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REPORT NO. DOT-UMTA-71-2

ANALYSIS OF HYDRAULIC SERVO EQUATIONS FOR WRDRF PROTOTYPE CONTROL SYSTEM VOLUME I

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OCTOBER 1971
PRELIMINARY MEMORANDUM

Prepared for:
DEPARTMENT OF TRANSPORTATION
TRANSPORTATION SYSTEMS CENTER
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CAMBRIDGE, MA. 02142

1. Report No. DOT-TSC-UMTA-71-2	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Analysis of Electro Hydraulic Servo Equation for WRDRF Prototype Control System		5. Report Date October 1971
		6. Performing Organization Code
7. Author(s) Joseph E. Picardi		8. Performing Organization Report No. UM204
9. Performing Organization Name and Address Department of Transportation Transportation Systems Center 55 Broadway Cambridge, MA		10. Work Unit No.
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address Department of Transportation Urban Mass Transportation Admin. 400 Seventh Street Washington, D.C. 20580		13. Type of Report and Period Covered Preliminary Memorandum
		14. Sponsoring Agency Code
15. Supplementary Notes		

16. Abstract

A set of dynamic performance equations derived by Wylie Labs., Huntsville, Alabama, were independently re-derived and checked. These equations describe the performance of the prototype electro hydraulic servo actuator system selected by Wylie as representative for the preliminary design for the Wheel/Rail Dynamic Research Facility to be installed at the High Speed Ground Test Center at Pueblo, Colorado. Stability, frequency response and various transient response test runs were made using the M-Delta computer program. Results from DOT/TSC computer runs correlate very closely with Wylie's results. Denominator Root Printout of DOT/TSC computer runs show that one case studied by Wylie had unstable roots even though closed loop frequency response appears stable. It is advised that complete analysis including roots printout and transient response always be included to supplement any closed loop frequency response analysis.

17. Key Words	18. Distribution Statement This Preliminary Memorandum is an informal communication or working paper, with limited distribution.

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	48	

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INTRODUCTION

Solution of Electrohydraulic Servo Equations for the Wheel/Rail Prototype Control System

The purpose of this report was to (1) verify the mathematical model given in Wylie report Tech Brief 9.1.3 against the schematic diagrams given in the same report; (2) verify the solutions of the mathematical model; and (3) verify whether there is agreement between the servo analysis computer program used by Wylie Labs with the MDELTA program available at DOT/TSC.

The mathematical models were independently formulated for the servo loops that are shown in figs. 5 and 6 of Appendix I. The resultant set of coupled transformed differential equations as shown on page 13 Appendix I and also shown as 14 first order differential equation on page 17 of Appendix I were found to be identical to the Wylie derived equations of Tech. Brief 9.1.3.

The MDELTA program was used to obtain computer run solutions including equations and responses for the following cases.

1. Closed loop frequency response for the case of exciter position gain, G_e, equal to 0.9.

2. Step function response using state variables for the case of G_e, equal to 0.03.

3. Finite pulse response using state variables for the case of G_e, equal to 0.03.

Responses for these cases are shown in figures 7 and 8, Appendix I, page 16 and 17 for case 1, Case 2 response is shown in figure 9 and case 3 response is plotted in figure 12 of Appendix I.

As shown in these figures the results from DOT/TSC closed loop analysis digital computer program correlate with Wylie's results but also showed that one case selected by Wylie exhibited stable closed loop frequency response data output but was actually unstable. The DOT/TSC computer runs printout the characteristic equation denominator roots that exhibit the degree of stability of instability that may not be evident for closed loop frequency response tests. It is advised that complete analysis of selected servo system configuration should include roots printout and that transient response test runs should always be included to supplement any closed loop frequency response analysis.

At the present time the DOT/TSC computer program is available for fast turn around to obtain frequency response, root-locus, time response, and two-loop gain boundary evaluations when up to date data is received from Wylie labs.

DERIVATION OF WRDRF ELECTRO HYDRAULIC SERVO SYSTEM EQUATIONS

Each roller module of the WRDRF facility test machine is driven by a set of 6 linear Electro-Hydraulic Servo Actuators that provide for 6 degrees of freedom spatial motion as shown in Fig. 1. An analogue-digital computer control system converts desired track motions into command signals that position each actuator. Track signals recorded on magnetic tape and deterministic signals are coordinate converted and transformed from digital to analoge signals to drive the magnetic torquers on each electro hydraulic actuator. The servo electronics are of an analoge configuràtion in order to achieve rapidity of response. This hybrid computer control system is shown in Fig. 2. The hydraulic servo chain consists of a pilot valve driving a slave valve that in turn hydraulically actuates the exciter position drive units. In the prototype unit selected for analysis, pilot valve velocity feedback is used, mixed with position feedback from the slave valve and position feedback from the execiter piston. A diagram of the electro hydraulic network is shown in Fig. 3.

The performance equations for the electro hydraulic drive are derived by writing the expressions for each output to input quantity transcending each power amplification unit in order. These equations are as follows:

<u>PILOT VALVE</u>

The input voltage driving the magnetic torque motor on the pilot valve developes the torque motor current as:

$$E_p = i_p r_e + L_e \bar{i}_p + (BL)\,\dot{x}_p \qquad 1.0a$$

where $(BL)\ x_p$ is the velocity induced back e.m.f. The magnetic force is related to coil current by the equation

$$F_p = (BL) i_p \qquad 1.0b$$

This force drives the pilot spool by the relationship

$$F_p = M_p\,\ddot{x}_p + R_p\,\dot{x}_p + \frac{1}{C_p} x_p \qquad 1.0c$$

Neglecting initial conditions the Laplace transforms of these equations are given by

$$E_p = i_p\ (r_e + L_e s) + (BL)\ s\ x_p$$

$$F_p = (BL)\ i_p$$

$$F_p = x_p \left(M_p s^2 + R_p S + \frac{1}{C_p} \right) \qquad 1.0d$$

Whence the pilot spool velocity is derived to be related to the input voltage by the equation:

$$\frac{x_p}{E_p} = \frac{(BL)\ C_p S}{(M_p C_p S^2 + R_p C_p S + 1)\ \ (r_e + L_e S) + (BL)^2\ C_p S} \qquad 1.0e$$

HYDRAULIC COUPLING OF PILOT VALVE TO SLAVE VALVE

Slave valve motion is related to pilot valve flow output by the fuid continuity equation. The flow rate output from the pilot valve is given by the equation:

$$Q_p = K_p X_p - K_c P_L \qquad 2.0a$$

For first order effects the back pressure coefficient term, KcP_L, is neglected and the pilot valve flow rate output is written in transform notation as:

$$\frac{Q_p}{A_s} = \frac{X_p}{S} \times \frac{K_p}{A_s} \qquad 2.0b$$

SLAVE VALVE TRANSFER FUNCTION

The flow rate into the slave valve cause the slave valve spool to move according to the relationship:

$$Q_p = A_s \dot{X}_s + \frac{P_s}{R_{LS}} + \frac{C_s}{2} sP_s \qquad 2.0c$$

The developed pressure in the slave valve accelerates the spool as determined by:

$$P_s\ A_s = M_s \ddot{X}_s + R_s \dot{X}_s \qquad 2.0d$$

The three equations above are combined to yield the slave output as a function of pilot valve motion as given by:

$$\frac{X_s}{X_p} = \frac{K_p/A_s}{s\left[1 + \frac{R_s}{(A_s)^2 R_{LS}} + \frac{M_s C_s}{2(A_s^2)} s^2 + \frac{R_s C_s}{2A_s^2} + \frac{Ms}{R_{LS}\ As^2}\ s\right]} \qquad 2.0e$$

HYDRAULIC COUPLING OF SLAVE VALVE INTO EXCITER PISTON

Neglecting pressure feedback the flow output of the slave valve is related to slave valve spool position by:

$$\frac{Q_s}{A_c} = \frac{X_s}{S} \times \frac{K_s}{A_c} \qquad 3.0a$$

The slave valve output flow rate induces motion of the exciter piston coupled with leakage and fluid compressibility effects as given by the equation:

$$Q_s = A_e \dot{X}_e + \frac{P_e}{R_{Le}} + \frac{C_e}{2} P_e \qquad 3.0b$$

The pressure build-up in the exciter piston produces a hydraulic force, F_{ex}, that actuates a complex mechanical load impedance by the relationship:

$$F_{ex} = A_e P_e = Z_L \dot{X}_e \qquad 3.0c$$

Combining these equations yield the relationship of exciter position force, F_{ex}, to slave and exciter spool positions derivatives as follows:

$$K_s X_s - A_e S X_e - \left(\frac{C_e}{2A_e} s + \frac{1}{R_{Le} A_e}\right) F_{ex} = 0 \qquad 3.0d$$

EXCITER PISTON MECHANICAL LOAD IMPEDANCE

A representative diagram of a vehicle and truck suspended on the exciter piston is shown in Fig. 4. The dynamic equations for load motions are given by:

$$F_{ex} = M_L \ddot{X}_e + R_L \dot{X}_e + K_L X_e + R_D(\dot{X}_e - \dot{X}_d) + K_D(X_e - X_D)$$

$$0 = M_D \ddot{X}_D + R_D(\dot{X}_D - \dot{X}_e) + K_D(X_D - X_e) \qquad 4.0a$$

In transform form these equations are derived as:

$$F_{ex} = (M_L s^2 + R_L s + K_L + R_D s + K_D)Xe - (R_D s + K_D)X_D$$

$$0 = (M_D s^2 + R_D s + K_D)X_D - (R_D s + K_D)Xe \qquad 4.0b$$

The load impedance function can be derived as

$$Z_1 = \frac{F_{ex}}{SXe} = \frac{\left[M_L s^2 + (R_L + R_D)s + K_L + K_D\right]\left[(M_D s^2 + R_D s + K_D) - (R_D s + K_D)^2\right]}{s(M_D s^2 + R_D s + K_D)}$$

A block diagram of the electro-hydraulic servo actuator system is shown in Fig. 5. The electrical feedback signals from the pilot, slave and exciter valves are combined as summations signals into an operational amplifier along with the drive signal as shown in Fig. 6. The summation of voltages thru the shaping networks is given by the relationship:

$$0 = E_1 + \frac{E_p}{2} \times \frac{R_G}{R_G + R_{15}} \times \frac{R_{17}}{R_9}(T_5 s+1) + F_{ex} \times F_f \times G_f \times \frac{R_{17}}{R_5}$$

$$+ XeFeGe\frac{R_{17}}{R_1} \frac{(T_1 S+1)}{(T_{13}S+1)} + X_s F_s G_s \frac{R_{17}}{R_3} + \dot{X}_p F_p G_p \frac{R_{17}}{R_4}$$

$$\times \frac{T_{15}S}{(T_{15}S+1)} \qquad 5.0a$$

In order to investigate the stability and dynamic response of the selected representative electro-hydraulic system, the above set of equations were used with parameters from Wylie's report as inputs to the MDELTA computer program at DOT. This computer program can be used to print out almost any servo loop analysis feature desired. The input data cards are submitted with matrix coefficients derived from the system equations in polynomial transform form or in transform state variable notation. Both input data methods were used in this analysis and checked against one another by comparing the characteristic roots of each matrix form for identity.

In the analysis computer runs the polynomial form was used for frequency response and step function transient response. It was necessary to revert to a state variable notation in order to provide a finite pulse input response test. The input equations for these methods are described as follows:

MDELTA - W/R EQUATIONS IN POLYNOMIAL FORM

An accumulated set of performance equations in polynominal form that adequately specify the electro-hydraulic servo system are shown in Table 1. These equations are six in number with a seventh equation added to provide dummy loops for the MDELTA program manipulation to work properly. Table 2 shows the "A" matrix parameters of the polynominal equations for insertion into the input data card deck. In the polynominal form, a closed loop frequency response of the servo systems was run for the exciter position output as a function of input drive voltage into the pilot valve. The results of this frequency response are shown plotted as superimposed points on the Wylie derived frequency plot shown in Fig. 7. Notice that in this run the force feedback gain is set to zero and the exciter piston displacement gain, G_e, is set equal to 0.9. Also shown plotted in Fig. 8 is the displacement of the truck load, X_D, as a function of frequency. Both of these frequency plots that coincide with Wylie data appear to be stable systems which is misleading information. The MDELTA computer program prints out the characteristic roots of the matrix and these roots, as shown in Table 3 for the frequency response, shows the occurence of poles in the right hand plane and consequent instability.

Reduction of exciter piston displacement feedback gain from 0.9 to 0.03 is necessary to produce zero system stability. Of course, with this reduction of gain, the frequency response bandwidth and transient response time of the servo loops deteriorate. A step function transient response case was run for the stable system with $G_e = 0.03$. The transient time response values are shown printed out in Table 4 and the graphical superimposition of points on the Wylie curve is shown in Fig. 9. The fourteen root printout for the stable transient response case with $G_e = 0.03$ is shown in Table 5. From the step function response curve the system characteristic is roughly shown to be 0.2 seconds implying that stability margin has reduced the bandwidth to approximately 5 Hz instead of 30 Hz. To maintain stability and bandwidth, Wylie tripled the number of actuators driving the same load for the vertical excitation case.

In order to exercise a computer solution for a finite pulse input the MDELTA program requires state variable notation form. From the root printout in the previous polynominal runs it is known that at least 14 state variables are needed to specify performance. Two sets of state variables have been derived from two methods using the analogue computer block diagram approach and the method of breaking down the system equations into a set of linear first order differential equations. These methods are briefly described as follows:

State VARIABLES USING COMPUTER DIAGRAM

In this method an analogue computer diagram is configured using single integrators, gain coefficients and summers. The initial step is to rewrite the set of equations in differential form equating the highest derivative of each state variable parameter in terms of other derivative functions. The set of equations as written in this form are shown in Table 6. From this formulation of equations the analoge block diagrams are shown in Fig. 10. The output of each integrator in the block diagram is specified to be a state variable. A list of state variables from the block diagram is shown in Table 7.

State VARIABLES USING DIFFERENTIAL EQUATION TECHNIQUE

A set of fourteen state variables can be derived by breaking down the system equations into first order linear differential relationships as follows. This development shows the the selection of state variables and the method of decomposition of the original equation. For the equation that contains all the feedback voltage in summation form it was necessary to revert to the original electrical network as shown in Fig. 6, in order to derive the completed set of state variables. A matrix showing both sets of state variables is shown in Fig. 11. Notice that the first 10 state variables from each derivation are identical. Using the set of state variables derived from the differential equations as inputs to the MDELTA program, a pulse response forcing function case was run. The results of this test run are printed out in Table 8, and are plotted on the Wylie pulse response run as shown in Fig. 11. These results were obtained in the computer run using state transition technique. By requesting a frequency response from the state variable input run it was possible to have the computer print out a set of fourteen roots as shown in Table 9. These roots derived from the state variable run were compared to the roots derived from the polynominal run and found to be substantially identical.

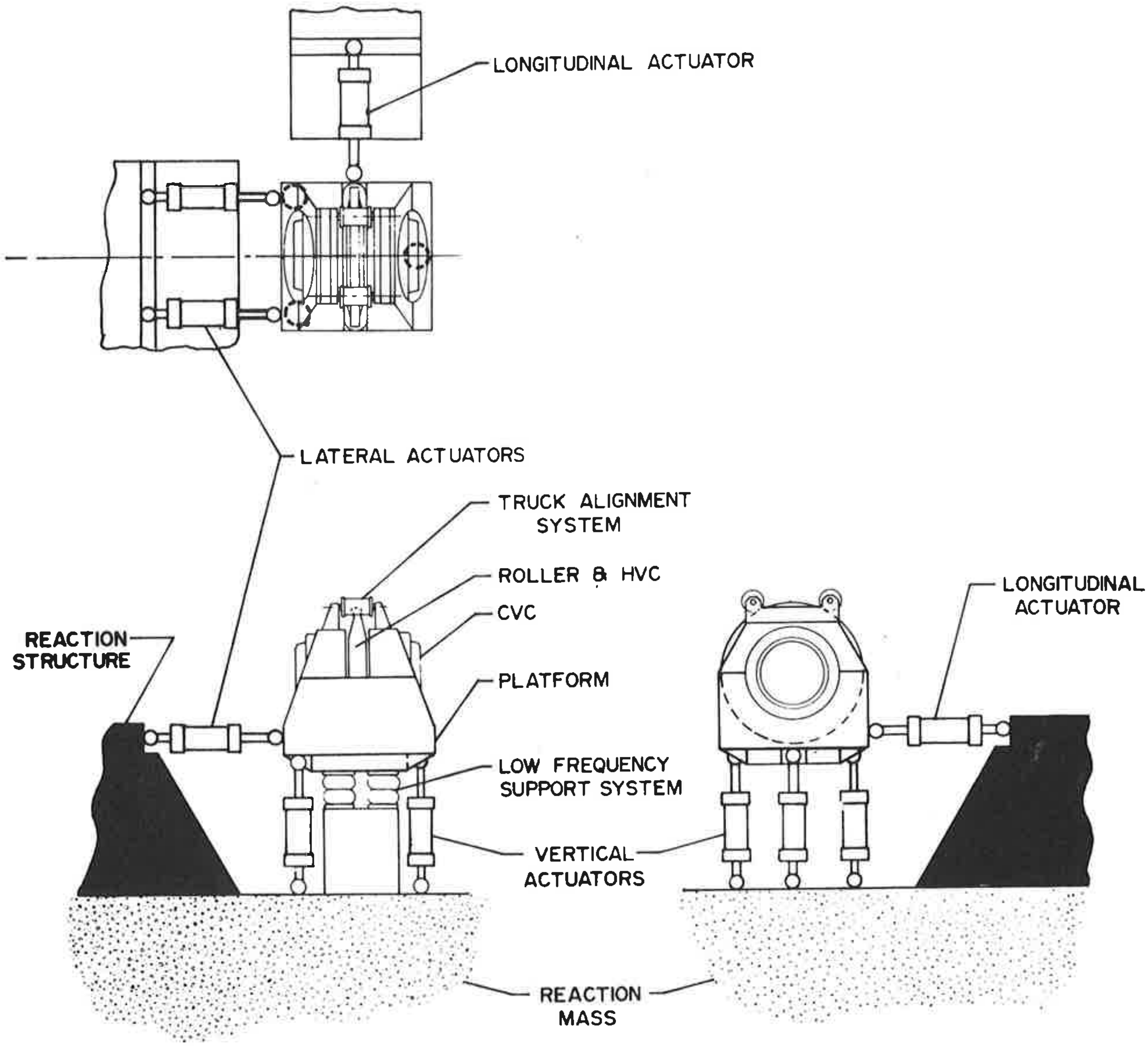

Figure 1. Single Wheel Module Configuration
(Wylie Figure 1, Ref. 1)

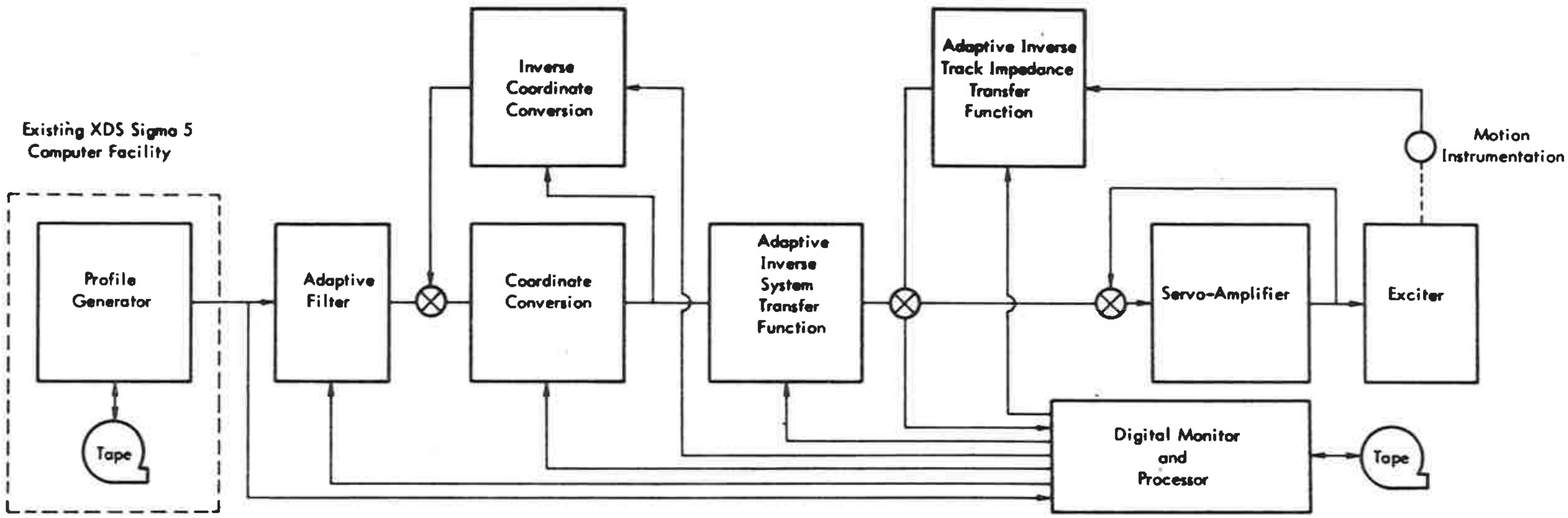

Figure 2. Schematic Block Diagram of Hybrid Control System Concept (Wylie Figure 7-Ref. 1)

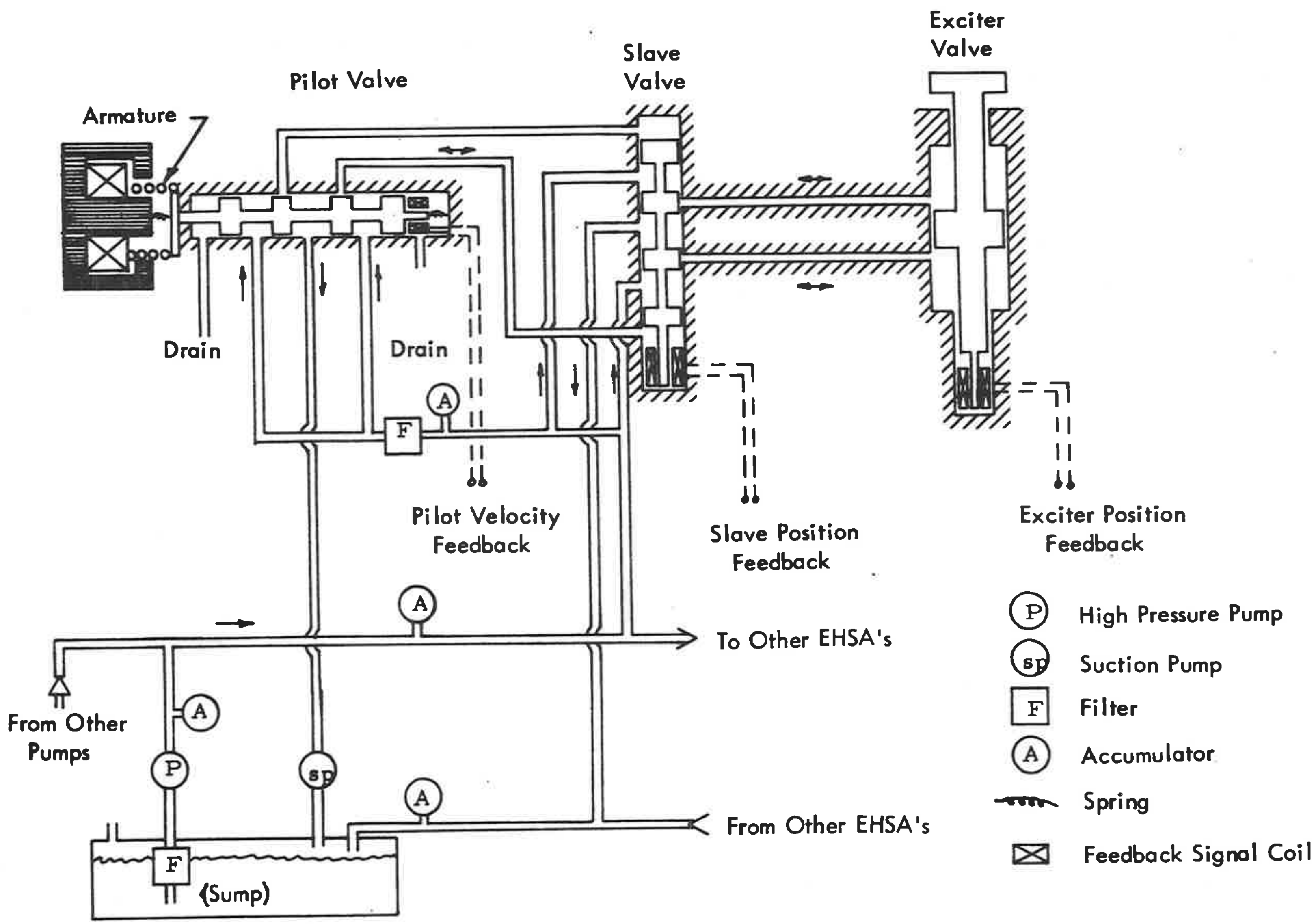

Figure 3. Schematic of Electro-Hydraulic Servo Actuator (EHSA) System (Wylie Figure 1, Appendix D, Ref. 1)

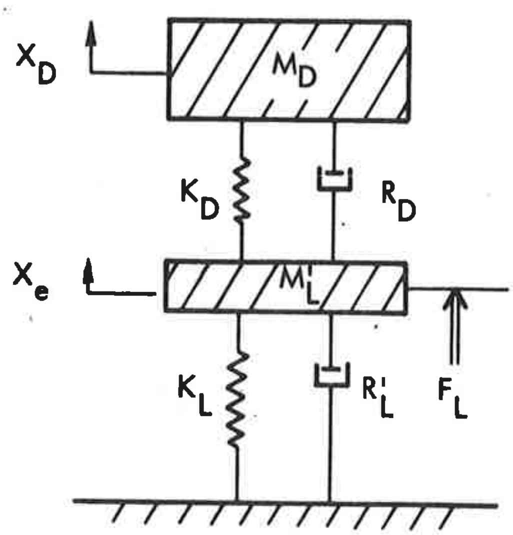

Figure 4. Schematic of Vibration System Complex Load
(Wylie Figure 6, Appendix D, Ref. 1)

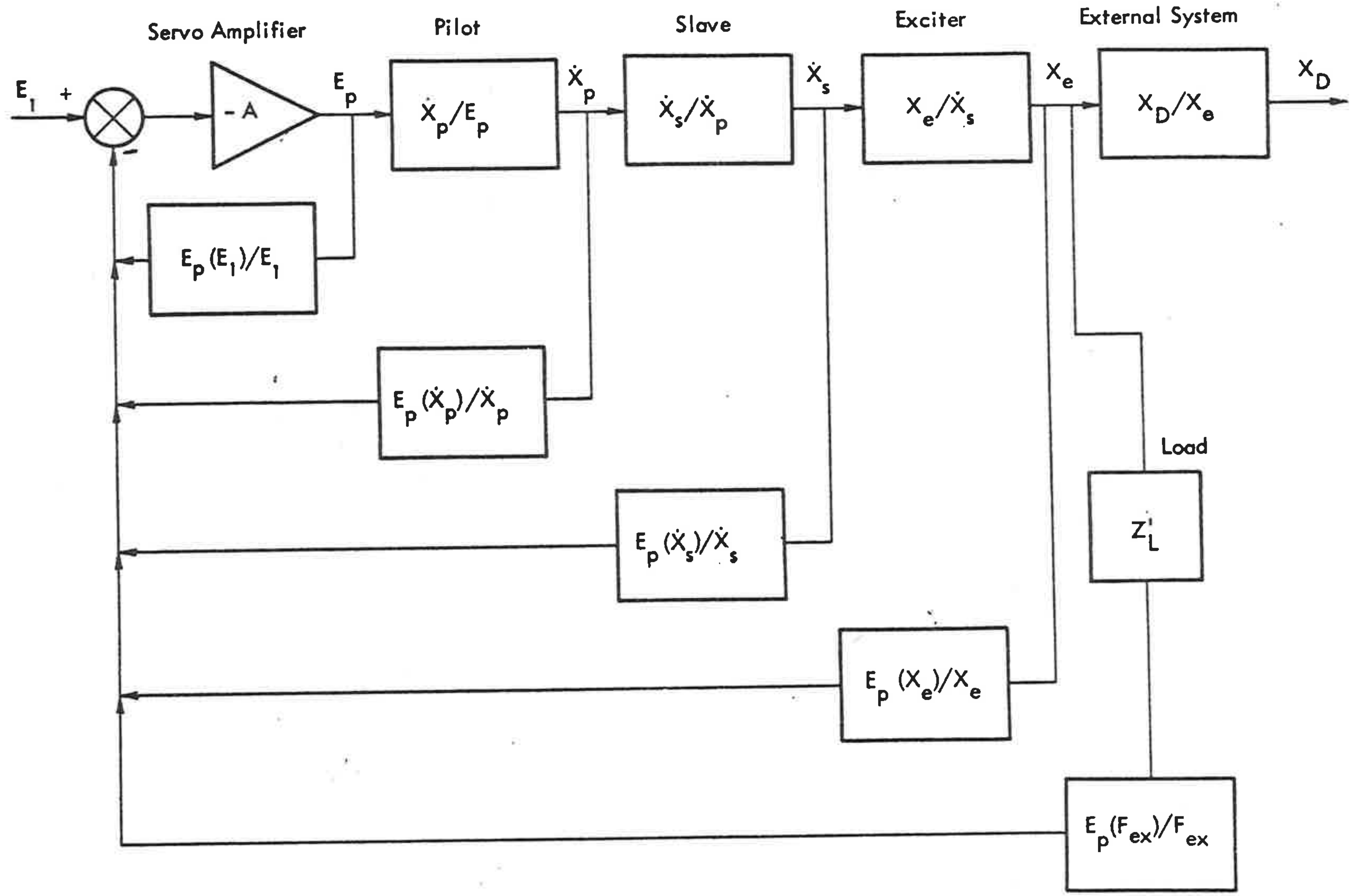

Figure 5. Block Diagram for the Electro-Hydraulic Servo Actuator System (Wylie Figure 9, Appendix D, Ref. 1)

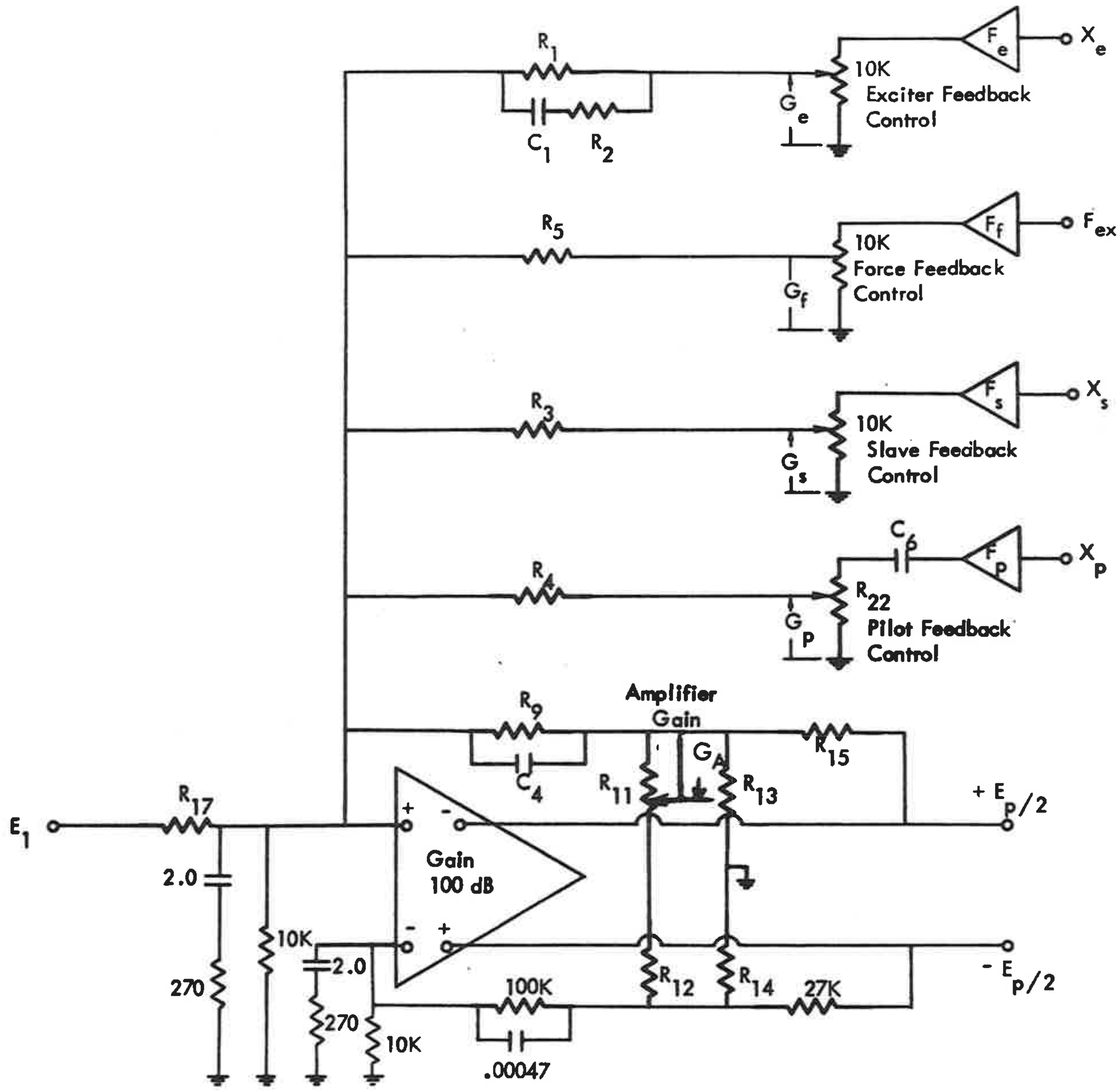

Figure 6. Revised Diagram of Servo Amplifier and EHSA Feedback Circuit (Wylie Figure 8, Appendix D, Ref. 1)

TABLE 1

MDELTA-W/R EQS. FOR ELECTROHYDRAULIC SERVOSYSTEMS IN POLYNOMIAL FORM

1.0 $\left(M_D s^2+R_D s+K_D\right) X_D - \left(R_D s+K_D\right) X_e = 0$

2.0 $F_{ex} - \left(M_L s^2+\left(R_L+R_D\right)s + \left(K_L+K_D\right)\ \right)X_e + \left(R_D s+K_D\right) X_D = 0$

3.0 $K_s \dot{X}_s - A_e s^2 X_e - \left(\frac{C_e}{2A_e} s^2 + \frac{1}{R_{Le} A_e} x\ s \right) F_{ex} = 0$

$K_s X_s - A_e s X_e - \left(\frac{C_e}{2A_e} s + \frac{1}{R_{Le} A_e} \right) F_{ex} = 0$

4.0 $\left[\left(\frac{R_s C_s}{2A_s{}^2} + \frac{M_s}{R_{Ls} A_s{}^2} \right) s^2 + \frac{M_s C_s}{2A_s{}^2} s + \left(1+\frac{R_s}{A_s{}^2 R_{Ls}}\right)s\right] \dot{X}_s - \frac{K_p}{A_s} \dot{X}_p = 0$

5.0 $(BL) \frac{C_p}{R_e} \cdot s \cdot E_p - \left[M_p C_p s^2 + \left(R_p C_p + \frac{(BL)^2 C_p}{8.85 R_e}\right) s + 1\right] \dot{X}_p = 0$

6.0 $-\left[T_{13}T_{15}s^3+\left(T_{13}+T_{15}\right)s^2+s\right] E_1 - \frac{E_p}{2} \frac{R_G}{R_G+R_{15}} \times \frac{R_{17}}{R_9} \left[T_5 T_{13} T_{15} s^4\right.$

$\left. + \left(T_{13}T_{15}+T_5T_{15}+T_5T_{13}\right)s^3 + \left(T_5+T_{13}+T_{15}\right) s^2+s\right]$

$- F_{ex} \times F_f G_f \times \frac{R_{17}}{R_5} \left[T_{13}T_{15}s^3+\left(T_{13}+T_{15}\right)s^2+s\right]$

$- 1x_e F_e G_e \times \frac{R_{17}}{R_1} \times \left[T_1 T_{15} s^3+\left(T_1+T_{15}\right) s^2+s\right]$

$- \dot{x}_s \times F_s G_s \times \frac{R_{17}}{R_3} \left[T_{13}T_{15}s^2+\left(T_{13}+T_{15}\right)s+1\right]$

$- \dot{x}_p\ F_p G_p \frac{R_{17}}{R_4} T_{15} \left(T_{13}s^3+s^2\right) = 0$

$$7.0 \quad + E_1 + (KDUM)x_C + (KDUM2)x_D + (KDUM3)\dot{x}_s + (KDUM4)\dot{x}_p = F_1$$

$$6.0 \quad 0 = E_1 + \frac{E_p}{2} \times \frac{R_G}{R_G+R_{15}} \times \frac{R_{17}}{R_9} \left(T_5 s+1\right) + F_{ex}F_f G_f \frac{R_{17}}{R_5}$$

$$+ x_e \times F_e G_e \frac{R_{17}}{R_1} \frac{\left(T_1 s+1\right)}{\left(T_{13} s+1\right)} + x_s F_s G_s \frac{R_{17}}{R_3}$$

$$+ \dot{x}_p \, F_p G_p \frac{R_{17}}{R_4} \frac{T_{15} s}{\left(T_{15} s+1\right)}$$

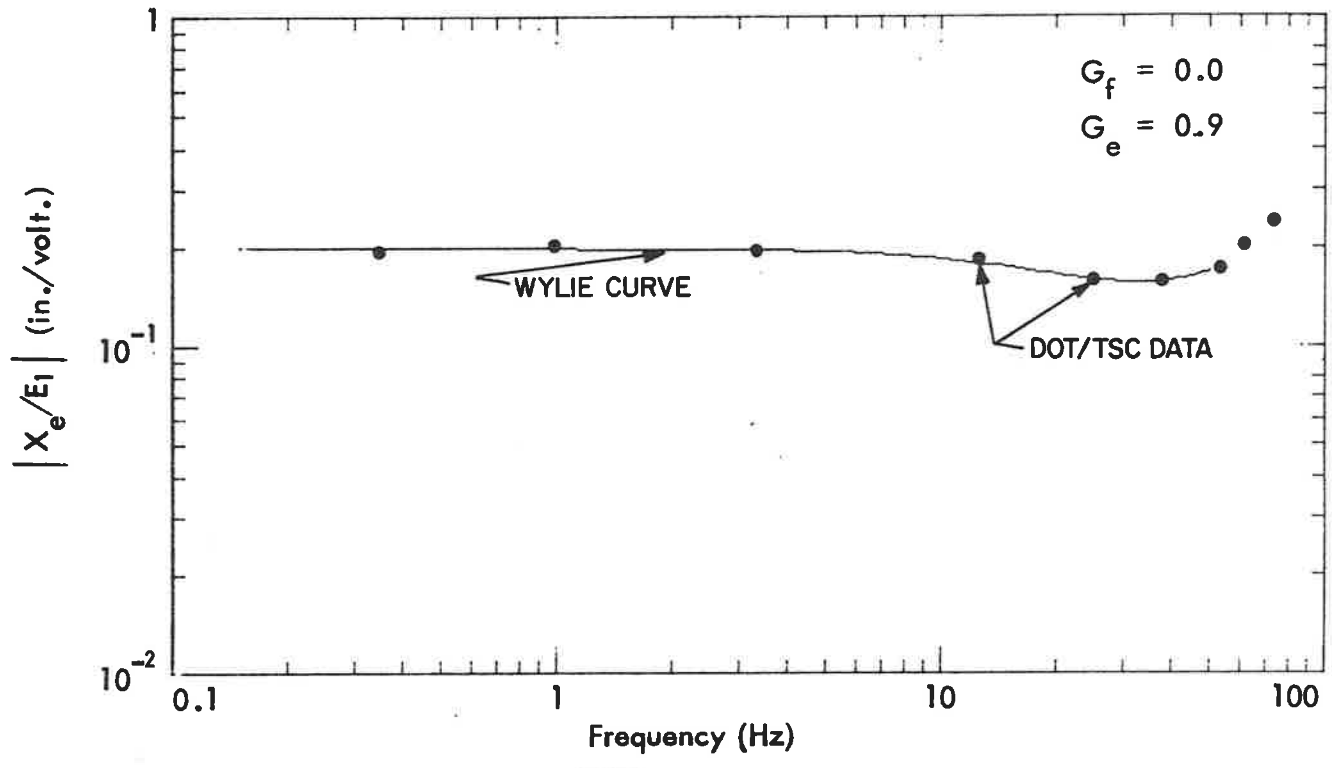

Figure 7. Amplitude of the Overall Transfer Function X_e/E_1 with Exciter Displacement Feedback (Wylie Figure 14, Appendix D, Ref. 1)

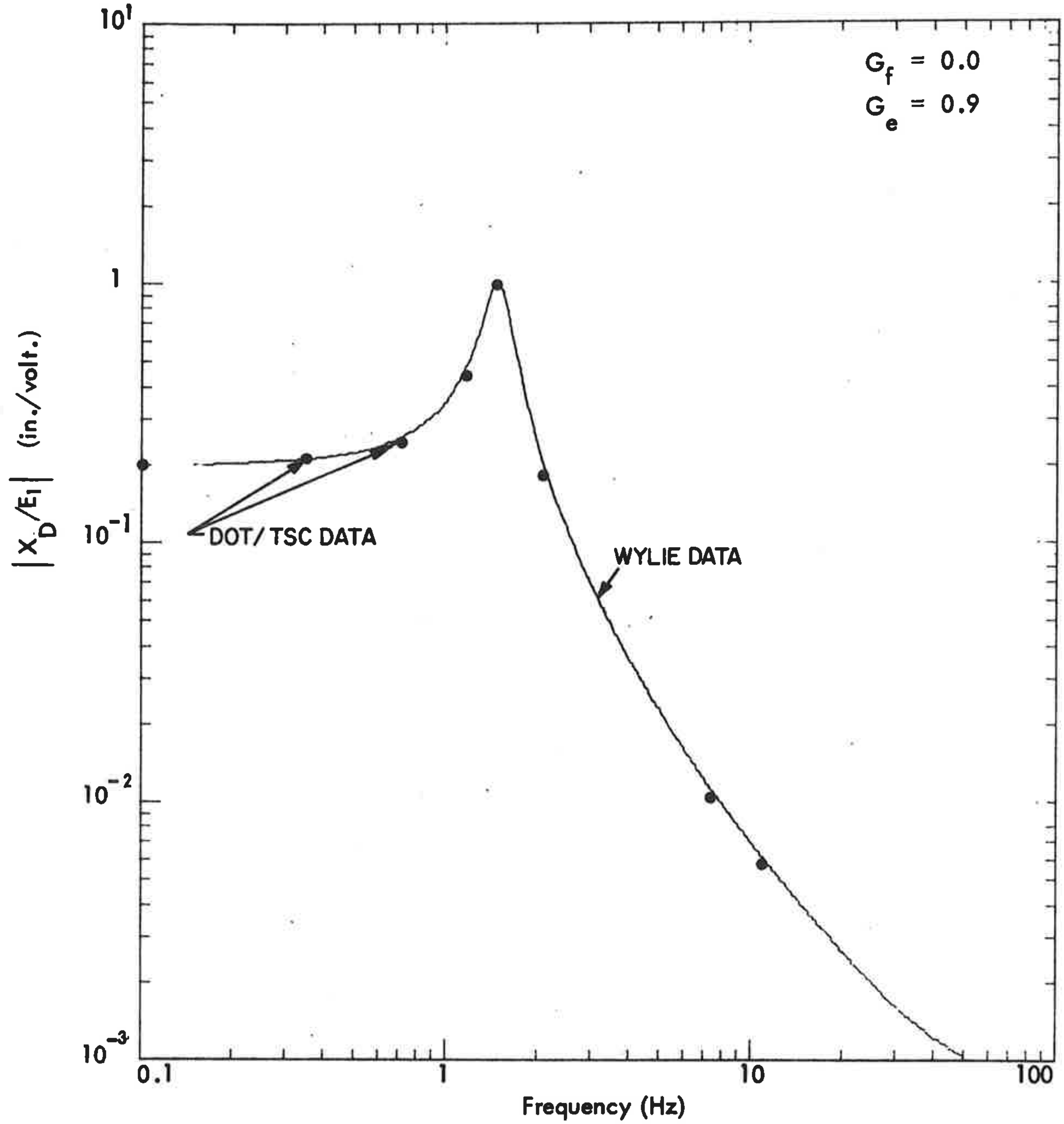

Figure 8. Amplitude of the Overall Transfer Function X_D/E_1 with the Exciter Displacement Feedback (Wylie Figure 15, Appendix D, Ref. 1)

TABLE 2. "A" MATRIX FOR POLYNOMIAL INPUT OF W/R EQS.

2#/PARAMETER	X_D	X_e	F_{es}	$\dot{X}_s$	$\dot{X}_p$	F_p	E_1
1.0	$M_D s^2 + R_D s + K_D$	$-\left(R_D s + K_D\right)$					
2.0	$R_D s + K_D$	$-\left[M_L s^2 + \left(R_L + R_D\right)s + K_L + K_D\right]$	1				
3.0		$-A_e s^2$	$-\left(\frac{C_e}{2A_e} s^2 + \frac{s}{R_{Le} A_e}\right)$	K_s			
4.0				$+\left[\frac{M_s C_s}{2A_s^2} s^3 + \left(\frac{R_s C_s}{2A_s^2} + \frac{M_s}{R_{Ls} A_s^2}\right) s^2 + 1 + \left(\frac{R_s}{A_s^2 R_{Ls}}\right)\right] s$	$\frac{-K_p}{A_s}$		
5.0					$-\left[M_p C_p s^2 + \left(R_p C_p + \frac{(BL)^2 C_p}{8.85 R_e}\right) s + 1\right]$	$+\frac{(BL) C_p}{R_e}$	
6.0		$-F_e G_e \frac{R_{17}}{R_1}\left[T_1 T_{15} s^3 + \left(T_1 + T_{15}\right) s^2 + s\right]$	$-F_f G_f \frac{R_{17}}{R_5}\left[T_{13} T_{15} s^3 + \left(T_{13} T_{15}\right) s^2 + s\right]$	$-F_s G_s \frac{R_{17}}{R_3}\left[T_{13} T_{15} s^2 + \left(T_{13} + T_{15}\right) s + 1\right]$	$-F_p G_p \frac{R_{17}}{R_4}\left(T_{15}\ T_{13} s^3 + s^2\right)$	$-\frac{1}{2} \frac{R_G}{R_G + R_{15}} \times \frac{R_{17}}{R_9}\left[T_5 T_{13} T_{15} s^4 + \left(T_{13} T_{15} + T_5 T_{15} + T_{13} T_{15}\right) s^3 + \left(T_5 + T_{13} + T_{15}\right) s^2 + s\right]$	$-\left[T_{13} T_{15} s^3 + \left(T_{13} + T_{15}\right) s^2 + s\right]$
7.0	KDUM2	KDUM		KDUM3	KDUM4		

TABLE 3
FREQUENCY RESPONSE
WYLIE FIG. 14, Ge = 0.9
SYSTEMS IS UNSTABLE

DENOMINATOR ROOTS

EIGENVALUE EVALUATION BY TARNOVES METHOD

DEGREE OF POLYNOMIAL ELEMENTS= 4
REAL MATRIX ORDER= 7

ROOT NUMBER	LAMBDA REAL	LAMBDA IMAGINARY
1	-2.167653E-07	-2.328306E-07
2	-9.466508E-01	-9.394233E 00
3	-9.466508E-01	9.394233E 00
4	-1.160816E 02	-8.680502E-09
5	-3.399001E 02	3.141917E 02
6	-3.399001E 02	-3.141917E 02
7	5.773792E 01	5.368186E 02
8	5.773792E 01	-5.368186E 02
9	-9.285073E 02	-1.724691E-03
10	-7.932402E 02	4.564016E-05
11	-1.116989E 04	2.893741E 03
12	-1.116989E 04	-2.893741E 03
13	-1.607565E 04	2.106845E 03
14	-1.607565E 04	-2.106845E 03

TABLE 4
STEP FUNCTION INPUT
TIME RESPONSE FOR XE

	ROOT REAL	ROOT IMAGINARY	RESIDUE REAL	RESIDUE IMAGINARY	EXP
1.)	9.999999E-04	0.0	2.975834E 00	-1.655243E-05	0
2.)	-9.999999E-04	0.0	2.976671E 00	-1.656268E-05	0
3.)	-7.349698E 00	0.0	-6.100557E 00	3.516357E-05	0
4.)	-9.456432E-01	-9.393666E 00	-4.872193E-04	-1.787531E-06	0
5.)	-9.456432E-01	9.393666E 00	-4.872193E-04	1.793029E-06	0
6.)	-2.720078E 02	0.0	1.845930E-01	-1.029916E-06	0
7.)	-2.363294E 02	0.0	-9.586626E-02	5.376951E-07	0
8.)	-1.374303E 00	-6.249041E 02	1.872918E-02	-1.633651E-03	0
9.)	-1.374303E 00	6.249041E 02	1.872920E-02	1.633442E-03	0
10.)	-9.418142E 02	2.790020E 02	1.142217E-02	-1.135723E-02	0
11.)	-9.418142E 02	-2.790020E 02	1.142230E-02	1.135710E-02	0
12.)	-1.116992E 04	2.893815E 03	5.211498E-07	-7.519825E-07	0
13.)	-1.116992E 04	-2.893815E 03	5.211593E-07	7.519764E-07	0
14.)	-1.607567E 04	2.106841E 03	1.553045E-07	-2.168716E-08	0
15.)	-1.607567E 04	-2.106841E 03	1.553047E-07	2.168522E-08	0

IME	XE	TIME	XE	TIME
	4.176678E-06	2.999998E-01	5.295280E 00	6.000000E-01
00E-02	1.762175E 00	3.499997E-01	5.497773E 00	6.999999E-01
96E-02	3.058354E 00	3.999997E-01	5.636742E 00	7.999999E-01
99E-01	3.954513E 00	4.499996E-01	5.731890E 00	8.999999E-01
99E-01	4.573626E 00	4.999996E-01	5.796903E 00	9.999998E-01
98E-01	5.000870E 00	5.000000E-01	5.796907E 00	

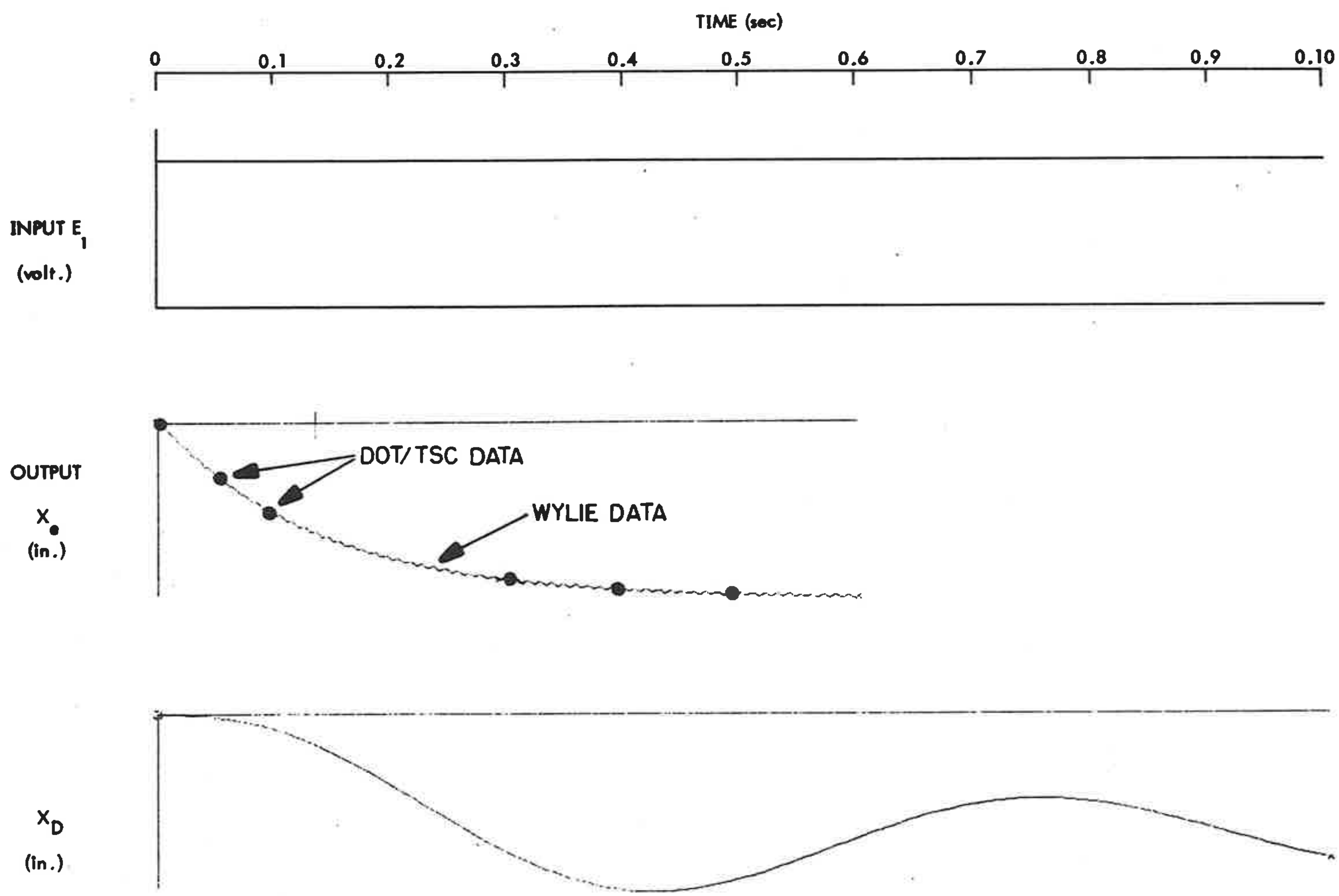

Figure 9. Transient Response to a Unit Step Input for a Typical EHSA System (Example 1) (Wylie Figure 18, Appendix D, Ref. 1)

TABLE 5
ROOTS FOR STEP FUNCTION RESPONSE POLYNOMIAL INPUT METHOD
WYLIE FIG. 18 (Ex #1) Ge = 0.03 SYSTEM IS STABLE

DENOMINATOR EIGENVALUES

EIGENVALUE EVALUATION BY TARNOVES METHOD

DEGREE OF POLYNOMIAL ELEMENTS= 4
REAL MATRIX ORDER= 6

ROOT NUMBER	LAMBDA REAL	LAMBDA IMAGINARY
1	-5.124602E-07	1.343869E-07
2	-7.349698E 00	-1.864391E-07
3	-9.456432E-01	-9.393666E 00
4	-9.456432E-01	9.393666E 00
5	-2.720078E 02	1.004357E-03
6	-2.363294E 02	-1.560917E-05
7	-1.374303E 00	-6.249041E 02
8	-1.374303E 00	6.249041E 02
9	-9.418142E 02	2.790020E 02
10	-9.418142E 02	-2.790020E 02
11	-1.116992E 04	2.893815E 03
12	-1.116992E 04	-2.893815E 03
13	-1.607567E 04	2.106841E 03
14	-1.607567E 04	-2.106841E 03

TABLE 6

1) $$X_d = -\frac{R_D}{M_D}\dot{X}_D - \frac{K_D}{M_D}X_D + \frac{R_D}{M_D}\dot{X}_e + \frac{K_D}{M_D}X_e$$

2) $$X_e = -\frac{R_L+R_D}{M_L}\dot{X}_e - \frac{K_L+K_D}{M_L}X_e + \frac{R_D}{M_L}\dot{X}_D + \frac{K_D}{M_L}X_D + F_{ex}$$

3) $$K_sX_k - A_esX_e - \left(\frac{C_e}{2A_e}s + \frac{1}{R_{Le}A_e}\right)F_{ex} = 0$$

$$\dot{F}_{ex} = -\frac{2}{R_{Le}C^e}F_{ex} - \frac{2A_e^2}{C_e}\dot{X}_e + \frac{2A_e}{C_e}K_s$$

4) $$\frac{R_sC_s}{2A_s{}^2}\dddot{X}_s + \frac{M_sC_s}{2A_s{}^2}\ddddot{X}_s + \ddot{X}_s - \frac{K_p}{A_s}\dot{X}_p = 0$$

$$\ddddot{X}_s = -\frac{R_s}{M_s}\dddot{X}_s - \frac{2A_s{}^2}{M_sC_s}\ddot{X}_s + \frac{2A_s}{M_sC_s}K_p\dot{X}_p$$

5) $$(BL)\frac{C_p}{R_e}\dot{E}_p - M_pC_p\dddot{X}_p - R_pC_p + \frac{(BL)^2C_p}{8.85R_e}\ddot{X}_p - \dot{X}_p = 0$$

$$\dddot{X}_p = -\frac{1}{M_p}\left(R_p + \frac{(BL)^2}{8.85R_e}\right)\ddot{X}_p - \frac{1}{M_pC_p}\dot{X}_p + \frac{(BL)}{M_pR_e}\dot{E}_p$$

TABLE 6 (Cont)

$$6)\quad -E_1 = \frac{E_p}{2} \times \frac{R_G}{R_G+R_{15}} \; \frac{R_{17}}{R_9} \left(T_5 s+1\right) \dot{E}_p = -2 \times \overbrace{\frac{R_G+R_{15}}{R_G} \; \frac{R_p}{R_{17}}}^{C'} \times \frac{1}{T_5} \times E_1$$

$$- G_p/T_5$$

$$+ F_{ex} \, F_f G_f \, \frac{R_{17}}{R_5} \qquad - C' F_f G_f \, \frac{R_{17}}{R_5} \, F_{ex}$$

$$+ X_e \, F_e G_e \, \frac{R_{17}}{R_1} \, \frac{(T_1 s+1)}{(T_{13} s+1)} \qquad - C' F_e G_e \, \frac{R_{17}}{R_1} \, \frac{(T_1 s+1)}{(T_{13} s+1)} \, X_e$$

$$+ X_s \, F_s G_s \, \frac{R_{17}}{R_3} \qquad - C' F_s G_s \, \frac{R_{17}}{R_3} \, X_s$$

$$+ \dot{X}_p \, F_p G_p \, \frac{R_{17}}{R_4} \, \frac{T_{15} s}{(T_{15} s+1)} \qquad - C' F_p G_p \, \frac{R_{17}}{R_4} \, \frac{T_{15} s}{T_{15} s+1} \, \dot{X}_p$$

$$M_D\ddot{X}_D + R_D\dot{X}_D + K_DX_D - R_D\dot{X}_e - K_DX_e = 0$$

$$\ddot{X}_D = -\frac{R_D}{M_D}\dot{X}_D - \frac{K_D}{M_D}X_D + \frac{R_D}{M_D}\dot{X}_e + \frac{K_D}{M_D}X_e$$

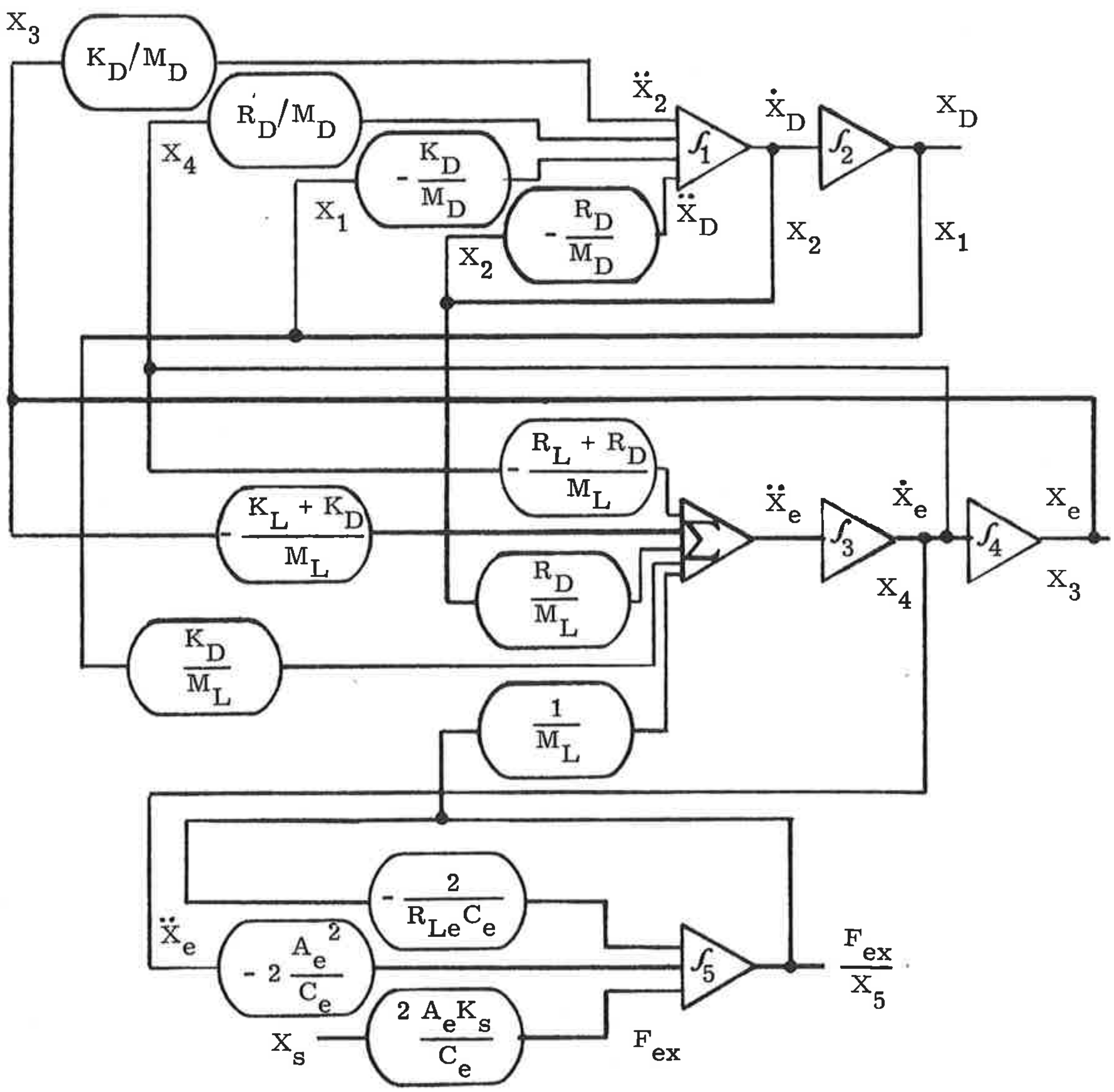

Figure 10. State Variables by Block Diagrams

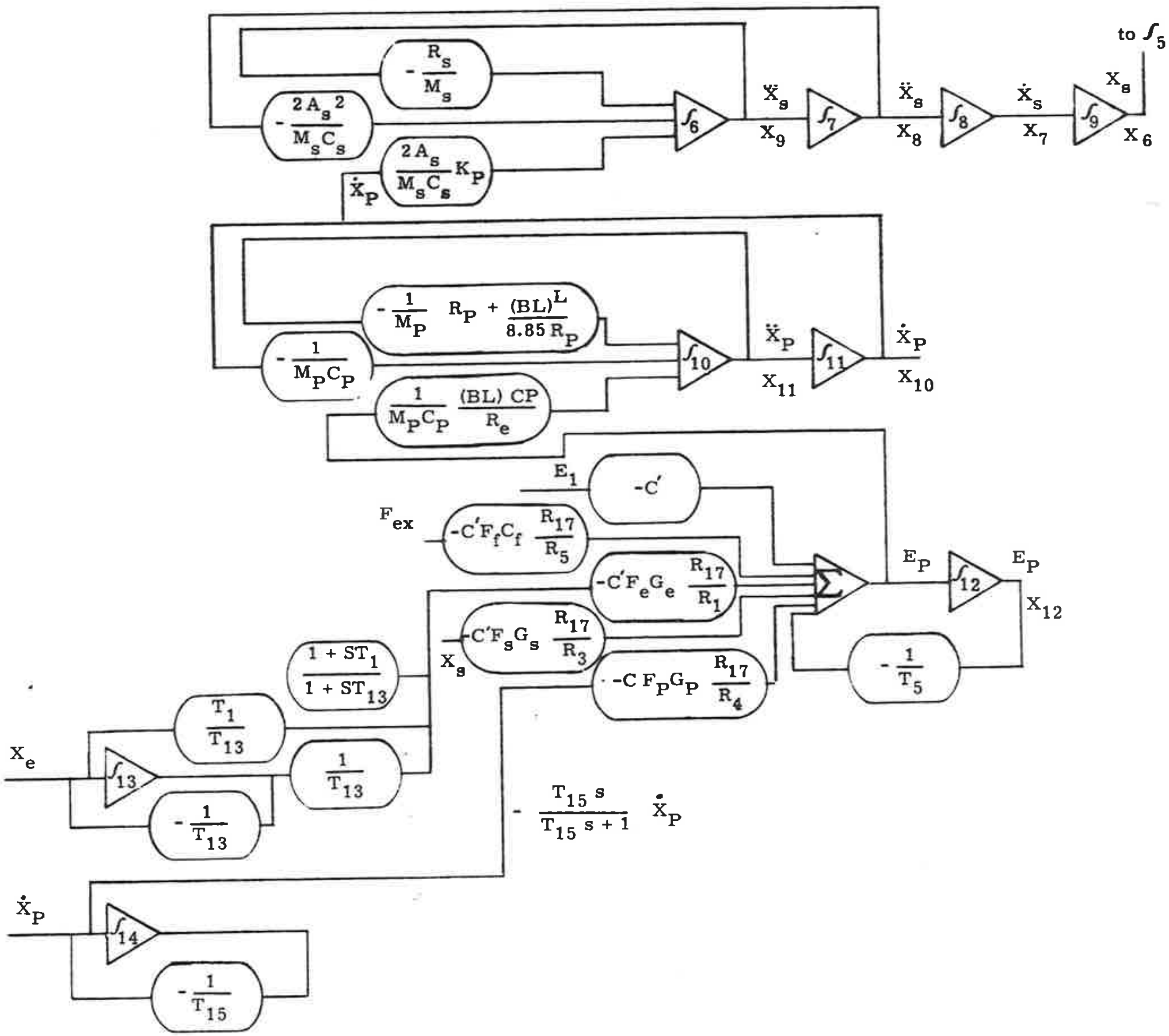

Figure 10. State Variables by Block Diagram.

TABLE 7. ANALOGUE DIAGRAM DEVELOPED LIST OF STATE VARIABLES RELATIONSHIPS

State Variable Rate / State Variable		(X_D) X_1	$(\dot{X}_D)$ X_2	(X_e) X_3	$(\dot{X}_e)$ X_4	(F_{ex}) X_5	(X_A) X_6	$(\dot{X}_A)$ X_7	$(\ddot{X}_A)$ X_8	(X_B) X_9	$(\dot{X}_P)$ X_{10}	$(\ddot{X}_P)$ X_{11}	K_P X_{12}	X_{13}	X_{14}	[illegible]
From Dif. Eq's	$\dot{X}_1$		1													
	$\dot{X}_2$	$-K_D/M_D$	$-R_D/M_D$	$+K_D/M_D$	$+R_D/M_D$											
	$\dot{X}_3$				1											
	$\dot{X}_4$	K_D/M_L	R_D/M_L	$-(K_L+K_D)/M_L$	$-(R_L+R_D)/M_L$	$1/M_L$										
	$\dot{X}_5$				$-2A_e^2/C_e$	$-2/C_e \cdot R_{Le}$	$+2A_e/C_e \times K_a$									
	$\dot{X}_6$							1								
	$\dot{X}_7$								1							
	$\dot{X}_8$									1						
	$\dot{X}_9$								$-2A_a^2/M_aC_a$	$-R_a/M_a$	$+\frac{2A_a}{M_aC_a} \times K_P$					
	$\dot{X}_{10}$											1				
	$\dot{X}_{11}$			$-\frac{C'}{R_{17}} \times K_{DUM}$		$C \times \frac{G_fF_f}{R_5}$	$C \times \frac{G_aF_a}{R_3}$				$-\frac{1}{M_PC_P}$	$-\frac{1}{M_PC_P} \times \left(R_PC_P + \frac{(BL)^2 C_P}{8.85\, R_e}\right)$	$C \times \frac{R_G}{R_G + R_{15}} \times \frac{1}{2R_9}$	C	C	$\frac{C}{R_{17}}$
	$\dot{X}_{12}$			$\frac{C}{R} \times K_{DUM}$	$\frac{T_1}{T_{13}}$	$-\frac{C' \times R_{17}}{R_5} G_fF_f$	$-\frac{C'R_{17}}{R_3} G_aF_a$						$-\frac{1}{T_5}$	$-C' \times R_{17}$	$-C' \times R_{17}$	$-C'$
	$\dot{X}_{13}$			$\frac{1}{R_1R_2C_1} \times G_eF_e$	$\frac{R_1 - R_3}{R_1R_3} G_eF_e$									$+\frac{1}{R_{20}}$		
	$\dot{X}_{14}$											$\frac{G_PF_P}{R_4}$			$-\frac{1}{R_{23}C_8}$	
From Block Diagram	$\dot{X}_{11}$			$-\frac{C}{R_1} F_eG_e \frac{T_1}{T_{13}}$ $-\frac{C}{R_{17}} \times K_{DUM}$		$\frac{C}{R_5} G_fF_f$	$\frac{C}{R_3} F_aG_a$				$-\frac{1}{M_PC_P} + \frac{C}{R_4} F_PG_P$	$-\frac{1}{M_PC_P} \times \left(R_PC_P - \frac{(BL)^2 C_P}{8.85\, R_e}\right)$	$-\frac{1}{M_PC_P} \frac{BL\, C_P}{R_e} \frac{1}{T_5}$	$\frac{C}{R_1} F_eG_e \frac{1}{T_{13}} \left(1 - \frac{T_1}{T_{13}}\right)$	$-\frac{C'}{R_4} F_PG_P \left(\frac{1}{T_{15}}\right)$	$\frac{C}{R_{17}}$
	$\dot{X}_{12}$			$C' \times K_{DUM} -$ $C' \times F_eG_e \frac{R_{17}}{R_1} \frac{T_1}{T_{13}}$		$-C'F_fG_f \frac{R_{17}}{R_5}$	$-C' \frac{R_{17}}{R_3} F_aG_a$				$-C'F_PG_P \frac{R_{17}}{R_4}$		$-\frac{1}{T_5}$	$+C'F_eG_e \frac{R_{17}}{R_1} \times \frac{1}{T_{13}} \left(1 - \frac{T_1}{T_{13}}\right)$	$+C'F_PG_P \frac{R_{17}}{R_4} \frac{1}{T_{15}}$	$-C'$
	$\dot{X}_{13}$													$-\frac{1}{T_{13}} X'_{13}$		
	$\dot{X}_{14}$														$-\frac{1}{T_{15}} X'_{14}$	

NOTES

C' is called F1 in Parameter Values

Watch out that F1 is FP input notation

$$C' = \frac{R'_G - R_{15}}{R_G} \times \frac{2}{C_4} = \frac{1}{R_{17}} \cdot \frac{20.1}{2.1} \times \frac{2}{470 \times 10^{-6}} \times \frac{1}{.056} = 1.05 \times 10^{6}$$

$$C' \times R_{17} = 1.05 \times 10^{6} \times .056 = .059 \times 10^{6}$$

$$C = -\frac{1}{M_P} \times \frac{B_L}{R_e} \times \frac{R_G + R_{15}}{R_G} \times \frac{2}{C_4} = -\frac{1}{1.3 \times 10^{-4}} \times \frac{40}{400} \times \frac{20.1}{2.1} \times \frac{2}{470 \times 10^{-6}}$$

$$C = -27.1 \times 10^{6}$$

DEVELOPMENT OF STATE VARIABLES FROM DIFFERENTIAL EQUATIONS

1) $M_D\, X_D + R_D\, \dot{X}_D + K_D\, X_D - R_D\, \dot{X}_e - K_D\, X_e = 0$

let $X_1 = X_D \quad X_2 = \dot{X}_1,\ X_3 = X_e,\ X_4 = \dot{X}_3$

$M_D\, \dot{X}_2 + R_D\, X_2 + K_D\, X_1 - R_D\, X_4 - K_D\, X_3 = 0$

$$\underline{\dot{X}_1 = X_2}$$

$$\underline{\dot{X}_2 = \frac{1}{M_D}}\left(R_D X_2 + K_D X_1 - R_D X_4 - K_D X_3\right)$$

$$\underline{\dot{X}_3 = X_4}$$

2) $F_{ex} - \left(M_L s^2 + \left(R_L + R_D\right)s + K_L + K_D\right) X_e + \left(R_D s + K_D\right) X_D = 0$

let $X_5 = F_{ex}$

$$X_5 - M_L \dot{X}_4 - \left(R_L + R_D\right) X_4 - \left(K_L + K_D\right) X_3 + R_D X_2 + K_D X_1 = 0$$

$$\dot{X}_4 = + \frac{K_D}{M_L} X_1 + \frac{R_D}{M_L} X_2 - \left(\frac{K_L + K_D}{M_L}\right) X_3 - \left(\frac{R_L + R_D}{M_L}\right) X_4 + \frac{X_5}{M_L}$$

Eq3) $K_s \dot{X}_s - A_e s^2 X_e - \left(\frac{C_e}{2A_e} s^2 + \frac{1}{R_{Le} A_e} s\right) F_{ex} = 0$

also $K_s X_s - A_e s X_e - \left(\frac{C_e}{2A_e} s + \frac{1}{R_{Le}}\right) F_{ex} = 0$

$X_6 = X_s$

$$K_s X_6 - A_e\, X_4 - \frac{C_e}{2A_e} \dot{X}_5 - \frac{1}{R_{Le} A_e} X_5 = 0$$

$$\dot{X}_5 = \left[K_s X_6 - \frac{1}{R_{Le} A_e} X_5 - A_e X_4\right] \frac{2A_e}{C_e}$$

$$\dot{X}_5 = \frac{2A_e}{C_e}\left[-A_e\left(+\frac{K_D}{M_L} X_1 + \frac{R_D}{M_L} X_2 - \frac{(K_L+K_D)}{M_L} X_3 - \frac{(R_L+R_D)}{M_L}\right.\right.$$

$$\underbrace{\left.\left.\frac{1}{R_{Le}A_e} X_5\right) + K_s X_6\right]}_{\text{for } R_{Ls} = \infty}$$

4) $$\left(\left(\frac{R_s C_s}{2A_s^2}\right) s^2 + \frac{M_s C_s}{2A_s^2} s^3 + s\right) \dot{X}_s - \frac{K_p}{A_s} \dot{X}_p = 0$$

let $X_6 = X_s$, $X_7 = \dot{X}_6$, $X_8 = \dot{X}_7$, $X_9 = \dot{X}_8$, $X_{10} = \dot{X}_9$

$$\frac{M_s C_s}{2A_s^2} \dot{X}_9 + \frac{R_s C_s}{2A_s^2} X_9 + X_8 - \frac{K_p}{A_s} X_{10} = 0$$

$$\dot{X}_6 = X_7$$

$$\dot{X}_7 = X_8$$

$$\dot{X}_8 = X_9$$

$$\dot{X}_9 = -\frac{2A_s^2}{M_s C_s} X_8 - \frac{R_s}{M_s} X_9 + \frac{2A_2}{M_s C_s} K_p X_{10}$$

5) $$\frac{(BL)C_p}{R_e} s E_p - \left(M_p C_p s^2 + \left(R_p C_p + \frac{(BL)^2 C_p}{8.85 R_e}\right) s + 1\right) \dot{X}_p = 0$$

$X_{10} = \dot{X}_p$, $X_{11} = \dot{X}_{10}$, $\dot{X}_{11} = s^2 \dot{X}_p$, $X_{13} = E_p$, $\dot{X}_{12} = \dot{E}_p$

$$\dot{X}_{10} = X_{11}$$

$$\dot{X}_{11} = \frac{1}{M_p C_p}\left[\frac{(BL)C_p}{R_e} \dot{X}_{12} - \left(R_p C_p + \frac{(BL)^2 C_p}{8.85 R_e}\right) X_{11} - X_{10}\right]$$

from eq. 6

$$\dot{X}_{11} = \frac{1}{M_pC_p}\left\{\frac{(BL)C_p}{R_e}\left(-\frac{R_G+R_{15}}{R_G}\frac{2}{C_4}\right)\left[X_{13}+\frac{G_fF_f}{R_5}X_5-\frac{G_sF_f}{R_3}X_6+X_{14}\right.\right.$$

$$\left.\left.+\frac{R_G}{R_G+R_{15}}\frac{1}{2R_9}X_{12}+\frac{E_1}{R_{17}}\right]-\left(R_pC_p+\frac{(BL)^2C_p}{8.85R_e}\right)X_{11}-X_{10}\right\}$$

for eq. 6 derive by equating i s = 0 into mode.

$$V_e = G_eF_e \times X_e \quad , \quad i_e = V_e \Big/ \frac{R_1\left(R_2+\frac{1}{Gs}\right)}{R_1+R_2+\frac{1}{Gs}}$$

$$R_1\left(R_2+\frac{1}{Gs}\right)i_e = V_c\left(R_1+R_2+\frac{1}{Gs}\right)$$

$$\left(R_1R_2C_1s+R_1\right)i_e = V_e\left(\left(R_1+R_2\right)Gs+1\right)$$

$$\dot{i}_e = -\frac{1}{R_2C_1}i_e+\frac{1}{R_1R_2C_1}V_e+\frac{R_1+R_2}{R_1R_2}\dot{V}_e$$

let $X_{13} = i_e$, $\dot{X}_{13} = \dot{i}_e$

$$\dot{X}_{13} = -\frac{1}{R_2C_1}X_{13}+\frac{1}{R_1R_2C_1}G_eF_eX_3+\frac{R_1+R_2}{R_1R_2}G_eF_eX_4$$

$$V_f = G_fF_f \times F_{ex} \quad , \quad i_f = \frac{V_f}{R_5} = \frac{G_fF_f}{R_5}F_{ex} = \frac{G_fF_f}{R_5}X_5$$

$$i_s = \frac{G_sF_s}{R_3}X_s = \frac{G_sF_s}{R_3}X_s$$

$$V_p = \frac{R_{22}}{R_{22} + \frac{1}{C_6 s}} G_p F_p \dot{X}_p \quad , \qquad i_p = \frac{V_p}{R_4}$$

$$i_p = \frac{1}{R_4} \frac{R_{22} C_6 s}{1 + R_{22} C_6 s} G_p F_p \dot{X}_p$$

$$i_p + R_{22} C_6 i_p = \frac{R_{22}}{R_4} C_6 G_p F_p \, s \, \dot{X}_p$$

$$\text{let } X_{14} = \dot{X}_p \qquad X_{14} = i_p$$

$$\dot{X}_{14} = \frac{1}{R_4} G_p F_p X_{11} - \frac{1}{R_{22} C_6} X_{14}$$

$$i_1 = \frac{E_1}{R_{17}}$$

$$V_g = \frac{R_G}{R_G + R_{15}} \times \frac{E_p}{2} \quad , \qquad i_g = \frac{V_g}{\dfrac{R_9 \times \frac{1}{C_4 s}}{R_9 + \frac{1}{C_4 s}}}$$

$$i_g \times \frac{R_4}{C_4 s} = V_g \left(R_9 + \frac{1}{C_4 s} \right) \quad , \qquad i_g = V_g \left(\frac{1}{R_9} + C_4 s \right)$$

$$i_g = \frac{R_G}{R_G + R_{15}} \times \frac{1}{2} \times \frac{1}{R_9} X_2 + \frac{R_G}{R_G + R_{15}} \times \frac{C_4}{2} \times \dot{X}_{12}$$

$$\Sigma \, i \, s = 0 \quad , \; i_e + i_f + i_s + i_p + i_g + i_1 = 0$$

$$X_{13} + \frac{G_f F_f}{R_5} X_5 + \frac{G_s F_s}{R_3} X_6 + X_{14} + \frac{R_G}{(R_G + R_{15})} \frac{1}{2R_9} X_{12}$$

$$+ \frac{R_G}{R_G + R_{15}} \frac{C_4}{2} \dot{X}_{12} + \frac{E_1}{R_{17}} = 0$$

$$\dot{X}_{12} = \underbrace{- \frac{R_G + R_{15}}{R_G}}_{-C' \times R_{17}} \frac{2}{C_4} \left[X_{13} + \frac{G_f F_f}{R_5} X_5 + \frac{G_s F_s}{R_3} X_6 + X_{14} + \frac{R_G}{(R_G + R_{15})} \frac{1}{2R_9} X_{12} + \frac{E_1}{R_{17}} \right]$$

TABLE 9
STATE VARIABLE METHOD
PULSE INPUT
TIME RESPONSE EVALUATION

X1	=-5.088396E-07	X3	=-8.423063E-04	TIME= 2.000000E-03 .002
X1	=-3.091950E-05	X3	=-2.160228E-02	TIME= 3.999997E-03 .004
X1	=-2.483400E-04	X3	=-9.859586E-02	TIME= 5.999994E-03 .006
X1	=-8.904601E-04	X3	=-2.196671E-01	TIME= 7.999990E-03 .008
X1	=-2.052718E-03	X3	=-3.237454E-01	TIME= 9.999987E-03 .010
X1	=-2.586845E-01	X3	=-1.526415E-01	TIME= 1.500000E-01 .15
X1	=-3.221382E-01	X3	=-1.049034E-01	TIME= 1.999999E-01 .2
X1	=-3.313633E-01	X3	=-7.187831E-02	TIME= 2.499999E-01 .25
X1	=-2.834721E-01	X3	=-4.904082E-02	TIME= 2.999998E-01 .3
X1	=-1.895629E-01	X3	=-3.326092E-02	TIME= 3.499998E-01 .35

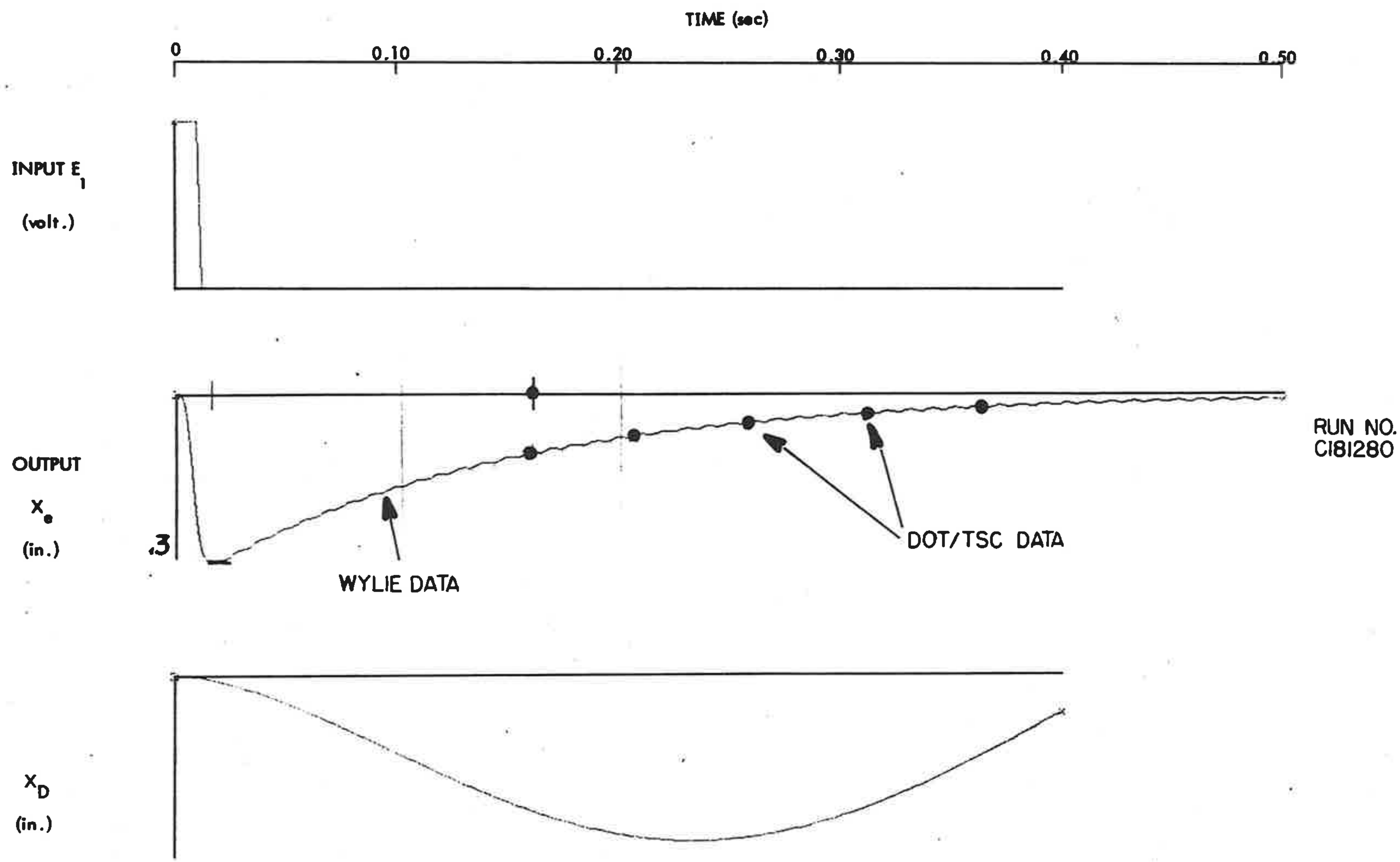

Figure 12. Transient Response to a Unit Impulse Input Signal (Example 1)
(Wylie Figure 17, Appendix D. Ref. 1)

TABLE 10

ROOTS FOR PULSE INPUT STATE VARIABLE METHOD

Ge = 0.03 WYLIE FIG. 17

DENOMINATOR ROOTS

EIGENVALUE EVALUATION BY TARNOVES METHOD

DEGREE OF POLYNOMIAL ELEMENTS= 1

REAL MATRIX ORDER= 14

ROOT NUMBER	LAMBDA REAL	IMAGINARY
1	-2.729141E-02	-6.225663E-05
2	-7.365518E 00	2.512501E-07
3	-9.456441E-01	9.393661E 00
4	-9.456441E-01	-9.393661E 00
5	-2.730332E 02	4.770387E-03
6	-2.345472E 02	-3.578602E-04
7	-9.399922E 02	2.836006E 02
8	-9.399922E 02	-2.836006E 02
9	-1.372005E 00	6.248970E 02
10	-1.372005E 00	-6.248970E 02
11	-1.117550E 04	2.916989E 03
12	-1.117550E 04	-2.916989E 03
13	-1.607169E 04	2.108917E 03
14	-1.607169E 04	-2.108917E 03

REFERENCES

1. Technical Proposal, Prototype Control System for WRDRF, to to FRA, DOT: by Wylie Labs. East Operations, Huntsville, Ala., Feb. 12, 1971.

APPENDIX A

APPENDIX A

TABLE 1

COMPONENT DESCRIPTION AND VALUES FOR EHSA MECHANICAL ANALOG CIRCUITS

Pilot Valve (Figure 2)

E_p	Load Voltage Across Armature	Volts
BL'	Electrodynamic Coupling Constant	lb/amp
C_p	Compliance of Armature Coil	in/lb
L_e	Inductance of Armature Coil	henries
M_p	Mass of Armature and Pilot Valve	lb-sec^2/in
r_e	Electrical Resistance of Armature Coil	ohms
R_p	Mechanical Resistance of Pilot Valve	lb-sec/in
X_p	Velocity of Pilot Valve	in/sec
K_p	Flow Control Constant for Pilot Valve	in^3/sec-in

Slave Valve (Figure 3)

A_s	Effective End Area of Slave Spool	in^2
C_s	Hydraulic Compliance of a Single end Cavity	in^5/lb
M_s	Mass of Slave Spool	lb-sec^2/in
R_{Ls}	Hydraulic Leakage Resistance around Slave Spool	lb-sec/in^5
R_s	Mechanical Resistance of Slave Spool	lb-sec/in
X_s	Velocity of Slave Spool	in/sec
K_s	Flow Control Constant for Slave Valve	in^3/sec-in

TABLE 1 (Continued)

Exciter (Figure 6)

A_e	Effective End Area of Exciter Piston	in^2
C_e	Hydraulic Compliance of a Single End Cavity	in^5/lb
F_{ex}	Mechanical Force Output of Exciter	lb
R_{Le}	Hydraulic Leakage Resistance Around Exciter	$lb\text{-}sec/in^5$
M_e	Mass of Exciter Piston	$lb\text{-}sec^2/in$
R_e	Mechanical Resistance of Exciter Piston	lb-sec/in
X_e	Velocity of Exciter Piston	in/sec
F_L	Mechanical Force on Load	lb
M_L'	Mass of External Load	$lb\text{-}sec^2/in$
R_L'	Mechanical Resistance of External Load	lb-sec/in
$1/K_L$	Compliance of External Load	in/lb
M_D	Mass of External Load	$lb\text{-}sec^2/in$
R_D	Mechanical Resistance	lb-sec/in
$1/K_D$	Compliance of External Load	in/lb
X_D	Displacement of External Load	in

TABLE 2

PARAMETERS TO BE USED FOR SERVO-AMPLIFIER ANALYSIS

Circuit Elements in Figure 8

R_1 = 100.0 kilohms	R_{13} = 2.5 kilohms
R_2 = 6.8 kilohms	R_{14} = 2.5 kilohms
R_3 = 100.0 kilohms	R_{15} = 27.0 kilohms
R_4 = 100.0 kilohms	R_{17} = 56.0 kilohms
R_5 = 56.0 kilohms	R_{22} = 10.0 kilohms
R_9 = 100.0 kilohms	C_1 = 0.68 microfarads
R_{11} = 10.0 kilohms	C_4 = 0.00047 microfarads
R_{12} = 1.8 kilohms	C_6 = 0.22 microfarads

Time Constants

$T_1 = (R_1 + R_2)\,C_1 = 0.00726$ sec $T_{13} = R_2C_1 = 0.0046$ sec

$T_5 = R_9C_4 = 0.000047$ sec $T_{15} = R_{22}C_6 = 0.0022$ sec

Transducer Constants

F_f = 3.67 volt/lb

F_e = 10 volts/in

F_s = 60 volts/in

F_p = 0.7 volts/in/sec (including transducer amplifier gain of 26)

Nominal Gain Settings

G_f = 1.0

G_e = 0.9

G_s = 0.5

G_p = 0.3

G_A = 0.2

TABLE III

COMPONENT VALUES FOR EHSA SYSTEM

Pilot Valve	Example 1	Example 2
BL' (lb/amp)	40	
C_p (in/lb)	0.0005	
r_e (ohms)	490	
R_p (lb-sec/in)	0.24	
M_p (lb)	0.05	
K_p (in^3/sec-in)	3200	
Slave Valve		
A_s (in^2)	0.375	
C_s (in^5/lb)	0.625×10^{-6}	
M_s (lb)	0.75	
R_{Ls} (lb-sec/in^5)	$\delta_s \simeq 1.0$	
R_s (lb-sec/in)		
K_s (in^3/sec-in)	1.5×10^{-4}	
Exciter		
A_e (in^2)	20.5	
C_e (in^5/lb)	5.5×10^{-5}	
R_{Le} (lb-sec/in^5)	∞	
M_e (lb)	150	
R_e (lb-sec/in)	3.44×10^{-8}	

TABLE III (Continued)

External Loads	Example 1	Example 2
M_L (lb)	1.5×10^4	0.5×10^4
R_L (lb-sec/in)	0.549×10^2	0.183×10^2
K_L (lb/in)	0.777×10^4	0.259×10^4
M_D	2.0×10^4	0.667×10^4
R_D (lb-sec/in)	0.977×10^2	0.326×10^2
K_D (lb/in)	0.460×10^4	0.153×10^4
Nominal Gain Settings		
G_f	0.0	0.0
G_e	0.03	1.0
G_s	0.5	0.5
G_p	0.3	0.3
G_A	0.2	0.2

NOTE: In Example 2, the same EHSA system was used as in Example 1, except the external loads which were excited by three servo actuators simultaneously.