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OCCUPANT MOTION SENSORS

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16. Abstract

An analysis has been made of methods for measuring vehicle occupant motion during crash or impact conditions. The purpose of the measurements is to evaluate restraint system performance using human, anthropometric dummy, or animal occupants. A detailed Fourier frequency analysis is made of the sensor requirements. Potential candidate systems are evaluated, and five of these recommended for further development and field testing.

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N O T I C E

OCCUPANT MOTION SENSORS

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1.0 INTRODUCTION

The purpose of this project is the design of an instrumentation system that can be used to gather data on occupant motions during vehicle impact. This includes a broad study of measurements required, an evaluation of all presently known measurement techniques, development of new techniques, and the selection of a combination to satisfy uniquely the requirements of this program. The long-range goal of obtaining more accurate measurements is to understand better the mechanisms of human injury and to use this understanding in developing more effective passenger restraint systems. It is the hope of the participants in this program that this knowledge can be effectively used to reduce the alarming number of passenger injuries and fatalities occurring annually in this country.

2.0 SUMMARY

Before the subject of sensors could be studied, it was necessary to devise a list of General Specifications to which any candidate system would have to perform. Implicit in these specifications is a body and fixed space reference coordinate system.

The General Specifications, worked out in cooperation with the sponsor, include the maximum expected values of the measurements to be taken and the importance or priority of each measurement. The General Specification list is included in Appendix A. This list of measurements and their magnitudes is very important. The sensors finally selected depend to a great extent on the velocities and accelerations called for in the General Specifications.

The most significant way in which the peak accelerations and velocities determine the sensors is through the bandwidth and frequency response. Consequently, following the establishment of the General Specifications, a bandwidth and frequency analysis was carried out. A simple mathematical model was made for the occupant motion at crash impact, and the results Fourier analyzed. This Fourier analysis, coupled with the accuracy requirements (5% goal) resulted in a sensor bandwidth specification. This is a very important number to have, since many candidate sensor systems are electromechanical in nature. This means they have relatively narrow bandwidth, on the order of a hundred Hz, typically.

With a knowledge of the bandwidth required, a survey was made of all possible systems for occupant motion sensing. Manufacturers were contacted to determine the current state-of-the-art for each system. In addition, where the sensor systems were close to meeting the specifications, the possibility of a development program was explored. Although bandwidth is an important criterion, there are many others, some of which are as important as bandwidth. A list of twelve weighted criteria was put together and each candidate sensor system evaluated. From this evaluation, five sensor systems were chosen for hardware prototype development and eventual field testing with human subjects at a deceleration sled facility. The final part of this report describes the survey and evaluation phase of this effort.

3.0 MATHEMATICAL MODEL AND BANDWIDTH STUDIES

In order to apply the mathematical tools of Fourier analysis, one must know the time history of the signal. The simple mechanical model of an occupant during vehicle crash, to be discussed, provides a pathway from the maximal values given in the general specifications to the time history of the sensor signal.

Because the motions of the head/neck are obviously the most important, from the injury standpoint, and, therefore, have the highest priority (1 in the General Specifications), we will analyze the head motion, assuming it to take place in one plane, as shown in Figure 1.

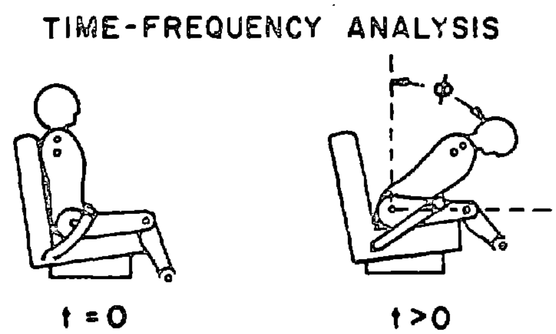


Figure 1.- Model of Vehicle Occupant During Impact

The following simplifying assumptions are made:

(1) The final head angle ϕ_f, is 90° and the initial head angle is zero.

(2) The head is subjected to a uniform acceleration, $\ddot{\phi}$, at time t = o.*

The following Newtonian equations of motion apply:

$$\phi_f = 1/2 \ \ddot{\phi} \ T^2 \qquad (1)$$

where T = the total event time during which the acceleration is applied, as compared to "t", the running or instantaneous time variable.

Solving Eq. (1) for T:

$$T = \left[2\phi_f/\ddot{\phi}\right]^{1/2} = \left[\pi/\ddot{\phi}\right]^{1/2} \qquad (2)$$

In Figure 2, this equation is plotted; T as a function of the acceleration $\ddot{\phi}$. The maximum value shown for $\ddot{\phi}$ is the value given in the General Specifications, Appendix A, for forward head acceleration. Originally, this was taken to be 10^7 deg/sec^2, but was later reduced to 10^6 deg/sec^2. This latter value is more realistic, in terms of actual test results and known human tolerance levels.

From Eq. (1), we can now predict the time function of the velocity and acceleration signals, by differentiation:

$$\frac{d\phi_1}{dt} = \dot{\phi} = \ddot{\phi}t = \text{angular velocity} \qquad (3)$$

$$\frac{d^2\phi}{dt^2} = \ddot{\phi} = \text{angular acceleration} \qquad (4)$$

*A dot above a variable means differentiation with respect to time, d/dt. Two dots indicate the second derivative, d^2/dt^2.

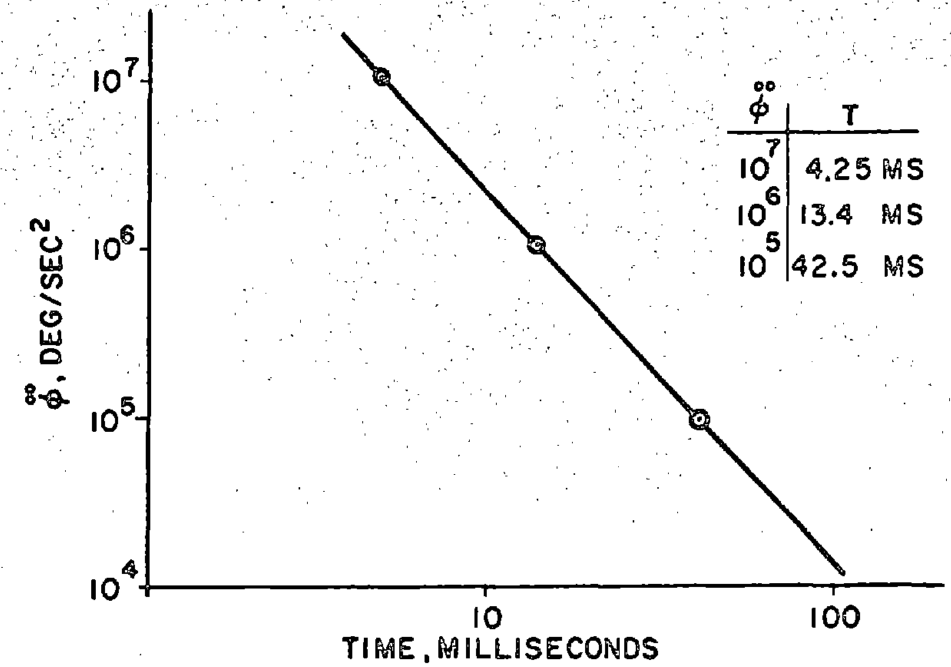


Figure 2.- Peak Acceleration vs Event Time

Figure 3 shows a plot of these equations, and represents the input motional signal to accelerometer, velocity, and displacement sensor, respectively. Constants of integration have been set equal to zero, as they will not affect the frequency analysis to follow.

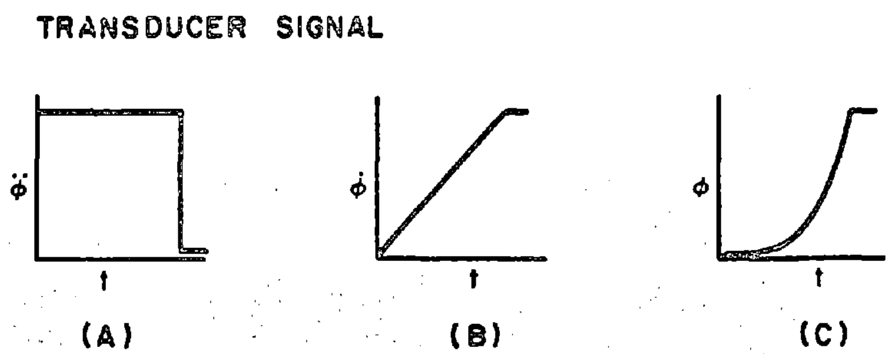


Figure 3.- Idealized Transducer Signals

3.1 Fourier Analysis

Now that we have a model which postulates the sensor signal's time history, we can proceed to Fourier analyze this signal into its component frequencies and determine the bandwidth specifications for the sensors.

Because a vehicle impact is a single event in time and not a periodically repeating function, the Fourier integral form (ref. 1), as opposed to the Fourier series form, must be used.

The Fourier spectrum* is,

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}\, dt \qquad (5)$$

and its inverse

$$f(t) = \int_{-\infty}^{\infty} F(\omega)e^{i\omega t}\, d\omega \qquad (6)$$

where $f(t)$ = a real or complex time signal

$i = \sqrt{-1}$

ω = angular frequency = $2\pi\nu$

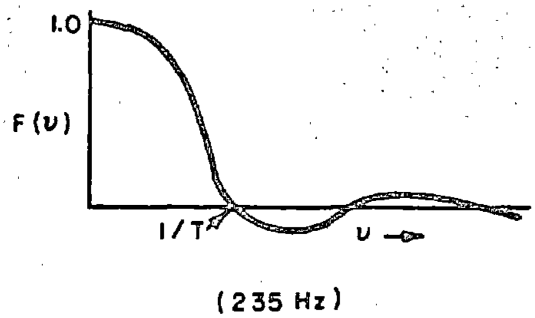
ν = frequency in Hertz

The spectrum of the accelerometer signal, is by equation (5),

$$F_{(\omega)} = \frac{\sin(\omega T/2)}{(\omega T/2)} = \mathrm{sinc}(\omega T/2) \qquad (7)$$

*Capital letters will be used to designate Fourier spectral functions, and lower case letters the corresponding functions in the time domain. That is, $f(t)$ and $F(\omega)$ constitute a Fourier transform pair.

This function is plotted in Figure 4. The first zero crossing is at a frequency of 1/T, which for an angular acceleration of 10^6 deg/sec^2 corresponds to 75 Hz. The accompanying table gives the peak values of each successive side lobe. Note that the spectrum decays rather slowly, as $1/\omega$.



LOBE #	MAX. VALUE
1	- 0.217
2	+ 0.128
3	- 0.091
4	+ 0.071
5	- 0.058
6	+ 0.049

Figure 4.- Frequency Spectrum of Accelerometer Signal

The spectrum of the velocity signal, the ramp function of Figure 3b, is:

$$F(\omega) = \int_{\infty}^{\infty} te^{-i\omega t}\, da \qquad (8)$$

$$= \left[\frac{T\sin(\omega T)}{\omega} + \frac{\cos(\omega T)}{\omega^2} - \frac{1}{\omega^2}\right]$$

$$+ i\left[\frac{T\cos(\omega T)}{\omega} - \frac{\sin(\omega T)}{\omega^2}\right] \qquad (9)$$

This is a complex quantity, containing both real and imaginary parts. Since most electronic instrumentation is not sensitive to the phase, we will adopt the convention of plotting the absolute value or modulus of the complex function. This quantity is also equal to:

$$\left[F(\omega)F^*(\omega)\right]^{1/2} \tag{10}$$

where the star in the second factor stands for the complex conjugate.

The plot of this is shown in Figure 5.

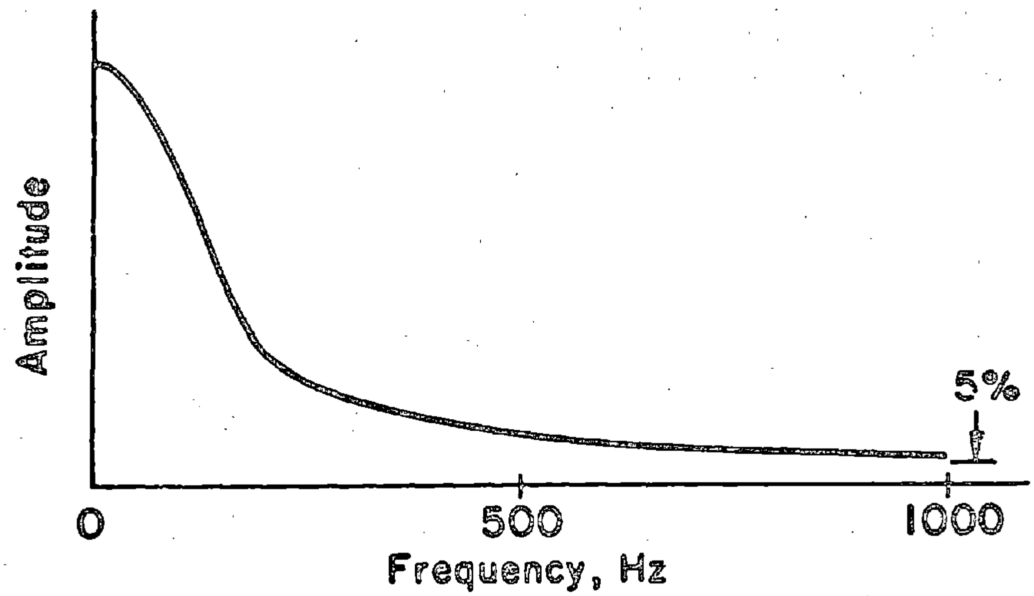


Figure 5.- Frequency Spectrum of Velocity Signal

The duration of the event, T, was chosen to correspond to a peak angular acceleration of 10^6 deg/sec^2.

The fourier analysis of the position signal, (Figure 6) the quadratic function of Figure 3c, is:

$$F(\omega) = \int_{\infty}^{\infty} t^2 e^{-t\omega t} \, d\omega \qquad (11)$$

$$= \left[\frac{T^2 \sin(\omega T)}{\omega} + \frac{2T\cos(\omega T)}{\omega^2} - \frac{2\sin(\omega T)}{\omega^3}\right]$$

$$+ i\left[\frac{T^2\cos(\omega T)}{\omega} - \frac{2T\sin(\omega T)}{\omega^2} - \frac{2\cos(\omega T)}{\omega^3} + \frac{2}{\omega^3}\right] \qquad (12)$$

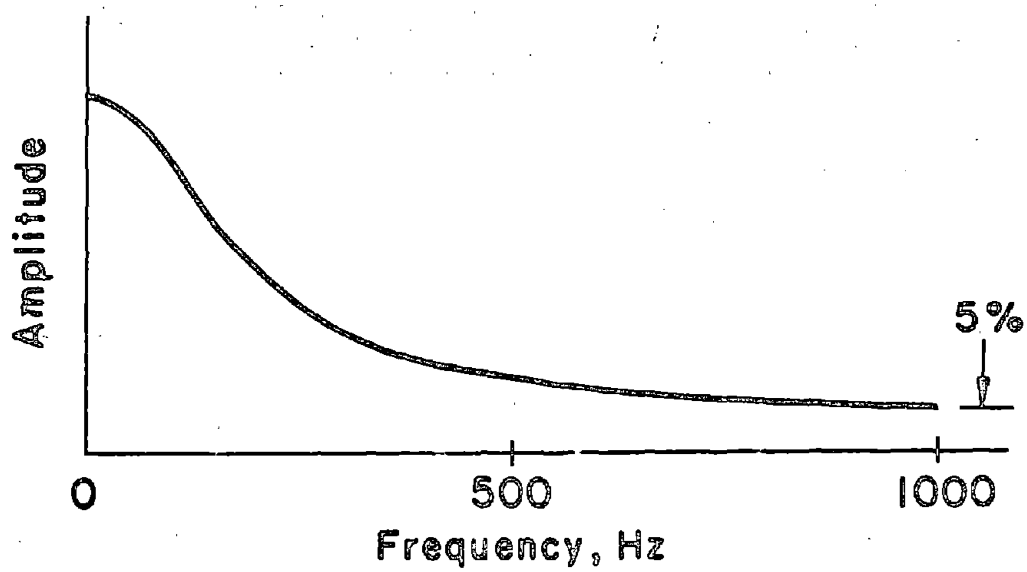


Figure 6.- Frequency Spectrum of Displacement Signal

3.2 Error Analysis

We have Fourier analyzed the expected time signals from the sensors and have found that the frequency spectrum extends to infinity in a continuous, steadily decreasing manner.

Obviously, no physically reliazable transducer is going to be capable of an infinite bandwidth. Indeed, most mechanical and electromechanical sensors are limited to one hundred Hertz or less. When a motional signal of infinite extent is passed through a transducer of finite bandwidth, an error is introduced into the measurement, because information has irretrievably been lost in the process. The accuracy of the measurement, therefore, depends in some way on the bandwidth. From the General Specifications we see that a measurement of accuracy of 5% is desired. We will now explicitly derive a relationship between the bandwidth and accuracy.

Consider a signal, f(t), from the accelerometer containing no error, and one containing a small error, $f^1(t)$. This is shown in Figure 7.

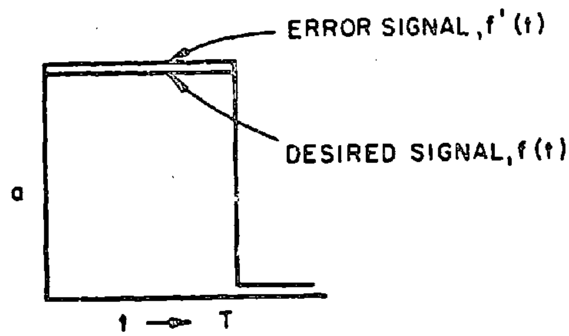


Figure 7.- Error Signal

$$f(t) = K \qquad 0 < t < T \qquad (13)$$
$$f^1(t) = K^1$$

Taking the Fourier transform of each of these,

$$F(\omega) = K\text{sinc}(\omega T/2)$$
$$F^1(\omega) = K^1\text{sinc}(\omega T/2) \tag{14}$$

The percent of error, ε, introduced into the Fourier spectrum by this error in the time domain is

$$\varepsilon = 100\ \frac{K^1\text{sinc}(\omega T/2) - K\text{sinc}(\omega T/2)}{K\text{sinc}(\omega T/2)} = 100\left[\frac{K^1}{K}\right] - 1 \tag{15}$$

Therefore, for the sinc function, a 5% error in the time domain introduces a 5% error in the frequency plane. This means that the bandwidth of the sensor must be such that the Fourier spectrum is down to 5% of its DC or zero frequency value. Reexamining the table in Figure 4, this means that the accelerometer bandwidth for the 5% error is approximately 425 Hertz.

A similar error analysis for the velocity and position sensor leads to the following results shown in Table 1.

TABLE 1.- ERROR ANALYSIS

	Bandwidth 5% error (Hz)
Accelerometer	425
Velocity Sensor	380
Position Sensor	665

These bandwidths are derived under the condition of a uniform acceleration of 10^6 deg/sec^2. To scale these to any other acceleration, note that:

$$T \text{ (varies as) } \left[1/\ddot{\phi}\right]^{1/2} \tag{16}$$

and

$$\text{F.T.}\left[f(kt)\right] = F(\omega/k) \tag{17}$$

where F.T. stands for Fourier transform, and k is any arbitrary scaling constant. Taking Eqs. (16) and (17) together means that reducing $\ddot{\phi}$ by a factor of 10 reduces the bandwidth requirement, for a given accuracy, by a factor of $\sqrt{10}$, or 3.16.

How does this simple model, Figure 7, compare to actual experimental data? Very little of the raw data have been published. After the above model was almost finished, a report on sled tests at Holloman AFB (ref. 2) (using baboons) was called to our attention. This report showed rotational accelerometer data from several sled runs. Figure 8a shows one such run. We approximated their data by a damped sine wave, $e^{-bt}\sin(\omega_{\circ}t)$, as shown in Figure 8a. The computed absolute value of the Fourier spectrum is:

$$|F(\omega)| = \frac{1}{\sqrt{2}}\left[\frac{a^2+\omega_{\circ}+\omega^2}{a^2+\omega_{\circ}^2+\omega^2 - 2\omega\omega_{\circ}{}^2} - \frac{a^2-\omega_{\circ}^2+\omega^2-2\omega_{\circ}\omega}{a^2-\omega_{\circ}^2+\omega^2-2\omega_{\circ}\omega{}^2 + 2a\omega_{\circ}{}^2}\right]^{\frac{1}{2}} \tag{18}$$

The plot of Eq. (18) is shown in Figure 8b. The 5% level occurs at approximately 900 Hz. This is in reasonable agreement with the predicted value of 425 Hz, considering the simplicity of the model.

In the future, time histories taken by our own instrumentation in the field will be Fourier analyzed on our computer to obtain a more precise picture of the bandwidth requirements. Toward this end, we have obtained a copy of IBM's "FØRT" scientific subroutine.[3] This program will extract the complex Fourier transform of up to 8,192 sample data points in 0.175 minutes on the #7094 computer. This is a very powerful tool in data analysis and processing, since it enables the user to do not only frequency and spectral analysis, but also filtering, integration, differentiation, etc. These studies will enable us to make recommendations to NHTSA on the best or optimum ways to process crash test data and insure uniform, accurate results by all groups doing field testing.

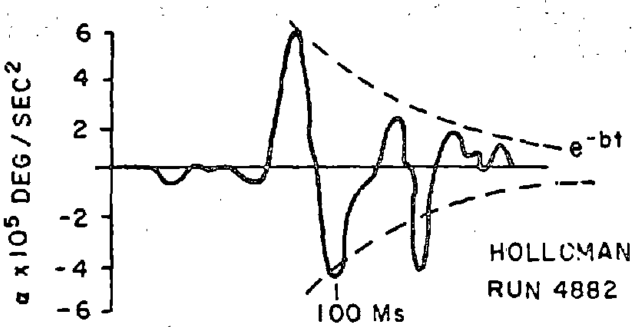


Figure 8a.- Published Test Run Data, Holloman AFB

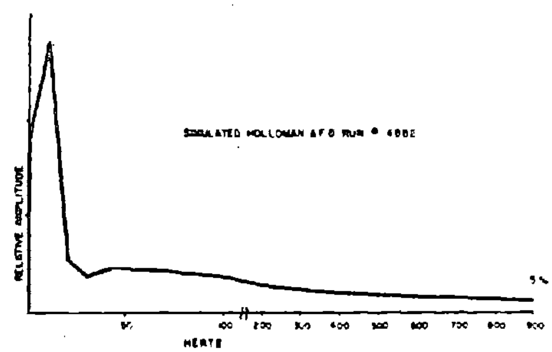


Figure 8b.- Frequency Spectrum of Test-Run Data

In conclusion then, it is apparent that a minimum sensor bandwidth of at least 425 Hz is required, and that probably twice that bandwidth, or 900 Hz, is desirable.

Although bandwidth is an important consideration, it is by no means the only one. Crash testing places severe physical constraints on a potentially useful system. We now examine some of these other requirements.

4.0 ANALYSIS OF SYSTEMS

In addition to the electrical characteristics just discussed, there are a number of other considerations to be evaluated for any sensor or transducer used in occupant motion sensing during vehicle impact. A list of these criteria is shown in Table II, together with the relative importance or weighting for each category. An ideal sensor would have a score of +100. Since crash survivability is a necessary condition, it is scored as positive or negative. A negative score, of whatever magnitude, is unacceptable.

TABLE II.- CRITERIA LIST

		Weight
1.	Crash Survivability	+,-
2.	Accuracy & Calibration Stability	15
3.	Freedom from Spurious Outputs	15
4.	Unique Advantages or Disadvantages	10
5.	Data Reduction Requirements	10
6.	Reliability	10
7.	Development Costs	9
8.	Maintenance Required	9
9.	Signal/Noise Ratio	8
10.	Level of Personnel Required to Operate System	6
11.	Measurement Taken	5
12.	Power Requirements	3
		±100

4.1 Discussion of Criteria

Each of these will be discussed in order.

4.1.1 Crash Survivability.- Obviously a measuring system must be able to withstand the g force at crash impact to be acceptable. Systems that work adequately in a laboratory environment might be unacceptable in the field. The laser velocimeter is a good example of this.

4.1.2 Accuracy and Calibration Stability.- From the General Specifications, Appendix A, the desired accuracy of the total system is 5%. This includes not only the sensor transducer itself, but any auxiliary electronics required to process or demodulate the signal. In addition, if a sensor must be attached to the body the errors inherent in the attachment, such as straps, bands, tape, bite-bars, etc., must be considered. The weight of the sensor, if it is attached to the body, will cause an error due to an inertial lag or loading of the body member.* A sensor taped to the skin will stretch the skin and introduce error. If a sensor is too massive it could materially change the motion of the member being measured, for example head motion. The sensor must hold its calibration over a reasonable period of time to be practical.

4.1.3 Freedom from Spurious Outputs.- There are two aspects to this problem. First, the motional response must represent pure mode; a rotational sensor should not respond to linear motions and vice versa. If a measurement is to be made in one plane, or axis, the sensor must be insensitive to other planes or axes. Second, a transducer should be sensitive only to a single input signal. For example, a motional transducting system was proposed using a small magnetic field transducer mounted on the body. The idea was that the transducer would produce an electrical output proportional to the angle between its own axis and the direction

*See Appendix B for a derivation of this.

of a stationary magnetic field. However, tests with a sample of this transducer, borrowed from the manufacturer, revealed that a spurious electrical output signal was obtained each time the transducer was mechanically shocked or jarred. Such a characteristic is obviously fatal for crash testing.

4.1.4 Unique Advantages or Disadvantages.- This is a catch-all category to include any important considerations not taken care of by the rest of this list.

4.1.5 Data Reduction Requirements.- This consideration is due to the fact that some types of sensors put out direct analog signals, i.e., acceleration in, volts out, and others put out signals that need to be filtered, demodulated, or computer processed to obtain the motional information. A system whose signal requires processing is generally less desirable than one that does not. This is because the processing inevitably adds errors of its own, and in the case of the digital computer, it adds to the total cost of the system. The most familiar example of the disadvantages and errors introduced by data processing is in the reduction of the universally used high speed camera data. When the positional data is numerically differentialed once, or twice, to obtain velocity, or acceleration, large errors are unavoidably introduced into the data.*

4.1.6 Reliability.- A deceleration sled run or car crash test is an expensive proposition. It is doubly costly if it must be repeated because a key sensor failed. Therefore, we require the sensors to be reliable.

4.1.7 Development Costs.- The budget, while generous, is finite. Most instrumentation problems can be solved given enough time and funds. A reasonable compromise must be maintained between the other criteria and cost of development.

*A digital computer study of this problem, quantitatively, is under way, and will be included in the next report.

4.1.8 Signal/Noise Ratio.- This category is also related to Par. 4.1.2, the accuracy. The sensor should not be particularly susceptible to external interference, either magnetic, electro-static, mechanical, or RF. The test site represents a relatively noisy environment in all four departments. For a 5% accuracy, a minimum signal/noise ratio of 20:1 must be maintained.

4.1.9 Measurement Taken.- The question here is does the sensor measure the desired quantity directly. For example, a first priority measurement is foreward head acceleration. A sensor that measures rotational displacement, velocity, or the derivative of acceleration (such as the so-called "Jerkmeter" does) is less desirable than a sensor directly measuring the desired motion. This category is obviously related to Par. 4.5, Data Reduction Requirements, since most motional measurements can be transformed from one type to another within the limitations already discussed.

4.1.10 Power Requirements.- This refers to any on-board sled requirements of the sensor system. Since the sled is connected to the ground-based instrumentation via a long umbilical cable, typically over 100 ft, it would be preferable for the sensor to be a passive device. Because the sensor output signal is travelling through the same cable, any power required on the sled must be analyzed from the point of view of interference it will induce into the signal channels; in 100 feet, inner channel capacity is significant. Sending well-filtered DC over the umbical cable could probably be tolerated, but AC could not. If AC were required, it should be low-current, to reduce magnetic interference, and a separate shielded cable should be used.

With these criteria in mind, we next examine in detail existing state-of-the-art sensors and transducing systems, together with several new approaches to the problem.

4.2 Evaluation of Candidate Systems

An examination of the General Specifications shows that of the six major headings, four are for rotational motions. In the first and second priority measurements, three fourths are for rotational measurements. This is consistent with a growing body of evidence that rotational, not linear, accelerations of the head-torso system are responsible for injuries to this region of the body (ref. 4,5). Therefore, in the main, we are looking for sensors that measure rotational, as opposed to linear, motions.

We also will want to examine the applicability of relatively new technological achievements such as the laser velocimeter, doppler radar, and holography. Is it possible that the laser could do for occupant motion measurements what it has done for astronomers in measuring minute earth-moon motions?

Table III shows a list of some possible candidate sensor systems. Systems currently in use at sled and car-crash test facilities are linear accelerometers (piezoelectric, and strain gauge types) and high-speed cameras. Rotational motions are difficult to infer from both these types of sensors. In the case of converting linear to rotational acceleration, a radius of curvature must be assumed. This is a nebulous quantity in the head, neck, torso system, as it is constantly changing with time. In the case of the high-speed camera, the difficulty is that when the positional raw data are differentiated the measurement error is substantially increased, as previously discussed. Therefore, we can conclude that presently used techniques are inadequate to measure rotational occupant motions accurately.

We will now go through the list of sensors and systems of Table III and evaluate them from the point of view of the Criteria of Table II. The assessments of the advantages or

TABLE III.- CANDIDATE SENSOR SYSTEMS

A. Mechanical

1. Gyro
2. RVDT
3. Potentiometers
4. Strain Gauge
5. Linear Velocity Transducer
6. Accelerometer

B. Optical

1. High Speed Photography
2. Laser Doppler System
3. Laser Range Finder
4. Holography
5. Ellipsometry

C. Electromagnetic, High Frequency

1. Doppler Radar
2. FM Phase Lock

D. Ultrasonic or Acoustic

1. Doppler Shift
2. FM Phase Lock
3. Signal Strength
4. Interferometry

E. Electromagnetic, Low Frequency

1. Capacitive
2. Magnetometer
3. Radio Direction Finding (RDF)

disadvantages of a sensor or system is based on many things: calculations of performance requirements; discussions with manufacturers and their engineering personnel; a survey of the manufacturers; discussions with people in the field of deceleration sled testing; personal experience of members of the TSC staff, and books, reports, and monographs.

4.2.1 Mechanical

1. Gyro.- These are available to measure angular position or angular velocity. The large mass (~ 70 gm), low frequency response (~ 100 Hz), and limited maximum measurement make them unacceptable for occupant motion sensing.

2. Rotary Variable Differential Transformer (RVDT).- This class of transducers measures angular position of a rotating shaft relative to a stator. A direct application of this sensor would require a mechanical linkage from the head or torso to the RVDT. This is undesirable. In addition, the mass (~ 50 grams) is too large and the bandwidth (~ 100 Hz) is too low. Another approach to using this device would be to allow either the stator or rotor to be free wheeling, and attach the other member rigidly to the head, for example. Inertia of the free member, say the rotor, would tend to keep it aligned in its original position, while the stator would move relative to it with head motion. Residual forces and friction between rotor and stator would degrade performance at the low-frequency end of the spectrum. Unfortunately, the weight and bandwidth limitations of currently available RVDT's rule out this approach.

3. Potentiometers.- These devices are potentially attractive for torso rotational measurements because they have an inherently wide bandwidth, 10^5 Hz,

typically. They do require a direct mechanical link to the occupant, which is not desirable-on two counts. First, errors in measurement are unavoidably introduced at the point attachment due to relative motion between the body and the attachment; typically, a taut band or strap system. Second, there is an inherent vector error in this type of attachment. The transducer measures motion, or displacement, along its shaft or axis, which may not be the direction of the instantaneous velocity vector. It should be noted that for some region in space this vector error is zero. In restraint system testing, one is primarily concerned with <u>peak</u> accelerations and velocities which also tend to be localized in space, e.g., just in front of the air bag. Therefore, by judicious alignment this vector error can be minimized.

There are two useful potentiometer configurations for occupant motion monitoring.

One is the rectilinear potentiometer, which can be obtained in lengths of up to two feet - approximately what is required to track torso motion. The other is the standard rotary multi-turn potentiometer equipped with a spring-loaded roller and cable to convert a linear motion to the required rotary motion. It is a window-shade type of action. Both types can be obtained with excellent electrical accuracy (0.1%). The rotary type is also available with a velocity output, obtained by coupling a tachometer to the potentiometer shaft. The maximum velocity is set by the strength of the cable which is limited to 50 g.*

*Based on the torso acceleration, $\beta = 10^6$ deg/sec^2, given in the General Specifications, the torso moves with an acceleration of 820 g's. This is for a human with a 1.5-foot torso-hip distance.

4. Linear Velocity Transducer.- In appearance, these, transducers resemble rectilinear potentiometers. They work on Faraday's law of electromagnetic induction: a permanent magnet moving with a uniform velocity inside a solenoidal coil generates an induced EMF in the coil. Therefore, they are passive devices and require no excitation. The windings of the coil, together with the between turn distributed capacity, form an LRC resonant circuit which limits the frequency response. The longer the coil length, the lower the resonant frequency. However, we were able to find a manufacturer who makes a linear velocity transducer with a stroke length of 20 inches, probable frequency response of 68 KHz, and a linearity of better than 1%. All the considerations of attachment problems and vector errors discussed in connection with the rectilinear potentiometer are relevant to the linear velocity transducer.

5. Strain Gauges.- These gauges are incorporated electrically into a bridge circuit, and loaded mechanically with a mass, to make them sensitive to linear acceleration. Frequency limitations are imposed by problems with the bridge circuit, primarily stray and distributed capacity. Most available models' response are limited to a few hundred Hertz. A survey of thirty transducer and instrumentation manufacturers revealed that nobody makes a rotational accelerometer using the strain gauge principle.

6. Accelerometer.- These sensors are basically a combination of a force transducer, and Newton's Second Law of Motion: $\vec{A} = \vec{F}/M$. This statement is true for transducers sensitive to linear accelerations. A survey of manufacturers revealed that only

one manufacturer makes rotational accelerometers. One type utilizes a fluid principle, has a band-width of 40 Hz, and a weight of 242 grams. This is unacceptable for occupant motion sensing.

A second type of rotational sensor, made by the same manufacturer who produces the fluid principle rotational accelerometer, uses a sensor system force balance principle. Although very accurate (0.1%), the bandwidth and weight are inadequate by a factor of 10.

In the course of the survey of the manufacturers, it quickly became apparent that the only wideband, (> 1 KHz) light-weight, (< 10 grams) accelerometer is the pieyoelectric crystal accelerometer. Unfortunately, it responds only to linear acceleration. However, a method was devised to combine two of these devices into a single unit that responds only to rotational acceleration. A simple variation of the principle produces a transducer responding only to rotational velocity squared. The mathematical derivation of this is shown in Appendix C.

4.2.2 Optical

1. High Speed Photography.- This method, currently in use at all sled-testing facilities, is used because photographic film, on a cost-per-bit basis, is still the cheapest and quickest way to store large amounts of information. The accuracy is limited. Most workers in the field put this number somewhere between 10% and 20% when measuring displacements. Although data recording is fast, retrieval of the data is time consumming, costly, and prone to more error. Semi-automatic film readers are now available

which, to a limited extent, alleviate these problems. Another problem, already discussed, is that when the raw data (position) are differentiated (velocity) and differentiated again (acceleration) the original error is magnified. A digital simulation study is currently underway to predict this increased error.

Part of the error is inherent in the design of the high-speed camera shutter. Additional non-lens elements are placed in the optical path and degrade the image. Part of the error is inherent in the lens design itself, such as pin-cushion and barrel distortions. Even if these above mentioned errors could be eliminated, there would still be errors inherent in the photographic film, grain noise, base fog levels, limited dynamic range, non-linear response and the relative motion between the emulsion and base which takes place during development.

In conclusion, high-speed photography is invaluable in obtaining a qualitative understanding of the overall system performance, but should not be used for occupant motion sensing, particularly for high velocities and accelerations set forth in the General Specifications, Appendix A.

2. Laser Doppler System.- In a typical doppler system velocimeter, a return signal-frequency shifted by a moving target-is heterodyned in a non-linear mixing element to product a beat or difference frequency (ref. 6). The conversion of target velocity to beat frequency is 1.0 MHz per foot/sec., for the red 6328 Angstrom line of a He-Ne laser. From the numbers given in Appendix A for the rotational velocities of the head and torso, the corresponding linear velocities can be obtained by:

$$v_1 = \dot{\theta} r \tag{19}$$

with

v_1 = linear velocity

$\dot{\theta}$ = appropriate rotational velocity

r = radius of rotation

For the head:

$\dot{\theta} = \dot{\phi} = 10^4$ deg/sec = 175 rad/sec

r = 4 inches = 0.33 feet

v_1 = 60 ft/sec (60 MHz/sec)

For the torso:

$\dot{\theta} = 10^4$ deg/sec = 175 rad/sec

r = 1.5 feet

v_e = 262 ft/sec (262 MHz/sec)

This is about the state-of-the-art in wideband amplifiers. In a system with this wide a bandwidth, noise could be a potential problem, since the rms noise voltage developed in a system is proportional to the square root of the bandwidth.

Another problem with a laser velocimeter system is the fact that a standard laser, mounted on the sled, would not survive the crash impact, taken to be a half sine with a 100-g peak. This means one of two things: either a specially hardened laser must be developed, or a standard laser on the ground with its beam piped on board by a mirror/prism system must be used. The latter would probably be a better course to pursue, but the mirror/prism system is an added complication.

Finally, a problem more serious than the two already discussed is the problem of a target. The General Specifications call for limits of +120° to -180° in forward head motion, a total of 300°. In order to reflect the laser beam back to the receiving optics coherently (required to produce the heat frequency in the detector), a corner cube reflector system would have to be used. This immediately transforms the measurement into a linear velocity, not the desired rotational velocity. A single corner cube is only effective over 90° in two orthogonal planes, i.e., a solid angle of $\pi/2$. Therefore, an array or collection of corner cubes would have to be used. Since coherence must be maintained, a high quality of optical components must be used, probably glass. This raises serious problems with weight limitations for a head-mounted target, and human occupant safety if the target were to smash or shatter at crash impact.

In conclusion, therefore, we must say that the laser velocimeter is an unacceptable occupant motion sensing system.

3. Laser Range Finder.- This is an optical adaption of the RADAR principle. All the problems of a suitable optical target and system crash-worthiness, discussed in connection with the laser doppler system, apply here. In addition, there is a problem of the required resolution time in the electronic circuits decoding the basic positional information. For an on-board transmitter/receiver one meter from the subject, the receiver timing circuit would have to be capable of resolving approximately 50 picoseconds for a 5% accuracy. This would be very difficult requirement in a portable system.

Therefore, it is concluded that the laser rangefinder is unacceptable for occupant motion sensing.

4. Holography.- Holography is a method of recording a three-dimensional scene on high-resolution photographic film. Because of the extremely tight requirements on spatial and temporal coherence, a high-quality pulsed laser light source must be used. Holography avoids the use of photographic lenses, with their inherent spatial distortions. Interferometric holography, a recently invented variation, allows a means of recording and detecting small changes in the body being holographed.[7] This might be useful for occupant motion change detection, were it not for the fact that the technique is really too sensitive for this application. A dark fringe band appears on the reconstructed holographic image each time the body is moved the order of a wavelength of the laser light source roughly, 0.5×10^{-6} meters. If the occupant were to move 1 meter, 2×10^{6} fringes would be produced. This would make interpretation of the results impossible.

There are a host of other problems which make holography impractical for occupant motion sensing. For example, the required high-resolution film is relatively slow, which means that a very intense pulsed laser source must be used, typically in the magawatt range. The question of safety to the human occupant's eyes, should his head be inadvertently thrown in the direction of the laser beam is a real one. Commercially available systems are expensive (~ $30,000) and are of less than desired reliability.

It is concluded, therefore, that holography is not a practical solution to the occupant motion sensing problem.

5. <u>Ellipsometry</u>.- This concept, which is an original approach to occupant motion sensing, is basically a high-speed photographic method, but without some of the problems, already discussed, of high-speed photography.

The basic system is shown in Figure 9.

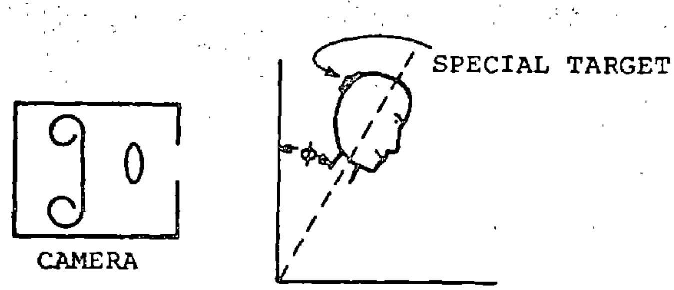
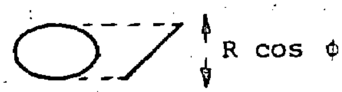


Figure 9.- Elliptic Determination of Angular Position

A high-speed camera records a small, light weight target taped to the head. The target, shown in Figure 10, is simply a circle and sphere alligned coaxically. The film records the projection of this circle, which is an ellipse of varying eccentricity, depending on ϕ, the angle of tilt. For a rotation about one axis, the ratio of the axes is:

$$\epsilon = \frac{R \cos \phi}{R} = \cos \phi$$

The appeal of this system is that we presently have a computer-based optoelectronic system that can automatically measure the eccentricity of a high-contrast disc on photographic film. This system was developed as part of a remote sensing occulometer, or eye direction tracking system. This system eliminates one source of error on conventional semi-automatic film readers: a human operator is required to line up a set of cross-hairs on a fiducial point on the occupant's image. This, of course, is a serious source of error. In ellipsometry, no human judgement is required.



Ratio of Axes: $\frac{R \cos \phi}{R} = \cos \phi$

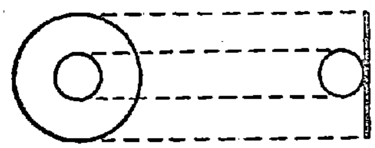


Figure 10.- Ellipsometry Target

The target is a simple diffusely reflecting device, whose weight can easily be kept less than 10 grams*. Coherence is not a requirement here, as with the laser measuring systems. A small sphere can be included in the target to give linear displacement of the head, by measuring the sphere's diameter.

A further advantage of this system is that the computer used to measure the eccentricity of the ellipse can also be used to correct systematic displacement errors produced by the camera's optical system, once the high-speed camera has been calibrated. This eliminates a second significant source of error over the conventional high-speed photographic system.

*A target size of approximately 1 inch will be required to keep the measurement error below the 5-% level.

4.2.3 Electromagnetic, High Frequency

1. Doppler Radar.- This is similar to the laser doppler system already discussed, except for the frequencies involved. A transmitted signal is beamed at the occupant. A suitable target reflects a portion of the outgoing beam. This reflected signal is mixed with a portion of the transmitter in a nonlinear element to produce a difference frequency. The difference frequency is related to the velocity of the moving target.

The beat frequency is given as:

$$f_d = \frac{v}{V} f_t \tag{21}$$

where

f_d = beat or difference out of detector

v = target velocity

V = velocity of light

f_t = transmitter frequency

The lowest permissible value for the transmitter frequency is set by the lowest velocity to be measured, and the fact that at least one full cycle of f_d is required to measure its frequency. In fact, regardless of f_t, in the limit of v going to 0, an infinitely long time is required to measure f_d. Since a crash or sled deceleration run lasts in the order of 100 milliseconds, the time to measure one cycle is very important. Taking all these factors in account, we calculate a minimum transmitter frequency, for an error of 5%, of:

$$f_t \geq 60 \times 10^9 \text{ Hz or } 60 \text{ GHz}$$

In addition, the required stability of the source must be 50 Hz. A microwave klystron source for f_t is stable to about 1 part in 10^4. 50 Hz in 60 GHz represents 1 part in 10^9. This means an on-board klystron transmitter would not be stable enough. Therefore, a ground-based crystal-controlled 60-GHz generator would have to be built, and piped to the sled over flexible cables. This would be a very expensive, cumbersome system.

There is also a problem providing a safe, simple, light-weight microwave reflector for mounting on the occupant, one that will respond only to angular orientation over an angle of 300°.

There is also the problem of stray reflections from an air bag, or other parts of the occupant, and vehicle compartment. These reflections will produce extraneous signals and lower the effective signal/noise ratio.

In conclusion, then, a microwave doppler system is judged to be inadequate for occupant motion sensing.

2. FM Phase-Lock Systems.- This is a system[8] for making very precise measurements of the doppler shifted return signal, such as from a doppler radar system. As discussed above, the problems with a doppler radar system for occupant motion sensing occur before the detection process. Hence, the detector system's pros and cons are not relevant.

3. Signal Strength Measurements.- This is an amplitude sensitive, CW system consisting of receiver and transmitter, either one of which could be stationary, the

remaining one mounted on the subject. The biggest problem would be in making the system sensitive only to angular changes. If we break down any antenna system into a multipole expansion, even the simplest term, the dipole, has a response which varies with angular orientation and a distance vector. In addition, stray reflection from an expanding air bag or other moving body components would be a source of error.

This system is judged to be unacceptable for occupant motion sensing.

4. Interferometry.- This is a positional detection system based on the interference between a reference wave and a reflected wave. Each time the target moves through a distance of half a wavelength, $\lambda/2$, the detector produces a zero response. Problems with spurious response, target mode response, carrier frequency stability, and general system complexity are about the same as with the doppler radar system.

Therefore, this instrumentation approach is judged to be unacceptable.

4.2.4 Ultrasonic or Acoustic

(1) Doppler Shift

(2) Signal Strength Measurement

(3) Interferometry

All these systems will be taken as a class, since there are some basic physical consideration that apply to all three.

The first problem is localization of the return signal. It would be virtually impossible to separate a signal returned from the head from one returned by the neck or torso. This is the familiar targ . problem.

A second problem is that it is difficult to imagine a system sensitive purely to rotational motion - the first priority measurement. Most of the conceivable systems would either measure linear displacement or linear velocity.

And finally, at the time of impact, initial accelera-tion of the sled, or deployment of an air bag, large acoustical signals are generated, whose Fourier component could easily extend to the frequency of the measurement, conservatively estimated to be 20 KHz. This would cause erroneous signal and, therefore, uncertainties and errors in the measurements.

Hence, for the foregoing reasons we do not recommend pursuing ultrasonics as a practical way of measuring occupant motions.

4.2.5 Electromagnetic, Low Frequency

1. Capacitive.- Displacement measuring devices of this type are based on the change in resonance of an RLC circuit when a displacement changes the capacitance of the circuit. A survey of manufacturers of these devices reveals that while these devices are capable of resolving displacements in the microinch region, the measurements do not extend beyond a few tenths of an inch. When asked about the possibility of developing a unit capable of less resolution, but with a range of up to several feet, all manufacturers gave a negative response.

In addition, there is the target problem. While it would probably be possible to devise a capacitance geometry that would be sensitive to rotational motions exclusively, even when translation is also present, there would be the problem of target localization. One side of the capacitance is at ground potential, and one side is "hot" or above ground. Typically, a small metal disc target is placed on the moving body which preferably is nonconductive. The hot side of the motion-sensing capacitance is stationary. In the case of the human body, electrically it consists of a relatively high resistance sheath (the skin) surrounding a highly conductive medium. Therefore, it is difficult to use a metal disc to form the capacitor for the resonance circuit uniquely. To state it another way, all the field lines from the hot side of the capacitor do not end on the target disc, but penetrate the surrounding skin to the conducting viscera underneath. Since these parts, in general, are all moving relative to each other, it is uncertain just what a change of capacity means under these conditions. This manifests itself as error.

Therefore, it must be concluded that capacitance techniques are unacceptable for occupant motion sensing.

2. Magnetometer.- A system was proposed that would consist of a magnetic field, either DC or low-frequency AC, created externally by a set of Helmholtz coils*,

*So called for historical reasons. It consists of two coaxial coils separated by a distance equal to their radius. Helmholtz was first to point out that this configuration is optimum from the standpoint of spatial field variations at the center of the system and a small magnetic field sensor placed on the occupant. This is a angular displacement measurement, where the raw data would have to be processed by an inverse sine or cosine function to obtain the angle.

A survey of instrumentation manufacturers turned up one source with a potentially useful sensor. Its current bandwidth and mass are 10 Hz, and 70 grams, respectively, although the manufacturer thought the bandwidth could be extended to 500 Hz, and the mass trimmed somewhat. A unit was borrowed for testing. In laboratory tests, it became apparent that the transducer not only responded to changes in the external magnetic field, but also to mechanical accelerations. The manufacturer sent a representative to check the unit, which was pronounced normal in all respects. Repeating the previous tests showed that the output signal was still responsive to rapid mechanical motions, as well as magnetic field.

In view of the problems with this sensor, and the uncertainties in the outcome of a development program to make this sensor acceptable in all respects, it was decided that this system was not a good candidate for occupant motion sensing.

3. Radio Direction Finding (RDF).- This is the application of direction finding techniques used in vehicle navigation to determine angular orientation or heading. In the usual application, a low-frequency (100 to 500 KHz) transmitter and antenna established a reference field. A receiver with a ferrite loop antenna is used to receive the signal and measure the heading.

This approach is attractive for occupant motion sensing because only the ferrite core antenna coil - which could be made very small and with low mass - would be placed on the occupant.

A proposed system is shown in Figure 11. A low-frequency RF driver supplies power to a set of coils to create a reference field. The sensor consists of a pair of ferrite core coils rigidely held 90° with respect to each other. The reason two coils

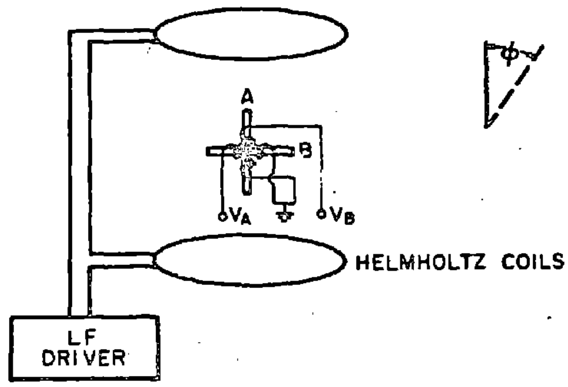


Figure 11.- RDF System

are used is to eliminate errors arising from changes in the field strength as the pick up coils are moved off the axis. As will be proved, only changes in the reference field direction will limit the accuracy of this system.

The terminal voltages of the coils, $V_{a,b}$, are:

$$V_a = KNH_o\omega_o \cos(\omega_o t) \cos(\phi) \tag{22}$$

$$V_b = KNH_o\omega_o \cos(\omega_o t) \sin(\phi) \tag{23}$$

where:

N = number of turns

ω_o = angular frequency of RF driver

K = a factor taking into account the geometry and magnetic properties of the ferrite core

H_o = peak RF field produced by the Helmholtz coils.

In processing the signals, V_a and V_b are summed and the resultant divided by the square root of the sum of the squares of V_a and V_b.

$$V = \left(V_a + V_b\right) / \left(V_a^2 + V_b^2\right)^{1/2} \tag{24}$$

$$= \sqrt{2} \sin \left(\phi + \frac{1}{\sqrt{2}}\right) \tag{25}$$

Therefore, the response depends only on the angle between H_o and the pick-up coil array.

Achieving a bandwidth of 425 Hz or 900 Hz presents no problem. For a center frequency of several tens of kHz, a few kHz should be easily obtainable. In fact, the only reason to make it smaller would be to reduce broad-band noise response.

In the course of evaluating this system, it was apparent that the Helmholtz coil configuration could not be kept, and still have the occupant seated between them. This would have meant a coil diameter of at least 8 feet mounted on the sled. A general mathematical expression was developed for the magnetic field at any point from two identical coils of any spacing. The resulting elliptic integrals were computer evaluated using a procedure based on Simpson's rule. The derivation was based on Ampere's Law for DC currents, but will be valid for AC and low-frequency RF currents, as long as the radiation from the coils is small, and the distant to the sample point is much less than a wavelength. This condition is easily met, since at 50 kHz, for example, the wavelength is 6000 meters.

What systems, then, are left for occupant motion sensing? The results of the evaluation just completed indicate that there are five systems or sensors that seem to pass the list of criteria. However, this endorsement is based, in large part, on a mathematical model and manufacturer's data and recommendations. All this is a necessary, but possibly not a sufficient set of criteria. The final hurdle will be field testing of these instruments and systems on humans and anthropometric dummies in crash and sled deceleration tests. Unforseen problems may develop, or problems known but thought to be minor may prove to be significant.

We conclude this report with a brief description of each of the five candidate systems selected for prototype assembly.

5. RECOMMENDED CANDIDATE SYSTEMS

5.1 Rotational Accelerometer

Using the principle demonstrated in Appendix C, all linear accelerometers must be considered as possible candidates for rotation measurements. However, the list quickly decreases when the full list of specifications, Table II, is considered. The choice is between the piezoelectric or strain gauge linear accelerometer. Of the two, the piezoelectric would seem to have definite advantage, at least in the matter of frequency response. The piezoelectric type, being passive, does not require excitation. The strain guage type does, and stray capacitance eventually sets the upper limit on the excitation frequency. It was, therefore, decided to build the rotational accelerometer from existing piezoelectric linear accelerometers. The acceleration configuration was used, as opposed to the velocity-squared (v^2) design. This is probably the more useful measurement, especially if one is measuring rotational severity index.

We have now received, from a manufacturer, a specially made rotational accelerometer, consisting of two linear piezoelectric accelerometers with the following characteristics:

Mass	:	5 grams
Size	:	0.8 x 0.8 x 3.0 cm
Sensitivity	:	0.4 millivolts/radian/sec^2
Bandwidth	:	1.5 Hz to 8 kHz

It is planned to reduce the size by a factor of two, and fabricate a bite-bar type mounting. This combination should result in a system capable of making more accurate rotational measurements than have heretofore been possible. In addition, there is the possibility of including a miniature FM telemetry transmitter on the bite-bar, eliminating all wires to or from the subject.

5.2 Potentiometer

The second sensor recommended for occupant motion sensing is the rectilinear potentiometer. The electrical circuit is shown in Figure 12, together with the mounting and attachment methods. The bandwidth is limited by the distributed capacity, shown by dashed lines. The manufacturer estimates the bandwidth to be 100 kHz, with a stroke of 24 inches. This device, while not suitable for head motion measurement, will be suitable, within the limitations already discussed, for torso measurements. A gimbaled mounting allows the potentiometer to follow the moving subject freely. On anthropomorphic dummies, one piece of a ball-and-socket joint can be bolted rigidly to the back. On human subjects, a strap arrangement would be required.

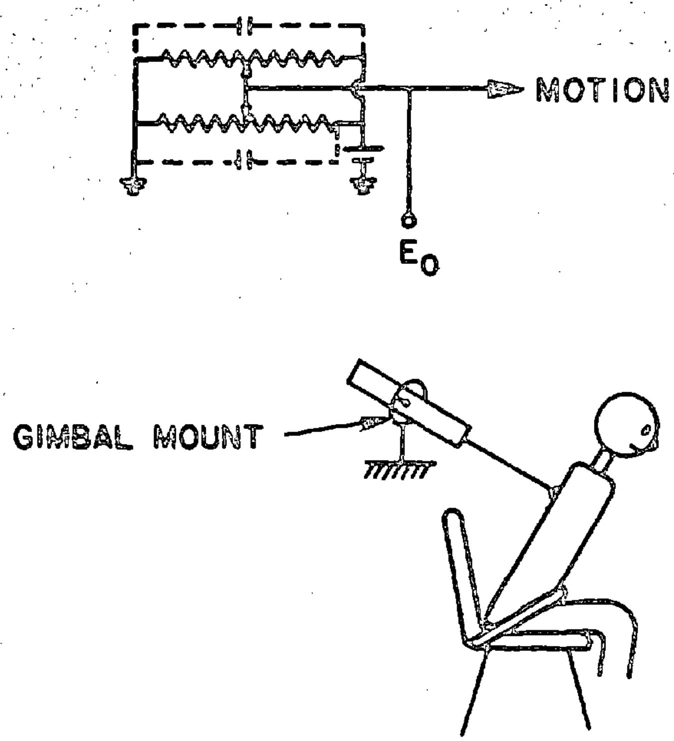


Figure 12.- Rectilinear Potentiometer - Displacement Measurement

5.3 Linear Velocity Transducer

Mechanically and application-wise, this transducer is like the rectilinear potentiometer. It is useful only for torso measurements and will be mounted on a gimbaled platform behind the subject. The electrical equivalent circuit is shown in Figure 13.

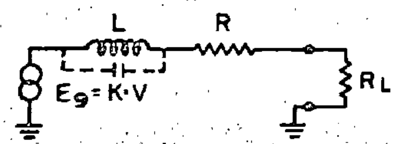


Figure 13.- Electrical Equivalent Circuit

The electrical transfer function is:

$$G(\omega) = \frac{1}{1 + R/R_L + i\omega L/R_L} \tag{26}$$

when

$$R_L >> R$$
$$\omega_c = R_L/L \tag{27}$$

A unit has been purchased with the following characteristics:

Stroke: 20 inches

L: 0.035 Henries

R: 3 k ohms

For

$$R_L = 15 \text{ k ohms},$$
$$\nu_c = \frac{\omega_c}{2\pi} = \text{kHz} \tag{28}$$

This 68-kHz bandwidth, if true, in more than adequate. However, the electrical equivalent circuit does not include the distributed, inner turn capacity, as this information was unavailable from the manufacturer. Test are under way now to measure this quantity on the unit received. It is anticipated that the bandwidth will still be in excess of the 900 Hz requirement, even when capacitive effects are included.

5.4 RDF System

This system, shown in Figure 11, is included because of the wide bandwidth capability, and low sensor coil mass. In fact, the receiving coil assembly is probably small enough to include on a bite-bar mounting plate.

One potential problem that only field testing will elucidate is that the RF reference field produced by the Helmholtz coil may cause interference in the other instrumentation channels, particularly ones like the high-impedance rotational accelerometers.*

5.5 Ellipsometry

This system is shown in Figure 9. A high-speed camera capable of on-board mounting has been ordered for use with this system. It may be necessary to use an active target to get enough illuminance for a proper exposure. This is because the light level, at the time of test, is generally high to enable the high-speed ground based cameras to function properly.

Figure 14 shows the complete instrumentation package. It will be portable and be sent to the sled test facility for field evaluation of the five systems just discussed. Two standard, triaxial, linear, accelerometers will be included in the head and torso cavity in dummy testing to aid in the calibration and

*After this evaluation phase was concluded, a possible improvement in this RDF system was conceived. This would be to use a single coil placed off the sled. Our field calculations reveal that if a coil-to-subject distance of 30 feet is used, the measurement error can be held to less than 5%. The trade-off for this is a lower signal received at the pick up coil on the subject. We are currently redesigning the electronics for the pick up coil to work with this lower level signal. This arrangement, if used, should eliminate any interference problem, as well as greatly simplify matters by not requiring large on-board Helmholtz coils.

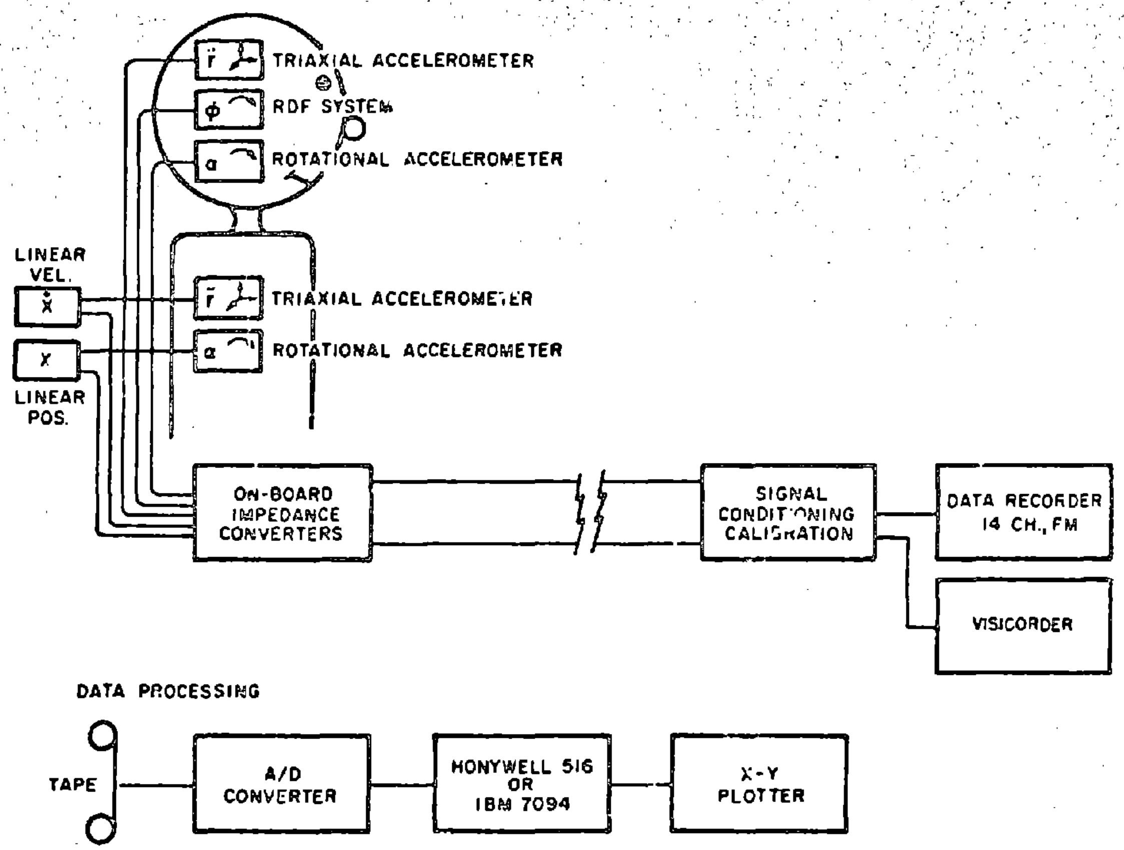


Figure 14.- Instrumentation Package

validation of the other sensors. After thought and discussions with workers in the field, it was decided to use on-sled impedance converters to transform all transducer signals to a low impedance level. This will help to maintain a high signal/noise ratio through the 120-foot umbilical cable connecting the sled to the recording instrumentation. The data processing equipment chain is shown in the lower portion of the figure.

6. CONCLUSION

These, then, are the five systems, based on the General Specifications, the mathematical model, and the Criteria for Occupant Motion Sensors, that have been selected as probable solutions to the occupant motion sensing problem. Although it is felt that a reasonably exhaustive survey of possible systems was made in the time allowed, it is always possible that a potential system has been overlooked. In addition, field testing may dictate that some of the systems selected may not be adequate to the problem. Therefore, we present the results of the studies to date as a reasonable first approximation to the problem's solution, not as an iron-clad list of systems and criteria that must be strictly adhered to.

In the reports to follow, and particularly in the final report, we hope to have preliminary test results back from field evaluation of some of these proposed systems.

REFERENCES

1. Bracewell, R. M.: The Fourier Transform and its Applications. McGraw-Hill, New York, 1965.

2. Clarke, T. D.: Daisy Track Lethal Tolerance Tests. Final Report, June 1970, Holloman Air Force Base, New Mexico, for U. S. Department of Transportation.

3. This program is based on the fast Fourier transform (FFT) algorithm developed by J. W. Cooley (IBM) and J. W. Tukey (Bell Labs). For a description of its operation, see: The fast Fourier Transform. E. O. Brigham, R. E. Morrow, IEEE Spectrum, December 1967.

4. Holbourne, A. H. S.: Lancet vol. 2, p. 438, October 9, 1943,

5. Hirsch, A. E., Ommaya, A. K., and Mahone, R. M.: Report 2876, Dept. of Navy, Naval Ship Research and Development Center, Washington, D. C., August 1968.

6. Ross, M.: Laser Receivers. Devices, Techniques, Systems. John Wiley & Sons, Inc., New York, 1966.

7. Powell, R. L., and Stetson, K. A.: Journal Optical Society of America, vol. 55, December, 1965, p. 1593.

8. Gardner, F. M.: Phaselock Techniques. John Wiley & Sons, Inc., New York, 1966.

Appendix A

GENERAL SPECIFICATIONS

For purposes of establishing a uniform coordinate system to allow a systematic analysis of the measurement problem, a hip, upper torso, and head model is used. The hips are allowed one degree of translational movement. The upper torso is allowed one degree translation and one degree rotation within the sagital plane. The head is allowed three degrees rotation and two degrees of translation in the sagital plane. This model can readily be expanded to include additional degrees of freedom. However, this simple model does take into consideration neck extension and compression, neck twist relative to the torso, and torso acceleration. For simplicity the back is considered a rigid member. The coordinate system is shown below in Figure A-1.

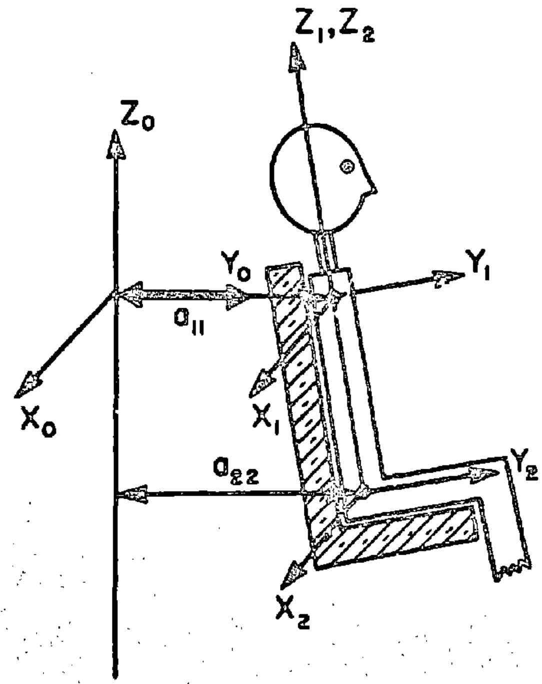


Figure A-1.- Coordinate System

Below is a list of desired body motion measurements, together with the importance or priority of each (rated 1 to 3 - high to low) and the maximum value expected for each.

		Maximum Value	Priority
(a)	Head Rot-tions		
	ϕ	+120°, -180°	
	$\dot{\phi}$ *	10^4 deg/sec	
	$\ddot{\phi}$ **	10^7 deg/sec^2	
	(Measured around x_1 Axis)		1
(b)	Head-Neck Extension or Compression		
	$(z_1 - z_2)$	± 5 cm	
	$(\dot{z}_1 - \dot{z}_2)$	1 meter/sec	
	$(\ddot{z}_1 - \ddot{z}_2)$	10^3 meter/sec (100 g's)	1
(c)			
	θ_A	±90°	
	$\dot{\theta}_A$	10^2 deg/sec	
	$\ddot{\theta}_A$	10^4 deg/sec^2	
	(Measured around y_1 Axis,		2
(d)			
	θ_B	±90°	
	$\dot{\theta}_B$	10^2 deg/sec	
	$\ddot{\theta}_B$	10^4 deg/sec	
	(Measured around z_1 Axis)		

(e) Rotation of Shoulder Chest)

(Center of $x_1y_1z_1$ System) $\dot{\beta} = 10^4$ deg/sec

$\ddot{\beta} = 10^6$ deg/sec^2 3

(f) Translation of Hips

(Center of $x_2y_2z_2$ System)

$\dot{y}_2$ 20 meters/sec

$\ddot{y}_2$ 10^3 meters/sec^2 (100 g's) 3

---

*A single dot indicates a velocity (d/dt)
**A double dot indicates acceleration (d^2/dt^2)

A fixed coordinate system, $x_oy_oz_o$, is also shown in Figure A-1. It is the reference system against which all other measurements will be made. It will either be the frame of the sled or crash vehicle, or an earth based system. The choice will be determined by the types of transducers finally selected. The motions of a_{11} and a_{22} will allow one to convert measurements in the $x_1y_1z_1$ or $x_2y_2z_2$ system to the absolute $x_oy_oz_o'$ system.

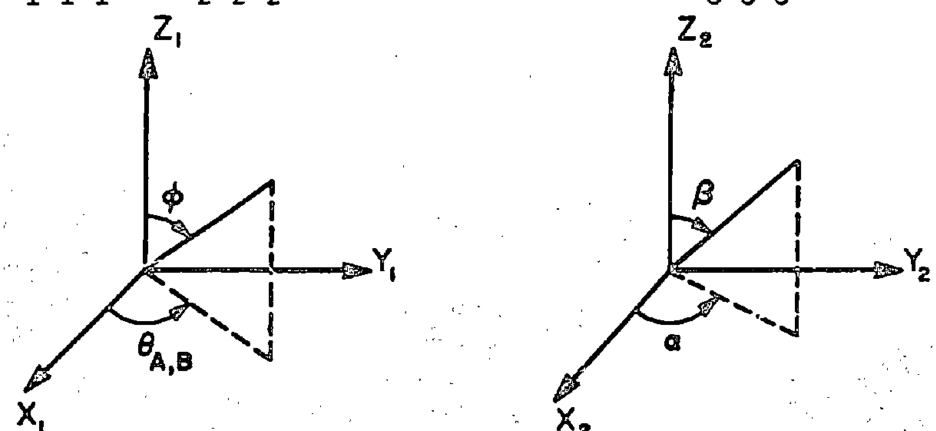


Figure A-2.- Fixed Coordinate System

MEASUREMENT PRECISION

(1) Absolute - 10%

(2) Trial to Trial - 5% Repeatability

CALIBRATION

(a) Laboratory calibration traceable to NBS standards.

(b) Field calibration - limited sensor self-calibration; complete electronic field calibration before and after test; validation tests, comparison with accelerometer and photo data from non-air-bag dummy tests.

Appendix B

SENSOR MASS CONSIDERATIONS

Consider the head undergoing acceleration with a sensor of mass M_s attached.

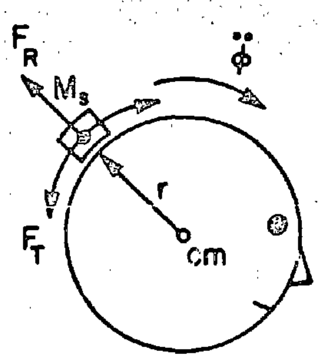


Figure B-1.- Head Under Acceleration

The forces acting on the sensor can be resolved into a tangential component F_T, and a radial component, F_r. The net force will be the vector sum of these two. Let us calculate each separately using the values of $\ddot{\phi}$ and $\dot{\phi}$ given in the General Specifications, Appendix A.

$$F_T = M_S A_T = M_S r\ddot{\phi} \text{ (dynes)}$$

r = distance from center of mass to sensor, taken as 10 cm.

$$\ddot{\phi} = 10^6 \text{ deg/sec}^2 = 1.75 \times 10^4 \text{ rad/sec}^2$$

To convert F_T, dynes, to grams force, divide by g=980 cm/sec^2;

$$F_T = M_S \cdot r \cdot \ddot{\phi}/g = 1.8 \times 10^2 \text{ grams force/gram sensor mass.}$$

For F_R:

$$F_R = M_S \dot{\phi}^2 r/g = M_S \left(1.75 \times 10^2\right)^2 \cdot 10/g$$

$$= 3.1 \times 10^2 \text{ grams force/gram transducer mass.}$$

The vector sum of this is F_S,

$$F_S = \left(F_R^2 + F_T^2\right)^{1/2} = 358 \text{ grams force/gram sensor mass.}$$

Each gram of sensor mass exerts 358 grams of force, or 12.6 pounds, on the mounting straps or whatever holds the device in place.

While these values might be somewhat higher than encountered in the field, particularly in the case of testing human occupants, they do show the need for extremely light-weight sensors.

We will, therefore, place the upper limit of sensor mass as 14 grams (1/2 ounce) and a desired mass of 1 gram (1/28 ounce).

Appendix C

PROOF OF ROTATIONAL MEASUREMENTS FROM LINEAR TRANSDUCERS

It will now be shown that a pure rotational motion either acceleration ($\ddot{\phi}$) or velocity squared ($\dot{\phi}^2$), can be obtained by properly placing two linear accelerometers.

Consider Figure C-1. A and B are two points on a body rotating around point C with a clockwise angular acceleration of $\ddot{\phi}$.

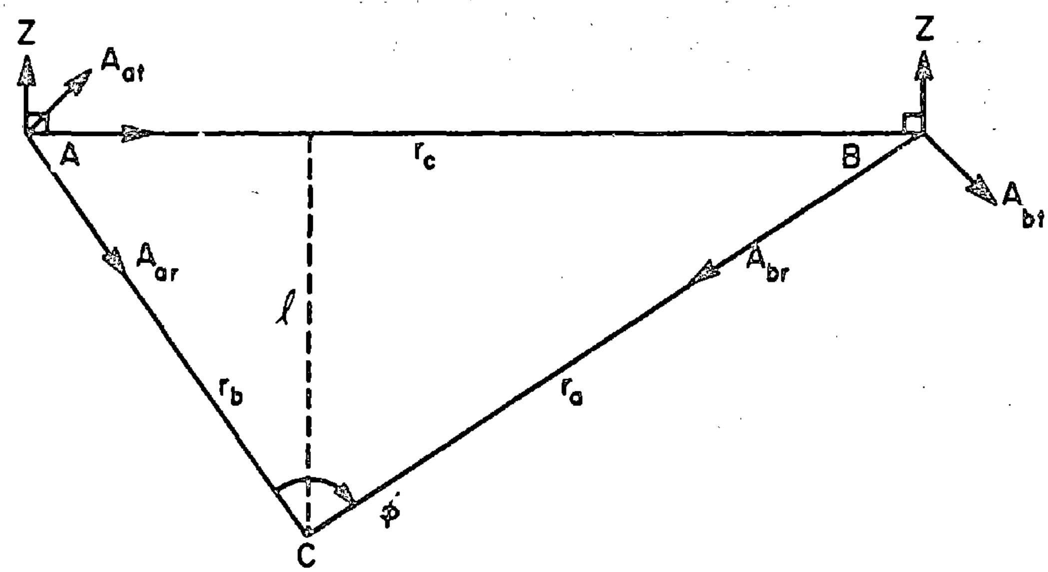


Figure C-1.- Rotational Motion from Two Linear Accelerators

No special relationship is assumed about triangle ABC. The Z direction is defined as being perpendicular to line segment AB. $A_{a,b}$ refer to the linear accelerations at point a and b, respectively. The subscripts, t and r, refer to the tangential and radial components of the acceleration.

The components of acceleration parallel to the Z direction[11] at points A and B are:

$$A_{az||} = A_{at} \sin A + A_{ar} \cos A \tag{29}$$

$$= \ddot{\phi} r_b \sin A + \dot{\phi}^2 r_b \cos A \tag{30}$$

and

$$A_{bz||} = A_{bt} \sin B - A_{br} \cos B \tag{31}$$

$$= \ddot{\phi} r_a \sin B - \dot{\phi}^2 r_a \cos B \tag{32}$$

Subtracting Eq. (32) from Eq. (30),

$$A_{az} - A_{bz} = \dot{\phi}^2 \ (r_b \cos A + r_a \cos B) + \ddot{\phi} \ (r_b \sin A - r_a \sin B) \tag{33}$$

But,

$$r_b \sin A = r_a \sin B = \ell \tag{34}$$

and

$$r_b \cos A + r_a \cos B = r_c \tag{35}$$

Putting Eqs. (34) and (35) into Eq. (33) and rearranging terms gives:

$$\dot{\phi}^2 = \frac{A_{az} - A_{bz}}{r_c} \tag{36}$$

This proves the first part of an assertion; rotational velocity squared is equal to the difference in the linear component of acceleration parallel to a line joining two points of the body divided by the perpendicular distance between these same two points.

To obtain rotation acceleration, we start by taking the components of acceleration perpendicular to the Z direction (⊥).

$$A_{az} = A_{at}\cos A - A_{bt}\sin A \quad (37)$$

$$= \ddot{\phi} r_b \cos A - \dot{\phi}^2 r_b \sin A \quad (38)$$

$$A_{bz} = -A_{bt}\cos B - A_{br}\sin B \quad (39)$$

$$= -\ddot{\phi} r_a \cos B - \dot{\phi}^2 r_a \sin B \quad (40)$$

Subtracting Eq. (40) from Eq. (38):

$$A_{az\perp} - A_{bz\perp} = \ddot{\phi}(r_b\cos A = r_a\cos B) + \dot{\phi}^2 \; (r_a\sin B - r_b\sin A) \quad (41)$$

Using the identities of Eqs. (34) and (35), and rearranging gives:

$$\ddot{\phi} = \frac{A_{az\perp} - A_{bz\perp}}{r_c} \quad (42)$$

This proves the second part of our assertion: a pure rotational acceleration can be obtained from two linear acceleration measurements by measuring the difference in the components perpendicular to the line joining them, and dividing by that distance.