastern bank of the valley. Around this area, there is a ring of densely populated old neighbourhoods which is the zone corresponding to the first expansion of the city outside its walls. The population density decreases to the periphery so that one proceeds little by little from a dominance of compact apartment buildings to a dominance of single family houses with much open space. To the South, a beautiful old beech-grove is one of the most attractive spots of the Agglomeration.

For Brussels, there is no information available about any critical values of the <u>input</u> variables corresponding to the social efficiency thresholds of the model. Thus we are forced to leave it to the imagination of the reader to interpret the features of the map described above by trying to locate estimates of these thresholds in space.

We also have no information about the corresponding values for the <u>internal</u> variables. As a matter of fact, the only information we have is the spatial distribution of the socio-economic characteristics of the Brussels' population. This could be interpreted as a spatial image of the values reached by the behavioural equations of the model. Consequently, here again, we are forced to choose arbitrarily the real values corresponding to the hypothetical thresholds.

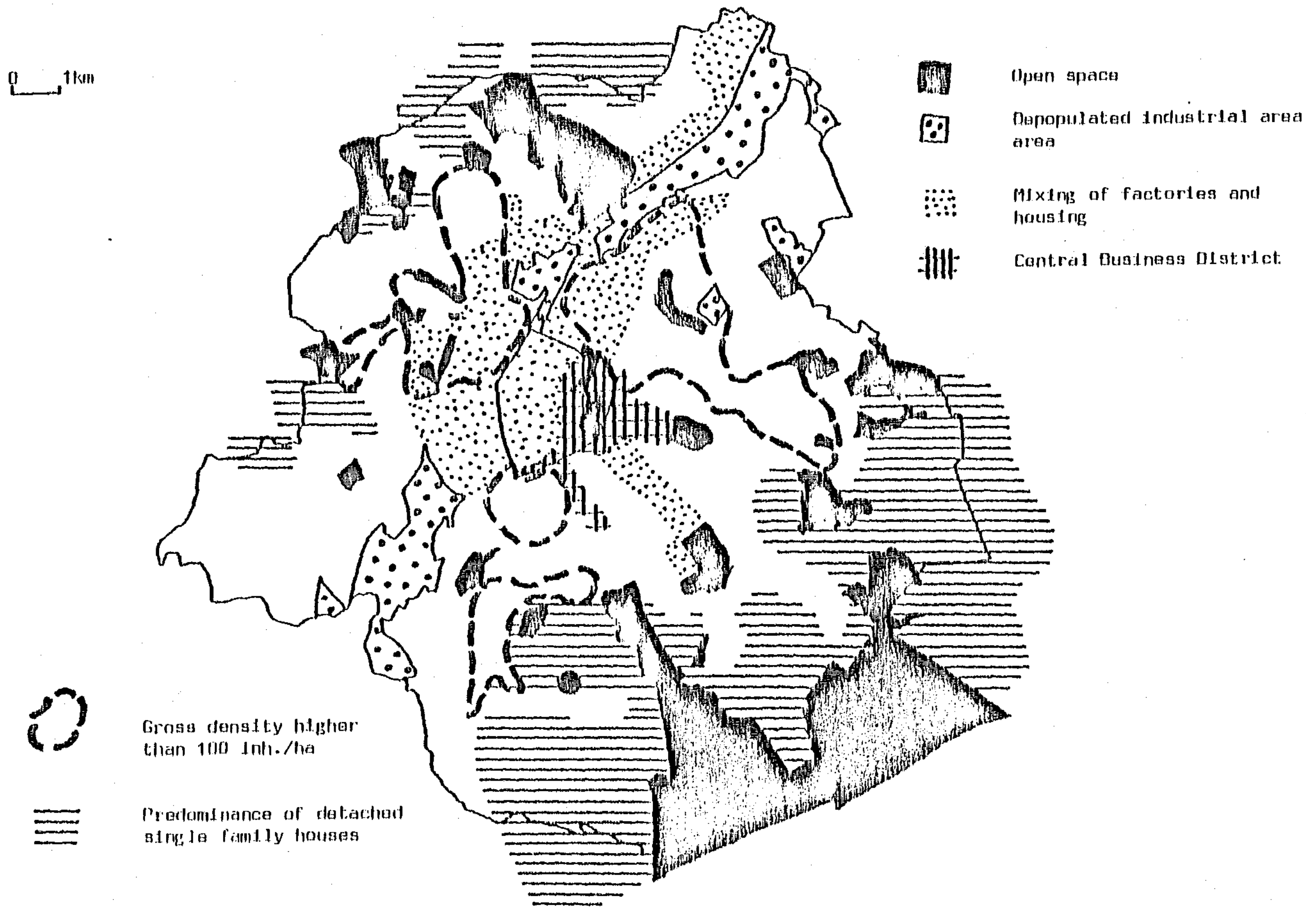


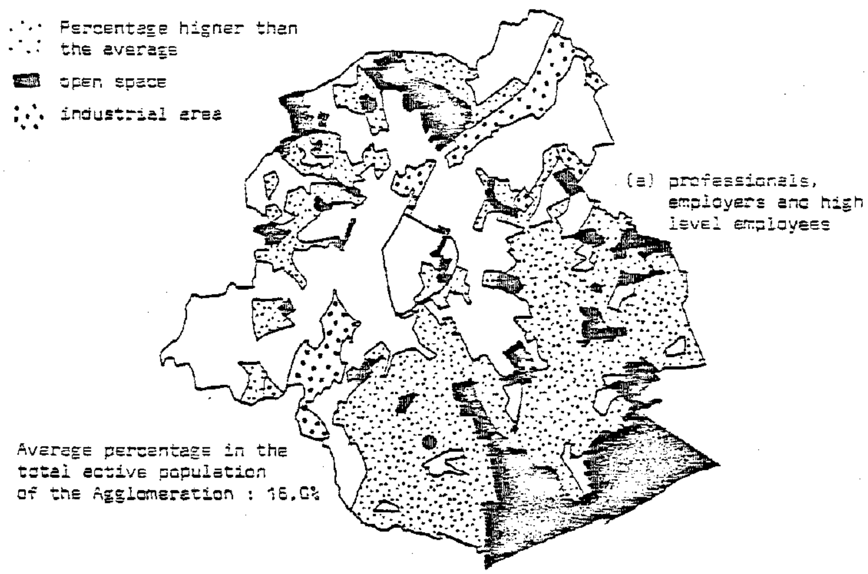
FIGURE 23. AGGLOMERATION OF BRUSSELS, BELGIUM - 1975 BASIC ELEMENTS ALLOWING IMAGINATION OF THE LOCATION OF THE SOCIAL EFFICIENCY THRESHOLDS OF THE INPUT VARIABLES

Because of the limitations of the 1970 populations census,[6] we have approximated income classes with occupational classes:

- high income residents correspond to professionals, employers and high level employees;
- middle income residents to the other types of employees;
- low income residents to workers.

Figures 24 a, b, c, show the census tracts having a proportion of each of these occupational classes higher than its average proportion in the Agglomeration (percentage of the total active population). As to the (active and non active) foreigners, the census gives only their proportion in the total population, whatever their activity may be (Figure 24d).

The internal structure of the theoretical city is vaguely consistent with the reality of the Agglomeration of Brussels which does not have any peripheral satellite inside its administrative limits. Indeed high income families (Figures 20a and 24a) carefully avoid the industrial axis on both maps; in Brussels, they seem to concentrate around public parks and in the peripheral neighbourhoods where single family houses are predominant. One finds them also in the restored and renewed parts of the center. It seems that they are more sensitive to density than indicated in the model. The middle income residents (Figures 20b and 24b) mix with industrial activities only where the population density is relatively low; they effectively seem to be rather sensitive to population density. The workers in Brussels (Figure 24c) are clearly limited to the areas close to the industrial axis. A comparison with the foreigners' map (Figure 24d) shows that the foreign workers concentrate around the C.B.D, while the indigenous workers have a tendency to spread along the industrial axis. This difference in the spatial distribution of workers, according to their nationality, does not appear on the theoretical maps (Figure 20c,d) which, in other respects, reproduce quite well the overall tendency of workers to locate close to the industrial areas.



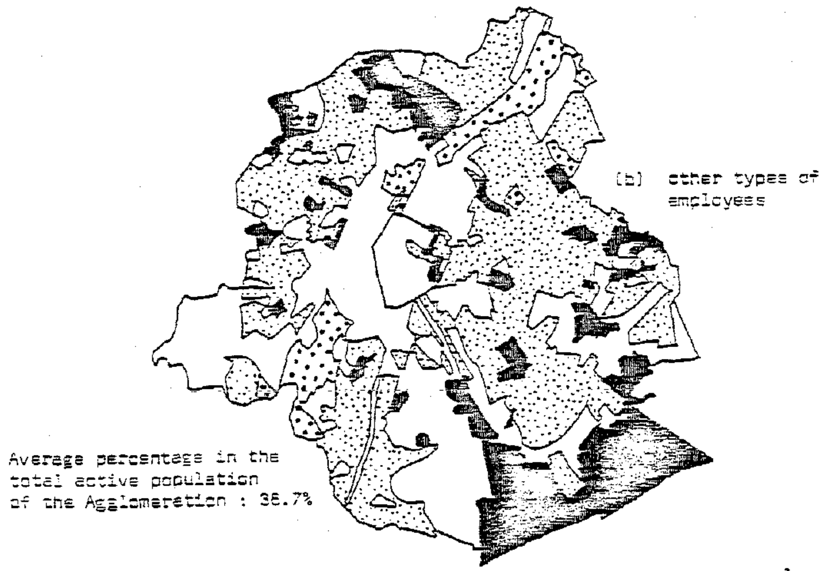
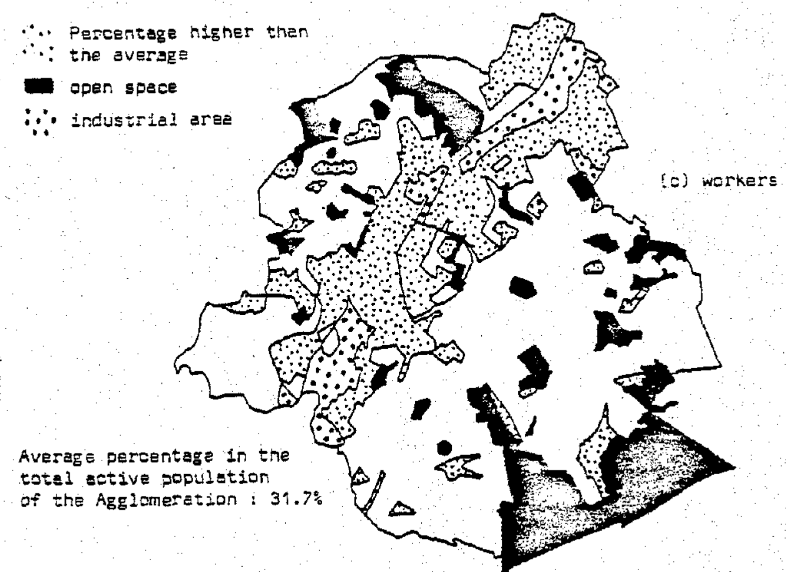


FIGURE 24a,b. AGGLOMERATION OF BRUSSELS - LOCATION OF RESIDENTS BY OCCUPATIONAL CATEGORIES - 1970: (a) PROFESSIONALS, EMPLOYERS AND HIGH LEVEL EMPLOYEES, (b) OTHER TYPES OF EMPLOYEES



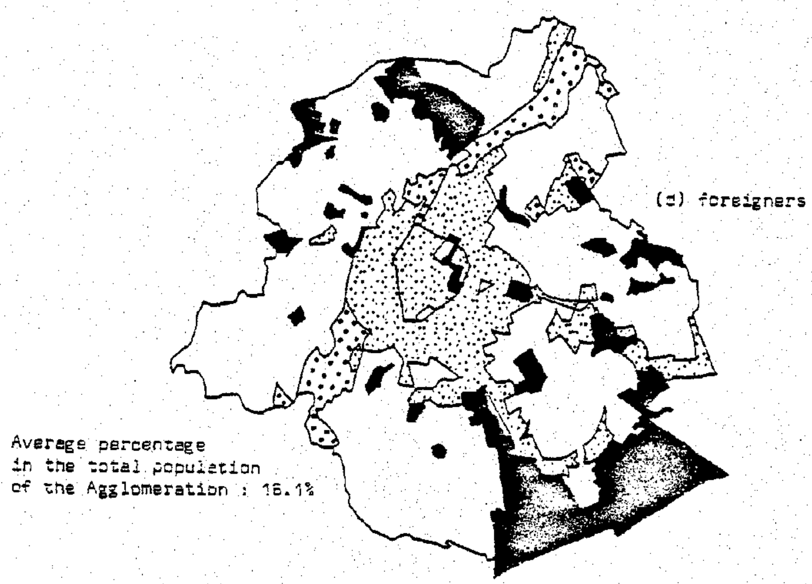


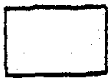
FIGURE 24c,d. AGGLOMERATION OF BRUSSELS - LOCATION OF RESIDENTS BY OCCUPATIONAL CATEGORIES AND NATIONALITY - 1970: (c) WORKERS, (d) FOREIGNERS

With the same data, let us now draw a map of the 1970 residential cohabitations (Figure 25). Arbitrarily again, two classes of population cohabit in a neighbourhood when the proportion of each of them in the neighbourhood oversteps its average in the Agglomeration, diminished by 20 percent. These cohabitations in 1970 are not always stable states. Some areas, we know, have started to undergo a deep transition process. It is the case, for instance, of the vast South-Western area where high income families mix with workers: it is the last rural area included in the Agglomeration limits which has undergone the classical process of urbanization by expansion of the urban fringe. Other areas have been stable for some time: the Southern wealthy neighbourhoods, for instance, and the central foreign neighbourhoods which will stay foreign and poor for a long time, even though they are submitted to a strong demographic pressure and because of that changing quickly. But information is missing about this problem of neighbourhood stability.

When we compare the map of all the possible stable states in the theoretical city (Figure 19) with the map of existing states in the Agglomeration of Brussels (Figure 25), we notice that the real situation looks much simpler than the theoretical one. Whatever the area, a selection seems to operate among the possible stable states in order to promote one or two of them. In other words, in any area, each possible stable state does not have the same frequency of occurrence. In order to make this type of model more realistic, we should then associate a probability to each possible stable state. This is what we have done with the help of computer simulations, as we will see later on. For the moment, let us analyze in detail the dynamics of the model in order to appreciate how the Boolean formalism could help in making planning decisions.

2.3 BOOLEAN FORMALISM, A TOOL FOR DECISION MAKING?

The Boolean formalism, when it is applied to the study of our hypothetical city, allows a complex situation to be clarified, Legend of Figure 25

 Uninhabited area

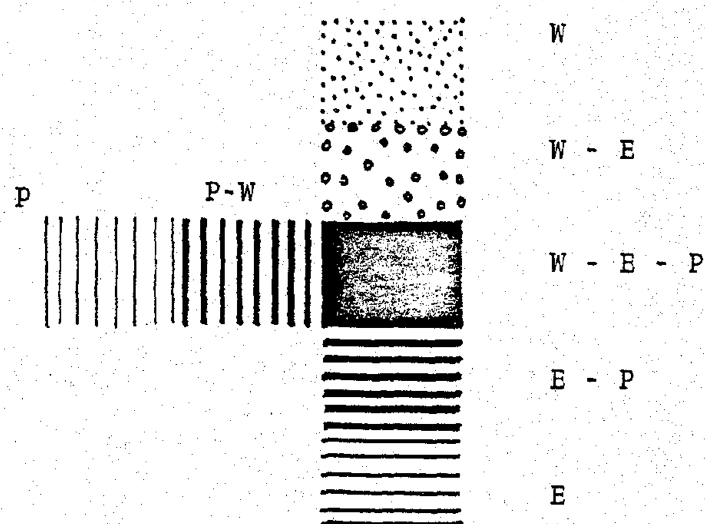
 High percentage of foreigners in the total population

Occupational cohabitations

P Professionals, Employers and high level Employees

E Other types of Employees

W Workers



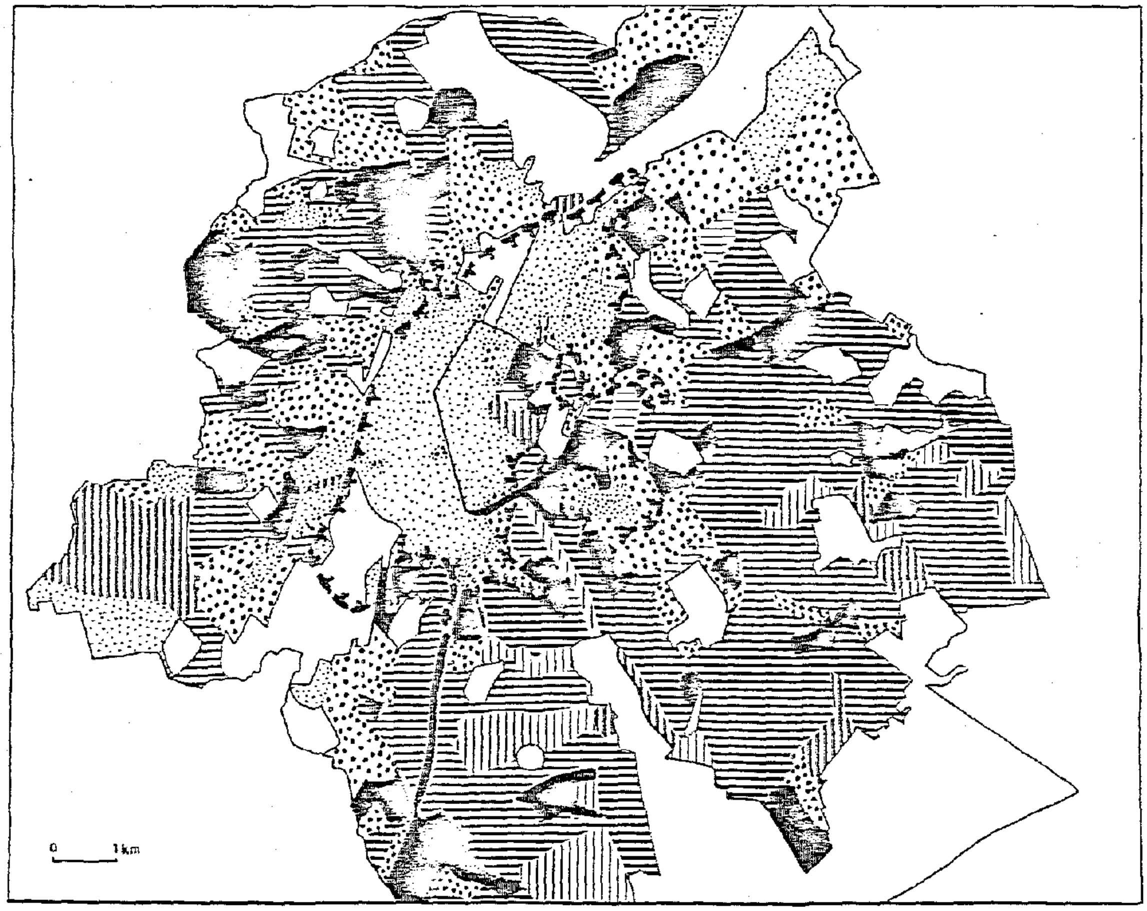


FIGURE 25. AGGLOMERATION OF BRUSSELS - OCCUPATIONAL COHABITATIONS - 1970

and according to which one can adopt two types of attitudes:

- on the one hand, one can wish to maximize one's individual interest: for example, a household looking for the best residential location or an entrepreneur trying to maximize his profit;
- on the other hand, one can wish to promote the collective interest of the entire urban community, knowing well that there is often some conflict between the indiviual and the collective points of view. This will be the goals of the urban planner and, sometimes, of the politician.

Even though the Boolean formalism allows us to adopt either of these points of view, we will consider here only the second case: the case of the urban planner who is confronted with the necessity of adapting the urban space to the development constraints.

2.3.1 A General Development Policy or the Search for Coherence Between Goals and Means

Let us imagine the case of an urban planner, taking as a goal, the integration of the different social classes in the theoretical city. Implicitly he tries to reach his goal with a strategy which would eliminate a maximum of undesirable effects. Furthermore, he wishes his goal to be maintained in time: so this has to be a stable state. Consequently, he wants to reach a stable state which is as close as possible to 01111 or 11111. The tendency of the three indigenous classes being to exclude foreigners, it is not surprising to find that in the state table (Figure 16), there is no stable state integrating all social classes. The stable state being the closest to this goal is the one that integrates the three indigenous classes: 01110. It can be reached only in a low density and nice neighbourhood which is close to the employment areas (column 010) and where housing prices are low.

In these conditions, the urban planner's reasoning will be the following:
For the integration of the foreigners, there is only one solution: change the behaviour of the three indigenous classes with regard to them and this will lead to a change in their behavioural equations. For the integration of the three other classes, there are a lot of possible solutions, dominated by these two requirements: put the neighbourhood in the input state (column 010) (D,H,T) and let the housing price be low ($\pi = 0$)

- fix a maximum to the population density which is lower than the threshold for which the inter-class rejection takes place. Employment continuing to increase spontaneously in the center, the home-to-work travel time will tend to increase and make the variable T turn to 1;
- there are two possibilities of maintaining T at the value 0:

  1) if one wants to keep centralized the internal structure of the city (which is its spontaneous tendency), one should stop the growth of the city and carry forward the overall urban growth on other cities of the region;

  2) on the contrary, if one does not want to stop the city growth, it becomes necessary to change its internal structure. Here again there are two possibilities:

     a) decentralize the employment to the periphery
     b) or improve the transportation system

- this being ensured, maintain the housing price at a low level. A new choice appears:

  1) stop land speculation
  2) or increase the income of the low income classes.

Obviously the means suggested by the stable table are not all equivalent; some of them are feasible for the planner, others are not. Everything depends on the room he has for maneuver in a specific socio-economic system. It falls to him to appreciate his constraints. This information about the coherence existing between goals and means is of a crucial importance for the planner.

2.3.2 The Planning of a Specific Neighbourhood or the Search for the Best Strategy of Investments

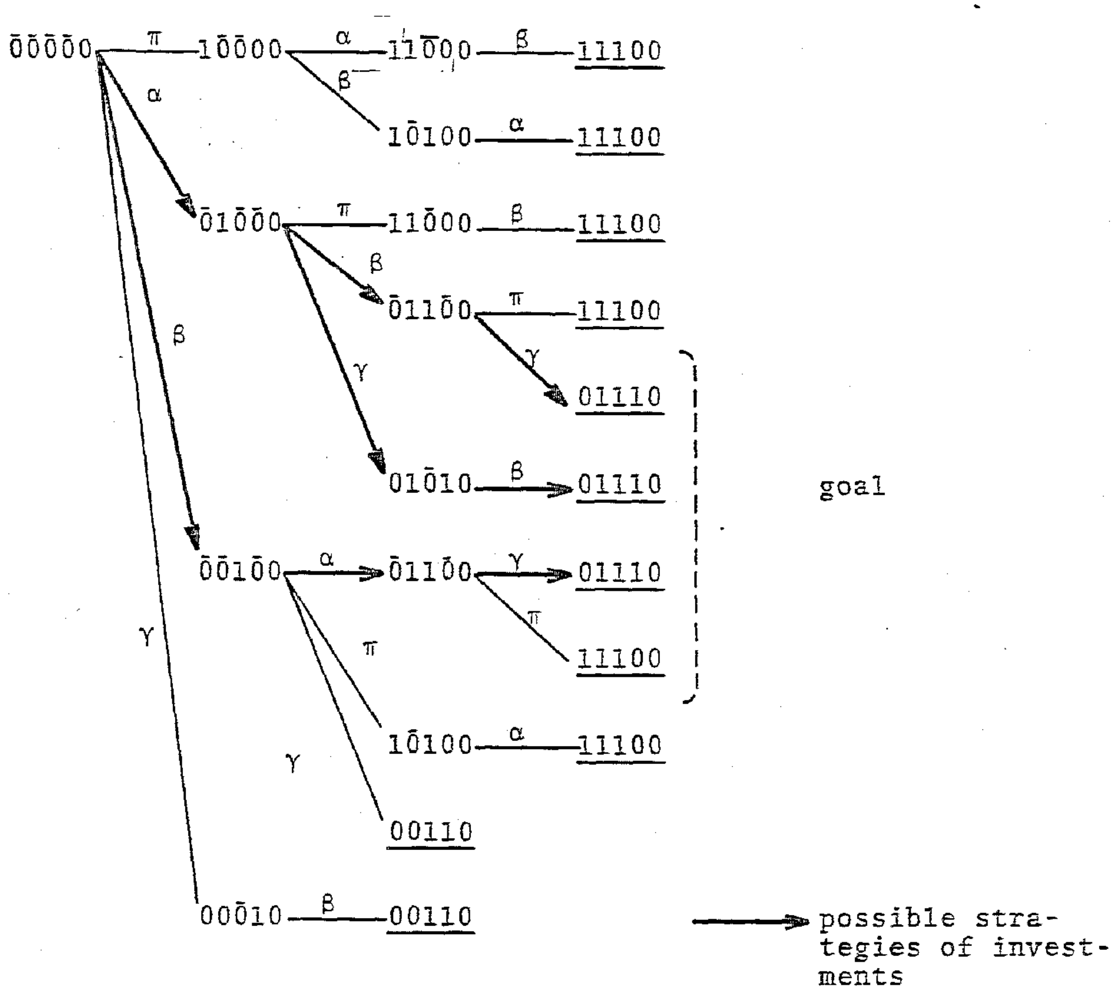
Maintaining the same goal of social integration - 01110 - let us imagine a nice neighbourhood (H = 1), close to the employment area (T = 0) and let us consider this case at low and high densities, starting from the initial state 00000. How could a planner program the right sequence of investments?

a) at low density

The detailed description of all the possible paths that the system can follow gives the entire spectrum of possible investment strategies. All paths do not lead to the goal 01110 and, among the ones that reach it, some are better then others.

Input state (D, H, T): 010

Initial state (π, α, β, γ, δ): 00000



Indeed it is only the building of wealthy or middle income family residences first that will lead to the goal. The building of low income family houses first would definitely divert the system from the goal. But in these conditions the decision maker is confronted with a possible increase in housing price during the execution of his project, and because of this, the system can be diverted from the goal at any time. How could the planner evaluate the chances of success of the chosen strategy?

Computer simulations are able to give some elements of the solution (for details of these simulations, see Reference 7). Let us suppose that the immigration (ε) and emigration (σ) delays of the social groups, on the average and in any neighbourhood, obey the following relationships:

$$\varepsilon(\delta) < \varepsilon(\beta) < \varepsilon(\alpha) < \varepsilon(\gamma)$$

$$\varepsilon(\alpha) = \sigma(\alpha)$$
$$\varepsilon(\beta) = \sigma(\beta)$$
$$\varepsilon(\gamma) = \sigma(\gamma)$$
$$\varepsilon(\delta) = \sigma(\delta)$$

Thus we assume that the most mobile residents in space are the foreign low income families and the most stable ones the indigenous low income families. Let us maintain constant these migration delays (arbitrary chosen as 20, 30, 40 and 50 with a random variation of 20 percent around these averages) while allowing the time delay of the housing price to vary (turn-on delay).

The probability for the occurrence of a stable state strongly varies as a function of the housing price rise delay (Figure 26). When the land speculation is intense and the price rise is rapid, the threshold value is quickly exceeded and the only possible state is 11100. The wealthy families are the only ones who can afford land speculation of such intensity and middle income families who are intent on clibing the social ladder. When land specualtion becomes weaker, the rise in the price of housing takes longer and the most probable state is still 11100. Two other stable states appear however: 01110 (the goal) and 00110 (the mixing of middle income families with indigenous low income families). A further lengthening of the delay strongly increases the probability of occurrence of the goal (01110), gives the 00110 state with a low probability and gives zero probability for the state 11100.

In this theoretical example, we see that the probability of occurrence of the goal is a function of the rapidity with which the price of housing rises. In reality (Figure 25), the frequency of occurrence of a state depends on the location of the neighborhood in the city. In other words, this would mean that the intensity of land speculation varies in the urban space and that computer simulations of that kind would probably be able to give some information about the spatial distribution of the speed of variation of the housing price.

Investment strategy should include a variable fraction of its costs used to cut down on land speculation. Here again, the urban planner is free to choose the best way to do so, according to the economic system involved. He could even try to make a cost-benefit analysis of different development strategies.

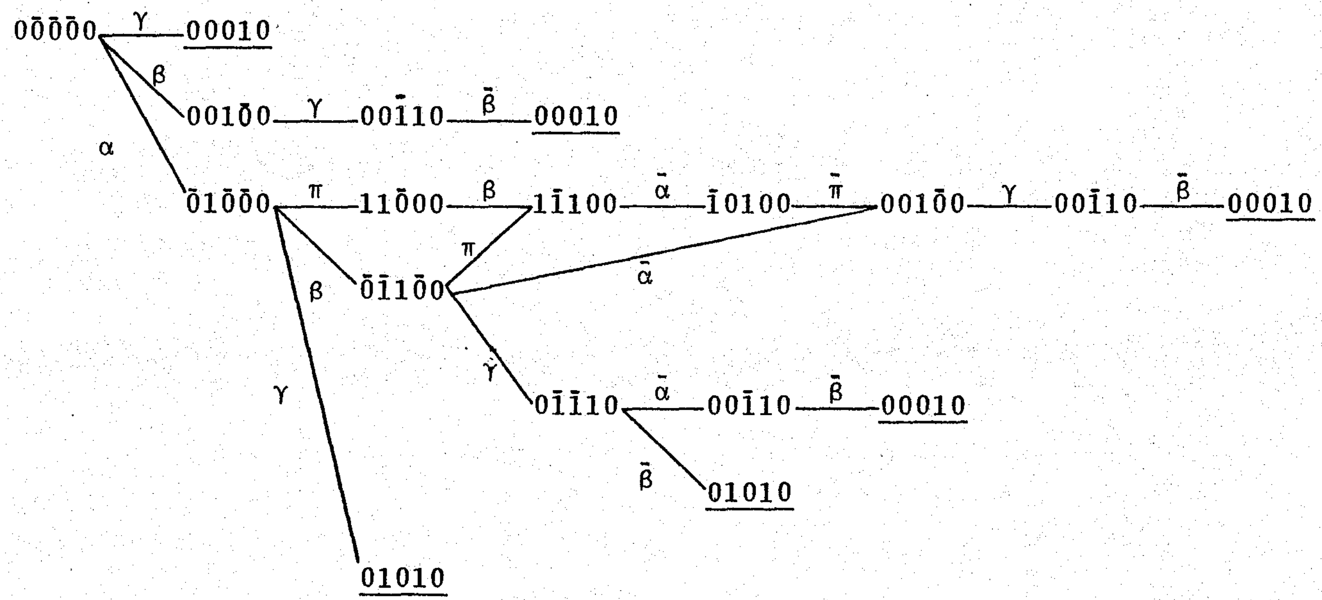
Other initial conditions - 11111 - still at low density (Figure 27), give a very different probability distribution of the goal 01110 which can be interpreted as the influence of history on the final state of the system. In this case, the influencing factor is no longer the rate at which the price of housing rises, but that of its fall (turn-off delay). The temporal strategy will be modified. On the other hand, it is only from this initial state that one sees the formation of ethnic neighbourhoods. Foreign immigrants must be imposed on the system explicity; they do not appear spontaneously in the neighbourhood by the internal dynamics of the system.

b) <u>at high density</u>

At high density, whatever the initial state may be, the goal 01110 is unattainable (Figures 26 and 27). (See next page.) Residential segregation greatly increases, as one would expect from the equations. Here again, the initial conditions strongly influence the stable states (Figure 27).

Input state (D, H, T): 110

Initial state (π, α, β, γ, δ): 00000



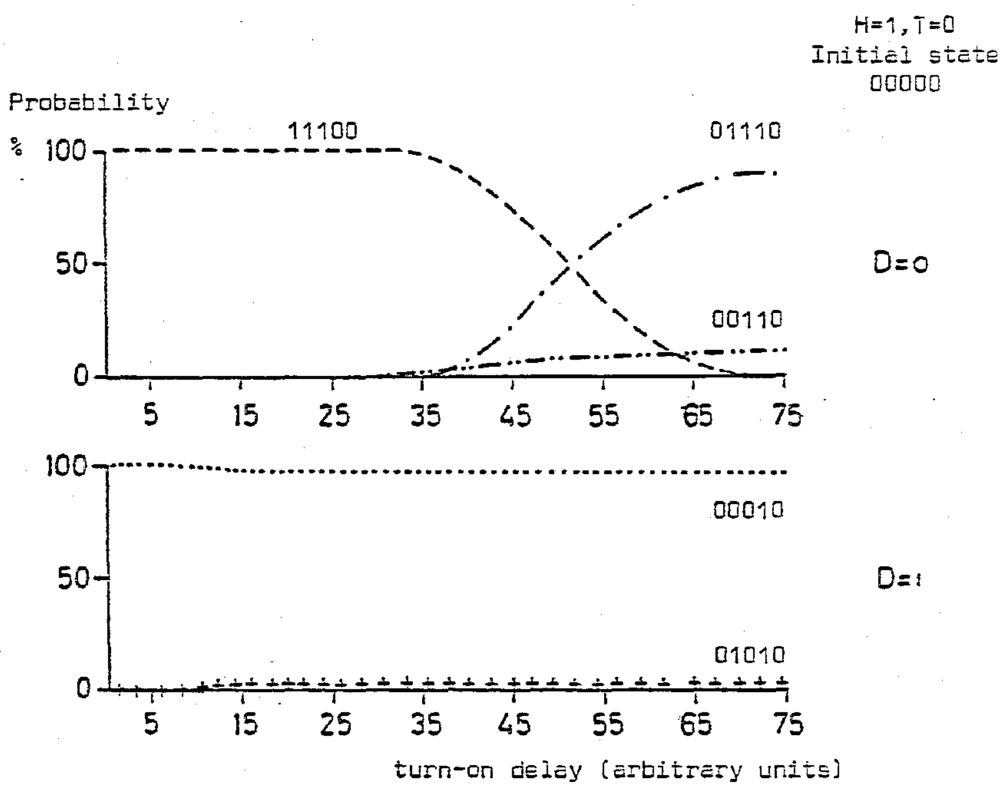


FIGURE 26. PROBABILITY OF OCCURRENCE OF THE STABLE STATES AS A FUNCTION OF THE HOUSING PRICE RISE DELAY (TURN-ON DELAY)

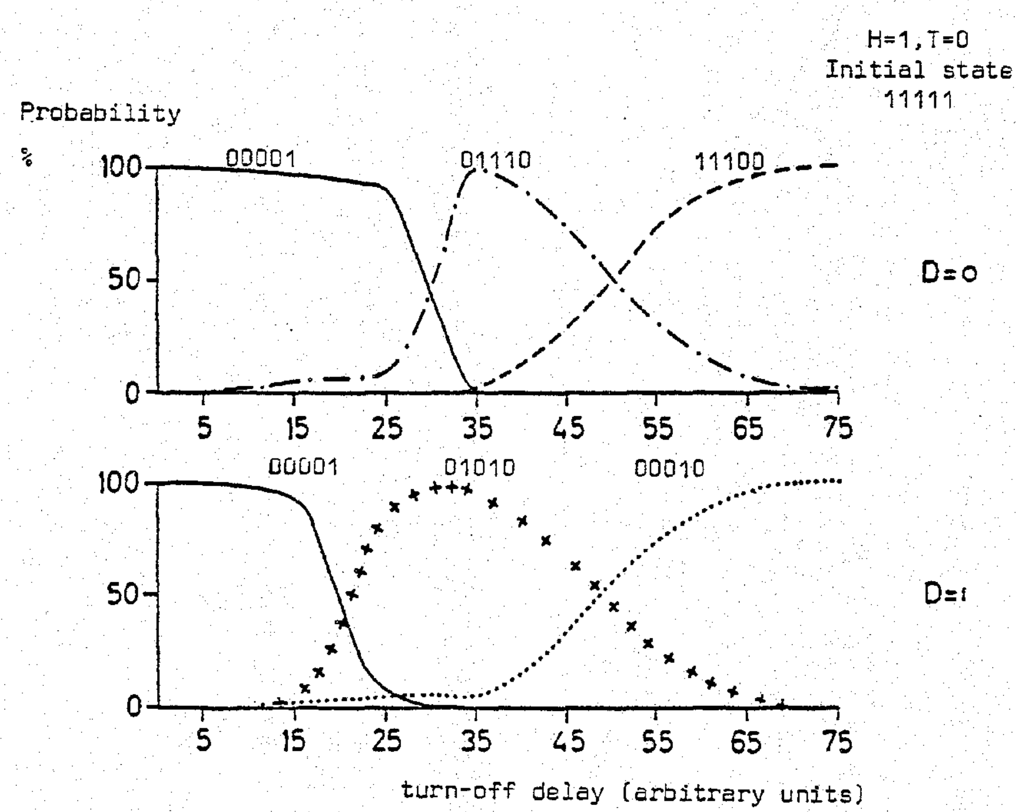


FIGURE 27. PROBABILITY OF OCCURRENCE OF THE STABLE STATES AS A FUNCTION OF THE HOUSING PRICE FALL DELAY (TURN-OFF DELAY)

CONCLUSION

The Boolean formalism has the great advantage of being easily adaptable to the dynamic analysis of complex systems. It remains manageable when a continuous formalism, although attractive because of its greater analytical power, becomes too complex to handle. Its flexibility, its simplicity and the rapidity of analysis that it allows could make it a precious help to decision making in socio-economic matters.

Acknowledgment

We warmly thank Professors Jean-Pierre Boon and René Thomas and Dr. Philipe Van Ham for fruitful discussions and suggestions.

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7. The computer program used for the simulations is the PRAN 2 program, realized by Philippe Van Ham. Van Ham, P., A random simulation of deferred actions logical systems dynamics, IFAC, Congress on discrete systems, Dresden (DDR), March 1977, Vol. 5, p. 27-35.

Details of the simulations

Initial state 00000
Each point of the curves is the average calculated from eight series of one hundred simulations. These series are characterized by variable turn-off delays of the housing price (arbitrary units from 5 to 85). For each simulation, the values of the delays are randomly chosen in a range of 20 percent around the values.

Initial state 11111

The simulation method is the same but here the average is computed from variable turn-on delays of the housing price.

3. DYNAMIC MODELS OF COMPETITION BETWEEN TRANSPORTATION MODES

3.1 INTRODUCTION

In the preceding reports,[1] we have attempted to analyse models describing the creation and evolution of urban structure. However, transportation within the city was treated in a very summary fashion, where only the geometric distance between the trip origin and destination was used. This is of course a vast oversimplification, and as has been shown (Adams, Figure 28) the development of a particular form of transport results in a particular type of urbanization. Trams, trains and highways promote a directional urban development, while the initial stage of pedestrian travel, and the later one of automobile trips promote a compact circular form.

While it is true that the type of transportation available will influence the urban structure, the inverse is also true since the construction and operation of a public transport facility, (train, subway, bus, etc.) require a certain transport of population density and transit demand along its path. Thus, the location of employment and residences in the urban structure will depend on the availability of urban transport, and vice versa. Ultimately, our aim is to include this mutual dependence in our equations for the evolution of the urban structure, but here, initially, we will consider the simpler problem for which the transit demand is given, and the different modes of transportation are in "competition symbiosis" or "parasitism".

The dependence of the optimal usage of a means of transportation on the distance involved (Figure 29), leads us to suppose that the solution to the problem of urban transportation has the form of a complex hierarchy made up of inter-dependent modes. The complexity of such a problem may be gauged from the study of the competition between two transport modes for the clientele of a given trip, where such factors as the "quality" of the transportation offered, and the imitative behaviour of people may play a decisive role.

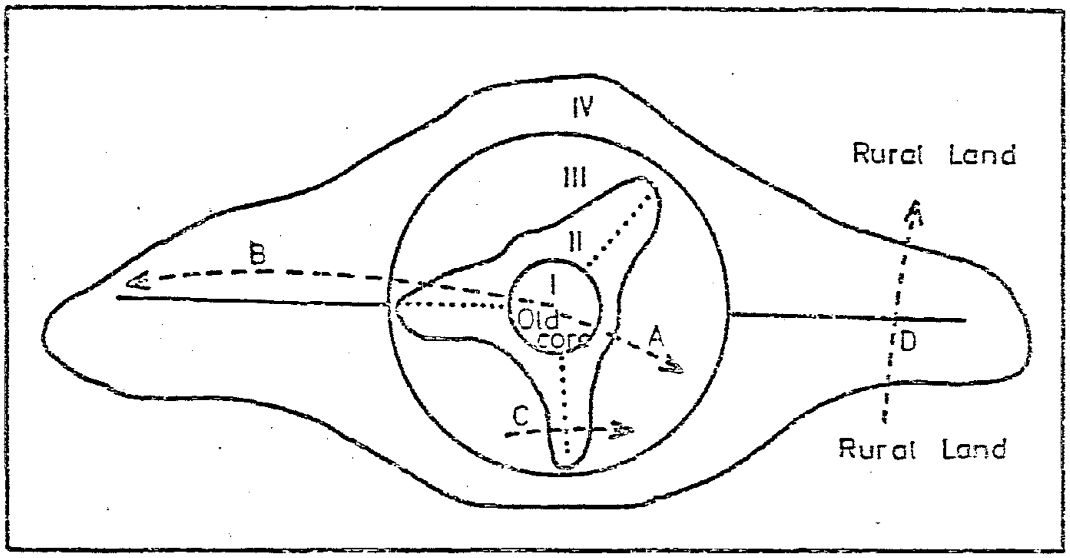


FIGURE 28. EXPECTED DISTORTIONS FROM GROWTH PATTERNS. THE HIGHLY ARTICULATED URBAN TRANSPORT NETWORKS OF TRANSPORT ERAS II (STREET-CAR LINES) AND IV (FREEWAYS) PROMOTED TRANSPORT SURFACES AND COMPACT, CIRCULAR URBAN FORMS. TRAVERSES A THROUGH D INDICATE THE VARIETY OF CONTRASTING AGE GRADIENTS.

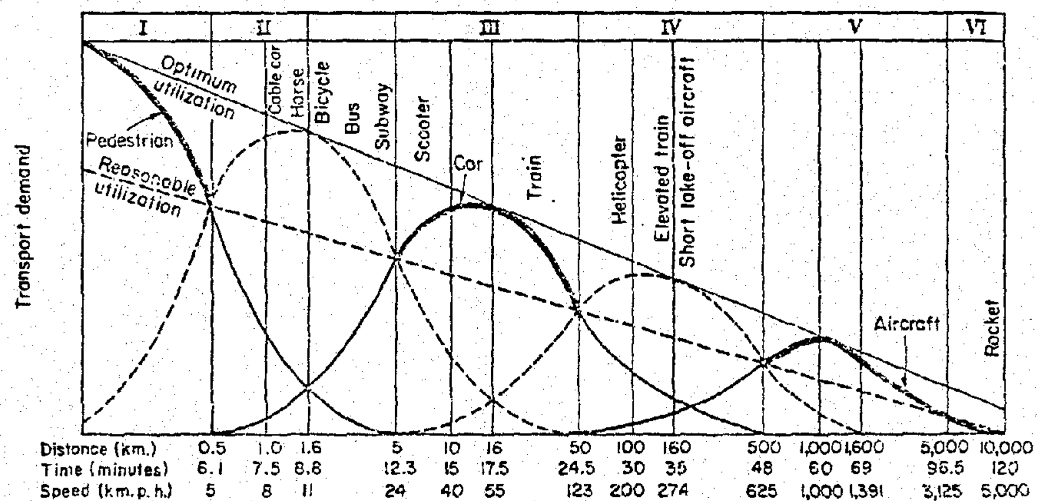


FIGURE 29. TRANSPORT GAPS. WHEN DEMAND FOR TRANSPORT (VERTICAL AXIS) IS PLOTTED AGAINST THE SPEED AND OPTIMUM RANGE OF EXISTING TRANSPORT SYSTEMS, WE SEE THAT THE TRANSPORT RANGE HAS THREE AREAS (I, III, AND V) WHICH ARE WELL TAKEN CARE OF BY PEDESTRIAN, CAR, AND AIR TRANSPORT. MAJOR GAPS OCCUR IN AREAS II AND IV (ADAPTED FROM BOULADON, FIG. 1).

3.2 MODELS

3.2.1 Introduction

The models which we shall present are models of choice among different transportation modes. Our purpose here is not to develop a model of transportation choice which reflects the actual and complex decisions which go into such choices by different groups of individuals. Rather it is to present a methodology which is dynamic and which allows inherent fluctuations in the behaviour of the individuals to play a role (and often it is a fundamental one) in the determination of the way the system responds to different mode choices.

We consider the case of choice between two modes of transportation. Let x and y be the number of individuals who choose transportation mode 1 and 2 respectively. Let A_1 and A_2 be the attractivities of transportation modes 1 and 2. Let D be the estimated number of people who want to go from point A to point B, and assume in this first approach that D is constant (though in reality D is a function of the locational processes). When the system reaches stationary state we have

$$\frac{x_s}{D} = \frac{A_1}{A_1 + A_2} = P_1 \text{ and } \frac{y_s}{D} = \frac{A_2}{A_1 + A_2} = P_2 \,. \tag{8}$$

In general A_1 and A_2 are functions of x and y. System (8) can have more than one solution. It is not possible to know without some additional information which solution the system will adopt. In fact we must provide the system with information on its dynamic evolution in order for it to integrate different historical occurrences which will determine the final solution adopted by that system. In general the system will remember its initial conditions (x_0 and y_0) and the perturbations, both external and internal, which have occurred in its history. In this case only the densities of x and y will be subjected to perturbations.

3.2.2 Development of the Dynamic Equations

Taking an approach often used in ecology, we write the law of evolution of the variable z, defined by

$$z = x + y, \tag{9}$$

as

$$\dot{z} = D - z, \tag{10}$$

where $\dot{z} = dz/dt$ (t denotes time). In equation (10) the parameter D, which is a constant, is usually called the "carrying capacity". The value of the carrying capacity determines the final state reached by the system.

For the variables x and y we assume the same form for the equations of evolution:

$$\dot{x} = D_1 - x \text{ and } \dot{y} = D_2 - y, \tag{11}$$

D_1 and D_2 being known functions which must be determined. Since equation (9) holds for all times t, we have

$$D_1 + D_2 = D. \tag{12}$$

The equations of evolution of the variables x and y, given equations (8), will therefore be

$$\dot{x} = \frac{DA_1}{A_1 + A_2} - x \quad \text{and} \quad \dot{y} = \frac{DA_2}{A_1 + A_2} - y \tag{13}$$

Note that the carrying capacities in the equations for the evolution of the variables x and y are functions of time.

In order to illustrate the behaviour of the equations (13) we have sketched the evolution of the variables x and y for the case when $P_1 = P_1(x,y)$ is given (Figure 30). In general $P_1(x,y)$ is not known. Figure 30 shows graphically that z tends asymptotically to D, as is obtained by integration of equation (10) over time. The curve represented by $DP_1(t)$ is taken as given and the curve labelled $DP_2(t)$ is obtained from this by use of the fact that $P_1 + P_2 = 1$. The curves for x(t) and y(t), which may be obtained by numerical integration of equations (13), are also sketched in the figure. It is apparent that the solutions x(t) and y(t) tend asymptotically to DP_1 and DP_2 respectively, and, for all times t, $x + y = z$.

Note that when P_1 and P_2 are given by equations (8) it is not possible in general to compute analytically the solutions x(t) and y(t) of equations (13). However, mathematical techniques do exist which provide information on the evolution of the system. For our purposes we may use bifurcation analysis, which yields information on which possible final solutions are accepted by the system as well as information on their stability properties when subjected to perturbations in the densities of x and y.

3.2.3 The Theoretical Models

The models presented here do not pretend to take into consideration all decisions affecting mode choice. We prefer instead to develop simpler models which may be computed analytically. This is done in order to be able to show some of the interesting properties which appear when P_1 and P_2 are functions of the state of the system. (See equations (8).)

3.2.3.1 The First Model

In order to define the model we must give a particular form to the attractivity functions, A_1 and A_2. We suppose that these functions depend solely on the speed of transport. We assume that

$$A_1 = v_1^p \text{ and } A_2 = v_2^q \qquad (14)$$

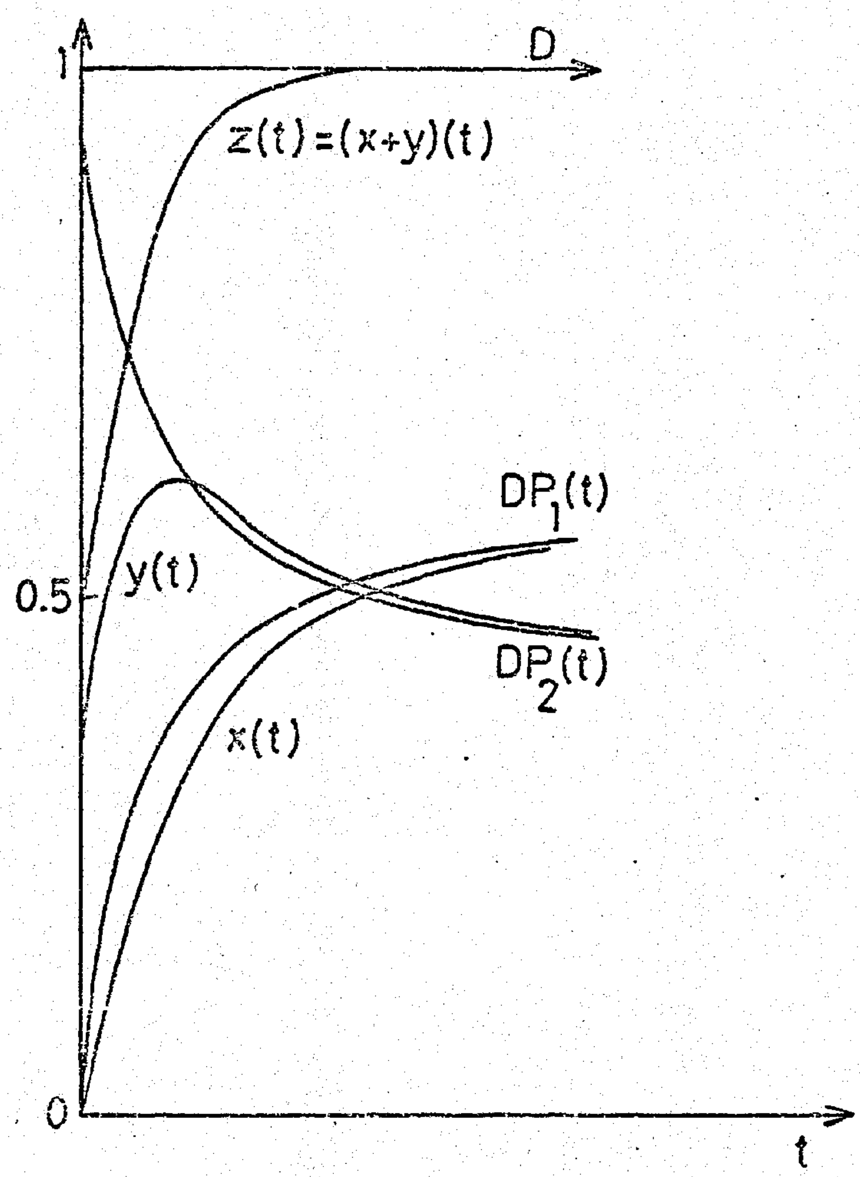


FIGURE 30. EVOLUTION OF x AND y WITH TIME

where v_1 and v_2 are the velocities of modes 1 and 2 respectively, and p and q are positive exponents. We suppose, for example, that mode 1 corresponds to the automobile and mode 2 to the bus. We further assume, in this example, that there is no interaction between the two transportation modes (it is not difficult, however, to remove this restriction).

Figure 31a shows the assumed dependence of the velocity of cars on their density (the congestion effect - see Haight, 1963, Ref.2). For the velocity-density relationship for buses, we assume, in addition, that the supply tends to adjust to the demand; that is, as more people demand bus transit, more buses are put into service, which results in a reduced overall time of transit (waiting plus riding time). This relationship is sketched in Figure 31b.

We may fit the curves of Figures 31a and 31b respectively by the following analytic expressions:

$$v_1 = \frac{1}{a + bx} \quad \text{and} \quad v_2 = \frac{dy^n}{c + y^r s} \tag{15}$$

where a, b, c, d, s, n, and r are positive constants and $n \leq r$. For example, if we take for the velocities of mode 1 (the car) and mode 2 (the bus) the following:

$$v_1 = \frac{1}{a + x} \quad \text{and} \quad v_2 = \frac{dy}{c + y} \tag{16}$$

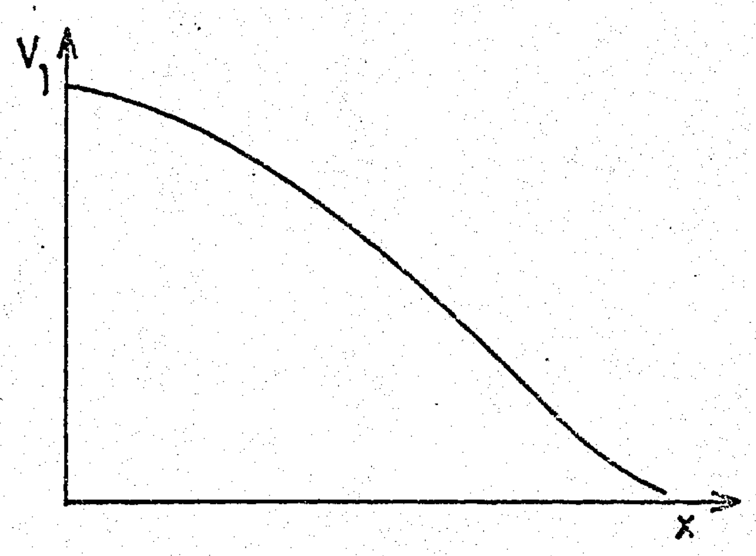
and for the exponents of equations (14) the following:

$$P = q = 1; \tag{17}$$

the equations for the time evolution of transportation modes 1 and 2, equations (13) then become

$$\dot{x} = \frac{D}{a + x} \Big/ \left(\frac{1}{a + x} + \frac{dy}{c + y}\right)^{-1} - x, \quad \dot{y} = \frac{-Ddy}{c + y} \Big/ \left(\frac{1}{a + x} + \frac{dy}{c + y}\right) - y. \tag{18}$$

(a)



(b)

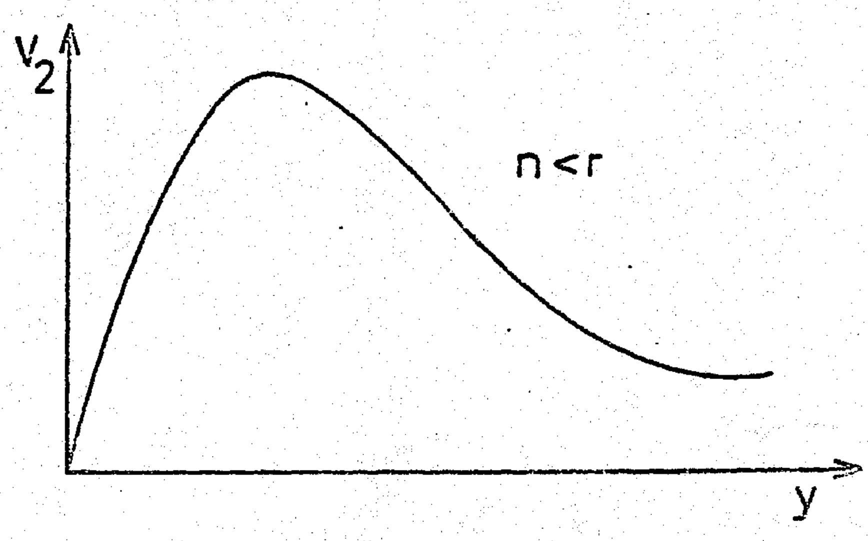


FIGURE 31. VELOCITY-DENSITY RELATIONSHIPS (a) FOR THE CAR MODE AND (b) FOR THE BUS MODE

It is easily verified that the final stationary states of the system ($\dot{x} = 0$, $\dot{y} = 0$) are such that

$$x_s + y_s = D. \qquad (19)$$

Using this relationship to find the values of the stationary states, we see from equations (18) that the state ($x = D$, $y = 0$) is a stationary state. We can therefore write the equation giving the values of the other stationary states in the following form:

$$dx^2 + (1 + da)x - (c + D) = 0 . \qquad (20)$$

Then system (18) has the following three stationary solutions:

$$x' = D, \qquad y' = 0. \qquad (21)$$

$$x^{\pm} = \frac{-(da + 1) \pm ((da + 1)^2 + 4(c + D)d)^{1/2}}{2d} \qquad (22)$$

$$x^{-} < 0. \qquad (23)$$

The solution (x^{-}, y^{-}) is physically not acceptable because x is negative (or zero). The solution (x^{+}, y^{+}) is physically acceptable only if (see equation (19)

$$D \geq x^{+}. \qquad (24)$$

We may put this condition in the form

$$D \geq 1/2 \; (-a + (a^2 + 4c/d)^{1/2}). \qquad (25)$$

Solutions (21) and (22) are represented graphically in Figure 32.

We thus see that, for a sufficiently large transit density, D, the system accepts a solution other than given by $x' = D$, $y' = 0$ (all cars). In fact, in this example, the system can tell us

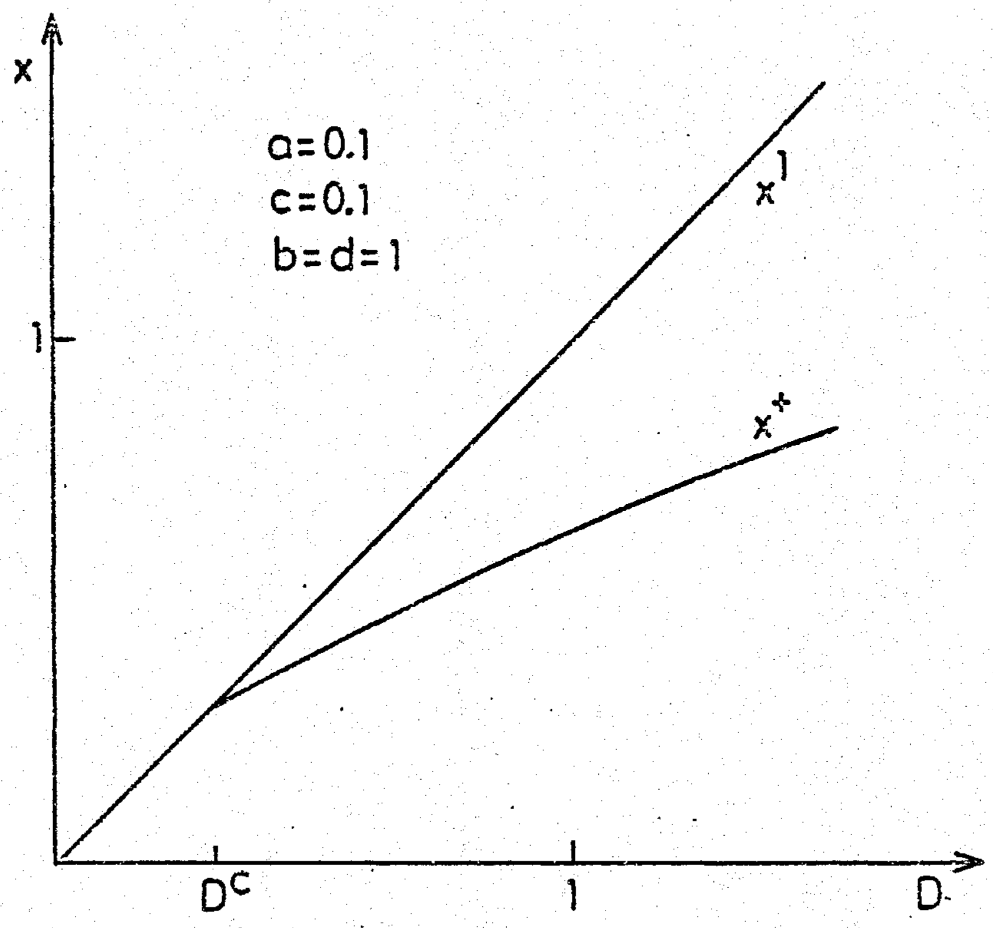


FIGURE 32. SOLUTIONS x^1 AND x^+ OF EQUATIONS (21) AND (22)

which solution will be adopted even if we do not know its historical evolution. The system will adopt a solution only if it is sufficiently stable to fluctuations in the densities x and y. The laws introduced describe only the average behaviour of the densities, but perturbations around this average behaviour are inevitable. In our example a stability calculation[3] shows that the solution (x', y') becomes unstable if the density, D, becomes large enough (whether caused by fluctuations or by other means). All perturbations (δx, δy) around a stationary state are assumed to vary with time according to the function exp (ut). The stability of the stationary state will depend on the sign of u. If it is positive the system is unstable to perturbations; if it is negative the system is stable to perturbations. Then the solution (x', y') becomes unstable when

$$D \geq 1/2 \ [-a + (a^2 + 4c/d)^{1/2}] = D^c \ . \qquad (26)$$

Note that this stability condition is identical to the condition for existence of the solution x^+ in this case.

Figure 33 presents the bifurcation diagrams showing the different final solutions the system may adopt and their stability as a function of the parameter D. We see in Figure 33 (b) that if $D < D^c$, then the transit density is not sufficiently large for the initiation of a bus service: the only stationary state permitted by the system is the state ($x' = D$, $y' = 0$). However, for higher densities $D > D^c$ the share of people taking the bus mode, y/D, increases.

Note that internal perturbations near a stationary state (x_s, y_s) are such that

$$\delta_x \ll x_s \qquad \delta_y \ll y_s \ . \qquad (27)$$

In the case where $y_s = 0$, the perturbation must be introduced as an external factor (corresponding to a new transportation mode). The theory developed here thus tells us the conditions under which

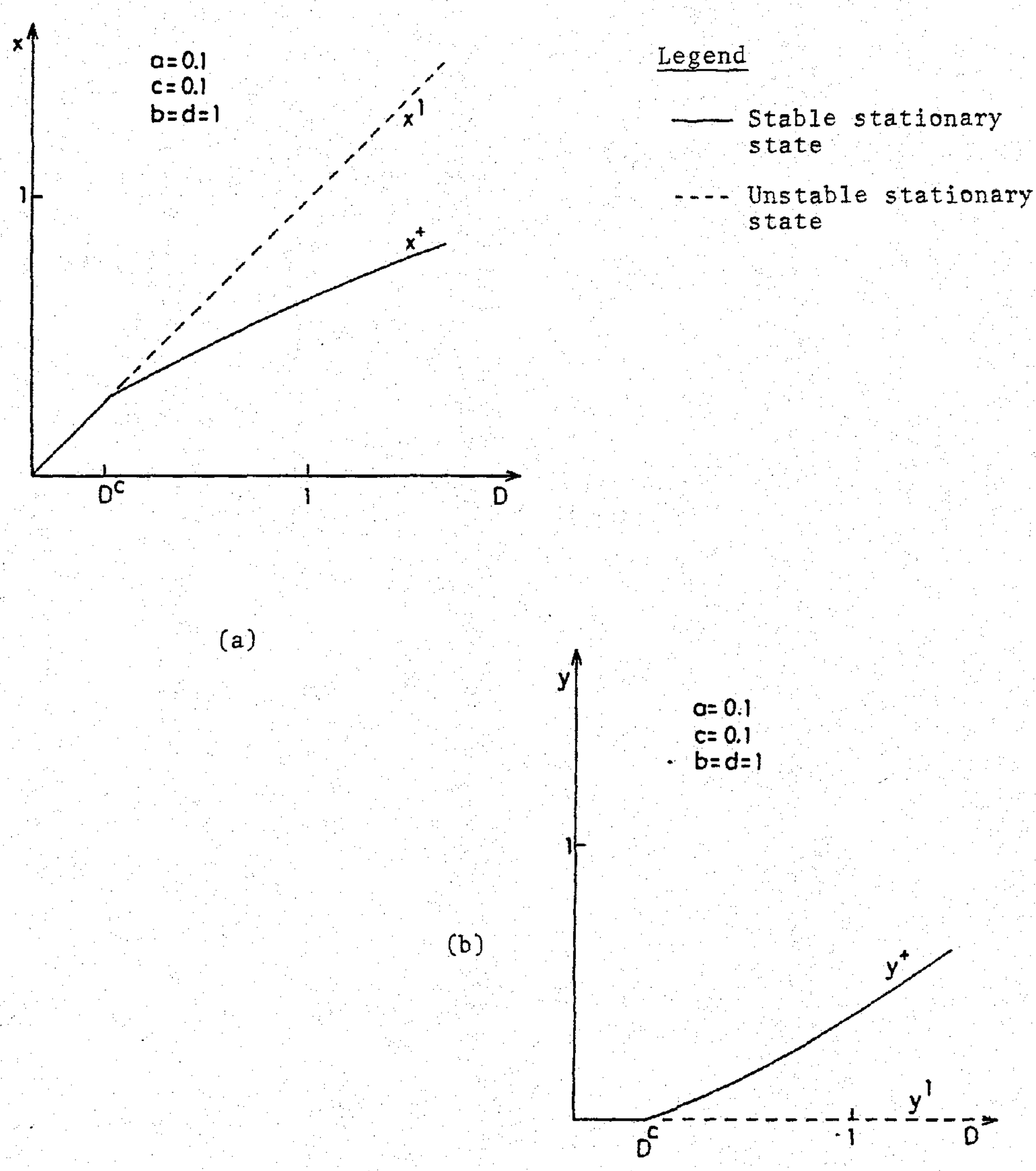


FIGURE 33. BIFURCATION DIAGRAMS OF (a) x VERSUS D AND (b) y VERSUS D

the system becomes unstable with regard to the introduction of a new transportation mode. The condition in which the system accepts this new mode of transportation ($D > D^c$) obviously depends on the characteristics of the existing transportation mode (the parameter a) and on the characteristics of the new one (the parameters c and d). (See equations 16 and 26). The fundamental role played by the bifurcations has been illustrated by this example.

Figure 34 sketches the evolution of the velocities of each transportation mode and the average velocity in the system as functions of the density of transit, D (computed when the system has reached the stationary state).

3.2.3.2 The Second Model

In the first model, we introduced a classical effect for the attractivity function, namely that as the speed of a transportation mode increases, the attractivity of that mode increases. There are other factors, psychological for example, which also influence the choice. Publicity and increased information about a particular mode, for instance, may influence an individual's choice. In a similar vein the process of imitation for people taking a particular mode may partially explain some existing situations. We will show in this section that these kinds of effects can considerably increase the richness of the behaviour of the system.

We now introduce into the attractivity functions, psychological factors, F_i, to obtain

$$A_1 = v_1^p F_1 \qquad \text{and} \qquad A_2 = v_2^q F_2 . \tag{28}$$

For the functions F_i we take the following simple forms:

$$F_1 = \theta_1 + \alpha_1 x \quad \text{and} \quad F_2 = \theta_2 + \alpha_2 y, \tag{29}$$

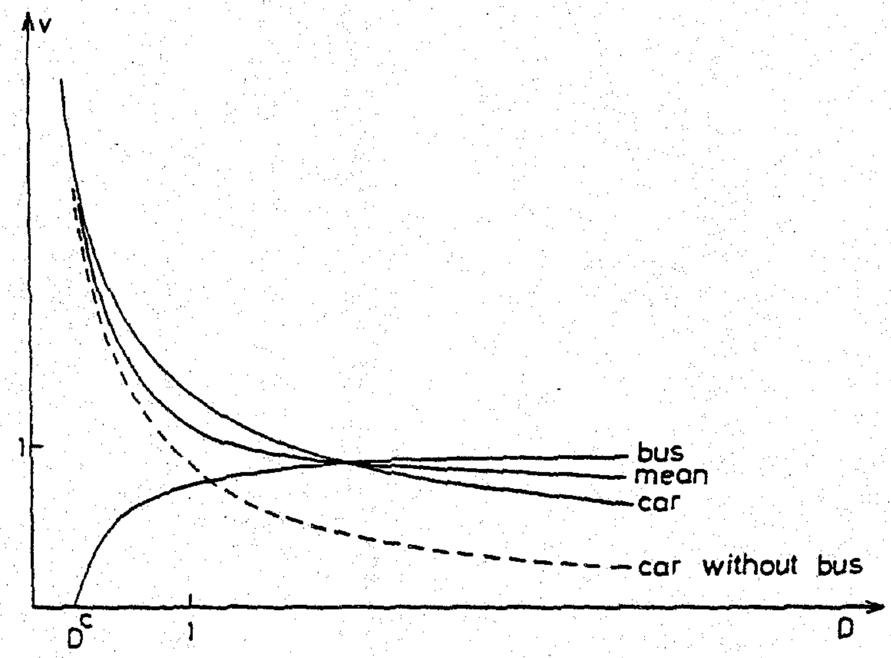


FIGURE 34. CAR, BUS, AND AVERAGE VELOCITIES IN THE SYSTEM AS FUNCTIONS OF THE TRANSIT DENSITY, D

Where θ_1 and θ_2 are publicity terms and $\alpha_1 x$ and $\alpha_2 y$ are imitation terms. For the dependence on the velocity, we use the same forms as before (equations 15), and in order not to complicate the problem these velocities will in this case be simplified to

$$v_1 = 1/x \quad \text{and} \quad v_2 = y. \tag{30}$$

We also take

$$p = q = 1 \tag{31}$$

in equations (28). With these values the equations of evolution, equations (13), become

$$\left.\begin{aligned} \dot{x} &= D\left(\frac{\theta_1}{x} + \alpha_1\right) \Big/ \left(\frac{\theta_1}{x} + \alpha_1 + \theta_2 y + \alpha_2 y^2\right) - x \\ &\text{and} \\ \dot{y} &= D(\theta_2 y + \alpha_2 y^2) \Big/ \left(\frac{\theta_1}{x} + \alpha_1 + \theta_2 y + \alpha_2 y^2\right) - y. \end{aligned}\right\} \tag{32}$$

Note that we have taken all parameters to be positive (it is in fact certainly possible to have, for example, negative publicity terms). Figure 35 shows the attractivities of the two modes as functions of x and y.

We shall now discuss the case in which $\theta_1=0$ (no publicity for the car). As can be seen from equations (28), (29), (30), and (31), this case yields a constant attractivity for the car mode, $A_1 = \alpha_1$.

Using equation (19) and the fact that $(x' = D,\ y' = 0)$ is a stationary state, we find that the other stationary state of system (32) will be given by

$$\alpha_2 y^2 + (\theta_2 - \alpha_2 D)y + (\alpha_1 - D\theta_2) = 0 \, . \tag{33}$$

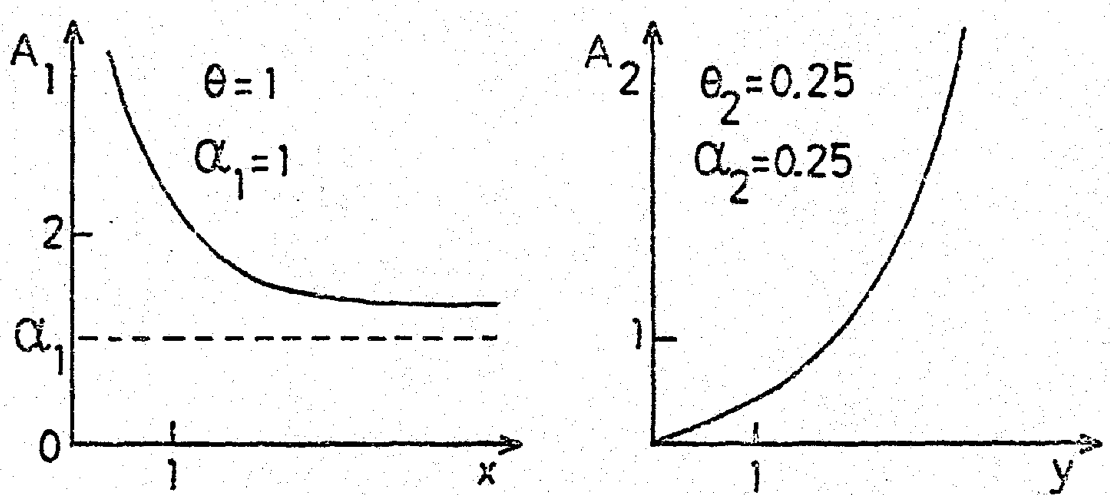


FIGURE 35. THE ATTRACTIVITY FUNCTIONS, A_1 AND A_2

This equation will have no, one, or two physically acceptable solutions with the following properties.

1. If the publicity term for the bus is large enough, $\theta_2 > (4\alpha_1\alpha_2)^{1/2}$, equation (33) will have two real roots. For $\theta_2 < (4\alpha_1\alpha_2)^{1/2}$, equation (33) will have two real solutions only if the transit density is high enough, $D > D^c$, where

$$D^c = \frac{(4\alpha_1\alpha_2)^{1/2} - \theta_2}{\alpha_2}. \qquad (34)$$

2. If equation (33) has two real solutions, y^+ and y^-, then the sign of these roots will depend upon the relative magnitude of the transit density with respect to the two critical densities D_1^c and D_2^c defined by

$$D_1^c = \theta_2 \,/\, \alpha_2 \text{ and } D_2^c = \alpha_1 \,/\, \theta_2. \qquad (35)$$

If $D < D_c^1$ and $D < D_c^2$, equation (33) has two negative roots; $D_c^1 < D < D_c^2$, equation (33) has two positive roots; $D > D_c^2$, equation (33) has one positive root and one negative root.

3. $D^c < D_2^c$ whatever the values of the parameters.

4. For large values of the publicity parameter for the bus, $\theta_2 > (\alpha_1\alpha_2)^{1/2}$, we have

$$D^c < D_1^c \text{ and } D_1^c > D_2^c. \qquad (36)$$

For small values of the bus publicity parameter, we have

$$D_1^c < D_2^c \text{ and } D_1^c < D^c. \qquad (37)$$

Figure 36 summarizes these conditions, showing the various conditions for the solutions of equation (33).

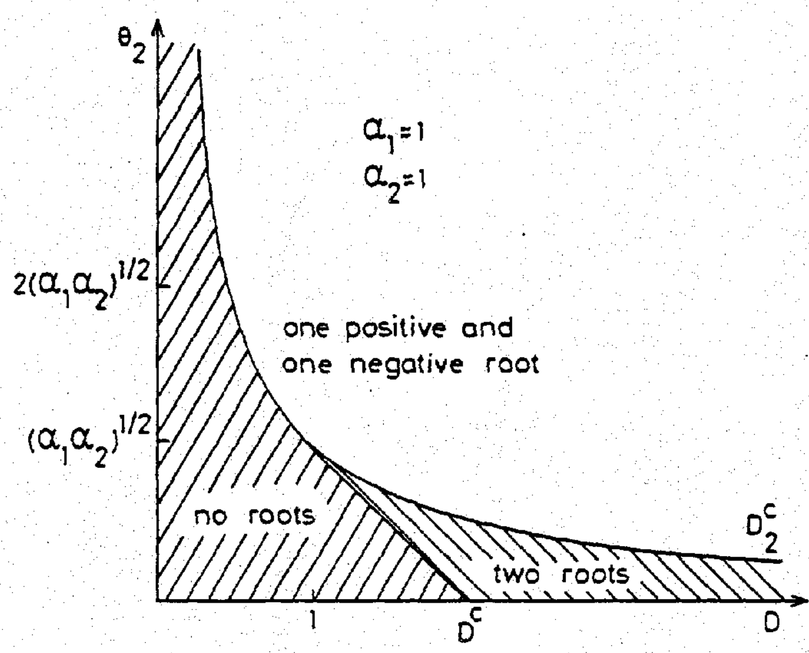


FIGURE 36. CONDITIONS FOR SOLUTIONS OF EQUATION (33)

There are qualitatively two different bifurcation diagrams for the solutions y of equation (33). These are shown in Figure 37. Figure 37a is similar to Figure 33b and so will not be discussed further. Figure 37b, where $\theta_2 < (\alpha_1\alpha_2)^{1/2}$, however, represents a qualitatively new situation. In this case we might say that the nonlinear term is more important than the linear term. For a density $D^c < D < D_2^c$, the system can accept two statinnary states, y^1 and y^+. (The stationary state y^- is unstable and cannot therefore be considered physically as a final state since perturbations will always cause the system to move away from this state.)

Let us say that a perturbation in the density y, of value Δy_i (for a given value of the traffic density, D_i), is needed to bring the system from the stationary state (x^1,y^1) to the stationary state (x^+,y^+). In Figure 37b we see that the value of the perturbation, Δy_i, for $D^c < D < D_2^c$, needed for this transformation decreases as density of transit, D_i, increases. This points to the role of history in determining which stationary state the system adopts. Further, if $D_i > D_2^c$, whatever the value of the perturbation $(\Delta y > 0)$, the system will spontaneously go to the stationary state (x^+,y^+) since the state (x^1,y^1) is unstable when $D > D_2^c$. We note that the bifurcation parameter D measures the feedback effect in the system. When the feedback parameter is sufficiently small, $D < D^c$, the system has only one stationary state. However, if D is sufficiently large, $D > D^c$, a qualitatively new stationary state appears in the system. In general, as D increases, the number of possible stationary states of the system increases.

Finally, Figure 38 shows under what conditions there may be co-existence between the two modes of transportation. For constant α_1 and α_2, a state of co-existence between the bus and the car will appear first (for small values of D) for the case when there is good publicity, $\theta_2 > (\alpha_1\alpha_2)^{1/2}$, and only later for the case when there is little bus publicity, $\theta_2 < (\alpha_1\alpha_2)^{1/2}$, which requires larger values of the traffic density, D for there to be co-existence.

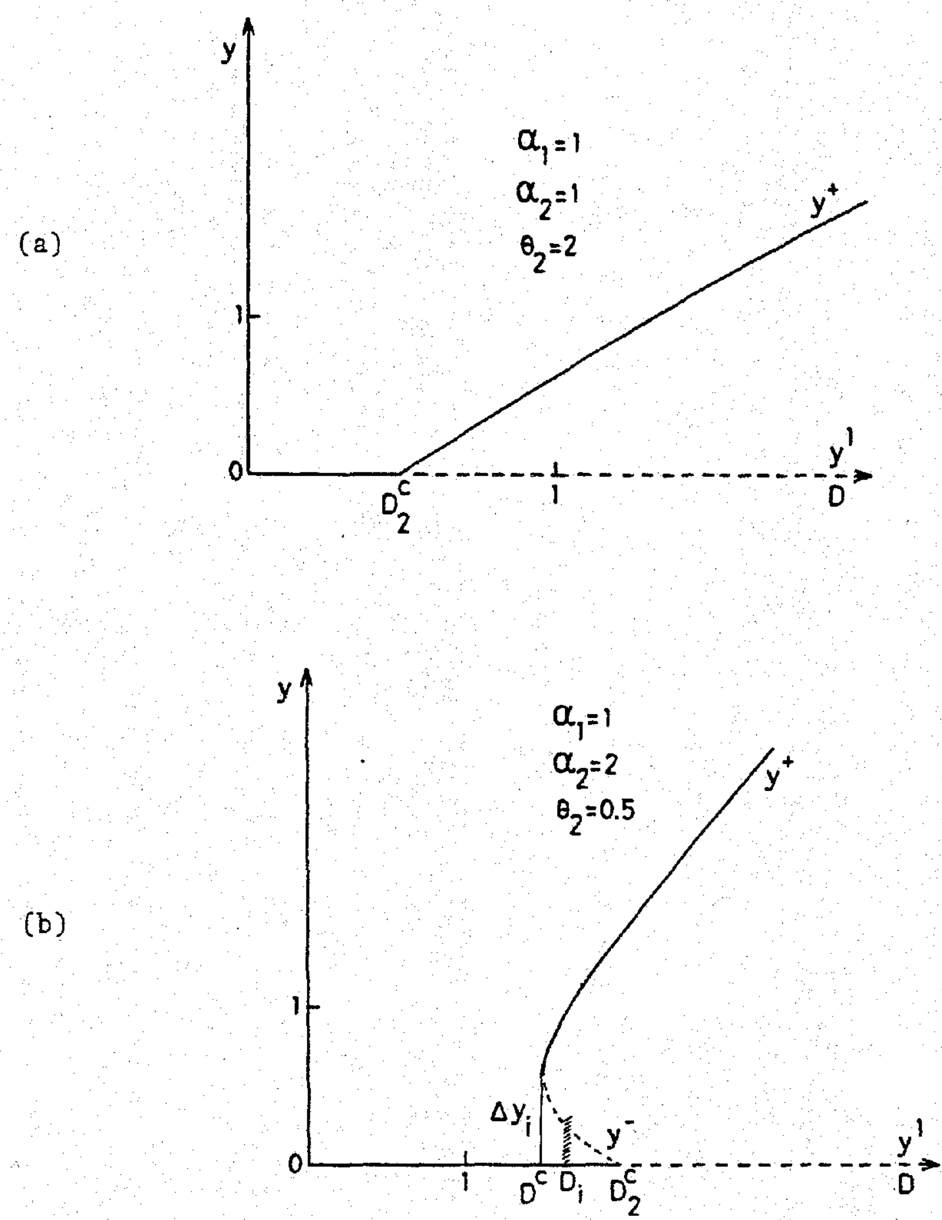


FIGURE 37. BIFURCATION DIAGRAMS FOR THE CASES WHEN (a) $\theta_2 > (\alpha_1\alpha_2)^{1/2}$ AND (b) $\theta_2 < (\alpha_1\alpha_2)^{1/2}$

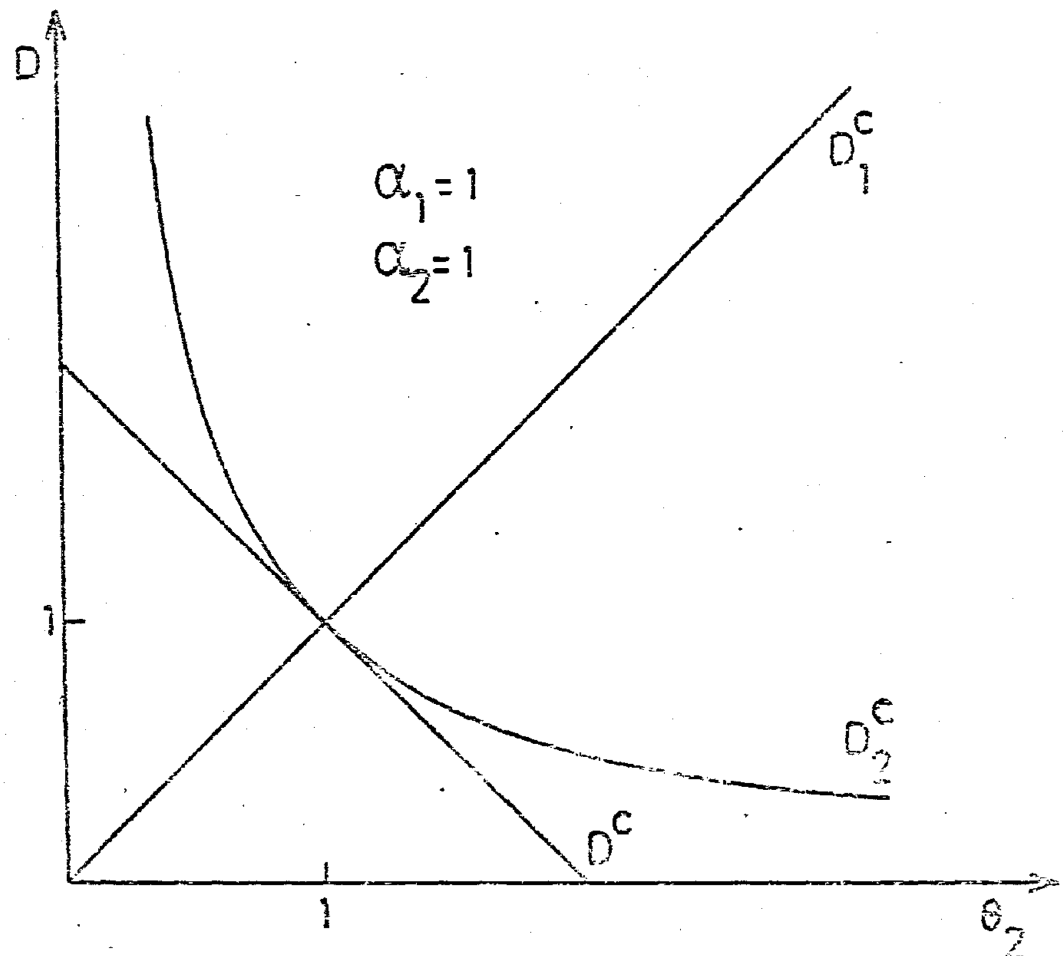


FIGURE 38. DENSITY , D, VERSUS THE BUS PUBLICITY FACTOR θ_2

3.2.4 Conclusions

The methodology and models presented in this section have illustrated the importance of behavioural fluctuations in determining the stability of competing modes of transportation. The bifurcation diagrams introduced in the text to illustrate the feedback effects resulting when travel choice is allowed to be a function of the state of the system provide information on the stability of the system to such fluctuations in human behaviour. Some stationary states are seen to be unstable even to small fluctuations, whereas others, though locally stable, would become unstable if a sufficiently large fluctuation occurred. The system would then adopt a new solution which is stable to perturbation. This adaptative emergence is one example of the concept of order by fluctuation (Nicolis and Prigogine, 1977), whereby a system reorganizes itself into a new mode of behaviour when critical size thresholds for stability are exceeded.

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APPENDIX

BASIC ELEMENTS OF BOOLEAN FORMALISM

The systems analyzed in this appendix are equivalent to a black box, A, provided with inputs and outputs. By definition, it is possible to choose arbitrarily the value of each input variable, the value of each output variable being determined by the internal dynamic of the system. Two basic hypotheses are made:

- each variable may take only two values 0 and 1 (binary variable).

- the value of the output variables at time t depends on the values taken by the input variables at previous moments (we assume that the system has some memory).

In the first part we will make some remarks concerning Boolean algebra. In this language we summarize our hypotheses, while in the second part, we will specify the concept of a system with memory.

BOOLEAN ALGEBRA - COMBINATORY SYSTEM

Systems considered in this paragraph have no memory effects. They are called combinatory systems.

a) Definitions

State space: let a system have n input variables. Each state has two values 0 or 1. We have then 2^n different states. If n = 2, the 4 different states are: (0,0); (0,1); (1,1); (1,0). Functions space: Let f be defined by:

$$(x_1 \cdots x_n) \longrightarrow z = f(x_1 \cdots x_n)$$

z may have two different values 0 or 1. The number of different functions f will be 2^{2^n}. If n = 1, we have 4 different functions.

x	f_1	f_2	f_3	f_4
0	0	0	1	1
1	0	1	0	1

b) Logical functions of a combinatory system

In a combinatory system in which we know the input variables, we can compute the values of the output variables (z). We can define the following operations of the Boolean algebra $\oplus$, +, and.

x_1	x_2	z
0	0	1
0	1	1
1	1	0
1	0	1

$z = x_1 \oplus x_2$

Rules of multiplication and addition.

x_1	x_2	$x_1 + x_2$	$x_1 \cdot x_2$
0	0	0	0
0	1	1	0
1	1	1	1
1	0	1	0

We define also the following operation:

x_1	$\bar{x}_1$
0	1
1	0

The following rules can be easily shown:

$$\overline{x_1 + x_2} = \bar{x}_1 \cdot \bar{x}_2$$

$$\overline{x_1 \cdot x_2} = \bar{x}_1 + \bar{x}_2 \quad .$$

SEQUENTIAL SYSTEMS

a) In the above systems, no dynamical effects are included. Input and output were supposed to adjust themselves instantaneously. In the case of urban systems, such an hypothesis cannot be accepted. For example, let $[$ be the average transportation time from A to B. $\tilde{[}$ will be the threshold of perception of the transportation time for the people moving from A to B. The phenomena will be discretized in the following manner (Figure 39).

b) $[$ can be the explicative variable of another phenomenon, for example $[= 0$ could be the condition to switch or maintain a phenomenon of migration, measured by the variable P_{ab}. $\tilde{P}_{ab}$ will be the threshold of the variable P_{ab}. The situation is represented in Figure 40.

Δ is the growth time delay for the variable P_{ab}

$$P_{ab}(t) = g\ ([_{t-\Delta}) \quad .$$

In the following section we will write the general equations of a sequential system.

c) We do have a sequential system if the knowledge of the internal variables (at time t) is not sufficient to determine the value of the output variables (at time t). In this case we will take into account the history of the system by including a new kind of variable in the system: the memory variables. A sequential system can be represented in the following manner:

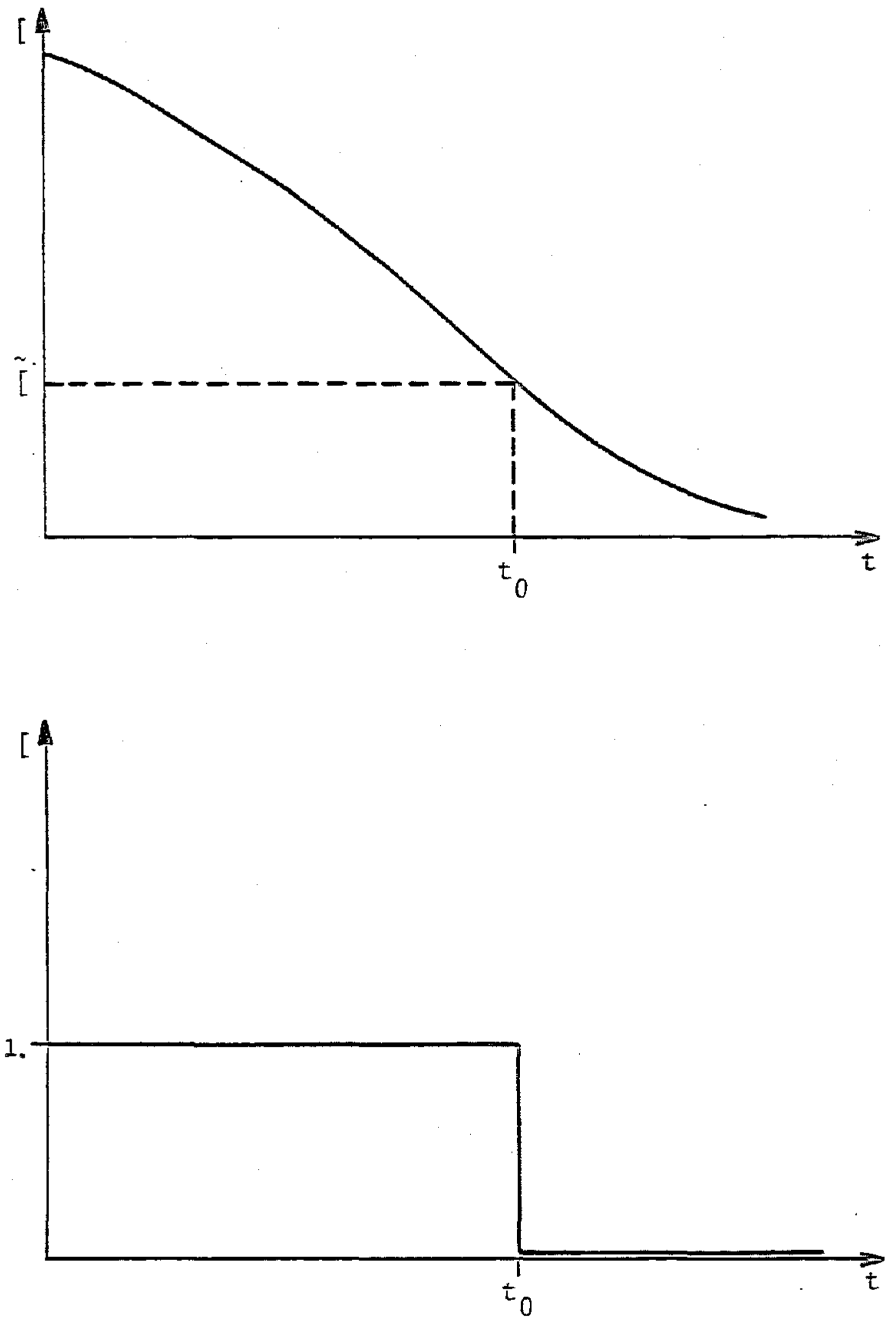


FIGURE 39. SEQUENTIAL SYSTEMS a

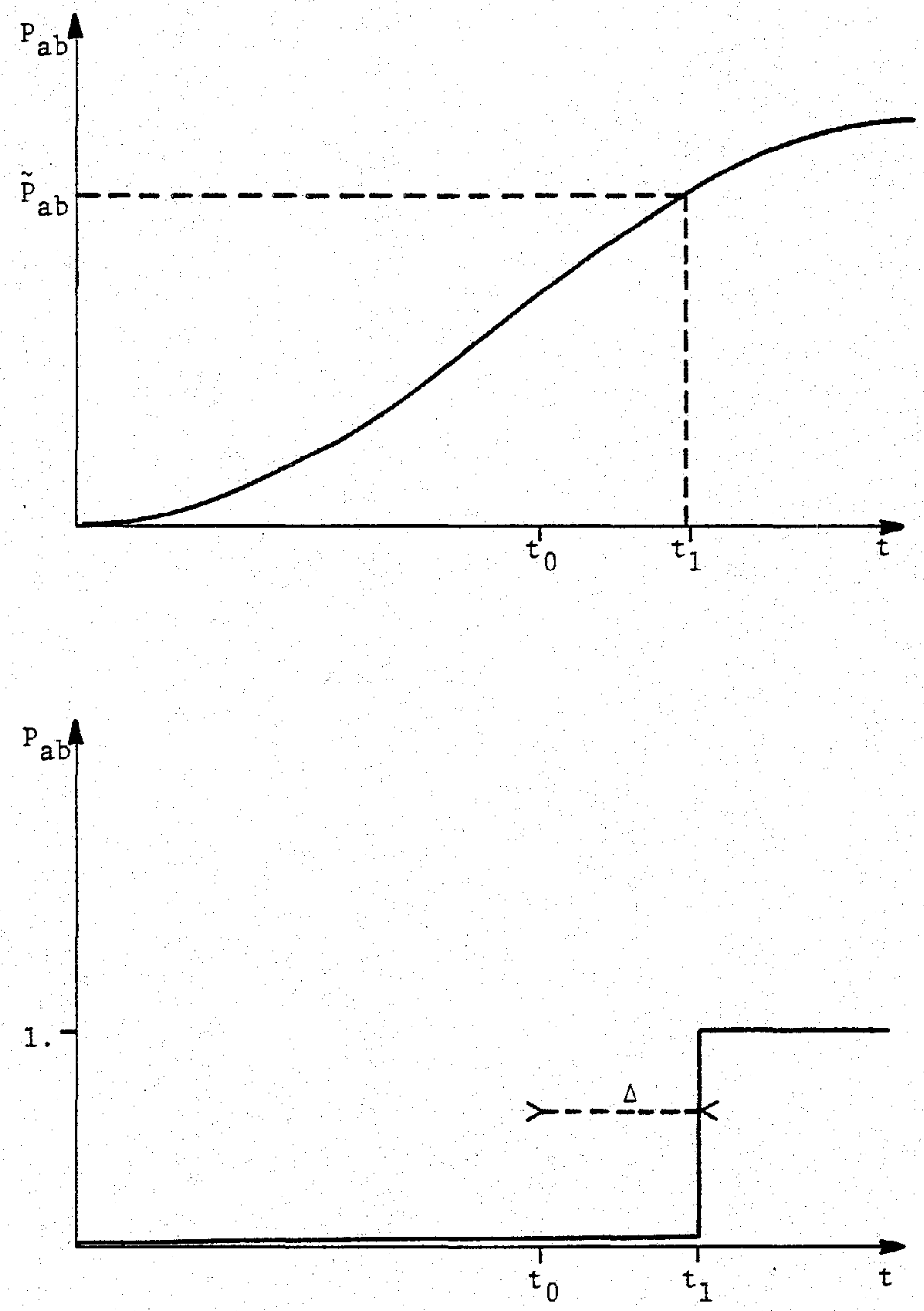
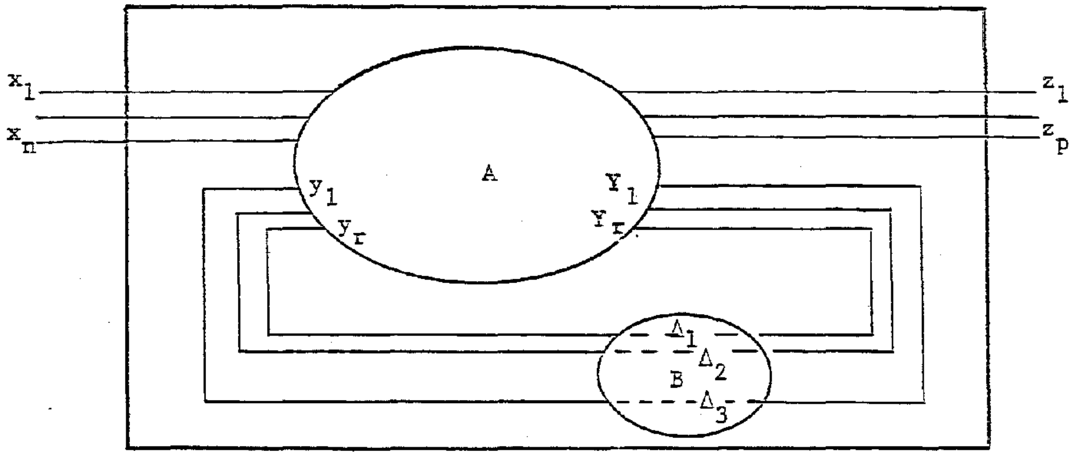


FIGURE 40. SEQUENTIAL SYSTEMS b



x_1 --- x_n	input variables.
z_1 --- z	output variables.
y_1 --- y_r	memory variables.
Y_1 --- Y_r	memory functions.

A is a combinatory system. When the signals go into the system B, they are retarded. The equations of the whole system are:

$$z_j(t) = f_j(x_1(t),\ \ldots,\ x_n(t);\ y_1(t),\ \ldots,\ y_r(t)) \tag{38}$$

$$Y_j(t) = g_j(x_1(t),\ \ldots,\ x_n(t);\ y_1(t),\ \ldots,\ y_r(t)) \tag{39}$$

In general we have different growth and decay time delays for a given variable. We have

$$y_i(t) = Y_1(t-\Delta_i)\ . \tag{40}$$

Then equation (39) becomes:

$$Y_j(t) = g_j(x_1(t), \ldots x_n(t); y_1(t - \Delta_i), \ldots y_r(t - \Delta_r)). \quad (41)$$

This is a system of implicit equations in Y. The values of the variables $x_1 \ldots x_n$ are given; these variables can be considered as parameters. The knowledge of the initial conditions and of the values of the input variables, allows us (41) to compute the value of the memory variables for each time. For fixed input variables, when we have $y_i(t) = Y_i(t)$, for each i, then we are in a stable state. If not, we are in an unstable state. The system evolves to a stable state or to a cycle.

d) Let us assume for example that:

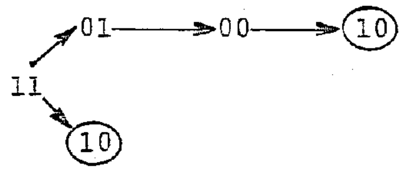
		D = 0		D = 1	
p_{ab}	c	p_{ab}	[	p_{ab}	[
0	0	1	0	1	1
0	1	0	0	(0 1)	
1	1	0	0	0	1
1	0	(1 0)		1	1

For D = 0, the system has one stable state (10) and for D = 1 the system has also one stable state (01) . If we start from the unstable state p_{ab} = 1; c = 1 for D = 0, we have two possible different evolutions.

11 ⟶ 01 ⟶ 00 ⟶ (10)

or 11 ⟶ (10)

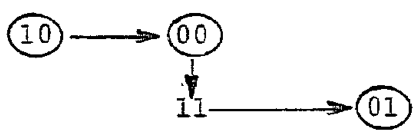
The system will take the first path if the decay time delay of the variable P_{ab} is less than the decay time delay of the variable D. Such a phenomenon is called a critical course phenomenon and will be represented in the following manner:



In general, when for a set of given values of the input variables, different final stable states are possible, the system evolution will be very important in the determination of the stable state finally adopted. We will clarify this remark by the following example. Let us define the system by the following diagram:

a	b	A	B
0	0	0	0
0	1	0	1
1	1	0	0
1	0	0	1

If we start from the state a = 1 and b = 0, the system can, depending on the values of the time delays, evolve to the stable state (01) or to the stable state (00). The different possible system evolutions (starting from 10) may be represented in the following diagram:



For some values of the time delay, the system can run around (w) before reaching a stable state.*

For systems in which the equation (41) is given, there are some rapid algorithms to find the stable states and the cycles.

*This happens only if the system has some memory of the time at which the process leading to its growth or decay began. There is always a demand for the decay of the variable a, then the system will reach a stable state after a finite period of time. In the other case (no memory effect) if the system takes the cycle (w) once, it will remain there.