REPORT NO. DOT-TSC-FHWA-73-3

LANE BLOCKAGE EFFECTS ON FREEWAY TRAFFIC FLOW

D. Kahn
M. Kierstead
W. Stevens

JANUARY 1974
FINAL REPORT

Prepared for
DEPARTMENT OF TRANSPORTATION
FEDERAL HIGHWAY ADMINISTRATION
Office of Research
Washington DC 20590

Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
DOT-TSC-FHWA-73-3		

4. Title and Subtitle

LANE BLOCKAGE EFFECTS ON
FREEWAY TRAFFIC FLOW

5. Report Date

January 1974

6. Performing Organization Code

7. Author(s)

D. Kahn, M. Kierstead, W. Stevens

8. Performing Organization Report No.

DOT-TSC-FHWA-73-3

9. Performing Organization Name and Address

Department of Transportation
Transportation Systems Center
Kendall Square
Cambridge MA 02142

10. Work Unit No. (TRAIS)

R4102/HW-408

11. Contract or Grant No.

12. Sponsoring Agency Name and Address

Department of Transportation
Federal Highway Administration
Office of Research
Washington DC 20590

13. Type of Report and Period Covered

FINAL REPORT
July 1 - Sept. 30,1973

14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The traffic-density buildup following a lane blockage on a four-lane freeway carrying low-density traffic is determined for several different densities (0.0065, 0.0100, and 0.0106 vehicles per foot) characterizing the freeway. The time for the traffic to return to normal after the blockage is removed is also calculated. The traffic-density buildup following a lane blockage on a four-lane freeway carrying high-density traffic has been considered.

17. Key Words

· LANE BLOCKAGE
· TRAFFIC SHOCK WAVE
· FREEWAY TRAFFIC FLOW

18. Distribution Statement

DOCUMENT IS AVAILABLE TO THE PUBLIC THROUGH THE NATIONAL TECHNICAL INFORMATION SERVICE, SPRINGFIELD, VIRGINIA 22151.

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	38	

Form DOT F 1700.7 (8-72)

PREFACE

The problem of analysis of shock wave phenomena for freeway control has been undertaken as part of an overall freeway corridor traffic-improvement program. The effort herein is an attempt to shed some light on the understanding of how disruptive shock phenomena are on the freeway, and what actually happens to vehicles as a result.

This final report covers the work which has been performed at the Transportation Systems Center (TSC) during the first quarter of fiscal year 1974 for the Traffic Systems Division of the Federal Highway Administration, Department of Transportation.

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1. INTRODUCTION

The concept of traffic flow modification to produce improvement in traffic flow is becoming more widely accepted today. This is not so much so because of theoretical studies which have proved the efficacy of traffic flow modification but rather because of practical benefits which have resulted from the implementation of traffic flow modification. An example of such traffic flow improvements is the increased through-put and decreased congestion time which results from the real time traffic flow modification techniques used in the Lincoln and Holland tunnels by the Port Authority of New York. Other examples include traffic flow modification procedures used in traffic networks of city streets, as for example, in Metropolitan Toronto. In the case of freeways, success has perhaps been less pronounced with the main thrust taking the form of ramp metering. On-freeway control such as variable message signs which indicate downstream conditions is, understandably, more difficult. On the other hand since theoretical studies have indicated that traffic flow modifications could yield significant improvement in traffic flow by reducing the severity of shock wave phenomena through adjustments of traffic speed and traffic density, the use of On-freeway controls is potentially very useful.[1-10]

One problem associated with on-freeway traffic control is incident detection, for example, lane blockage. In order to detect the lane blockage, particular parameters affected by the lane blockage have to be determined. For example, the increase in lane density upstream of the blockage. If these parameters can be determined, a detection procedure could be developed and a traffic flow modification technique could be devised.

Under light freeway traffic conditions, a theory was developed which allowed calculation of traffic densities following a lane blockage on a multi-lane freeway.[11] In the present note we provide further examples of this previous theory by calculating the traffic density in each lane of a four-lane freeway as a function of time since the onset of the blockage. These examples of different initial traffic flow conditions are provided in Section 2.

In Section 3, the time for traffic conditions to return to normal after the blockage is removed is estimated. This information is useful for the development of freeway incident removal strategies.

Finally, in Section 4 the case of heavy initial traffic densities on a four-lane freeway is discussed.

2. TRAFFIC DENSITIES FOLLOWING A LANE BLOCKAGE

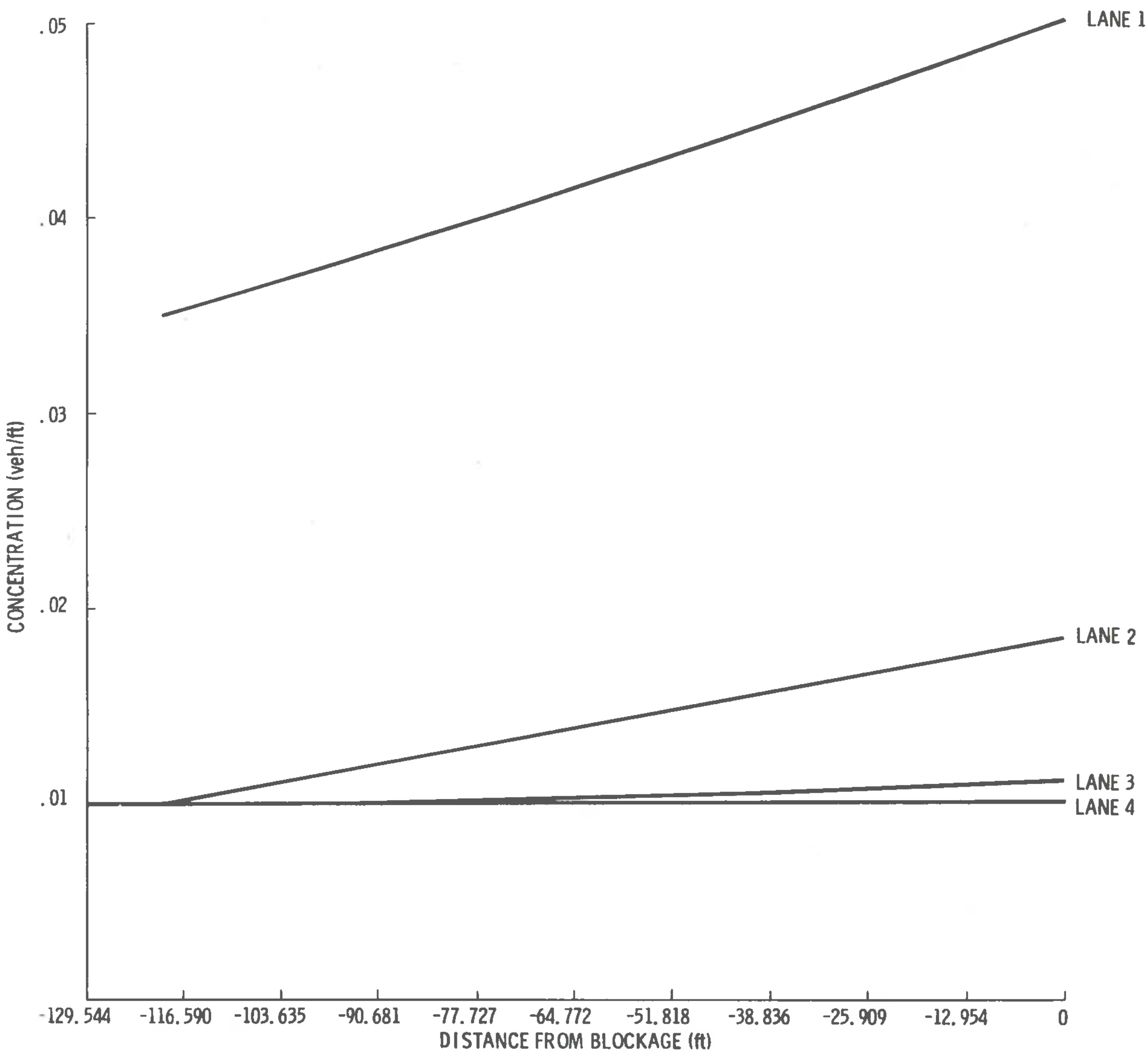
2.1 STEADY STATE

In the light traffic density situation considered in this section, after a given amount of time following a lane blockage, a steady state is reached.[11] We examine the traffic densities following a lane blockage on two different freeways (characterized by different K_j and K_m) after steady state has been reached. K_j is the jam concentration and K_m is the concentration at which the flow is maximum. In one case the initial density before the blockage occurred is K_o=0.0065 vehicles per foot, and in the other case it is K_o=0.01 vehicles per foot.

After steady state is reached, the concentration K does not increase with a further increase in time. We show the traffic densities that would be measured by detectors placed in each lane of a four-lane freeway upstream of the blockage point. For details of the theory, see reference 11.

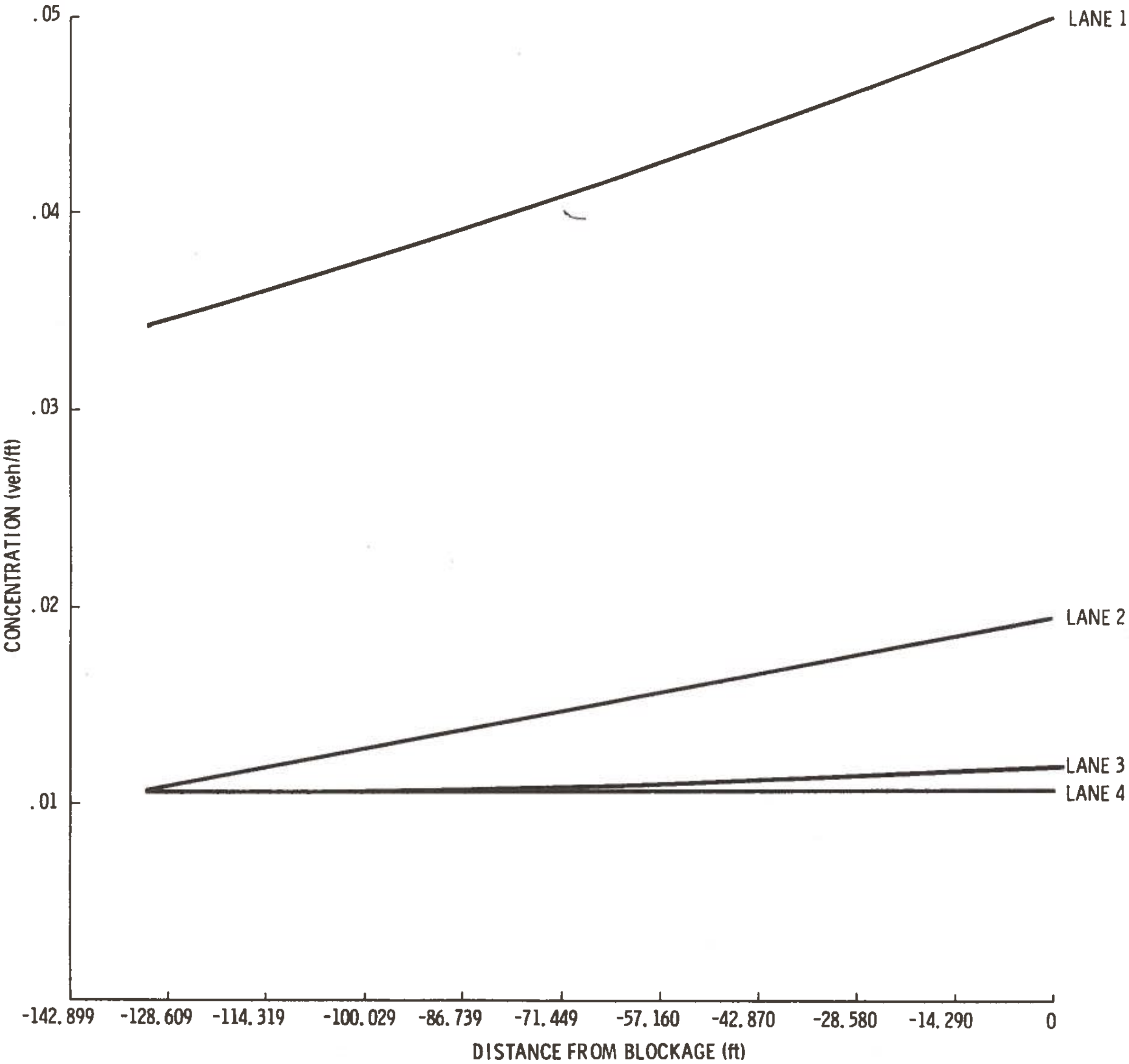
At x=0 the traffic density in the blocked lane is at jam concentration because of the blockage, assumed to occur at x=0. The traffic densities that would be measured at different upstream locations from the blockage in each freeway lane are shown in Figure 1. In lane 1, the density at the steady state position, x=-120 feet, is reduced to a value of 0.035 vehicles per foot (beyond that it is at the normal 0.01 vehicles per foot). In the other lanes the density is gradually reduced to 0.01 vehicles per foot at the steady state position. The density is greatest in all four lanes at the blockage point, at x=0; is greatest in the blocked lane and diminishes in intensity from lane 1 outward. In lane 2, the lane closest to the blocked lane, the density is higher than in lanes 3 or 4 since most cars switching from the blocked lane end up in lane 2. Lanes 3 and 4, as can be seen in the figure, are also affected but less so than lane 2.

Figure 2 shows the same freeway (same K_j and same K_m) as figure 1, but with a higher initial traffic density. Because of



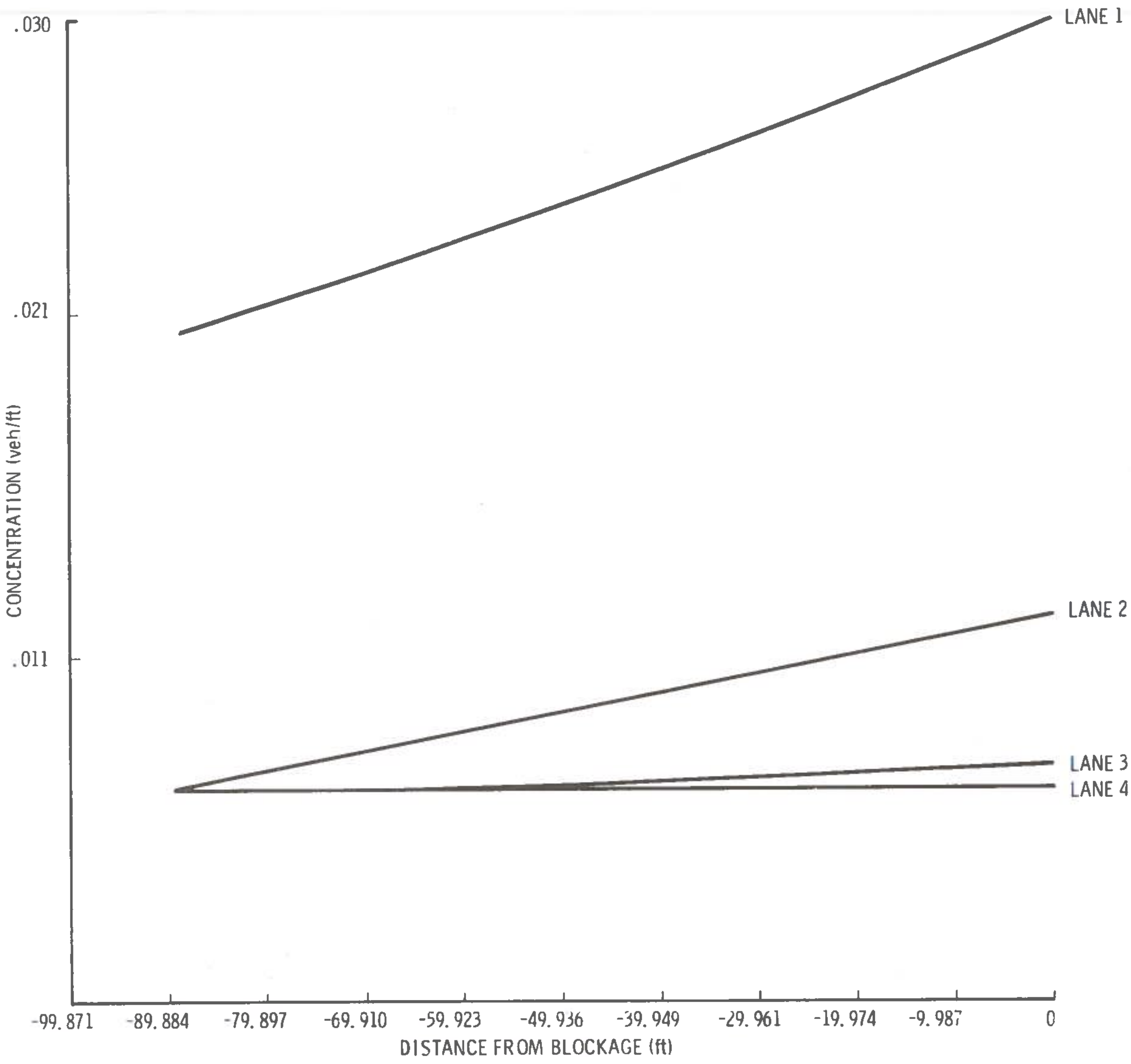
K_o = 0.01 vehicles/foot
K_m = 0.02 vehicles/foot
K_j = 0.05 vehicles/foot
q_m = 1 vehicle/second

Figure 1. Steady-State Traffic Concentration as a Function of Distance from Blockage



K_o = 0.0106 vehicles/foot
K_m = 0.02 vehicles/foot
K_j = 0.05 vehicles/foot
q_m = 1 vehicle/second

Figure 2. Steady-State Traffic Concentration as a Function of Distance from Blockage



K_o = 0.0065 vehicles/foot
K_m = 0.0121 vehicles/foot
K_j = 0.0303 vehicles/foot
q_m = 0.4 vehicle/second

Figure 3. Steady-State Traffic Concentration as a Function of Distance from Blockage

the higher initial density the point upstream at which steady state is reached is further out (the shock wave generated at the blockage has a higher initial speed and is maintained longer in the heavier density case). Again, higher traffic densities are observed in lanes 1, 2, 3 and 4 at the blockage point, x=0, and lower ones further upstream from this point. At the steady state location, the traffic densities in lanes 2, 3 and 4 reduce to the initial value (0.0106 vehicles per foot) while in the blocked lane, lane 1, the density diminishes to a value of 0.0341 vehicles per foot leaving a predicted concentration discontinuity in lane 1 of 0.0235 vehicles per foot (the difference between the concentration at the steady state location and that beyond). To approaching traffic this discontinuity appears as a bottleneck, that is, as a sudden increase in traffic density.

For the freeway and initial traffic densities shown in figure 3, the steady state location is seen to be closer to the blockage point than it is in the freeway shown in figure 1. This is because the initial shock wave speed is less in this case and the shock dissipates more quickly in this lower density case. However, other than the shorter distance traveled by the traffic discontinuity (shock wave), the same functional behavior of the traffic density with distance upstream from the blockage is exhibited in both cases as seen in comparing figures 1 and 3.

It is emphasized that the traffic densities shown in figures 1 through 3 as a function of position on the freeway are those densities which would be measured after a sufficiently long time has passed for steady state to be reached.

In the next subsection we show the traffic densities that would be measured at various points in the roadway as a function of time since the blockage occurred.

2.2 NON-STEADY STATE

The non-steady state theory developed in Reference 11 which allowed calculation of traffic concentrations only along the shock wave front has been extended to allow calculation of traffic concentrations for any upstream position and for any time following the blockage (not necessarily only along the shock wave front).

In figures 4, 5 and 6 we show the densities that would be measured in each lane of a four-lane freeway as a function of time following a lane blockage which occurred in lane 1 at x=0 and at time t=0. Again, the reader is referred to reference 11 for the general theory.

Figures 4, 5 and 6 are for different K_j, K_m and K_o (corresponding to the values shown previously in the steady state cases of figures 1, 2 and 3). The three sets of figures show similar functional dependence of concentration on time and distance though different numerical values are obtained depending upon the different K_j, K_m and K_o characterizing the freeway and initial traffic conditions.

The figures illustrate the very rapid traffic density build up as a function of the time since the start of the blockage. These traffic density build ups are shown at three different upstream locations. We discuss only figure 4 in detail. Similar discussions apply to the other two sets of figures corresponding to the different K_j, K_m or K_o used to obtain those figures.

Referring to figure 4, we notice that the shock wave (or discontinuity) reaches x=-40 feet approximately three seconds after it started in lane 1 at x=0 at time t=0. In these three seconds the concentration in lane 1 reaches a value of 0.044 vehicles per foot (from a value of 0.01 vehicles per foot before the blockage started). In comparison, at x=-120 feet (the steady state position of the discontinuity) the effects of the shock wave are not felt until about 38 seconds after the occurrence of the blockage, and at this time the concentration in lane 1 increases from K_o (0.01 vehicles per foot) to 0.035 vehicles per foot.

That the effects of the blockage are greater at x=-40 feet agrees with our intuition since this position is closer to the blockage (which means that fewer vehicles in lane 1 have had the chance to switch into lane 2). This principle that the concentra-

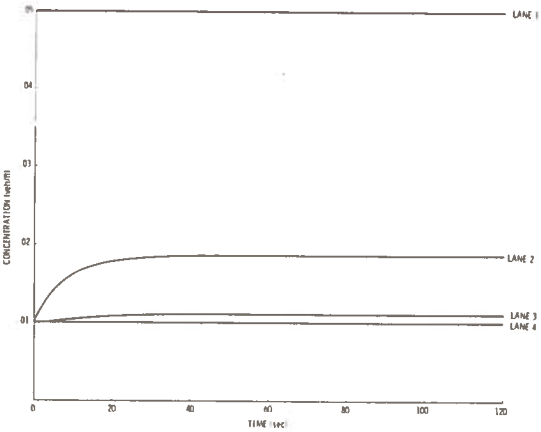


Figure 4a.
x=0

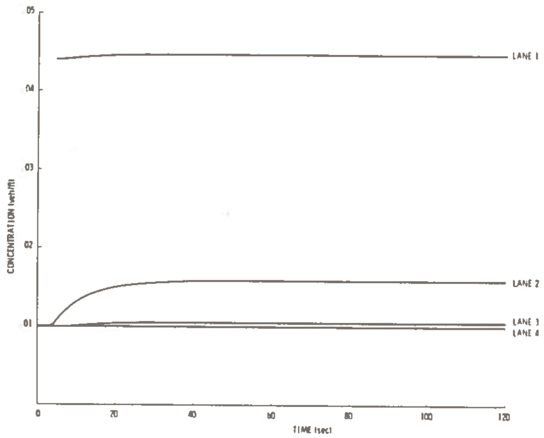


Figure 4b.
x=-40 ft.

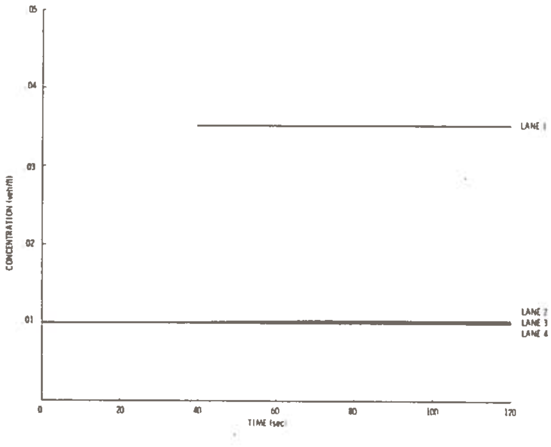


Figure 4c.
x=-120 ft.

Figure 4. Non-Steady-State-Traffic Concentrations as a Function of Time Since Lane Blockage Occurred. K_o=0.01, K_m=0.02, K_j=0.05 (vehicles/foot), q_m=1 vehicle/second

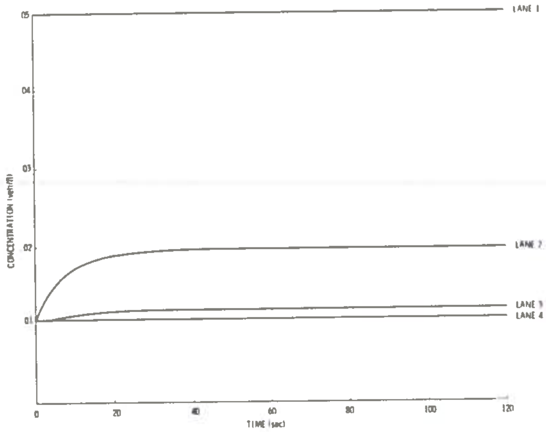


Figure 5a.
x=0

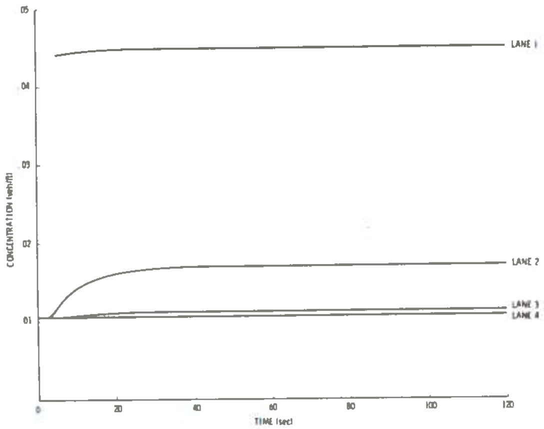


Figure 5b.
x=-40 ft.

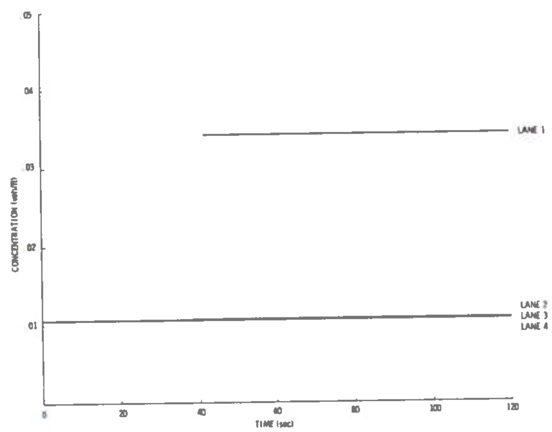


Figure 5c.
x=-135 ft.

Figure 5. Non-Steady-State-Traffic Concentrations as a Function of Time Since Lane Blockage Occurred. K_o=0.0106, K_m=0.02, K_j=0.05 (vehicles/foot) q_m=1 vehicle/second

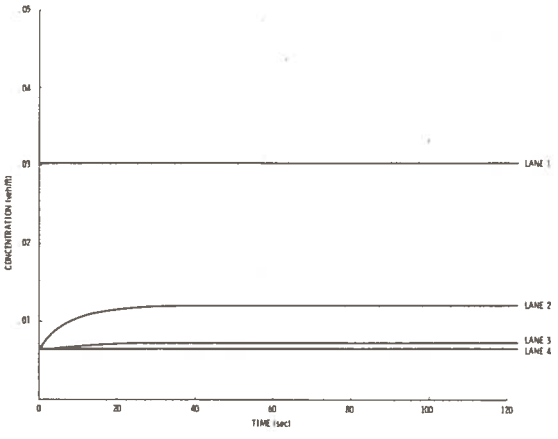


Figure 6a.
x=0

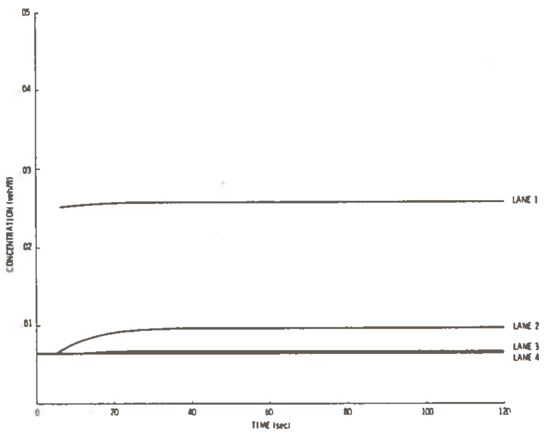


Figure 6b.
x=-40 ft.

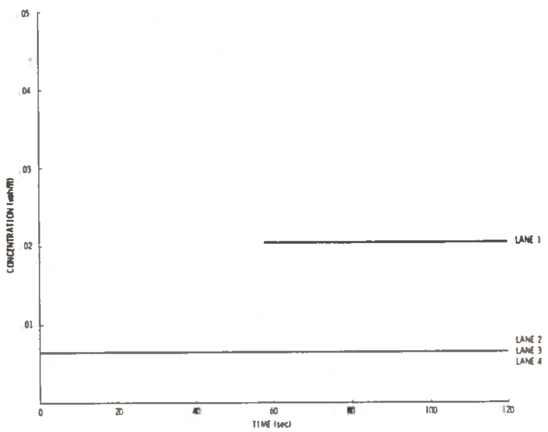


Figure 6c.
x=-92 ft.

Figure 6. Non-Steady-State-Traffic Concentrations as a Function of Time Since Lane Blockage Occurred. K_o=0.0065, K_m=0.0121, K_j=0.0303 (vehicles/foot), q_m=.4 vehicle/second

tions are less affected at positions further from the blockage is illustrated again by the fact that the concentrations in lane 2, 3 and 4 at x=-120 feet are hardly affected by the blockage while the concentrations in lanes 2 and 3 at x=-40 feet are affected significantly.

As mentioned above, the results for different K_j, K_m or K_o are similar. Figures 5 and 6 show these results for the different traffic density parameters. We again note the very rapid initial traffic density build up following the lane blockage and the much slower increase in traffic density as steady state is approached.

It should be re-emphasized that these results are for light-density traffic. This is the reason that a steady state is reached at such a relatively short distance from the point of blockage. As a practical matter of traffic flow improvement, the lane blockage under these light traffic densities causes only minor traffic congestion and the main reason for timely removal of the lane blockage is much more related to safety than to traffic flow improvement. The theory for the more interesting case of heavy freeway traffic density has not yet been treated in detail though steady state results for this case are discussed in Section 4.

Bearing in mind the limited traffic-density buildup associated with these light traffic density conditions we may still note under what conditions the blockage would be detected. If detectors happened to be placed beyond the steady state location (here in figure 4, 120 feet upstream of the blockage), the incident would not be detected, as normal (K_o) concentrations would be measured. If the detectors were located approximately 100 feet upstream of the blockage, the incident would be detected but not until approximately 40 seconds after its occurrence, (a concentration of 0.035 vehicles per foot would be measured in lane 1). For detectors located 75 feet upstream of the blockage the incident would be detected in 10 seconds and a concentration of 0.04 vehicles per foot would be measured in lane 1. Detectors located 40 feet upstream of the blockage would measure a concentration of 0.044 vehicles per foot 3 seconds after the occurrence of the blockage.

As will be seen in section 4 for heavier traffic densities, the traffic density build up will extend over far greater distances upstream of the blockage, and hence its detection becomes much more reasonable and realistic.

3. BLOCKAGE REMOVAL

In order to obtain an idea of how quickly traffic flow improvement will occur on removal of the lane blockage, estimates were made of the time needed for the traffic congestion following a lane blockage to return to normal. Lane switching was not included in the estimates so that these estimates will yield longer times than would actually be observed. Again, we stress that these estimates were made for light traffic conditions where the congestion extends for only very short distances from the blockage.

The congestion due to the lane blockage is determined from the theory presented in reference 11 (shown here in figures 1 through 6). Now assume that at time t=0 the blockage is removed and the first car at the blockage point, x=0, is assumed to accelerate uniformly to a speed $V(K_o)$. For this illustrative example we use a V-K relationship as shown in figure 7 where it has been assumed that the vehicular velocity remains constant for concentrations below $K_m/4$, decreases linearly for higher concentrations up to K_m and finally decreases exponentially for concentrations above K_m. This V-K relationship is used here rather than the previous one[11] in which V remained constant for all values of K below K_m, in order to be able to obtain the time it takes for the concentration to decrease to values below K_m, to K_o. With the original V-K relationship only times for which the concentration is reduced to K_m are obtainable; however, for either V-K relationship the times found for the traffic concentration to return to K_m were approximately the same.

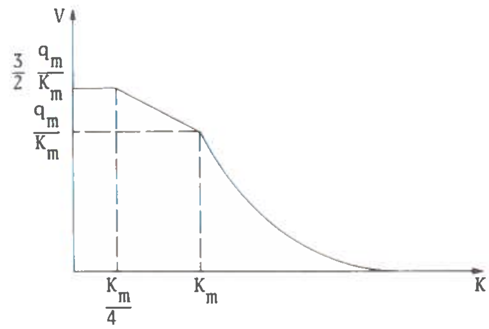


Figure 7. Velocity vs. Vehicle Concentration Diagram

The velocities of all other vehicles are determined by the concentration each experiences. For $K \geq K_m$ the velocities are given by[11]

$$V = q_m(K_j/K - 1)/(K_j - K_m) \qquad (3.1)$$

and for $K < K_m$ the velocities are given by (see figure 7),

$$V = \frac{q_m}{K_m}\left(\frac{5}{3} - \frac{2}{3}\frac{K}{K_m}\right) \qquad (3.2)$$

The concentration at the blockage is K_j; the concentration at the steady state location D is (see Ref. 11 for method of derivation)

$$K(D) = K_j + \frac{K_o}{K_m}\left(K_j - K_m\right)\left(\frac{2}{3}\frac{K_o}{K_m} - \frac{5}{3}\right) \qquad (3.3)$$

and q_m is the maximum flow. The concentration both upstream of D and downstream of x=0 before the blockage is removed is K_o.

Further, we assume steady state to exist and that the interval between x=0 and x=D contains the maximum number of vehicles possible, distributed uniformly by concentration.

The results are presented in figures 8, 9 and 10 for different K_j, K_m and K_o corresponding to the values used previously in the analyses of section 2. We discuss only figure 8 since, except for numerical differences, figures 9 and 10 for the different initial traffic density parameters exhibit similar behavior. The plots are of concentration as a function of position on the freeway for lane 1.

Upon removal of the blockage at time t=0, the traffic density is distributed as shown in figure 8a: The concentration is everywhere K_o except within the congestion area between x=0 and x=-120 feet where it ranges from a high of just under K_j to a low of K(D).

Five seconds later the traffic has become redistributed as shown in figure 8b. The congestion area has increased further along the roadway but the maximum level of the density has decreased.

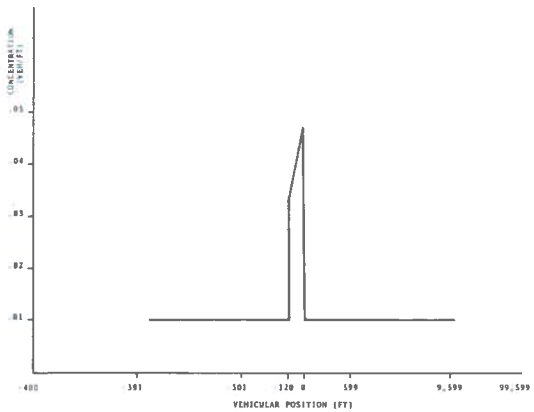


Figure 8a.
t=0 sec.

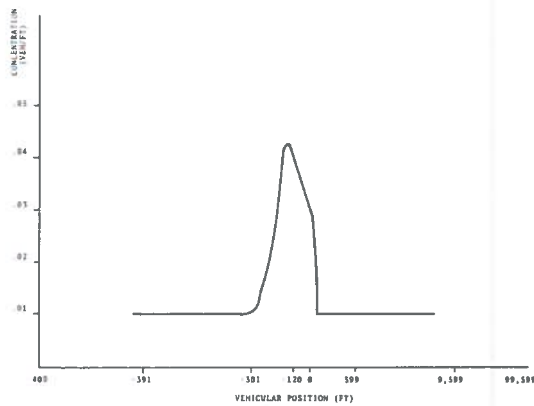


Figure 8b.
t=5 sec.

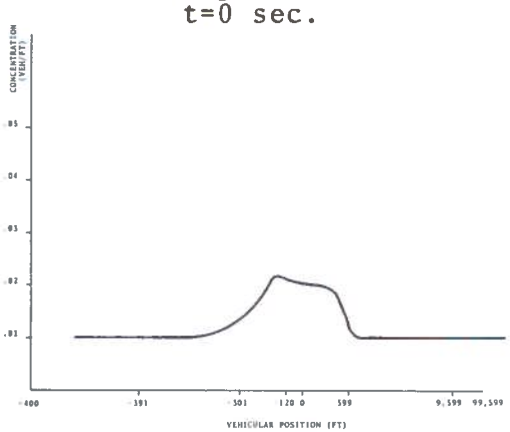


Figure 8c.
t=20 sec.

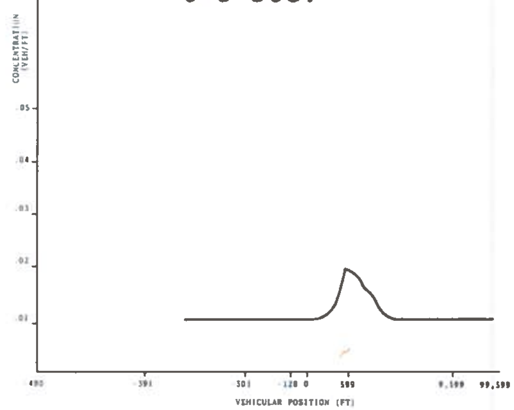


Figure 8d.
t=40 sec.

Figure 8. Traffic Density as a Function of Position Following the Removal of a Lane Blockage. $K_j=0.05$, $K_m=0.02$, $K_o=0.01$

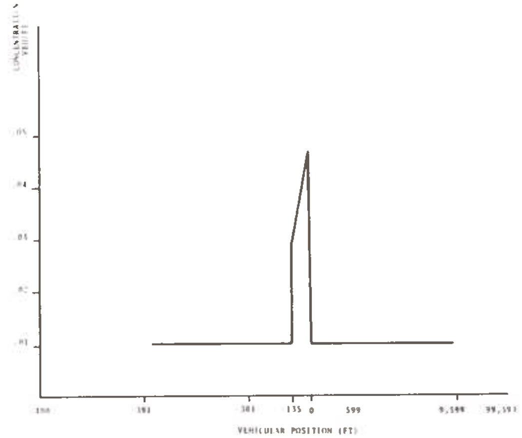


Figure 9a.
t=0 sec.

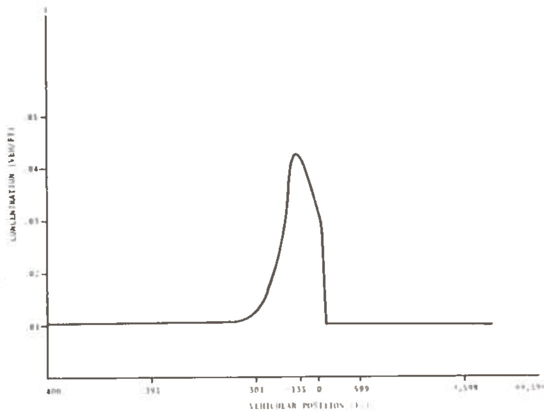


Figure 9b.
t=5 sec.

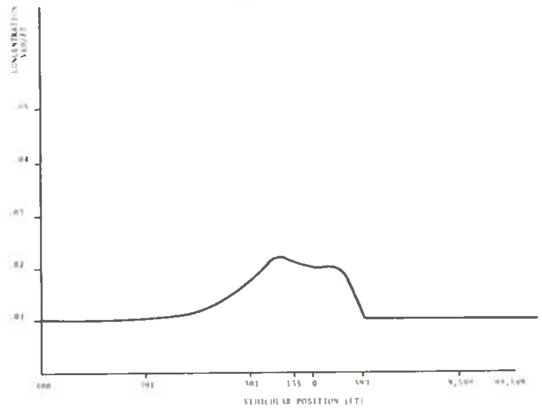


Figure 9c.
t=20 sec

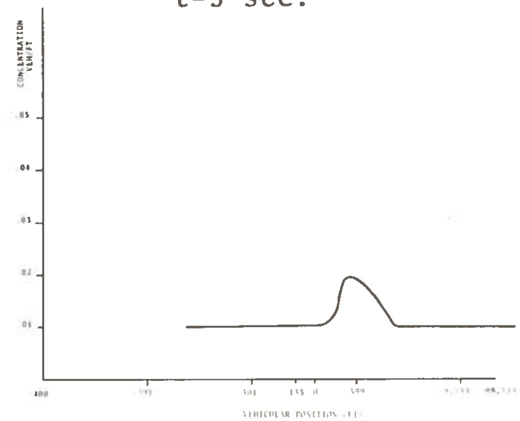


Figure 9d.
t=40 sec.

Figure 9. Traffic Density as a Function of Position Following the Removal of a Lane Blockage. K_j=.05, K_m=0.02, K_o=0.0106

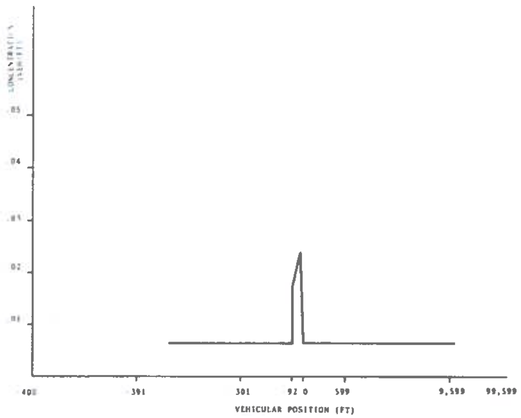


Figure 10a.
t=0 sec.

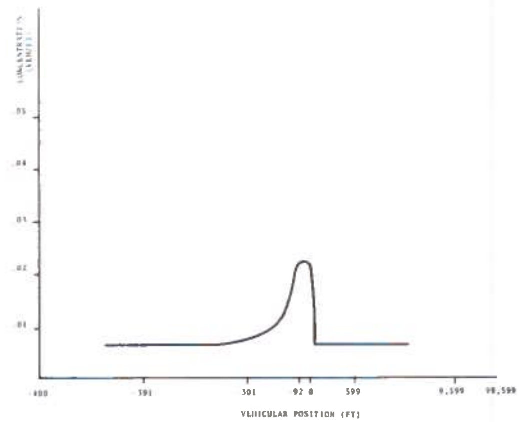


Figure 10b.
t=5 sec.

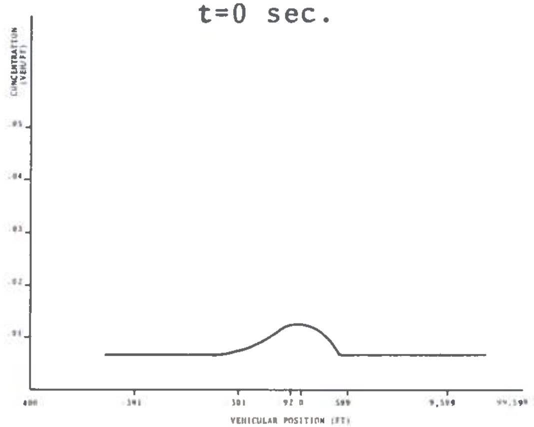


Figure 10c.
t=20 sec.

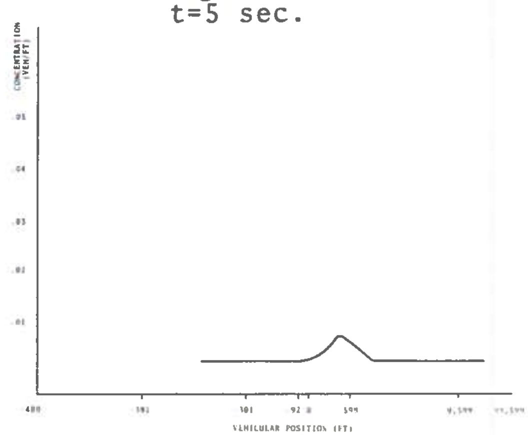


Figure 10d.
t=40 sec.

Figure 10. Traffic Density as a Function of Position Following the Removal of a Lane Blockage. $K_j = 0.0303$, $K_m = 0.0121$, $K_o = 0.0065$

The congestion region has increased because fast vehicles traveling with speeds appropriate for upstream portions of the freeway enter the congested area (and are slowed down). The maximum level of the density has decreased because the blockage has been removed with a resulting decrease in traffic density.

After 20 seconds (figure 8c) the density has been reduced to values just over K_m (so that traffic is close to ideal in the sense that K_m is the concentration at which the flow is maximum). By 40 seconds the concentration has already gone below K_m. The value of concentration K_o appropriate for the light traffic conditions assumed to exist on the freeway prior to the blockage is not reached until minutes later. However, once the traffic density is reduced to values below K_m, the congestion can be said to have effectively disappeared.

In an attempt to corroborate these results, we used a theory similar to that presented by Lighthill and Whitham[12] to approximate the times needed for the concentrations to fall below certain levels after blockage removal (see Appendix).

We found that although in using Lighthill's methods we assumed that the concentration in the interval between the blockage and the shock wave was K_j (which would make the times greater) these results differed from our original results by only 20-25% on the average. And in fact, when we used the same assumption (that $K=K_j$ in the interval between blockage and shock wave) in our original method, the results agreed to better than 10%.

The following summarizes the results: (vehicle/feet)

	$K_j = 0.05$ $K_o = 0.01$	$K_j = 0.05$ $K_o = 0.0106$	$K_j = 0.0303$ $K_o = 0.0121$
Time to return to K_m	27 sec	29 sec	30 sec

4. FREEWAY WITH HIGH-DENSITY TRAFFIC

In this case traffic conditions are so heavy that $K_o > K_m$ in all lanes. Assuming that steady state is reached, we have from Munjal et al[13] that:

$$K(x,t) = MB(x)\, M^{-1}\, K(0,t - x/c_1) \tag{4.1}$$

where c_1 is the velocity of the shock wave upstream, and,

$$B(x) = \begin{bmatrix} d_1(x) & & & \\ & d_2(x) & & \\ & & \ddots & \\ & & & d_n(x) \end{bmatrix}$$

where $d_\ell(x) = \exp\,[-\,\lambda_\ell\; a\; x/c_1]$

For the four-lane freeway the eigenvalues are $\lambda_1 = 0$, $\lambda_\ell > 0$, $\ell = 2, 3, 4$, where ℓ is the freeway lane number. The reader is referred to reference 13 or to reference 11, appendix 2 for details on the eigenvalues, λ_ℓ, the functions a, and the matrices M.

Since $K_\ell(0,0)$ is known for lanes one through four, we can calculate the concentrations in these lanes for any x of the form $x = c_1 t$. At any other x, though, the concentration depends on $K_\ell(0,t)$ $(t \neq 0)$, and $K_\ell(0,t)$ depends on downstream as well as upstream conditions since for some distance downstream of the blockage (in lane one) the density will remain below K_m. It is our intent to solve for the concentration at any x and t taking into account downstream conditions.

Initially we have a four-lane highway with a concentration $K_o > K_m$ in all lanes. At t = 0 a blockage occurs at x = 0 in lane 1. Downstream of the blockage (at t=0) in all lanes we have a concentration equal to K_o and a vehicular velocity v equal to:

$$v = \frac{q_m}{K_j - K_m}\left(\frac{K_j}{K_o} - 1\right)$$

At any time, t, those vehicles originally downstream of the blockage will be unaffected by the blockage. Therefore, at any $t = t_1$ the concentration downstream of the point $x = vt_1$ is at the known value K_o.

At this $t = t_1$ we know also the position of the upstream shock wave ($x = c_1t_1$) and the concentrations at this position, which are given by Munjals solution, (equation 4.1). The situation at $t = t_1$ is shown in figure 11.

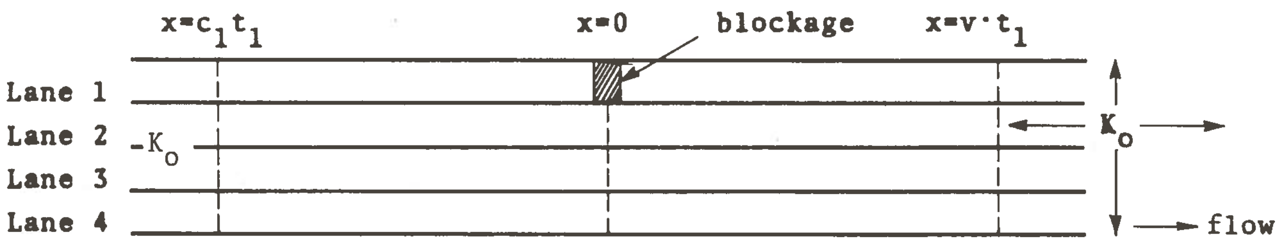


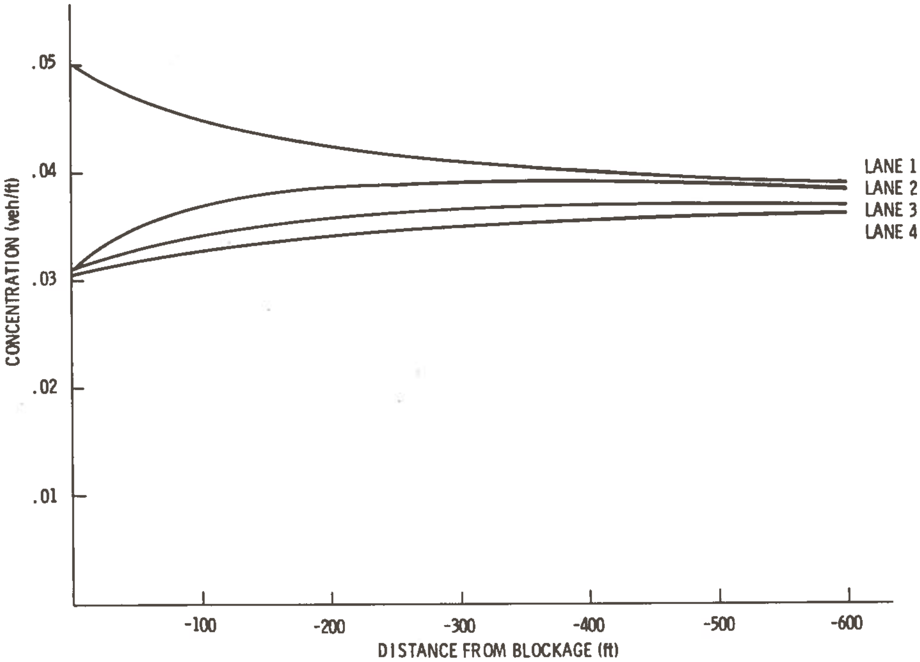
Figure 11. Sketch of Four-Lane Freeway for High-Density Traffic Calculations, $t = t_1$

By equating the net flow into an interval or roadway with the flow out, we obtain three equations with three unknowns (the concentrations $K_\ell(0,t_1)$, $\ell = 2$, 3, and 4). We have assumed for simplicity that K vs. x is approximately linear (since we took the average concentration in an interval $[x_1,x_2]$ in lane ℓ at time t to be $\frac{1}{2}\ [K_\ell(x_1,t) + K_\ell(x_2 + t])$.

Thus Munjal's solution (equation 4.1) can be used to obtain estimates of concentration at any x and t, and these estimates now will take into account both downstream and upstream conditions.

This has been done and figure 12 shows K vs. x in the four-lanes at steady state. However, we note here that equation 4.1 and therefore these results assume that a steady state in fact exists. The existence of a steady state has not yet been proven for this high density case.

Figure 12 shows that the concentration in lane 1 which is at jam density at the point of the blockage (x=0) decreases upstream of the blockage to a constant value which is higher than the original traffic density before the blockage occurred. The decrease in concentration is the result of cars switching from the blocked lane 1 to the other lanes. The concentration in the other lanes increases from the value of concentration before the blockage to some higher constant value upstream of the blockage (these lanes receive vehicles from the blocked lane). The concentration in all four lanes approaches the same constant which is given by the average of the concentrations at the blockage point over all four lanes.[11] We note that unlike the light-traffic density cases previously treated (see figures 1, 2 and 3) here the traffic density in each freeway lane has increased to values above the original concentration that existed before the blockage occurred and remains at this higher level for all points upstream of the blockage so that the total flow rate past this bottleneck is reduced.



K_j = 0.05 vehicles/foot

K_m = 0.02 vehicles/foot

K_o = 0.03 vehicles/foot

q_m = 1 vehicle/second

Figure 12. Traffic Concentration as a Function of Distance from the Blockage for High-Density Traffic Conditions

5. CONCLUSIONS

We demonstrated that certain traffic flow parameters could be used to characterize a lane blockage on a four-lane freeway. These parameters were all measurable quantities. We thus conclude that a lane blockage incident on a multi-lane freeway may be detected with suitably placed detectors. This opens the way for affecting an overall traffic flow improvement by traffic flow modification procedures (for example, by ramp metering and on-freeway controls).

The step from showing theoretical feasibility to practical feasibility is the next one that should be undertaken.

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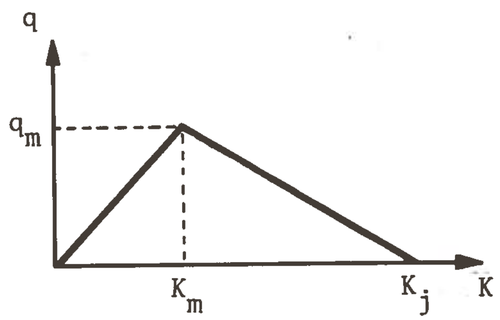
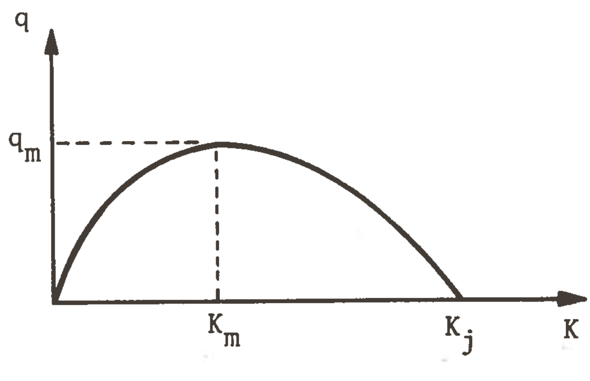
APPENDIX A
ESTIMATION OF TIME FOR TRAFFIC DENSITIES TO RETURN TO K_m USING LIGHTHILL'S METHOD

Lighthill's method as well as the method discussed in section 3 was used to obtain an estimation of the time needed for congestion to clear upon removal of the lane blockage. The two methods yielded essentially the same results. In this appendix we outline our use of Lighthill's method.

In using Lighthill's method it was necessary to "approximate" our original q-K curve, sketched below,



with the smooth curve shown below



since Lighthill's method depends on the fact that $dq/dK = c(K)$ changes continuously as K changes. (We will see that the results are relatively insensitive to this change.) Greenberg's relationship is used, namely, $q(K) = q_m K[\ln(K_j/K)]/K_m$.

The lane blockage problem is approximated with Lighthill's "red light-green light" problem. That is, at t=0 and at x=0 a traffic light turns red causing a shock wave to propagate upstream, as soon as this wave reaches the steady state position of the shock wave we let the light turn green. We note that this method will predict longer times for the concentration to return to K_m since the concentration in the interval between the light and the shock wave is K_j (whereas in the method presented in section 3 it is only K_j at the blockage and diminishes to the steady state value further upstream).

The following diagram is a plot of the characteristics and the shock locus in x-t space, figure A-1.

The trajectory of the shock before the effects of the green light are felt (or before the shock intersects the expansion fan) is given by

$$x = ut = \frac{[q(K_o) - q(K_j)]t}{K_o - K_j} = \frac{q(K_o)\ t}{K_o - K_j} \qquad \text{(A-1)}$$

Once the shock intersects the expansion fan, its trajectory is determined by

$$u = dx/dt = \frac{q(K_o) - q(K_{EF})}{K_o - K_{EF}} \cong \left[c(K_o) + c(K_{EF})\right]/2 \qquad \text{(A-2)}$$

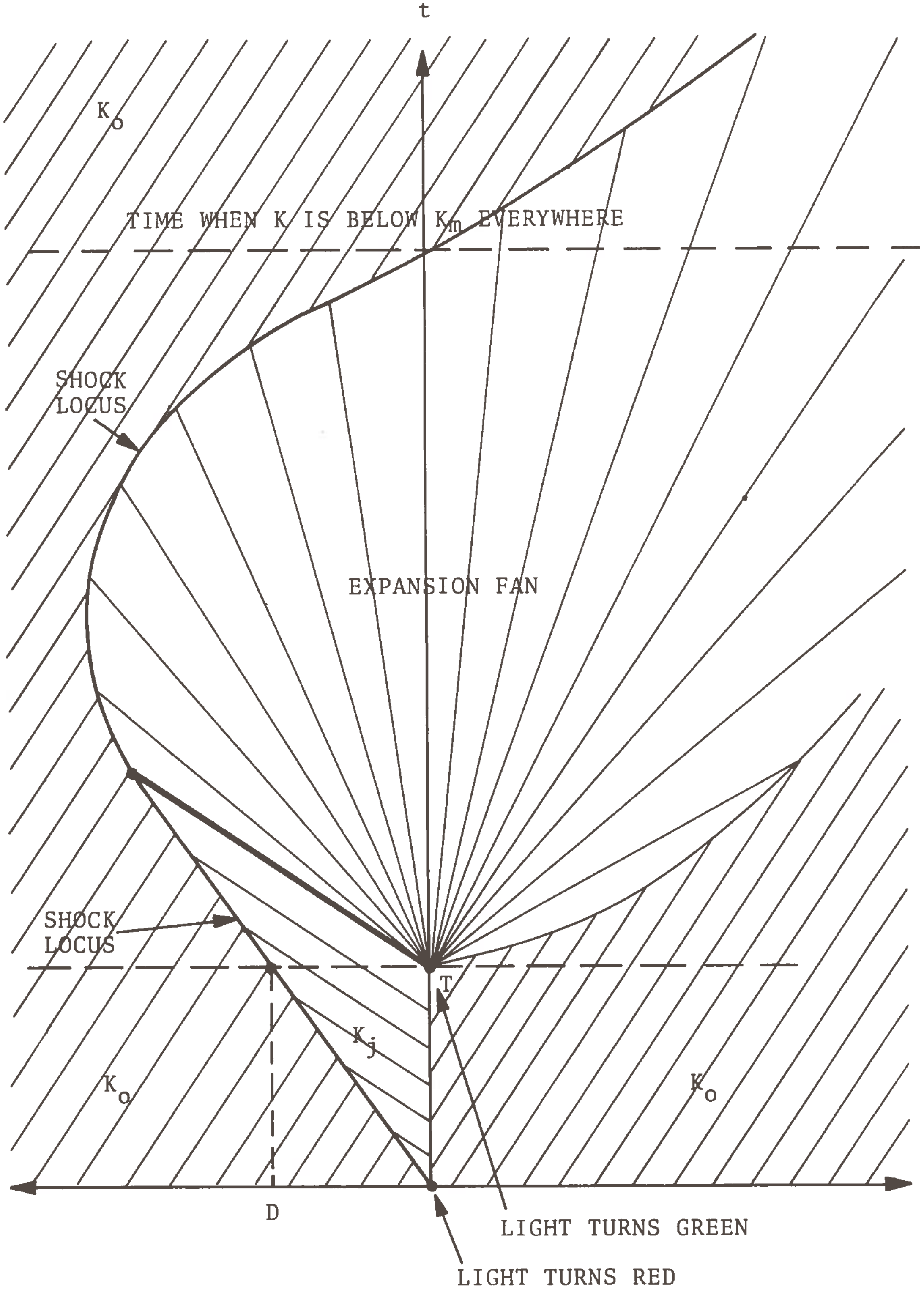
where K_{EF} is the concentration at (x,t) determined by the expansion fan. The approximation indicated in equation (A-2) is valid if K is not allowed to approach zero (which it does not in our problem).

From figure A-1, slope $c(K_{EF})$ is given by

$$c(K_{EF}) = x/(t-T) \qquad \text{(A-3)}$$

where T is the time when the light turns green, and

$$c(K_o) = (dq/dK)_{K=K_o} = q_m\left[\ln(K_j/K_o) - 1\right]/K_m \qquad \text{(A-4)}$$



A-1. Diagram of Characteristics and Shock Locus in a Space-Time Plot where we used Greenberg's relationship.

We thus obtain from equation (A-2)

$$dx/dt = q_m\left[\ln(K_j/K_o) - 1\right]/2K_m + x/\left(2(t-T)\right) \qquad \text{(A-5)}$$

The solution is

$$x = q_m\left[\ln(K_j/K_o) - 1\right](t-T)/K_m + b(t-T)^{1/2} \qquad \text{(A-6)}$$

where b is a constant of integration.

We now wish to solve for the time needed for the concentration everywhere to fall below K_m. For the sake of illustration we will solve for the case in which K_j=.05 vehicles per foot, K_m=.02 vehicles per foot, K_o=.01 vehicles per foot, q_m=1 vehicle per second and the steady state distance D is -120 feet.

We first determine the point where the shock wave first intersects the expansion fan. The equation of the shock up to this point is given by equation (A-1) while the equation of the left hand boundary of the expansion fan (see heavy line in figure A-1) is given by equation (A-3) with $c(K_{EF})$ replaced by $c(K_j)$.

From equation (A-1) we obtain

$$t = x\ (K_o-K_j)/q(K_o) = -0.05\ x \qquad \text{(A-7)}$$

where $q(K_o)$, using Greenberg's relationship, is 0.8 vehicles per second.

From equation (A-3) we obtain

$$t = x/c(K_j) + T = -0.02\ x + 6 \qquad \text{(A-8)}$$

where $c(K_j)=(dq/dK)_{K=K_j}$ = -50 feet per second, and T, the time the light turns green is 6 seconds obtained from equation (A-7) with x = D (see figure A-1).

The intersection of the shock locus with the expansion fan occurs therefore at $x \cong -200$ feet and at time $t \cong 10$ seconds. With these values the constant of integration b in equation (A-6) turns out to be -160.9 (feet/(second)$^{1/2}$). Equation (A-6) is then used to obtain the time when the concentrations drop below K_m. From the figure (A-1) this happens for vertical slope lines (corresponding to zero slope on a q-K curve where $K=K_m$), or when $x=0$. Solving equation (A-6) for t-T we find that once the light turns green it takes 28 seconds for the concentrations to drop to K_m. This is in good agreement with the results obtained by the method used in section 3, where we obtained a time of 27 seconds for the time after blockage removal for the concentration to drop to K_m.