ORT NO. UMTA-MA-06-0025-73-15

WHEEL/RAIL NOISE AND VIBRATION CONTROL

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MAY 1974

INTERIM REPORT

DOCUMENT IS AVAILABLE TO THE PUBLIC THROUGH THE NATIONAL TECHNICAL INFORMATION SERVICE, SPRINGFIELD, VIRGINIA 22151.

Prepared for

U.S. DEPARTMENT OF TRANSPORTATION
URBAN MASS TRANSPORTATION ADMINISTRATION
OFFICE OF RESEARCH AND DEVELOPMENT
Rail Programs Branch
Washington DC 20590

Technical Report Documentation Page

1. Report No. UMTA-MA-06-0025-73-15	2. Government Accession No. PB 237-012/AS	3. Recipient's Catalog No.
4. Title and Subtitle WHEEL/RAIL NOISE AND VIBRATION CONTROL		5. Report Date May 1974
		6. Performing Organization Code
7. Author(s) Paul J. Remington, et al.	Report no.	8. Performing Organization Report No. DOT-TSC-UMTA-74-7
9. Performing Organization Name and Address Bolt Beranek and Newman, Inc.* 50 Moulton Street Cambridge MA 02138		10. Work Unit No. (TRAIS) UM-404/R4721
		11. Contract or Grant No. DOT-TSC-644
12. Sponsoring Agency Name and Address U.S. Department of Transportation UMTA/Office of Research and Development Rail Programs Branch Washington DC 20590		13. Type of Report and Period Covered Interim Report July to December 1973
		14. Sponsoring Agency Code

15. Supplementary Notes

*Under contract to: U.S. Department of Transportation, Transportation Systems Center, Kendall Square, Cambridge MA 02142

16. Abstract

Reported here are the interim results of a program under the UMTA Urban Rail Supporting Technology Program to develop a basic understanding of urban transit wheel/rail noise generation for application to the evaluation and improvement of wheel/rail noise control devices. The report critically reviews existing analytic models and related experimental findings for the wheel/rail dynamic system and for the three categories of wheel/rail noise generation: squeal, impact, and roar. The limitations found result in recommendations for the remaining work required. A compilation is presented of existing or promising wheel/rail noise control devices, their acoustic and non-acoustic effects. The relative severity of the three noise categories is compared by examining wayside noise data from numerous transit systems and railroads around the world, and by using a scale recommended here for rating urban transit wheel/rail noise, i.e., the peak A-weighted sound pressure level to which the receiver of interest is exposed. Squeal produces the most annoying noises followed closely by impact and roar. Lastly, methodology is presented for assessing the non-acoustic performance of wheel/rail noise control devices. The method is applied to an example in which it is assumed that resilient wheels are installed on all New York City Transit Authority cars.

17. Key Words Noise, Transportation Noise Wheel/Rail Interaction Acoustic Rating Scale, Noise Control	18. Distribution Statement DOCUMENT IS AVAILABLE TO THE PUBLIC THROUGH THE NATIONAL TECHNICAL INFORMATION SERVICE, SPRINGFIELD, VIRGINIA 22151.

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	176	

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PREFACE

This interim report presents the results of the first three tasks under a seven-task program to study wheel/rail noise control technology in urban transit systems. The report has been prepared by Bolt Beranek and Newman Inc. (BBN) under contract DOT-TSC-644 as part of the Urban Rail Supporting Technology Program managed by the Transportation Systems Center, Cambridge MA, under the sponsorship of the Rail Programs Branch, Urban Mass Transportation Administration, Washington DC.

A summary to the report is provided for those who wish to obtain an overview of the major results and findings of these first three tasks without going into the details contained in the main body of the report.

This effort was technically coordinated at the Transportation Systems Center by Robert Lotz, Code TMP. The work was performed under the direction of Paul J. Remington of BBN with significant contributions from Erich K. Bender, Gene Fax, Michael Rudd, Steven Swanson, Eric E. Ungar, Istvan Vér, and Larry Wittig, all of BBN, and Manfred Heckl of Mueller-BBN. The cooperation of the New York City Transit Authority is also gratefully acknowledged.

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SUMMARY

Mass transit systems offer an attractive means for transporting large numbers of people rapidly, safely, and economically within urban areas. However, any plan to extend or add transit systems in urban areas must account for the major drawback of existing systems - the intense noise generated by the interaction between the rail and the wheel while the vehicle is moving. Accordingly, the Urban Mass Transportation Administration has contracted with Bolt Beranek and Newman Inc. to study the wheel/rail dynamic system, associated noise-generating mechanisms, and methods for evaluating the effectiveness, both acoustic and economic, of proposed noise control devices and procedures. This summary presents the major findings and conclusions contained in the interim report submitted under Contract No. DOT-TSC-644. The entire report is summarized here for those who wish a concise overview of our results and conclusions.

The interim report presents the results of the first three tasks of this contract: (1) a review of the existing theoretical and experimental knowledge of wheel/rail noise; (2) development of an acoustic rating scale to assess people's reaction to wheel/rail noises and a cost analysis method for assessing the non-acoustic impact of a particular noise control device; and (3) a comparison of the severity of the mechanisms that generate three characteristic wheel/rail noises — squeal, impact, and roar. The work accomplished during this phase of the program provides information for allocating priorities in the remaining phases.

Wheel/Rail Noise - The State of the Art

In examining the state of knowledge of wheel/rail noise we searched both the domestic and foreign technical literature and

contacted researchers in the field and transit authority engineers. We had two major goals during the search: information to enable us to predict the noise when a flanged metal wheel runs over a metal rail and information on existing techniques to control that noise.

To be able to predict the noise produced by wheel/rail interaction, we must understand not only the three mechanisms producing squeal, impact, and roar but also the dynamic and sound radiation characteristics of the wheel and rail. Figure S.1 is a schematic representation of the noise generation process. The interaction at the wheel/rail interface produces a force at that interface that causes both the wheel and the rail to respond. Mechanical vibration of both wheel and rail radiates noise which is heard by transit system patrons, operators, and the community at large. The interaction of the wheel and the rail generates squeal noise if a stick-slip mechanism is involved, impact noise if there are track discontinuities, and roar noise if there are microroughnesses of the wheel or rail.

Wheel and Rail Response and Radiation

Dealing with the interaction at the wheel/rail interface requires finding a suitable measure of the wheel's and rail's resistance to motion. Such a measure is the *mechanical impedance,* i.e., the ratio of applied force to resulting velocity at the point of application of the force. The impedance is generally a function of the frequency of the applied force and is usually complex (reflecting that there is a phase shift between force and velocity). As an example of the effect of impedance on the wheel/rail interactions, consider the case in which the wheel impedance greatly exceeds the rail impedance. In such a case, if the wheel encounters a bump on the rail, the wheel itself will not move but will push the rail aside. If the opposite were true, the rail would not move but the wheel would deflect or move up as it encountered the bump.

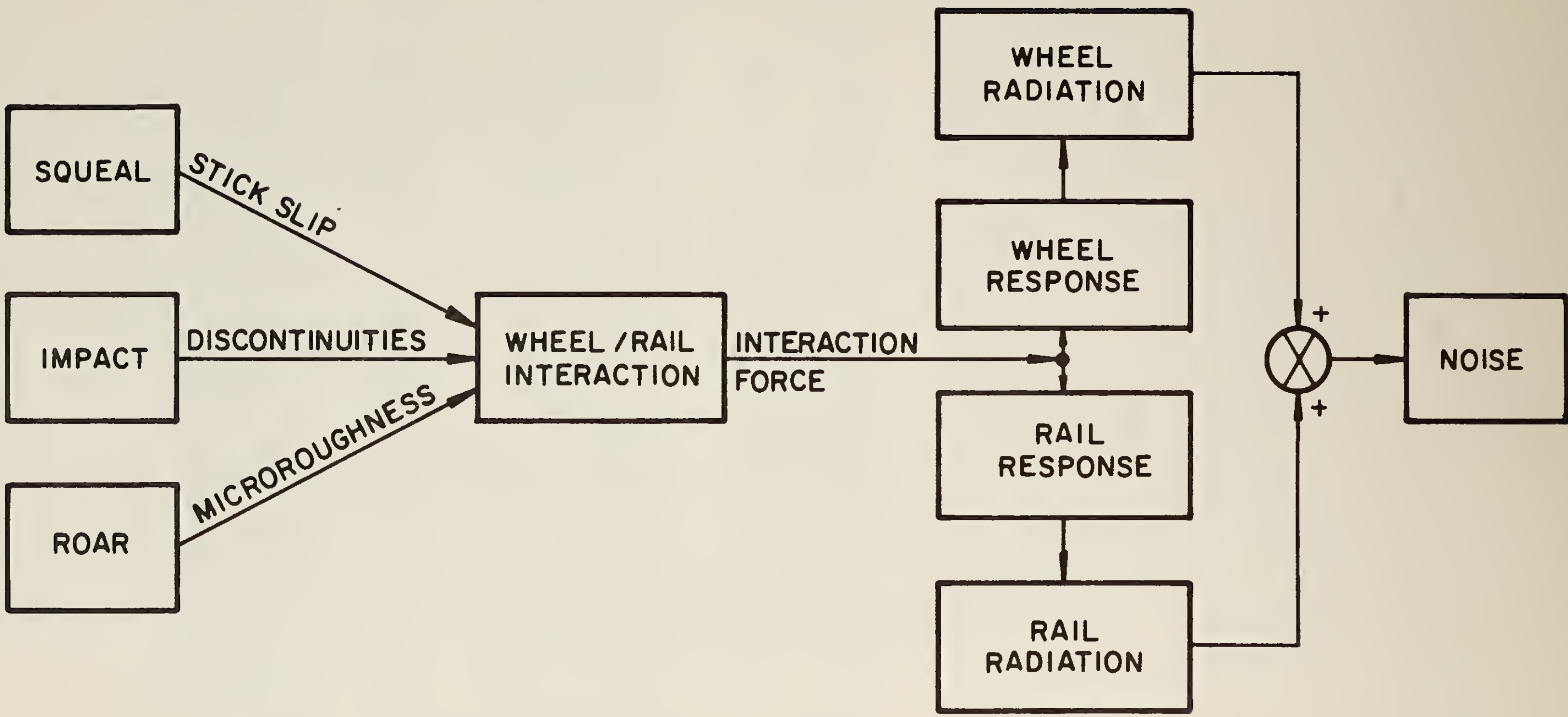

FIG. S.1. PREDICTIVE FORMULA DEVELOPMENT

Clearly then the wheel and rail response as well as the force of interaction depend strongly on the wheel and rail point impedance. Of course, of equal importance is how the wheel and rail respond at points well removed from the excitation point — for example, the decay of vibration in the rail as one moves away from the excitation point.

As the wheel hits the rail, mechanical vibration is transformed into acoustic radiations. To find out how much noise is produced, three facts must be known. The most important is "radiation efficiency", the characteristic which allows an observer to predict the amount of radiated sound power once he knows the mean square velocity, averaged over the wheel surface or along the rail length.

The second is "directivity", a measure of how the wheel or rail radiates sound in different directions. The third is the distance of the observer himself from the track. With these three facts, a prediction can be made: how much noise an observer will hear.

One aspect of our literature search, then, was to find predictive formulas (and corroborating data) for the dynamic and radiation properties of wheels and rails described above which would account for change in relevant parameters such as wheel radius, wheel type, track bed geometry, truck geometry, etc. Section 2 of this report details our findings which show that such information is quite limited. We summarize these findings below:

- There are analytical models and supporting data for the impedance of rails on resilient fasteners. However, for rails on tie and ballast, additional measurements and models are required. How well or poorly the track is maintained determines whether rail impedance is affected by static load.

- Measurements of rail response, i.e., of the decay of vibration away from the excitation point on the rail, are available, but additional measurements are required for the relative response of different parts of the rail cross section and for the effect of rail joints.
- No measurements were found of rail radiation efficiency or directivity. Information on wheel radiation efficiency and directivity is likewise limited, although a simple disk model exists for the radiation efficiency.
- There are no measurements on the point impedance of wheels. A simple mass model exists, but it is invalid above the first resonance frequency of the wheel (about 300 Hz). Thus, a need exists for a model based on wheel resonances.
- Not many data exist on wheel response, although there are enough to suggest that the wheel can be modeled fairly simply as a circular ring. Further measurements are required to validate such a model.

The Mechanisms

Referring again to Fig. S.1, we see that the interaction at the wheel/rail interface is a result of three mechanisms, stick-slip, track discontinuities, wheel and rail microroughnesses, which respectively produce squeal, impact, and roar noise. Further examination of the literature was carried on in search of a quantitative description of each of these mechanisms that would account for changes in relevant parameters such as wheel and track condition, truck wheel base, curve radius, etc. on the noise produced. The very limited information that was found is described in detail in Sec. 3. We present a brief summary here.

Squeal, the very intense noise composed of a few pure tones that occurs when transit vehicles enter short radius curves, is believed to be generated by a stick-slip mechanism at the wheel/rail interface. Because of the finite-length wheel base of a two-axle transit car truck, the wheel is not tangent to the rail in a curve. As a result, the wheel "crabs", i.e., there is a finite relative velocity between the wheel and the rail in the direction of wheel axis, resulting in alternate sticking and slipping of the wheel on the rail. It is presently believed that this alternate sticking and slipping excites the resonant modes of the wheel which then radiate as squeal. But at present, there are no measurements corroborating the above, nor has the theory been quantified.

Impact occurs when wheel flats hit the rail or when the wheel interacts with rail joints, signal junctions, switches, and other track discontinuities. There is currently no model which accurately describes this mechanism; however, it is known that measures such as limiting the permissible height of wheel flats, using welded rail, and using resilient wheels can substantially reduce impact noise levels.

The mechanism accounting for roar noise is believed to be wheel and rail roughnesses that give rise to unsteady loads and vertical motion of the wheel and rail. The major problem in the characterization of the roar mechanism is lack of measurements of the roughness spectrum in the 1/2 in. to 1 ft wavelength region in both wheels and rails. As of now, there are no devices available for measuring this portion of the spectra.

Devices and Procedures for the Control of Wheel/Rail Noise

In addition to searching for basic predictive formulas for wheel/rail noise, we reviewed a number of techniques, some of which have been tested and others which have not, for reducing the noise. These techniques are presented in Sec. 4. The following is a summary of the conclusions we have reached about each:

- *Wheel truing and rail grinding.* Grinding wheels and rails smooth has a definite beneficial effect on roar noise. Noise reductions of 6 dB(A) have been observed, and even higher reductions can probably be achieved.
- *Antilock devices.* No experimental evidence exists to confirm the theory that antilock devices on train brakes would completely eliminate flat spots on wheels.
- *Rail welding.* Welded rail joints can result in a 4 to 5 dB(A) reduction in wayside impact noise, lower maintenance costs, and possibly better fuel economy.
- *Lubrication.* The application of a lubricant (usually oil) to the track has proved effective for reducing or even eliminating screech on curves. However, this advantage must be weighed against the loss in braking ability and the noise-producing wheel flats which sliding can cause.
- *Resilient wheels.* Four designs of resilient wheels are presently available which differ only in how an elastomer connects the wheel tread and hub together. Use of these wheels can lead to significant reduction of squeal noise on curved track [reductions of up to 18 dB(A) have been measured] but only slight reductions [3 dB(A)], if any,

in noise from trains on tangent track. Resilient wheels are more than twice as expensive as standard wheels, but these costs tend to be offset by operational savings associated with reduced wear on rails and rolling stock.

- *Resilient rail fasteners.* Although these mountings reduce the transmission of vibration from the rail to the ground and to surrounding structures, they have little effect on the sound radiated from the rail itself.
- *Wheel damping.* Properly designed damped wheels have been found to reduce squeal noise on short radius curves by up to 24 dB(A). However, on tangent track no significant reductions in wayside noise have been obtained.
- *Rail damping.* The evidence on the effectiveness of the use of damping compounds on the non-running surfaces of rails is conflicting. However, reductions in wayside noise of up to 4 dB(C) have been measured in one instance.
- *Track maintenance.* Although there are no quantitative measurements of the noise reduction achievable through good track maintenance, it is expected that if the tracks are kept straight and parallel and if the joints are kept in good repair, impact noise due to either flange or joint impact is less likely to occur.
- *Barriers.* Barriers constructed as close to the track as possible can reduce wayside wheel/rail noise by about 12 to 14 dB(A).
- *Wheel skirts.* Wheel skirts of questionable design have been tested and show little ability to attenuate noise from wheels. We feel, however, that a properly designed skirt would be quite effective if the wheel proves to be a major source of sound.

- *Noise deadening rings.* These rings act to add extra damping to the wheel tread. They are a very promising noise control device and should be investigated further.
- *Resilient rail heads or wheel treads.* Although some good results have been reported using elastomeric layers, many practical problems will have to be solved before this technique can be implemented.
- *Titanium wheel treads.* Using a thin layer of titanium on wheel treads is a promising technique for reducing roar noise, because the lower stiffness of titanium will lead to a larger contact patch between the wheel and rail and hence a tendency to average out the roughness on the wheel and rail. In addition, the friction and wear characteristics of titanium on steel are expected to be superior to those of steel on steel.
- *Pneumatic tires.* Because of conflicting evidence, it is difficult to say at this time whether or not well-designed and maintained pneumatic tires are significantly quieter than well-designed and maintained metal wheels.
- *Track bed noise absorption.* The use of an acoustically absorptive layer on the track bed does not appear to be an effective noise control treatment for wayside noise.
- *Wheel web vibration absorbers.* These devices have been reported to be effective in suppressing squeal for about one year. Then they deteriorate.

Relating the Acoustic Effectiveness and Non-Acoustic Performance of Noise Control Measures

Acoustic Rating Scale

An important factor to consider in allocating noise control efforts is the reaction of system operators, patrons, and the surrounding community to different types of urban transit noise. In a separate report (Schultz, 1974) prepared under this contract we have adapted a scale for rating this reaction, based on similar scales for other types of noises and the results of new social and physical noise measurement surveys on the impact of train noise.

Some typical results from two of those studies, a Japanese survey and a French survey, are shown in Figs. S.2 and S.3, respectively. The Japanese survey plots annoyance on a one ("not disturbed at all") to five ("very frequently disturbed") scale versus the peak A-weighted SPL during a train passage for the the eight-year-old Tokaido line (T) and the four-month-old Sanyo line (S). The neutral point (3) for the Tokaido line is about 70 dB(A) and about 62 dB(A) for the Sanyo line, showing what appears to be the effect of habituation. The French survey uses a seven point scale to rate annoyance to train noise. Figure S.3 groups all responses from 1 to 3 which are taken to mean that the noise is "acceptable", groups all responses from 5 to 7 which are taken to mean that the noise is "intolerable", and plots the percent of responses in each group out of the total responses versus L_{EQ} (the Equivalent Noise Level).* The French study concludes

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T.J. Schultz, 1974. "Development of an Acoustic Rating Scale for Assessing Annoyance to Wheel/Rail Noise in Urban Mass Transit," Department of Transportation Report No. UMTA-MA-06-0025-74-2, February, 1974.

*L_{EQ} is the A-weighted SPL average over a 24-hour period. Increases in traffic or increases in the level from a single train passage will increase it.

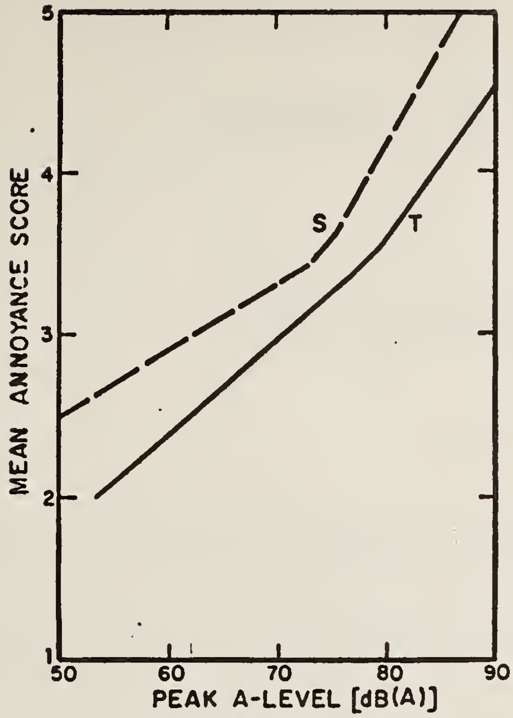

FIG. S.2. JAPANESE SURVEY

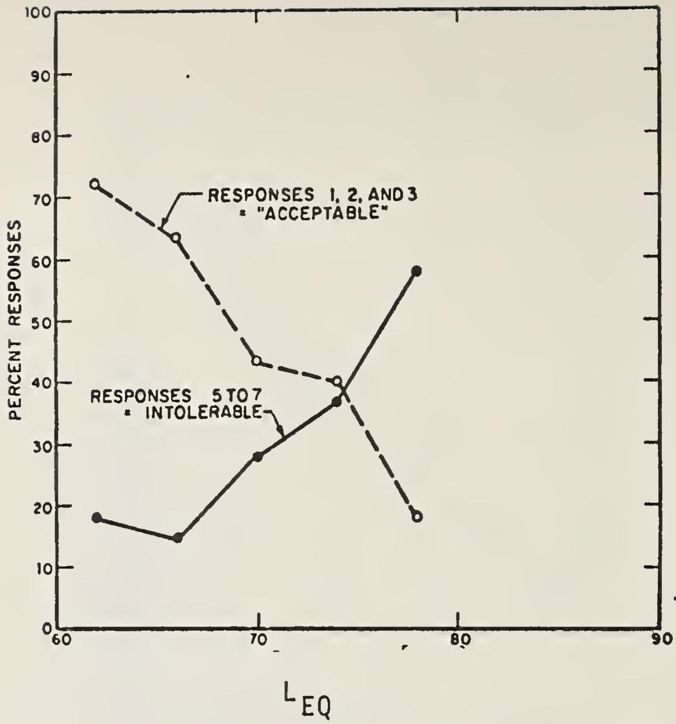

FIG. S.3. FRENCH SURVEY

that an L_{EQ} of 72 dB(A) represents a maximum acceptable exposure to train noise. Schultz, 1974 discusses these surveys in detail.

Using the rating scale derived from the above studies, as well as others, we find the rated level, L_R, by

$$L_R = L_A + 5 \left\{\begin{matrix}\text{if pure tone} \\ \text{present}\end{matrix}\right\} + 10 \log \left(\frac{T}{1 \text{ sec.}}\right) \qquad \text{(S.1)}$$

where L_A is the peak A-weighted sound pressure level and T is the duration of the noise in sec during a 24-hour period. The 5 dB(A) correction reflects the fact that noises with pure tone content (squeal noise) are more annoying and the exposure corrections allows for the fact that how long people are exposed to noise affects their annoyance.

Non-Acoustic Performance

An evaluation of the feasibility of a proposed noise control measure on a rapid transit system must take into account the economic impact on the system of the safe reliable use of that measure. Accordingly, in Sec. 6, to obtain a single number representing the overall cost impact of the noise control measure, we have set up a "discounted cash flow" approach. This method involves identifying the amount and timing of every expenditure or revenue associated with the proposed modification, and using this information to construct a projected cash flow profile which gives, for each future year, the change in the net flow of money into or out of the enterprise. Once a "change-in-cash-flow" schedule is created, the cash flow for each future year is discounted to the present at an appropriately chosen rate of interest.

$$\Delta NPV = \sum_{n=1}^{m} \frac{R_n}{(1+i)^{n-1}} , \qquad \text{(S.2)}$$

where ΔNPV = change in net present value

m = expected lifetime of the system being modified from the date of modification

R_n = change in cash flow for year n (sign convention: a net disbursement is a positive ΔR_n)

i = opportunity cost of capital.

In our case, the R_n's are determined by the costs associated with retrofitting rail transit systems with noise control treatments. System lifetime m is the lifetime of the component being treated (track or cars). Opportunity cost of capital i is obtained from appropriately chosen interest rates (see Appendix A for detailed discussion).

The cost categories that must be computed to obtain the R_n's in the above equation include initial costs when the noise control treatment is installed as well as operating costs incurred during the lifetime of the treatments. A detailed discussion of each of these cost categories is given in Sec. 6. In addition, Sec. 6 presents an example in which it is assumed that the New York City Transit system is fitted with resilient wheels.

Relative Severity of the Wheel/Rail Noise Mechanisms

We assembled data on the three characteristic wheel/rail noises — squeal, impact, and roar — to rank-order their severity. All the data (peak A-weighted SPL as required by the above rating scale) were normalized to a single moving car at 50 ft. Details may be found in Sec. 5. Roar data were compiled by collecting data on wayside noise from trains passing over welded tangent track. These data are presented in Fig. S.4 as a function of train velocity. The noise from the quieter transit systems closely fit a line whose equation is

$$L_A = 60 + 30 \log_{10} (V/15) \tag{S.3}$$

where V is in miles per hour and L_A is the peak A-weighted sound pressure level (SPL) at the wayside (50 ft). Unfortunately, there is sufficient scatter in the data so that some transit systems (such as the MBTA in Boston) can be as much as 15 dB(A) higher than predicted by that equation.

Data were also compiled for wayside noise from bolted track to characterize impact noise. These data are plotted in Fig. S.5 and a line defined by

$$L_A = 67 + 30 \log_{10} (V/15) \tag{S.4}$$

is used to fit the impact data. Although the scatter is considerable, we feel that 7 dB(A) is a good average number for the increased noise due to bolted track.

Squeal noise data are presented in Fig. S.6 as a function of curve radius for a variety of speeds. The peak A-weighted SPL is seen to be on the order of 90 dB(A) and is independent of curve radius although the likelihood of squeal occuring seems to increase with decreasing curve radius.

On the surface, the data seem to indicate that squeal noise is the most severe. However, such variables as track construction and the extent to which people are exposed to and annoyed by each kind of noise made difficult any conclusion about which noise mechanism should be attacked first.

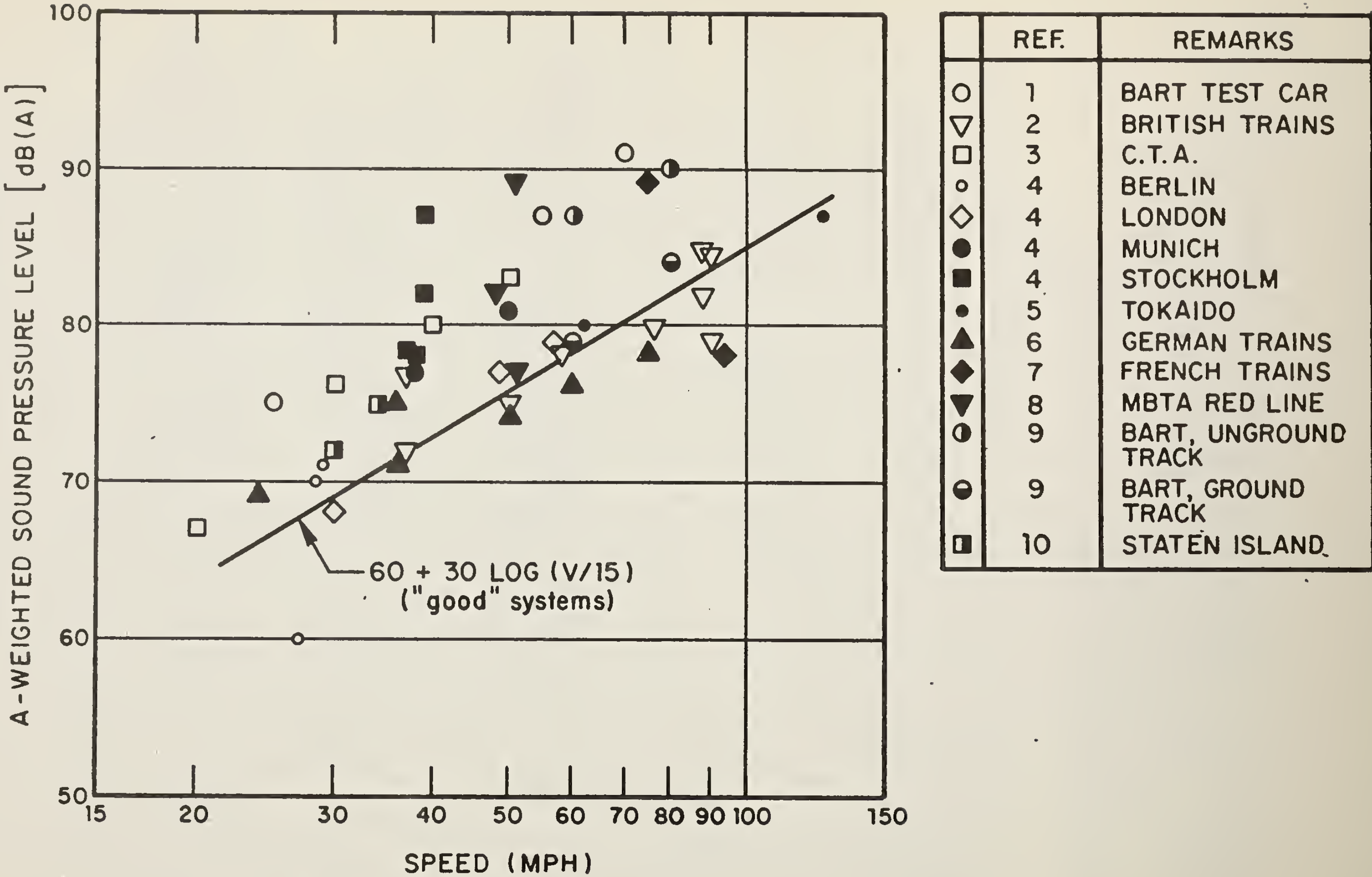

FIG. S.4. NORMALIZED DATA FROM SEVERAL WELDED TRACK SYSTEMS. Ref. 1 = Parsons, Brinckerhoff, 1968; 2 = Peters, 1973; 3 = Patterson, 1972; 4 = Bender and Heckl, 1970; 5 = Franken, 1969; 6 = Stüber, 1973; 7 = Rapin, 1972; 8 = Rickley and Quinn, 1972; 9 = Wilson, 1972; 10 = Wittig; 1973.

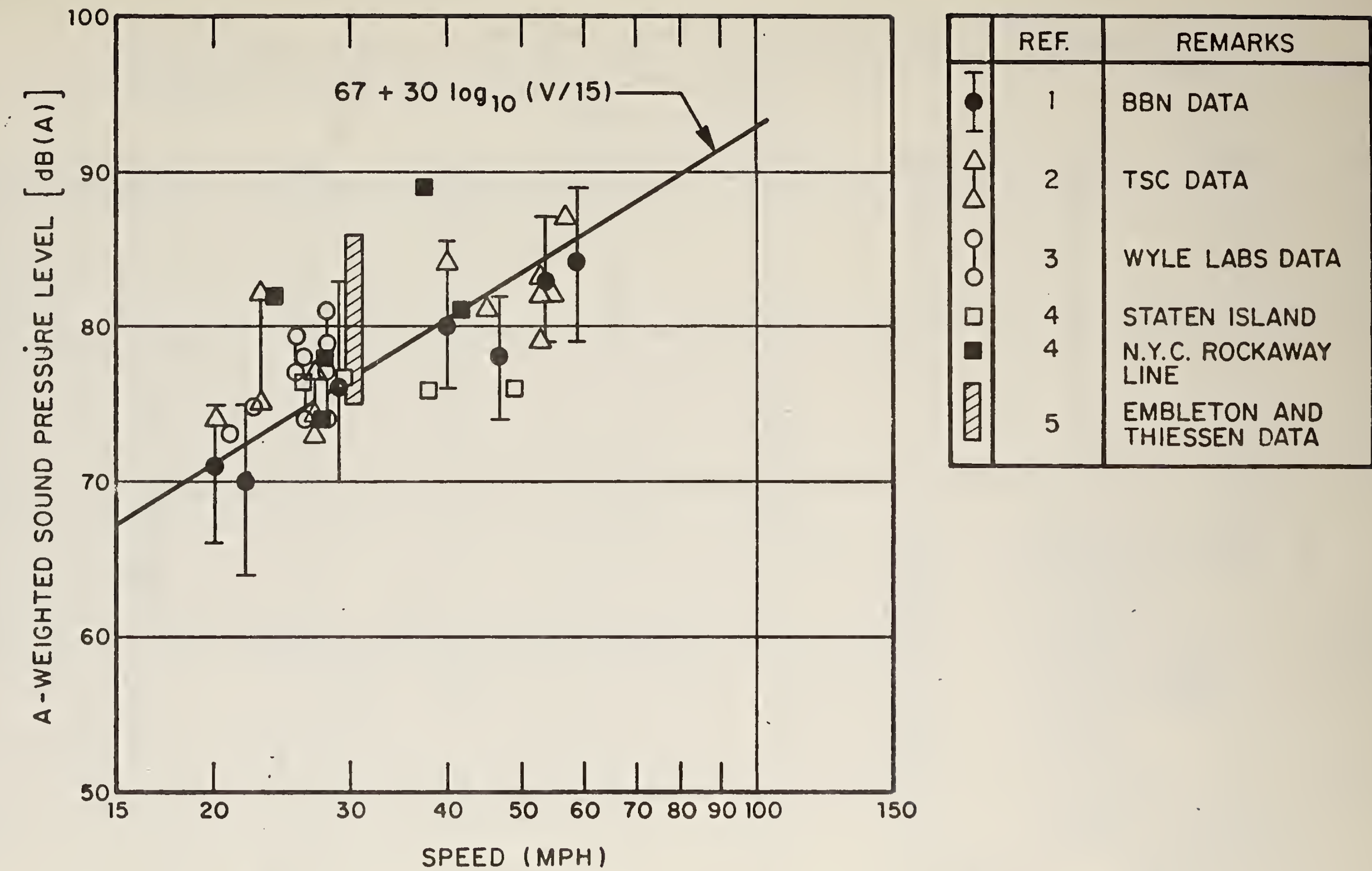

FIG. S.5. NORMALIZED DATA FROM SEVERAL JOINTED TRACK SYSTEMS. Ref. 1 = Bender and Heckl, 1970; 2 = Ely, 1973a; 3 = Ely, 1973b; 4 = Wittig, 1973; 5 = DOT, 1970.

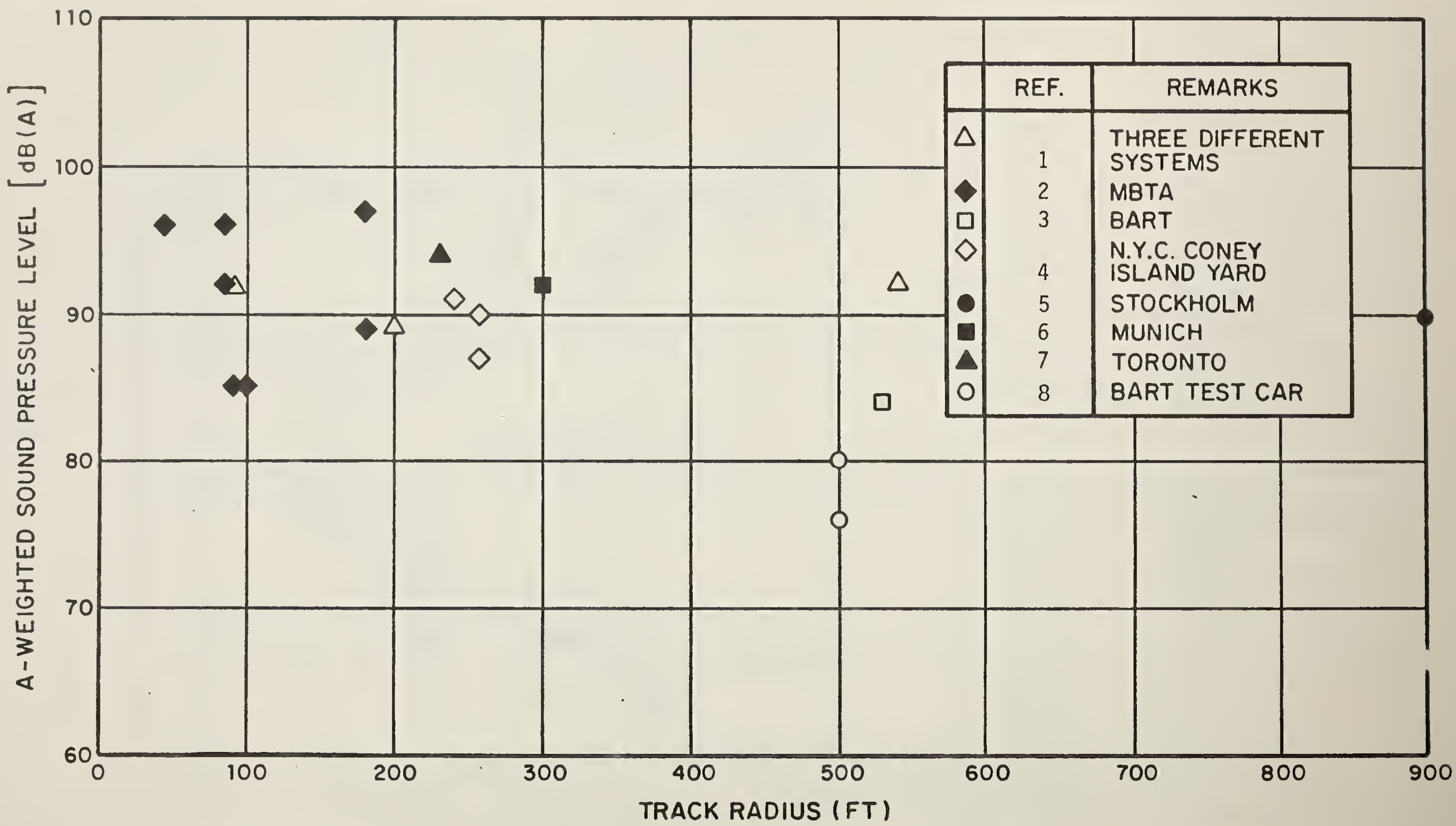

FIG. S.6. NORMALIZED SQUEAL DATA FROM SEVERAL SYSTEMS. Ref. 1 = Kirschner, 1972; 2 = Bender and Hirtle, 1970; 3 = Wilson, 1972; 4 = Wittig, 1973; 5 = Berglund, 1972; 6 = Mueller BBN, 1973; 7 = Swanson, 1966; 8 = Parsons, Brinckerhoff, 1968.

We conclude that work on reducing all three noise mechanisms should proceed simultaneously with approximately equal resources allocated to each.

1. INTRODUCTION

As population and traffic densities within our cities continue to increase, the need for fast, efficient, comfortable urban mass transportation is becoming more and more critical. Existing surface transportation systems designed to fulfill this need consist traditionally and almost exclusively of vehicles with flanged metal wheels running on metal rails. Such an arrangement offers a number of attractive features for transporting large numbers of people rapidly, safely, and economically within our cities. For example, the vehicles are self-guiding, rolling resistance is low, riding quality is potentially quite good, steel wheels and steel rails are inherently very durable, and the very fact that such an arrangement has been commonly used in mass transit systems for many years implies the existence of a well-established manufacturing and operational technology.

The major drawback of the present flanged wheel-steel rail guidance and support systems for use in urban areas is the intense noise generated by the interaction between the wheel and the rail while the vehicle is in motion. This noise is commonly divided into three very general categories: squeal (or screech), impact, and roar. Squeal is generally the term used to describe the intense noise consisting of one or more pure tones associated with transit cars rounding small radius curves. Impact describes the noise associated with wheels rolling over their own flat spots, over rail joints, and over other track discontinuities. Roar describes the continuous noise most noticeable on tangent track in the absence of discontinuities.

To date, many suggestions have been made for the control of wheel/rail noise, and some related hardware has been built. Unfortunately, most of these efforts have been based on, at best, a sketchy understanding of the wheel and rail as dynamic systems.

It is unlikely that any useful cost-effective advances in controlling wheel/rail noise can be achieved without a considerable advance in the understanding of the important noise generating mechanisms.

This interim report delineates the results of the first three tasks of a seven-task study to develop a quantitative understanding of the wheel/rail dynamic system and the associated noise generating mechanisms and to specify means for evaluating the effects of various noise components in order to provide a firm basis for the development of cost-effective wheel/rail noise control devices and procedures. The seven tasks and their interrelation are shown in Fig. 1.1.

Although this report is concerned primarily with only the first three tasks of this study, we can better explain the motivation for these three tasks if we describe in some detail what our ultimate goal is and in what form we anticipate it will be fulfilled.

As a major goal, we wish to be able to predict the noise generated when a flanged metal wheel rolls over a metal rail. As described above, this noise is produced by one or more of three mechanisms — squeal, impact, and roar. Thus, we need to develop a mathematical model of the excitation produced by each. The excitation would probably be the force at the wheel/rail interface for impact and squeal and the roughness on the wheels and rails for roar. Each model should, of course, account for the effect of changes in relevant parameters, such as speed, curve radius, rail joint gap spacing, wheel radius, truck wheel base, wheel and track condition, etc.

Once the excitations from the three mechanisms have been modeled with their relevant parametric dependences, it is still

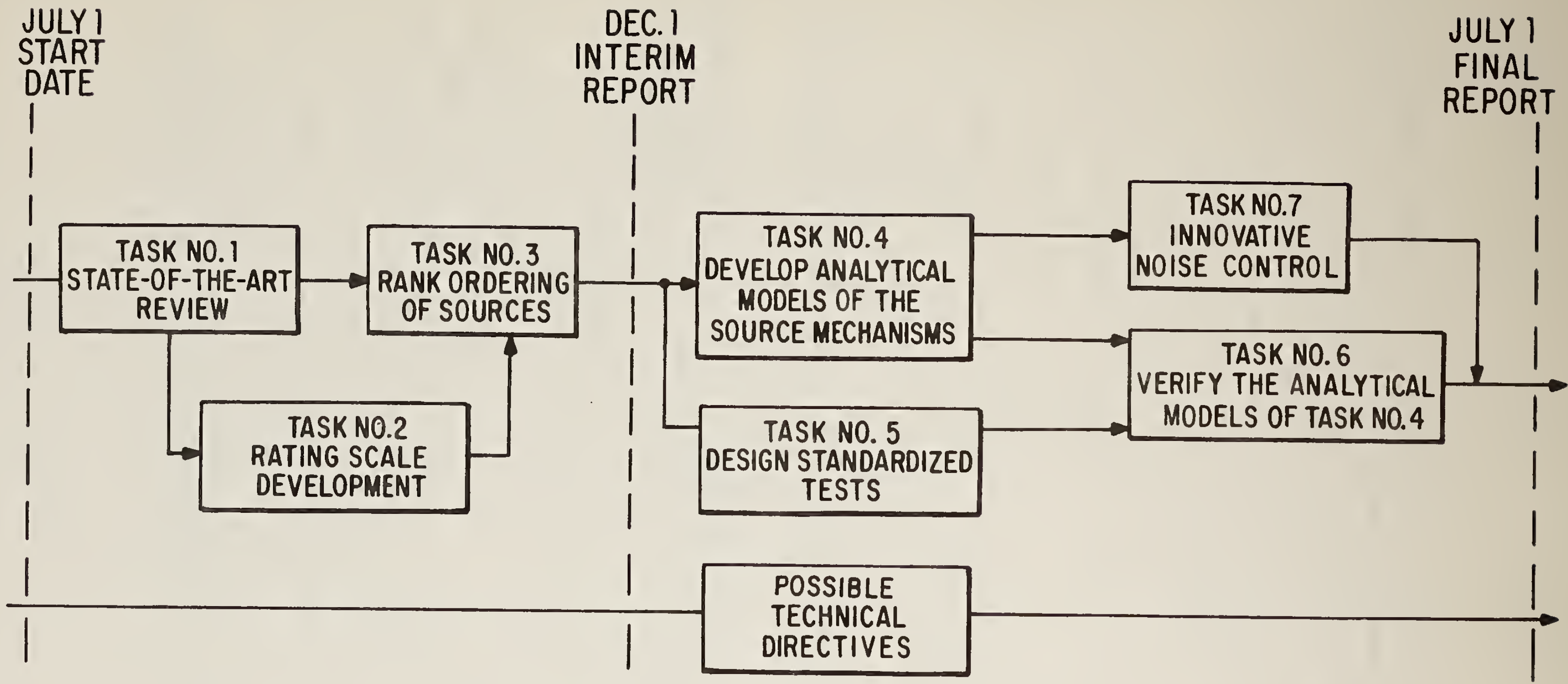

FIG. 1.1 ARRANGEMENT OF THE WHEEL/RAIL NOISE PROGRAM BY TASKS

necessary to determine what response these excitations produce in the wheels and rails and also how this response, i.e., mechanical vibration, is transformed into acoustic noise. In order to do this, we must develop mathematical models of various dynamic and radiation properties of wheels and rails, such as mechanical impedance, radiation efficiency, and directivity. These models must also be able to take into account the effect of various parameter changes such as rail fasteners, stiffness, rail cross-section, wheel damping, and rail damping. Knowing the mechanical impedance of wheels and rail and having some knowledge of how the wheel and the rail respond at positions removed from the point of excitation, one can determine the wheel and rail response from the excitation. When the radiation efficiency is known, the acoustic power radiated can be calculated from the response, and when the directivity is known, the noise (sound pressure level) can be calculated from the radiated acoustic power at any position relative to the wheel and the rail.

The mathematical models should be able to predict the change in noise radiated by each of the mechanisms when individual wheel and track parameters are varied. This means that the effectiveness of various noise control devices and procedures could be determined by exercising the mathematical models.

The development of these mathematical models basically constitutes Task 4 in Fig. 1.1. Task 7, the development of innovative noise control measures, will be a natural outgrowth of Task 4. The other tasks support these two main tasks. For example, Tasks 5 and 6 are essentially field testing to validate the mathematical models developed in Task 4. Tasks 1, 2, and 3, the subjects of the report, provide a foundation for Task 4.

Task 1 calls for a thorough review of existing knowledge, from both domestic and foreign sources, on wheel/rail interaction and noise generation. In carrying out this task, our intent is not to compile an exhaustive list of reference material but rather to review critically that information which is essential to forming a firm foundation for the Task 4 work. Of particular interest are analytical models and corroborating experimental evidence for the dynamic and radiation characteristics of wheels and rails, as well as the mechanisms involved in the generation and radiation of squeal, impact, and roar. In addition, we have sought information on existing wheel/rail noise control devices and procedures.

To embark on a cost-effective program for the abatement of wheel/rail noise, one must have some means for rating the severity of the noise generated by the various sources. Further, once a noise control device or procedure has been selected, some means for rating both its acoustic and nonacoustic (cost, reliability, safety, etc.) performance must be found. Under Task 2 we have developed an acoustic rating scale to assess the annoyance of system patrons, operators, and the community to wheel/rail noise. The recommended scale, essentially the peak A-weighted sound pressure level to which the particular receiver of interest is exposed, was selected based on a number of recent studies of community annoyance to train noise and represents the best available state-of-the-art rating scale for wheel/rail noise. Our work under Task 2 also sets up a methodology for assessing the cost impact on a rapid transit system of incorporating a particular noise control device or procedure. By putting all tangible costs, as well as such intangibles as reliability and safety, on a comparable basis, use of the "net present value analysis" allows one to assess readily the nonacoustic performance of a noise control measure. To clarify the application of the methodology, we give an example in which it is assumed that all New York City Transit Authority cars are fitted with resilient wheels.

Task 3 is concerned with using the acoustic rating scale of Task 2 to compare the relative severity of the three mechanisms for wheel/rail noise generation. To do this we have gathered data from more than nine transit systems in the United States, Canada, and Europe,* as well as various railroads, on the peak A-weighted sound pressure level produced by a moving train [normalized to a single car at grade at 50 ft (15.2 m)]. Data are presented for jointed rail at various speeds to assess the magnitude of impact noise; for welded rail at various speeds to assess the magnitude of roar noise; and for curved track at various curve radii to assess the magnitude of squeal noise. It appears that squeal produces the most annoying noise, followed closely by impact and then roar. However, in allocating abatement resources, one must not follow this rank-ordering too closely, because there is considerable scatter in the data and because many factors were not accounted for in the rating scale (such as the number of people affected).

The efforts under Task 1 are reported in Secs. 2, 3, and 4. Section 2 summarizes existing knowledge concerning the wheel/rail dynamic system, and Sec. 3 reviews the existing information on the mechanisms for the generation of wheel/rail noise. Section 4 contains a compilation and evaluation of existing devices and procedures for the control of wheel/rail noise.

The results of Task 3, to compare the severity of the three wheel/rail noise generating mechanisms, are reported in Sec. 5. The acoustic rating scale and the methodology for assessing non-acoustic performance of noise control measures are described in Secs. 6 and 7, respectively.

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*Part of the data was measured specifically for this program by BBN on the New York City Transit Authority.

2. THE WHEEL/RAIL DYNAMIC SYSTEM

In view of the importance of the dynamic and radiation characteristics of railroad wheels and rails to the generation and control of wheel/rail noise, it is surprising that there is very little information on them in the literature. Information we looked for, with little success, includes data on wheel and rail impedance, response, and radiation. Although it is well known that for speeds typical of transit vehicles the speed of travel of the wheel over the rail (Ludwig, 1968; Birmann, 1965-66; Evansen and Kaplan, 1969) and the speed of rotation of the wheel (Evansen and Kaplan, 1969) has no effect on the interaction between the wheel and the rail*, other basic information is

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Ludwig, K., 1968. "The Deformation of a Roadway Elastically Bedded without Limit on Both Sides by Loads with Constant Velocity," *Proceedings of the 5th International Congress for Applied Mechanics*, Cambridge, Mass. Also NASA Technical Translation, NASA T7-13, 619, April 1971.

Birmann, F., 1965-66. "Track Parameters Static and Dynamic," *Interaction Between Vehicle and Track*, Proceedings of the Institute of Mechanical Engineers, 180: Part 3F, Birdcage Walk, Westminster, London, SW3.

Evansen, D.A. and Kaplan, A., 1969. "Some Problems of Wheel/Rail Interaction Associated with High-Speed Trains," *Bulletin of the International Railway Congress Association*, pp. 513-541.

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*If one applies a moving load to a beam on an elastic foundation (which we will see is a good mathematical model of a rail) at a critical speed, a resonant phenomenon occurs leading to higher beam response than would occur if the load were stationary. For parameters typical of transit authority rails and track beds, this speed is around 1000 mph. A similar phenomenon is expected to occur with wheels at speeds approaching 1500 mph.

sparse. For example, if one is to understand the interaction between the wheel and the rail as the wheel rolls over the rail, the point impedance* of both the wheel and the rail is an important dynamic property to know. Some information exists for the rail (Naake, 1953; Volberg, 1972; BBN internal information) but virtually none for the wheel. Equally important is the manner in which the different parts of the wheel and rail respond relative to the velocity of excitation at the wheel/rail interface. Except for some measurements by Stappenbeck (1954) and Taschinger (1951) on the resonance frequencies of a typical transit wheel and some measurements by Naake (1953) of the longitudinal decay of vibration on a rail, we again find no information.

Naake, H., 1953. "Experimental Investigation of the Vibration of Railroad Rails" (in German), *Acustica,* 3, pp. 139-147.

Volberg, G., 1972. "Comparative Measurements of Structureborne Vibration on Various Track Beds" (in German) *Proceedings of the Conference on Acoustics and Vibration Technology,* Stuttgart, VDI-Vertag GMBH, Berlin, pp. 386-389.

Stappenbeck, H., 1954. "Street Car Curve Noise" (in German), *Z. VDI,* 96(6), pp. 171-175.

Taschinger, 1951. "The Present State in the Investigation of Noise Problems due to Moving Railroad Cars" (in German), *Glasers Annalen,* pp. 242-249.

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*The point impedance is the ratio of exciting force to resulting velocity at the point of excitation when the exciting point force consists of a single frequency. The impedance may be a function of the exciting frequency and may be complex if there is a phase shift between force and velocity.

The lack of information is even more acute when we consider the radiation characteristics of wheels and rails. With the exception of some very cursory work by Ungar *et al* (1970) on wheel directivity, nothing is known about wheel or rail directivity or radiation efficiency.

Although the above discussion presents a bleak picture, there is both quantitative and qualitative information to be gleaned from the literature that will, in fact, be helpful in determining where the emphasis is necessary in the wheel/rail noise control program to fill many of the voids in our understanding of the wheel/rail dynamic system. In the remainder of the section, we will examine this information in some detail.

2.1 The Wheel

In this section, we discuss what analytical and experimental information is available on the dynamic and radiation characteristics of railroad wheels. In cases of extremely sparse information, we allow ourselves the luxury of some speculation.

Point impedance. To the best of our knowledge there are no measurements available on the point impedance of railroad wheels. In general, we would like to know the real and imaginary parts of this impedance for a point force applied at the tread both radially and axially. Bender (1972) has theoretically modeled

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Ungar, E.E. *et al*, 1970. "An Investigation of the Generation of Screech by Railway Car Retarders," BBN Report No. 2067.

Bender, E.K., 1972. "Rail Fastener Design for Noise and Vibration Control," BBN Report No. 2485.

the wheel impedance as if it were a simple mass (the mass of the wheel plus one-third the mass of the axle) for a radial force. This model, although useful for Bender's purpose of obtaining a first-cut comparison between wheel impedance and rail impedance, is invalid for frequencies at or above the first resonance frequency of the wheel. Measurements made on a 30-in. (0.765 m) transit car wheel by Stappenbeck (1954) and similar measurements made by Taschinger (1951) on an unspecified wheel* show a first resonance around 350 to 400 Hz, suggesting that the simple mass model will, in fact, be invalid above about 300 Hz.

The resonance frequencies measured by Taschinger and Stappenbeck are fairly sparse. Individual resonance peaks can be easily picked out in Stappenbeck's data to above 5000 Hz (a total of only 16 peaks from 0 to 5000 Hz). The lack of data presents some difficulties in analytically modeling the wheel impedance. Since the geometry of common transit car wheels varies greatly (web geometry, wheel diameter, etc.), the resonance frequencies of the wheels will also vary.† The impedance will depend on these resonance frequencies, their location, and the damping associated with them. To predict analytically the location of wheel resonances for a range of wheel geometries is probably not possible without involved and costly computer modeling. We anticipate that a statistical modeling technique in which only the approximate location of the resonances and the modal densities are known will

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* Both measurements were made by striking a wheel with a hammer and analyzing the resulting vibration or noise radiation.

† Taschinger measured a first resonance at 350 Hz while Stappenbeck measured the resonance associated with the same mode shape at 400 Hz.

probably have to be used even though the low modal density, about 4 modes per 1000 Hz, will inevitably cause some difficulties.

Wheel Response. By the term wheel response we mean in particular how the different parts of the wheel (tread and web) respond to axial and radial point forcing at the tread. Again, Taschinger and Stappenbeck provide the only available information. Both determined the location of the nodal lines at each resonance. These mode shapes, along with the response as a function of frequency, are reproduced from Stappenbeck's paper in Fig. 2.1. Curiously enough, the modes with only radial node lines (no circumferential node lines) seem to respond the most. This behavior suggests a fairly simple model of the wheel, e.g., a circular ring, in which the web simply follows the motions of the tread. Further measurements of both response and impedance are required to validate such a model.

Taschinger also measured the reverberation time of a wheel. His intent was to determine the effect of various damping treatments.* He found that the modes at 365 Hz and 950 Hz had reverberation times of 21 and 7 sec, respectively, giving a loss factor of about 10^{-4} for both modes on an untreated wheel. Clearly, wheels are extremely lightly damped structures.

Wheel Radiation Efficiency. In order to calculate the noise radiated by the vibrating wheel, we must have some measure of how effectively the wheel transforms its mechanical vibration into acoustic radiation. The radiation efficiency, σ, is such a measure and is defined by

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*Discussed further in Sec. 4.

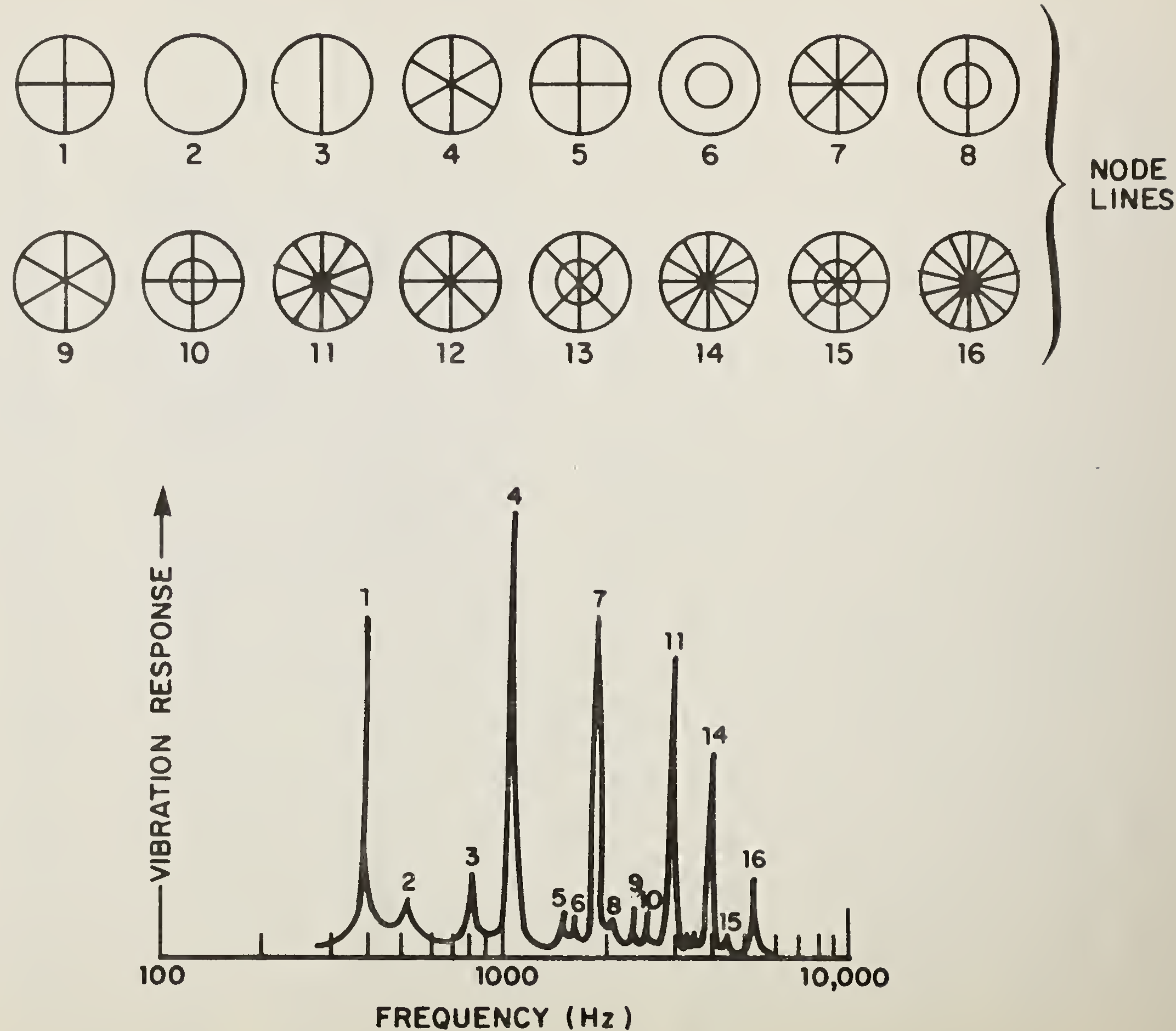

FIG. 2.1 RESONANCE FREQUENCIES AND MODE SHAPES OF A 765 mm TRANSIT CAR WHEEL (from Stappenbeck, 1954).

$$\sigma = \frac{W}{\rho c A \langle u^2 \rangle} \quad ,$$

where W is the total acoustic power radiated by the wheel, ρc is the acoustic impedance of air, A is the area of the wheel, and $\langle u^2 \rangle$ is the wheel's space-time averaged vibratory velocity squared. No analytical models or measurements of this quantity exist for transit car wheels.

As a very simple model of the wheel radiation efficiency one may take the wheel as a uniformly vibrating, unbaffled disk. Morse and Ingard (1960) have made an approximate calculation for such a model by assuming uniform pressure across the face of the disk. Taking the area of both sides of the disk to be $2\pi a^2$, where a is the radius, we obtain

$$\sigma = 1, \ ka >> 1$$

$$\sigma = \frac{\frac{3}{2}(ka)^4}{[1 + \frac{(ka)^3}{2}]^2 + \frac{(ka)^4}{4}} \simeq \frac{3}{2}(ka)^4 \quad , \quad ka << 1$$

where k is the acoustic wavenumber. The model for the two ranges of ka intersect around ka = 0.9, which implies a radiation efficiency of about 1 for a 28-in. (0.71 m) wheel above a frequency of 140 Hz. This result is consistent with the opinion generally held that the wheel is the primary radiator of sound in wheel/rail interaction.

Directivity. The directivity of radiation from the wheel is an important aspect of its dynamic characteristics and is useful for

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Morse, P.M. and Ingard, K.U., 1960. *Theoretical Acoustics*, McGraw-Hill Book Co., New York.

purposes of noise control and prediction of wayside noise. Unfortunately, the only information available is from measurements made by Ungar *et al* (1970) by striking a wheel with a wooden stick. Ungar attempted to correlated this data with a simple model of the wheel (a baffled, uniformly vibrating disk). The correlation of the data with the model predictions is shown in Fig. 2.2 and is not very good.

2.2 The Rail

Point Impedance. There is more information on the point impedance of rails than on any other aspect of the dynamic characteristics of the wheel on the rail. In general, the rail can be modeled as a beam on an elastic foundation and many analytical studies of such a model have been made (Crandall, 1959; Ludwig, 1968; Dörr, 1948). This simple rail model has generally been confirmed for rails on resilient fasteners through measurements by Bender (available at BBN) and Volberg (1972). For rails on ties and ballast, Naake (1953) has shown that the periodic support given by the ties to the rail can lead to impedance minima at frequencies corresponding to a half wave length between the ties such as one would expect from a periodically supported beam.

Crandall, S.H., 1959. "The Timoshenko Beam on an Elastic Foundation," AMFDC-TR-59-8, pp. 79-106.

Dörr, J., 1948. "The Dynamics of a Resiliently Supported Infinite Beam," *Z. Ing. Archiv.* XVI No. 5 and No. 6, pp. 287-298 (in German).

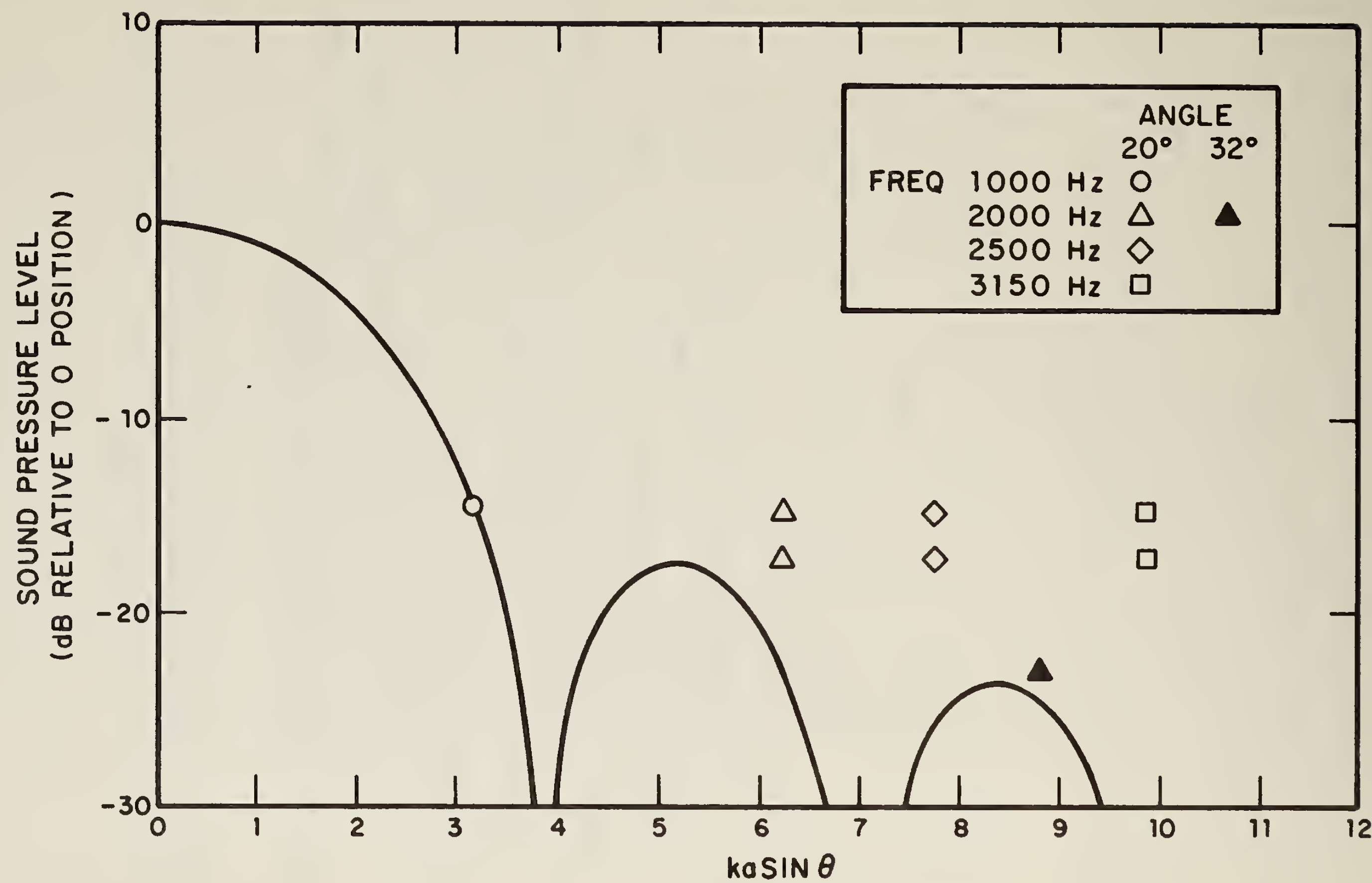

FIG. 2.2. THEORETICAL AND EXPERIMENTAL DIRECTIVITY PATTERNS FOR RAILROAD WHEEL RADIATION

There have been numerous theoretical papers dealing with the dynamics of a beam on an elastic foundation. If we take the simplest model, a Bernoulli-Euler beam on a linearly elastic foundation, we obtain for the impedance

$$Z_R = j\, 2\sqrt{2}\,(EI)^{\frac{1}{4}}\; K^{\frac{3}{4}}\; \omega^{-1}\; [1-(\omega/\omega_0)^2]^{\frac{3}{4}} \quad , \omega \leq \omega_0$$

$$Z_e = 2(EI)^{\frac{1}{4}}\, K^{\frac{3}{4}}\, \omega^{-1}\; [(\frac{\omega}{\omega_0})^2-1]^{\frac{3}{4}}\; (1-j) \quad , \omega \geq \omega_0$$

where $\omega_0 = \sqrt{\frac{K}{\rho_\ell}}$ and E is the modulus of the rail material, I the bending moment of inertia, K the foundation stiffness, and ρ_ℓ the rail density per unit length. The amplitude and phase of this theoretical vertical rail impedance for AREA 100 rail* on resilient fasteners are shown as the solid lines in Fig. 2.3. The dots give values measured by Bender (1972) on tracks supported on NYCTA resilient fasteners, and the agreement between theory and measurement is seen to be quite good for the amplitude and somewhat poorer for the phase. The fact that damping was not included in the theoretical model accounts in part for the discrepancy. Additional data taken by Volberg (1972) on resiliently supported tracks are shown in Fig. 2.4. The measurements were performed with a tapping machine and, as a result, only the amplitude of the impedance is shown. Volberg modeled the rail as a beam on an elastic foundation and found excellent agreement with measured data, except possibly above 2000 Hz.

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*Cross sectional area 10-in.^{2} (6.45 10^{-3} m^2), vertical moment of inertia 45-in.^{4} (1.86 10^{-5} m^4).

Both measurements and theoretical calculations show that at very low frequencies the rail impedance is controlled by the stiffness of the rail fastener and, hence, decreases as $1/\omega$. In the mid-frequency range, there is a strong minimum in the impedance because the mass impedance of the rail cancels the stiffness impedance of the fasteners. At high frequencies, the rail moves essentially independently of the fastener stiffness and for all practical purposes is essentially a freely vibrating beam with the impedance increasing as $\omega^{1/2}$.

Impedance measurements of rails on ties and ballast have also been performed by Naake (1953). He measured both the real and the imaginary part of the impedance on an unspecified rail, (but one whose properties were similar to an AREA 100 rail) for forcing the rail both vertically and horizontally. Naake's results showed a strong minimum at 800 Hz in the vertical impedance and at 430 Hz in the horizontal impedance. He concluded that these minima were a result of the periodic supports that the ties provide the rail [every 25.5 in. (0.65 m)] and which cause passbands at these frequencies (such as one gets with a periodically supported beam) and hence minima in the impedance. Curiously enough, one can find a zero in the phase in Bender's data (see Fig. 2.3) at 800 Hz, which corresponds approximately to a passband for the configuration that Bender measured, i.e., a rail on resilient pads spaced 2 ft (0.61 m) apart.

Unfortunately, Naake's results only span the frequency range from 100 to 1500 Hz. Furthermore, he made no attempt to provide a mathematical model of the rail impedance. Nevertheless, taken as a whole, his information will be of great help in developing such a model.

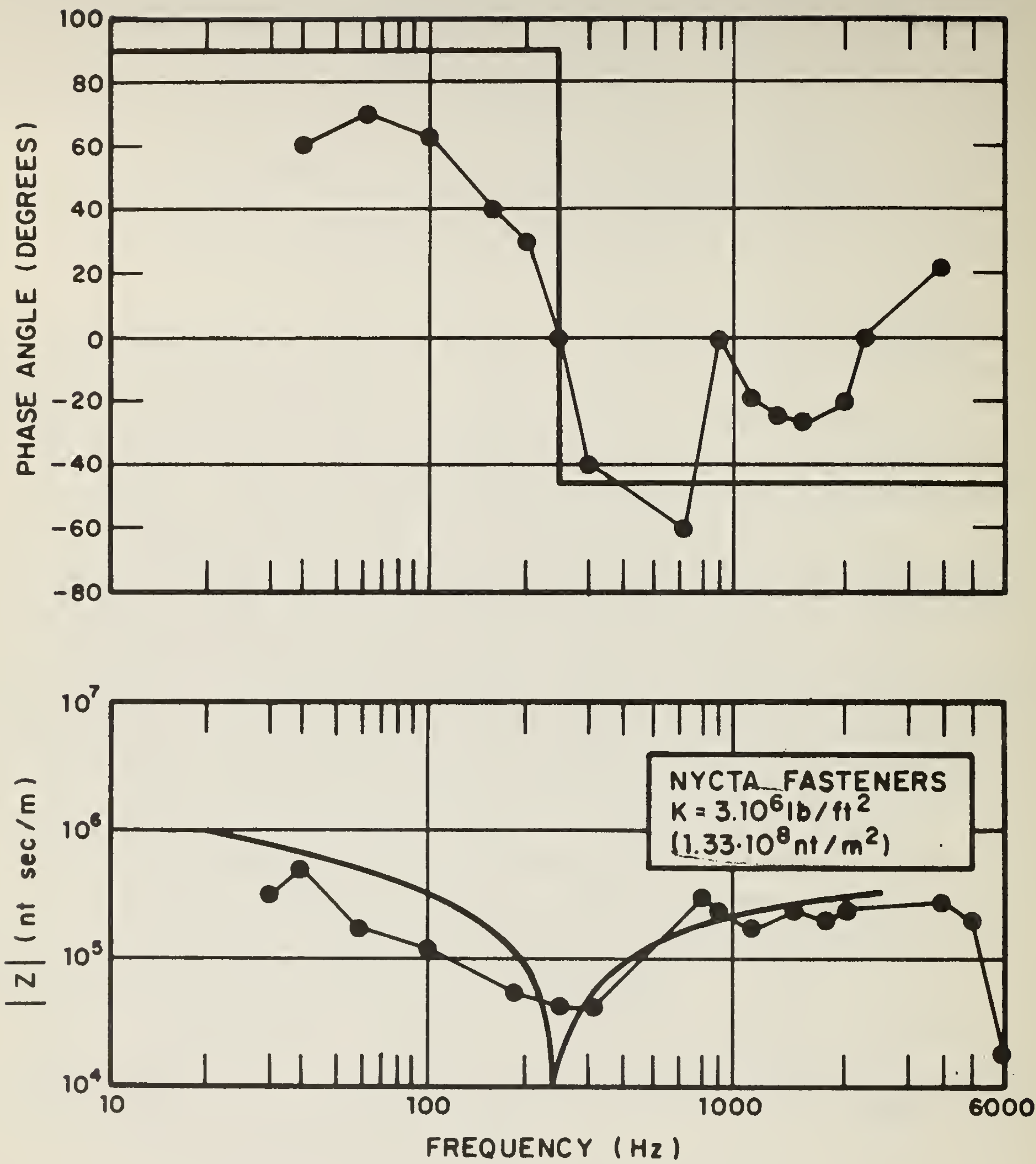

FIG. 2.3 COMPARISON OF MEASURED AND PREDICTED RAIL IMPEDANCE (Bender, 1972).

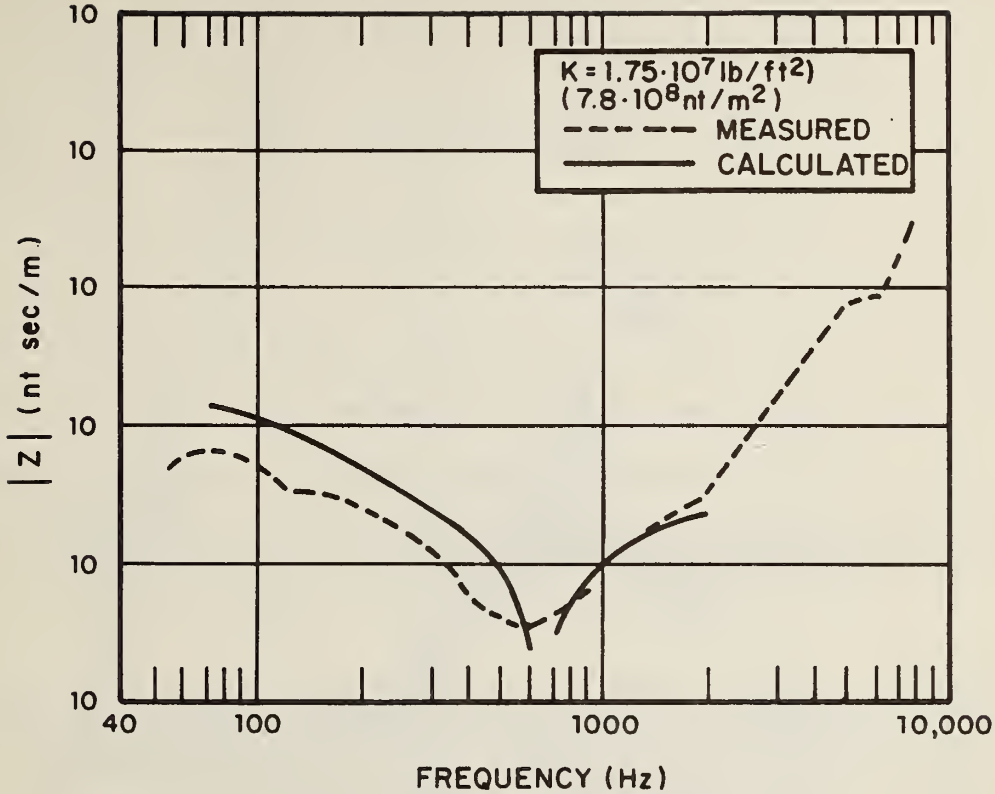

FIG. 2.4 COMPARISON OF MEASURED AND PREDICTED RAIL IMPEDANCE (from Volberg, 1972)

The Effect of Static Load on the Point Impedance. The question of the effect of static load on the impedance has not as yet been completely resolved. Certainly, if the track bed behaves in a linear manner, i.e., the applied force and resulting deflection are linearly related, then one would anticipate that static load would have no effect on the track impedance, at least under a vertical load. Track bed stiffness measurements made by Meacham (1970) by comparing the tie plate force with track deflection during the passage of a train (see Fig. 2.5) give somewhat conflicting results. Measurements on the Penn Central test track far from a joint give essentially linear stiffness. Measurement on that same track near a joint and measurements far removed from a joint on a main track of the B&O/C&O Railroad show a decidedly nonlinear stiffening character. Meacham concludes that the nonlinear character is due to looseness in the track joint in one case and in the ballast in another, but that the basic character of the trackbed stiffness should be linear.

The foreign literature contains numerous calculations and measurements of the natural frequency of the rail on tie and ballast under load (Birmann, 1965-66; Birmann, 1957; Nöthen, 1953).

Meacham, H.C., 1970. "The Influence of Track Dynamics on the Design of Advanced Track Structures," The Dynamics and Economics of Railway Systems and Management Association, pp. 137-154.

Birmann, F., 1957. "New Measurement on Rails with Various Ties" (in German), Eisenbahntechnisch Rundschau 6 (7) July 1957, pp. 229-245.

Nöthen, J., 1953. "Stiffness and Vibration Mass of Vertically Elastic Rails" (in German), *Glasers Annalen.*

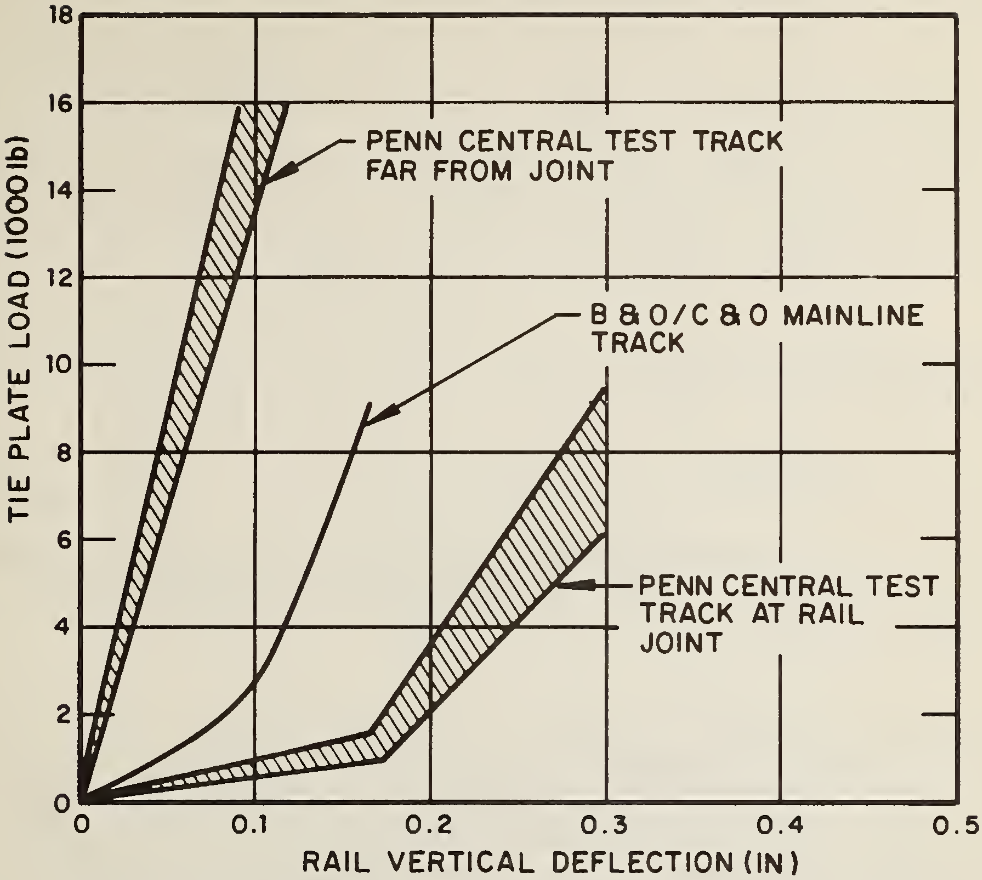

FIG. 2.5 TRACK BED STIFFNESS (from Meacham, 1970).

If the impedance is not sensitive to static load, then this natural frequency should be independent of the applied load. However, since these studies have generally been oriented towards determining the differences due to tie materials (concrete, steel, wood), they have included the considerable mass of the wheel set or shaker that provides the static load. As a result, these studies are of little use to us.

So far in our discussion we have not taken into account local deformation of the wheel and rail, by which we mean the nonlinear local indentations of the rail by the wheel and vice versa. These deflections were first studied by Hertz (1895) and recently extended by Nayak (1972) and Nayak and Tanner (1972) to the contact vibrations of steel wheels on steel rails.

The interaction is nonlinear with the force P related to the resulting deflection z by

$$P = Cz^{3/2} \quad ,$$

where C is a constant depending on Young's modulus of the wheel and rail material and the radius of curvature of the surface of the wheel and rail.

Hertz, H., 1895. *Gesammelte Werke*, Vol. 1, Leipzig.

Nayak, P.R., 1972. "Contact Vibrations," *J. Sound Vib.* 22;2.

Nayak, P.R., and Tanner, R.B., 1972. "Investigations of the Frictional and Vibratory Behavior of Rolling and Sliding Contacts," BBN Report No. 2402.

There is a dynamic stiffness associated with these contact deformations that is strongly dependent on the static load. For a railroad wheel on a rail where the static load on the wheel from the car is P_0 and the local deflection between the wheel and rail is z_0, the dynamic stiffness for dynamic deflections z may be given by

$$K \cong \frac{3}{2} \frac{P_0}{z_0} \quad .$$

As a result, if these local deformations are comparable to the deflections of a rail on its "elastic" foundation, then the impedance and, hence, the static load could be significantly affected by these local deflections.

For typical wheel loads of 10,000 lb (44,500 nt) and typical Hertzian deflections under those loads of 1 mil ($2.54 \cdot 10^{-5}$ m) one obtains a stiffness of $K = 1.5 \cdot 10^7$ lb/in. ($2.63 \cdot 10^9$ nt/m). The impedance to local deflections Z_L can then be written

$$Z_L = \frac{K}{j\omega} = \frac{1.5 \cdot 10^7}{j\omega}$$

As long as $Z_L >> Z_R$, where Z_R is the rail impedance, one can ignore the local Hertzian deflections. However, when Z_L becomes comparable to Z_R, Hertzian deflections will become comparable to rail deflections. For the AREA 100 rail on resilient mounts shown in Figs. 2.3 or 2.4, one finds that the Z_L and Z_R are equal at about 1800 Hz, quite near the high-frequency limit for roar noise. As a result, near 1800 Hz, Hertzian deflections may significantly affect the impedance. Unfortunately, there are no rail impedance measurements under static load to show such an effect.

Rail Response. In understanding the radiation from the rail, it is important to know how rapidly the vibration decays away from the excitation point. It is equally important to know how the different parts of the rail, head, web, and foot respond relative to one another. Only Naake provides any measurements on the subject. Figure 2.6, reproduced from his paper, shows the decay for both vertical vibration and horizontal vibration with distance along a rail on tie and ballast. It is clear from the data that horizontal vibration is more lightly damped than vertical vibration. Again, however, the published data is limited to below 1500 Hz. Naake also provides some information on the isolation provided by rail joints although only for horizontal vibration of the rail. Table 2.1 summarizes his results.

TABLE 2.1 HORIZONTAL VIBRATION REDUCTION ALONG THE RAIL AND ACROSS A JOINT

Freq.	Spatial Decay Along the Rail	Reduction in Level Across a Rail Joint
430	0.46 dB/ft (1.5 dB/m)	0
1500	0.23 dB/ft (0.77 dB/m)	12 dB
2350	0.094 dB/ft (0.31 dB/m)	22 dB

Naake points out that whether the joint is open or closed appears to have no effect on the losses across the joint. In general, though, it is apparent that at low frequency the spatial decay along the rail limits the length of rail that effectively radiates. For horizontal motion, at intermediate and high frequencies, it is the losses across the rail joint that are

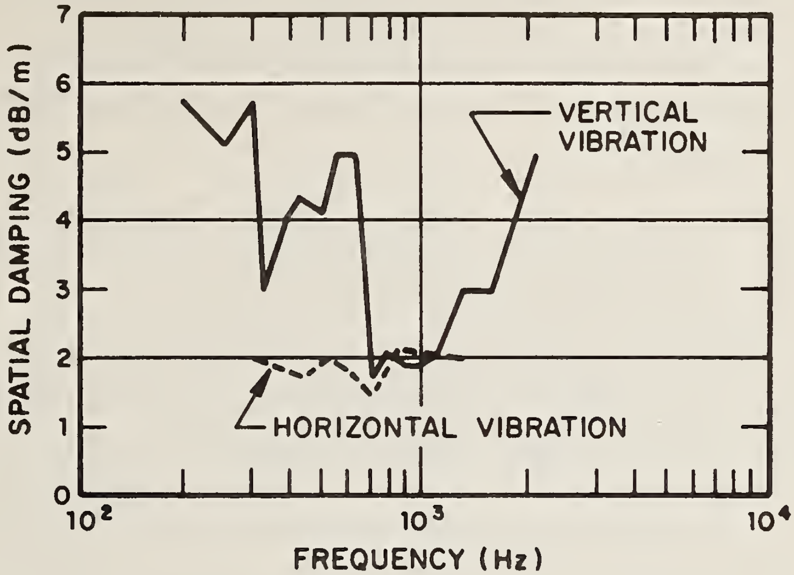

FIG. 2.6 SPATIAL DECAY ALONG THE RAIL (after Naake, 1953).

controlling. By contrast, Haeske (1957) reports almost no losses across welded joints.

Naake also examined the internal cross-sectional resonances of the rail. He found the following:

(1) at 1400 Hz the head and foot resonated on the bending stiffness of the web,

(2) at 4500 Hz the foot vibrates like a beam in which the center and ends are out of phase, and

(3) at 6300 Hz the head and foot vibrate out of phase with the compressional stiffness of the web acting as a spring.

Unfortunately, Naake made no attempt to model analytically either the spatial decay or the internal resonances of the rail, and we know of no other attempts to do so.

Rail Radiation Efficiency. As described in Sec. 2.1, knowledge of the radiation efficiency of a vibrating body allows one to predict the sound radiation from that body. Unfortunately, at the present time, there are no published measurements of rail radiation efficiency, and the only analytical model that has so far been suggested is to treat the rail as if it were a vibrating cylinder (Bender, 1972; Peters and Hemsworth, 1973). The radiation

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Haeske, H., 1957. "Vibration Investigation on Railroad Rails in The Ultrasonic Region," *Acoustica* 7, 146-150.

Peters, S. and Hemsworth, B., 1973. "A Comparison of the Sound Levels Radiated by a Vibrating Rail and Measured Rail Wheel Abuse," British Railway Board, Derby, England.

properties of such a cylinder have been worked out in detail by Bailey and Fahy (1972). Unfortunately, it is not clear how to size the cylinder diameter from the rail cross-sectional geometry.

Rail Directivity. There is essentially no data in the literature on rail directivity. For an analytical model, one can use the cylinder model described above. This model, when combined with the well-known fact that above 100 Hz the wave speed on a rail exceeds the speed of sound in air, yields a cosine directivity pattern with the maximum in the direction of rail motion. However, the model is certainly much too simple, for we know from Naake's work that above 1400 Hz the rail cross-section does not move uniformly, a fact that will certainly have an effect on the directivity pattern. In addition, for vertical motion of the rail, the reflectivity characteristics of the surface (ballast, concrete, etc.) on which the rail is mounted must have an effect on the directivity pattern, because we know that a dipole (which is essentially what the vibrating cylinder model is) mounted near a perfect reflecting surface with its major lobe normal to the surface becomes a quadrupole with a radically different directivity pattern.

2.3 Conclusions

As described above, the information on the dynamic characteristics of wheels and rails is rather sparse. We see the need for the generation of additional information during this program in the following areas:

Bailey, J.R., and Fahy, F.J., 1972. "Radiation and Response of Cylindrical Beams Excited by Sound," *ASME Trans, J. of Engineering for Industry* 94, Series B, No. 1, pp. 139-147.

Rail Impedances. There appears to be sufficient agreement between analytical models and data for the impedance of rails on resilient fasteners, at least for vertical forcing, to make further studies on that configuration unnecessary (Bender, 1972; Volberg, 1972). However, for rails on ties and ballast, additional measurements and mathematical models of the impedance for both vertical and horizontal forcing are required.

Static Load. The effect of static load on the rail impedance is difficult to determine, since for tie and ballast, at least, it seems to depend on the state of maintenance of the track (Meacham, 1970). In fact, the impedance of different parts of the same track might or might not be affected by static load depending upon the condition of individual ties and rail joints. It appears that if one is to make a valid assessment of the effect of static load, the impedance at a large number of joints along a track would have to be taken. This would involve a very expensive effort, the results of which would be of questionable utility.

For resilient fasteners, it seems reasonable to expect that knowing the spring stiffness of the fastener at the given static load (load deflection curve) is sufficient for determining the impedance.

In our discussion we have neglected the effects of Hertzian contact stiffness and the resulting contact resonances that, in fact, are static load dependent (Nayak, 1972). The most reasonable approach is to construct an analytical model of the effects of the contact resonances. If these effects are significant, then measurements of impedance under static load should be made to verify the analysis.

Rail Response. Measurements of the decay of vibration away from the excitation point on the rail are available, including the losses across a rail joint for horizontal motion (Naake, 1959) of the rail. New measurements are required which will provide the losses across a rail joint for vertical motion of the rail and also the relative response of different parts of the rail cross-section (head, foot, and web) for both vertical and horizontal forcing.

Rail Radiation Efficiency and Directivity. Since no measurements of rail radiation efficiency or directivity are extant, experimental measurements and analytical models of these quantities must be made.

Wheel Impedance. The impedance of a transit car wheel attached to a track should be measured for a point force at the tread in the radial and axial directions. Analytical models are also required.

Wheel Response. There are little data on the relative response of the wheel tread and web to forcing at the tread in the axial and radial direction (Stappenbeck, 1959). Measurements are required to aid in developing a model of wheel response.

Wheel Radiation Efficiency and Directivity. The lack of information on wheel radiation efficiency and directivity requires that measurements and analytical models be made of these quantities.

3. THE WHEEL/RAIL NOISE MECHANISMS

The kinds of noise generated by flanged metal wheels running on metal rails are generally categorized as "squeal", "impact", and "roar". Squeal is the very intense, intermittent noise (usually composed of one or two frequencies) that generally occurs when transit vehicles pass through short radius curves, although it has been known to occur on straight track. Impact noise is the "clickety-clack" sound caused by wheel flats, rail joints, and other rail discontinuities. Roar noise, which generally dominates on welded tangent track, is usually assumed to be due to the microroughness on wheels and rails. In this section, we examine what is presently known about these three mechanisms: We investigate both analytical models and confirming experimental data, point out deficiencies in this knowledge, and suggest new research to correct these deficiencies. As with the whole report, we intend this section to be a critical review of the state of knowledge rather than an exhaustive compilation of reference material. As a result, only those references which contribute significant information to our knowledge of the mechanisms are included.

3.1 Wheel Squeal

Wheel squeal, the very high pitched and very unpleasant noise emitted from train wheels, is a well-recognized phenomenon. However, there is very little published information on the subject. In this section, we review the rather sparse literature and evaluate the available information. Our comparison of the various possible models indicates that the model based upon the crabbing of the wheels of a truck is the most likely description of this mechanism. We, therefore, elaborate upon this model, use it to develop certain predictions, and suggest some means for reducing wheel squeal noise levels.

3.1.1 Occurrence of wheel squeal

Measurements of wheel squeal are difficult to make because the noise is highly intermittent and not easily reproducible. Wheel squeal occurs at a few discrete tones, with only one tone or, at most, a few tones occurring at a time. These discrete tones can appear anywhere within the range of about 500 Hz to 4 kHz.

In Sec. 5 of this report, where the contributions to wheel/rail noise are rank-ordered, Fig. 5.5 shows squeal levels as a function of curve radius for a wide range of both curve radius and train speed. The data are taken from measurements of a number of different transit systems in the United States, Europe, and Canada. If one discounts the BART test car data, for which the analysis technique appears to have given unrealistically low results, the range of squeal noise levels is 84 dB(A) to 97 dB(A) for measurements taken at 50 ft (15.2 m) from the track. This data covers a 10:1 range in curve radius and a 2:1 range in speed.

Wheel squeal, as shown in Fig. 5.5, was found to occur over a large range of radii – from 90 ft (27.4 m) to 900 ft (274 m). A 90-ft (27.4 m) radius is about as tight a curve as a 50-ft (15.2 m) passenger car can negotiate so there are not likely to be any data for curves sharper than this. Also, there are no reports of squeal for radii greater than 900 ft (274 m), probably because near this point the curve in the track becomes slight enough that significant forces are no longer applied in the turn. The main factor determining train speed in a curve is passenger comfort rather than capability to negotiate the curve. Transit authorities generally try to keep lateral acceleration below about 6% of gravity. The speed range investigated in the literature shows very little correlation between speed in the curve and noise levels.

3.1.2 Possible mechanisms for wheel squeal

It is generally accepted, although there is little proof, that wheel squeal is generated by a "stick-slip mechanism", i.e., a force on the wheel tends to make the wheel slide, but static friction tends to make the wheel stick on the rail. As the sliding force increases, it finally reaches the level required to break static friction and the wheel starts to slide. The sliding friction is generally less than the static friction, so the wheel continues to slide until the force which causes the sliding drops to the sliding friction value. Then, as static friction builds, the wheel sticks again. This sticking and sliding occurs in very rapid succession.

There are different ways in which the wheel can slide and, hence, there are three models for the mechanism of wheel squeal:

1. Differential slip between inner and outer wheels on a solid axle,

2. Rubbing of the wheel flanges against the rail, and

3. Crabbing of the wheel across the top of the rail.

Differential slip occurs when two wheels on the same axle go round a curve and the outer wheel has to travel further and hence rotate faster than the inner wheel. If the axle is solid, then there is a torque on the wheel tending to make it slip on the rail. *Flange rubbing* occurs when the wheel flange, a device used to prevent derailment by allowing the rail to guide the wheel, rubs against the outside rail. Since there is sliding of the flange against the rail, there is an opportunity for sticking and slipping to occur. *Wheel crabbing* occurs when a truck has more than one axle. Most transit cars have two 2-axle trucks, each truck having two parallel axles. When the truck enters a curve, since the axles are parallel, they cannot both lie upon the radius of the curve. In general, the front axle tends to run out of the curve and the trailing axle tends to run into the curve, causing the axles to have radial as well as a tangential velocity. Thus, as well as rolling around the curve, the wheels slide across the rail, again with the possibility of inducing squeal. This crabbing motion is solely a consequence of the finite length of the truck, and a truck with a single axle would not crab.

To distinguish between these models, we must look into the nature of friction, the stresses and strain in the wheel, and the dynamics of the wheel and truck.

3.1.3 Friction

The friction between two surfaces is determined by the microstructure and the elastic and plastic deformation of those surfaces. The theory of friction is well-known (Bowden and Tabor, 1950), so there we avoid a detailed discussion addressing only the two areas that are relevant to the problem of squeal: rolling contact with an applied torque and rolling contact with an applied transverse force. The friction force is divided by the wheel load to give the coefficient of friction and the slip velocity is divided by the rolling velocity to give the differential slip.

Rolling Contact with an Applied Torque

Reynolds (1875) noticed that when he applied a torque to a rubber roller, the circumferential velocity was not equal to

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Bowden, F.B. and Tabor D., 1950. *The Friction and Lubrication of Solids*, Clarendon Press, Oxford.

Reynolds, O., 1875. *Phil. Trans. of Royal Society of London*, 166:155.

the forward velocity of the roller. This differential velocity is called creep, an important phenomenon when a train is accelerating, braking, or traversing a curve. Creep initially arises from elastic deformation of the tread of the wheel.

Let us consider an accelerating train driving wheel: Ahead and inside the contact patch between the wheel and the rail, the wheel tread is in circumferential compression. For a heavily loaded wheel, the strain can be up to 1%. Hence, the length of the tread in contact with the rail is shorter than it would be unstrained. The wheel then has to rotate faster for the same forward velocity and, hence, the creep. In practice, the whole of the contact patch does not adhere to the rail. The compression in the tread tends to be relaxed in the aft portion of the contact patch, so that only the front part adheres to the rail and slip occurs over the aft part of the contact patch (see Fig. 3.1). This small-scale slipping, or microslip, occurs over a large part of the contact patch as the torque on the wheel increases. When a total slip of about 1% of forward velocity is reached, the creep starts to increase rapidly as plastic creep sets in. Here, creep arises from plastic deformation of the tread (see again Fig. 3.1). Creep in this region can be very substantial, going as high as 10%. Finally, true slipping occurs and there is a drastic reduction in the coefficient of friction.

The theory of the elastic creep of railroad wheels was given by Carter (1926). The theoretical solution, based on elasticity theory, for two cylinders may be written as

$$\text{Creep} = \mu\sqrt{\frac{4(1-\sigma)}{\pi G}\, N'\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}\; 1 - \sqrt{1 - \frac{F}{F_{max}}}$$

Carter, F.W., 1926. "On the Action of a Locomotive Driving Wheel," *Proc. Roy. Soc. A.*, 112:151.

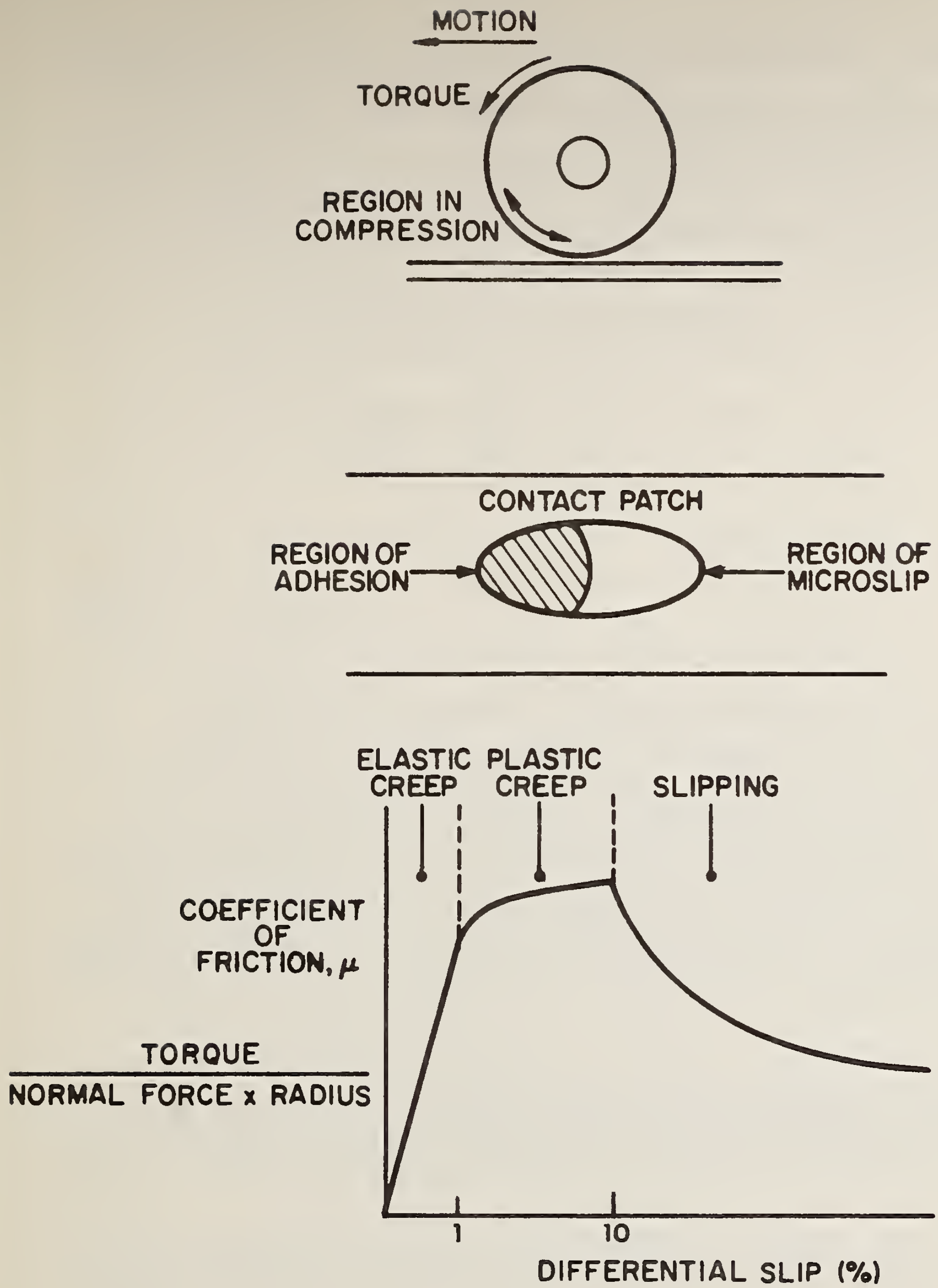

FIG. 3.1. ROLLING CONTACT WITH APPLIED TORQUE MOTION.

where creep is the distance slipped divided by the distance rolled, σ is the Poisson ratio for material of cylinders, G is the shear modulus for material of cylinders, N' is the normal load per unit width of cylindrical wheels with radii R_1 and R_2, F is the tangential force (F_{max} being its maximum value), and μ is the coefficient of friction between the surfaces. If appropriate elastic constants are used for steel and if R_1 = 6.25 in. (0.16 m) and R_2 = 13.25 in. (0.337 m),

$$\text{Creep} = 1.35 \times 10^{-4} \ \mu\sqrt{N'}\left(1 - \sqrt{1 - \frac{F}{\mu N}}\right) ,$$

where N is the total loading on the wheel. Note that creep increases with the square root of the wheel loading N'. Maximum creep occurs when F = μN. Then, for N' = 10,000 lb/in., creep = 1.35μ %. This is the maximum of the elastic creep, but plastic deformation and vibration can both increase total creep.

Because of this creep, it is possible for an axle to go around a curve without either of its wheels slipping. The friction on the rail supplies a torque to the wheel so that the tread elasticity can take up the differential motion. A 1% creep allows the axle to go around a 236-ft (72 m) radius curve without slipping. Plastic creep allows the wheel to pass around even tighter curves: 2.5% creep for a 94-ft (28.6 m) radius curve without slipping. Thus, it is quite possible for a solid axle wheel-set to pass around very tight curves without any large-scale slipping.

Rolling Contact With an Applied Transverse Force

Carter (1926) also considered the case of creep with a transverse force applied to the wheel. Here a similar situation occurs as with longitudinal creep. For small transverse forces applied to a *rolling* wheel, the wheel will tend to crab sideways rather than slip. This creep is due to the elastic deformation of the contact patch and only occurs during rolling. We have then a similar relationship between transverse creep and the coefficient of friction (transverse force/wheel loading) as for longitudinal creep. Also at some value of creep of a few percent, the coefficient of friction drops abruptly as sliding occurs. This abrupt drop was found by Frederich (1970) to occur at 1-2% creep.

Effect of Humidity

Barwell and Woolcott (1963) found that humidity reduces the static friction while leaving the sliding friction unaltered. Thus, there was an abrupt drop between static and sliding friction below 80% relative humidity (RH). Above 80% RH, there was a very gradual reduction from static to sliding friction as the sliding velocity increased; at 100% RH sliding friction increased steadily with velocity. The value of the friction coefficient for large amounts of sliding was always about the same.

Frederich, F., 1970. "Viraftschlussbeanspruchungen am Schrägrolenden Schienfahrzeugrad" (Interaction Forces in Wheels of Rail Vehicles Rolling Obliquely), *Glas. Ann*, 94: 86-94.

Barwell, F.T. and Woolcott, R.G., 1963. "The N.E.L. Contribution to Adhesion Studies," *Proc. Instr. Mech. Engrs.*, 178: 145-160.

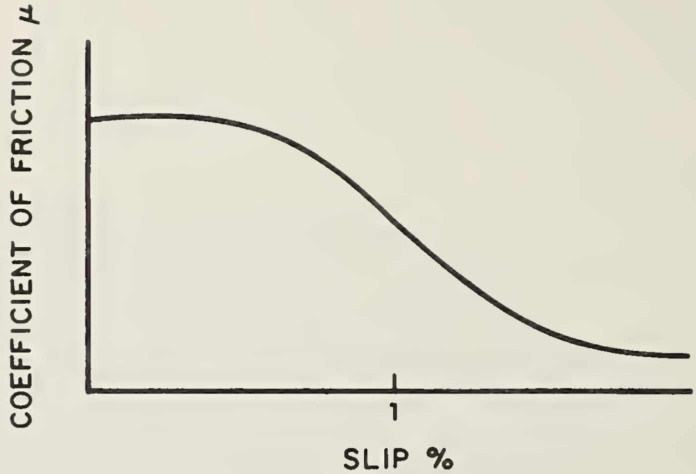

FIG. 3.2. FRICTION-SLIP CURVE FOR TRANSVERSE MOTION.

Reason for Reduction of Friction With Sliding

There are at least three possible explanations for the abrupt reduction in friction when sliding occurs:

1. Momentum carries the body over bumps in the contact surface;

2. Local melting acts as a lubricant;

3. Electrostatic force drops because of increased separation.

The second of these explanations applies only to very high sliding velocities. The third possibly describes the effectiveness of water and corrosive additives such as caprylic acid in eliminating stick-slip motion by eliminating the electrostatic forces of friction (Schurmann, 1962); however, these lubricants also have a serious side effect of rapidly accelerating wear by encouraging a tearing action between the surfaces. Hence, they are not to be recommended.

3.1.4 Dynamics of a truck in a curve

The wheels of a railroad truck generally have tapered rather than cylindrical treads. This type of tread provides a centering action for the axle when it is running on tangent (straight) track so that the flanges of the wheels will not rub. When these tapered wheels enter a curve, the axle moves outward slightly, enabling the truck set to roll around the curve on the tapered treads. A typical wheel has a taper of 1 in 20 and a clearance of 3/4 in. (19 mm). Thus, on well aligned curves with radii greater than 2000 ft (608 m), the flanges will

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Schurmann, R., 1962. "Friction and Wear," *Wear*, 5:31-42.

not generally rub against the rail, but on tighter curves they will. In fact, at low speeds, the outside-leading and the inside-trailing wheel flanges will rub because the front axle is tending to run out of the curve and the rear axle is tending to run into the curve. At medium speeds, the centrifugal force throws the whole truck slightly outwards so that the trailing inner-wheel flange is no longer in contact with the rail; only the leading outer-wheel flange rubs - the most common situation (see for example Minchin [1956]). At higher speeds still, the centrifugal force throws the truck out further so that both outer wheels rub, but this case is not very common. Figure 3.3 shows diagrams of the three cases.

Since the axles of the truck are parallel (except for the generally small effect of truck elasticity), they cannot both pass through the center of curvature of the track. Hence, in general, the wheel will make some small angle with the rail. At high speeds (when both outer flanges rub), this angle of attack will be $L/2R$ where L is the wheel base of the truck and R is the radius of the curve. At very low speed, when both the leading outer and trailing inner flanges rub, the angle of attack is approximately $L/2R + C/L$, where C is the total clearance between the flange and the rail.

Since the wheel is not parallel to the rail, it must slide normal to its plane as well as roll to traverse the curve. This crabbing velocity is given approximately by $VL/2R$, where V is the speed of the train in the curve.

R.S. Minchin, 1956, "The Mechanics of Railway Vehicles on Curved Track," The Journal of the Institution of Engineers, Australia, Vol. 28, pp. 179-185.

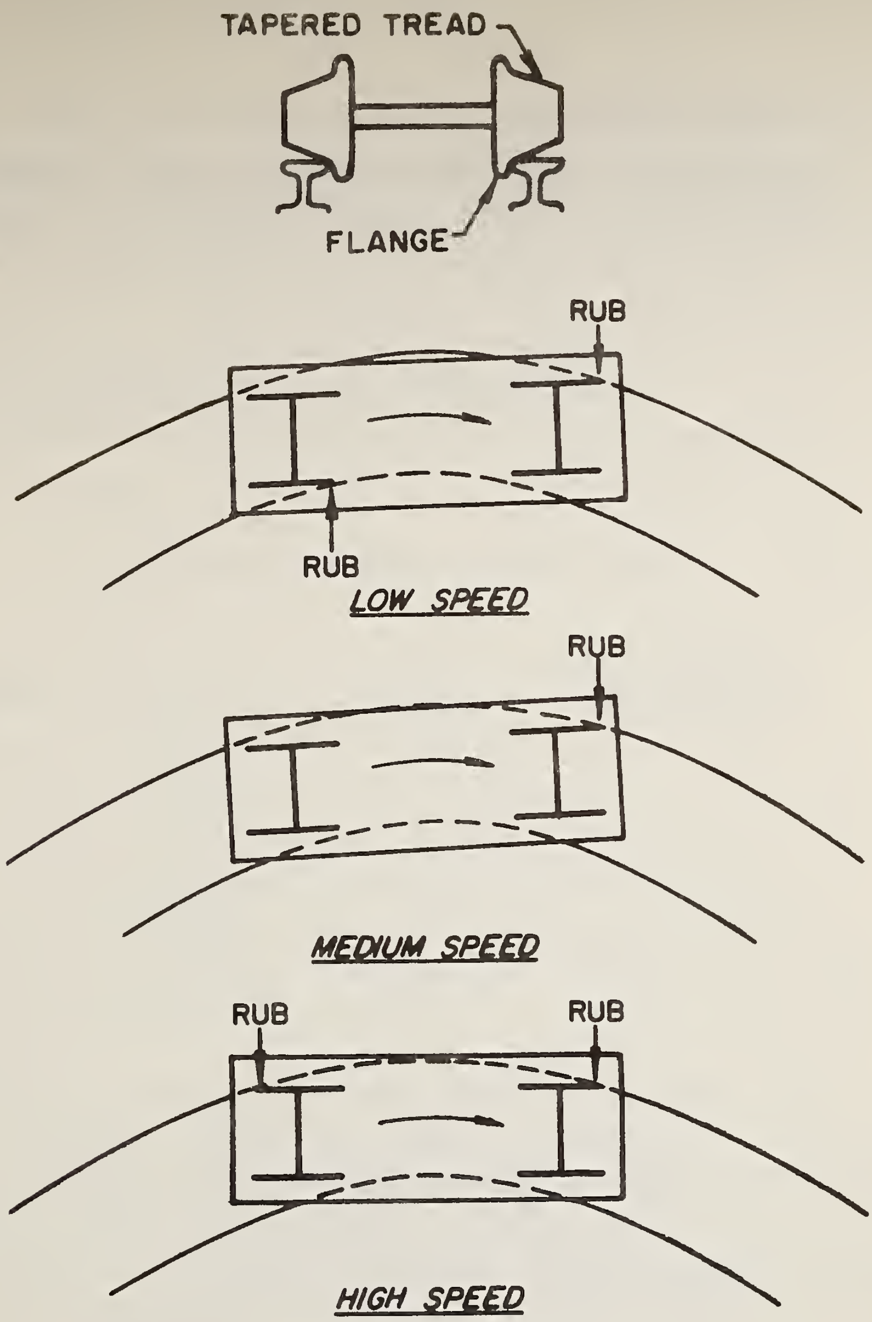

FIG. 3.3. TRUCK IN A CURVE.

3.1.5 Wheel resonances

Stappenbeck (1954) conducted measurements on wheel response and identified the resonant modes (see Sec. 2). From his study, we find that when the wheel is being excited at the rail, we should expect this point to be an antinode. Thus, any modes in which the circumference at the wheel tread is a node will not be excited. Stappenbeck found the strongest modes to be the ones with radial node lines. The frequencies of these modes are listed below in Table 3.1.

TABLE 3.1 RESONANT MODES OF WHEEL RESPONSE

Mode Circumferential Node Circles	Radial Node Lines	Frequency	Approximate Ratio
0	2	400 Hz	0.4
0	3	1060 Hz	1
0	4	1930 Hz	2
0	5	2940 Hz	3
0	6	3940 Hz	4

These frequencies are typical for street car wheels. Since the tread of the wheel contains two-thirds of the mass, the main character of these modes are flexural waves in the tread, normal to the plane of the wheel. The web of the wheel plays little part in determining the response. Further, it should be noted that, except for the lowest mode, the frequencies of the others are all approximately in a harmonic ratio.

Swanson (1966) and Kirschner (1972) similarly found that the modes important in wheel squeal were modes with radial node lines. In general, the modes with 2 and 3 node lines are the most common, although those with 4 and 5 nodes have also been observed.

3.1.6 Choice of model to describe wheel squeal

Of the models discussed above, the one which assumes an applied torque suggests that differential slip between inner and outer wheels on an axle is not likely to cause squeal, because the elastic compression of inner wheel and extension of outer wheel tread can compensate for the differential velocity around a curve. Further experiments in Stockholm (Berglund, 1972) showed that lubricating the outer rail on a curve did *not* eliminate squeal. Also, Pullman Standard has built a car with independently driven wheels, so that differential rotation is possible, but the vehicle still squeals in tight curves. Thus, the basic conclusion then is that the differential slip is unlikely to be a sufficient source of wheel squeal on curved track. However, it is still possible that the wheel slip which occurs on rapid acceleration or braking on straight track could produce squeal.

Swanson, R.C., 1966. "Acoustic Performance of Tread Damping on Steel Transit Wheels," B.F. Goodrich Company.

Kirschner, F., 1972. "New Developments in the Control of Railroad Wheel Screech Noise," The Soundcoat Company, Presented at Internoise 1972 Conference.

Berglund, H., 1972. "Stockholm Tackles the Noise Problem," *Railway Gazette International*, pp. 254-259, July.

Experimental evidence also suggests that flange rubbing alone is not a sufficient source mechanism for squeal. Berglund (1972) reports that lubricating the outer flange alone did *not* eliminate squeal; Stappenback (1954) found that it was the inner wheels which squealed.

This evidence suggests that differential wheel motion or flange contact alone is not sufficient to produce wheel squeal. However, a model for squeal, which involves a crabbing motion of the wheel, it consistent with the above evidence. Another possible mechanism is flange contact in combination with crabbing. Here the flange of the leading outer wheel will not be parallel to the rail, but tend to run into it. This can lead to "spragging" or amplification of the normal force between flange and rail and hence give rise to a stick-slip motion and hence squeal.

It is possible that all three mechanisms do in fact take place, but eliminating differential slip or flange contact does not eliminate squeal. Thus, in order to cure squeal, the mechanism involving the crabbing of the wheel has to be considered primarily. When this mechanism of squeal has been cured, it is still possible, however, that the others could take place.

Thus we feel the basic model of wheel squeal on a curve is due to the crabbing motion of the wheels, with or without flange contact, which arises as a consquence of the finite length of the truck. This induces a stick-slip motion transverse to the rail, which in turn excites a natural wheel resonance. The behavior is similar to a violin bow which excites the natural resonances of the violin string.

This mechanism, where the exciting forces are normal to the plane of the wheel, explains why the wheel is such an efficient radiator of sound. The other two mechanisms, differential slip and flange rubbing, excite the wheel in its own plane, where it is a relatively inefficient radiator of sound.

Negative Impedance of Stick-Slip Mechanism

The impedance of a mechanical system is the ratio of the force applied to the velocity of the system. When both are harmonic functions of time at the same frequency ω.

Dash Pot: $F = CV$, where F = force, V = velocity, C = constant of dash pot.

Impedance $F/V = C$.

Mass: $F = MA = i\omega MV$, where M = mass, A = acceleration, ω = frequency of harmonic motion, i = square root of -1.

Impedance $F/V = i\omega M$.

Spring: $F = KX = V/i\omega K$, where K = spring constant, X = displacement

Impedance $F/V = K/i\omega = -iK/\omega$.

Stick-Slip: $F_S + \Delta F = N\ (\mu + \nu\ \Delta V/V)$ from friction slip curve, where F_S = static friction force, ΔF = change on sliding, N = load on wheel, ΔV = sliding velocity, V = rolling velocity, ν is a coefficient related to the slope of the friction-sliding velocity curve and μ is the coefficient of static friction

But $F_S = N\mu$,

therefore, $\Delta F = +N\nu\ \Delta V/V$

$$\text{Impedance } \Delta F/\Delta V = +\ N\nu/V.$$

Thus, the impedance of stick-slip is a function not only of the slope ν of the friction-sliding velocity curve, but also of the wheel load N and sliding velocity V. Note that since friction generally decreases with sliding velocity, the coefficient ν is generally negative, and we have a negative real impedance. To conclude,

Dash Pot:	Positive real impedance	C
Mass:	Positive imaginary impedance	$i\omega M$
Spring:	Negative imaginary impedance	$-iV/\omega$
Stick Slip:	Negative real impedance	$-\lvert\nu\rvert N/V.$

At a velocity resonance, the imaginary parts of the impedance cancel; i.e.,

$$i\omega M = iK/\omega \text{ or } \omega = \sqrt{K/M},$$

and the response is determined by the real part of the impedance. If $C > \lvert\nu\rvert N/V$, the impedance is positive; the vibrations at frequency $\sqrt{K/M}$ are damped, and there is no squeal. However, if the impedance is negative, the vibrations grow in amplitude and squeal occurs. When the impedance is zero, the vibration has a constant amplitude. Thus, the conditions for squeal to occur are

$$C \leq -\ \nu N/V \quad .$$

The coefficient ν will be determined solely by the state of the wheel and rail. The parameter C will be determined by the amount of damping in the wheel, and this will be a function of the mode.

Frequency of Squeal

The models for wheel squeal discussed so far claim that the wheel will be excited into a resonant mode by the stick-slip mechanism. However, there is no indication so far as to which or how many of the many modes will be excited. We know that generally only one of the wheel modes is excited at a time, but that the particular mode will change upon circumstances. A similar situation arises with an organ pipe where the frequency can be raised or lowered an octave by under-blowing or over-blowing.

The rate of growth of the resonance is given by

$$\exp\left(-\frac{\nu N}{v} - C\right) t \quad .$$

Since damping varies with mode, that mode which has the least damping will grow fastest and dominate all other modes. According to Stappenbeck (1954), this is the mode with 3 diametral node lines which has a frequency of 1 to 2 kHz. However, it is well-known that squeal occurs at other modes as well. Thus, we must seek some mechanism for describing which modes of the many possible modes will be excited by the stick-slip mechanism.

There is a difference here between wheel squeal and the action of a violin. The possible modes of a violin string are all harmonically related, so a harmonic excitation from the bow will excite all modes. However, the modes in a wheel are not harmonically related, so that a harmonic excitation can only excite one mode at a time. Which of these modes will be excited under what conditions? This question remains the subject of further work.

However, we can say that the relevant parameters for wheel squeal are

- Curve radius
- Speed of train
- Wheel load
- Rail and wheel surface conditions
- Damping of wheel.

3.1.7 Conclusions

A thorough study of the phenomenon of wheel squeal was performed by Stappenback in 1954. He proposed a mechanism as one of stick-slip as the wheels slide transversely across the rail, rather than one of differential slip of inner and outer wheels or flange rubbing, as had previously been thought. However, flange contact may play a part in squeal. This work does not seem to be widely recognized in the United States today, perhaps because it was written in German. This transverse, or crabbing, velocity of the wheels arises from the geometry of a truck of finite wheel base negotiating a curve.

Squeal has generally only been considered on curves, although it can occur on tangent track as well, when a train accelerates or brakes hard.

The level of squeal has been found to vary from 84 dB(A) to 97 dB(A) at 50 ft (15.2 m). Squeal occurs on curves between 90 ft (27.4 m) and 900 ft (274 m) radius and at speeds up to 30 mph (48 km/hr).

Authors have found that *lubricating* the head and sides of only the outer rail of a curve does not eliminate squeal, although lubricating both rails does eliminate it. Also, the application of damping treatment to wheels will eliminate the squeal.

The noise has been found to radiate primarily from the wheel at frequencies corresponding to resonances of flexural waves running around the tread of the wheel with displacements normal to the plane of the wheel. Consequently, any damping applied to the web of the wheel has little effect. The damping must be applied to the rim.

The literature has suggested a basic mechanism for squeal, but there are many aspects which have yet to be determined, including:

- What determines the level of the squeal?
- What determines the resonant mode at which squeal occurs?
- How much damping is needed to eliminate squeal?
- What is the effect of surface roughness or grain size?
- Why is squeal intermittent?

It is expected that these questions can be answered by further development of the theoretical model described in this section and by further experimental studies. In this latter case, measurements will be made on rolling contact and on the surface conditions of steel wheels; rail surfaces should also be studied. Further theoretical and experimental studies must be made before quantitative predictions can be developed from the model described here.

3.2 Impact Noise

Impact noise is the "clickety-clack" sound resulting from flats on the wheels and from the interaction of wheels with rail joints, signal junctions, switches, and other track discontinuities. In this section, we discuss the present state of knowledge of the impact noise phenomenon, including existing mathematical models and relevant data.

3.2.1 Relative importance of impact

Section 5 of this report provides a detailed compilation and comparison of data on the noise generated by the three known wheel/rail noise generating mechanisms: squeal, impact, and roar. Here, we discuss in somewhat more detail a number of measurements of particular relevance to the levels generated by the impact phenomenon.

The results in Sec. 5 and other recent experimental investigations have indicated that the noise generated by wheel impacts at rail joints dominates the passby noise of rail vehicles running on properly maintained steel wheels. Rapin (1972) has found 4 to 5 dB(A) higher noise levels when a train passes over a well maintained track with joined rails than those obtained for a track with welded rails. The Hungarian Railways (MAV) has carried out extensive investigations on the effect of rail joints on rolling resistance. A summary of this work is reported by Kerkapoly (1965). In the framework of this study, the effect of rail joints on the interior noise of the car was also evaluated. Figure 3.5, taken from Kerkapoly's summary, compares the time histories of the interior noise levels obtained on jointed rail and welded rail respectively. Both recordings were made in the same car (a 4-axle, 2nd-class, passenger train car) at 51 mph (80 km/hr) speed. Observing the upper level trace obtained for jointed rail, one notices a sudden 9 dB(A) to 10

Rapin, J.M., 1972. "Noise in the Vicinity of Rail Lines: How to Characterize and Predict It," Centre Scientifique, Technique du Batiment Chaier, Library Trans. 1737, Building Research Establishment WD27JR.

Kerkápoly, E., 1965. "Welded Rail from the Viewpoint of Economy," Eisenbahntechnische Rundschau (in German).

dB(A) increase of the interior noise level every time the wheel rolls over a joint.* The lower trace obtained for welded rails shows only random fluctuations with the same average level as one observes on jointed rail midway between joints. Figure 3.5 illustrates that the impact noise created by the wheel rolling or jumping over a rail joint not only disturbs the population along the roadway because of its impulsive character, but for straight track (i.e., no squeal) also constitutes the main noise disturbance to passengers in the car. Bender and Heckl (1970) have analyzed particular time sequences of car passage noise tapes recorded with a microphone positioned in the vicinity of a rail joint. They found that the noise recorded at the time of the wheel impact is on the order of 8 to 10 dB(A) higher than that recorded at a time halfway between impacts. Unfortunately, this statement has been frequently misinterpreted by other investigators, who have concluded that impact at rail joints increases train noise by the same amount. This result, of course, does not follow, because source-receiver distance (i.e., distance between wheel and microphone) was larger for the time sequence recorded midway between impacts than for that recorded during the impact. Consequently, from their data, there is no way to separate what portion of the increase was due to the increased noise generation at the rail joint and what was due to the difference in source receiver distance.

The experimental evidence available at this time permits us to conclude that under normal circumstances (i.e., no wheel

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*The microphone in the car was positioned above the truck.

Bender, E.K., and Heckl, M., 1970. "Noise Generated by Subways Above Ground and in Stations," BBN Report No. 1898.

FIG. 3.4. CAR INTERIOR NOISE LEVELS ON WELDED AND JOINTED TRACK. TRAIN SPEED 51 mph.

flats, no shelling of the rail, and no squeal), the level of the farfield train noise for tracks with jointed rails is controlled by impacts at rail joints. This conclusion is not at all surprising, because irregularities found at rail joints are usually much larger than those found on other portions of the rail or on the wheel flange. Accordingly, for all tracks with jointed rails, one can obtain appreciable noise reduction only if one finds ways to reduce the impact.

3.2.2 Present state of knowledge of the generation of noise by the impact mechanism

From reviewing the domestic and European literature, we have found that the present understanding of the impact phenomenon is extremely poor. Some information does exist on measurements of impact vibration levels at joints (Cain, 1940), on the statistical distribution of wheel flats in wheels (Schramm, 1955; Rubin, 1952), and on the theoretical calculation of wheel trajectory over rail joints (Kerkapoly, 1965).

Cain reports on the research and development work of the Electric Railway Presidents' Conference Committee. The objective of this work was the reduction of noise and vibration in conventional street cars. The impact mechanism was investigated experimentally by means of a wheel test machine using a single car

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Cain, B.S., 1940. "Vibration of Rail and Road Vehicles," Chapter 15, Pitman Publishing Corporation, New York.

Schramm, G., 1955. "The Loading of the Rail by Railway Cars," *Glasers Annalen,* 79:317-323 (in German).

Rubin, 1952. "Contribution to the Problem of Rail Stresses due to Flat Wheels," Archiv für Eisenbahntechnik, 1:28 (in German).

wheel operating on a circular track of 25-ft (7.6 m) diameter and achieving a maximum speed of 30 mph (48 km/hr). The rail had three joints with gaps of 1/16 in. (1.6 mm), 1/8 in. (3.2 mm), and 1/4 in. (6.3 mm). The joints were supported by ties directly beneath them. The wheel loading could be varied from zero to 7000 lb (31,200 nt). The overall acceleration at the journal boxes and the noise at a distance of 6 ft (1.8 m) from the joint have been measured. The joint-impact experiments yielded the following results:

1. The overall acceleration on the journal box increased approximately linearly with speed of travel for conventional steel wheels reaching 50 g at 30 mph (48 km/hr).

2. Wheels of larger diameter produced smaller journal box acceleration than wheels of smaller diameter.

3. As compared with commercial steel wheels, resilient wheels produced considerably smaller journal box acceleration, the acceleration decreasing with increasing resiliency of the wheel.

4. The noise (characterized by the sound pressure level of a 1000-Hz pure tone of equal loudness) is plotted as a function of speed. Up to 20 mph (32 km/hr), the loudness increases with a slope of 9 dB per doubling of speed; above 20 mph the slope tends to flatten out. (This result is in contrast with the speed dependence of the journal box acceleration as well as with train noise measurements obtained at high speeds.)

5. Resilient wheels generate substantially less impact noise than do commercial steel wheels.

In addition to the joint-impact test, a "wedge jump" test was conducted to develop some understanding of impacts that occur when the wheel separates from and then returns to the rail. A steel wedge was mounted on the surface of the rail so that the wheel would climb the gradually sloping part of the wedge and jump off the thick end back to normal rail level. The thick end was 3/32 in. (2.4 mm) high. The important results of this test are:

1. Above a certain critical speed (where the tire member ceased to roll off the edge and actually took on a trajectory), an increase in travel speed does not appreciably effect the vertical acceleration of the journal box.

2. Observations made of solid wheels indicate that the wheel underwent up to seven successive secondary bounces and that the distance between the successive impact points decreased gradually.

3. No secondary bounces were observed for resilient wheels.

The usefulness of the data presented by Cain (1940) is limited because: (1) no spectral information is given. (2) There is no information given on the vertical misalignment of the rail heads at the joint. (The wedge-jump test indicates that this information would be essential to know whether the wheel was jumping up or down at the joint.) (3) No data are presented on how the gap lengths of 1/16 in. (1.6 mm), 1/8 in. (3.2 mm), and 1/4 in. (6.3 mm) affected the noise.

Information on wheel flats is given by Schramm (1955), Rubin (1952, 1953), Luber (1961), Popp (1952), and Suthoff (1959). Schramm reports on the results of a statistical survey carried out on 11,000 wheels checked for flats. Since this survey was conducted exclusively in Europe, caution should be used in applying the results to American transit systems. The statistical distribution of the flat heights (defined as the proper radius minus the actual radius of the flat) is given in the table below:

TABLE 3.2 DISTRIBUTION OF FLAT HEIGHTS ACCORDING TO SCHRAMM AND RUBIN

Flat Height	Probability
0 to 1 mm (0 - 0.04 in.)	64%
1 to 2 mm (0.04 - 0.079 in.)	23%
2 to 3 mm (0.074 - 0.118 in.)	7%
3 to 4 mm (0.118 - 0.157 in.)	4%

Similar information on flat heights is reported by Luber (1961) and Sauthoff (1959):

Rubin, H. 1953. "American Investigations on the Effect of Flat Wheels on Rail Stresses," *Archiv für* Eisenbahntechnik, 2 (in German).

Luber H., 1961. "Contribution to the Design of Track on Elastic Foundation for Vertical Load," Ph.D. Thesis, Institute for Railroad and Highway Engineering, Technical University, Munich (in German).

Popp, C., 1952. "Impact of Flat Railroad Car Wheels," *Archiv für* Eisenbahntechnik, 1 (in German).

Sauthoff, F., 1959. "Wheel Flats and Other Irregularities of Railroad Wheels," *Glasers Annalen,* 9 (in German).

TABLE 3.3 DISTRIBUTION OF FLAT HEIGHTS ACCORDING TO LUBER AND SAUTHOFF

Flat Height	Probability
up to 0.5 mm (0.02 in.)	30.3%
0.5 to 1 mm (0.02 - 0.04 in.)	29.8%
1 to 1.5 mm (0.04 - 0.059 in.)	14.3%
1.5 to 2 mm (0.059 - 0.079 in.)	10.5%
2 to 3 mm (0.079 - 0.118 in.)	10.3%
3 to 5 mm (0.118 - 0.196 in.)	2.8%
over 5 mm (0.196 in.)	2.0%

An interesting comparison of the statistical distribution of flats for passenger and freight trains is provided by Luber (1961) and Popp (1952):

TABLE 3.4 COMPARISON OF WHEEL FLAT STATISTICS FOR PASSENGER AND FREIGHT TRAINS

Flat Height	Probability	
	Passenger Train	Freight Train
below 1 mm (0.04 in.)	69%	50%
1 to 2 mm (0.04 - .079 in.)	24%	26%
over 2 mm (0.079 in.)	7%	24%

As expected, one finds a substantially larger percentage of wheels with flats above 2 mm (.079 in.) on freight cars than on passenger cars. To protect the track from high intensity impacts causing excessive rail stresses and noise, there is an international agreement (Luber, 1961) to limit the permissible height of wheel flats to 2 mm (.079 in.). As one sees in the previous tables, most wheel flats found in periodic maintenance check-ups have flat heights below 2 mm (.079 in.); however, there are flats which substantially exceed this limit. The largest flat height reported was 31 mm (1.22 in.).

Researchers of the Hungarian Railways (MAV) and staff members of the Technical University of Budapest carried out an extensive theoretical and experimental program to compare the economic advantages of replacing jointed rails by welded rail (Kerkapoly, 1965). The main thrust of the study was directed toward the track geometry, maintenance cost, and rolling resistance problems. Certain aspects of the theoretical and experimental studies regarding rolling resistance are of particular interest to the impact noise problem. However, the main conclusion of the economic study is also of great practical importance. A careful cost analysis has revealed that the cost of converting the main line tracks from jointed to welded rails is amortized within 3 years for lines of relatively light use and within less than 1-1/2 years for lines with heavy use (16×10^6 ton/year or above). Considering that the rolling resistance on a welded track is substantially smaller (20% for freight trains and 10% for fast passenger trains) than that on a jointed track, the welded track provides substantial savings in needed locomotive power - a fact which deserves special consideration in times of energy shortages. This is especially true for long-haul freight traffic where there are no frequent periods of acceleration and braking which is realistic for rapid transit.

To analyze the importance of the rail joints on rolling resistance, Kerkapoly (1965) developed a mathematical model to study the dynamics of a wheel passing over a rigid rail joint. Two typical situations have been analyzed. For the simple gap between the otherwise level rail ends, there is a distinct difference between low speed and high speed travel. At low speeds, the wheel rolls over the joint maintaining contact with at least one of the rail ends. If the train speed exceeds a certain critical speed, the wheel separates from the forward end of the back rail. In this case, the wheel follows a trajectory and impacts on the forward rail. The vertical component of the wheel velocity at the time of impact has been calculated for both cases.

The next rail joint geometry considered by Kerkapoly (1965) was that of a vertical misalignment at the rail joint. It was found that this vertical misalignment, which forces the wheel to jump up on the ramp, is by far more severe than the level gap. Accordingly, the Hungarian Railways limits the maximum permissible vertical misalignment at rail joints to 2 mm (0.079 in.) for tracks carrying slow traffic [less than 50 mph (80 km/hr)] and to 1 mm (.04 in.) for tracks carrying fast traffic [over 50 mph (80 km/hr)].

Using the theoretical relationships derived by Popp (1952), Luber (1961) calculated the bending moment of the rail and the dynamic force acting on the rail due to impacts caused by a flat wheel rolling over the rail at various speeds. Generally both the dynamic force and the bending moment depend on (1) flat heights, (2) resiliency of the rail bed, (3) train speed, and (4) tie spacing. Dynamic force and bending moment increase very strongly with increasing train speed up to approximately 18.7 mph (30 km/hr). Above 18.7 mph (30 km/hr) train speed, the increase of speed has only negligible effect. Unfortunately, Luber does not explain the leveling off of the dynamic force above 18.7 mph (30 km/hr).

Timoshenko (1926) calculated the excess dynamic deflection of the rail due to irregularities of rail or wheel. The simplified model, which neglects the inertia of the rail, is based on the differential equation of motion of a rigid wheel in the vertical direction, namely:

$$M \frac{d^2(y+\eta)}{dt^2} + \alpha y = 0 \tag{3.1}$$

where M is the mass of the wheel, y is the deflection in the vertical direction, η is the depth of the irregularity of the rail, α is the vertical load necessary to produce unit rail deflection, and t is time. Assuming for instance that the shape of the low spot is given by

$$\eta = \lambda/2 \ (1-\cos \frac{2\pi x}{\ell}) \tag{3.2}$$

where ℓ is the length of the low spot and $\lambda/2$ is the maximum depth at $x = \ell/2$, x being the point of wheel contact.

Assuming x = 0 at the time t = 0, denoting the train speed by v, and substituting x = vt in Eq. 3.2, one obtains

$$\eta(t) = \lambda/2 \ (1-\cos \frac{2\pi vt}{\ell}) \quad . \tag{3.3}$$

Substituting Eq. 3.3 into Eq. 3.1 and solving for y(t), one obtains the deflection due to the dynamic load:

$$y = \lambda/2 \ [1-(\frac{\tau_1}{\tau_2})^2]^{-1}(\cos \frac{2\pi t}{\tau_1} - \cos \frac{2\pi t}{\tau_2}) \quad , \tag{3.4}$$

where $\tau_1 = \ell/v$ is the time required for the wheel to pass the low spot and $\tau_2 = 2\pi(M/\alpha)^{\frac{1}{2}}$ is the period of free vibration of the wheel on rail.

According to Eq. 3.4, the dynamic deflection is proportional to the depth of the spot and depends on the ratio τ_1/τ_2. The maximum deflection of $y = 1.47\ \lambda$ occurs at a train speed corresponding to $\tau_1/\tau_2 = 2/3$. The dynamic force produced on the rail under these conditions is

$$F(t) = \alpha y(t) \quad . \tag{3.5}$$

According to Eqs. 3.4 and 3.5, comparatively small low spots can produce sizable dynamic deflections and dynamic forces which are in addition to the semi-static force and deflection caused by the rolling load.

For an arbitrary shape of low spot described by

$$M \frac{d^2\eta}{dt^2} = F(t) \quad , \tag{3.6}$$

Eq. 3.1 has the following general solution:

$$y(t) = \frac{1}{M} \frac{\tau_2}{2\pi} \int_0^{t_1} F(t) \sin \frac{2\pi}{\tau_2} (t_1 - t)\, dt \quad . \tag{3.7}$$

Timoshenko (1926) reports that calculating the maximum deflection for several different shapes of low spots showed that the ratio of y_{max}/λ does not depend substantially on the particular shape of the continuous low spot.

The basic limitations of Eqs. 3.1 to 3.7 for use in estimating noise radiation caused by low spots in rails are due to the basic assumptions made in deriving them. The three basic assumptions were that (1) the wheel always remains in contact with the rail; (2) the inertial reaction of the rail is small compared with the spring-like reaction of the rail; and (3) the wheel is rigid. The second assumption is valid only if the frequency of excitation is below the resonance frequency of the fundamental vibration of the rigid rail on elastic foundation f_0, which is given by

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{K}{M}} \quad , \tag{3.8}$$

where K is the foundation modulus (i.e., the load per unit length required to produce a unit deflection of the foundation) and M is the mass per unit length of the rail. For standard rail on an average foundation [K = 1500 lb/in.^{2}($1.02 \cdot 10^7$ nt/m^2)], this fundamental resonance frequency is 60 Hz.

Since the dynamic rail deflection is controlled by the low frequency components of the dynamic wheel/rail interaction force, Timoshenko's formulae may provide useful results in calculating maximum rail stresses. However, the noise radiation from the wheel/rail interaction is dominated by the dynamic forces with frequencies well above the fundamental resonance of the resiliently supported rail (where Timoshenko's formulae do not give valid results).

3.2.3 Need for additional work

As the above review indicates, there is a considerable scarcity of information on the impact phenomenon. There is a strong need for some basic simplified mathematical modeling beyond that done by Kerkápoly, Timoshenko, Popp, and Luber to characterize both the magnitude and possibly the time history of the force at the wheel/rail interface during impact due to both rail discontinuities and wheel flats. Once this modeling is complete it should be evaluated using full-scale and/or model tests, carefully checking the result of varying such parameters as train speed, discontinuity dimension, wheel diameter, etc. In addition, further information on the geometry of wheel flats, rail joints, and other factors needs to be compiled, since it is anticipated that the force of impact at the wheel/rail interface will depend on this geometry.

Of course, development of the model of the force at the wheel/rail interface requires knowledge of the impedance and dynamic response of the wheel and rail, as well as knowledge of the radiation characteristics of wheels and rails, so that the model can be used to calculate the noise radiated due to impact. As a result, efforts to model the impact force must be carefully coordinated with efforts to characterize the dynamic and radiation properties of wheels and rails as described in Sec. 2. The result would ultimately be a mathematical model from which one could determine the effect on the noise radiated during a train passing of such measures as reducing the rail joint gap, rail misalignment, and so forth.

3.3 Roar Noise

Roar noise results from the rolling of steel wheels on continuous, straight steel rail. The mechanism accounting for the roar is believed to be wheel and rail roughnesses that give rise to unsteady loads and vertical motion of the wheel and rail. The force and velocity spectra were first related by Bender *et al* (1969) to wheel and rail roughness, wavenumber spectra, and to wheel and rail impedances as follows:

$$\Phi_{FF}(\omega) = \frac{\omega^2}{u}\left|\frac{Z_r Z_w}{Z_r + Z_w}\right|^2 [\Phi_{ww}(k) + \Phi_{rr}(k)] \quad , \tag{3.9}$$

$$\Phi_{v_w v_w}(\omega) = \frac{\omega^2}{u}\left|\frac{Z_r}{Z_r + Z_w}\right|^2 [\Phi_{ww}(k) + \Phi_{rr}(k)] \quad , \tag{3.10}$$

Bender, E.K. *et al*, 1969. "Effects of Rail-Fastener Stiffness on Vibration Transmitted to Buildings Adjacent to Subways," BBN Report No. 1832.

$$\Phi_{v_r v_r}(\omega) = \frac{\omega^2}{u}\left|\frac{Z_w}{Z_r+Z_w}\right|^2 [\Phi_{ww}(k) + \Phi_{rr}(k)] \quad , \qquad (3.11)$$

where the subscripts w and r refer to wheel and rail, Φ_{FF} is the force spectra, Φ_{VV} is the velocity spectra, ω is frequency, u is train speed, Z is impedance, and $\Phi(k)$ is the wavenumber of the roughness of the wheel or rail.

One of the objectives of the present study is to characterize the parameters of these equations in order to determine how to minimize the forces developed at the wheel/rail interface and the attendant motion and sound radiation. Presently, we will review prior work conducted to (1) characterize the wheel/rail interface phenomena insofar as it applies to the generation of unsteady forces, (2) determine wheel and rail roughness levels, and (3) develop roughness-measuring techniques.

3.3.1 Interface phenomena

Of the considerable amount of research performed to characterize the interface between two elastic bodies in rolling or static contact, most has been directed to an assessment of wear on electrical or thermal contact resistance. However, some work has been of a sufficiently general nature to be applied to the evaluation of wheel/rail dynamics.

Classical stress analysis by Hertz (1881) and Poritsky (1950)

Hertz, H., 1881. "On the Contact of Elastic Bodies," *J. für die reine und angewandte Mathematic*, 92.

Poritsky, H., 1950. "Stresses and Deflections of Cylindrical Bodies in Contact with Application to Contact of Gears and of Locomotive Wheels," *J. Applied Mechanics*.

(also Timochenko, 1951; Nayak, 1972) of the stress distributions of elastic bodies in contact show that one may regard the resulting deflections in terms of a nonlinear spring described by

$$P = C\,\delta^{3/2} \tag{3.12}$$

where P is the normal load, C is a constant that depends on material and geometrical properties, and δ is deflection. The concept of an effective spring between wheel and rail leads one to look for possible resonance effects of the wheel/rail system and also for attenuation of the response above resonance. Because the above force/deflection relation is nonlinear, the effective stiffness

$$dp/d\delta = 1.5C\delta_0^{1/2} = 1.5C^{2/3}P^{1/3}$$

is an increasing function of wheel/rail load. Accordingly, the contact resonance and the frequency at which attenuation due to contact deformation occurs are expected to increase gradually with wheel load.

Gray and Johnson (1972) and Nayak (1972) have investigated such contact resonance with two rigid bodies. These studies are only marginally applicable to the wheel/rail dynamics, because of the continuous behavior of the rail and multimodal behavior of the wheel at frequencies of interest. An

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Timoshenko, S., 1951. *Theory of Electricity*, McGraw-Hill, New York.

Nayak, P.R., 1972. "Contact Vibrations," *Sound and Vibration*, 22.

Gray, G.G. and Johnson, K.L., 1972. "The Dynamic Response of Elastic Bodies in Rolling Contact to Random Roughness of their Surfaces," *J. Sound and Vibration*, 22.

area of relevance to the present study that has been partially investigated is the effect of wheel and rail spectral components with wavelengths of the order of or less than the characteristic dimensions of the wheel/rail contact patch. Greenwood and Trip (1967) analyzed the effect of microroughness on the load/deflection characteristics of stationary bodies in contact. Their model of the surfaces involves a number of randomly spaced asperities, with statistically distributed heights. The contact between pairs of asperities is characterized by a Hertzian load/deflection relation. The result is that for light loads the pressure spreads out over a larger area than predicted by Hertzian theory but corresponds well to Hertzian theory for heavy loads. (There is no simple criterion to distinguish heavy from light.) Since railroad wheels are heavily loaded by almost any criterion, we assume the Hertzian effect will hold.

In addition to the matter of microroughness where only the tops of asperities are believed to contact each other, there is a flattening of smooth irregularities on each surface in continuing contact. Gray and Johnson (1972) hypothesize an averaging of load throughout the contact region, resulting in a wave-number filter of the form

$$\sin^2(ka)/(ka)^2$$

where a is the effective contact patch radius. This first-approximation amplitude is probably reasonably valid for small

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Greenwood, J.A. and Tripp, J.H., 1966. "The Elastic Contact of Rough Spheres," *J. Applied Mechanics*.

irregularities, but it cannot be precise because it assumes linearity, which Hertzian theory shows to be incorrect. This function provides 3 dB of attenuation for ka = 1.4 (more above and less below). Since $ka = 2\pi fa/u$, the frequency f_s at which wavenumber filtering becomes significant is

$$f_s = 1.4u/2\pi a \quad . \tag{3.13}$$

For train speed u to 60 mph (96 km/hr) and radius a of 1/4 in. (6.3 mm), f_c = 940 Hz, which is slightly above the peak of typical railroad noise spectra. In fact, this wavenumber filtering effect may account in large measure for high-frequency noise attenuation.

3.3.2 Wheel and rail roughness data

The wheel and rail roughness data spectra required for Eqs. 3.9 and 3.11 must be derived from profile measurements. Most of rail profile measurements made thus far have been for purposes of ride characterization. Several years ago, Gilchrist (1965) measured rail profiles and computed wavenumber spectra, for only for low wavenumbers. Also, the Department of Transportation has a track-survey car used in evaluations of low wavenumber components. Rudd and Brandenberg (1973) developed a rail profilometer that is useful up to about 12 Hz. They report profiles, but no spectra.

Gilchrist, A.O., 1965. "A Report on Some Power-Spectral Measurements of Vertical Rail Irregularities," British Railway Research Department, Derby, England.

Rudd, T.J. and Brandenburg, E.L., 1973. "Inertial Profilometer as a Rail Surface Measuring Instrument," *American Society of Mechanical Engineers.*

In summary, wheel and rail wavenumber spectra do not appear to exist in the range of interest and will be measured as part of the present wheel/rail program.

3.3.3 Measurement techniques

Certain theoretical and experimental techniques have been developed for profile measurement, most of which are only marginally relevant to the problem at hand. Nayak (1971) considered theoretically the problem of estimating the distribution of asperity heights in an isotropic surface with a Gaussian elevation distribution. He shows that the distribution estimated by measurements along a line underestimates high peaks, because the line tends to go over the shoulder of most peaks, only rarely encompassing a peak. This effect might become important at very high wavenumbers for wheels and rails, but it is likely to be of little consequence for the problem at hand, which is very much like a line profile along a two-dimensional surface.

Instrumentation for profile measurements has been developed for long and short wavelengths, but for nothing in between — the area of interest to us. Long wavelength rail profiles have been measured by Rudd and Brandenberg (1973) with inertial profilometers and by Gilchrist (1965) using survey techniques. Similar techniques have been used to measure roadway and airport runway pavement profiles. Very short wavelength roughness profiles of various surfaces have been measured by numerous

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Nayak, P.R., 1971. "Random Process Model of Rough Surfaces," *ASME Transactions, J. Lubrication Technology,* pp. 398-407.

investigations using a stylus connected to a strain gauge. These are useful over distances of about 1 in. We are interested in distances from about 0.1 in. to 10 ft.

3.3.4 Conclusions

A basic analytical model has been developed to characterize the roar mechanism by relating wheel and rail roughness spectra to spectra of force and wheel and rail (vertical) velocities at the wheel/rail interface. Most prior investigations are only peripherally related to the problem at hand. Hertzian stress analysis suggests a high-frequency attenuation due to localized deformations. Roughness profiles and associated spectra corresponding to very long and very short wavelengths have been measured, but no surface profiles have been measured corresponding to wavelengths that excite wheel and rail vibration in the acoustic frequency region.

Clearly, then, the major deficiency in our knowledge of the roar mechanism is measurements of the roughness spectrum in the 1/2 in. to 1 ft wavelength region on both wheels and rails. Presently, there are no devices available for making this measurement. As a result, the major contribution that can be made to a model of roar once the wheel and rail impedances have been properly modeled (see Sec. 2) is to develop a device, preferably portable, that will measure the required roughness spectra. Such a device will not only be useful as a research tool but could also (if of a simple design) be useful to transit authorities in deciding when wheel truing and rail grinding are required before excessive noise is produced.

4. DEVICES AND PROCEDURES FOR THE CONTROL OF WHEEL/RAIL NOISE

A number of devices and procedures have been suggested to reduce the noise from wheel/rail interaction in urban transit systems. Many of these ideas for noise control have been tested on real transit systems, but, unfortunately, in many cases operational constraints or lack of understanding of acoustics by the personnel involved has resulted in testing that is piecemeal or poorly controlled and, as a result, inconclusive. In this section, we review many of these techniques and discuss their effectiveness.

4.1 Wheel Truing and Rail Grinding

Grinding train wheels and rails smooth has a beneficial effect on roar noise simply by reducing the amplitude of the excitation mechanism. Bender and Heckl (1970) report differences of approximately 6 dB(A) between noise levels measured for ground and underground rails on the Munich Subway. These results are encouraging and by no means indicate the limits achievable by grinding.

The important parameter to be controlled in grinding is not the surface (micro) finish but rather the irregularities having wavelengths of the order of 1 in. (.025 m) to 1 ft. (0.304 m). Scratches or asperities as much as 0.1 in. (2.5 mm) apart correspond to frequencies of 10 kHz for a 60 mph (96 km/hr) train and are beyond the range of significance for wheel/rail noise. In addition, as described in Sec. 3.3, the finite size of the wheel/rail contact patch (~1/2 in. [12.5 mm]) tends to filter out such short wavelength roughness.

Medium wavelength [1 in. (.025 m) to 1 ft (0.304)] *wheel* irregularities can be controlled by spinning the wheel while grinding it. Grinding the *rail* is more difficult because running a vehicle with an attached grinding wheel slowly over the rails causes the grinder to move vertically in response to the vertical motion of the vehicle wheels, although there is a moderate averaging effect owing to the partial cancelling effect of several wheels at different places on a rail.

4.2 Antilock Devices

There is no information on the reduction of braking noise caused by antilock devices, i.e., devices that prevent the wheels from locking during braking. However, the device prevents locked wheels from sliding on the rails and causing flat spots to form on the wheels. Since the noise produced by wheel flats is considerable and procedures for removing the flat spots are often time-consuming and expensive,* their prevention is highly desirable.

4.3 Rail Welding

As pointed out in Sec. 3 of the report, welding all rail joints on a transit line can lead to a 4 to 5 dB(A) reduction in wayside noise. In addition to its contribution to noise control, welded rail has a number of other advantages. It has a potential for decreased maintenance, since there are no joints to be worn down by passing wheels. In addition, the Hungarian Railways (MAV) reports up to a 10% decrease in average rolling resistance with welded rail, indicating that one might expect better fuel economy. However, when on-site welding is done, the joint is seldom as

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*Too much wheel truing can also lead to the wheel having to be replaced.

smooth as the continuous part of the rail. In addition, the wear characteristics of the weld are often different from those of the continuous rail, which leads to differential wear and, finally, roughness at the joint with the passage of time. Maintenance difficulties are also increased somewhat by the use of welded rail, because replacing a damaged section of track no longer involves removing a few bolts and spikes and replacing the worn section with a new section, but rather requires that the rail be cut and a new section fitted and rewelded. In the absence of rail joints one might expect the incidence of damaged rails to decrease.

4.4 Lubrication

Applying lubricants comes to mind as one of the simplest means for eliminating screech. Indeed, it is well known that even rain tends to reduce the incidence of screech. Water lubrication has been tried on a number of railyard curves with some limited success.

Measurements taken by Bender and Hirtle (1970) compared noise measured at approximately 6 ft (1.82 m) from trains moving at 15 mph (24 km/hr) around a 180-ft (55 m) radius curve of dry track to the noise from trains traveling at 15 mph (24 km/hr) around a 200-ft (61 m) radius curve of lubricated track. On the average the lubricated track was approximately 9 dB(A) quieter.

Oil lubricators are used on the tight curves of the CTA Chicago loop but primarily for wear reduction rather than for

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Bender, E.K. and Hirtle, P.W., 1970. "The Acoustical Treatment of Stations to Alleviate Wheel-Squeal Noise," BBN Report No. 2052.

noise control. No screech noise is noted around the loop; however, the absence of screech may also be due to the very low speeds at which cars travel around the curves.

Transit engineers tend to frown on lubrication, primarily because even though the lubricant may be applied only to the side of the rail head it tends to spread to the top of the rail along the system, making braking more difficult and causing wheels to slide and form flat spots.

4.5 Resilient Wheels

Resilient wheels have undergone continuous development since their invention in 1899 (Rautenberg, 1899). However, there are available at present only four different designs:

1. "Penn Cushion" wheels, available in the U.S. from Penn Machine Co., Johnstown, Pa.
2. "Acousta Flex" wheels, marketed by the Standard Steel Division of Baldwin-Lime-Hamilton Corp., Burnham, Pa.
3. "SAB" resilient wheels, marketed in the U.S. by American SAB Company, Inc., Chicago, Ill.
4. "P.C.C." wheels, made by Penn Machine Co., Johnstown, Pa.

The Penn Cushion wheel was developed about 15 years ago by Bochumer Verein A.G., a West German subsidiary of Friedrich Krupp Hüttenwerke A.G., and is known in Europe as the "Bochum 54" wheel.

Rautenberg, W., 1899. "Employment of Resilient Wheels for Use Under Railbound Vehicles for Suburban Traffic and Main Line Railways," *Proc. Third International Wheelset Conference.*

The German manufacturer lists more than 73,000 wheels in urban transportation service - predominantly in Europe - as of the end of 1972. The economy and wear records of these wheels appear to have been excellent. The Acousta Flex wheel was developed by Standard Steel in about 1965. It appears so far not to have gained wide acceptance.

The SAB wheel was developed in Sweden nearly 30 years ago. The manufacturer, Svenska Aktiebolaget Bromsregulator of Malmö, Sweden, claims nearly 27,000 wheels in service as of the beginning of 1964, mostly on European rail and transit systems (with over 60% of the wheels installed on streetcars and subways). These wheels appear to have accumulated an extensive record of satisfactory performance.

The P.C.C. wheel was developed for the American Transit Association's President's Conference Commission streetcar in 1933. It proved successful for these streetcars, but use of this design on larger, more heavily loaded wheels led to early failures. P.C.C. wheels were tried on Chicago Transit Authority vehicles, but they were removed from service when slippage of the rubber elements resulted in large eccentricities.

The Penn Cushion and Acousta Flex wheels are similar in principle. Both involve an elastomeric "ring" between the wheel's rim and its central disc. In the Penn Cushion wheel, the ring is made up of a number of closely spaced rubber blocks; in the Acousta Flex wheel, the ring consists of a continuous, relatively thin, rubber layer with a sawtooth-like cross-section (in a plane that encompasses the wheel axis and a radius) (Wilson, 1972). In both cases, radial loading of the wheel leads primarily to compression of the rubber.

The SAB and PCC wheels also are similar to each other in principle. In these wheels, the rim is part of a steel disc, and the hub assembly consists of one or more parallel steel discs. The rim disc is connected to the hub assembly via rubber elements which deform in shear as the wheel is loaded radially.

All of the manufacturers' literature contains claims that their wheels produce considerable improvements in ride comfort and quiet operation, in addition to reduced wear and increased economy. However, such data as they present are difficult or impossible to interpret quantitatively.

Extensive tests were conducted by the TTC to compare Penn Cushion and standard wheels (Hendry, 1971; Toronto Transit Commission). The tests involved measuring noise and vibration both in and on the car and at the wayside, as well as monitoring the wheel temperatures and tread wear. The Penn Cushion wheel was found to yield significant reductions in journal box and bogie vibrations but to produce insignificant changes in the vibrations measured on the passenger compartment floor or at the wayside. The Penn Cushion wheel also produced no significant reduction in the wayside noise at tangent track. On the other hand, the Penn

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Hendry, I.G., 1971. "Toronto Transit Commission Experience With Penn Machine Company Resilient Wheel," Rail Transit Conference of the American Transit Association, San Francisco, Calif.

Toronto Transit Commission, 1969. "Interim Report on Penn Resilient Wheels."

Cushion wheels did not screech on certain short radius curves as the standard wheels did. Finally, the Penn Cushion wheels resulted in minor (1 to 3 dB(A)) reductions in in-car noise under a variety of nonscreeching operational conditions. Penn Cushion wheels have been installed on one MBTA car, and the Camden/ Philadelphia Port Authority Transportation Corp. is about to embark on an evaluation of them.

A series of measurements were made on a BART prototype car (Wilson, 1972) to provide information on the effects of rail grinding and of resilient and damped wheels on wayside and interior noise. These measurements led to the following general conclusions:

1. On tangent track, Penn Cushion and Acousta Flex wheels and damped wheels resulted in at most 2 dB(A) reductions in wayside and interior noise, compared to standard wheels.

2. On short-radius curves, resilient wheels produced significant noise reductions; the Penn Cushion wheels gave the best overall results, with reductions of up to 18 dB(A) (due primarily to screech elimination).

In July 1973, the Southeast Pennsylvania Transit Authority (SEPTA) initiated a test of Acousta Flex wheels on one car, with Boeing supplying instrumentation and analysis. Although Boeing's test report is not yet available, observations (Vigrass, 1973) indicated that during fast runs on bolted tangent track, the noise in the car with the Acousta Flex wheels was as much as 10 dB(A) lower than that in a similar car with standard wheels. The Acousta Flex wheels were also found to reduce or eliminate the screech on some curves.

Vigrass, J.W., 1973, "Resilient Wheels," PATCO Interoffice Communication to R.B. Johnson, dated 2 July.

In summary, resilient wheels appear to be useful devices for reducing wheel/rail noise. In spite of some negative experiences that some transit authorities have had, resilient wheels have gained wide acceptance and have accumulated an extensive record of safe and economical performance. The higher initial cost of such a wheel (as much as twice that of a standard wheel) tends to be offset by operational savings associated with reduced wear of track and rolling stock and with lower repair costs resulting from the feasibility of replacing only the rim instead of the entire wheel.

4.6 Resilient Rail Fasteners

Resilient rail fasteners are being used in transit systems to reduce vibration transmission to the ground and to structures surrounding the track. Although the fasteners are effective in reducing vibration transmitted to the structure supporting the rail, Bender (1972) has shown that they should have little effect on sound radiated from the rail. He gives the following reasons for this conclusion:

1. At low frequencies (< ~50 Hz), the rail impedance exceeds the wheel impedance, the rail response depends on the rail impedance and, hence, fastener stiffness; but the rail is an inefficient radiator at these frequencies.
2. At high frequencies (> 500 Hz), the rail impedance is independent of the fastener stiffness.
3. At intermediate frequencies (100 to 300 Hz), the wheel impedance exceeds the rail impedance; as a result, the rail response is independent of the rail impedance.

Wilson (1973) claims on the contrary that the use of resilient fasteners can, in fact, lead to increased under-car and wayside noise but presents no data to confirm this claim. This is in contrast to statements (unsupported by data) by the Toronto Transit Commission (1967) that the use of resilient rail fasteners has no effect on wayside noise although car interior noise may be reduced. In general we have been unable to find published data comparing noise generated during a train passage on rails with and without resilient fasteners. However, based on Bender's reasoning, we see no reason that sound radiation should be affected.

4.7 Wheel Damping

Although a large number of wheel damping treatments have been explored, none appears to have gained practical acceptance. Some of the early attempts at constructing damped wheels involved annular inserts of such materials as lead (Taschinger, 1951), hardwood, and steel, either pressed into matching depressions in the rim or loosely inserted into cut-outs. Filling hollowed-out steel tires with gravel was also considered (Stappenbeck, 1954).

Wilson, G.P., 1973. "Rail Fastener Analysis," Wilson Ihrig and Associates, Inc., Oakland, California. Prepared for the Baltimore Region Rapid Transit System.

Toronto Transit Commission, 1967. "Noise and Vibration Studies - Track Fasteners," Report, RD 106.

In 1965, the B.F. Goodrich Co. undertook the development of a wheel damping system. As tested on a two-car train of the Toronto Transit Commission in 1966, this system consisted of a layer of viscoelastic damping material bonded to the inside of the wheel rim and covered with a bonded steel "constraining" layer (Swanson, 1966; Swanson and Thrasher, 1967). The manufacturer claims that this treatment eliminates screech, reducing farfield noise obtained on tangent track by up to 2 dB(A) at high speeds, and also attenuates rail vibrations.

Some limited experiments made by B.F. Goodrich showed that the damping resulting from an "unconstrained" viscoelastic layer attached to the wheel web resulted in no significant noise reduction (Swanson and Thrasher, 1967). Similar layers on both sides of the web were evaluated by the German National Railway (Stüber, 1965), but also without much success. On the other hand, using unconstrained, efficient viscoelastic damping layers on one side of the wheel webs of Toronto Transit Commission (TTC) cars resulted in noise reductions of 12 to 15 dB(A) (Kirshner, 1972).

Kirschner (1968, 1972) also tested a 5-layer damping treatment ring attached to the inside of the wheel rim of a Port

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Swanson, R.C. and Thrasher, D.B., 1967. "Acoustic Noise and Vibration Control Systems for Rapid Transit," B.F. Goodrich Company.

Stüber, C., 1965. "Beispiele zur Lärmabwehr bei der Deutschen Burdesbahn", ("Examples of Noise Control in the German National Railway"), *Lärmbekämpfung*, Heft #1.

Kirschner, F., 1968. "Control of Railroad Wheel Screech Noise", Paper F-2-7, *6th* International Congress on Acoustics, Tokyo.

Authority Trans Hudson (PATH) system car. The ring eliminated screech on sharp curves, resulting in a 24 dB(A) overall reduction of noise.

The most recent test of damped wheels was made on a BART prototype car (Kirschner, 1972; Wilson 1972). The damping treatment consisted of a 4-layer configuration attached to the wheel's aluminum web. Compared to undamped wheels, the damped wheels were found to have no significant effect on interior and wayside noise on tangent track, but they eliminated some screeching on curved track [resulting in reductions of up to 20 dB(A) and reduced the nonscreeching noise on curved track by up to 4 dB(A)].

4.8 Rail Damping

The use of damping compounds on the nonrunning surfaces of the rails appears at first glance to be an effective method of reducing noise. Damping should shorten the length of rail that vibrates when a wheel passes over it by increasing the spatial decay along the rail, although the response in the vicinity of the excitation point would not be significantly affected. Just how effective the technique can be depends strongly on what the rail contribution to radiated noise is relative to the contributions from other sources.

The results of experiments on rail damping effectiveness are presently somewhat mixed. Anderson (1964) reports a 4 dB (C scale) reduction in radiated sound after applying "sound deadening material" to 400 ft (122 m) of rail on the Chicago Transit System.

Anderson, L.G., 1964. "Rapid Transit Car Noise," *Metropolitan Transportation and Planning Magazine*.

Unfortunately he gives no details on the damping material or track structure. Van Os (1965) reports measurements on the Rotterdam Metro in which two kinds of damping material — "Trumpac" and "Spuitkit" — were applied to the inner side of the rail. Although the velocity level on the rail in both the horizontal and vertical directions was reduced by 5 to 10 dB above 500 Hz, the maximum octave band sound pressure levels during the train passage remained unchanged. This suggests that the rail is not a significant contributor to radiated noise and that rail damping should not be an effective noise control technique. Without more information on the Chicago experiment, it is difficult to tell whether the effectiveness of the damping measure there was caused by another effect (such as acoustic absorption) introduced by applying the damping coatings or whether the radiation characteristics of the wheels and rails in Chicago are sufficiently different from Rotterdam that damping the rails was indeed effective.

4.9 Track Maintenance

Track maintenance procedures can lead to some reduction of wheel/rail noise. Ballast is known to be an effective absorber of acoustic energy, and maintaining the ballast in good condition can enhance this property. The Stanford Research Institute (1966) has shown that the absorption coefficient of 6 in. (0.152 m) of ballast compares favorably with 2 in. (50 mm) fiberglass. Maintaining good alignments at rail joints can lead to reduced impact

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Van Os, G.J., 1965. "Noise Control in the Rotterdam Subway," Technische Physische Dienst, Report 62, 322 V.

Stanford Research Institute, 1966. "Noise Control in the Bay Area Rapid Transit System," Final Report, Menlo Park, California.

noise at the joint. Also, if the straightness and parallelism of the tracks are regularly maintained, the likelihood of flange impact noise can be reduced. As with rail grinding, however, all of these procedures are expensive and must be repeated at regular intervals. At the present time, there are no quantitative measurements of the noise reduction achievable through good track maintenance procedures.

4.10 Barriers

Relatively short barriers - about 4-ft (1.22 m) high - placed as close to a track as possible are among the most effective means for reducing wayside wheel/rail noise. Properly constructed barriers can reduce noise levels by 12-14 dB(A)* (Parsons *et al.*, 1968; Wilson, Ihrig and Associates, 1971). This type of barrier, currently being tested for use on the BART system, works best if the vehicle has a skirt covering about the top quarter of the wheels (see Sec. 4.11). For maximum effectiveness, the transmission loss through a barrier should be about 10 dB(A) greater than the loss around it.

Besides short barriers close to the track, high walls, hills, or earth berms several yards away can also be effective sound

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Parsons, Brinckerhoff, Tudor and Bechtel, 1968. "Technical Report No. 8, Acoustic Studies," San Francisco Bay Area Rapid Transit District Demonstration Project.

Wilson, Ihrig and Associates, 1971. "Noise and Vibration Characteristics of High Speed Transit Vehicles," Department of Transportation Report No. OST-ONN-71-7.

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*A single car at 50 ft.

barriers, and they are frequently used to reduce highway noise (Gordon *et al.*, 1971). Aesthetic and economic considerations often make it impractical to construct such highway-type barriers to block wheel/rail noise; however, if in the process of constructing a rail line a cut must be made, or if a natural barrier already exists, then highway barrier noise reduction methodology can be applied to wheel/rail noise.

The Japanese National Railways will be the first to implement acoustic noise barriers for trains on a large scale (*Railway Gazette International*, 1973). They plan to install about 180 miles of barrier to quiet noise from their high-speed trains to 80 dB(A). From experiments, they found it possible to reduce wayside noise by 5-7 dB(A) with nonabsorptive barriers and by 8-11 dB(A) with absorptive barriers (no measurement distance from track given). Their walls are about 6.5 ft (2-m) high and are not as close to the track as the walls being tested by BART.

One drawback of using barriers is the fact that the wheel/rail noise which is ordinarily radiated to the wayside may now be reflected back toward the car and result in increased car interior noise levels. Absorptive treatment on the track-side of the barrier can, of course, reduce this problem. However, we know of no measurements quantifying these effects.

Gordon, C.G., Galloway, W.J., Kugler, B.A., and Nelson, D.L., 1971. "Highway Noise - A Design Guide for Engineers," National Cooperative Highway Research Program (NCHRP), Report 117.

Railway Gazette International, 1973. "Noise Control on Shinkansen," 129, No. 7.

4.11 Wheel Skirts

Skirts to cover the exposed wheels of transit cars are a potentially effective noise control treatment, if in fact the wheel or bogie is an important source of noise radiation. The covers would have to have a high transmission loss and should probably be isolated from the vibrating car body; absorptive treatments should be applied to the wheel side of the cover and to as much of the wheel well area as possible. In effect, then, a wheel skirt is a barrier which is attached to a train. Although such a treatment does have practical problems associated with it, (wheel inspection is difficult, the attachment must be sufficiently fail-safe that the covers do not fall off during service, the absorptive treatment must not absorb contaminants such as oil or grease and become a fire hazard, etc.) it is an attractive retrofit method.

Present information on the effectiveness of wheel skirts is not, however, encouraging. Van Os (1965) tested a wheel skirt on the Rotterdam Metro. It consisted of a rubber pad (no details given) attached to the car body and yielded essentially no reduction in noise during a drive-by except above 4000 Hz. In fact below 200 Hz, Van Os measured up to a 5 dB *increase* in noise. Anderson (1964) tested skirts on Chicago Transit cars and claims to have obtained no reduction in noise, although he gives no details. However, we should emphasize that the best acoustic design, as described above, was probably not used in Van Os' and Anderson's tests and that a properly designed skirt might in fact be quite effective if the wheel is a major source of sound radiation.

4.12 Noise Deadening Rings

London Transport currently incorporates noise-deadening rings as standard devices on all the solid wheels of their rolling stock. A groove is cut in the plate fillet of the wheel, either when new or when it returns to the shop for refurbishing, and a 1/2-in-thick, 2-ft (.61 m) diameter ring is fitted into the groove. The noise-deadening ring is sprung into position in the groove and then the ends are welded together. The ring must not be welded to the wheel or fit too tightly; it must retain its own natural vibration periodicity. The ring appears to introduce extra damping to the wheel tread by means of coulomb friction between the ring and the tread; hence, the fit is quite critical.

London Transport claims that there is a significant noise reduction from wheels equipped with the ring and is sufficiently pleased with it to employ it as a standard item. The ring is a very promising noise control device, since it is cheap, easy to install, and can be retrofitted on older wheels.

4.13 Spoked Wheels

Another possibility for reducing wheel/rail noise is the use of spoked wheels. Spoked wheels have a smaller noise-radiating area than solid web wheels and as a result should be less efficient radiators of wheel noise. However, spoked wheels have some negative aspects which must be considered. For example, the spokes would probably need to be highly stressed and, therefore, would be subject to fatigue failure. Also, the wheels have a circumferentially nonuniform radial impedance which could lead to parametric excitation as the wheel rolls on a rail. Unfortunately, there are no data available at this time to shed light on any of these hypotheses, although London Transit presently uses spoked wheels on the motor bogies of its trains, ostensibly for noise control purposes (Davis, 1964). Curiously, the same bogie is fitted with both spoked and solid web wheels. We think it unlikely that such a configuration could provide much noise reduction.

4.14 Resilient Rail Heads

Resilient wheel treads or rail heads have been suggested as possible means for reducing wheel/rail noise. The use of an elastomeric layer on the rail head was tested by Anderson (1964) on a 90-ft (27.3 m) test track of the Chicago transit system. He claims a significant reduction in radiated sound (14 dB on the C scale). The considerable reduction in rail impedance seen by the wheel (which results in reduced wheel response to wheel roughness) is responsible for the lower sound level.

Although roar and impact noise may be considerably reduced by this technique, wheel squeal due to flange rubbing is probably less responsive. However, covering the side of the rail head to eliminate flange rubbing would cause considerable wear problems to the elastomer. Deterioration due to imbedded particles, increased rolling resistance, and heating caused by dissipation in the elastomer are some of the practical objections to this noise control technique.

4.15 Titanium Wheel Treads

The use of a thin titanium layer on the wheel treads is an extremely promising noise control technique. Titanium has a

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Davis, E.W., 1964. "Comparison of Noise and Vibration Levels in Rapid Transit Wheel Systems," Operations Research, Inc., Silver Spring, Maryland, Technical Report No. 216.

lower elastic modulus than steel but a higher tensile strength. Accordingly, wheels with titanium treads have the potential for attenuating noise in two ways. First, the lower modulus reduces the effective contact resonance frequency, thereby reducing high-frequency vibration. Second, the larger contact area provides greater wavenumber filtering. Unfortunately, no noise radiation measurements are available on titanium treaded wheels.

Another major advantage of a titanium tread is its adhesion capability. The friction characteristics of titanium on steel are superior to those of steel on steel. Under controlled laboratory tests, the friction coefficient of steel on steel with a thin oil film at the interface (wheels and rails are invariably contaminated) was found to vary between 0.17 and 0.22. For titanium on (oily) steel, the friction coefficient ranges from 0.54 to 0.7, a factor of three greater. This implies that a train fitted with titanium wheel treads would be able to stop in 1/3 the distance of a steel-wheeled train. The safety implications are very clear; however, wear and cost factors should be further investigated.

4.16 Pneumatic Tires

One technique that claims to sidetrack the whole wheel/rail noise problem is to eliminate the rail and the metal wheels and use pneumatic tires on a concrete guideway. Although such a system does present numerous guidance, switching, reliability, and maintenance problems, existing transit systems use them: Montreal, Paris, and Mexico City

There is some debate over whether pneumatic tires do in fact produce quieter operation. Wilson (1971) claims that there are negligible differences in wayside noise caused by a well maintained metal-wheeled car and a pneumatically tired car. Wilson probably bases his conclusions on data from the Paris Metro, which is in fact no quieter than the systems in Hamburg, Toronto, or Berlin (Davis, 1964) (all of which use metal wheels) in terms of car interior or station noise levels. Unfortunately, these measures must necessarily be contaminated by other factors such as differences in station acoustic treatment, noise from car auxiliary equipment, and transit car acoustic treatment. In short, it is difficult at this time to state categorically whether or not pneumatic tires are quieter than a steel on steel system designed from the start for quiet operation.

4.17 Track Bed Absorptive Treatment

The use of an acoustically absorptive layer on the track bed does not seem to be an effective noise control treatment by itself.* In fact, even when it is used in conjunction with other treatments such as barriers or wheel skirts, the increase in noise reduction is minimal. Tests were conducted at the BART test track at Mt. Diablo during which a 4-in. (0.1 m) blanket of fiberglass covering about one-half of the track bed was used in conjunction with a barrier. The combination treatment yielded only a 1.5-PndB decrease in the wayside noise level over barrier

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Wilson, G.P., 1971. "Rapid Transit Noise and Vibration" presented at the Rail Transit Conference of the American Transit Association.

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*It is known [Stanford Research Institute (1966)] that ballast can provide considerable acoustic absorption. However, applying absorptive treatments to the track bed does not seem to be an effective noise control measure.

alone (Stanford Research Institute, 1966). Similar tests by Van Os (1965) on the Rotterdam Metro, in which a car with wheel skirts was used together with a 5.2-in. (0.13 m) deep layer of a granular material called Hollith [0.4 (10 mm) - 0.18 in (4.5 mm) grain size], gave no change in the wayside noise with or without the Hollith.

4.18 Wheel Web Vibration Absorbers

"Tuned dampers" or "vibration absorbers" have been tried at least in one instance (Stappenbeck, 1954). They consisted of disc-shaped metal masses arranged in a circle and attached to the wheel web via rubber springs. The natural frequencies of these spring/mass systems were tuned to coincide with the wheel resonance frequencies. The absorbers were reported to suppress screech on a streetcar installation for about one year, but then became ineffective, as aging of the rubber changed the natural frequencies of the absorbers.

5. RELATIVE SEVERITY OF WHEEL/RAIL NOISE MECHANISMS

We have gathered and compared data from a wide variety of rail systems to assess the severity of the three characteristic wheel/rail noise mechanisms: roar, impact and squeal. The tangent track data presented here is for level grade, open field, wayside measurements with the rail on tie and stone ballast. Wooden ties are used in all systems reported here except the MBTA South Shore line where concrete ties are used. Most of the curved track data was gathered in tunnels, and the measurements were made close enough to the wheel to be considered direct field rather than reverberant field data. The data is classified into three groups: (1) welded tangent track data characteristic of roar noise, (2) jointed tangent track data characteristic of impact noise, and (3) curved track data characteristic of squeal. The tangent track data is plotted as a function of speed, and the curved track data is presented as a function of curve radius. Some information on the speed dependence of squeal noise is also presented. In all cases, the noise levels are reported in terms of the peak A-weighted sound pressure level. We consider this the best acoustic rating scale (see Sec. 6) and most of the data available in the literature is in this form.

The data presented for tangent tracks have been normalized for train length and distance from the center of the track. There are several schemes in the literature that can be used for this type of normalization (Bender and Heckl, 1970; Wilson, Irhig and Associates, 1971); however, we feel that there are errors in some of these methods. Therefore, we have included another normalization scheme here. If we model the train as a finite line of uncorrelated point sources radiating into a hemispherical half space (Fig. 5.1), then the mean square pressure at a distance d away on the perpendicular bisector of the train is given by:

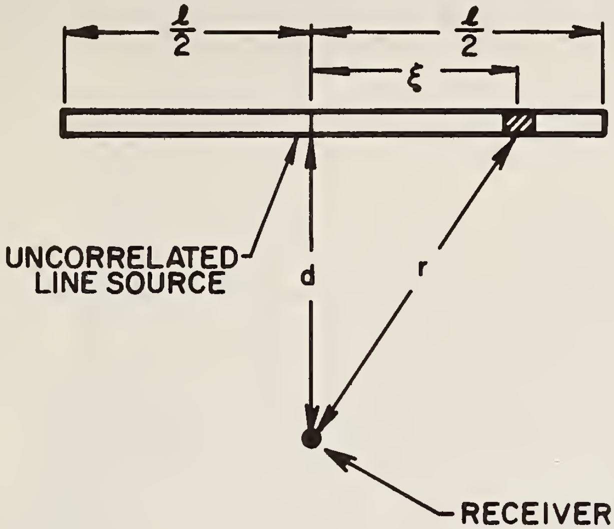

FIG. 5.1 MODEL USED FOR NORMALIZATION DEVELOPMENT.

$$\langle p^2 \rangle = \frac{\rho c}{2\pi} \int_{-\ell/2}^{\ell/2} \frac{W}{r^2} \, d\xi \quad , \tag{5.1}$$

where ρc is the acoustic impedance, W is the sound power per unit length, and r is the distance from an element of the line source to the receiver. Integrating Eq. 5.1 gives

$$\langle p^2 \rangle = \frac{\rho c W}{\pi d} \tan^{-1} \frac{\ell}{2d} \quad . \tag{5.2}$$

The difference in sound pressure level between an arbitrary train of length ℓ at distance d and a one-car train [ℓ = 75 ft (228 m)] at d = 50 ft (15.2 m) is

$$\Delta L = 10 \log_{10} \left[\frac{77.7}{d} \tan^{-1} \frac{\ell}{2d} \right] \quad , \tag{5.3}$$

where ℓ and d must now be expressed in feet. A graph of Eq. 5.3 is presented as Fig. 5.2. This graph was used to reduce all the tangent track data presented in this section for a single car at 50 ft (15.2 m). Close to the train ($d < \ell/2$), the peak sound pressure level is most sensitive to the individual wheel sets which act like point sources. Hence the peak level should drop 6 dB per doubling of distance in this range. However, for d > 20 ft (6.1 m), Fig. 5.2 is accurate to within 1 dB.

5.1 Roar Noise

An overview of the wheel/rail noise from several systems operating on welded track is presented in Fig. 5.3 together with a line representing 30 log V, the variation of sound level with

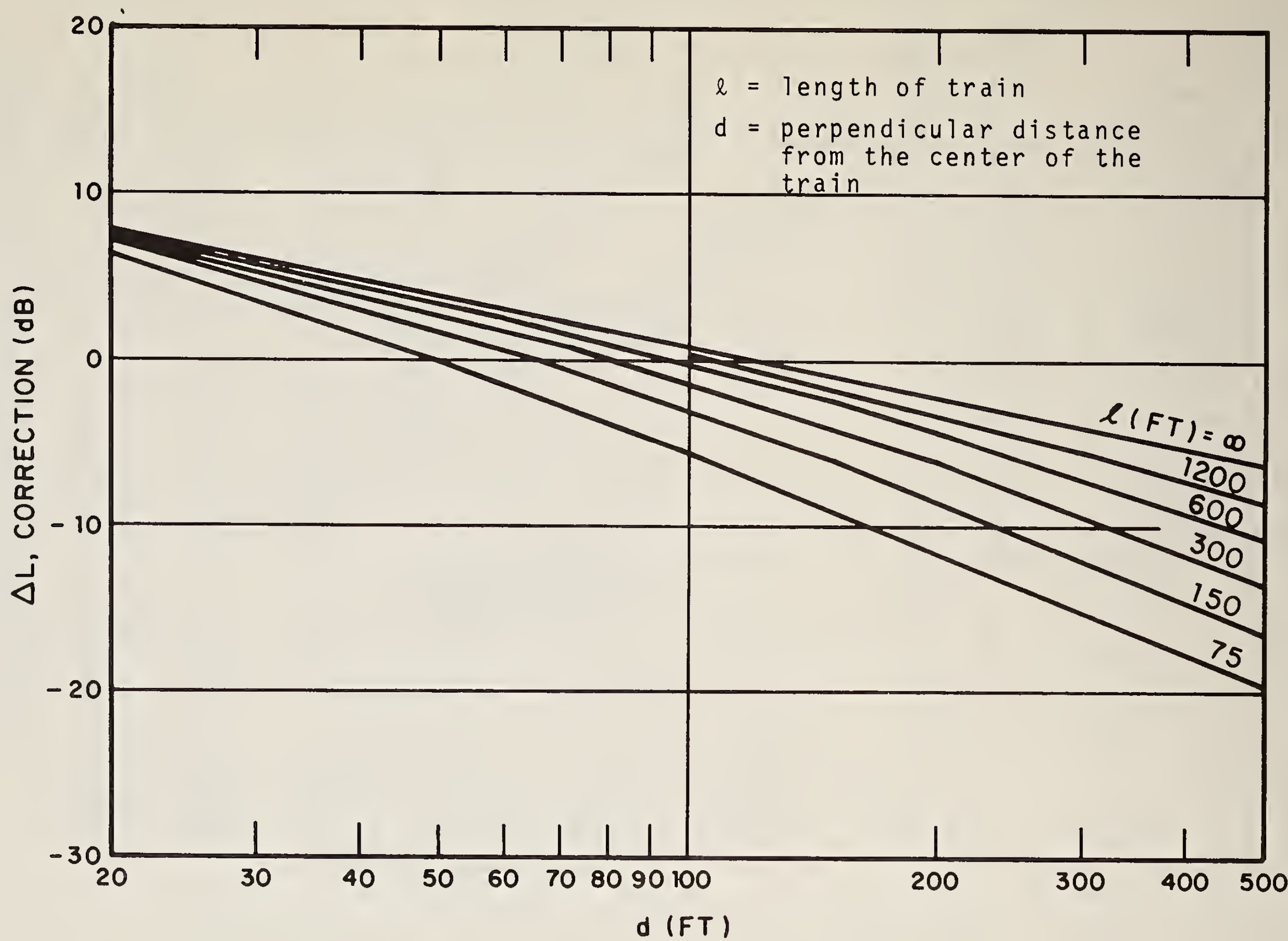

FIG. 5.2 PLOT OF EQ. 5.3 USED TO NORMALIZE ALL IMPACT AND ROAR DATA.

speed. Both passenger train and rapid transit data are included, and all the data are for trains at ground level on tie and ballast track. In addition all the data have been normalized as discussed earlier. To the best of our knowledge, none of the systems uses any special techniques to reduce noise, such as resilient wheels. The BART system does, however, use aluminum centered wheels.

About 90% of the A-weighted sound pressure levels from the better maintained systems (those which either report rail grinding or report that the rail is maintained in very good condition) fall within ±3 dB(A) of a value given by $L_A = 60 + 30 \log_{10} (V/15)$, where V is the speed in miles per hour. Noise levels from systems maintained less well or systems with unground rail are 5 to 15 dB(A) louder.

Figure 5.3 shows one data point that is about 7 to 8 dB(A) lower than all the others. This point represents the Berlin system line G, which produces 60 dB(A) at 27 mph (43 km/hr) (Bender and Heckl, 1970). The other Berlin points on the graph (about 70 dB(A) at 29 mph (46.5 km/hr) are from a different line. Additional data for line G trains on earthen embankments seem to verify that this line does indeed produce considerably less wayside noise. We are at present trying to obtain more information concerning this system, both to verify the low data point reported and to find out the characteristics which make it so quiet.

The next quietest train is the new French "Le Mistral" (Rapin, 1972). Following this, the next best measurements are

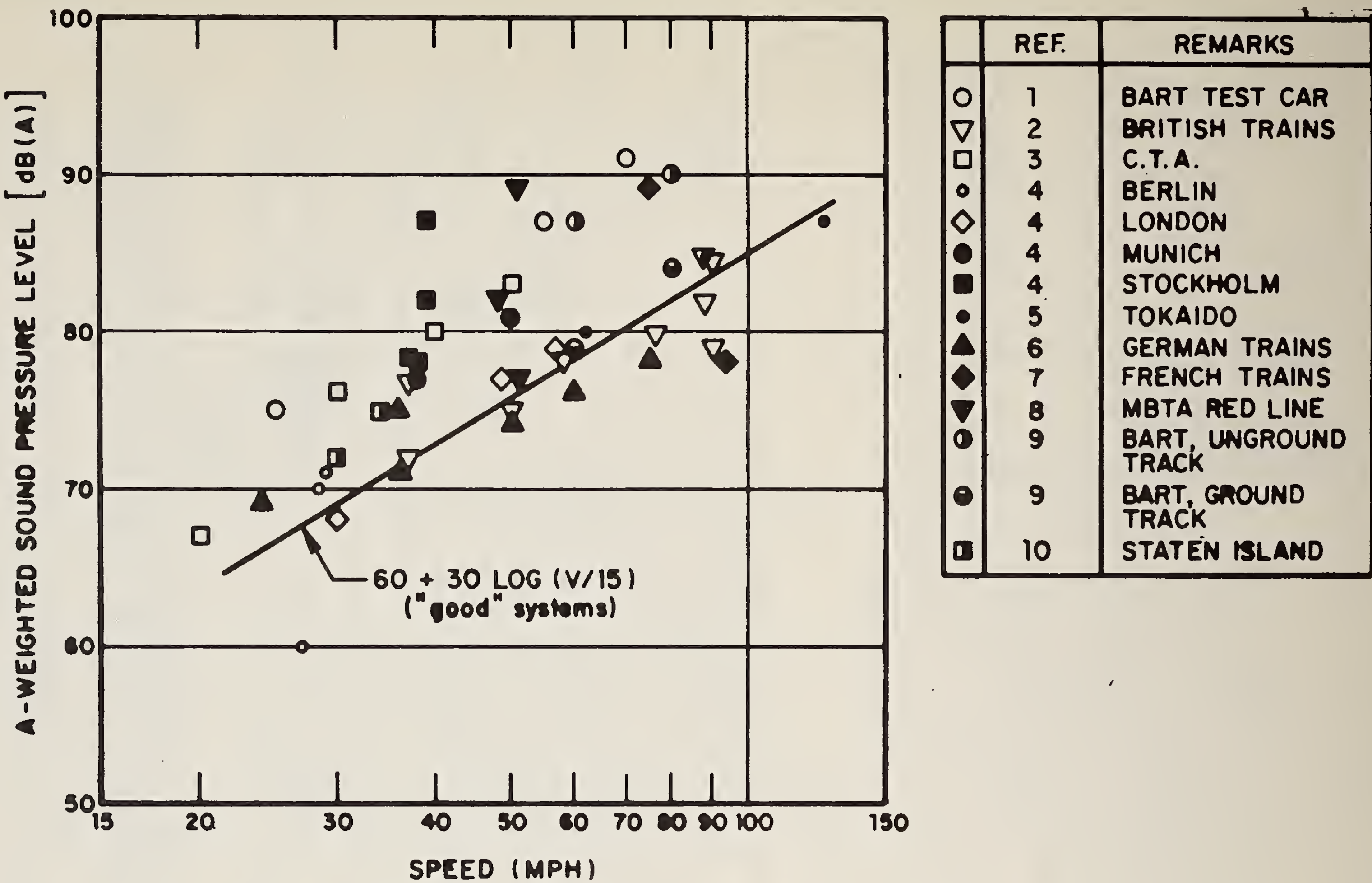

FIG. 5.3 NORMALIZED DATA FROM SEVERAL WELDED TRACK SYSTEMS. Ref. 1 = Parsons, Brinckerhoff, 1968; 2 = Peters, 1973; 3 = Patterson, 1972; 4 = Bender and Heckl, 1970; 5 = Franken, 1969; 6 = Stüber, 1973; 7 = Rapin, 1972; 8 = Rickley and Quinn, 1972; 9 = Wilson, 1972; 10 = Wittig; 1973.

those reported by Stüber (1973) for German S-bahn trains at speeds up to 76 mph 120 km/hr and those reported by Peters (1973) for British passenger trains. The best American data at higher speeds are for BART trains on ground track. The BART data are 3 to 5 dB higher than the S-bahn data. BART uses wheels with cylindrical treads which require flange-rail contact for guidance. It may be worthwhile to see if such treads are inherently slightly more noisy than tapered treads. The systems which make more noise than the BART system are generally those which do not use ground track.

5.2 Impact Noise

A graph of A-weighted sound pressure level measurements for bolted track is shown in Fig. 5.4. These data, except for the New York City measurements, are generally for very long freight trains, and the sound pressure levels tend to vary ±5 dB(A) during a passage (discounting the locomotive). For that reason the data presented here give both the mean value and the range. Since New York City rapid transit trains are generally much shorter than the freight trains, only their peak A-weighted sound pressure levels are reported here. The data presented in Fig. 5.4 were normalized as discussed earlier.

Measurements in New York City were made both on the poorly maintained Rockaway line and on the well maintained Staten Island line. On the Rockaway line, the range of data for trains at

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Stüber, C., 1973. "Noise Generation of Rail Vehicles," German Federal Railway, Munich Testing Laboratory, Electrophysics Department, Report No. P1/1973 (Translated by the Translation Center of New England, Arlington, Mass).

Peters, S., 1973. "Prediction of Rail-Wheel Noise from High-Speed Trains," *Acustica*, 28.

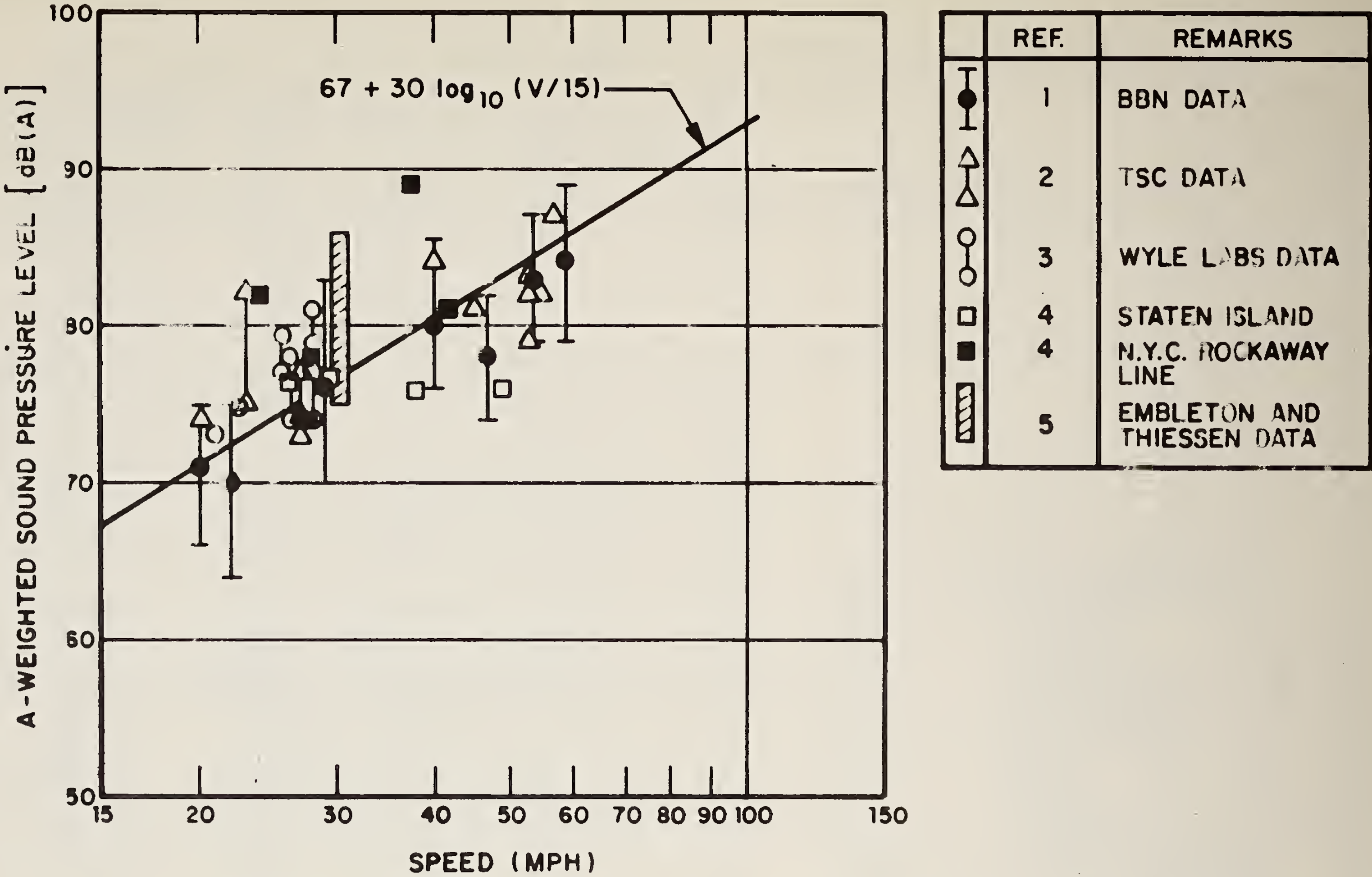

FIG. 5.4 NORMALIZED DATA FROM SEVERAL JOINTED TRACK SYSTEMS. Ref. 1 = Bender and Heckl, 1970; 2 = Ely, 1973a; 3 = Ely, 1973b; 4 = Wittig, 1973; 5 = DOT, 1970.

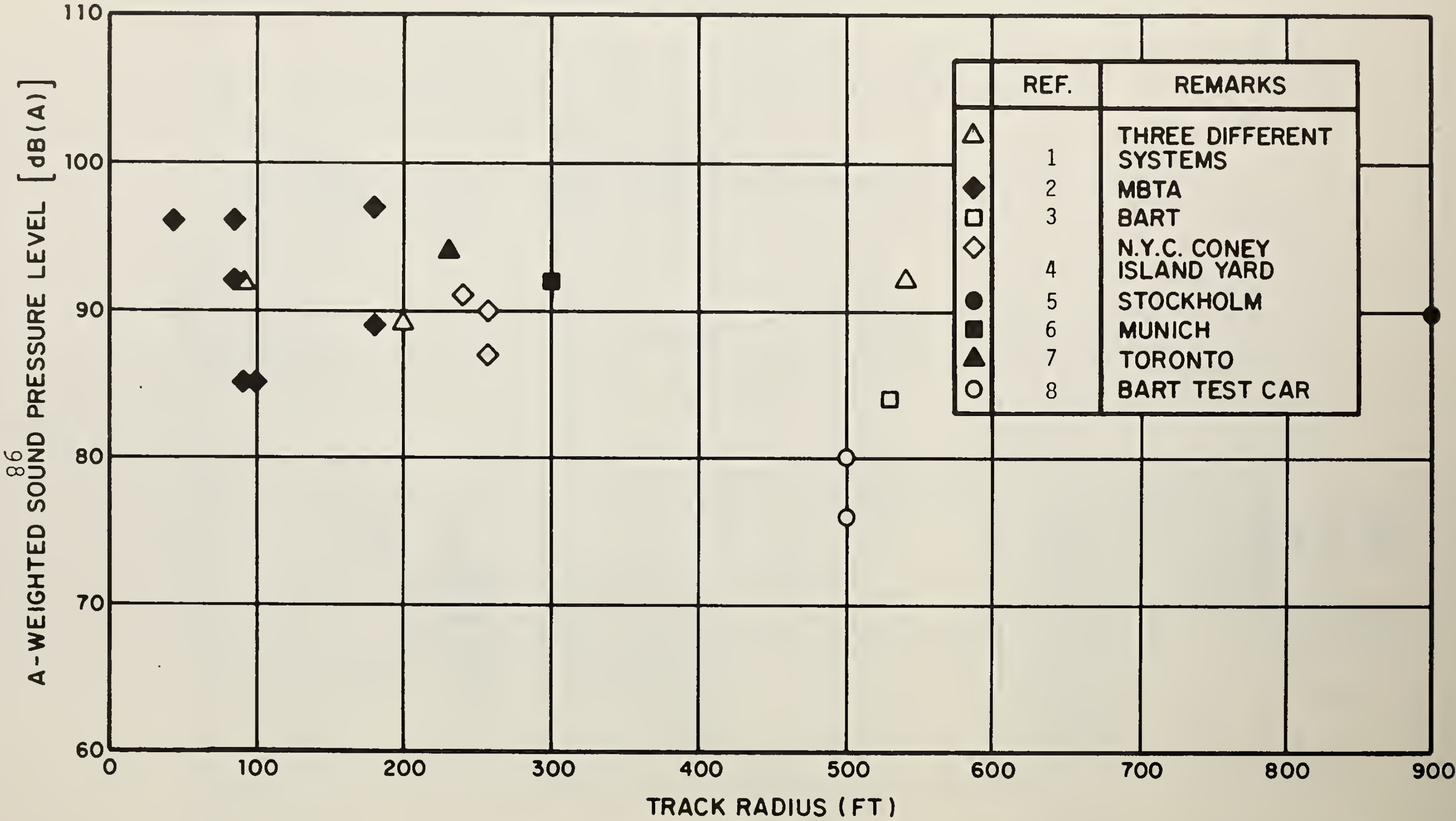

FIG. 5.5 NORMALIZED SQUEAL DATA FROM SEVERAL SYSTEMS. Ref. 1 = Kirschner, 1972; 2 = Bender and Hirtle, 1970; 3 = Wilson, 1972; 4 = Wittig, 1973; 5 = Berglund, 1972; 6 = Mueller BBM, 1973; 7 = Swanson, 1966; 8 = Parsons, Brinckerhoff, 1968.

the same speed and the same track location was greater than 10 dB(A). Very old cars (some of which were just being moved from one yard to another without carrying passengers) appeared to radiate sound from the car body as well as from wheel/rail interaction. Staten Island has sections of both welded and bolted track that are well maintained but not ground. The welded and bolted track appear to be in about the same condition - both sections are about five years old.

In general, the data from bolted track are about 7 dB(A) higher than the data for welded tracks [$L_A = 60 + 30 \log_{10} (V/15)$]. However, some of the more noisy normalized welded track data (even BART on unground track) are just as loud, if not louder, than the bolted track data.

The most direct comparison we have for the difference between well maintained (but not ground) welded and bolted track can be made using our Staten Island data (see Figs. 5.3 and 5.4). Similar trains measured on the near track at 30 mph (48 km/hr) showed the peak A-weighted sound pressure level to be 5 dB(A) lower on welded track. Rapin (1972) reports a difference of only 4 dB(A) for the same train on both welded and bolted tracks, but he says that this effect is often less pronounced. However, we feel that a 4 to 5 dB(A) improvement for welded track is probably more realistic than values of 8 to 10 dB(A) reported elsewhere (Bender and Heckl, 1970; Wilson, Ihrig and Associates, 1971). Of course, if one were to switch from very poorly maintained bolted track to well maintained welded track, improvements of the order of 10 dB(A) could probably then be easily obtained.

5.3 Wheel/Rail Squeal Noise

Wayside open field measurements of squeal noise are very rare. Most reported squeal data are from trains operating in tunnels or stations, and for this reason most data have been measured close to the wheels (from 3 to 5 ft). In order to normalize the data from several measurements, it has been assumed that only the noise from one wheel is measured at a time. The assumption seems reasonable for the close measurements, because only a few wheels are near the microphone at any time. Under this assumption, we can conclude that the noise level falls off at 6 dB(A) per doubling of distance from the tracks. Outdoor measurements made on the BART test car at two distances from the tracks (Parsons, Brinckerhoff, 1968) and observations made by Wittig (1973) confirm the assumption.

Squeal data from several systems are presented in Fig. 5.5 in which peak A-weighted sound level is plotted as a function of track curve radius. These data have been normalized to distance of 50 ft (15.2 m) using a 6 dB(A) per doubling of distance fall off rate. On the basis of these data, it appears that the noise level associated with squeal is independent of radius. However, from observations by Wittig (1973), it appears that the likelihood of squeal increases as the curve radius decreases. At the New York City Coney Island yard, many wheel sets negotiated 240-(73 m) and 255-ft (77.5 m) radius curves without squealing, while other wheel sets squealed intermittently. On other tighter curves in tunnels in New York City, almost continuous squeal can be heard.

The A-weighted squeal noise level also appears to be independent of speed. Measurements on BART trains on both 530-(161 m) and 540-ft (164 m) radius curves indicated the same sound pressure level for speeds of 18 (29 km/hr) and 35 mph (56 km/hr) (Wilson, 1972).

Wittig, L., 1973. Unpublished data gathered in New York City for Bolt Beranek and Newman Inc.

The only data that are considerably different are for the BART test car [76 and 80 dB(A) at a radius of 500 ft (152 m)]. These data may be low for two reasons: (1) The investigators (Parsons, Brinckerhoff, 1968) do not indicate that they have reported peak levels as have the other investigators. (2) The BART wheels have aluminum centers, whereas the other systems have solid steel wheels.

5.4 Conclusions

On the basis of the data presented in Fig. 5.3-5.5, we can determine approximate sound levels for the three wheel/rail interaction noise characteristics

Characteristic	Normalized Sound Level dB(A)	Conditions
Roar	70	Well maintained welded track, 30 mph (48 km/hr)
Impact	77	Normally maintained bolted track, 30 mph (48 km/hr).
Squeal	90	--

Such a simplification indicates that the problem of squeal is the most severe and should be solved straight-away, if possible.

However, the sound levels given above should be placed in perspective before a decision is made on which noise characteristic should receive the highest priority. First, we should find out how many people are affected by each sound. Secondly, although squeal noise levels are quite high and particularly annoying due to their pure tone content, squeal cannot be considered in the same context with roar and impact noise, because it is associated more with a fixed location (a curve) than with the moving train.

Furthermore, the 7 dB(A) difference between impact and roar noise is misleading. Comparative measurements indicate only a 4 dB(A) difference between well maintained welded and bolted track. The remaining 3 dB(A) appears to be the difference between normally maintained and well maintained bolted track. Furthermore, roar noise on normally maintained welded track can be reduced 6 dB(A) by grinding.

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Thus, comparisons given in this chapter are useful, but there is no clear-cut answer as to which kind of wheel/rail noise to attack first. The conclusion to be drawn is that equivalent resources should be applied to reducing each kind of noise.

6. A METHODOLOGY FOR ASSESSING THE NONACOUSTIC EFFECTS OF NOISE CONTROL PROCEDURES

A balanced assessment of any noise control procedure or device must include an analysis of its associated costs both to the rail transit operator and to the public at large. This section deals with the development of criteria by which the economic impact on these two sectors can be computed. Officials responsible for implementing noise control programs can then combine the cost measures with the previously calculated acoustical rating scheme to facilitate the selection of an "optimal" set of noise control techniques.

6.1 Impact of Costs on the Operator

This section describes the rationale and use of a cost criterion which allows the nonacoustic performance of alternate noise control techniques to be compared on a common basis. The criterion itself is fairly simple, being an adaptation of Net Present Value (NPV), an index which reduces future cash flows to a present equivalent amount by means of an appropriate interest rate (see Appendix A). The chief virtue of using NPV as an index is its generality: it considers all investments, whether they be items of machinery or common stocks, as a time series of cash payments and receipts, and reduces them all to a common basis.

Our purpose is to evaluate the overall effect on urban transit systems of a proposed hardware modification. The technical aspects of the modification are only of secondary interest at this point; what really concerns us is the extra costs to be incurred by the change, and the points in time at which they occur. In other words, if the operation has an initial NPV, what is the change in NPV which will be induced by the technical modification?

A number of alternative quieting schemes whose acoustical effect may be identical will, in general, have different cash flow time histories and, therefore, different ΔNPV's. Since the motive of cost/effective noise control is to maximize total NPV, we will choose that treatment program which, all things being equal, minimizes the decrease in NPV as being most economic. We see that Net Present Value thus presents both a relative rating of alternatives and an absolute measure of the economic effect of a given level of noise treatment. The significance of the ensuing discussion is in the careful identification and computation of the various costs which together constitute the net change in cash flow produced by a proposed abatement method.

The following discussion is divided into two parts. The first gives the logic and method for computing the change-in-Net-Present-Value (ΔNPV) criterion. The second gives an example, in which the ΔNPV of substituting resilient wheels on the cars operated by the New York City Transit Authority is calculated.

6.1.1 Description of the method

Criterion Definition

In general, any modification to an existing enterprise (rail transit system or any other) results in two categories of costs being incurred.* The first is an initial cost related to the implementation of the modification, and is usually incurred only once. The second is a change in the cost of operating the system in its new configuration; this cost recurs with time.

*The term "cost" here is used in the general sense and should not imply that all noise abatement schemes necessarily result in increased costs. Some techniques may result in actual savings; for our purposes, these are simply negative costs.

A classic problem in management accounting has been the combination of nonrecurring (initial) with recurring (operating) costs to obtain a single number representing the overall cost impact of the modification. The most acceptable solution to-date has been the "discounted cash flow" approach. This method involves identifying the amount and timing of every expenditure or revenue associated with the proposed modification, and using this information to construct a projected cash flow profile which gives, for each future year (or quarter), the change in the net flow of money into or out of the enterprise. (Note that depreciation is not a cash flow but merely an accounting convention — no actual funds change hands — and is therefore not included in discounted cash flow analyses.) Once a "change-in-cash-flow" schedule is created, the cash flow for each future year is discounted to the present at an appropriately chosen rate of interest using the expression:

$$\Delta NPV = \sum_{n=1}^{m} \frac{\Delta R_n}{(1+i)^{n-1}} , \qquad (6.1)$$

where ΔNPV = change in net present value

m = expected lifetime of the sytem being modified from the data of modification

ΔR_n = change in cash flow for year n (sign convention: a net disbursement is a positive ΔR_n)

i = opportunity cost of capital.

In our case, the ΔR_n's are determined by the costs associated with retrofitting rail transit systems with noise control treatments. System lifetime m is the lifetime of the component being treated (track or cars). Opportunity cost of capital i is obtained from the interest currently paid on Aaa commercial bonds (see Appendix A for detailed discussion).

The manner in which the ΔR_n series is constructed is affected by the time which the transit operator is allowed to take for compliance.* The following discussion shows how to include the effect of compliance period on ΔNPV.

Begin by determining the stock of capital to be retrofitted (miles of track, number of cars, number of wheels, as appropriate). Divide the capital stock into y equal segments corresponding to the number of years y allowed in the compliance period. Determine for each segment k (k = 1 to y), the series $\Delta R_{n',k}$ (procedure given in next section), where $n' = n - k + 1$, as if the k *th* segment were the only one being treated.**

Compute the series of ΔR_n's using the expression

$$\Delta R_n = \sum_{k=1}^{y} \Delta R_{n',k} \quad , \tag{6.2}$$

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* It is sometimes assumed that the effect of stretching a compliance period from, say, 2 to 5 years is merely to divide the same total initial cost by 5 rather than 2. This is not generally true, because a short compliance time has a greater probability of forcing the operator to acquire extra shop and labor capacity, thus increasing the total initial cost. This will be demonstrated in Sec. 6.1.2.

**What is being done here is to substitute, for the entire capital stock, that segment which would be retrofitted in any one year. For computation purposes, we introduce the dummy time variable $n' = n - k + 1$ reflecting the fact that the third segment (k = 3) is begun in the third year (n = 3). Relative to the items being retrofitted, however, this is the first year of the program, so $n' = 1$.

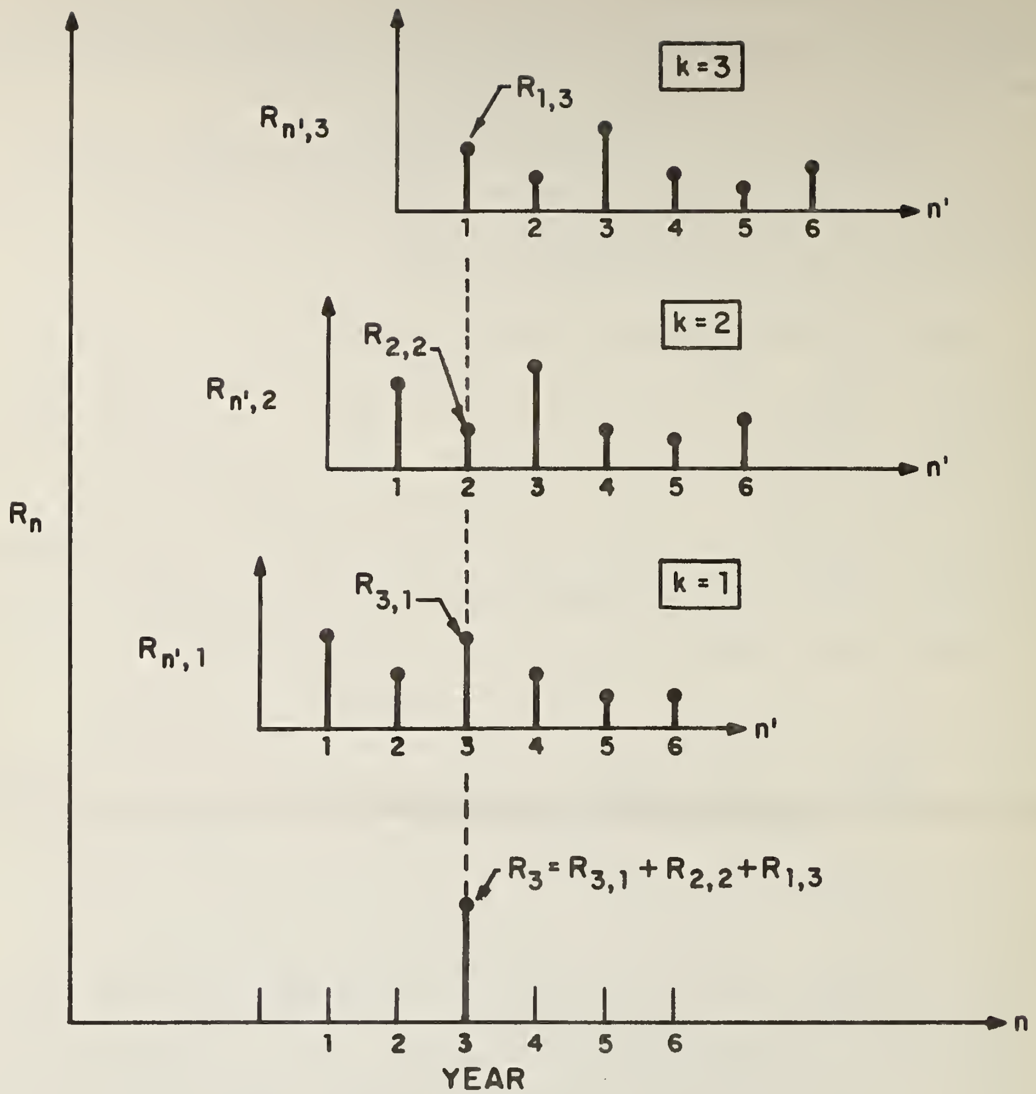

FIG. 6.1. ILLUSTRATION OF HOW R_n IS CONSTRUCTED FROM $R_{n',k}$.

where, as before, $n' = n - k + 1$ ($n' \geq 1$). Thus, assuming that $y = 3$, the total change in cash flow in the third year of the program, R_3, is given by

$$R_3 = \sum_{k=1}^{3} R_{(3 - k + 1),\, k}$$

$$= R_{3,1} + R_{2,2} + R_{1,3}$$

where $R_{1,3}$, for example, represents the amount spent on the *third* segment of the capital stock in the *first* year that segment comes due for treatment. The summation procedure is shown graphically in Fig. 6.1.

Cost Category Definition

This section defines the cost items which must be computed to obtain the R_n series* referred to above. The required parameters are summarized in Table 6.1. Each segment of the capital stock being treated incurs initial costs at the time of treatment and subsequent operating costs during its lifetime. For each of these two cost categories, there are two subcategories: direct costs, which are incurred as an immediate result of the treatment program, and indirect costs, which arise from secondary impacts made by the treatment program on the transit system's operations.

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*Recall that $\Delta R_{n',k}$ is the series of cash flows undergone by the *kth* segment of the capital stock. In this development, we have made the reasonable assumption that all the segments be equal in size, so all will undergo the same cash flow history (although the first segment will be treated in year $n = 1$ and the *kth* segment in $n = k$). We therefore drop the k subscript for the rest of Sec. 6.1.1.

TABLE 6.1. TABLE OF PARAMETERS FOR THE ECONOMIC METHODOLOGY

Initial Direct Cost	
U	Net materials costs per unit of capital stock being treated ($).
K	Number of units of capital stock being treated per year.
T	Labor required per unit of capital stock (man-hrs).
LR	Labor rate, including fringe benefits ($/man-hrs).
L_{NC}	Labor available at no charge ($).
C_{KT}	Capital (i.e., shop space and equipment) required to accomplish retrofit program ($).
K_{NC}	Capital available at no charge ($).
Initial Indirect Cost	
H	Time interval corresponding to a period of peak demand on system (hrs).
K	Number of cars in service during period H.
P	Peak number of passengers carried during period H, as obtained from historical data.
P'	Average number of passengers carried during period H.
h	Number of list car-hours during period H due to retrofit.
r	Average revenue per passenger ($/passenger).
W	Number of weekdays' duration of the retrofit program.

TABLE 6.1. (cont'd)

Direct Operating Costs

β_D	Increase in annual direct operating cost (power, operating labor) due to retrofit program ($).

Indirect Operating Costs

β_I	Increase in annual indirect operating costs (maintenance materials, maintenance labor, administrative labor) due to retrofit program ($).

Net Present Value

y	Number of years allowed for compliance.
k	Index designating each segment of capital stock according to the year in which it is treated (k = 1, ..., y).
n	Index designating real-time years from the start of the retrofit program (the first year is n = 1).
n'	Index designating years from the data of retrofit of the *kth* segment of the capital stock (n' = 1 represents the year in which the *kth* segment is treated).
m	Remaining economic lifetime of the capital stock being treated, counted from the year of treatment.
i	Interest rate for public projects.

TABLE 6.1. (cont'd)

Net Social Cost

K_1	Number of units of capital (cars, miles of track, etc.) taken out of service at any one time due to retrofit.
K_2	Number of "extra" or "spare" units of capital in the system.
P_K	Purchase price of a new unit of capital.
V	Resale value of a new unit of capital.
s	Economic lifetime of the capital stock item from its data of purchase.

If we designate initial costs as α_D and α_I (for direct and indirect, respectively) and annual operating costs as β_D and β_I, then we can compute the series $\Delta R_{n'}$ from

$$\Delta R_{n'} = \alpha_D + \alpha_I + (\frac{\beta_D + \beta_I}{2}), \text{ for } n' = 1$$

$$= \beta_D + \beta_I, \text{ for } n' > 1. \qquad (6.3)$$

Note that in the first year, half of the capital stock being retrofitted, on the average, incurs the additional operating costs resulting from the retrofit.

Initial Direct Costs. The chief initial direct costs are the materials, labor, and shop capacity needed for the treatment itself. The cost of materials is computed for the segment of the capital stock being retrofitted in one year:

$$C_M = U \times K \quad , \qquad (6.4)$$

where U is the net materials cost per unit of capital stock* (e.g., per car or per mile of track) and

K is the number of units of capital stock being treated per year (e.g., cars).

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*Materials cost per unit should include the cost of building the replacement device with a large enough safety factor to insure reliability. In the case of a service-proven unit, this will be included in the actual purchase price. In the case of an experimental device, the estimated purchase price should be increased by a factor representing an "overdesign" sufficient to insure compliance with safety and reliability requirements.

The cost of labor is a bit less straightforward. The total labor cost C_{LT} involved in the treatment program is computed from the expression

$$C_{LT} = T \times K \times LR$$

where T is the labor required per unit of capital stock (e.g., man-hours per car or per mile of track),

K is the number of units of capital stock being treated per year, and

LR is the labor rate, including fringe benefits.

But some of the necessary labor may be available at no charge to the treatment program because (a) there is some excess labor capacity in the system which is available for free, or (b) part of the program can be accomplished by substituting tasks associated with the treatment for other work presently going on, thus incurring no extra labor charge. To obtain the actual labor cost chargeable to the treatment program, therefore, we must subtract from the total labor requirement the portion which can be obtained at no cost:

$$C_L = C_{LT} - L_{NC} \tag{6.5}$$

where C_L = labor cost chargeable to the program (dollars)

C_{LT} = total labor requirement (dollars)

L_{NC} = labor available at no charge (dollars)

Costs of capital are treated in the same way as labor costs. As before, a certain amount of shop capacity may be available for free, and must be subtracted from the total shop effort required to do the job. The chargeable capital cost is given by

$$C_K = C_{KT} - K_{NC} \tag{6.6}$$

Where C_K = capital cost chargeable to the program (dollars)

C_{KT} = total capital requirement (dollars)

K_{NC} = capital available at no charge (dollars).

Total initial direct cost is given by

$$\alpha_D = C_M + C_L + C_K \tag{6.7}$$

Initial Indirect Costs. Performing noise control treatment on capital items frequently requires their removal from service for periods longer than their usual maintenance downtime. Because the equipment is not in service for some extra period, the system's service capacity is diminished. Depending on the degree of reduced capacity and the level of extra capacity with which the system normally operates, the possibility exists for fewer passengers and, hence, less revenue. This is an indirect cost associated with noise control treatments.

The computation of lost revenue is complex, since it depends on the available estimates of system excess capacity. Determination of the system's actual total capacity is difficult, since it is usually possible to "stretch" the system to accomodate peak loads by using spare equipment, deferring maintenance, and using other such short-term measures. Perhaps the fairest measure of total system short-term capacity is the amount of traffic (revenue passengers) carried per car-hour during a period when the system was known to be operating at its limit. We can then assume that average excess capacity is measured by the difference between the historical peak load and the average load on the system.*

To determine lost revenue, we must first determine the loss in passengers.† This can be obtained as follows. Determine, as described above, the total number of passengers P carried by the system during a heavy traffic time interval H, at a time when the system was known to be under stress. The peak unit capacity is then obtained from this historical data using the expression,

$$p = \frac{P}{KH} \quad ,$$

where p is the peak unit capacity in passengers per car-hour and K is the number of cars in service at the time. Similarly, determine from *average* traffic figures what the average passenger load P' is during the same time interval H. Average unit capacity is then

$$p' = \frac{P'}{KH} \quad .$$

where p' is the average unit capacity in passengers per car-hour. The average excess system capacity is then

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*Because the load on a transit system is highly time-dependent, comparison of peak vs average traffic should be made only for periods of heavy demand. These are usually on weekdays from 7 a.m. to 9 a.m. and from 4 p.m. to 7 p.m.

†The ensuing discussion applies only to the removal of cars from service. It is assumed that the removal of track from service would be considered infeasible if it interfered with rush-hour traffic.

$$E = (1 - \frac{p'}{p})\ KH \quad , \tag{6.8}$$

Where E = excess system capacity in car-hours per day.

We can now compute the loss of passengers. First, an estimate of extra lost car-hours per day during the interval h due to the treatment program must be obtained from operating personnel. This amount is designated h. If lost car-hours during the interval are less than the excess capacity, no passengers will be lost:

$$\Delta P = 0 \quad \text{for } h \leq (1 - \frac{p'}{p})\ KH \quad , \tag{6.9}$$

where ΔP is the change in passengers carried per day. On the other hand, if more car-hours will be lost than are available in excess system capacity,

$$\Delta P = [h - (1 - \frac{p'}{p})\ KH]\ p' \tag{6.10}$$

for $h > (1 - \frac{p'}{p})\ KH$.

The initial indirect cost of the program C_I is given by

$$C_I = r \cdot \Delta P \cdot W \quad , \tag{6.11}$$

where r = average revenue per passenger and

W is the number of weekdays* taken up by the program.

Note that if the entire retrofit program takes an integral number of years, W = 250. If it takes a non-integral number of years,

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*Daily Lost passengers P is multiplied by the number of weekdays in the year. It is assumed that there will always be excess capacity on weekends and holidays.

W for the fractional year must be estimated explicitly from the retrofit schedule.

Since loss revenue is the only element of initial indirect cost which we are considering,*

$$\alpha_I = C_I \tag{6.12}$$

Direct Operating Costs. The largest elements of direct operating costs are fuel (or power) and wages. The latter can be neglected, since it is unlikely that either the manpower per train or the average running time per train would be affected by noise control treatments. Fuel requirements, however, may change, especially when a noise abatement procedure results in modification of the wheel/rail system. The actual change in fuel requirement per car-mile must be obtained from test data or engineering estimates.

Indirect Operating Costs. The component of indirect operating costs which most concerns a noise reduction program is maintenance. A quantitative discussion of incremental maintenance cost depends on the details of the proposed abatement procedure, and so cannot be gone into here. We point out, however, that what is of interest is the net annual maintenance expenditure;

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*An alternate approach to the lost-capacity problem would be to compute the cost of adding extra capital stock to compensate for the loss. In the case of rail transit systems, however, this is rarely a feasible alternative due to: a) the very high initial cost of cars and track (and their unavailability on a rental basis); b) the long lead time of such items, which would considerably delay implementation of an attendant noise control program; and c) the problem of what to do with the excess capacity once the noise control treatment program is over. These considerations do not, however, preclude using the cost of temporary replacement capital as one measure of net social cost (see Sec. 6.2.1).

that is, projected annual maintenance costs for the quieted assembly must be reduced by what would have been spent even if the quieting program had not been implemented.

6.1.2 Application of the method

The general technique described in Sec. 6.1.1 requires some elucidation in the form of an example. We shall use the method to compute the ΔNPV associated with retrofitting resilient wheels on all of the 7,200 cars operated by the New York City Transit Authority. Each step of the computation of the example corresponds to a section of the discussion above.

To illustrate the effect of different compliance periods, the ΔNPV calculation is performed assuming alternative periods of one year and three years for the retrofit program.

Cost Computations

Direct Initial Costs. Table 6.2 summarizes the cost of materials C_M for the resilient wheel retrofit program.

Labor cost, as noted above, depends on the excess capacity available and the amount of retrofit work which can be done at zero cost by replacing other presently scheduled work. In the case of the NYCTA, all wheel shops are working at full capacity. Part of the retrofit program, however, could be taken up at no

TABLE 6.2. COMPUTATION OF C_M, ANNUAL DIRECT RETROFIT COST FOR MATERIALS

	1-Year	3-Year
1. Net cost of parts for resilient wheels, per pair*	$ 800	$ 800
2. Number of pairs to be retrofitted per year	28,800	9.600
3. Annual cost of materials (1 × 2)	$ 23.0M	$ 7.7M

*Assumes $550 per wheel purchase price, $30 per wheel recovery from sale of scrapped wheels, $120 per wheel credit for standard wheels which would otherwise be installed.

extra labor cost by the normal wheel replacement schedule. The man-hours available in this manner are summarized in Table 6.3. (Notice that our estimate of future volume of wheel replacement activity assumes that a presently-planned 20-pair-per-day shop will be available and that an existing 8-pair-per-day shop will be phased out as planned.) The table shows that 36,000 man-hours will be available at no extra charge.

TABLE 6.3. PRESENT MAINTENANCE SHOP LABOR AVAILABLE FOR WHEEL RETROFIT

	Single-Shift Throughput	No. of Workers Per Shift	Man-Hours Per Pair of Wheels	Man-Hours Available Per Year (1 Shift)
Present Shop	16 pairs/day	16	8	32,000
Planned Net Increase*	12 pairs/day	2	-	--
Planned Future Capacity	28 pairs/day	18	5.2	36,000

*Includes adding one 20-pair per day shop with (estimated) 10 employees and retiring one 8-pair per day shop with 8 employees.

Table 6.3 shows how C_L is computed. Total labor requirement C_{LT} is computed assuming a direct labor rate of $6.05 per man-hour.* The labor available at no charge, L_{NC}, is computed from Table 6.2 and subtracted from C_{LT} to give C_L, the labor cost to the program. For the one-year program C_L is $790K and for the three-year program it is $121K per year. It is assumed, based on conversations with the NYCTA, that extra wheel-shop labor is readily available for hire and involves negligible training and start-up cost.

Capital cost involves the cost of adding extra shop facilities to handle the projected maintenance overload. Table 6.4 shows that the overload imposed by a wheel retrofit program can be met by retaining the shop presently scheduled for closeout, adding shifts to the present work schedule, and treating 1800 wheel pairs on overtime. Since, in this case, no new shop facilities must be built or otherwise acquired, the capital cost is merely the present cost per unit time of operating the shop times the extra time on line. The NYCTA estimates the cost of operating a shop to be proportional to the man-hours of labor expended in the shop, the proportionality factor being 0.9 times the labor rate. Multiplying C_L by 0.9, therefore, gives the extra capital cost chargeable to the retrofit program. For a one-year program, C_K is $655K and for a three-year program, H is $109K. ✓

*Figure obtained from NYCTA. Note that, because of the limited shop capacity, 1,800 wheel pairs must be treated on overtime at 1.5 times the normal labor rate (see Table 6.4). This results in an equivalent extra man-hour requirement (at straight time) of 10,400 man-hours or $63,000.

TABLE 6.4. COMPUTATION OF C_L, LABOR COST CHARGEABLE TO RETROFIT PROGRAM

	Compliance Period	
	1-Year	3-Year
1. Marginal man-hours per pair of wheels	5.8*	5.8*
2. Number of pairs to be converted per year	28,800	9,600
3. Total man-hours required per year (1 × 2)	157K	56K
4. C_{LT}: Total labor cost required per year**	$1008K	$339K
5. L_{NC}: Labor available at no charge***	$ 218K	$218K
6. C_L: Labor cost chargeable to retrofit program (4-5)	$ 790K	$121K

* Assumes two 8-pair per day shops and one 20-pair per day shop are operating.

** Assumes a labor rate of $6.05 per man-hour. Note that, in the one-year case, 1800 wheel-pairs are treated on overtime (see text and Table 6.5).

*** See Table 6.2.

TABLE 6.5. COMPUTATION OF AVAILABLE SHOP CAPACITY USING PRESENT FACILITIES IF EXTRA SHIFTS ARE ADDED (Figures in Wheel Pairs Per Year)

	Compliance Period	
	1 Year	3-Years
Required annual wheel pair throughput	28,800	9,600
Planned future capacity one shift*	7,000	7,000
Feasible future capacity one shift†	9,000	9,000
Feasible future capacity three shifts†	27,000	27,000
Deficit - to be met using overtime work (new facilities not required)	1,800	--

*Assumes 250 working days per year.

†Includes retaining one shop presently scheduled for phaseout.

We can now compute $\alpha_D = C_M + C_L + C_K$:

	Compliance Period	
	1-Year	3-Year
C_M =	\$23,000 K	\$7,700 K
C_L =	790 K	121 K
C_K =	655 K	109 K
	\$24,445 K	\$7,930 K

Initial Indirect Costs. The resilient wheel retrofit program which we are using as an example would not result in any lost revenue. This is because the cars themselves are in the wheel shop just long enough to have the trucks removed and a set of replacement trucks put on. This process takes only a short time, with the result that system carrying capacity is reduced by a negligible amount. Therefore, $\alpha_I = 0$.

Direct Operating Costs. There is a possibility that the use of resilient wheels will result in higher unit power costs, since some work must be expended in deforming the resilient inserts in the wheels. No reliable estimates of this cost exist, however. We will therefore assume for computational purposes that a 2% increase in power consumption will occur. Total power consumed by the rail transit operations of the NYCTA cost $53.5M in 1972, so we can estimate that $\beta_D \approx \$1.07$ M.

Indirect Operating Costs. Some manufacturers of resilient wheels state that maintenance costs for this type of wheel are lower than for standard wheels, since once a tire wears out one can simply fit a new tire onto an existing wheel instead of throwing the entire wheel away. Present knowledge, however, is insufficient to substantiate this claim. For one thing, the service life of resilient inserts is not established under all conditions; in some applications, such wheels may require more frequent replacement than their standard counterparts. Another issue is that of inspection. At present, steel wheels are visually inspected for cracks about every six weeks. Checking a resilient layer for signs of failure may require more frequent or more sophisticated inspections. We therefore set $\beta_I = 0$, chiefly as a reflection of our lack of knowledge.

Criterion Computation

From the cash-flow elements computed in Sec. 6.1.2 and summarized in Table 6.6, we can compute the series $R_{n'}$ using Eq. 6.3.

For the one-year compliance period,

$$R_{1'} = \$24.98M$$

$$R_{n'} = \$\ 1.07M, \quad n' = 2,3,...$$

For the three-year compliance period,

$$R_{1'} = \$8.10M$$

$$R_{n'} = \$0.35M \quad n' = 2,3,...$$

Using the $R_{n'}$ series, we can compute the R_n series. For the one-year compliance period, the two series are identical. For the three-year plan, according to Eq. 6.2,

$$\begin{aligned}
R_1 &= R_{1',1} && = \$8.10M \\
R_2 &= R_{2',1} + R_{1',2} \\
&= 8.10M + .35M && = \$8.45M \\
R_3 &= R_{3',1} + R_{2',2} + R_{1',3} \\
&= 8.10M + .35M + .35M && = \$8.80M \\
R_4 &= R_{4',3} + R_{2'3} + R_{2',1} \\
&= .35M + .35M + 35M && = \$1.07M \\
R_n &= && \$1.07M \quad n = 5,6,...
\end{aligned}$$

TABLE 6.6. SUMMARY OF CASH-FLOW ELEMENTS FOR RESILIENT WHEEL RETROFIT PROGRAM (Figures in $ Millions)

	Compliance Period	
	1 Year	3 Years
Direct initial, α_D	\$24.45	\$7.93
Indirect initial, α_I	0	0
Direct operating, β_D	1.07	0.35
Indirect operating, β_I	0	0

The summation of the R_n series for the three-year compliance period is shown graphically in Fig. 6.2.

Using Eq. 6.1, we can compute the ΔNPV of the two R_n time series obtained above. We assume that the average remaining lifetime m of the cars being retrofitted is 15 years. The return on investment i is obtained from the rate on Aaa bonds, which at present is approximately 7.8% (see Appendix A). Using these figures, we obtain:

ΔNPV (one-year compliance period) = \$33.16M

ΔNPV (three-year compliance period) = \$29.81M

6.2 Social Impact

In order to have a complete and balanced analysis of the costs and benefits of a noise control retrofit program, one must take into account the impact of the program on the community at large. The benefit to society is measured in terms of decreased noise exposure; this has been dealt with in Schultz, (1974). The social cost is reflected in terms of possible decreases in quality of service while the noise control program is being implemented. The problem in quantifying social cost is that a decline in

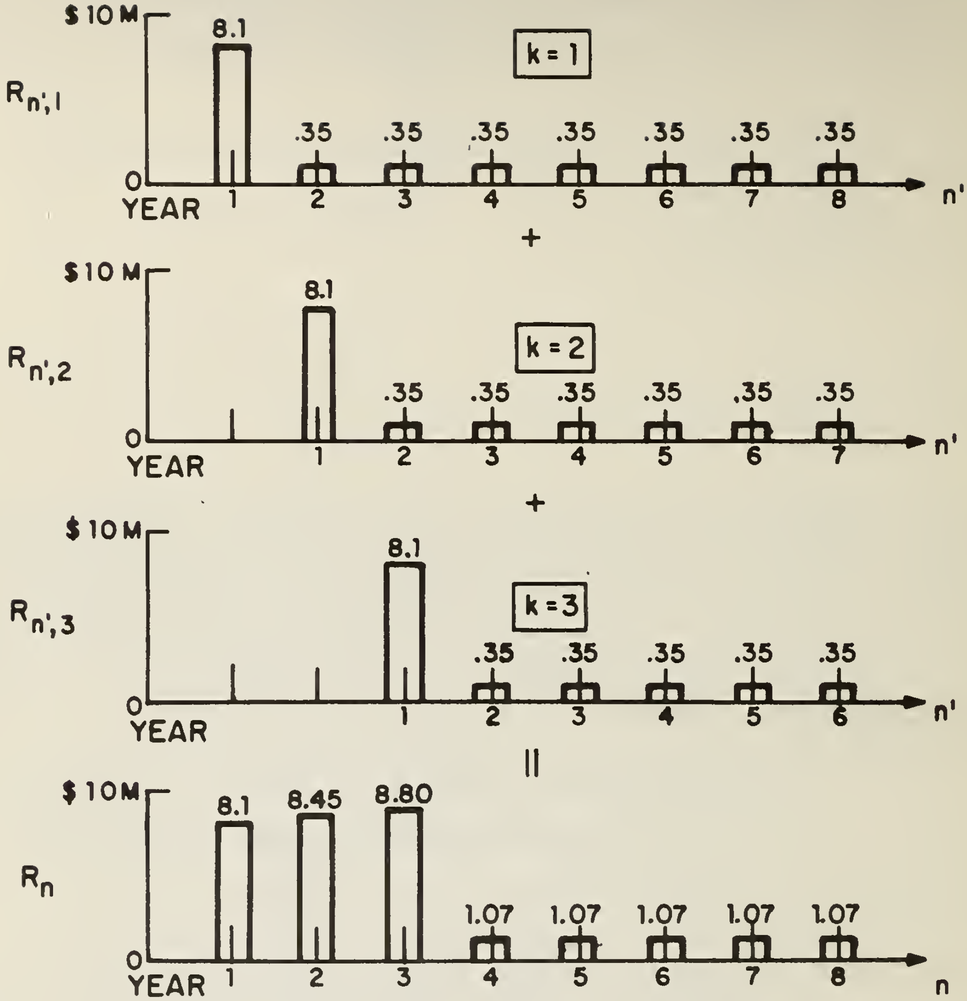

FIG. 6.2. CONSTRUCTION OF CASH FLOW SERIES R_n FOR THE THREE-YEAR RETROFIT PROGRAM.

transit service affects not only the users of the transit system but the non-users as well. This section presents a method for computing the net social cost (NSC) of a noise control program on the community as a whole.

6.2.1 Discussion

We may divide the goods which are produced in society into two groups: those which provide benefits only to the consumer of the goods and those which provide benefits to the consumer and affect third parties as well. These effects upon third parties may be either positive or negative. In the case of mass transit, benefits to non-users include reduced highway congestion, reduced auto pollution, more efficient use of energy, etc. Neo-classical economic theory defines the "optimal" quantity of a good or service produced as that quantity for which the equilibrium market price equals the cost of producing the last unit of that good or service (the "marginal cost"). The equilibrium market price under these conditions represents the benefit to the user of the last unit of that good or service consumed (the "marginal benefit"). If, however, there are third parties who do not pay for their benefits, the price (for the case of interest here, the transit fare) does not reflect the *total* benefit to society. In our case, the optimal quantity described above is that quantity for which marginal cost equals marginal benefit, where marginal benefit includes *both* user and non-user benefits.

This point can be illustrated by means of a schematic supply-demand curve such as in Fig. 6.3. The curve labeled MC (marginal cost) represents the cost to the supplier of a commodity (in this case transportation service) of providing one additional unit of that commodity, as a function of the number of

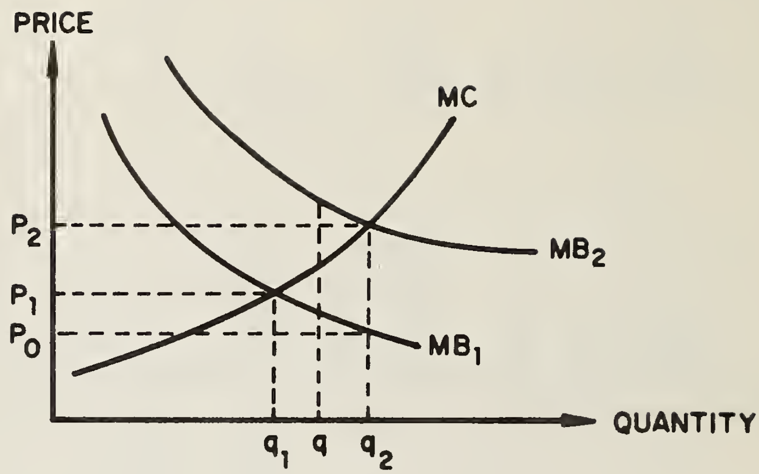

FIG. 6.3. SCHEMATIC MARGINAL COST CURVE ILLUSTRATING THE DEFINITION OF SHADOW PRICE

units produced. The curve can also be interpreted as the "supply" curve for that commodity, since it represents the unit price (P) at which the supplier is willing to sell any given quantity (q). The curves labeled MB (marginal benefit) represent the price which the consumer is willing to pay per unit of the commodity when a given quantity q is available. In the case of an optimally distributed commodity, the intersection of the MC and MB curves gives the price and quantity of the commodity (P,q); the total revenue from the sale of the commodity is then given by $P \times q$.

The demand curve (curve MB_1 or MB_2 in Fig. 6.3) for transit service can be obtained in two ways, depending on whether one considers the demand for transportation only, or for transportation *plus* the indirect benefits to society which transportation provides. The value of transit service to users is MB_1, which is the demand curve for transportation. If there were no external benefits to non-users, the level of service would be (see Fig. 6.3) q_1 and the unit would be price P_1 (which would represent the value of the last unit of service provided). When non-user benefits are added to MB_1, the marginal benefit becomes MB_2 and the cost of providing the $q_2{}^{th}$ unit of service is P_2 (see Fig. 6.3). The price which clears the market is P_0. This is the value of the marginal unit to users (the marginal unit being the last unit produced). The true value of the marginal unit to both users and non-users, P_2, is called the "shadow price."

Given these considerations, if the value of the marginal unit of service were established at P_0, this would underestimate the true value P_2. If we assume that the current level of service is optimal (q_2), the true marginal value P_2 can be measured by the marginal cost of providing the service. In addition, if the change in the level of service is small, we can assume that $dq \cdot P_2$ approximates the value of the lost service to the community at large. This would appear to be the net social costs. But, part of this quantity has already been included in the calculations of Sec. 6.1, i.e., the lost revenue which is a measure of the value of the lost service to users of the system. Referring to Eq. 6.11 we can readily see that lost revenue ($P_0 \cdot dq$ in Fig. 6.3) has been included as an initial indirect cost. As a result, for our purposes here we define the *net social cost as the value to nonusers of the transit system of service lost due to a retrofit program.* We calculate this quantity by subtracting lost revenue (a measure of the value of the lost service to system users) from the "shadow price" times the lost service (a measure of the true value of the lost service to both users and non-users). To put this in the terms of the parameters in Fig. 6.3 we write

$$NSC = dq \cdot (P_2 - P_0)$$

where dq is a measure of the lost service.

The major reservation in using this technique is that the current level of service may not be optimal. It seems most likely, given the current emphasis upon improving mass transit, that the level of service is less than optimal. That is, the existing service should be expanded. Under this condition our estimate of a shadow price would be lower than the true value. This can be shown in Fig. 6.3. Taking a point at which output is less than optimal, such as q, the shadow price is the marginal cost, the value of P on curve MC, while the true value is the marginal benefit, the value of P on curve MB_2. As a result, the marginal cost is less than the marginal benefit, and the shadow price underestimates the user-nonusers benefits.

Nevertheless, our methodology in this particular study is to assume that the cost of maintaining the same level of service is the marginal cost (i.e., the shadow price). Clearly, if the transit authority actually maintains the original level of service, then the lost revenue will be zero. Even so, the revenue which *would have been* lost would still represent the value of the incremental service to the transit user.

6.2.2 Procedure

The determination of the social cost of a retrofit program requires that the shadow price of the current output be estimated. The social cost is the difference between the shadow price (i.e., total cost to society) and the lost revenue (i.e., cost to the system users). In the following computation, shadow price is approximated by the cost of new capital necessary to maintain existing levels of service. The computation is given in outline form.

A. Compute the net increase in operating capital stock necessary to maintain service.

$$K_0 = K_1 - K_2 \quad , \qquad (6.13)$$

where K_1 = the number of units (cars, miles of track, etc.) taken out of service at any one time due to retrofit

K_2 = the number of "extra" or "spare" units in the system.

B. Compute the discounted cost of operating capital C_{K_0}*:

$$C_{K_0} = K_0 \left[P_K - V(1+i)^{-s} \right] \quad , \qquad (6.14)$$

*Expression 6.14 implies that the purchase price of the new capital is incurred in year 1 and that the resale value, which is obtained in year s, is discounted to year 1 at interest rate i. This is basically the net present value calculation of Eq. 6.1 where P_K is an expense in year 1 (positive R_n) and V is income (negative R_n) in year s.

where P_K = purchase price per unit of capital item

V = unit capital item resale value

i = interest rate (see Appendix A)

s = economic liftime of capital item

C. Calculate C_A, the annual cost of capital;

$$C_A = RF \times C_{K_0} \tag{6.15}$$

where RF, the capital recovery factor* is defined by

$$RF = \left[\frac{i}{(1+i)^s - 1} \right] \tag{6.16}$$

The economic lifetime s must be specified rather carefully. Three options exist, depending on how the newly acquired capital is used:

a) If the new equipment becomes excess capacity after the retrofit program, then s is the duration of the retrofit program.

b) If the new equipment is used to retire older but still productive assets, then s is the productive life of the new equipment. In this case, a term may be added to C_A to account for the increase in annual cost associated with early retirement of the older assets.

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*In effect C_K is the average amount of capital that must be recovered each year so that at the end of s years the amount recovered including interest equals C_{K_0} .

c) If the new equipment is used to expand the system after the retrofit program is completed, then s is the productive life of the new asset.

D. Compute net annual social cost:

$$NASC = C_A - C_I \quad , \tag{6.17}$$

where C_I is the indirect cost represented by lost revenue, as defined in Sec. 6.1.1.

E. Compute net total social cost:

$$NSC = s \cdot NASC$$

where s is the lifetime of the capital as used in Eq. 6.16.

APPENDIX A: DETERMINING DISCOUNT RATE FOR INVESTMENTS IN THE PUBLIC SECTOR

The question of the appropriate rate of discount (i.e., interest rate) to be applied to public investment has been considered by a number of authors. The critical issue in the selection of the discount rate is that it represents the opportunity cost of using resources in the public sector. Under conditions of full employment, investments made in the public sector will displace investment in the private sector. The optimal level of public investment can only be achieved if the return to public investment is compared with the rate of return in the private sector.

The position taken by Baumol (1958) is that the rate of interest which should be applied to public investment is h • r, where r is the rate of return in the private sector and h is a scalar which allows the comparison of the rates of return in the private and public sector. The value of h is between 1 and 2. Since the corporate tax rate is about 50%, the value of h should be about 2, since firms in fact earn twice the after tax rate of return. Clearly, if all capital expansion is not financed by equity but rather in part through debt, h is less than 2.

One fact seems to emerge from this discussion: The use of a rate of interest for the public investment which is lower than the current market rate in the private sector leads

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*Baumol, W., 1958. "On the Social Rate of Discount," *American Economic Review*, pp. 788-802.

to misallocations. For the purpose of this study we have used the current Aaa corporate bond rate as the rate of interest to be applied in public investment projects. This rate is somewhat lower than Baumol would recommend, but higher than the rate for 90-day bills or long-term government securities.

Some economists have rejected the higher rate of interest, because a higher discount rate applied to public investment would exclude a number of long-term government projects. These projects may be desirable for reasons other than efficiency (e.g., redistribution of income) and should be evaluated accordingly. However, if efficiency is to be the criterion by which projects are evaluated, then the relationship between private and public investment must be recognized. When private and public investment compete for resources, they must do so on an equal basis and this requires that the marginal yield in public investment equal the marginal yield in the private sector.

APPENDIX B: REPORT OF INVENTIONS

After a diligent review of the work performed under this contract, we have determined that to date no new innovation, discovery, improvement, or invention has been made.

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