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ANALYSIS OF POSITION ERROR HEADWAY PROTECTION

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INTERIM REPORT

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16. Abstract

An analysis is developed to determine safe headway on PRT systems that use point-follower control. Periodic measurements of the position error relative to a nominal trajectory provide warning against the hazards of overspeed and unexpected stop. A computer program has been developed to model these hazards for arbitrary safety system design parameters.

The results of computer runs indicate that the critical hazard on the main guideway is unexpected stop of a preceding car; on a station entry deceleration ramp, it is overspeed of a following car. The deceleration ramp headways are larger and are more sensitive to system parameters than are the main guideway headways.

Typical headways are five seconds on a 30 mph main guideway and 16 seconds on a deceleration ramp for state-of-the-art system parameters. With advanced system parameters and emergency decelerations applicable to well supported, seated passengers, required headways are 2.5 seconds on the main guideway and 3.5 seconds on the deceleration ramp.

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PREFACE

The work herein was performed under Contract No. DOT-TSC-421, Headway Safety Assurance Subsystem, by Alden Self-Transit System Corporation. It describes the background, analysis and generic application of the headway computer program developed under DOT-TSC-421, and used to design the headway system being demonstrated on the Alden Test Track.

Since the headway analysis and computer program were developed early in the contract and constitute a logically independent package, their description is submitted as a separate interim report.

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1. GENERAL INTRODUCTION AND ASSUMPTIONS

This report examines the application of position-error headway protection to Personal Rapid Transit (PRT) systems. PRT systems consist of small cars operating on exclusive guideways, under full automatic control. Stations are off-line so that a car can bypass unwanted stations and reduce trip time. A representation of such a system is shown on Figure 1-1.

A headway protection system is an independent subsystem of the PRT system. It determines when a potential collision hazard exists and initiates emergency signals. While this report is based on the Alden headway protection system, being demonstrated under Department of Transportation contract No. DOT-TSC-421, the techniques and results are applicable to any position error system.

Consideration is given to:

1. The framework of the problem;
2. The reference headway protection system;
3. The analysis of safe headway; and
4. Typical results.

The analysis is implemented through a computer program that determines safe headway at any point in the guideway, assuming either vehicle overspeed or unexpected stop failures.

Baseline headways consistent with the state-of-the-art are about 5 seconds on a 30 mph main guideway, and about 16 seconds on a station deceleration ramp. The prime hazard on the main guideway is unexpected stop, and on the deceleration ramp is overspeed (see Section I.B).

With more advanced system parameters, i.e., parameters felt to be feasible but beyond the present state-of-the-art, headways as short as 2.5 seconds on the main guideway and 3.5 seconds on the deceleration ramp are possible.

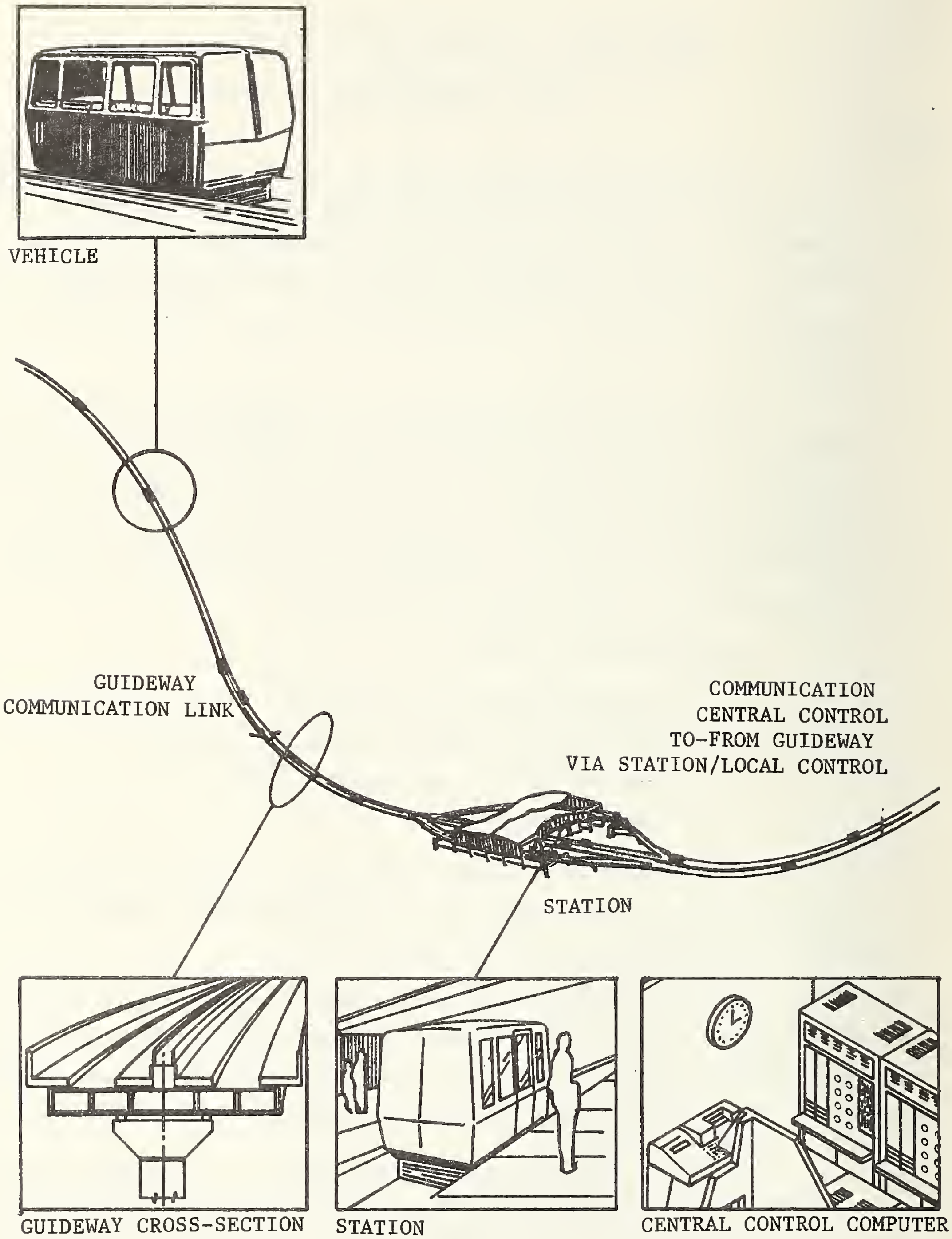

FIGURE 1-1 PRT System

1.1 INTRODUCTION

1.1.1. Significance

Successful PRT systems require short headways. With short headways more cars can pass down the guideway per unit time; and small, relatively private cars can provide:

1. Capacity,
2. Frequent service, and
3. Direct trips from origin to destination.

Headway is the time separation between equivalent points on successive vehicles. The actual distance between cars on the guideway and the headway are related, for a constant velocity, by

$$S = HV_C \qquad \text{I-1}$$

where: H is headway in seconds

S is distance separation between equivalent points on successive vehicles, in feet,

V_C is a constant velocity, in feet per second.

Distance separation as a function of headway and speed is plotted on Figure 1-2. As an example, with 4 second headway, cars traveling 30 mph (44 feet per second) would be 176 feet apart. The maximum capacity, or number of cars that can pass in an hour is given by:

$$C = \frac{3600}{H} \qquad \text{I-2}$$

where: C is maximum capacity in cars per hour.

Capacity versus headway is plotted on Figure 1-3. If cars are four seconds apart, 900 can pass a given point in an hour. If eight seconds must be provided, the maximum capacity (and hence potential revenue) is cut in half. Since the fixed cost of the guideway may be as much as 60 percent of the total cost of a system, there is a direct economic gain with the shorter headway.

1.1.2. Background

The problem of headway protection has been a recurring one in the PRT literature, e.g., references 1, 2, 3, and 4. The principal feature of the analysis herein is the use of an iterative computer program to determine

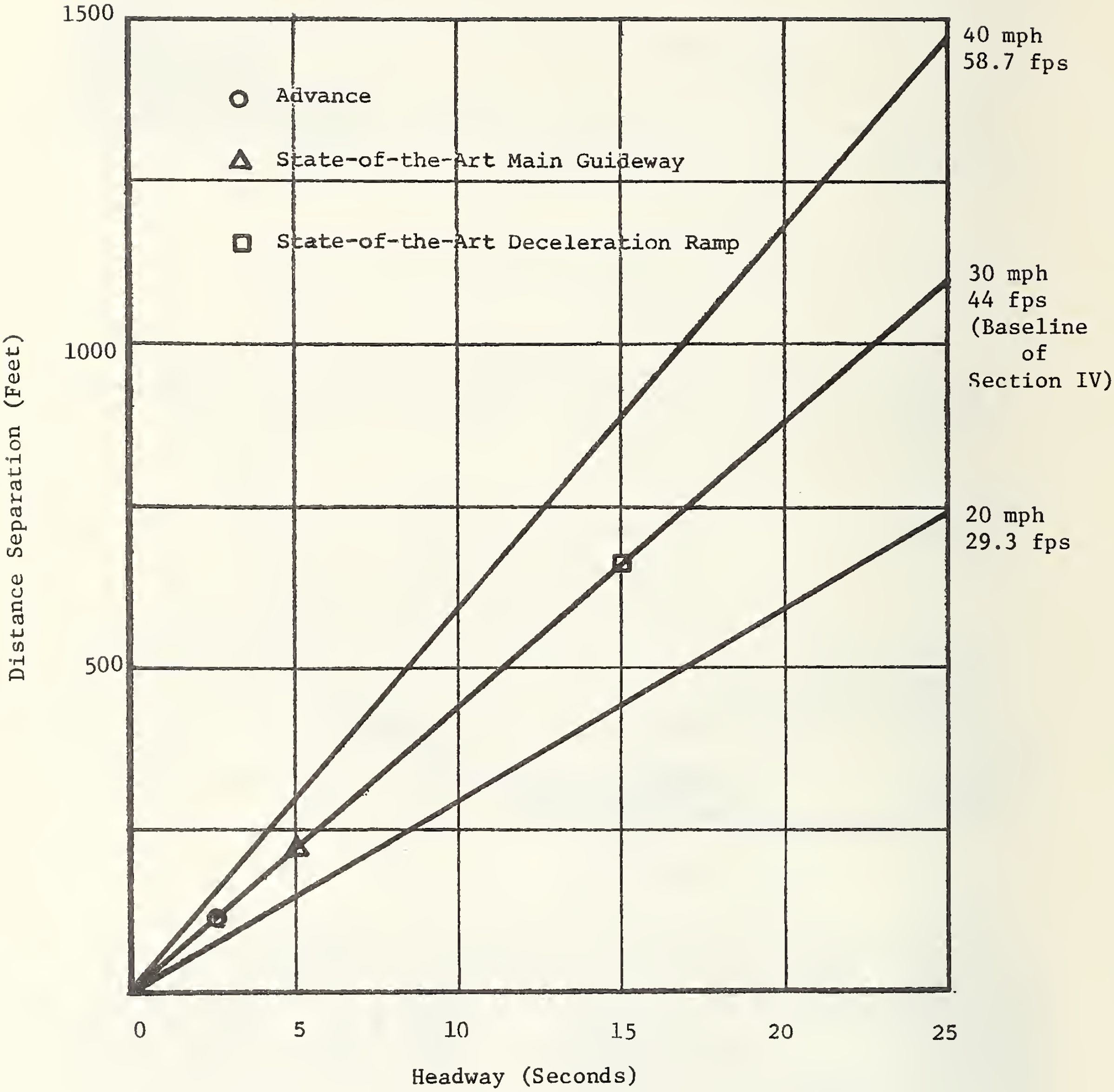

Figure 1-2. Distance Separation Between Vehicles

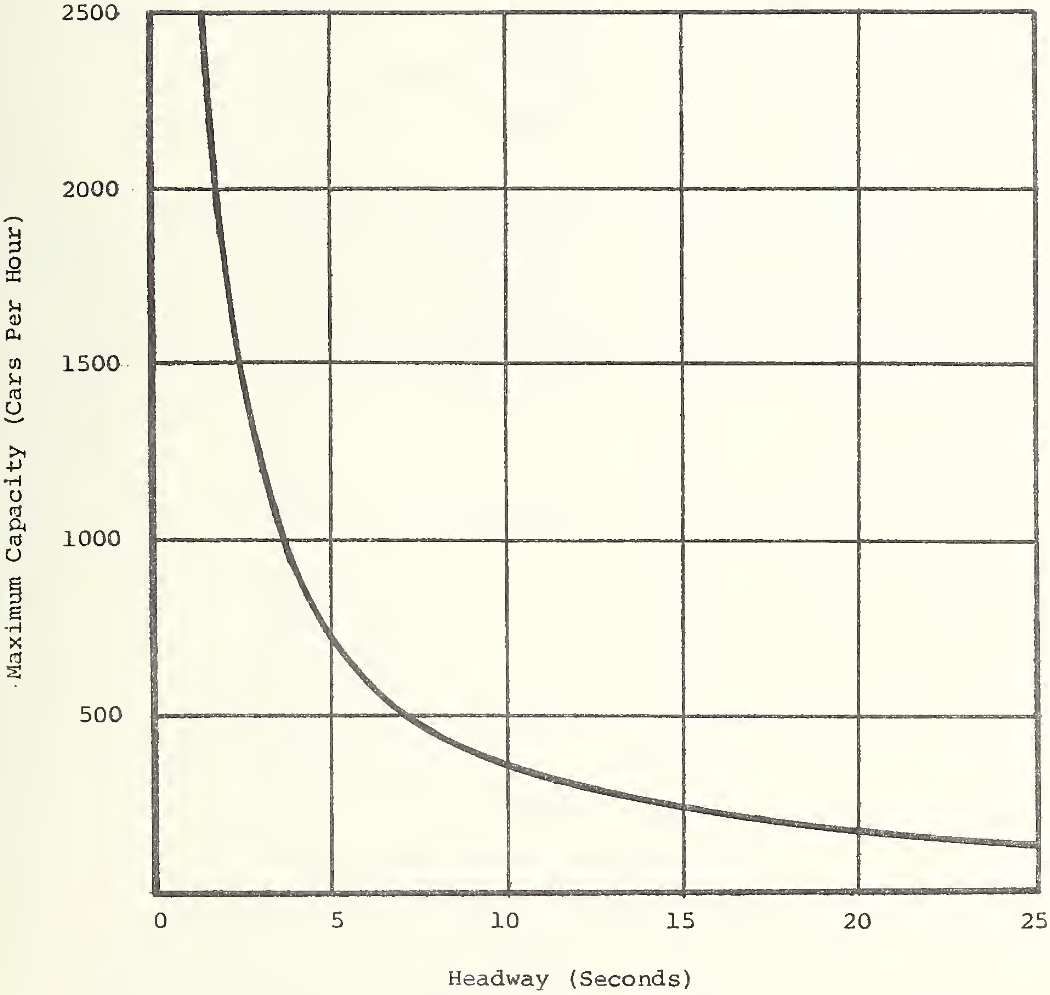

Figure 1-3. Maximum Capacity Versus Headway

the required headway for failures at each guideway sensor. Among the advantages over previous formulations are:

1. The velocity profile of the cars in normal operation is easily modified. Either acceleration and/or deceleration along arbitrary profiles is readily handled.
2. Cyclic velocity and distance deviations from the normal operating profile are directly incorporated.
3. Overspeed failures are covered as well as unexpected stops.

While the analysis and computer program are based on the Alden headway protection system, being demonstrated under the subject contract, the procedures and results are applicable to any position error or position-window headway protection scheme.

1.2 HAZARDS

The hazard considered herein is two cars colliding when one of the cars malfunctions. The case of two cars malfunctioning at the same time is not considered; nor is the case of some object, a tree for example, falling on the track.

There are two basic failures:

1. The following car overspeeds and collides with the preceding car.
2. The preceding car comes to an unexpected stop and the following car collides with it.

The function of the headway protection system is to provide warning of these hazards in time to prevent a collision under "worst" conditions. The choice of the worst conditions must consider:

1. Location on Guideway
   - A. Grade
   - B. Speed profile (acceleration, deceleration)
2. System Tolerances
   - A. Brake system
   - B. Time delays
   - C. Safety factor
3. Car Performance
   - A. Deviation from nominal profile
   - B. Failure acceleration or deceleration

The definitions and effect of these factors will be discussed in more detail in succeeding sections of the report. While rigorous analysis of worst conditions would consider the probabilistic combinations of parameter values, this level of analysis is not considered herein. The system worst case is assumed to be the direct, combined effect of the worst values of individual parameters. Thus the design worst case occurs at one point on the guideway. Under other conditions, there will be an additional safety factor.

It is generally assumed that in case of failure, there will be no collisions. Analytically, however, the effect of allowing collision can be simulated by assuming a negative safety factor in the equations developed in Section 3.

1.2.1. Overspeed Failure

A car will overspeed as a result of failure in the basic control or propulsion system. The car's velocity/time history after failure defines the magnitude of the hazard. It will depend on the characteristics of the control and propulsion system, the tire characteristics, load, whether the vehicle is on level or on a grade, and what the wind is.

A typical profile might assume that the vehicle accelerates at some constant acceleration until it reaches a maximum possible speed. It then continues at that speed until commanded to stop. With suitable parameters this profile will model most failures. It is the profile assumed in the analysis of Part 3 and the results of Part 4.

The net over-speed acceleration is given by:

$$a_{os} = 32.2 \left[\frac{T_{os} + \frac{1}{2}\rho (V_{rtw})^2 C_D S}{W} - G \right] \qquad \text{I-3}$$

where:

- a_{os} is the geometric over-speed acceleration, in feet per second^2
- T_{os} is the over-speed propulsion system thrust, in pounds (must not exceed tire traction limits)
- ρ is air density in slugs per cubic foot
- V_{rtw} is relative tail-wind velocity, in feet per second
- C_D is drag coefficient
- S is frontal area in square feet
- G is maximum grade in radians (positive up)
- W is the weight of the vehicle, in pounds

Down-grade, lightly loaded, with a tailwind, a given overspeed torque results in the greatest net acceleration and these conditions should be used to evaluate worst overspeed acceleration.

While rigorously V_{rtw} is a variable throughout the overspeed, its magnitude at the start of overspeed is conservative and can be used with confidence. From equation I-3 the most direct technique to limit overspeed acceleration is to limit the propulsion system thrust. Such limitation must be fool-proof, however, and still allow thrusts sufficient for the maximum requirements of normal control.

In a similar way, it is desirable to limit the maximum possible speed. The lower the limiting value that still allows normal correction maneuvers, the less will be the overspeed hazard. Detailed overspeed equations, incorporating both acceleration and limiting speed are developed in Section 3.

1.2.2. Unexpected Stop Failure

The more rapid an unexpected stop, the greater the headway must be. For the purpose of analysis it is reasonable to assume a constant deceleration during an unexpected stop. What that deceleration should be depends on the system and the risk the user is willing to take. The worst case is the so-called brickwall or infinite-g stop. While it is hard to devise a sequence of events that will result in a brickwall stop, it is completely conservative. A more realistic assumption is the constant friction-coefficient stop, which would correspond to locked wheels or a broken axle.

A complete definition of the stopping g force includes the effects of wind and grade, etc. as in the overspeed case. Thus, writing the geometric deceleration in terms of the failure friction coefficient:

$$a_{us} = 32.2 \left[\frac{\frac{1}{2}\rho (V_{rhw})^2 \, C_D \, S}{W} + f + G\right] \qquad \text{I-4}$$

where:

a_{us} is the geometric deceleration for unexpected stop, in feet per second

V_{rhw} is headwind, in feet per second

f is the failed car friction coefficient

0.8 is a reasonable upper bound on the friction coefficient that can be generated on a guideway. Worst case grade and wind assumptions will increase this value.

1.2.3. Baseline Values

The values of unexpected acceleration or deceleration and maximum possible speed used in the baseline of Section 4 are:

1. Overspeed Acceleration = 0.25 g
2. Maximum Possible Speed = 10% over guideway speed
3. Unexpected Deceleration = 1.0 g

Overspeed profiles of distance, speed, and acceleration are shown on Figure 1-4, for overspeed failures. Unexpected stop profiles are shown on Figure 1-5. Speed at failure is the baseline value of 30 mph, and baseline failure parameters are assumed.

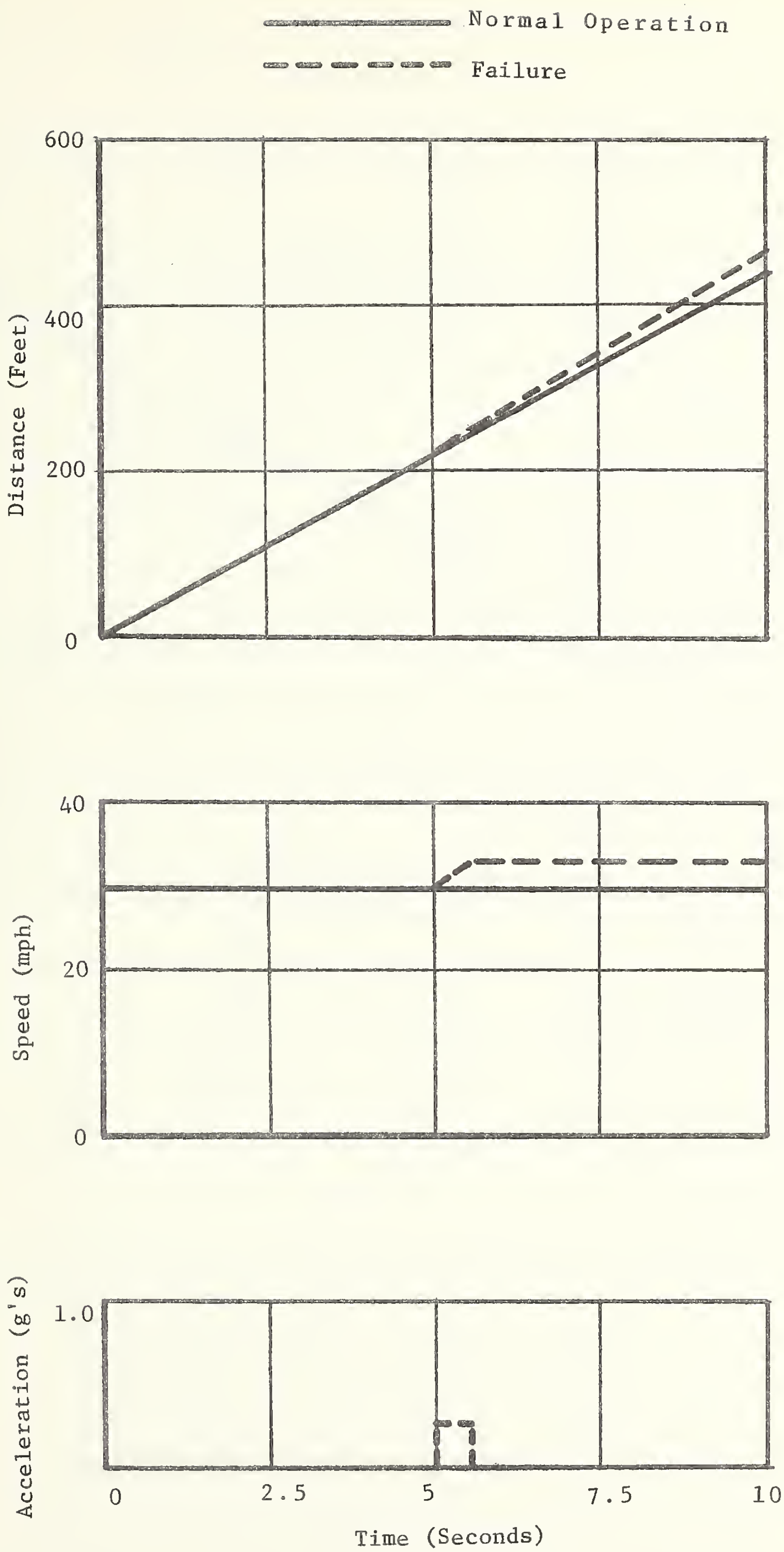

FIGURE 1-4. Overspeed Failure (Baseline Assumptions)

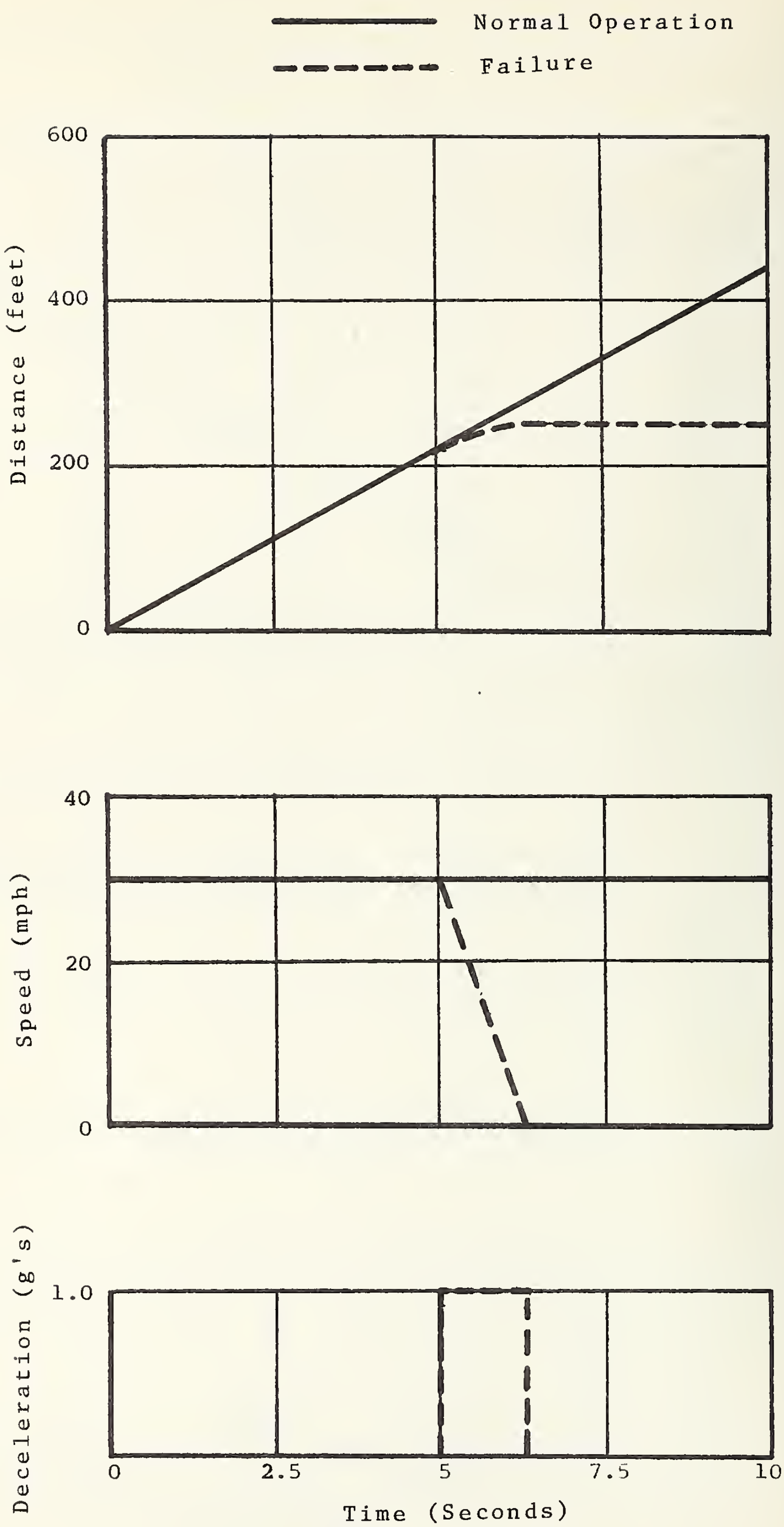

FIGURE 1-5 Unexpected Stop Failure (Baseline Assumptions)

1.3. EMERGENCY RESPONSE

If a car unexpectedly overspeeds or stops, the failure must be detected in terms of some measurable quantity, such as velocity, position, or acceleration. The failure "trigger" on the hazard warning must be set so that false warnings do not occur during normal operation.

1.3.1. Hazard Detection

This analysis assumes a position-error headway protection system. Periodic measurements are made to see if the car is within a specified position tolerance. The position tolerance is defined by the spacing of the position measuring sensors. The sensor could be either a loop antenna whose length is twice the tolerance or discrete "check-in/check-out" stations at the boundary of the tolerance area. The time between measurements is the time it takes to travel the length of the sensor spacing. In either case, if the car is within its proper area at the check time, the car is assumed to be performing properly. If it is not, an emergency is assumed to exist.

1.3.2. Potential Implementations

Action in case of an emergency depends on the detailed design of the headway protection and control system. One rationale that might be followed is presented below. It is assumed that headway protection and basic control are independent systems and that the headway protection system is broken into sectors.

If a car is found out of position in a sector, the basic control system is notified. It is given a short time to determine if it is possible to defer shutdown of the guideway. Shutdown might be deferred, if:

1. The sector were in a part of the system with "excess" headway (for example, a low-speed segment).
2. The guideway were very lightly loaded; i.e., there were no cars ahead or behind the apparently failed car.
3. If the system were being operated in a degraded mode, (for example, with twice normal separation).

If shutdown is not deferred by basic control, the headway protection system would bring the cars in the sector to an emergency stop.

The emergency stop signal might be any or all of the following:

1. a command signal
2. removal of command-signal carrier, or
3. shutting down of power

On receiving the emergency stop signal, all cars would brake to an emergency stop.

Initial shutdown would immediately affect only the local sector. Delayed commands independent of the main control system would, however, be sent to upstream headway units. These secondary stop commands would also be subject to delay and override from the main control.

Main control would, of course, attempt to minimize the extent of the emergency by delaying shut down or diverting cars to upstream stations or alternate guideway paths. In all cases, upstream sectors would continue to be shutdown until the emergency was controlled.

The important point relative to the analysis of Section 3 and the baseline of Section 4 is that <u>all</u> cars in a given area come to an emergency stop in case of a hazard warning.

1.4 EMERGENCY STOP

Given a hazard signal, cars are brought to an emergency stop. The higher the allowable emergency deceleration and jerk, the shorter the headway can be.

The analysis herein always considers geometric deceleration; i.e., deceleration relative to the guideway. However, in determining appropriate values of geometric deceleration, the perceived deceleration felt by a passenger must be considered. A car on a grade induces a component of perceived deceleration opposed to the direction of the grade. The passenger cannot differentiate between this component and geometric deceleration along a level guideway. Thus, if a car is decelerating at 0.25 g while going downhill, the passenger will feel 0.25 g plus (for small angles) the grade angle in radians; which defines the component of gravity felt by the passenger as a deceleration. If the grade angle is 0.05, the perceived g experienced by the passenger will be 0.05 g greater or 0.30 g.

The mathmatical relationship between geometric and perceived deceleration, for small grade angles, is given by:

$$a_g = a_p - (32.2)\ G \qquad \text{I-5}$$

where:

a_g is the geometric deceleration, in feet per second^2.

a_p is the deceleration perceived by the passenger, in feet per second^2.

g is the guideway grade, in radians (positive upward).

Reference values of emergency jerk and deceleration, synthesized from references 5, 6, and 7, are shown on Table 1-1. A typical 5 percent grade is assumed to determine allowable geometric deceleration. The baseline of Section 4 assumes the values for standing passengers with ready support.

TABLE 1-1

NOMINAL EMERGENCY JERK DECELERATION

(Geometric g's assume 5% Grade)

	Allowable Perceived Deceleration (g)	Allowable Geometric Deceleration (g)	Geometric Jerk (g/second)	
Standing Passengers (no ready support)	0.20	0.15	0.225	
Standing Passengers (ready support)	0.30	0.25	0.375	(Baseline)
Seated Passengers (well supported)	0.45	0.4	0.6	

An emergency stop profile with baseline geometric jerk and deceleration limits is shown schematically on 1-6, together with associated velocity and distance profiles. Tolerance values of 5 percent, as applied in the baseline of Section 4 are also shown.

It will be noted that the emergency profile is assumed to start at 0 acceleration. Thus the profile is independent of existing deceleration or acceleration at the time the brakes are applied. This assumption is consistent with a completely independent emergency brake system; one that starts its emergency stop profile with the instantaneous veolocity of the car when the emergency stop signal is received. If the car is already accelerating or decelerating, there may be a discontinuous transition to the emergency profile.

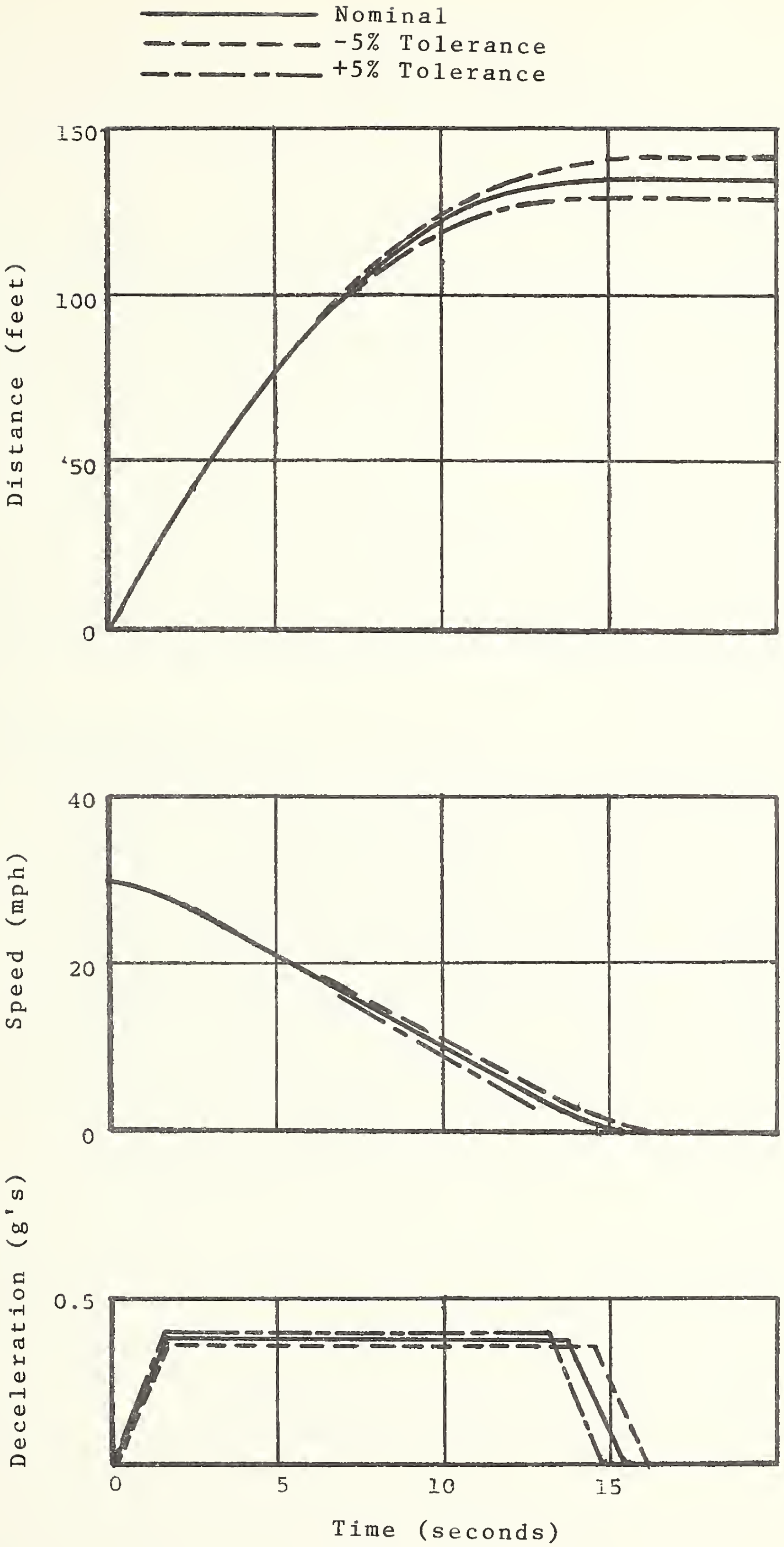

FIGURE 1-6. Emergency Deceleration Profiles

(Baseline Conditions)

2. POSITION-ERROR HEADWAY PROTECTION

In this section the Alden Headway Protection System is used as a discussion model for position-error headway protection. Its key parameters are analysed, as well as the total system parameters that affect its performance. While the discussion is based on the Alden system, it is generally applicable to any position-error system.

2.1. SYSTEM CONFIGURATION

A brief description of the Alden headway protection system is given below. More extensive descriptions are contained in Reference 8 and the final report under the subject contract. A schematic is shown in Figure 2-1

The basic geometric constraints are:

1. The car is assumed to follow a nominal trajectory from the time it leaves an origin station until it arrives at a destination station.
2. Sensor loops are laid out so that a car moving on its nominal trajectory will traverse each loop in a fixed time called the interval time. (This interval time may change, however, for different segments of the guideway, depending on the precision required.)
3. So that all cars will nominally be over loops at the same time, simplifying practical installation, the time separation (headway) between cars is an integral multiple of the interval time.
4. As a corollary, there are an integral number of loops separating cars.

Since loops are traversed in a fixed time interval, the length of individual loops is directly proportional to the guideway velocity. Thus, on a deceleration ramp the loops get shorter and shorter as one moves down the ramp. The opposite is true on acceleration ramps.

Each car contains a transmitter that activates the loop the car is over.

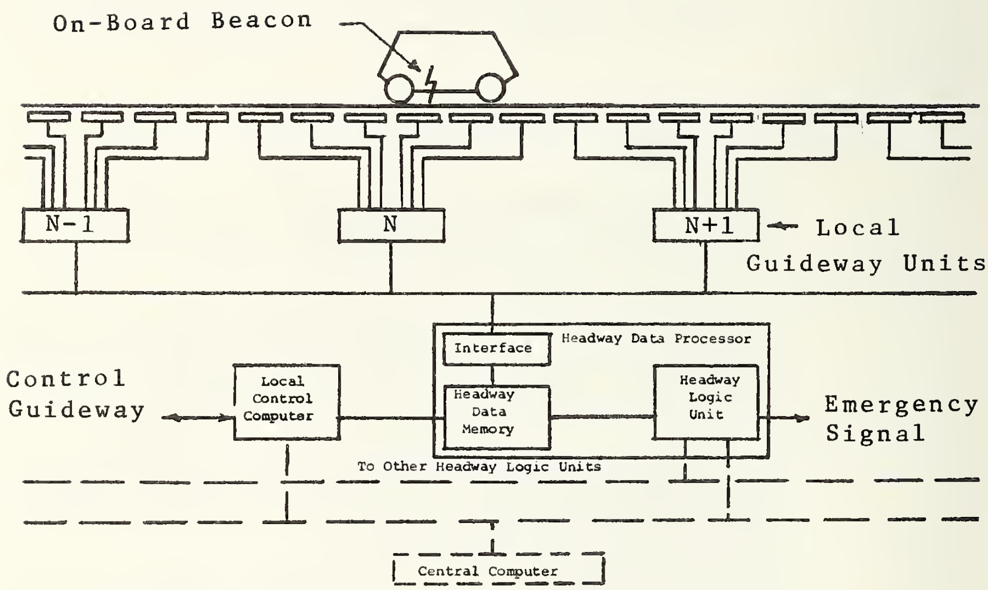

FIGURE 2-1 Headway Protection System

Phase-locked receivers in local guideway units (shown on Figure 2-1 will lock onto an active signal. A local guideway unit keeps track of what loop is "locked on" the car and stores the loop location. The number of loops in a guideway unit must not be greater than the number of loops separating cars, so that in normal operation, only one car will be in the jurisdiction of a given receiver.

When each car is scheduled over the center of a loop, the addresses of the "locked on" loops, as stored in the local guideway units, are collected by the headway data processor which monitors that section of the guideway. The collection time is controlled by a master clock whose frequency is proportional to system-wide guideway speed. The location of the occupied loops is then processed within the headway logic unit to determine if a car is out of position. Basically, a check is made to insure that each car has moved one loop since the last check. If a car either fails to appear, or is out of position, an emergency is indicated.

The system described above is thus a position error system. The allowable position error is equal to half the size of the loops.

While the above discussion assumes that loops, activated by an on-board transmitter, are used to sense the position of the car, the system may be implemented with passive sensors or with check-in -- check-out sensors placed at the boundary between the loops. In terms of the analysis that follows, these alternatives are equivalent.

2.2. OVERALL SYSTEM PARAMETERS

A number of overall system parameters are "given" to the headway protection system. They would be specified either by the user of the system or as a result of higher level trade-offs. They are listed and discussed below:

2.2.1. Guideway Speed

Guideway speed is very important to total system design. The faster the cars can operate, the shorter trip times will be and the fewer cars will be required. On the other hand, the cars will be more expensive, longer deceleration ramps will be required, and required curve radii will be greater. Thus the change in headway, and hence, capacity, with speed is crucial to some very basic system trade-offs.

A general curve showing changes in headway with speed is shown on Figure 2-2 Visualize a preceding car stopping unexpectedly, the failure being detected, and after a delay,the following car coming to an emergency stop. At very low speed the cars length and the safety margin have the dominant effect on headway. Stopping distance is short relative to the length of the car. Thus, as speed goes down, a term in car length plus safety margin divided by speed is introduced into the headway time and headways increase drastically.

This relationship places a limit on minimum speed, (approaching a station, for example) since the preceding car, must, at the very least, get out of the way of a following car. For example, if headway is six seconds and car length is 24 feet, the speed must be at least four feet per second, otherwise successive cars will touch.

At higher speed the major term in the headway is the increased time and distance it takes the following car to come to an emergency stop. This distance tends to increase with the velocity squared. However, since the separation for a fixed headway increases linearly with velocity in any case, the time headway required will only increase linearly at higher speeds.

In addition to the two effects above, which vary with speed, a simplified headway model will include a constant time term which is also shown schematically on Figure 2-2

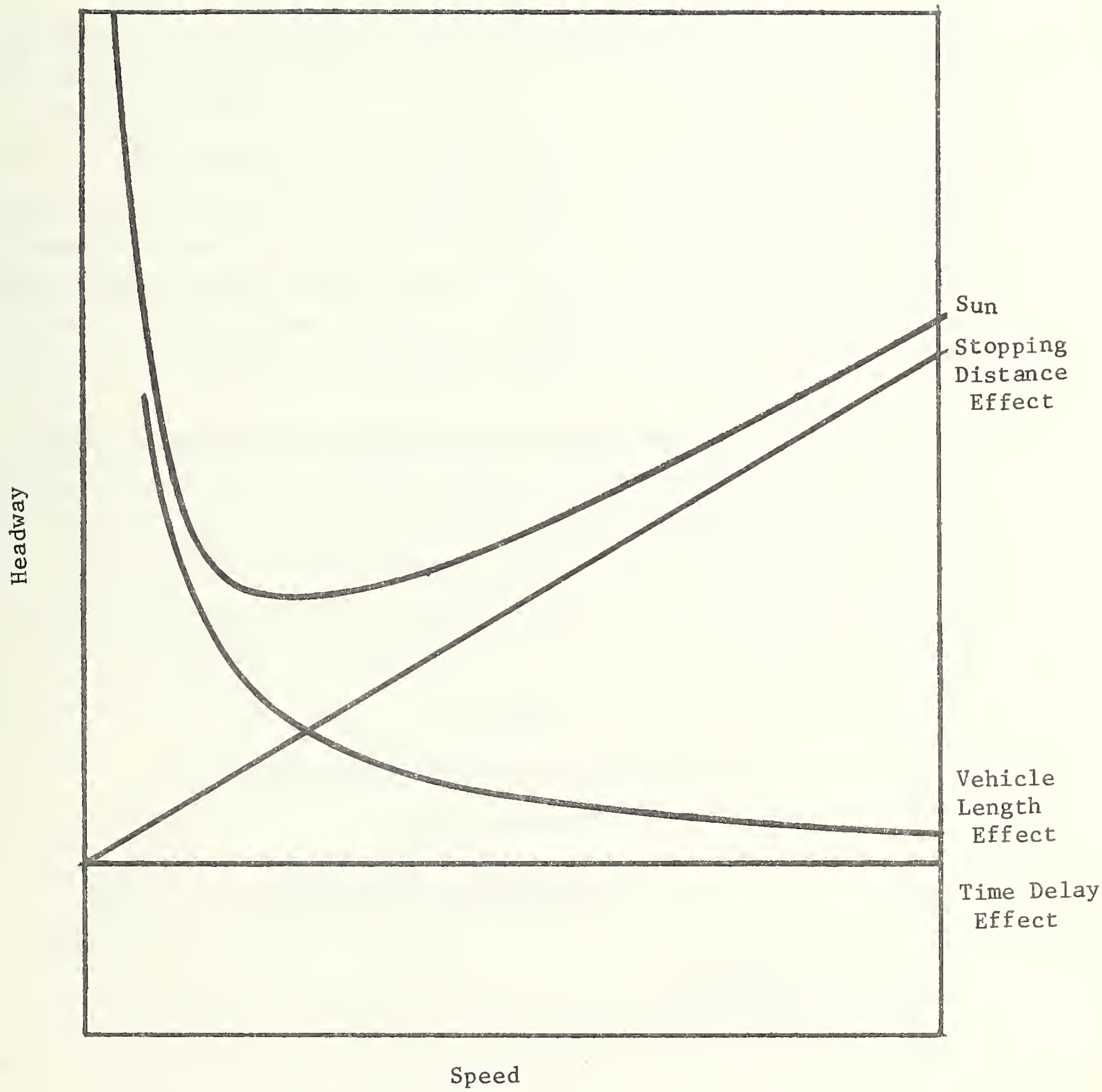

Figure 2-2 Generalized Variation of Headway With Speed

For the baseline of Section 4, it is assumed that

Main guideway speed = 30 mph

2.2.2. Speed Transitions

Guideway speed throughout this report generally refers to maximum design speed on the main guideway. Due to grades, limited space, etc., it will be desirable to operate some parts of the guideway at lower speeds. Transitions must then exist from maximum speed to lower speed and back. In addition, when entering and leaving stations, cars must decelerate from local guideway speeds to station platform speed, and then accelerate back. Since station deceleration is the critical headway condition, the baseline of Section 4 considers deceleration to:

Station Speed = 5 mph

The allowable deceleration/acceleration and jerk for guideway transitions is set by passenger comfort criteria. These limits should be well below the emergency limits discussed in Section 1.4, and will differ for standing and seated passengers. Based on data from references 5, 6, and 7, the following values are recommended. In the baseline runs of Section 4, the standing passenger (ready support) values are used.

TABLE 2-1

ALLOWABLE TRANSITION ACCELERATIONS

(Assume 0 Grade)

	Geometric & Perceived Acceleration/Deceleration (g)	Geometric Jerk (g/second)	
Standing Passengers (no ready support)	0.05	0.05	
Standing Passengers (ready support)	0.1	0.1	(Baseline)
Seated Passengers (well supported)	0.2	0.2	

The discussion of perceived and geometric deceleration contained elsewhere also applies to transition decelerations. The values of Table 2-1 assume 0 grade; hence, geometric and perceived g's are equal.

2.2.3. Vehicle Control System

The car's control system, coupled with guideway operating conditions and the car's dynamics, determines the position error of the car. The position error places a lower limit on the size of the headway loops. There would be false shutdowns if normal control errors carry the car outside of the loop boundary, i.e., if position error were greater than 1/2 the loop length.

The maximum position error must be determined for some worst case. The favored technique to determine worst case is to simulate the vehicle longitudinal dynamics. Combinations of deviations due to wind, guideway grade, acceleration profile, controls gains, car loading, etc., can then be tested to determine a maximum control deviation. In the absence of simulation, realistic upper bounds have to be estimated.

In the baseline case of Section 4

Maximum control Error = $\pm$ 5 feet at 30 mph

2.2.4. Car Length

Since the car "overhangs" its assumed point location, the length of the car must always be added to the required vehicle separation. As discussed in Section 2.2.1, this length has a great effect on headways when speeds are low. For the base line:

Car Length = 15 feet

2.2.5. Vehicle-Guideway Operating Conditions

A number of vehicle-guideway operating conditions indirectly affect the headway protection system. The effect is manifest through:

1. The control response of the vehicle, which, as discussed above, affects minimum loop size.

2. The performance of the emergency braking system, which affects the tolerance on emergency stopping distance. Open loop brake systems, i.e., systems without feed-back to control decelerations, are more strongly affected then controlled systems.

The most important of the operating factors are given below:

1. Grade - Grade will affect the vehicle lead or lag about its nominal trajectory. In general, it will lag uphill and lead downhill. Grade also affects the vehicle stopping distance tolerance; particularly with an open loop braking system since the grade induces a direct thrust proportional to the size of the grade. Although grade will also cause errors with a controlled deceleration brake system, the tolerances will be much smaller.

2. Wind - Head or tail winds, particularly gusts, will generate thrusts that will affect control precision. Wind has a direct affect, similar to grade, on an open-loop braking system; and less effect on controlled deceleration braking.

3. Surface Conditions - Design surface conditions, (wet pavement, ice, oil, etc.) will affect both control and braking systems. Basically, they will limit the friction coefficient that can be developed by the tires, and hence limit control corrections and braking force. It is important that checks be made to insure that control and braking accelerations are consistent with the minimum friction coefficient that can be generated by the tires.

In such computations, rear and front wheels should be considered separately and allowance made for the effect of weight shift due to grade, wind and accelerations.

Assuming a concrete surface and reasonably good tires, a minimum friction coefficient of 0.5 is probably appropriate - maximum might be as high as 0.9, For other conditions, see, for example, reference 9.

4. Car Weight - Variations in car weight with different passenger loads have an effect similar to wind and grade. Varying mass affects the response of the vehicle to control and brake commands, and hence affects control errors and emergency stopping distance.

The effect of the above operating conditions is manifest in the analysis, through worst case control deviations, nominal emergency brake profile and a tolerance on the normal brake profile. For example, maximum braking distance with an open-loop brake system would consider minimum friction coefficient,maximum car weight, a tailwind and a down-grade. Minimum brake distance would consider maximum friction coefficient, minimum weight car, a headwind and an up-grade. The two conditions would be specified through an average nominal deceleration and a plus or minus tolerance on the deceleration.

2.3 HEADWAY SYSTEM PARAMETERS

The parameters of the headway protection system itself are discussed below.

2.3.1. Loop Length

As noted in Section 2.2.3 , the control tolerance specifies the minimum loop length. As the length of the loops is increased from this minimum, the achievable headway will increase. This increase occurs for several reasons. The longer the loops, the longer it takes to sense that a failure has occurred, and emergency reactions are delayed. In addition, since an integral number of loops must be provided between cars, there may be larger round-off errors in matching the precise headway requirement. Lastly, the possibility of larger control errors must be considered with longer loops.

Note that since loop length is directly proportional to velocity, it is expressed in Section 3 and 4 as a fraction of the velocity. The baseline of Section 4 assumes:

Loop Length = 10 feet at 30 mph

2.3.2. Beam Width

The term beam width defines the effective width of the car's transmitting-antenna pattern. It, in effect, increases the loop length sensed by the headway protection system, and will depend on antenna configuration, transmitter power, etc. In the Section 4 baseline

Beam Width - 0.5 feet

2.3.3. Position Sample Time

This is the time required for the car beacon to trigger (or be lost by) the headway loop. It depends on the characteristics of the phase-lock receivers and the loop search pattern followed by the wayside logic units.

The distance traveled in the position sample time adds to the loop length perceived by the headway system in the direction of travel and subtracts from it in the opposite direction. In the baseline of Section 4

Position Sample Time = 0.01 seconds

In the analytical development of Section 3, the beacon beam-width and position sample time are best thought of as arbitrary parameters that may be used to define the effective loop length for any types of position error sensors. For the Alden system, practical values of the parameters have a small effect on headway system performance.

2.3.4. Delay Time

If there is a malfunction, the system receives first warning when a car fails to appear on schedule over a loop. The time a normally performing car would have been over the center of that loop defines the start of the delay time. The total delay is then the sum of the following increments.

1. Recognition Time - Time to logically judge that the car is not over its proper loop.
2. Delays for Possible Interrupt by Central Processor - Under some guideway conditions, it might be desirable to delay a guideway stop command (discussed in Section 1.3.2). This delay would require a specific override command from the central system control within the time specified for the interrupt. In the absence of such a command, the local headway protection system would shut down the guideway.
3. Time to Shut Down Power - or send alternate stop commands to the car.

Delay for Application of Brakes - given that the brake command has been received.

Typical total delays might range from 0.1 to 0.6 seconds. In the Section 4 baseline:

Reaction Delay Time = 0.2 seconds

2.3.5. Safety Margin

Safety margin is the minimum allowable distance between a failed car and a car that has come to an emergency stop. Its value will depend on judgement and the interaction between the designer and the system user. For the base-line of Section 4

Safety Factor = 5 feet

A negative safety factor implies the cars might collide, in the worst case.

2.4. EMERGENCY STOP BRAKES

The brake system must minimize emergency stopping distances within jerk and acceleration limits. Two types of brake systems will be discussed; a controlled deceleration brake, with velovity or acceleration feedback, and a "clamp" or open-loop brake. Either system would be activated by the system emergency stop command, and be subject to a stopping distance tolerance which must be accounted for in the headway calculations.

In the analysis of Section III, emergency geometric decelerations are specified by a nominal jerk and acceleration, amended by specified tolerances. It is assumed that by properly adjusting these parameters, the characteristics of any brake system can be adequately represented.

2.4.1. Clamp Brake

The clamp brake essentially applies a constant normal force to the brake pad through hydraulic or spring pressure. This force, in turn, is converted into a relatively constant deceleration torque on the wheel. While the rate of application of the force my be controlled to account for jerk limitations, it is hard to do so precisely. Tolerance on the torque force with a clamp braking system is at best, plus or minus 10 percent. Much greater overall tolerance must be assumed, however, since the brake force is not modulated as a function of grade, wind, and varying mass of the vehicle.

Under high headwind, light-load conditions with maximum-tolerance broke-torque very little braking force my be required to bring the car to a stop within the allowable g limitations. The brake force that gives this g force (at its lower torque tolerance) may result in large stopping distances when going downhill with full load and a tail wind. While the clamp brake system is not entirely satisfactory if minimum headways are desired, it is simple and readily available. Considerable inprovement is possible with the controlled braking discussed below.

2.4.2. Perceived g or Geometric g Controlled Braking

With perceived or geometric g braking, the force on the brake drums is modulated to match a specified jerk/deceleration profile. An accelerometer would be used in the perceived g case, and a wheel rotation sensor in the geometric g case. In each case a desired profile would be stored within the brake system. If the desired profile deceleration is exceeded, the brake force is reduced. If deceleration falls below the prescribed profile then the force on the brakes is increased. The braking system therefore automatically accounts for the effect of varyin grade, vehicle weight, wind, friction coefficient, etc., and the net braking tolerance is much less than with the clamp brake system. This ability to adjust to wind, load and brake tolerance reduces stopping distances considerably below the "worst" conditions with the clamp brake system.

It is convenient for the standpoint of analysis to assume a geometric g braking system. In terms of headway computations, it is identical to the perceived g system if the perceived g limit is not exceeded on the maximum system downgrade.

Three operational advantages also accrue:

1. All cars will come to an emergency stop in essentially the same distance, simplifying startup from an emergency stop.

2. The perceived g felt by passengers at all points other than maximumdown grades, will be less than the perceived g limit, since the perceived gravity component of deceleration will be less.

3. If a sensor on the wheels is used to control deceleration, some anti-skid protection is built into the system. (See Reference 10.)

These factors tend to faveo geometric g braking over preceived g braking for actual applications, as well as from the standpoint of convenient analysis. For the baseline case of Section 4, controlled g brakes are assumed, and

Brake Tolerance = 0.05

A clamp-style brake requires different assumption for the following, as opposed to the preceding car, in order to find worst case response. In that case, much larger tolerance need be considered.

3. ANALYSIS

The analysis develops the safe headway for two cars, each following the same point-to-point nominal trajectory.

3.1 NOMINAL TRAJECTORIES

It is assumed that the nominal trajectory of a car (the idealized, no deviation, trajectory) is defined by the continuous functions:

$$X = f_x(T) \qquad \text{III-1}$$

$$V = f_V(T) = \frac{dX}{dT} = f_x'(T) \qquad \text{III-2}$$

where: X is the distance along the guideway, in feet;

T is the time associated with a given car, in seconds;

V is the velocity, in feet per second.

T = 0 is associated with a fixed location on the guideway, such as the start of a deceleration ramp. X is assumed to be zero at the same point. While $f_x(T)$ and $f_V(T)$ are interrelated, since V is the derivative of X, they are, for convenience, treated as separate functions.

Two nominal trajectories are now assumed to be separated by a constant headway time, H.

Thus

$$T_1 = T_2 + H \qquad \text{III-3}$$

where: T_1 is the T associated with the lead car, in seconds

T_2 is the T associated with the following car, in seconds

The nominal separation between cars, which will vary with the speed profile, is given by:

$$S_n = f_x(T + H) - f_x(T) \qquad \text{III-4}$$

where: S_n is the nominal separation between cars, in feet.

It is now assumed that a car moves from the edge of one loop to the edge of another in a fixed time, called the interval time,

$$T_{in} = \frac{H}{N} \qquad \text{III-5}$$

where: T_{in} is the time it takes to traverse a loop, or roughly, the time from the center of one loop to the center of the next.*

N is the number of loops making up a headway time. It is an integer.

The exact length of an N^{th} loop is given by:

$$L_n = f_x\ (T_{n+1}) - f_x\ (T_n) \qquad \text{III-6}$$

where: L_n is the length of a loop, in feet; and,

$$T_{n+1} = T_n + T_{in} \qquad \text{III-7}$$

The boundaries between loops are defined by the successive application of Equation III-6. If the position of one loop is fixed, the position of all others follows. The computer program allows arbitrary positioning of the "starting" loop.

The length of a loop is approximately given by:

$$L = T_{in}\ V_{cl} \qquad \text{III-8}$$

where: V_{cl} is the velocity at the center of the loop, in feet per second.

Length is exactly given by Equation III-8 when the velocity is constant. Equation III-8 is used in Section 3.2, below, to determine velocity and position deviations at the start of emergency maneuvers.

---

* For constant velocity it is exactly the time between loop centers.

3.2 DEVIATIONS FROM NOMINAL TRAJECTORY

The vehicle control system constantly attempts to keep the vehicle on its nominal trajectory. Due, however, to tolerances in the control system and external disturbances, such as wind gusts and grades, the vehicle will not follow the nominal trajectory exactly. Rather, it will deviate by an amount dX. The maximum deviation at a loop is limited to 1/2 the loop length, i.e., any greater deviation and the car will be found out-of-position. Thus:

$$dX_{ml} = \frac{L_{el}}{2} \qquad \text{III-9}$$

where: dX_{ml} is the maximum deviation from a nominal trajectory, at a loop, in feet.

L_{el} is the electronic length of the loop, in feet.

The electronic length of the loop is the length that the headway logic "sees." As noted in Sections 2.3.3 and 2.3.4, it depends on the beacon beam-width and position sample time, thus

$$dX_{ml} = \frac{L_{el}}{2} = \frac{L}{2} - t_s V_{cl} + \frac{w_b}{2} \qquad \text{III-10}$$

where: L is the geometric length of the loop, in feet

t_s is the "position sample time," in seconds

w_b is the "beam-width," in feet

While the quantities t_s and w_b refer to particular physical parameters of the Alden headway protection system, they can be thought of as general parameters; one absolute and one proportional to velocity. Through them, the relationship between geometric loop length and effective loop length can be defined.

It is now assumed that there is a corridor defined by loop length which limits all deviation motions of the car. Substituting Equation III-8 into Equation III-10, and letting V_{cl} be any velocity, V, the following expression is obtained for the assumed maximum deviation anywhere on the trajectory.

$$dX_m = \frac{L_{eff}}{2} = V\,\frac{[T_{in} - t_s]}{2} + \frac{w_b}{2} \qquad \text{III-11}$$

where: dX_m is the maximum distance deviation from a nominal trajectory, in feet;

L_{eff} is the width of the loop corridor, in feet.

A schematic displacement corridor, defined by Equation III-11 is shown on Figure 3-1. Deviation from this corridor at a position-sample time will result in an emergency signal. The nominal position of the car is shown as a function of time by the heavy line. The slope of this line is the nominal velocity. The steeper the slope, the higher the velocity. Thus a decrease in nominal velocity is shown.

Each loop spans the time interval T_{in} as shown by the marks along the abscissa. Loop dimensions are shown on the ordinate; the loops get shorter as the velocity decreases.

Clearly, a whole family of possible deviations from the nominal trajectory may take place within the corridor of Figure 3-1. A typical curve of maximum position deviation is superimposed on the nominal position line. These deviations can be "slipped" relative to the time scale, so that maximum plus or minus deviations might occur anywhere on the trajectory.

It is quite possible that, for practical design reasons, the position corridor may be wider than the actual expected position deviation. This difference raises the philosophical question of whether to use the full loop length or some smaller corridor in determining maximum velocity deviation. The choice is ultimately up to the user of the program who must keep in mind that even if the expected velocity and position deviations are less, the headway protection system will not know it, and the worst case remains full corridor deviation.

Deviation motions are, in any case, limited by the acceleration that the vehicle will sustain in normal operations. For example, allowable acceleration might be limited by the internal logic of the vehicle control, to insure passenger comfort. At the very least there is a limit defined by the acceleration capability of the vehicle power plant.

Limits on the acceleration in the corridor imply a relationship between distance and velocity deviations. Assuming a sinusoidal variation in acceleration, the quasi-steady maximum velocity deviation is specified by:

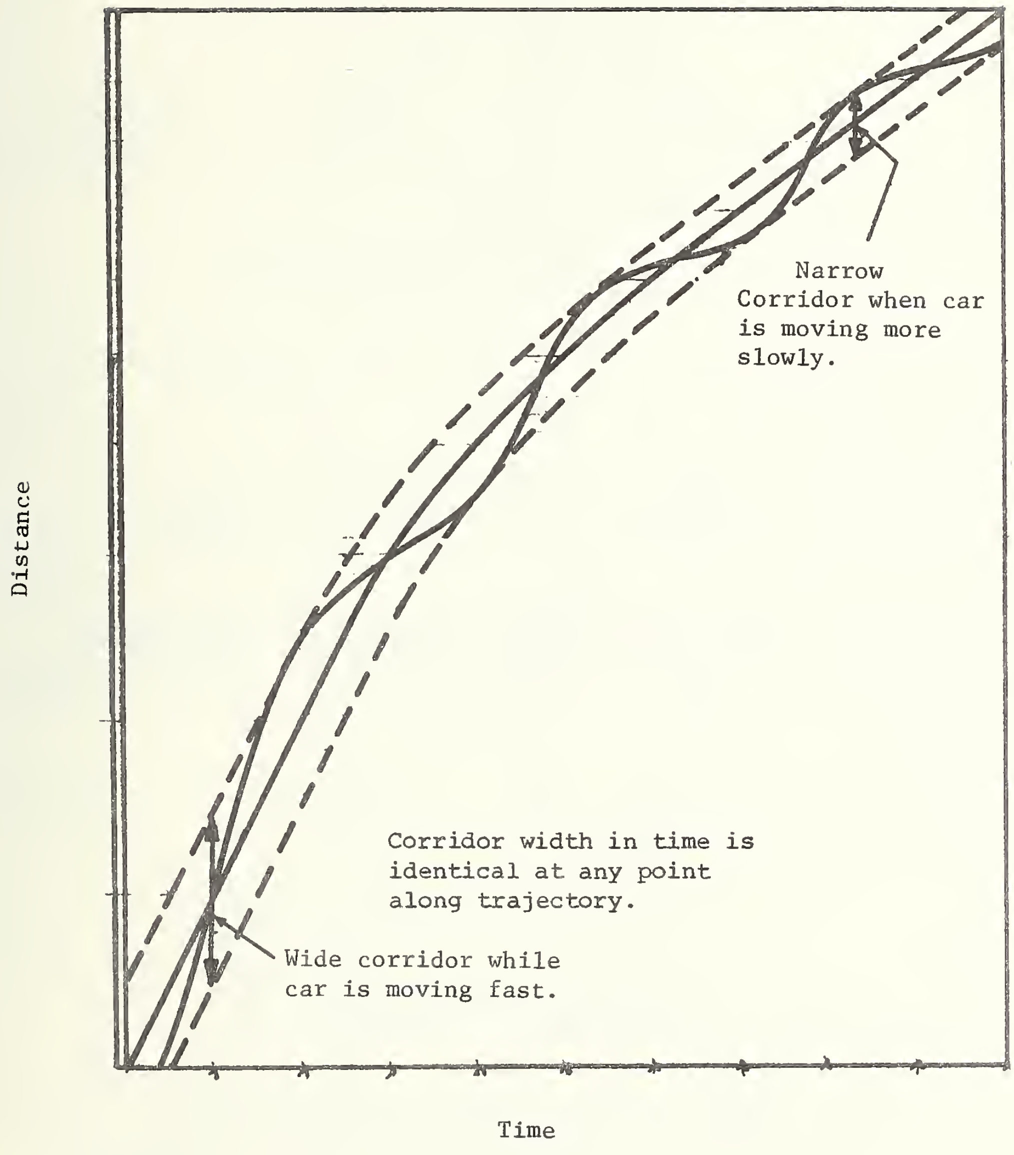

FIGURE 3-1 Schematic of Loop Corridor

$$dV_m = \sqrt{dA_{c_m}\ dX_m} \qquad \text{III-12}$$

$$= \sqrt{\frac{dA_{c_m}\ L_{eff}}{2}}$$

where: dV_m is the maximum velocity deviation, in feet per second.

dA_{c_m} is the maximum control acceleration, in feet per second^2.

The equations for deviation acceleration, velocity, and distance become:

$$dA_c = dA_{c_m} \sin wt \qquad \text{III-13}$$

$$dV = -dV_m \cos wt = -\frac{A_{c_m}}{w} \cos wt = -\sqrt{\frac{A_{c_m}\ L_{eff}}{2}} \cos wt \qquad \text{III-14}$$

$$dX = -dX_m \sin wt = -\frac{A_{c_m}}{w^2} \sin wt = -\frac{L_{eff}}{2} \sin wt \qquad \text{III-15}$$

where:

$$w = \sqrt{\frac{dA_{c_m}}{L_{eff}} \cdot 2} \qquad \text{III-16}$$

dA_c is the control acceleration, in feet per second^2.

dV is the velocity deviation, in feet per second.

The phase relationship among velocity, acceleration, and position; together with their dimension values is shown on Figure III-2 for:

L_{eff} = 10 feet

A_{c_m} = 3.22 feet/second^2 (o.1g)

The important point of Figure 3-2 is the interrelation between velocity and distance deviations. For example, it is inconsistant to have a car at the forward limit of its corridor, moving at ten percent over its nominal speed. In the next instant it would be outside of the corridor. At the forward limit of the corridor, the control system must be applying control dedeleration to bring the car back to nominal.

- 10 Foot Loops
- 0.1 g Correction Acceleration

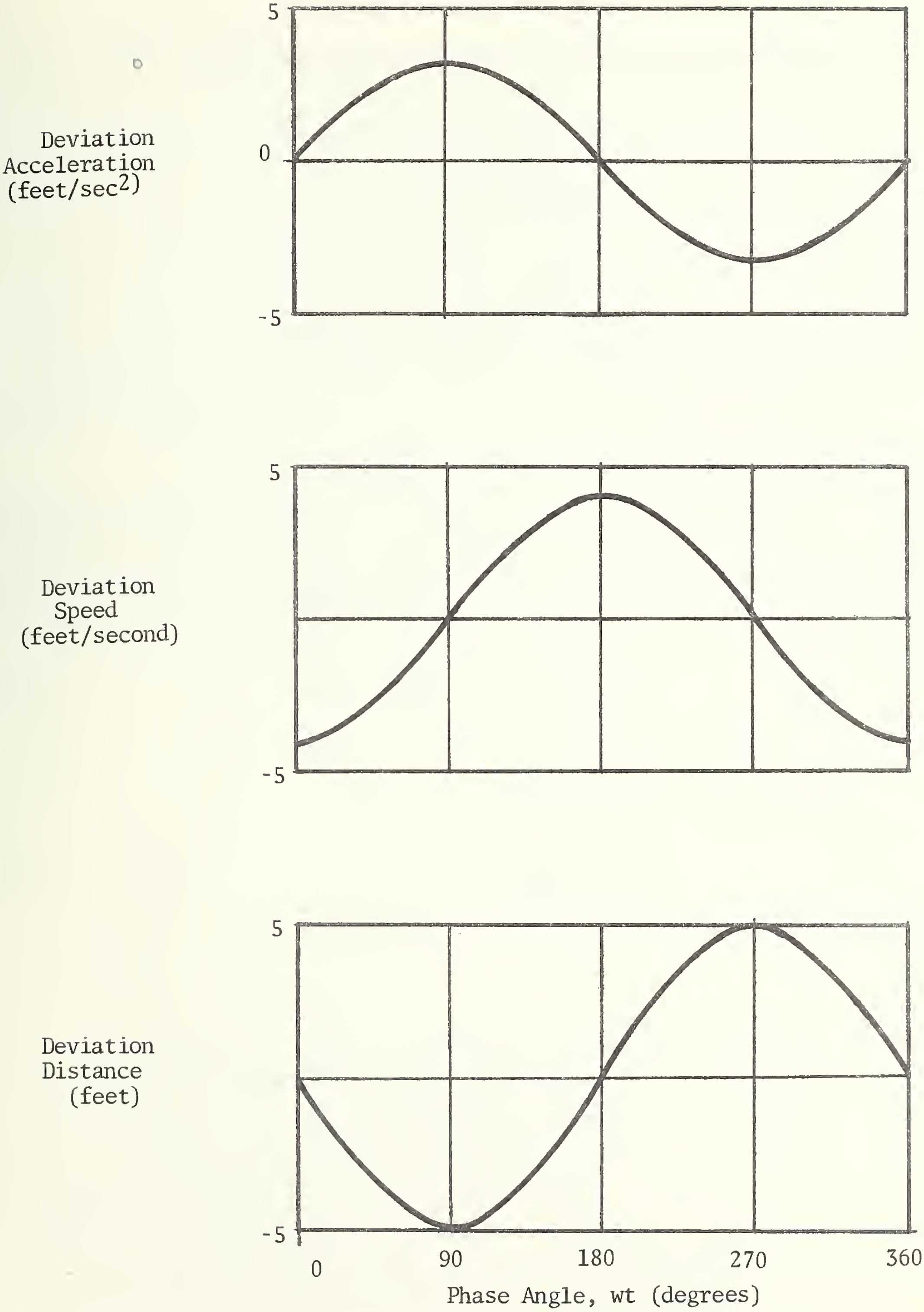

FIGURE 3-2. Typical Deviation Acceleration, Speed and Distance

Given Equations III-13 through III-16, the maximum jerk is defined by:

$$dJ_{c_m} = \sqrt{\frac{dA_{c_m} \cdot 2}{L_{eff}}}\; dA_{c_m} \qquad \text{III-17}$$

where: dJ_{c_m} is the maximum control jerk in feet per second^3.

Using Equation III-17, the ratio of jerk to acceleration ranges from 0.5 to 2 for values of maximum acceleration from 3 to 4 feet per second^2, and loop lengths from 2 to 15 feet. Thus a sinusoidal representation is consistent with typical limits on the values of control jerk.

There are, of course, alternatives to assuming a sinusoidal variation in correction acceleration. A reasonable alternative would be trapezoidal acceleration pulses, limited by both acceleration and jerk. The expression for deviation velocity in the case of trapezoidal control and accelerations becomes:

$$dV_m = -\sqrt{Z + Z^2 + 2dA_{c_m} \cdot dX_m} \qquad \text{III-18}$$

$$\text{for} \quad dV_m > \frac{(dA_{c_m})^2}{dJ_{c_m}}$$

where:

$$Z = \frac{(dA_{c_m})^2}{2 \cdot dJ_{c_m}} \qquad \text{III-19}$$

Thus, for the case of infinite jerk, i.e., a square acceleration pulse, the deviational velocity is $\sqrt{2}$, or 1.41 times the velocity given by Equation III-12 for the sinusoidal correction acceleration. For finite jerk, the agreement is closer. If jerk is 3.22 feet per second^3, acceleration 3.22 feet per second^2, and dX_m is 5 feet; dV_m is 4.28 feet per second with a trapezoidal pulse. This agrees closely with Equation III-12, which, for the sinusoidal variation gives a value of 4.01 feet per second.

The computer program, and results of Section 4 assume a sinusoidal acceleration variation, and hence the deviation velocity relationship of Equation III-12. While the trapezoidal formulation may be desirable under some conditions, computations with it are considerably more complex, due to the discontinuities in the acceleration pulse. Since either remains only an approximation to some real variation, the added complexity does not appear warranted. It is particularly convenient to use sinusoidal variations when investigating "worst case" initial conditions, using Equations III-13 through III-15 to define consistent deviations in speed and distance.

3.3 BASIC RELATIONSHIPS

In this section, the geometric relationships that define unexpected stop and over-speed emergencies are developed. These relationships are used in the computer program described in Section 3.4.

3.3.1. Unexpected Stop of Preceding Car

The unexpected stop emergency is shown schematically on the distance time grid of Figure 3-3. The lead car is constrained to fail so that it "just catches" the loop it is scheduled over at the time T_{jc}. At the next loop check, it will be out of position. To just catch the loop at the time T_{jc}, implies some initial earlier failure at time T_f, and distance X_f. The failure at T_f must take place at the worst initial conditions, $V + dV$ and $X + dX$, i.e., at the consistent combination of deviations that results in the smallest value of X when the car comes to a final stop. In order to accommodate consistent dX and dV and allow for arbitrary $f_x(T)$ and $f_v(T)$, an iterative, trial and error solution is used.

The failure point solution converges to satisfy the following constraints:

Failure Distance:

$$X_f = f_x\,(T_f) - dX_m \sin wt \qquad \text{III-20}$$

where: X_f is the distance coordinate at the time the preceding car starts to fail, in feet.

T_f is the time the preceding car starts to fail, in seconds.

Failure Speed:

$$V_f = f_v\,(T_f) - dV_m \cos wt \qquad \text{III-21}$$

where: V_f is the speed of the preceding car when it starts to fail, in feet per second.

Deviations:

$$dX_m = \frac{L_{eff}}{2} \qquad \text{III-22}$$

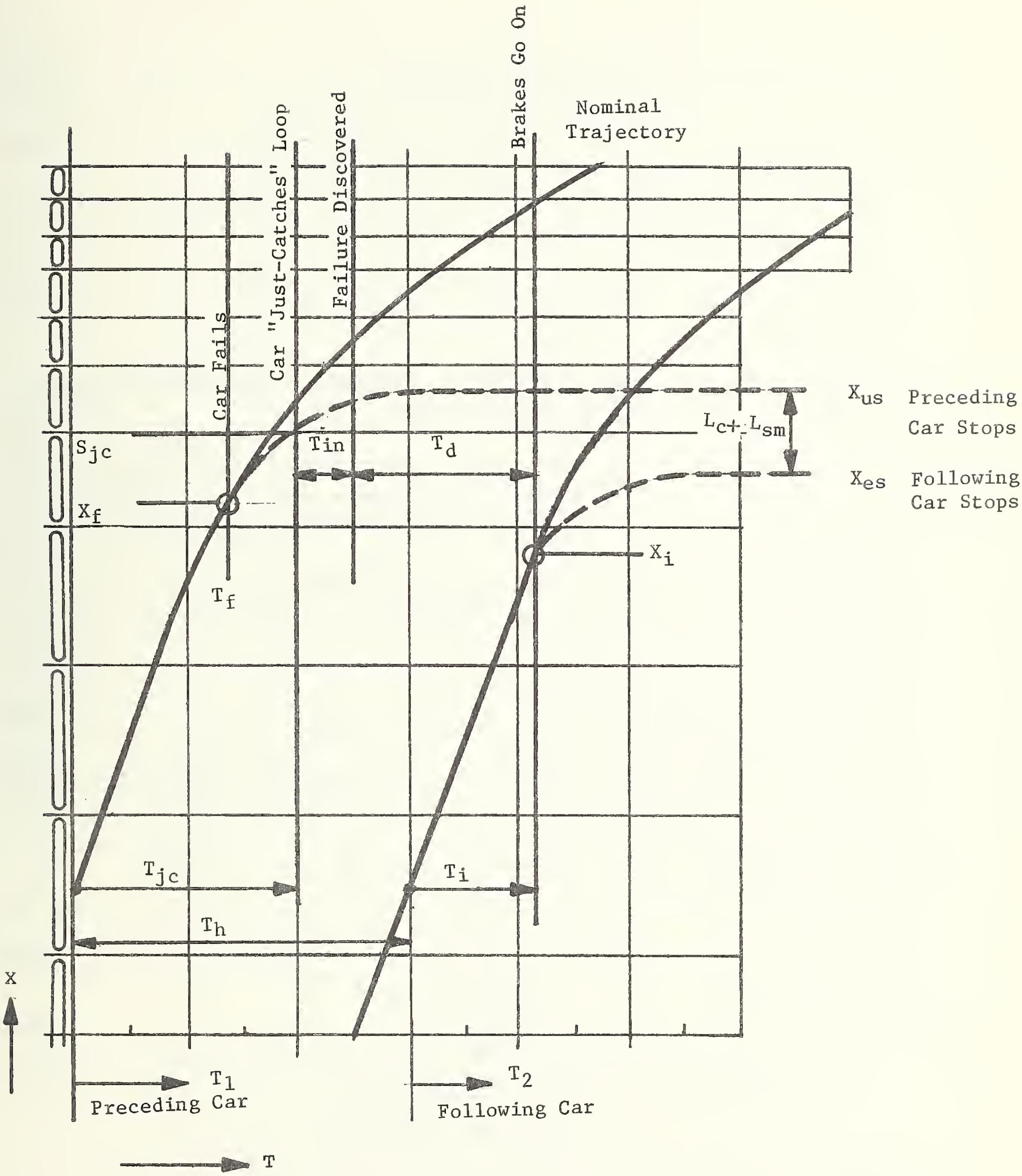

FIGURE 3-3. Unexpected Stop of Preceding Car

$$dV_m = \sqrt{dA_{c_m} \cdot dX_m} \qquad \text{III-23}$$

$$L_{eff} = V_f \left(\frac{T_{in}}{2} \cdot -t_s\right) + \frac{W_b}{2} \qquad \text{III-24}$$

"Just Catch" Point:

$$X_{jc} = f_x \left(T_{jc} - \frac{T_{in}}{2}\right) + V_{jc}\, t_s + \frac{W_b}{2} \qquad \text{III-25}$$

where:

X_{jc} is the distance coordinate when the car just catches its loop, in feet.

T_{jc} is the time the cars presence in a loop is checked, in seconds. It is the average of the times the car is due over the ends of the loop.

V_{jc} is the speed at the "just catch" point, in feet per second.

Failure Profile:

$$V_{jc} = V_f - (T_{jc} - T_f) \cdot a_{us} \qquad \text{III-26}$$

$$X_{jc} = X_f + (T_{jc} - T_f) \cdot V_f - \frac{1}{2}\, a_{us}\, (T_{jc} - T_f)^2 \qquad \text{III-27}$$

$$\text{for } (T_{jc} - T_f) > \frac{V_f}{a_{us}}$$

$$V_{jc} = 0 \qquad \text{III-28}$$

$$X_{jc} = X_f + \frac{V_f^2}{2\, a_{us}} \qquad \text{III-29}$$

$$\text{for } (T_{jc} - T_f) < \frac{V_f}{a_{us}}$$

where: a_{us} is the expected stop deceleration, in feet/second^2.

Assumed for a given computation are the system parameters: a_{us}, t_s, w_b, A_{cm}; the normal trajectory; $f_x(T)$ and $f_v(T)$; and the interval time, T_{in}.

The "just catch" time, T_{jc} is varied in intervals of T_{in} as successive loops are considered. Solved for in the iteration are the quantities:

T_f, X_f, V_f, dX_m, dV_m, V_{jc}, X_{jc} and L_{eff}

The method used to solve the equations is to assume a value of T_f, compute X_f and V_f; and from them determine trial values of X_{jc} and V_{jc}. Latest values of V_{jc} and V_f are then used to recompute the geometric X_{jc} and the maximum deviation, dX_m and dV_m. The process is then repeated, using a linearly adjusted value of T_f, until the geometric X_{jc} matches the failure profile of the car at T_{jc}, within 0.1 foot.

The quantity wt of Equation III-20 and III-21 defines the phase angle of the velocity and position deviations. It would either be set for a given computation, or varied to investigate "worst case" initial conditions. In the baseline runs of Section 4, it is equal to 270 degrees, i.e., the car is at the forward end of its corridor, moving at nominal speed. This position maximizes the time prior to discovery of a failure and is the "worst case" for the range of parameters considered in the baseline.

Given definition of the "just catch" profile, through the parameters X_f and V_f, it is possible to determine the stopping point for the unexpected stop, X_{us}. It is given by:

$$X_{us} = X_f + \frac{1}{2} \frac{(V_f)^2}{a_{us}} \qquad \text{III-30}$$

where: X_{us} is the stopping point of the failed car, in feet.

The following car must now come to a stop behind the failed car. The required distance is equal to the length of the car, plus the safety margin, thus

$$X_{es} = X_{us} - L_c - L_{sm} \qquad \text{III-31}$$

where:

X_{es} is the required stopping point for the following car, in feet.

L_c is the length of the car, in feet

L_{sm} is the safety margin, in feet.

Both L_c and L_{sm} are inputs to the analysis.

An iteration procedure, similar to that followed for the failed car is now performed to determine the initial conditions for the following car that will bring it to an emergency stop at the point X_{es}. The following equations are used.

Limiting emergency deceleration:

$$a_l = a_e \ (1 - e_b) \qquad \text{III-32}$$

where:

a_l is the lower limit emergency deceleration, in feet per second^2.

a_e is the nominal emergency deceleration, in feet per second^2.

e_b is the fractional brake tolerance

Limiting emergency jerk:

$$j_l = j_e \ (1 - e_b) \qquad \text{III-33}$$

where:

j_l is the lower limit emergency jerk, in feet per second^3.

j_e is the nominal emergency jerk, in feet per second^3.

(It will be noted that the emergency deceleration and jerk are set at their lower limits, i.e., the values that will give the longest stopping distance.)

Initial Position:

$$X_i = f_x\ (T_i) - dX_m \sin wt \qquad \text{III-34}$$

where: X_i is the distance coordinate when the following car starts to come to an emergency stop, in feet.

T_i is the time the following car starts to come to an emergency stop, in seconds.

Initial Speed:

$$V_i = f_v\ (T_i) - dV_m \cos wt \qquad \text{III-35}$$

where: V_l is the following car's speed when it starts to come to an emergency stop, in seconds.

Deviations:

$$dX_m = \frac{L_{eff}}{2} \qquad \text{III-36}$$

$$dV_m = \sqrt{dA_{cm} \cdot dX_m} \qquad \text{III-37}$$

$$\frac{L_{eff}}{2} = V_i \left(\frac{T_{in}}{2} - t_s\right) + \frac{w_b}{2} \qquad \text{III-38}$$

Stopping Point:

(Both jerk and acceleration limited -- trapezoidal deceleration profile.)

$$X_{es} = X_i + \frac{V_i}{2}\left[\frac{V_i}{a_1} + \frac{a_1}{j_1}\right] \qquad \text{III-39}$$

$$\text{for } V_i > \frac{{a_1}^2}{j_1}$$

(Jerk limited -- triangular deceleration profile.)

$$X_{es} = V_i\sqrt{\frac{V_i}{j_1}} \qquad \text{III-40}$$

$$\text{for } V_i < \frac{{a_1}^2}{j_1}$$

The given quantities in Equations III-32 through III-40 are:

$$a_e,\ j_e,\ e_b,\ t_s,\ w_b,\ A_{c_m}$$

the nominal trajectories, f_x (T) and f_v (T); and the interval time, T_{in}.

The quantity wt should be chosen to minimize T_i, which will maximize the headway. For the parameters of the baseline case, this "worst case" wt is 195 degrees. The car is near the middle of the loop corridor, at close to maximum speed.

The variables solved for in the iteration are:

$$a_1,\ j_e,\ T_i,\ X_i,\ V_i,\ dX_m,\ dV_m,\ \text{and}\ L_{eff}$$

The solution proceeds in much the same way as for the failed car. A starting value of T_i is chosen. X_i and V_i are determined and a trial stopping point X_{es} is computed. New values of dV_m and dX_m are determined, and together with an adjusted value of T_i, they are used to compute a new trial stopping point. The process then cycles until the stopping point is matched within 0.1 foot.

The output of the iteration is the variable T_i. It permits the required headway to be determined. The fact that the preceding car has failed is discovered one interval time, T_{in}, after the car "just catches" a loop. After the delay time T_d has passed, the brakes on the following car go on, at the time T_i for the following car.

Referring now to Figure 3-3, the following equality holds:

$$T_h + T_i = T_{jc} + T_{in} + T_d \qquad \text{III-41}$$

where: T_h is the "non-integer" headway, in seconds.

Solving Equation III-41 for T_h

$$T_h = T_{jc} + T_{in} + T_d - T_i \qquad \text{III-42}$$

The "non-integer" headway is the raw headway required; before applying the condition that there be an integral number of loops in a headway time. An integral number of loops implies that the headway must be a multiple of the interval time, T_{in}. Thus the true headway, H, is given by:

$$H = T_{in} \left[\begin{array}{l} \text{The integer} \\ \text{just greater} \\ \text{than} \end{array} \quad \frac{(T_{jc} + T_{in} + T_d - T_i)}{T_{in}} \right] \qquad \text{III-43}$$

3.3.2. Overspeed of Following Car

The case of overspeed by a following car follows a procedure similar to that just described for an unexpected stop. The trajectory diagram of Figure 3-4 illustrates the overspeed case. The first step is to determine the failure point that will cause the overspeeding car to "just catch" the front edge of a loop. This condition defines the worst failure for each loop. The failure will be discovered one interval time, T_{in}, later, when the car is due over the next loop. After the delay time, T_d, the car is braked to an emergency stop. The point at which the car comes to a stop is then determined. The preceding car, which has also been commanded to stop must stop a safe distance ahead of the following car.

The car is assumed to fail at a constant overspeed acceleration, until it reaches a specified maximum speed. It then continues at that maximum speed until commanded to brake (Section 1-2-1).

The iteration to determine the failure profile that just catches a loop must satisfy the following conditions:

Overspeed Point:

$$X_{fo} = f_x (T_{fo}) - dX_m \sin wt \qquad \text{III-44}$$

where: X_{fo} is the distance coordinate at the time the following car starts to overspeed, in feet.

T_{fo} is the time the following car starts to overspeed, in seconds.

Speed at Start of Overspeed:

$$V_{fo} = f_w (T_{fo}) - dV_m \cos wt \qquad \text{III-45}$$

where: V_{fo} is the speed of the following car when it starts to fail, in feet per second.

Deviations: $$dX_m = \frac{L_{eff}}{2} \qquad \text{III-46}$$

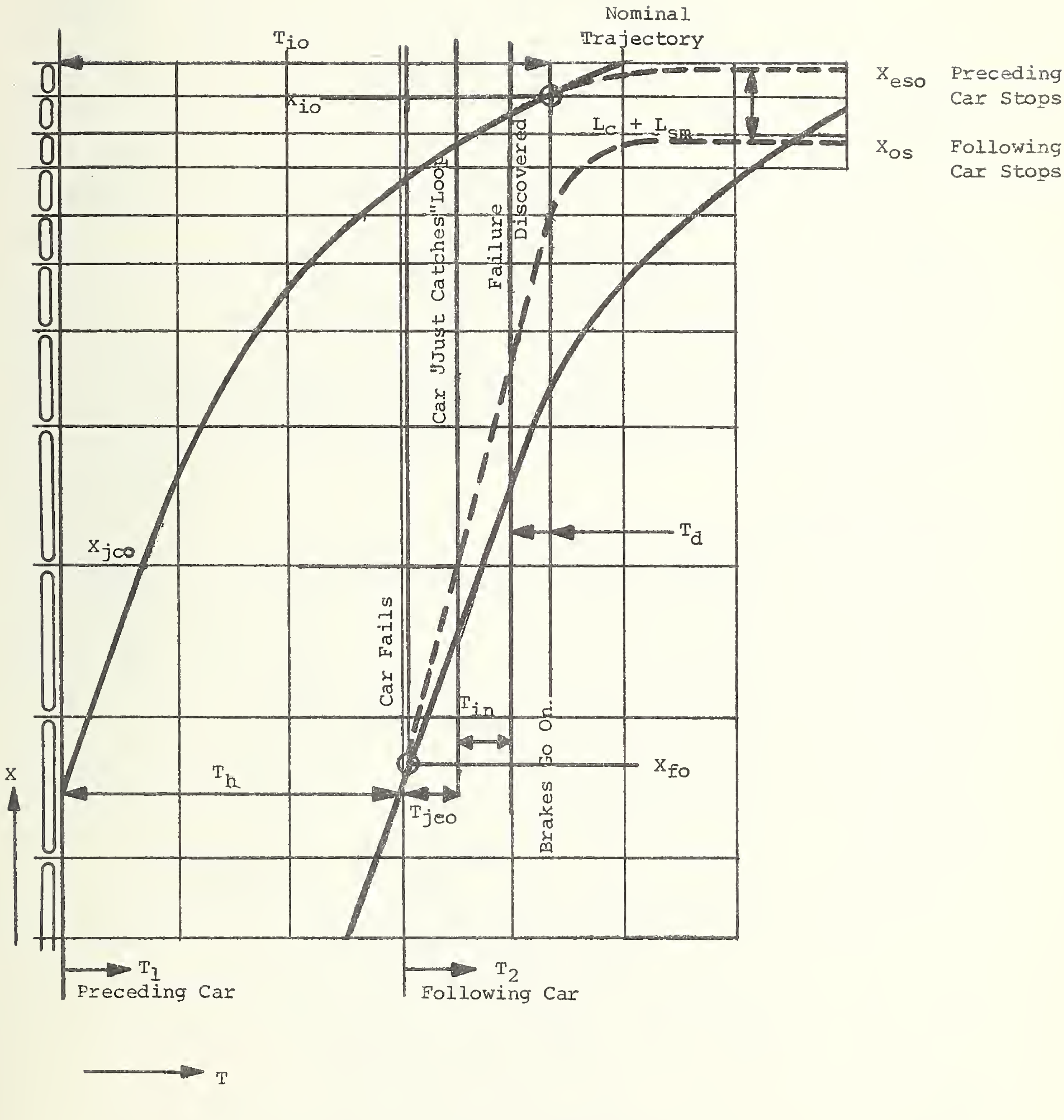

Figure 3-4. Overspeed of Following Car

$$dV_m = \sqrt{dA_{c_m} \cdot dX_m} \qquad \text{III-47}$$

$$\frac{L_{eff}}{2} = V_{fo} \cdot \left(\frac{T_{in}}{2} - t_s\right) + \frac{w_b}{2} \qquad \text{III-48}$$

"Just Catch" Point:

$$X_{jco} = f_x \left(T_{jco} + \frac{T_{in}}{2}\right) + V_{jco}\, t_s + \frac{w_b}{2} \qquad \text{III-49}$$

where:

X_{jco} is the distance coordinate when the car just catches its loop, in feet.

V_{jco} is the speed at the just catch point, in feet per second.

T_{jco} is the time the car's presence in the loop as checked, in seconds. It is the average of the times the car is due over the ends of the loop.

Failure Profile:

$$V_{jco} = V_{fo} + (T_{jco} - T_{fo}) \cdot a_{os} \qquad \text{III-50}$$

$$X_{jco} = X_{fo} + V_{fo} \cdot (T_{jco} - T_{fo}) + (T_{jco} - T_{fo})^2 \cdot \frac{a_{os}}{2} \qquad \text{III-51}$$

$$\text{for } (T_{jco} - T_{fo}) < \frac{V_{max} - V_{fo}}{a_{os}}$$

$$V_{jco} = V_{max} \qquad \text{III-52}$$

$$X_{jco} = X_{fo} + V_{fo} \cdot \frac{(V_{max} - V_{fo})}{a_{os}} + \frac{(V_{max} - V_{fo})^2}{2 \cdot a_{os}} +$$

$$V_{max} \cdot \left(T_{jco} - \frac{V_{max} - V_{fo}}{a_{os}} - T_{fo}\right) \qquad \text{III-53}$$

$$\text{for } (T_{jco} - T_{fo}) \geq \frac{V_{max} - V_{fo}}{a_{os}}$$

where:

a_{os} is the overspeed acceleration, in feet per second^2.

V_{max} is the maximum possible speed, in feet per second.

The system parameters:

$$a_{os},\ V_{max},\ t_s,\ w_b,\ \text{and}\ A_{c_m}$$

are inputs to a given computation; along with the nominal car trajectories, $f_x(T)$ and $f_v(T)$, and the interval time T_{in}. The iteration solves for

$$T_{fo},\ X_{fo},\ V_{fo},\ dX_m,\ dV_m,\ V_{jco},\ X_{jco},\ L_{eff}$$

As in the two previous cases, the quantity wt is used as a parameter to define the worst case initial conditions for the failure. The worst case for the baseline runs of Section 4 is wt = 90 degrees, i.e., the car is at the back edge of its corridor moving at nominal velocity. From this position it will have gathered maximum speed prior to being discovered out of position.

The iterative solution follows the pattern described for the unexpected stop case. Trial values of T_{fo} are sequentially adjusted until the computed position of the car matches the just catch point within 0.1 foot.

Given the "just catch" trajectory, the position and velocity of the car when emergency brakes are applied is determined. The brakes go on T_{in} plus T_d seconds after the just catch time. Thus, a total time, T_{ac} elapses from the time the car starts to overspeed until it starts to decelerate.

$$T_{ac} = T_{jc} - T_{fo} + T_{in} + T_d \qquad \text{III-54}$$

where: T_{ac} is the total duration of the overspeed, in seconds.

The terminal point of the overspeed is thus:

$$X_b = X_{fo} + V_{fo} \cdot (T_{ac}) + \frac{a_{os}}{2} \cdot (T_{ac})^2 \qquad \text{III-55}$$

$$V_b = V_{fo} + a_{os} \cdot (T_{ac}) \qquad \text{III-56}$$

$$\text{for } T_{ac} < \frac{V_{max} - V_{fo}}{a_{os}}$$

$$X_b = X_{fo} + V_{fo} \cdot \frac{(V_{max} - V_{fo})}{a_{os}} + \frac{(V_{max} - V_{fo})^2}{2\, a_{os}} +$$

$$V_{max} \cdot \left(T_{ac} - \frac{V_{max} - V_{fo}}{a_{os}}\right) \qquad \text{III-57}$$

$$V_b = V_{max} \qquad \text{III-58}$$

$$\text{for } T_{ac} > \frac{V_{max} - V_{fo}}{a_{os}}$$

where: X_b is the position of the overspeed car at the start of emergency braking, in feet.

V_b is the speed at the start of emergency braking, in feet per second.

The stopping point for the overspeeding car is now obtained by adding the stopping distance to the distance X_b, i.e.,

$$X_{os} = X_b + \frac{V_b}{2}\left[\frac{V_b}{a_1} + \frac{a_1}{j_1}\right] \qquad \text{III-59}$$

$$\text{for } V_b > \frac{a_1^2}{j_1}$$

$$X_{os} = X_b + V_b\sqrt{\frac{V_b}{j_1}} \qquad \text{III-60}$$

$$\text{for } V_b < \frac{a_1^2}{j_1}$$

where: X_{os} is the stopping point for the overspeeding car, in feet.

Note that the emergency deceleration and jerk are the lower limit values, which result in the largest stopping distance.

The preceding car must come to a stop a distance ahead of the stopped overspeeding car. It is assumed that both cars receive the same emergency stop signal at the same time, i.e., no attempt is made to differentiate between the failed and unfailed cars, or between an overspeed emergency and an unexpected stop emergency. While such differentiation is conceptually possible, implementation raises safety conflicts.

The closest point at which the preceding car can stop is:

$$X_{eso} = X_{os} + L_c + L_{sm} \qquad \text{III-61}$$

where: X_{eso} is the required stopping point for the preceding car, in feet.

Given the required stopping point, the time the preceding car starts to brake, T_{io}, is determined. The procedure followed is almost exactly the same as that described by Equations III-32 through III-40, for the unexpected stop case. The exceptions are:

1. The emergency deceleration and jerk are the upper tolerance values; giving the minimum stopping distance, i.e.,

$$a_u = a_e \ (1 + e_b) \qquad \text{III-62}$$

where: a_u is the upper limit emergency deceleration, in feet per second^2,

and $$j_u = j_e \ (1 + e_b) \qquad \text{III-63}$$

where: j_u is the upper limit emergency jerk, in feet per second^3.

2. The worst case phase angle, wt, must be chosen to minimize stopping distance. This occurs, for the baseline of Section 4, when wt is equal to 20 degrees, i.e., when the car is near the middle of its corridor and moving close to its lower tolerance limit speed.

Having determined the earliest time, T_{io}, the preceding car can start to brake, it is possible to determine the headway required to protect against an overspeed failure. Referring to Figure 3-4, it is seen that:

$$T_{io} = T_h + T_{jc} + T_{in} + T_d \qquad \text{III-64}$$

where: T_{io} is the time the preceding car starts to come to an emergency stop, in seconds.

Solving for T_h, $$T_h = T_{io} - T_{jco} - T_{in} - T_d \qquad \text{III-65}$$

It will be noted that this equation is the mirror image of the overspeed expression, Equation III-42. Now applying the condition that there must be an integral number of interval times in the full headway,

$$H = T_{in} \left[\begin{array}{l} \text{The integer just} \\ \text{greater than} \end{array} \left(\frac{T_{io} - T_{jco} - T_{in} - T_d}{T_{in}} \right) \right] \qquad \text{III-66}$$

3.3.3 Complete Computations

The procedures outlined in the last two sections apply to one loop on the system. A car either overspeeds and just catches the front edge of the loop, or it unexpectedly stops and just catches the back edge of the loop. In the computer program described in the next section, successive loops within an area of the guideway are automatically examined in sequence. At a higher level, all areas of the guideway must be examined to determine the worst case headway for a total system. In practice, this examination is not as difficult as it might seem, since critical areas are usually easy to pinpoint. The inputs to such an analysis are the nominal velocity profiles, minimum loop size, the overall system parameters and the parameters of the headway protection system. These latter values must be adjusted to obtain, if possible, the headway objectives sought for the system. The architecture of computer program that mechanizes this process is described in the next section.

3.4. HEADWAY COMPUTER PROGRAM

The headway computer program performs the computations outlined in the preceding section: for a range of

1. Loop lengths
2. Main Guideway speeds, and
3. "Just Catch" loops

Its basic features are:

1. It handles arbitrary nominal trajectories.
2. Both overspeed and unexpected stop emergencies are considered
3. Consistent initial conditions can be specified
4. Running cost is low

The program is written in FORTRAN IV, for the PDP-10. Coding was done by Alan Waltner of Kentron Hawaii Ltd. Input is through an interactive console and output is on a line printer. Inputs are logically grouped so that on successive runs a group can be repeated without keying the individual inputs. This approach simplifies parametric design runs.

3.4.1. Inputs

A sample initial input is recorded on the console print-out shown on Figure 3-5. The program contains prestored inputs for the "baseline" case. These values are the inputs to the first run, in the absence of a "Y" answer to any CHANGE GROUP questions. For the run illustrated on Figure 3-5, all baseline inputs have been reentered, as if they were to be changed. When inputs are actually changed, the new values become the stored values for the next run, so that changes on successive runs are cumulative.

While the console interaction is to a great extent self-explanatory, formal definitions are given below. Where applicable, the associated variables from Sections 3.2 and 3.3 are shown in parenthesis.

```
PROGRAM MODE: N=CONSTANT SPEED, R=RAMP, S=QUIT  R


LOOP LENGTH: INITIAL= 0.33   FINAL= 0.33   INCREMENT= 0.00
SPEEDS: INITIAL=  30.00   FINAL=  30.00   INCREMENT=   0.00
OUTPUT DEVICE IS: PRINT
OPERATING MODE = WORST CASE : OUTPUT MODE = SHORT

CHANGE GROUP 1 ? (Y=YES, N=NO) Y

LOOP LENGTH: INITIAL=0.33333
 FINAL=0.33333
 INCREMENT=0.0

SPEED: INITIAL=30.0
 FINAL=30.0
 INCREMENT=0.0


DEFINE OPERATING AND OUTPUT MODE
MODE-1  NOMINAL *WORST CASE* INITIAL CONDITIONS
    -2  SPECIFIED INITIAL CONDITIONS
    -3  SEARCH FOR WORST INITIAL CONDITIONS

OUTPUT MODE-1  LONG DIAGNOSTIC OUTPUT
           -2  SHORT OUTPUT
OPERATING MODE=2
OUTPUT MODE=2

OUTPUT DEVICE (3=PRINTER, 5=TTY) 3
```

Figure 3-5. Input For Baseline Case - 1

(Baseline Inputs Respecified for Illustration)

```
POSITION IN LOOP AT START
UNEXPECTED STOP: PRECEEDING CAR = 1.000
                 FOLLOWING CAR = 0.259
OVERSPEED: PRECEEDING CAR =-0.342
           FOLLOWING CAR =-1.000
VELOCITY DEVIATION
UNEXPECTED STOP: PRECEEDING CAR = 0.000
                 FOLLOWING CAR = 0.966
OVERSPEED: PRECEEDING CAR =-0.940
           FOLLOWING CAR = 0.000

CHANGE GROUP 2 ? (Y=YES,N=NO) Y

POSITION IN LOOP
UNEXPECTED STOP: PRECEEDING CAR =1.000

FOLLOWING CAR =0.259

OVERSPEED: PRECEEDING CAR =-0.342

FOLLOWING CAR =-1.000

VELOCITY DEVIATION
UNEXPECTED STOP: PRECEEDING CAR =0.000

FOLLOWING CAR =0.966

OVERSPEED: PRECEEDING CAR =-0.940

FOLLOWING CAR =0.000

CAR LENGTH=15.0 FEET: BEAMWIDTH=  0.50 FEET
BRAKE TOLERANCE=  0.05 POSITION SAMPLE TIME= 0.010 SECONDS
SAFETY MARGIN=  5. REACTION TIME DELAY=  0.20 SECONDS
DECELERATION OF FAILED CAR=  1.0000 G
ACCELERATION OF OVERSPEED CAR=  0.2500 G
NOMINAL EMERGENCY DECELERATION=  0.2500 G
NOMINAL EMERGENCY JERK=  0.3750 G/SECOND
CORRECTION ACCELERATION=  0.1000 G
NOMINAL DECELERATION ON RAMP=  0.1000 G
NOMINAL JERK ON RAMP=  0.1000 G/SECOND
MAXIMUM SPEED FACTOR=   1.10
```

Figure 3-5. Input for Baseline Case - 2 (Cont.)
(Baseline Inputs respecified for Illustration)

```
CHANGE GROUP 3 ? (Y=YES,N=NO) Y

CAR LENGTH=15.0

BEAM WIDTH=0.50

BRAKE TOLERANCE=0.05

SAMPLE TIME=0.010

SAFETY MARGIN=5.

REACTION TIME DELAY=0.20

DECELERATION OF FAILED CAR=1.0000

ACCELERATION OF OVERSPEED CAR=0.2500

NOMINAL EMERGENCY DECELERATION=0.2500

NOMINAL EMERGENCY JERK=0.3750

CORRECTION ACCELERATION=0.1000

NOMINAL DECELERATION ON RAMP=0.1000

NOMINAL JERK ON RAMP=0.1000

MAXIMUM SPEED FACTOR 1.10


 RAMP PROFILE SELECTOR TRAPIZOID
INITIAL TRANSITION FACTOR     1.00
FINAL TRANSITION FACTOR      -5.00
INITIAL 'JUST-CATCH' FACTOR  -0.50
FINAL 'JUST-CATCH' FACTOR     1.50
LOOP MULTIPLIER =   1

CHANGE GROUP 4 ? (Y=YES,N=NO) Y

RAMP PROFILE SELECTOR (1=TRAPIZOID) 1

INITIAL TRANSITION FACTOR= 1.00

FINAL TRANSITION FACTOR= -5.00

INITIAL 'JUST-CATCH' FACTOR= -0.50

FINAL 'JUST-CATCH' FACTOR= 1.50

LOOP MULTIPLIER (2 DIGITS) 01
```

Figure 3-5. Input for Baseline Case - 3 (Cont.)
(Baseline Inputs Respecified for Illustration)

INPUT DESCRIPTIONS

PROGRAM MODE	CONSTANT SPEED	Specifies a short computation of main guideway, constant-speed headways.
	RAMP	Specifies use of arbitrary trajectory mode.
GROUP 1		
LOOP LENGTH	INITIAL FINAL INCREMENT	Specifies the range of main guideway loop lengths to be investigated; loop lengths are specified in feet, as a fraction of the main guideway speed in mph. Thus for a main guideway speed of 30 mph, INITIAL = 0.333, FINAL = 0.667, and INCREMENT = 0.333, would investigate main guideway loop length of 10 and 20 feet.
SPEED	INITIAL FINAL INCREMENT	Specifies, in mph, the range of main guideway speeds to be investigated.

OPERATING MODE - 1 NOMINAL - WORST CASE - INITIAL CONDITIONS

Defines position and velocity deviations at start of emergency maneuvers; in sequence, 1.0, 0.0, 0.0, -1.0, 0.0, 1.0, -1.0 and 0.0. (See definitions under SPECIFIED INITIAL CONDITIONS, below.) Definitions correspond closely to worst case assumptions for baseline case.

Baseline values of initial conditions are entered under SPECIFIED INITIAL CONDITION (option)

\- 2 SPECIFIED INITIAL CONDITIONS

Calls for an arbitrary set of initial conditions, through GROUP 2 inputs, below.

\- 3 SEARCH FOR WORST INITIAL CONDITIONS

Not a valid input with existing program. Meant, in the future, to call for search on initial conditions to determine worst case.

OUTPUT MODE - 1	LONG DIAGNOSTIC OUTPUT	Used when details of interim computations are desired.
- 2	SHORT OUTPUT	Normal, compact output (shown on Figure 3-6).
OUTPUT DEVICE	3 PRINTER	Output is printer. Must be specified in RAMP program mode.
	5 TTY	Output is teletype; only specified if in CONSTANT SPEED PROGRAM MODE.

<u>GROUP 2</u>

POSITION IN LOOP AT START		This group is asked for if OPERATING MODE has been set to SPECIFIED INITIAL CONDITIONS.
UNEXPECTED STOP	PRECEDING CAR FOLLOWING CAR	Car's position in loop corridor; 1.0 furthest forward, -1.0 furthest aft.
OVERSPEED	PRECEDING CAR FOLLOWING CAR	(Same as value for -sin wt in Equation III-20, III-34, and III-44.)
VELOCITY DEVIATION		
UNEXPECTED STOP	PRECEDING CAR FOLLOWING CAR	Cars velocity deviation, relative to maximum deviation; +1.0 for maximum plus;
OVERSPEED	PRECEDING CAR FOLLOWING CAR	-1.0 for maximum negative (same as value for -cos wt in Equations III-21, III-35 and III-45).

<u>GROUP 3</u>

CAR LENGTH	(L_c)
BEAM WIDTH	(w_b)
BRAKE TOLERANCE	(e_b)
SAMPLE TIME	(T_s)
SAFETY MARGIN	(L_{sm})
REACTION TIME DELAY	(T_d)
DECELERATION OF FAILED CAR	$(\frac{a_{us}}{g})$
ACCELERATION OF OVERSPEED CAR	$(\frac{a_{os}}{g})$
NOMINAL EMERGENCY DECELERATION	$(\frac{a_e}{g})$

NOMINAL EMERGENCY JERK	$\left(\frac{J_e}{g/sec}\right)$
CORRECTION ACCELERATION	$\left(\frac{A_{cm}}{g}\right)$
NOMINAL DECELERATION ON RAMP	Parameter for defining allowable deceleration for nominal trajectory in g's.
NOMINAL JERK ON RAMP	Parameter for defining allowable jerk for nominal trajectory, in g's/sec.
MAXIMUM SPEED FACTOR	Defines the maximum possible speed. If positive, it is maximum speed as a multiple of main guideway (reference) speed. If negative, it is the absolute maximum speed, in mph. $\left(-\frac{V_{max} : 60}{88}\right)$

<u>GROUP 4</u>

RAMP PROFILE SELECTOR (1 = TRAPEZOID)	Only one arbitrary trajectory, a trapezoid profile velocity change is available at present. The option is programmed, however, in anticipation of adding more nominal trajectory profiles in the future.
INITIAL TRANSITION FACTOR	If positive, defines the initial speed of the transition, as a multiple of main guideway (reference) speed. If negative, its absolute value is the initial speed, in mph.
FINAL TRANSITION FACTOR	If positive, defines the final speed of the transition, as a multiple of main guideway (reference) speed. If negative, its absolute value is the final speed, in mph.
INITIAL "JUST CATCH" FACTOR	The initial "just catch" time, defined as a multiple of speed transition time. Time is zero at the start of the transition. Thus, if the transition takes 10 seconds, and the value is -0.5, the first "just catch" time is -5 seconds.

FINAL "JUST CATCH" FACTOR	Defined the same way as the initial "just catch" factor. Specifies the upper limit on the "just catch" times.
LOOP MULTIPLIER	The program will skip the specified number of loops for successive computations. For preliminary investigations, this feature reduces computation and output time.

3.4.2 Output

The output that results from the input on Figure 3-5 is shown in Figure 3-6. It is the baseline case of Section 4. With the possible exception of the quantity "implied precision of velocity control," the output is self-explanatory. Implied precision of velocity control is the maximum deviation speed, dV_m, divided by V, and is a measure of the control accuracy required to stay within the loop corridor.

The output first repeats the input; it then goes on to print, for each "just catch" time:

1. Speed and car positions at the "just catch" time.
2. Speed and position when car starts to fail.
3. Speed and position when failed car's brakes go on (overspeed failure only).
4. Stopping point for failed car.
5. Speed and position of avoiding car when it starts to emergency stop.
6. Stopping point of avoiding car.
7. Headway required, both raw headway and headway with integral number of loops.

```
EXECUTE RAMP PART OF PROGRAM

INVESTIGATE LOOP LENGTH FRACTION FROM 0.33  THROUGH 0.33
  IN INCREMENTS OF 0.00
INVESTIGATE SPEED FROM  30.00  THROUGH  30.00  IN INCREMENTS OF   0.00
OPERATING MODE = SPEC. INITIAL COND. 1
OUTPUT MODE = SHORT

INPUTS
CAR LENGTH=15.0 FEET1 BEAMWIDTH=  0.50 FEET
BRAKE TOLERANCE=  0.05 POSITION SAMPLE TIME= 0.010 SECONDS
SAFETY MARGIN=  5. REACTION TIME DELAY=  0.20 SECONDS
DECELERATION OF FAILED CAR=  1.0000 G
ACCELERATION OF OVERSPEED CAR=  0.2500 G
NOMINAL EMERGENCY DECELERATION=  0.2500 G
NOMINAL EMERGENCY JERK=  0.3750 G/SECOND
CORRECTION ACCELERATION=  0.1000 G
NOMINAL DECELERATION ON RAMP=  0.1000 G
NOMINAL JERK ON RAMP=  0.1000 G/SECOND
MAXIMUM SPEED FACTOR=   1.10

FRACTIONAL INITIAL POSITION
PRECEEDING UNEXPECTED STOP 1.000
FOLLOWING UNEXPECTED STOP  0.259
PRECEEDING OVERSPEED      -0.342
FOLLOWING OVERSPEED       -1.030
FRACTIONAL INITIAL VELOCITY
PRECEEDING UNEXPECTED STOP 0.000
FOLLOWING UNEXPECTED STOP  0.966
PRECEEDING OVERSPEED      -0.940
FOLLOWING OVERSPEED        0.000

 RAMP PROFILE SELECTOR TRAPIZOID
INITIAL TRANSITION FACTOR     1.00
FINAL TRANSITION FACTOR      -5.00
INITIAL 'JUST-CATCH' FACTOR  -0.50
FINAL 'JUST-CATCH' FACTOR     1.50
LOOP MULTIPLIER =  1
```

Figure 3-6. Output for Baseline Case Program Schematic -1

MAIN GUIDEWAY SPEED= 30.00 MPH
MAIN GUIDEWAY LOOP LENGTH= 10.00 FEET
~~MAIN GUIDEWAY LOOP INTERVAL= 0.227 SECONDS~~
MAXIMUM POSSIBLE SPEED= 33.00 MPH
IMPLIED PRECISION OF MAIN GUIDEWAY VELOCITY CONTROL= 0.09

TRAPIZOIDAL ACCELERATION PROFILE
INITIAL VELOCITY= 32.00 MPH
IMPLIED PRECISION OF VELOCITY CONTROL= 0.09
FINAL VELOCITY= 5.00 MPH
IMPLIED PRECISION OF VELOCITY CONTROL= 0.25
TRANSITION TIME= 12.39 SECONDS
TRANSITION DISTANCE= 317.94 FEET

PRECEEDING CAR COMES TO UNEXPECTED STOP

CAR JUST CATCHES LOOP			PRECEEDING CAR STARTS TO FAIL		LEAD CAR STOPS	FOLLOWING CAR STARTS TO STOP		FOL. CAR STOPS	NON-INTEGER	INTEGER
SECONDS	POSITION (FT)	SPEED (MPH)	POSITION (FT)	SPEED (MPH)	POSITION (FT)	POSITION (FT)	SPEED (MPH)	POSITION (FT)	HEADWAY (SEC)	HEADWAY (SEC)
-6.19	-277.58	12.77	-302.24	30.00	-272.18	-457.51	32.59	-292.18	4.66	4.77
-5.97	-267.58	12.77	-292.24	30.00	-262.18	-447.51	32.59	-282.18	4.66	4.77
-5.74	-257.58	12.77	-282.24	30.00	-252.18	-437.51	32.59	-272.18	4.66	4.77
-5.51	-247.58	12.77	-272.24	30.00	-242.18	-427.51	32.59	-262.18	4.66	4.77
-5.28	-237.58	12.77	-262.24	30.00	-232.18	-417.51	32.59	-252.18	4.66	4.77
-5.06	-227.58	12.77	-252.24	30.00	-222.18	-477.51	32.59	-242.18	4.66	4.77
-4.83	-217.58	12.77	-242.24	30.00	-212.18	-397.51	32.59	-232.18	4.66	4.77
-4.60	-207.58	12.77	-232.24	30.00	-202.18	-387.51	32.59	-222.18	4.65	4.77
-4.38	-197.58	12.77	-222.24	30.00	-192.18	-377.51	32.59	-212.18	4.65	4.77
-4.15	-187.58	12.77	-212.24	30.00	-182.18	-367.51	32.59	-272.15	4.66	4.77
-3.92	-177.58	12.77	-202.24	30.00	-172.18	-357.51	32.59	-192.18	4.66	4.77
-3.69	-167.58	12.77	-192.24	30.00	-162.18	-347.51	32.59	-182.18	4.66	4.77
-3.47	-157.58	12.77	-182.24	30.00	-152.18	-337.51	32.59	-172.18	4.66	4.77
-3.24	-147.58	12.77	-172.24	30.00	-142.18	-327.51	32.59	-162.18	4.66	4.77
-3.01	-137.58	12.77	-162.24	30.00	-132.18	-317.51	32.59	-152.18	4.66	4.77
-2.78	-127.58	12.77	-152.24	30.00	-122.18	-327.51	32.59	-142.18	4.66	4.77
-2.56	-117.58	12.77	-142.24	30.00	-112.18	-297.51	32.59	-132.18	4.66	4.77
-2.33	-107.58	12.77	-132.24	30.00	-102.18	-287.51	32.59	-122.18	4.66	4.77
-2.10	-97.58	12.77	-122.24	30.07	-92.18	-277.51	32.59	-112.18	4.66	4.77
-1.88	-87.58	12.77	-112.24	30.02	-82.18	-267.51	32.59	-122.16	4.66	4.77
-1.65	-77.58	12.77	-102.24	30.00	-72.18	-257.51	32.59	-92.18	4.66	4.77
-1.42	-67.58	12.77	-92.24	30.00	-62.18	-247.51	32.59	-82.18	4.66	4.77
-1.19	-57.58	12.77	-82.24	30.00	-52.18	-237.51	32.59	-72.18	4.66	4.77
-0.97	-47.55	12.77	-72.24	30.00	-42.18	-227.51	32.59	-62.18	4.65	4.77
-0.74	-37.58	12.77	-62.24	30.00	-32.18	-217.51	32.59	-52.18	4.66	4.77
-0.51	-27.58	12.77	-52.25	30.00	-22.18	-227.51	32.59	-42.18	4.66	4.77
-0.28	-17.58	12.77	-42.25	30.00	-12.18	-197.51	32.59	-32.18	4.66	4.77
-0.06	-7.58	12.77	-32.25	30.00	-2.18	-167.51	32.59	-22.18	4.66	4.77
0.17	2.42	12.77	-22.24	30.00	7.92	-177.51	32.59	-12.18	4.66	4.77
0.40	12.40	12.76	-12.26	30.00	17.90	-167.53	32.59	-2.20	4.66	4.77
0.62	22.34	12.71	-2.37	30.00	27.70	-157.62	32.59	7.72	4.66	4.77
0.85	32.20	12.58	7.38	30.00	37.43	-147.90	32.59	17.43	4.67	4.77
1.08	41.93	12.36	17.09	29.91	46.98	-138.35	32.59	26.98	4.68	4.77
1.31	51.50	12.07	26.80	29.72	56.31	-129.22	32.59	36.31	4.69	4.77
1.53	60.90	11.72	36.53	29.42	65.44	-119.89	32.59	45.44	4.71	4.77
1.76	70.14	11.36	46.32	28.99	74.40	-112.93	32.59	54.40	4.74	4.77
1.99	79.21	11.00	56.11	28.48	83.21	-102.12	32.59	63.21	4.76	4.77
2.22	88.12	10.63	65.73	27.97	91.85	-93.48	32.59	71.85	4.80	5.00

Figure 3-6. Output for Baseline Program Schematic - 2 (cont.)

2.44	96.36	10.27	75.16	27.45	100.34	-84.99	32.59	80.34	4.83	5.00
2.67	105.43	9.91	84.43	26.94	108.67	-76.66	32.59	88.67	4.87	5.00
2.90	113.84	9.55	93.52	26.42	116.84	-68.49	32.59	96.84	4.91	5.00
3.12	122.08	9.19	102.43	25.91	124.86	-60.47	32.59	104.86	4.95	5.00
3.35	130.15	8.83	111.17	25.40	132.72	-52.61	32.59	112.72	5.00	5.23
3.58	138.36	8.47	119.74	24.88	140.42	-44.91	32.59	120.42	5.26	5.23
3.81	145.80	8.12	128.13	24.37	147.96	-37.36	32.59	127.96	5.11	5.23
4.03	153.38	7.77	136.35	23.85	155.35	-29.98	32.59	135.35	5.17	5.23
4.26	160.79	7.41	144.40	23.34	162.58	-22.75	32.59	142.58	5.23	5.45
4.49	168.03	7.26	152.26	22.82	169.66	-15.67	32.59	149.66	5.32	5.45
4.72	175.10	6.71	159.96	22.30	176.57	-8.75	32.59	156.57	5.37	5.45
4.94	182.71	6.37	167.48	21.79	183.33	-2.00	32.59	163.33	5.44	5.45
5.17	188.76	6.02	174.82	21.27	189.93	4.73	32.58	169.93	5.52	5.68
5.40	195.34	5.68	181.99	20.76	196.38	11.69	32.53	176.38	5.59	5.68
5.62	201.75	5.34	188.98	20.24	202.66	19.23	32.40	182.66	5.64	5.68
5.85	207.99	5.02	195.80	19.72	208.79	27.44	32.18	188.79	5.68	5.68
6.08	214.07	4.66	202.44	19.21	214.76	36.66	31.84	194.76	5.69	5.91
6.31	219.98	4.32	208.91	18.69	220.58	47.59	31.29	200.58	5.66	5.68
6.53	225.73	3.99	215.20	18.17	226.23	59.12	30.66	206.23	5.62	5.68
6.76	231.31	3.66	221.31	17.65	231.72	72.24	30.03	211.72	5.57	5.68
6.99	236.72	3.33	227.25	17.14	237.06	81.39	29.41	217.06	5.52	5.68
7.22	241.97	3.00	233.02	16.62	242.24	91.65	28.79	222.24	5.48	5.68
7.44	247.05	2.68	238.62	16.17	247.26	101.91	28.17	227.26	5.44	5.45
7.67	251.96	2.36	244.01	15.58	252.12	111.37	27.56	232.12	5.40	5.45
7.90	256.71	2.04	249.25	15.06	256.82	121.53	26.95	236.82	5.36	5.45
8.12	261.29	1.72	254.30	14.54	261.37	130.90	26.35	241.37	5.32	5.45
8.35	265.71	1.41	259.18	14.02	265.75	139.97	25.75	245.75	5.29	5.45
8.58	269.96	1.12	263.89	13.50	269.98	148.73	25.15	249.98	5.26	5.45
8.81	274.24	0.83	268.41	12.98	274.24	157.20	24.56	254.24	5.23	5.45
9.03	277.96	0.52	272.76	12.46	277.95	165.34	23.98	257.95	5.22	5.23
9.26	281.71	0.20	276.93	11.94	281.69	173.22	23.41	261.69	5.18	5.23
9.49	285.29	0.00	280.92	11.42	285.28	180.78	22.84	265.28	5.16	5.23
9.72	288.72	0.00	284.74	10.90	288.71	188.02	22.28	268.71	5.15	5.23
9.94	291.97	0.00	288.46	10.36	292.25	195.13	21.71	272.25	5.13	5.23
10.17	295.06	0.00	291.88	9.84	295.12	201.63	21.18	275.12	5.12	5.23
10.40	297.99	0.00	295.12	9.32	298.23	207.91	20.66	278.23	5.12	5.23
10.62	300.74	0.00	298.19	8.80	300.77	213.57	20.16	280.77	5.14	5.23
10.85	303.33	0.00	301.06	8.29	303.35	219.35	19.66	283.35	5.15	5.23
11.08	305.76	0.00	303.75	7.77	305.77	224.64	19.18	285.77	5.17	5.23
11.31	308.02	0.00	306.27	7.25	308.02	229.76	18.70	288.02	5.19	5.23
11.53	310.11	0.00	308.60	6.74	310.11	234.27	18.28	290.11	5.23	5.45
11.76	312.04	0.00	310.75	6.22	312.05	238.42	17.86	292.05	5.28	5.45
11.99	313.85	0.00	312.75	5.74	313.86	242.37	17.46	293.86	5.33	5.45
12.22	315.59	0.00	314.62	5.37	315.58	246.35	17.05	295.58	5.38	5.45
12.44	317.26	0.00	316.39	5.13	317.26	250.11	16.65	297.26	5.44	5.45
12.67	318.93	0.00	318.09	5.02	318.93	253.86	16.25	298.93	5.49	5.68
12.90	320.59	0.00	319.76	5.00	320.59	257.64	15.82	300.59	5.54	5.68
13.12	322.26	0.00	321.43	5.00	322.26	261.43	15.38	302.26	5.58	5.68
13.35	323.93	0.00	323.09	5.00	323.93	265.24	14.93	303.93	5.61	5.68
13.58	325.59	0.00	324.76	5.00	325.59	269.08	14.46	305.59	5.63	5.68
13.81	327.26	0.00	326.43	5.00	327.26	272.93	13.97	307.26	5.65	5.68
14.03	328.93	0.00	328.09	5.00	328.93	277.06	13.41	308.93	5.64	5.68
14.26	330.59	0.00	329.76	5.00	330.59	281.06	12.86	310.59	5.63	5.68
14.49	332.26	0.00	331.43	5.00	332.26	285.12	12.26	312.26	5.60	5.68
14.72	333.93	0.00	333.09	5.00	333.93	289.24	11.62	313.93	5.55	5.68
14.94	335.59	0.00	334.76	5.00	335.59	293.44	10.93	315.59	5.48	5.68
15.17	337.26	0.00	336.43	5.00	337.26	297.72	10.16	317.26	5.39	5.45
15.40	338.93	0.00	338.09	5.00	338.93	302.12	9.30	318.93	5.25	5.45
15.62	340.59	0.00	339.75	5.00	340.59	307.11	8.20	320.59	5.01	5.23
15.85	342.26	0.00	341.43	5.00	342.26	312.15	6.94	322.26	4.68	4.77
16.08	343.93	0.00	343.09	5.00	343.93	315.01	6.41	323.93	4.55	4.77
16.31	345.59	0.00	344.76	5.00	345.59	317.12	6.21	325.59	4.49	4.55
16.53	347.26	0.00	346.43	5.00	347.26	318.83	6.19	327.26	4.48	4.55
16.76	348.93	0.00	348.09	5.00	348.93	320.52	6.19	328.93	4.48	4.55
16.99	350.59	0.00	349.76	5.00	350.59	322.18	6.19	330.59	4.48	4.55
17.22	352.26	0.00	351.43	5.00	352.26	323.85	6.19	332.26	4.48	4.55
17.44	353.93	0.00	353.09	5.00	353.93	325.40	6.19	333.93	4.49	4.55
17.67	355.59	0.00	354.76	5.00	355.59	327.18	6.19	335.59	4.48	4.55
17.90	357.26	0.00	356.43	5.00	357.26	328.85	6.19	337.26	4.48	4.55
18.12	358.93	0.00	358.09	5.00	358.93	330.52	6.19	338.93	4.48	4.55
18.35	360.59	0.00	359.76	5.0	360.59	332.19	6.19	340.59	4.48	4.55
18.58	362.26	0.00	361.43	5.00	362.26	333.85	6.19	342.26	4.48	4.55

Figure 3-6. Output for Baseline Case Program Schematic - 3 (Cont.)

FOLLOWING CAR OVERSPEEDS

CAR JUST CATCHES LOOP			FOLLOWING CAR STARTS TO OVERSPEED		FOLLOWING CAR STARTS TO STOP		FOL. CAR STOPS	PRECEEDING CAR STARTS TO STOP		PRECEEDING CAR STOPS	NON-INTEGER	INTEGER
SECONDS	POSITION (FT)	SPEED (MPH)	POSITION (FT)	SPEED (MPH)	POSITION (FT)	SPEED (MPH)	POSITION (FT)	POSITION (FT)	SPEED (MPH)	POSITION (FT)	HEADWAY (SEC)	HEADWAY (SEC)
-6.12	-266.78	33.00	-394.79	30.00	-246.10	33.00	-76.81	-166.32	27.48	-56.81	2.02	2.05
-5.97	-256.78	33.00	-384.79	30.00	-236.10	33.00	-66.81	-156.32	27.48	-46.81	2.02	2.05
-5.74	-246.78	33.00	-374.79	30.00	-226.10	33.00	-56.81	-146.32	27.48	-36.81	2.02	2.05
-5.51	-236.78	33.00	-364.79	30.00	-216.10	33.00	-46.81	-136.32	27.48	-26.81	2.02	2.05
-5.29	-226.78	33.00	-354.79	30.00	-206.10	33.00	-36.81	-126.32	27.48	-16.81	2.02	2.05
-5.06	-216.78	33.00	-344.79	30.00	-196.10	33.00	-26.81	-116.32	27.48	-6.81	2.02	2.05
-4.83	-206.78	33.00	-334.79	30.00	-186.10	33.00	-16.81	-106.32	27.48	3.19	2.02	2.05
-4.60	-196.78	33.00	-324.79	30.00	-176.10	33.00	-6.81	-96.29	27.48	13.19	2.02	2.05
-4.38	-186.78	33.00	-314.79	30.00	-166.10	33.00	3.19	-86.32	27.48	23.19	2.02	2.05
-4.15	-176.78	33.00	-304.79	30.00	-156.10	33.00	13.19	-76.32	27.48	33.19	2.02	2.05
-3.92	-166.78	33.00	-294.79	30.00	-146.10	33.00	23.19	-66.32	27.48	43.19	2.02	2.05
-3.69	-156.78	33.00	-284.79	30.00	-136.10	33.00	33.19	-56.32	27.48	53.19	2.02	2.05
-3.47	-146.78	33.00	-274.79	30.00	-126.10	33.00	43.19	-46.32	27.48	63.19	2.02	2.05
-3.24	-136.78	33.00	-264.79	30.00	-116.11	33.00	53.19	-36.32	27.48	73.19	2.02	2.05
-3.01	-126.79	33.00	-254.79	30.00	-106.11	33.00	63.19	-26.32	27.48	83.19	2.02	2.05
-2.78	-116.79	33.00	-244.79	30.00	-96.11	33.00	73.19	-16.32	27.48	93.19	2.02	2.05
-2.56	-106.79	33.00	-234.79	30.00	-86.11	33.00	83.19	-6.32	27.48	103.19	2.02	2.05
-2.33	-96.79	33.00	-224.79	30.00	-76.11	33.00	93.19	3.81	27.46	113.19	2.03	2.05
-2.10	-86.79	33.00	-214.79	30.00	-66.11	33.00	103.19	14.86	27.33	123.19	2.05	2.27
-1.88	-76.79	33.00	-204.79	30.00	-56.11	33.00	113.19	27.03	27.03	133.19	2.10	2.27
-1.65	-66.79	33.00	-194.79	30.00	-46.11	33.00	123.19	41.20	26.46	143.19	2.20	2.27
-1.42	-56.79	33.00	-184.79	30.00	-36.11	33.00	133.19	56.74	25.69	153.19	2.35	2.50
-1.19	-46.79	33.00	-174.79	30.00	-26.11	33.00	143.19	72.30	24.87	163.19	2.50	2.73
-0.97	-36.79	33.00	-164.79	30.00	-16.11	33.00	153.19	87.87	24.35	173.19	2.67	2.73
-0.74	-26.79	33.00	-154.79	30.00	-6.11	33.00	163.19	103.43	23.19	183.19	2.85	2.95
-0.51	-16.79	33.00	-144.79	30.00	3.89	33.00	173.19	118.99	22.30	193.19	3.04	3.18
-0.28	-6.79	33.00	-134.79	30.00	13.89	33.00	183.19	134.56	21.38	203.19	3.26	3.41
-0.06	3.21	33.00	-124.79	30.00	23.89	33.00	193.19	150.13	20.42	213.19	3.49	3.64
0.17	13.20	33.00	-114.67	30.00	33.88	33.00	203.17	165.68	19.42	223.17	3.74	3.86
0.40	23.14	33.00	-104.08	30.00	43.82	33.00	213.11	181.07	18.38	233.11	4.01	4.09
0.62	33.00	33.00	-92.64	30.00	53.68	33.00	222.97	196.50	17.28	242.97	4.31	4.32
0.85	42.73	33.00	-79.97	30.00	63.41	33.00	232.70	211.68	16.14	252.70	4.63	4.77
1.08	52.31	33.00	-65.73	30.00	72.99	33.00	242.28	226.49	14.94	262.28	4.98	5.00
1.31	61.72	33.00	-49.41	30.00	82.35	33.00	251.65	241.25	13.65	271.65	5.37	5.45
1.53	70.96	33.00	-32.19	30.00	91.63	33.00	260.92	255.72	12.27	280.92	5.80	5.91
1.76	80.24	33.00	-12.73	30.00	100.69	33.00	269.98	269.82	10.78	289.98	6.30	6.36
1.99	[illegible]	33.00	8.27	29.91	109.53	33.00	278.82	283.85	9.09	298.82	6.89	7.05
2.22	97.69	33.00	24.57	29.51	118.43	33.00	287.73	297.90	7.46	307.73	7.66	7.73
2.44	106.27	33.00	39.45	28.87	126.96	33.00	296.25	311.54	4.46	316.25	8.73	8.86
2.67	114.69	33.00	51.86	28.22	135.37	33.00	304.66	320.99	3.84	324.66	9.75	9.77
2.90	122.93	33.00	63.15	27.62	143.62	33.00	312.91	329.15	3.84	332.91	10.64	10.69
3.12	131.01	33.00	73.35	27.07	151.77	33.00	321.06	337.30	3.84	341.06	11.52	11.59
3.35	138.92	33.00	83.54	26.51	159.58	33.00	328.87	345.11	3.84	348.87	12.36	12.50
3.58	146.66	32.84	92.77	25.98	167.36	33.00	336.65	352.89	3.84	356.65	13.19	13.41
3.81	154.24	32.26	101.85	25.46	174.87	33.00	344.16	360.47	3.84	364.16	14.00	14.09
4.03	161.64	31.67	110.77	24.93	182.12	33.00	351.41	367.65	3.84	371.41	14.75	14.77
4.26	168.88	31.07	119.52	24.41	189.10	33.00	358.39	374.63	3.84	378.39	15.48	15.69
4.49	175.95	30.48	128.10	23.88	195.82	32.82	363.37	379.61	3.84	383.37	15.93	16.14
4.72	182.86	29.88	136.50	23.35	202.35	32.22	364.14	380.38	3.84	384.14	15.81	15.91
4.94	189.60	29.28	144.71	22.83	208.72	31.62	364.84	381.08	3.84	384.84	15.67	15.68
5.17	196.17	28.68	152.75	22.30	214.91	31.02	365.46	381.70	3.84	385.46	15.53	15.68
5.40	202.58	28.08	160.60	21.77	220.94	30.42	366.00	382.24	3.84	386.00	15.38	15.49
5.62	208.82	27.48	168.29	21.24	226.81	29.82	366.46	382.71	3.84	386.46	15.21	15.23

Figure 3-6. Output for Baseline Case Program Schematic - 4 (Cont.)

5.83	214.90	26.87	175.77	20.71	232.50	29.22	366.85	383.09	3.84	386.85	15.04	15.23
6.08	220.82	26.27	183.08	20.18	238.03	28.61	367.17	383.41	3.84	387.17	14.86	15.00
6.31	226.55	25.66	190.21	19.66	243.39	28.01	367.41	383.65	3.84	387.41	14.66	14.77
6.53	232.12	25.05	197.15	19.13	248.59	27.40	367.57	383.81	3.84	387.57	14.46	14.55
6.76	237.53	24.45	203.92	18.59	253.61	26.79	367.65	383.89	3.84	387.65	14.24	14.32
6.99	242.77	23.83	210.50	18.06	258.47	26.18	367.66	383.90	3.84	387.66	14.01	14.09
7.22	247.85	23.22	216.90	17.53	263.17	25.57	367.60	383.84	3.84	387.60	13.78	13.86
7.44	252.76	22.61	223.11	17.00	267.69	24.95	367.46	383.70	3.84	387.46	13.53	13.64
7.67	257.50	21.99	229.14	16.47	272.05	24.34	367.25	383.49	3.84	387.25	13.28	13.41
7.90	262.08	21.37	234.99	15.94	276.24	23.72	366.96	383.20	3.84	386.96	13.01	13.19
8.12	266.49	20.76	240.66	15.40	280.26	23.10	366.60	382.84	3.84	386.60	12.73	12.95
8.35	270.74	20.13	246.14	14.87	284.11	22.49	366.17	382.41	3.84	386.17	12.45	12.50
8.58	274.82	19.51	251.43	14.33	287.80	21.85	365.66	381.92	3.84	385.66	12.15	12.27
8.81	278.73	18.89	256.55	13.80	291.32	21.23	365.08	381.32	3.84	385.08	11.84	12.05
9.03	282.47	18.25	261.47	13.24	294.67	20.60	364.42	380.66	3.84	384.42	11.53	11.59
9.26	286.05	17.62	266.21	12.73	297.85	19.97	363.70	379.94	3.84	383.70	11.20	11.36
9.49	289.47	16.99	270.77	12.19	300.87	19.33	362.90	379.14	3.84	382.90	10.86	10.91
9.72	292.71	16.35	275.14	11.65	303.72	18.70	362.03	378.27	3.84	382.03	10.52	10.68
9.94	295.79	15.71	279.32	11.11	306.39	18.06	361.08	377.32	3.84	381.08	10.16	10.23
10.17	298.71	15.07	283.31	10.57	308.90	17.41	360.07	376.31	3.84	380.07	9.80	10.00
10.40	301.45	14.42	287.12	10.03	311.25	16.77	358.99	375.23	3.84	378.99	9.42	9.55
10.62	304.04	13.77	290.73	9.49	313.42	16.12	357.83	374.07	3.84	377.83	9.24	9.29
10.85	306.45	13.12	294.16	8.94	315.42	15.46	356.61	372.85	3.84	376.61	8.64	8.86
11.08	308.70	12.46	297.42	8.40	317.26	14.80	355.31	371.55	3.84	375.31	8.24	8.41
11.31	310.78	11.79	300.45	7.85	318.92	14.14	353.95	370.19	3.84	373.95	7.83	7.95
11.53	312.71	11.13	303.29	7.31	320.43	13.48	352.57	368.81	3.84	372.57	7.41	7.52
11.76	314.51	10.51	305.89	6.77	321.84	12.84	351.37	367.61	3.84	371.37	7.22	7.05
11.99	316.23	9.95	306.18	6.26	323.22	12.32	350.59	366.84	3.84	370.59	6.69	6.82
12.22	317.76	9.57	310.18	5.81	324.64	11.91	352.43	366.68	3.84	372.43	6.44	6.59
12.44	319.56	9.31	311.97	5.46	326.14	11.65	350.94	367.18	3.84	372.94	6.28	6.36
12.67	321.23	9.17	313.63	5.21	327.72	11.51	352.02	368.24	3.84	372.03	6.23	6.36
12.90	322.89	9.11	315.24	5.04	329.35	11.45	353.43	369.64	3.84	373.43	6.16	6.36
13.12	324.56	9.09	316.87	5.00	331.00	11.44	354.99	371.23	3.84	374.99	6.15	6.36
13.35	326.23	9.99	318.67	5.00	332.50	11.33	356.11	372.35	3.84	376.11	6.27	6.14
13.58	327.89	9.99	320.34	5.00	334.16	11.33	357.77	374.01	3.84	377.77	6.27	6.14
13.81	329.56	9.99	322.00	5.00	335.83	11.33	359.44	375.68	3.84	379.44	6.27	6.14
14.03	331.23	9.99	323.67	5.00	337.50	11.33	361.11	377.35	3.84	381.11	6.27	6.14
14.26	332.89	9.99	325.34	5.00	339.16	11.33	362.77	379.01	3.84	382.77	6.27	6.14
14.49	334.56	9.99	327.00	5.00	340.83	11.33	364.44	380.68	3.84	384.44	6.27	6.14
14.72	336.23	9.99	328.67	5.00	342.50	11.33	366.11	382.35	3.84	386.11	6.27	6.14
14.94	337.89	9.99	330.34	5.00	344.16	11.33	367.77	384.01	3.84	387.77	6.27	6.14
15.17	339.56	9.99	332.00	5.00	345.83	11.33	369.44	385.68	3.84	389.44	6.27	6.14
15.40	341.23	9.99	333.67	5.00	347.50	11.33	371.11	387.35	3.84	391.11	6.27	6.14
15.62	342.89	9.99	335.34	5.00	349.16	11.33	372.77	389.01	3.84	392.77	6.27	6.14
15.85	344.56	9.99	337.00	5.00	350.83	11.33	374.44	390.68	3.84	394.44	6.27	6.14
16.08	346.23	9.99	338.67	5.00	352.50	11.33	376.11	392.35	3.84	396.11	6.27	6.14
16.31	347.89	9.99	340.34	5.00	354.16	11.33	377.77	394.01	3.84	397.77	6.27	6.14
16.53	349.56	9.99	342.00	5.00	355.83	11.33	379.44	395.68	3.84	399.44	6.27	6.14
16.76	351.23	9.99	343.67	5.00	357.50	11.33	381.11	397.35	3.84	401.11	6.27	6.14
16.99	352.89	9.99	345.34	5.00	359.16	11.33	382.77	399.01	3.84	402.77	6.27	6.14
17.22	354.56	9.99	347.00	5.00	360.83	11.33	384.44	400.68	3.84	404.44	6.27	6.14
17.44	356.23	9.99	348.67	5.00	362.50	11.33	386.11	402.35	3.84	406.11	6.27	6.14
17.67	357.89	9.99	350.34	5.00	364.16	11.33	387.77	403.92	3.84	407.77	6.26	6.14
17.90	359.56	9.99	352.00	5.00	365.83	11.33	389.44	405.77	3.84	409.44	6.27	6.14
18.12	361.23	9.99	353.67	5.00	367.50	11.33	391.11	407.35	3.84	411.11	6.27	6.14
18.35	362.89	9.99	355.34	5.00	369.16	11.33	392.77	409.01	3.84	412.77	6.27	6.14
18.58	364.56	9.99	357.00	5.00	370.83	11.33	394.44	410.68	3.84	414.44	6.27	6.14

Figure 3-6. Output for Baseline Case Program Schematic - 5 (Cont.)

3.4.3 Program Architecture

The main program provides bookkeeping functions for the basic computation sequence. It indexes loop length, main guideway speed and "just catch" times. All loops are first checked for unexpected stops. The results are printed out and then loops are checked for overspeed. A block diagram of the program computations is shown on Figure 3-7. They follow the analytical development of Section 3.

It is seen that there is a parallel structure to the program that allows double use of the subroutines. The functions of the supporting subroutines are briefly discussed below.

SINC - Accepts time, T, as an input and outputs the nominal trajectory f_V and f_X. Alternate profiles are easily accommodated by branching within the subroutine, no other part of the program is affected.

TRIAL - determines initial trial value of failure time that will cause car to "just catch" the edge of a loop;

FAIL - Determines trial position of failed car at "just catch" time. Alternate failure profiles could be introduced here without affecting the rest of the program. At present unexpected stop failure is constant deceleration and overspeed failure is constant acceleration to a maximum speed. (See Page 3-10).

EST - Estimates new trial values of failure time (linear prediction).

EMERG - Computes emergency stop point, given initial conditions. At present assumes trapezoidal stopping profile (Section 2). Alternate profile. can be introduced by branching in the subroutine. There would be no affect on the rest of the program.

A program listing may be obtained through the Transportation Systems Center, Cambridge, Massachusetts.

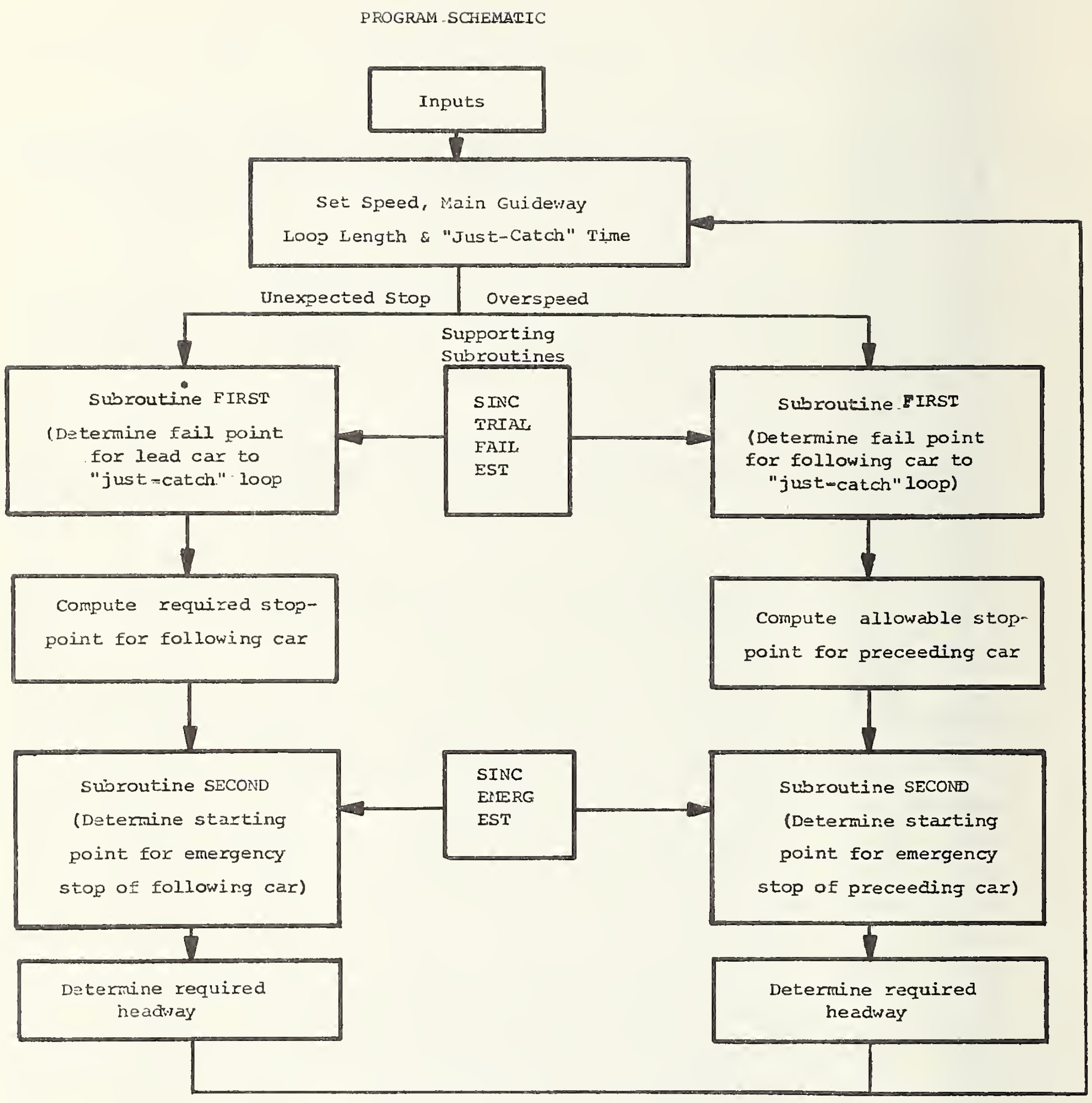

Figure 3-7. Program Schematic

4. TYPICAL RESULTS

In this section, the results of running the computer program described in Section 3.4 are given to illustrate variations in headway as system parameters are changed. The results are presented relative to the baseline case, illustrated in the computer printouts of Section 3.4.

4.1. BASELINE

The parameters of the baseline case are shown on Figure 4-1. They have been discussed in Sections 2 and 3. These parameters define a somewhat better than state-of-the-art system, that might, with some development, be installed in the next few years.

The baseline considers a constant speed of 30 mph followed by deceleration to a station entry speed of 5 mph. Deceleration is chosen as part of the baseline since it represents the worst case in terms of required headways. Less headway is required on the main guideway, and still less on acceleration ramps. Ramp deceleration and jerk are consistent with standing passengers who have ready support (Table 2-1), as are the emergency deceleration and jerk (Table 1-1). The main guideway loop length is 10 feet, implying a velocity control accuracy of about 10%. This accuracy is consistent with the capability of state-of-the-art control systems.

The results of the baseline runs are plotted on Figure 4-2. The plot shows the required headway as a function of the point at which a car starts to fail. Zero on the abscissa is the start of the deceleration transition. It takes about 315 feet to slow to 5 mph. Both overspeed failures and unexpected stop failures are plotted.

There are two forms of curves plotted on Figure 4-2. The continuous curves are the non-integer headways. The stepwise curves are the envelope curves of integer headways. When the non-integer headway is an integer multiple of the time required to traverse a loop (0.227 seconds for the baseline) the curves are coincident. If somewhat higher headway is required, a whole loop must be added and the integer headway jumps 0.227 seconds.

BASE LINE CASE

Decelerate from Main Guideway Speed		= 30 mph
To Transition Speed		= 5 mph
Nominal Deceleration on Ramp		= 0.1 g's
Nominal Jerk on Ramp		= 0.1 g's/sec
Correction Acceleration		= 0.1 g's
Main Guideway Loop Length		= 10 feet
Initial Conditions		
Unexpected Stop - Preceding Car	wt	= 270°
Following Car	wt	= 195°
Overspeed Preceding Car	wt	= 20°
Following Car	wt	= 90°
Safety Margin		= 15 feet
Car Length		= 15 feet
Brake Tolerance		= 0.05
Beam Width		= 0.5 feet
Sample Time		= 0.01 seconds
Reaction Time Delay		= 0.2 seconds
Deceleration of Failed Car		= 1.0 g's
Acceleration of Overspeed Car		= 0.25 g's
Maximum Speed Factor		= 1.1
Nominal Emergency Deceleration		= 0.25 g's
Nominal Emergency Jerk		= 0.375 g's/second

Figure 4-1. Baseline Case

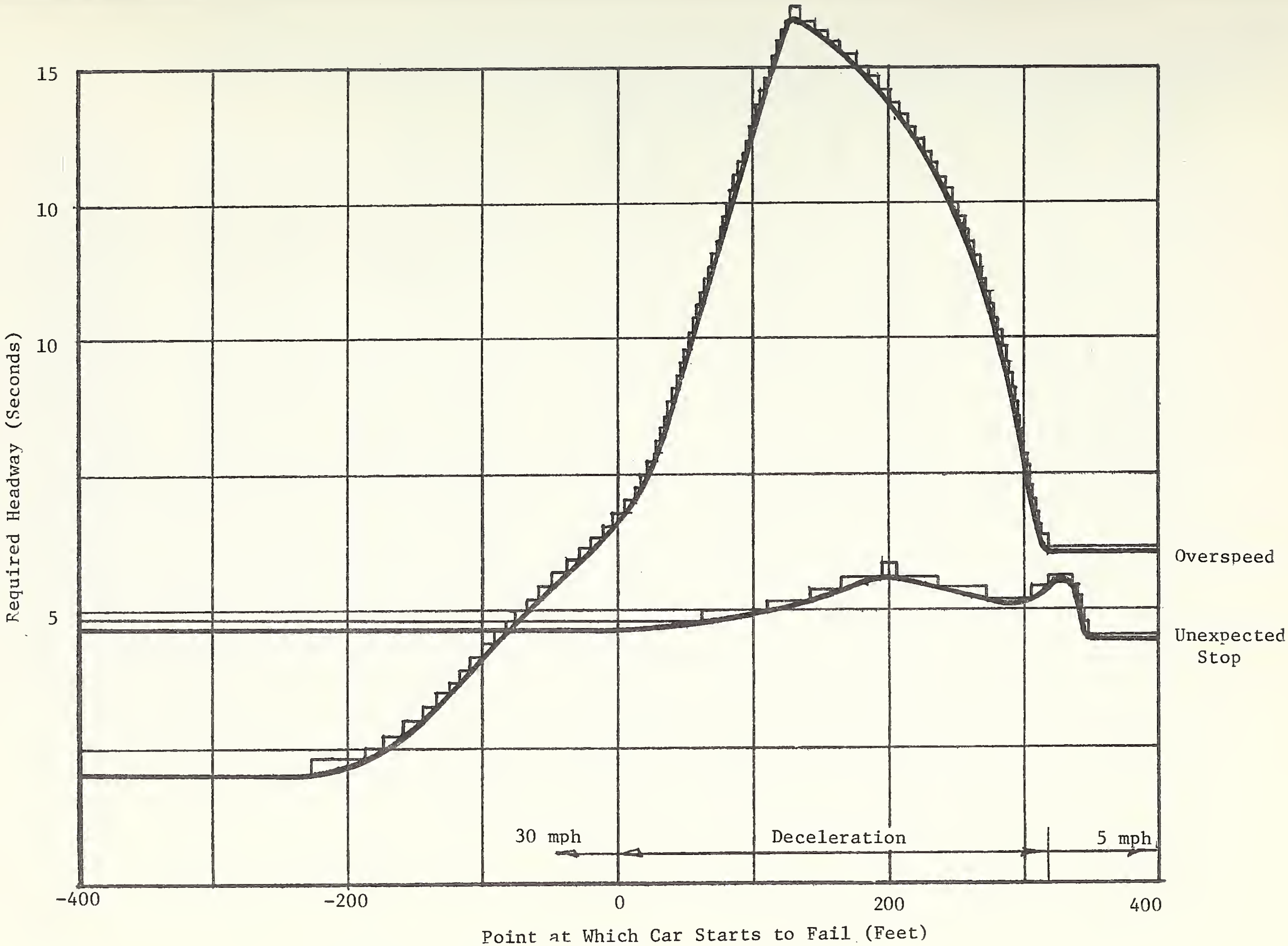

Figure 4-2. Required Headway for Baseline Case

The integer headway points of the baseline printout all fall on the stepwise "envelope" curves. They do not completely define the curve, however. Most of the stepwise curves are interpolated. In fact, the curves of headway versus failure point for a given loop configuration could be sawtoothed, i.e., much lower headways would be required for failures at points that did not "just catch" a loop. All of the printout points are, of course, worst cases that do "just catch" a certain set of loops.

Thus, curves of Figure 4-2 are actually what would be obtained if the results of a large number of runs were plotted together, and if for each run, the loop positions were "slipped" relative to the zero-distance point. Looked at in another way, the stepwise curves are upperbounds on headway required as function of failure point, assuming loop position and initial conditions are always adjusted to "worst case."

The main guideway headway is defined by the constant portion of the curves at the left of Figure 4-2. It is seen that protection against the unexpected stop hazard requires about twice as much headway as protection against the overspeed hazard (4.77 seconds versus 2.05 seconds). This conclusion changes, however, as the ramp starts to affect the required headways.

The effect of the ramp on unexpected stop headway is not apparent until the preceding car has actually entered the ramp, prior to failing. As the preceding car moves further down the ramp before failing, the headway increases to a peak of 5.91 seconds and then drops to a steady-state value of 4.55 seconds at the transition speed of 5 mph.

The ramp affects overspeed headways for failures well before the start of the ramp, since the preceding car will have already entered the ramp. The headway peaks at 16.14 seconds, which is required if the car fails about half-way down the deceleration ramp. It then drops off to 6.14 seconds, its value for the transition speed of 5 mph.

Thus, on the main guideway, the unexpected stop of a preceding car is the critical hazard while on the deceleration ramp, it is the overspeed of the following car. This general behavior is characteristic of the cases investigated herein. Needless to say, it would not be universally true. As a matter of fact, at the transition velocity of five mph, for the baseline parameters, the overspeed hazard is more critical than the unexpected stop hazard. This result is shown on the right hand side of Figure 4-2, and is primarily due to the high maximum-possible-speed (33 mph) versus the local guideway speed at that point (5 mph).

Based on Figure 4-2, if successive cars pass through a deceleration transition,there must be about 16 seconds between them, versus about five seconds for the main guideway. Overspeed during the deceleration is thus the determinant of system headway. The problem is, that if the system is run at 16-second headway, the main guideway is used inefficiently. There are several possible ways to maintain main guideway traffic; e.g., restricting station entry, improved system on the deceleration ramp, and warping the deceleration profile.

Assuming the deceleration is to a station entrance ramp, one solution is to run the main guideway at five-seconds headway and insure through systems management that only every fourth car enters a station. In this way, the headway between successive cars on the deceleration ramp will be a safe 20 seconds. The potential difficulty is that this approach restricts overall system management. Cars must be held in upstream stations, awaiting a space that will be able to enter the station. A solution that overcomes this restriction is to split the deceleration ramp. Successive cars would move down completely separate ramps. This solution is expensive, however, and only two successive cars could be accommodated.

Another approach is to selectively improve the parameters of the headway protection system only on the deceleration ramp, using shorter loops, shorter reaction times, etc. This method will bring the headway required on the deceleration ramp closer to the main guideway requirement. (On the other hand, similar improvements on the main guideway will always maintain its relative advantage.)

It is also possible to use something other than a trapezoidal deceleration profile. While there has been insufficient time to carefully explore this alternative, preliminary studies show it to have promise.

4.2 INITIAL CONDITIONS

Worst case initial conditions for the baseline case were determined through a series of runs that varied the phase angle of Equations III-14 and III-15. This phase angle varies the relative deviation in velocity and position within the loop corridor. The change in required headway with phase angle is plotted on Figures 4-3 and 4-4, for unexpected stop on the main guideway and for overspeed on the deceleration ramp. In both cases, non-integer headway is plotted, in order that the fine grain of the variation is not masked by the integral loop constraint.

In each case the phase angle wt for one car is varied while that for the other car is held constant. Since the worst case for one will not affect the worst case for the other, and the effects are additive, there is no loss of generality.

For unexpected stops on the main guideway (Figure 4-3) the worst case for the following car occurs when wt is 195 degrees. Referring to Figure III-2, the following car is slightly forward of its nominal position and moving at close to its maximum positive deviational speed. Worst case for a preceding car occurs when the phase angle is 270 degrees; i.e., when the car is most forward in its corridor at the time of failure.

When the following car overspeeds on the deceleration ramp, the required headway varies greatly with initial conditions (Figure 4-4). The worst case is when it is at the back of its corridor moving at nominal velocity; i.e., when wt is 90 degrees. This condition maximizes the time it takes to discover the failure. Worst case conditions for the preceding car are when it is slightly behind its nominal position, with close to maximum negative speed deviation. (wt = 20 degrees.)

The worst case phase angles shown on Figures 4-3 and 4-4 are used for all of the sensitivity runs which follow. While rigorously, worst case initial conditions should be determined for each of these runs, a preliminary investigation of widely separated cases indicates that no appreciable error is introduced by assuming the same constant values. A detailed examination of a particular case should, of course, reevaluate the worst case initial conditions.

"Worst Case" Initial Conditions

Baseline Parameters

Unexpected Stop on Main Guideway

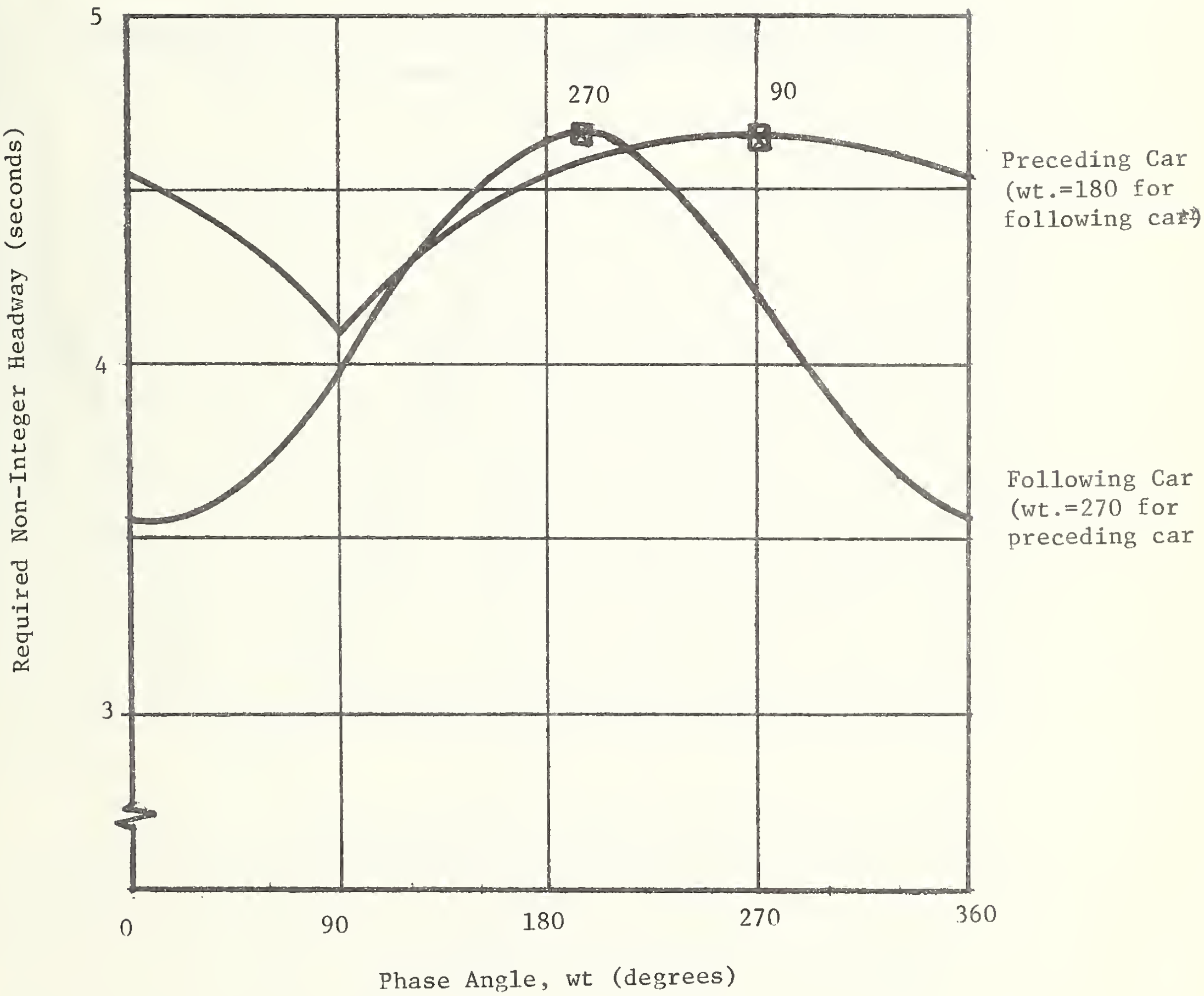

Figure 4-3. "Worst Case" Initial Conditions (First)

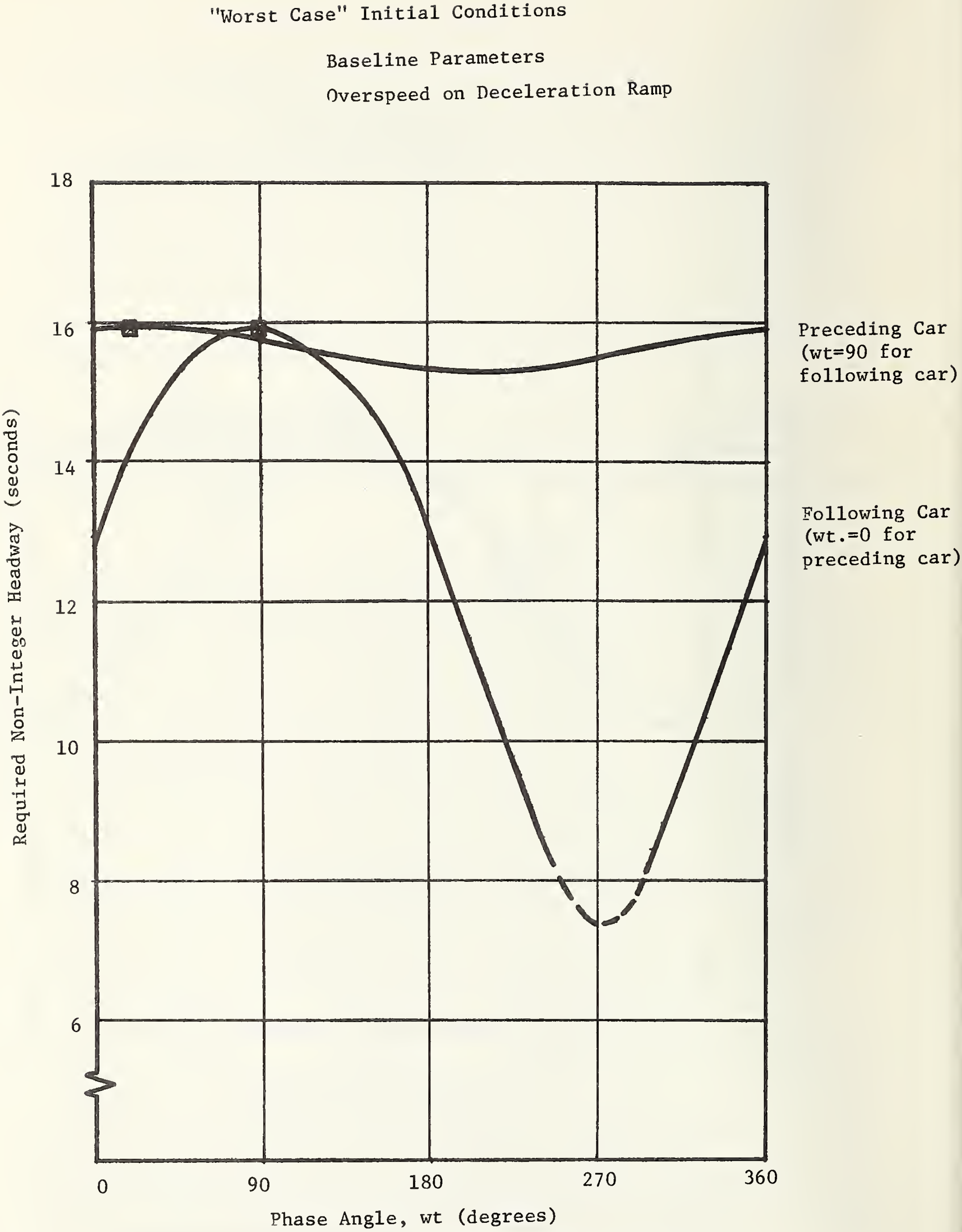

Figure 4-4. "Worst Case" Initial Conditions (Second)

4.3 SPEED AND LOOP LENGTH EFFECTS

The affect of speed and loop length on the headway required on the main guideway is shown on Figure 4-5. Unexpected stop is the critical hazard and required headways range from 3 seconds to something over 6 seconds. Headways increase with both loop length and speed. There is a cyclic variation in the required headway as loop length increases. This variation is due to the fact that an integral number of loops are specified for the headway distance. As an integral number of loops alternately match and mismatch the non-integer headway, a broken curve is generated.

Curves for the hazard of overspeed on the deceleration ramp are shown in Figure 4-6. In this case, however, the abscissa is the ratio of loop length to speed, instead of absolute loop length. This ratio is directly proportional to the time it takes the car to traverse a loop. For a deceleration ramp it is a more meaningful parameter, since, if the loop length to speed ratio is constant, the loop spacing near the end of a 40 mph ramp is the same as the loop spacing over most of a 20 mph ramp (independent of the absolute loop length at main guideway speed). Because of this effect, for low ratio of loop length to speed, the headway required does not depend on main guideway speed. At higher ratios of loop length to speed, overspeed failures take longer to discover, and the fact that the maximum possible speed increases with main guideway speed, makes for more serious failures as guideway speed increases; thus, required headways increase.

Headway on a deceleration ramp is much more sensitive to loop length than on the main guideway and also more sensitive to main guideway speed, as the loop-length ratio gets larger. Ratios of loop length to speed of 0.5 to 0.7 lead to very large required headways. This result clearly suggests reason to use, if possible, shorter loops on the deceleration ramp than on the main guideway.

A plot of variation of headway with speed, for the main guideway, is shown on Figure 4-7. The hazard is unexpected stop at all speeds. The ratio of loop length to speed is held constant at the baseline value of 0.33 feet/mph. It is seen that the curve has the general form discussed in Section 2.2.1 Minimum headways occur at about 12 mph.

Effect of Speed and Loop Length

Baseline Parameters

Unexpected Stop on Main Guideway

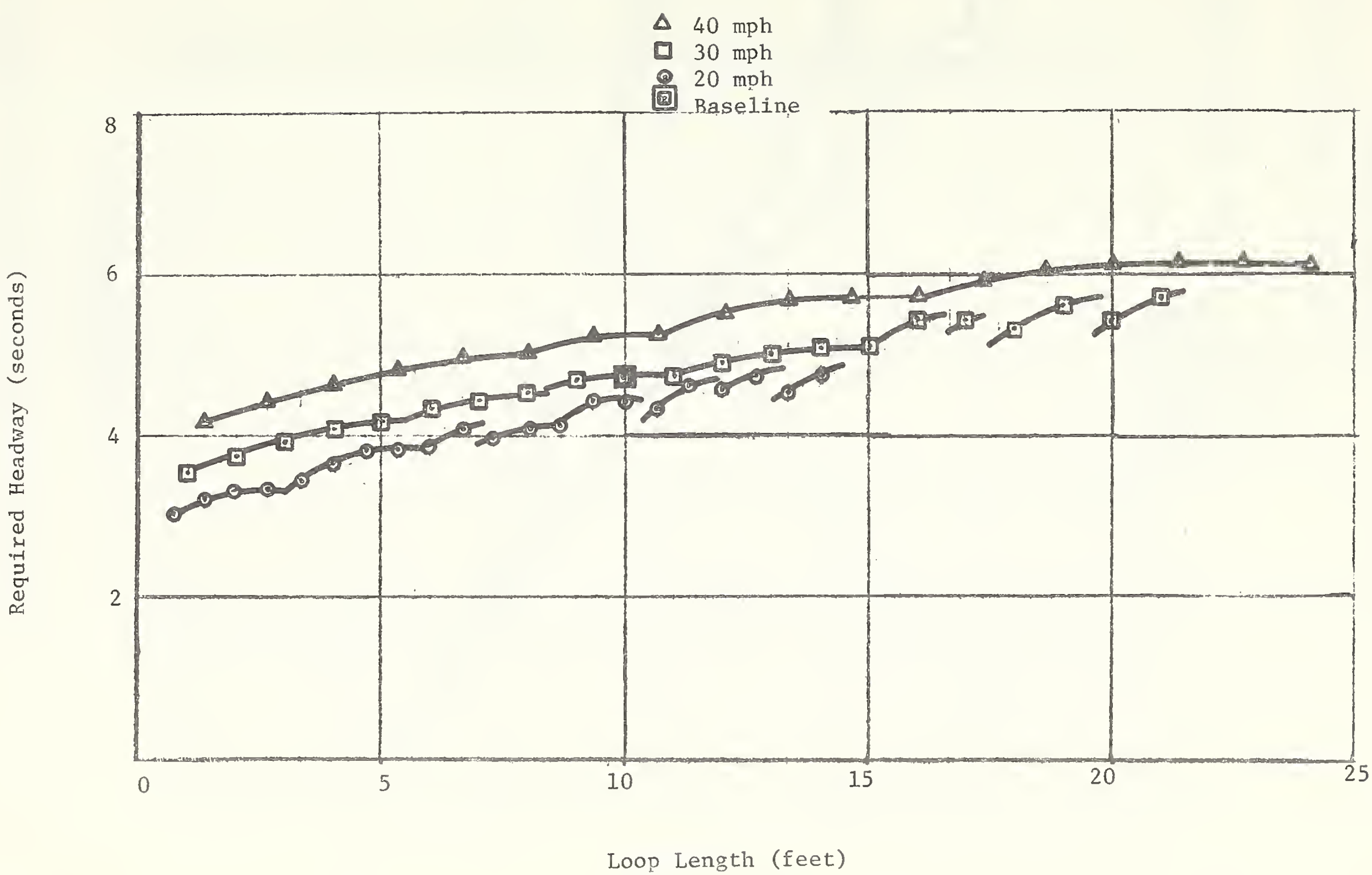

Figure 4-5. Effect of Speed and Loop Length (First)

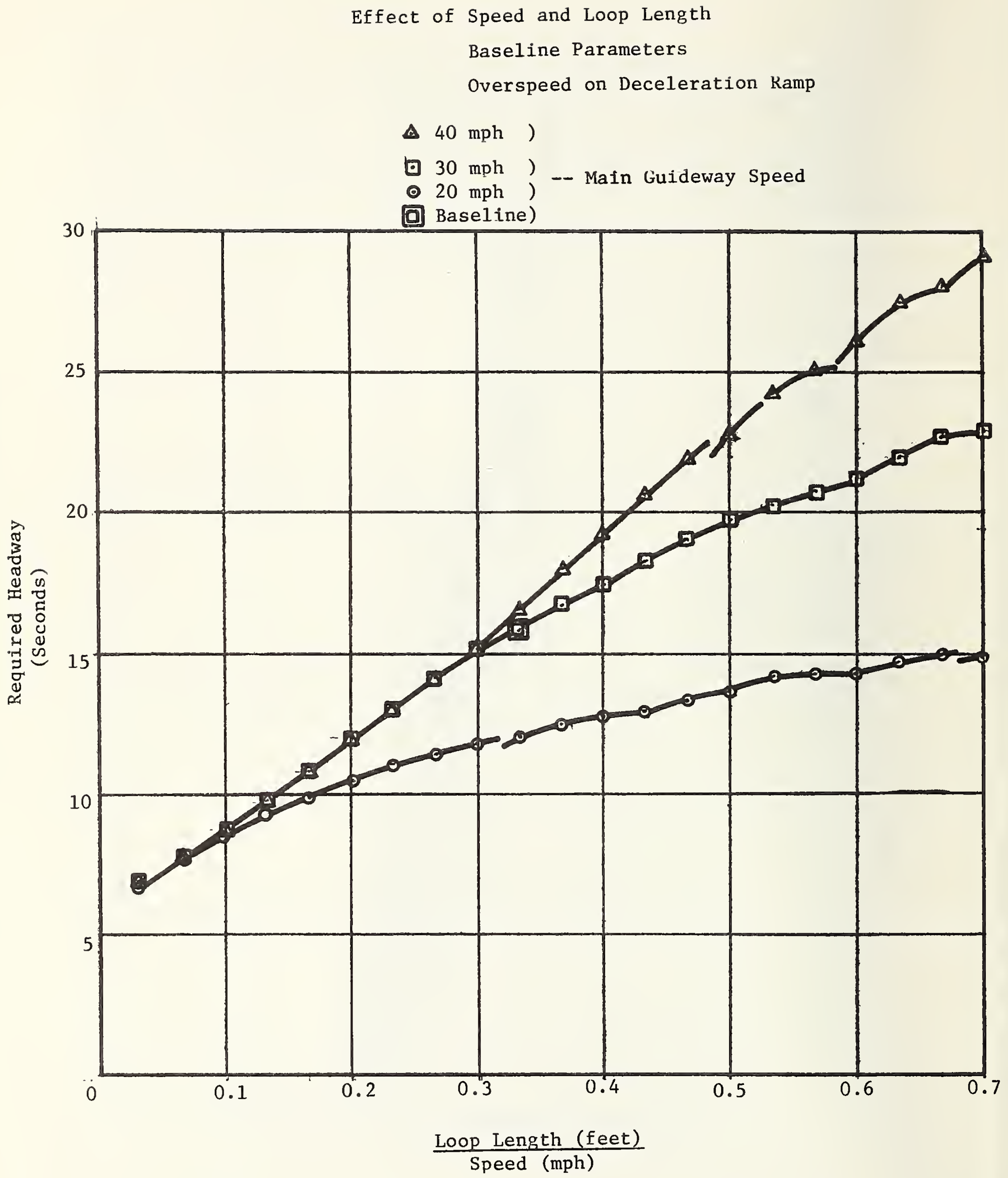

Figure 4-6. Effect of Speed and Loop Length (Second)

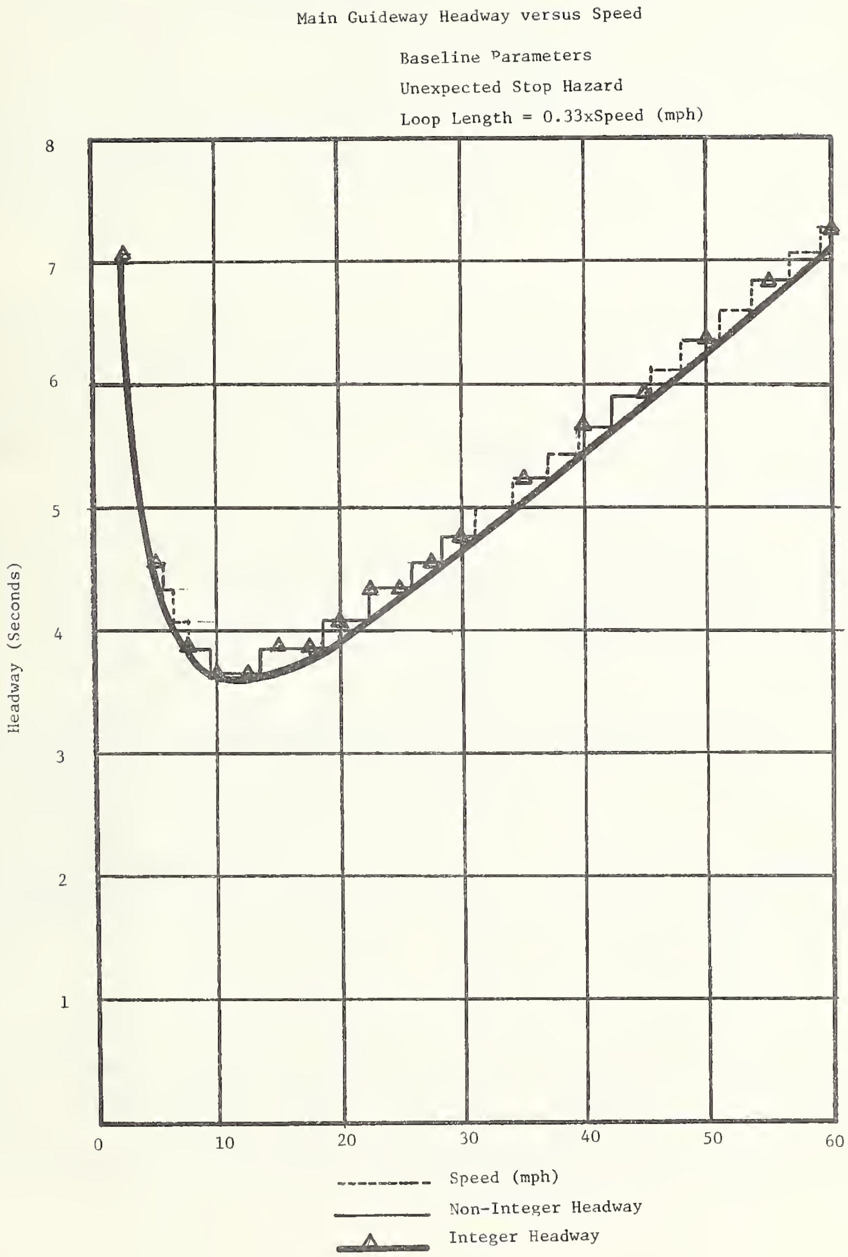

Figure 4-7. Main Guideway Headway Versus Speed

4.4 EFFECT OF VELOCITY DEVIATION

As pointed out earlier, a given loop length and control acceleration imply a value for velocity deviation. This implied velocity deviation, in percent, is plotted in Figure 4-8 as a function of loop length and speed. The percent velocity deviation goes up as the loop length goes up, i.e., less control accuracy can be insured with longer loops. The baseline control loops, at 30 mph, imply a velocity deviation of about 10 percent.

If loop length is reduced to 3 feet at 30 mph, it is necessary to control the velocity within plus or minus 5%, or else there will be false alarms. Thus, reduction of loop length, by itself, is not a satisfactory technique to obtain shorter headways. It is also necessary to provide a tighter system control.

On the other hand, it is possible to assume that control deviations are less than the full loop corridor (recognizing, as discussed in Section 3-B, that there may be no way to insure that they are).

The importance of restricted control deviations is illustrated on Figures 4-9 and 4-10. As loop length is increased it is assumed the vehicle maneuver corridor remains defined by one foot loops (at 30 mph). Required headways are less, but not that much less. Velocity deviations account for about 1/3 of the increase in headway with loop length while about 2/3's is due to other factors. (See Section 2.3.1).

It must be concluded that if short headways are required, both control deviations and loop lengths must be reduced. To reduce only loop length will lead to false shut downs. Reducing control deviations is only partially effective without commensurate reductions in loop length.

Implied Velocity Deviation as a Function
of Loop Length and Speed

Control Acceleration is 0.1 g

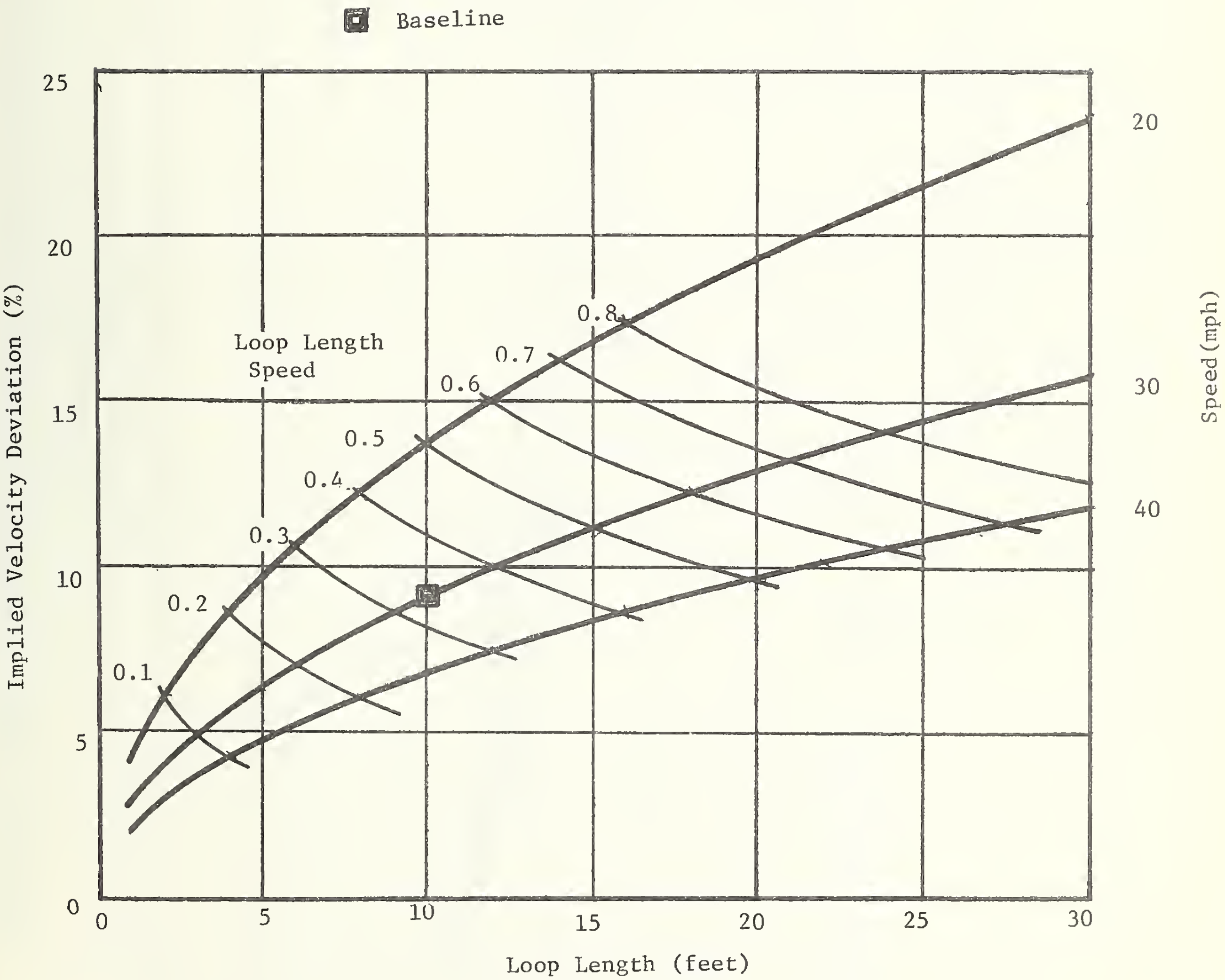

Figure 4-8. Implied Velocity Deviation as a Function
of Loop Length and Speed

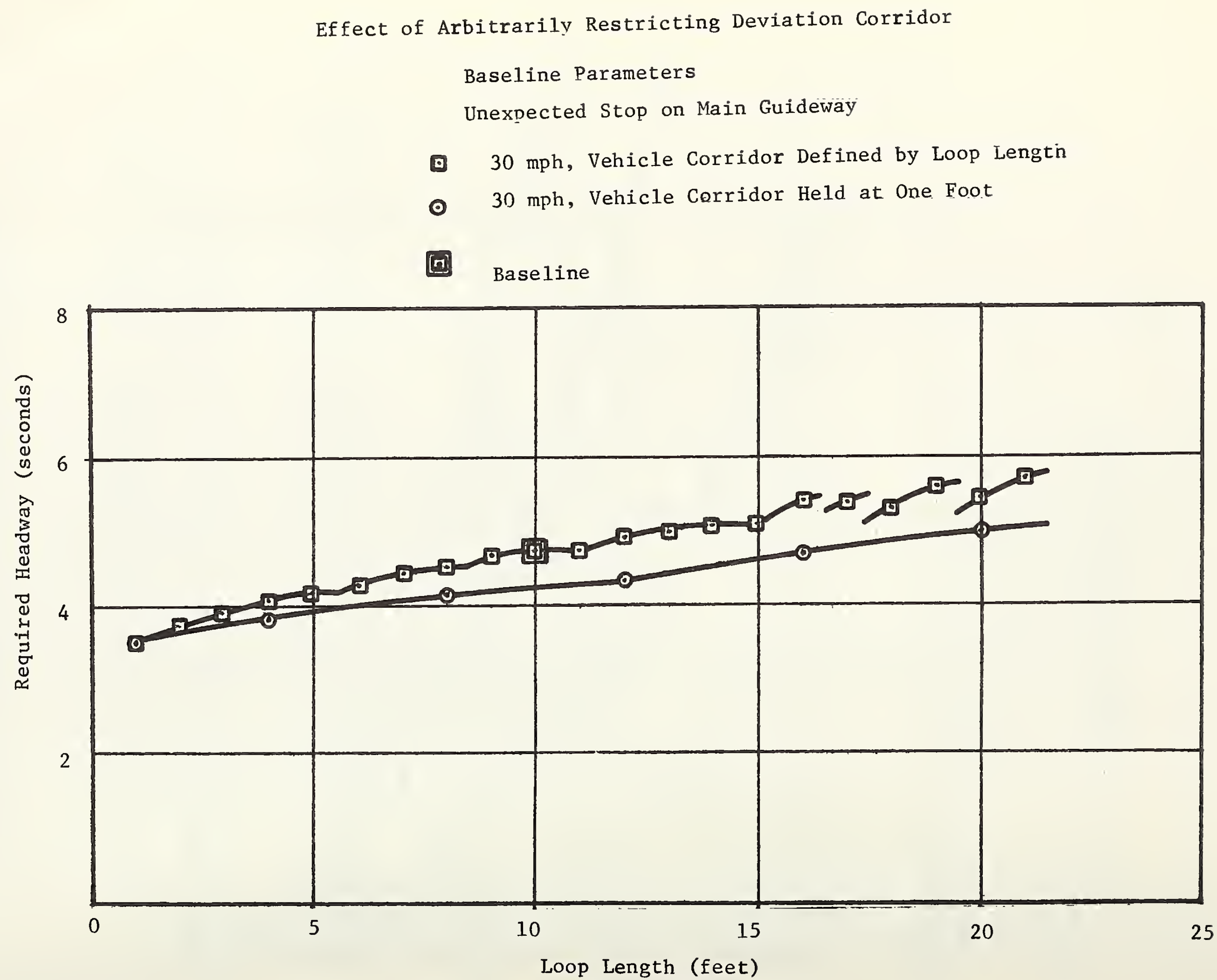

Figure 4-9. Effect of Arbitrarily Restricting Deviation Corridor (First)

Effect of Arbitrarily Restricting Deviation Corridor

Baseline Parameters

Overspeed on Deceleration Ramp

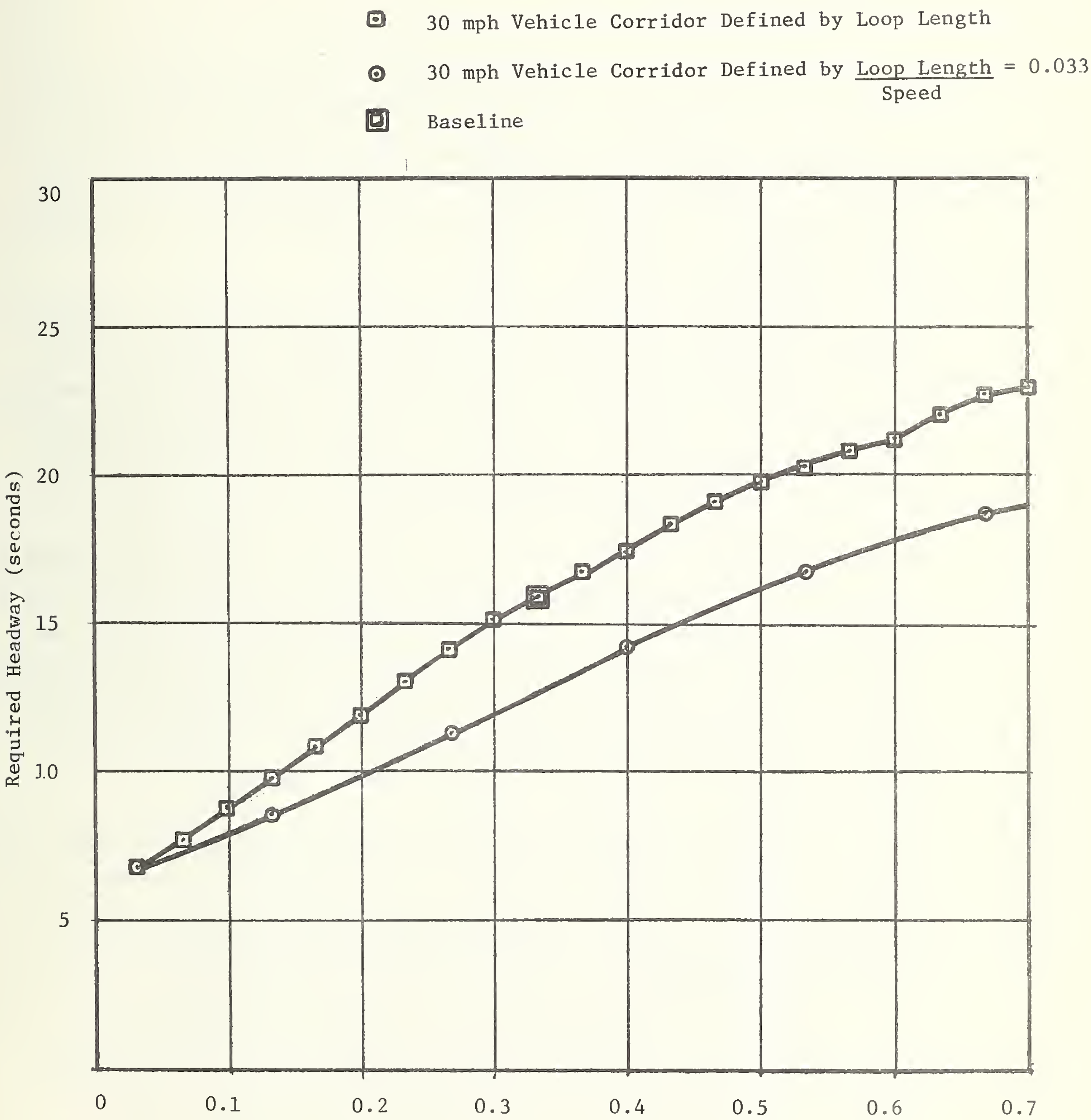

Figure 4-10. Effect of Arbitrarily Restricting Deviation Corridor (Second)

4.5 SENSITIVITY RUNS

The sensitivity of headways to changes in system parameters is tabulated on Figure 4-11. Incremental headways are tabulated relative to the baseline headways; 4.77 seconds for unexpected stop on the main guideway, and 16.14 seconds for overspeed on the deceleration ramp. The parameters that are varied are shown in the first column and in the second column the baseline values are shown.

Three groups of parameters are logically related and all parameters in these groups are varied together. Thus, car length and safety factor (which appear as added terms in the analysis) are grouped. Nominal emergency deceleration and jerk, which depend on the comfort criteria specified by the user form another group. The third group is correction acceleration and nominal deceleration and jerk on the ramp.

One set of modified parameter values on Figure 4-11 defines reduced headways, the other set defines increased headways. The incremental changes in headway are shown opposite each set of parameter values, under the major headings Reduced Headway and Increased Headway. The reduced headway parameter values are felt to be feasible, but would require considerable development work. On the other hand, parameters defining increased headway are not unreasonable upper tolerances on the state of the art.

The headway increments are based on integer headways, and thus, the headway increases in increments of 0.227 (0.23) seconds. Since this effect distorts small changes, for those cases where the non-integer increment differs appreciably from the integer increment, the non-integer increment is shown in parenthesis.

Those cases in which the headway increment is not affected by the parameter are shown with dashes. For example, the failed car deceleration does not affect overspeed on the deceleration ramp. Similarly the acceleration of an overspeed car does not affect unexpected stop on the main guideway.

Base Line Headway	Unexpected Stop on Main Guideway 4.77 Seconds Overspeed on Deceleration Ramp 16.14 Seconds

Parameter	Base Line Value	Reduced Headway: Parameter Value	Reduced Headway: Main Guideway Headway Increment	Reduced Headway: Deceleration Ramp Headway Increment	Increased Headway: Parameter Value	Increased Headway: Main Guideway Headway Increment	Increased Headway: Deceleration Ramp Headway Increment
[1] [Car Length (ft) Safety Factor]	[15.0 5.0]	[10.0 0.0]	-0.22	-1.37	[25.0 5.0]	+0.46	+2.72
Brake Tolerance	0.05	0.02	-0.00 (-0.10)	-0.69	0.10	+0.23	+1.13
Reaction Time Delay (Seconds)	0.2	0.10	-0.00	-1.37	0.40	+0.23	+2.27
Failed Car Decel. (g's)	1.0	0.6	-0.22	---	10.0	+0.23 (+0.13)	---
Acceleration of Overspeed Car (g's)	0.25	0.15	---	-3.19	0.35	---	+2.27
[Nominal Emergency Deceleration (g's) Nominal Emergency Jerk (g's/sec)]	[0.25 0.375]	[0.40 0.60]	-1.36	-6.82	[0.15 0.225]	+2.28	+13.41
[Correction Acc. (g's) Nominal Deceleration on Ramp (g's) Nominal Jerk on Ramp (g's/sec)]	[0.1 0.1 0.1]	[0.05 0.05 0.05]	-0.22 (-0.16)	-7.28	[0.15 0.15 0.15]	+0.23 (+0.13)	+4.31
Maximum Speed Factor	1.1	1.05	---	-0.46	1.3	---	+0.45
Transition Speed (mph)	5.00	7.00	---	-3.19	3.00	---	+5.91
Combined Parameter Increments		[2]	-2.38	-12.62	[3]	+4.32	+74.77
Combined-Parameter Headways			$\boxed{2.39}$	$\boxed{3.52}$		$\boxed{9.09}$	$\boxed{90.91}$

X.XX Integer Increment

(X.XX) Non-integer Increment

1) Brackets indicate parameters are changed together.

2) All reduced headway parameter values except correction acceleration, ramp deceleration and ramp jerk. Base line values are used for these parameters. In addition, main guideway loop length is 5 feet.

3) All increased headway parameter values except correction acceleration, ramp deceleration and ramp jerk. Base line values are used for these parameters. In addition, main guideway loop lengths is 20 feet.

Figure 4-11. Headway Sensitivity to Parameter Changes

The most striking headway increments occur when nominal emergency deceleration and jerk are changed. These parameters are primarily set by the user of the system and/or the passenger configuration. The baseline values are consistent with standing passengers who have a readily available support. Parameter values that go with increased headway (0.15 g and 0.225 g per second) imply no ready support. In this case, the headway increment on the deceleration ramp is 13 seconds, i.e., absolute headway increases to 30 seconds. On the other hand, if the emergency deceleration and jerk values are raised to 0.4 and 0.6 respectively (values which are consistent with well-supported, seated passengers) headway required on the deceleration ramp can be reduced a full 7 seconds. The effects on main guideway headways are relatively just as great, although absolute increments are not as large.

The ride comfort parameter group, which defines correction acceleration and deceleration and jerk on the ramp, also has a sizeable effect on headways. The baseline values are consistent with standing passengers with ready support. Reduced headway values are consistent with freely standing passengers, and the headway increment is 7.28 seconds on the deceleration ramp. The increased headway parameter values are consistent with seated passengers, and 4.31 seconds more headway is required than for the baseline. While lower deceleration limits decrease the required headways, it should be kept in mind that a price is paid in longer ramps. Higher limits conversely decrease ramp lengths.

Transition speed also has a major effect on the ramp headway. Higher transition speeds allow the preceding car to get out of the way of the overspeeding car more rapidly, and hence decrease required headways. The price in this case is a greater safety problem on the station platform. Lower transition speed appreciably increases headway required on the deceleration ramp.

Increments in headway, when parameters are combined, are shown in the next-to-the-last row of the table. In all cases, however, the "comfort group" is kept at the baseline value of 0.1 g. The reason for this decision, looking at the reduced headway set, is that the rest of the parameters define a high performance system, and it is unlikely that the reduced headway "comfort group" (which defines a low performance system) would be specified at the same time. A converse argument holds for the increased headway parameter values.

For three of the four cases, the combined increments are very close to the sum of the individual increments. For small changes in headway, this rule seems to be good. The very large increment for the increased-headway, deceleration-ramp case indicates that it is well out of an additive range, however. A more powerful cumulative degradation in performance is at work here. Unless parameters are controlled, things can get bad very quickly.

The combined-parameter headways (the numbers in the rectangles) demonstrate that headways of 2.5 seconds and 3.5 seconds are possible on the main guideway and deceleration ramp, using reduced headway parameters. If the increased-headway parameter values are used, headways on the main guideway roughly double, (9.1 seconds) but headways on the deceleration ramp are 10 times larger. Headway required on the deceleration ramp, besides being the critical condition, is clearly much more sensitive to parameter changes.

It is seen, therefore, that with advanced-system parameters, the reference headway protection design being demonstrated under DOT-TSC-421 is capable of insuring main guideway headways of 2.5 seconds.

5. SUMMARY AND CONCLUSIONS

An analysis is developed to determine safe headway on PRT systems that use point-follower control. It is assumed that successive vehicles follow a fixed nominal trajectory relative to the guideway, separated by a headway time. Periodic measurements of the position error relative to the nominal trajectory provide warning against the hazards of overspeed and unexpected stop. A computer program has been developed to model these hazards: for arbitrary nominal trajectories and periodic deviations from the nominal trajectories.

The results of computer runs indicate that the critical hazard on the main guideway is unexpected stop of a preceding car; on a station entry deceleration ramp, it is overspeed of a following car. The deceleration ramp headways are much more sensitive to system parameters than are the main guideway headways. In both cases, improvements in control errors must go hand-in-hand with improvements in sensor resolution in order to achieve minimum headways.

Typical headways are five seconds on a 30 mph main guideway and 16 seconds on a deceleration ramp for state-of-the-art system parameters and acceleration constraints suitable to standing passengers with ready support. With advanced system parameters and emergency decelerations applicable to well supported, seated passengers, required headways are 2.5 seconds on the main guideway and 3.5 seconds on the deceleration ramp.

The position error headway protection system being developed under the subject contract (DOT-TSC-421) has the capability to operate at 2.5 second headway on a 30 mph guideway.

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APPENDIX

REPORT OF INVENTIONS

After a diligent review of the work performed under this contract, we note that no new innovations, discoveries, improvements or inventions were made.