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PB-258 193

DYNALIST II. A Computer Program for Stability & Dynamic Response Analysis of Rail Vehicle Systems. Volume III

Wiggins (J H) Co, Redondo Beach, Calif

Prepared for

Federal Highway Administration, Washington, D C Office of Research and Development

Jul 76

PB 258 193

REPORT NO. FRA-OR&D-75-22.III

DYNALIST II
A COMPUTER PROGRAM FOR STABILITY AND DYNAMIC RESPONSE ANALYSIS OF RAIL VEHICLE SYSTEMS
Volume III: Technical Report Addendum

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JULY 1976
FINAL REPORT

Prepared for
U.S. DEPARTMENT OF TRANSPORTATION
FEDERAL RAILROAD ADMINISTRATION
Research and Development
Washington DC 20590

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Technical Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
FRA-OR&D-75-22.III		

4. Title and Subtitle	5. Report Date
DYNALIST II - A COMPUTER PROGRAM FOR STABILITY AND DYNAMIC RESPONSE ANALYSIS OF RAIL VEHICLE SYSTEMS, Volume III: Technical Report Addendum	July 1976
	6. Performing Organization Code
7. Author(s): Allen Bronowicki and T. K. Hasselman	8. Performing Organization Report No.: DOT-TSC-FRA-74-14.III
9. Performing Organization Name and Address: J. H. Wiggins Company* 1650 South Pacific Coast Highway Redondo Beach CA 90277	10. Work Unit No. (TRAIS): RR628/R6320
	11. Contract or Grant No.: DOT-TSC-990
12. Sponsoring Agency Name and Address: U.S. Department of Transportation Federal Railroad Administration Research and Development Washington DC 20590	13. Type of Report and Period Covered: Final Report February 1975 - March 1976
	14. Sponsoring Agency Code

15. Supplementary Notes

*Under contract to: U.S. Department of Transportation
Transportation Systems Center
Kendall Square
Cambridge MA 02142

16. Abstract

Several new capabilities have been added to the DYNALIST II computer program. These include: (1) a component matrix generator that operates as a 3-D finite element modeling program where elements consist of rigid bodies, flexural bodies, wheelsets, suspension elements and point masses assembled on a nodal skeleton; (2) a periodic and transient time-history response capability; (3) a component update capability for parametric studies; (4) an orthogonality check on component and system complex eigenvectors; (5) an option for improving low-frequency convergence under modal trunction; (6) a more general sine-amplitude forcing function capability; (7) automatic phase lag generation; (8) user-controlled scaling options on all response plots; and a number of additional minor improvements. A Technical Report Addendum and a completely revised User's Manual document these changes to the previous version of DYNALIST II.

17. Key Words	18. Distribution Statement
Rail Vehicle Dynamics, Rail Vehicle, Stability, Rail Vehicle Response, Ride Quality, Computer Modeling, Modal Synthesis	DOCUMENT IS AVAILABLE TO THE U.S. PUBLIC THROUGH THE NATIONAL TECHNICAL INFORMATION SERVICE, SPRINGFIELD, VIRGINIA 22161

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
Unclassified	Unclassified	74	4.50/3.00

Form DOT F 1700.7 (8-72) Reproduction of completed page authorized

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PREFACE

The Federal Railroad Administration (FRA) is sponsoring research, development, and demonstration programs to provide improved safety, performance, speed, reliability, and maintainability of rail transportation systems at reduced life-cycle costs. A major portion of these efforts is related to improvement of the dynamic characteristics of rail vehicles, track structures, and train consists.

Transportation Systems Center (TSC) is maintaining a center for resources to be applied to programs for improved passenger service, improved safety, and more cost-effective freight service. As part of this effort, TSC is identifying computer programs, analytic models, and analysis tools required to support the FRA objectives. In particular, TSC is acquiring, developing, and extending computer programs to provide realistic predictions of rail system dynamic performance under field conditions.

The DYNALIST II computer program was developed for the Department of Transportation by the J. H. Wiggins Company in 1974. Documentation was contained in two volumes, a technical report documenting the theoretical basis of the program and a user's manual. The present report also consists of two volumes. Volume III, entitled "Technical Report Addendum," is written as an addendum to the previous technical report. Volume IV, entitled "Revised User's Manual," is self-contained and supersedes the previous user's manual.

Numerous detailed comments provided by Dr. Russel Brantman, the TSC Technical Monitor, have been incorporated in both of these reports. The time which he and others at TSC have spent in reviewing the material from the standpoint of new users has, in the opinion of the authors, contributed significantly to its clarity. The authors wish to express their appreciation for this dedicated effort.

The Revised User's Manual reflects current modifications in output format which have been written into the program at TSC. These modifications were performed by Duncan Sheldon under the direction of Dr. Brantman.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

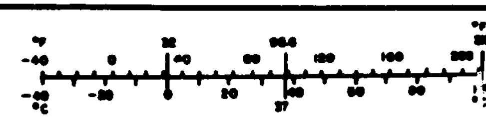
METRIC CONVERSION FACTORS

Approximate Conversions to Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
	AREA			
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
	MASS (weight)			
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
	VOLUME			
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
	TEMPERATURE (exact)			
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
	LENGTH			
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
	AREA			
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
	MASS (weight)			
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
	VOLUME			
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
	TEMPERATURE (exact)			
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F



1. SUMMARY

This report presents the results of continued development and application of the DYNALIST II computer program during the period February 1975 through March 1976. The following new capabilities have been added to the program:

- Element Synthesis - A third linear modeling capability has been added to the program which allows the user to synthesize three-dimensional components of a system from basic elements consisting of rigid bodies, flexural bodies, wheelsets, springs, dampers, and nodal masses. The building block methodology now proceeds from element to component to system.
- Periodic Response - The program now allows the user to specify periodic inputs in terms of a piecewise linear waveform with up to 100 points. A Fourier series representation is automatically generated, and response is computed in terms of RMS and periodic time histories.
- Transient Response - The periodic response capability is sufficiently general to allow the computation of response to transient type inputs such as rail discontinuity and bumps. A Fourier series approximation including up to 100 terms is used.

Beyond the addition of these major capabilities, several other significant capabilities and improvements to the program have been added:

- Component Update Capability - The user may now replace selected component data on the component data file, thereby facilitating parametric studies. Updated parameters are identified in the printed output.

- Orthogonality Check - An orthogonality check is now made automatically on all system eigenvectors to confirm the diagonalization of the state equations. When the test for orthogonality fails, the transformed dynamic matrix is printed so that the user may identify which eigenvectors are not independent.

- Low Frequency Convergence - An option for computing the residual truncation error in static response, and adding it to the truncated mode approximation of frequency response has been added to the program. This ensures convergence of frequency response functions at the low end of the spectrum when modes have been truncated.

- Sine Input - The amplitude of sinusoidal excitation as a function of frequency or wavelength can now be specified arbitrarily by the user using a tabular input, as opposed to the particular functional form previously built into the program.

- Phase Lag Input - Phase lags between wheelsets are now generated within the program based on user input wheel spacing and velocity. In addition, the phase input has been generalized so that sinusoidal response may be computed as a function of train velocity as well as the wavelength of track irregularity.

- Plot Scaling - User controlled scaling options have been provided for all response plotting. The purpose here is to provide for direct comparison between plots within the same run, or from one run to another.

Application of the program during the current period has focused on the ability to solve certain types of mathematically difficult problems. Situations involving multiply repeated roots and uncoupled equations have been investigated. Guidelines are provided to assist the user in the proper formulation of problems. To date, the program has demonstrated itself to be a very powerful and highly reliable analytical tool. The major part of the theoretical development is given in Reference [1]*.

*Numbers in square brackets designate references at end of report.

2. EXTENDED DEVELOPMENT

2.1 Component Matrix Generator

A major addition to DYNALIST II is the Component Matrix Generator, COMGEN. This modeling routine enables the user to acquaint himself more with the physics of the problems being solved and to avoid much troublesome detail. The topology of the system being modeled and the physical properties of its elements are defined by the user. The program then uses a small-deflection finite element type method to form the mass, damping, and stiffness matrices which describe the component's homogeneous equations of motion.

The element library includes rigid body, flexible body mode, lateral wheelset, spring-damper, and nodal mass elements. The elements are attached to a mesh of nodes defined in three-dimensional space. Displacement and rotational constraints may be applied to the motion of the nodes in any of the six coordinate directions. Thus, in addition to full three-dimensional dynamics problems, one and two-dimensional problems can be modeled.

The equations of motion at both the component and system levels are developed in terms of the dynamic p-coordinate system specified by the user. The dynamic p-coordinates are any set of generalized coordinates which completely specify the degrees of freedom of the system or component. They may correspond directly to a set of discrete physical u-coordinates or they may correspond to a set of distributed or modal coordinates. In addition to the dynamic p-coordinate system, the user also specifies a physical u-coordinate system. This coordinate system is the one in which the components are coupled together, the excitation forces are applied, and system response at selected locations is printed and plotted. Since the u-coordinates are not used to specify the degrees of freedom of the system, only as many u-coordinates need be defined as are required for input, output, and coupling. (As such, the physical u-coordinate system can be either larger or smaller than the dynamic p-coordinate system).

The user may assign u- and p-coordinates to any of the degrees of freedom of the nodes in a manner consistent with the system being modeled. The u- and p-coordinates are related by the transformation $\{u\} = [\phi]\{p\}$ which is automatically generated from the system topology and coordinate specification. A nodal degree of freedom may be given a u- and p-coordinate, in which case the two will be related identically. The rigid body elements serve to further define the ϕ matrix. The motion of several nodes may be tied to that of a master node, generating a linear transformation between the degrees of freedom of the master node and those of the slave nodes. Each flexible body mode constitutes a p-coordinate, and a column of the ϕ matrix is in effect defined by the user when he describes the mode shape. The equations of motion are built by adding the four basic elements to the nodal skeleton. The element properties are converted into the p-system in the manner $[k]_p = [\phi]^T[k]_u[\phi]$. Other elements may be easily added to this library since the transformation to the p-system occurs in a separate subroutine which is used by all of the elements. A detailed description of the functions of the Component Matrix Generator is given in Reference [2].

2.2 Component Update Capability

The component update capability is intended to serve as a convenience in performing parametric studies of the behavior of systems. This capability allows prior models of a component to be deleted from the component data file. A revised model of the component may then be replaced on the component data file and the file is thus kept down to a manageable size. Revisions to the mass, damping, stiffness and phi matrices are entered in a special Namelist block UPDATE, while the old model is entered in Namelist COMPO. The revisions to the model are displayed on the computer printout of the problem. Revisions to the model made in the Component Matrix Generator or the special truck and car matrix generator can be indicated on the printout by placing an Update Code of 1 in the last column of the input data card being revised. Details of this capability are presented in Reference [2].

2.3 Eigenvector Solution

Previous experience with DYNALIST II disclosed that under certain conditions, numerical difficulties were encountered during execution of the program, particularly in eigenvector computation. An investigation was therefore made to identify the source of the problem and take corrective action. Two things have resulted from this investigation. A coding error was discovered and eliminated. This solved the biggest part of the problem. In addition, some guidelines have been developed to help the user formulate his problem so as to avoid computational difficulty resulting from an ill-posed problem.

2.3.1 Problem Formulation

Because of the repetitive structure of train-type systems, and the physical isolation of parts of the system from each other (e.g., two trucks of a conventional rail car), there is a natural tendency for the eigenvalues to be repetitive. Sometimes very small differences among a subset of eigenvalues result from weak coupling. Sometimes the eigenvalues in a subset appear to be identical (within six digit accuracy). In certain cases, eigenvalues not only appear to be identical, but identity can be established on physical grounds. In the former two cases, correct eigenvector solutions have been obtained in all cases run to date. In the latter case, correct solutions have been obtained for all small problems (up to six degrees of freedom) but not for some larger problems (16 degrees of freedom). In all cases run to date, correct solutions have been obtained by formulating the problem properly, i.e., by avoiding situations where physical uncoupling between different parts of a system exists.

2.3.2 Diagonal Forms

The QR eigenvalue subroutine in DYNALIST II will not compute eigenvalues for a diagonal system of equations. However, there is no practical need to do so. If the mass, damping and stiffness matrices are in diagonal form to begin with, then the p-coordinates are totally upcoupled. Accordingly, the eigenvalues can be computed by hand. Thus, the problem is considered to be ill-posed for DYNALIST II. For Example, if a diagonal set of equations is given by

$$m_j\ddot{p}_j + c_j\dot{p}_j + k_jp_j = 0 : j = 1,2,\ldots,N \qquad \ldots(2\text{-}1)$$

the eigenvalues are given by

$$\lambda_j = -\frac{c_j}{2m_j} \pm \sqrt{\left(\frac{c_j}{2m_j}\right)^2 - \frac{k_j}{m_j}} \qquad \ldots(2\text{-}2)$$

Nothing could be gained by solving this problem using DYNALIST II. The only exception to this rule is for a one-degree-of-freedom oscillator. Because of its common application, a separate logic has been set up to handle this special type of diagonal system.*

2.3.3 Block Diagonal Forms

The block diagonal form is defined to be such that the mass, damping and stiffness matrices are of the form

$$[m] = \begin{bmatrix} m^1 & 0 & & 0 \\ 0 & m^2 & & 0 \\ & & \ddots & \\ 0 & 0 & & m^N \end{bmatrix} \qquad \ldots(2\text{-}3)$$

where the submatrices, m^ℓ, $\ell = 1,2, \ldots, N$, constitute the only non-zero partitions of [m]. The off-diagonal partitions

*Note that if a component is in diagonal form, but the system also includes nondiagonal components, then no synthesis problems will be encountered provided that the user does not attempt to calculate component modes for the diagonal component. Either the Direct System or Direct Sub-Systems approach may be used; or if Modal Synthesis is used, then the coordinates of the diagonal component should all be listed as constrained coordinates, rather than as free coordinates, so that modal conputation will be bypassed for that component.

are all null. Each submatrix m^{ℓ} is square and may have one or more columns. When equations of motion

$$[m]\{\ddot{p}\} + [c]\{\dot{p}\} + [k]\{p\} = \{0\} \qquad \ldots(2\text{-}4)$$

are such that [m], [c] and [k] are all of the same block diagonal form, the equations are said to be block diagonal.

An investigation was made to determine whether DYNALIST II could handle this kind of problem. The system shown below (Figure 2-1) consists of two two-degree-of-freedom systems isolated from each other by a rigid base.

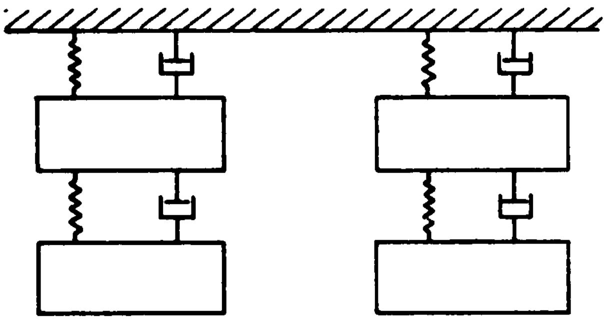


Figure 2-1. Illustration of System Exhibiting Block Diagonal Form

All four mass, damper and spring elements were identical. DYNALIST II was run using (1) the Direct System (DS) Method with all four degrees of freedom in one component, (2) the Direct Sub-Systems (DSS) Method combining two identical components without computing component modes, (3) the Modal Synthesis (MS) Method using the same two components without modal truncation, and (4) the Modal Synthesis Method with modal truncation. The first three cases produced identical results all of which were correct. Case (4) also worked properly, giving approximate results because of the modal truncation. This established the fact that block diagonal sets of equations can be handled by the program under all of its basic modeling options. (By contrast, a two-degree-of-freedom diagonal system will not execute.)

This of course left unanswered the question of whether other block diagonal systems could be handled, and in particular whether larger more representative problems will execute properly. The investigation was pursued in an attempt to answer this question.

The six-degree-of-freedom and 16-degree-of-freedom examples mentioned in Section 2.3.1 were a vertical car model and a three-car vertical model respectively. In the former, the six degrees of freedom constituted heave and pitch motion of a rigid car body and two rigid truck frames in a perfectly symmetric configuration. Only heave springs and dampers connected the trucks to the car body, no pitch springs or dampers. Therefore, pitch motion of the two trucks was completely uncoupled from the rest of the system (and of course from each other).

Considering first the single car model in Figure 2-2 (ignoring damping for the moment), one may develop homogeneous equations

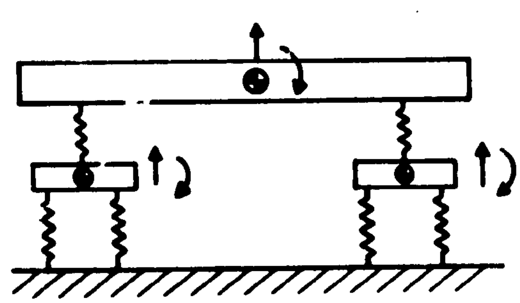


Figure 2-2. Vertical Car Model - Stationary

of motion wherein the mass matrix is diagonal and the stiffness matrix is block diagonal as follows:

$$[k] = \left[\begin{array}{cccc|c|c} X & 0 & X & X & 0 & 0 \\ 0 & X & X & X & 0 & 0 \\ X & X & X & 0 & 0 & 0 \\ X & X & 0 & X & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & X & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & X \end{array}\right]$$

Placement of dampers in parallel with the springs results in a damping matrix of the same form. The resulting equations of motion are in block diagonal form.

It should be recognized that a different ordering of the coordinate numbers might not produce a block diagonal form directly. But the equations are considered to be block diagonal if they can be reordered so as to produce this form. As previously stated, this problem did execute properly.

The next step was to couple three such cars together in a train. The intent here was to deliberately "push luck" in an attempt to test the program. Coupling between adjacent cars was effected by using the constraint equations to hinge the cars in a vertical plane. Three cars each having six degrees of freedom produced 18 degrees of freedom to begin with. Introducing two constraint equations reduced that number to 16. Using the DSS option, a correct set of eigen<u>values</u> was obtained (this was verified later). However, the eigen<u>vectors</u> were not independent.

Suspecting that part of the problem might be due to hinging the cars, we used very stiff vertical spring components (three orders of magnitude stiffer than secondary suspension) to couple the cars instead. This way the system retained 18 degrees of freedom. Using the DSS option again this time, independent eigenvectors were computed and the eigenvalues agreed with the previous case except that two additional conjugate pairs resulted. This case represents the most severe test passed to date. DYNALIST II successfully solved seven independent sets of second order differential equations simultaneously, six independent truck pitch equations and the remaining set of twelve coupled equations.

At this point it was desirable to check out the hinging without the added complication of uncoupled truck pitch motion. Pitch springs were therefore added to the secondary suspension and the 16-degree-of-freedom train case was rerun. This time the problem executed properly.

Finally, two runs were made on the 18-degree-of-freedom train using the Modal Synthesis (MS) option, first with and then without modal truncation. All of the oscillatory modes (two conjugate pairs for each car) were retained in the first case while the remaining real modes were deleted. Six conjugate pairs of eigenvalues were computed. The first four were almost identical to the car body modes obtained previously. The other two pair resembled the extra eigenvalues computed by the DSS option for the 18-degree-of-freedom model. The imaginary parts (frequency) were very close but the real parts (damping) were not only off in magnitude but sign also. In other words, the real positive parts of the complex roots implied negative damping or an instability. Even so, the frequency response plots obtained from the run appeared to be reasonable. One reason for this is that the fictitious modes (due to fictitious coupling springs) were in the high frequency range, beyond the region of interest. Another is that the real parts of the eigenvalues were still small compared to the imaginary parts, even though incorrect in sign. One would expect the sign error to cause a phase shift which the modulus of the frequency response function might not be sensitive to.

The last run on the 18-degree-of-freedom model using the MS option without truncation terminated during the system eigenvalue solution. The trace of the dynamic matrix did agree with that obtained by the DSS method. Another point worth mentioning is that the dynamic matrix obtained by the MS method under modal truncation was a partition of the dynamic matrix obtained in the last run, as expected. The trace obtained in the truncated MS case was however positive, consistent with the "unstable" roots.

2.3.4 Conclusions

The eigenvector investigation was not pursued further. Some interesting questions remain unanswered, at least from an academic point of view. From a practical standpoint, however, adequate guidelines can be suggested on the basis of this investigation.

- Never attempt to extract eigenvalues for a component whose equations of motion in the p-coordinate system are diagonal. It is unnecessary and will terminate execution.

- Try to avoid block diagonal systems of equations. If such equations represent a meaningful problem which must be solved, introduce negligibly small mass, damping or stiffness elements so that the equations are at least slightly coupled. This should result in solutions which are sufficiently accurate for practical purposes, and avoid potential numerical difficulties.

- Use spring-damper coupling elements between cars if possible rather than perfect hinges. Introduce coupling (small if desired) between rotational degrees of freedom as well as translational), e.g., pitch springs as well as vertical springs between cars.

- Use the Modal Synthesis option with caution. Be especially cautious of deleting low frequency modes or real modes associated with real roots. Until the consequences of this operation are better understood, it is advisable to avoid truncating real modes and low frequency modes altogether.

It is to be emphasized that the tests applied to the program and discussed in this section were very severe. All of the troublesome areas could have been avoided in the solution of practical problems. The eigenvalue/eigenvector subroutines incorporated in DYNALIST II appear to be very powerful when executed on the CDC machine which uses a large word size. As in the solution of any mathematical problem, the importance of proper formulation cannot be overstated.

2.4 Orthogonality Check

A major goal of solving a system eigenproblem is to diagonalize the system equations of motion so that the response of the system may be found. A set of linearly independent eigenvectors which correspond to the proper eigenvalues are required to diagonalize the equations. However, experience has shown that it is possible to find a set of eigenvectors which exhibit the correct eigenvalues but which are not linearly independent. This may happen in a system having several segments which are identical to each other but uncoupled from each other, giving rise to repeated eigenvalues. A certain way to determine if a correct set of mutually orthogonal modes has been found is to see if the equations of motion have actually been diagonalized.

The equations of motion may be written in p-coordinates as

$$[A_y]\{\dot{y}\} + [B_y]\{y\} = \{f_y\} \qquad \text{...(2-5)}$$

This leads to an eigenproblem posed in the form

$$\Lambda_j\{\Psi_{y_j}\} = -[A_y]^{-1}[B_y]\{\Psi_{y_j}\} = -[D_y]\{\Psi_{y_j}\} \qquad \text{...(2-6)}$$

where Λ_j is an eigenvalue and Ψ_j is its corresponding eigenvector, and where the matrix D_y is the dynamic matrix. The eigenvectors define a similarity transformation between the y-coordinates and the modal coordinates z. Applied to the dynamic matrix this similarity transformation should yield the diagonal eigenvalue matrix

$$-[\Psi_y]^{-1}[D_y][\Psi_y] = \lceil \Lambda \rfloor \qquad \text{...(2-7)}$$

In both of the eigenproblem routines of the program, at the component level and the system level, this similarity transformation is performed on the dynamic matrix to see if the diagonal eigenvalue matrix is obtained. The matrix I* is first computed

$$\lfloor I \rfloor^* = -\lfloor \Lambda \rfloor^{-1/2} [\Psi_y]^{-1} [D_y] [\Psi_y] \lfloor \Lambda \rfloor^{-1/2} \qquad \text{...(2-8)}$$

which should equal the identity matrix within a fine tolerance if the eigenvectors are mutually orthogonal. A tolerance equal to the number of eigenvalues squared times 10^{-9} is established. If each of the elements of the matrix I* is equal to the corresponding element of the identity matrix within this tolerance the orthogonality check is assumed to have been satisfied. If not the orthogonality check has failed.

The eigenvector orthogonality check lends a much higher level of confidence to the results of the program. It ensures that the eigenvectors correctly represent system behavior. It also ensures that the equations have been properly diagonalized and thus the response analyses are valid. Should the orthogonality check fail, the transformed dynamic matrix is printed. Large off-diagonal terms appearing in this matrix identify the eigenvectors which are dependent. This usually suggests an ill-posed problem which the user can then take steps to reformulate.

When computing system response, it is important that the user note whether the eigenvector orthogonality check is satisfied, since, if the orthogonality check is not satisfied, the generated system response may yield incorrect results.

2.5 Low Frequency Convergence

The use of the Modal Synthesis modeling option with component mode truncation is an approximate method of analysis. It leads to approximations in both the system modes obtained and in the response computed at the physical coordinates. The approximation is due to the fact that it is impossible to represent the exact motion of an n-degree of freedom system using less than n modes if all of the modes are contributing to the response. A means of improving the approximation in a certain frequency range is to compute the exact response at some particular frequency of interest and to make the approximate frequency response curve conform to the exact response at this one frequency. This remedy was incorporated into the program, and since it was chosen to make the frequency response curve conform to the exact response at zero frequency, it is known as low frequency, or static convergence.

The derivation of equations for low frequency convergence is given in Section 3.7 of Reference [1]. The exact response at a physical coordinate at zero frequency, H_{u_o}, can be obtained from the system stiffness matrix and the zero order force vector as

$$H_{u_o} = [\phi][\bar{\beta}][K]^{-1}[\bar{\beta}]^T[\phi]^T\{f_{u_o}\} \qquad \ldots(2\text{-}9)$$

The residual contribution of the unused modes to the response at low frequencies is then

$$H_{u_R} = H_{u_o} - H_u(i\Omega)\big|_{\Omega\to 0} \qquad \ldots(2\text{-}10)$$

Adding the residual term to $H_u(i\Omega)$ should result in an approximation, $\hat{H}_u(i\Omega)$, to the exact frequency response function which improves the accuracy at low frequencies. Thus,

$$\hat{H}_u(i\Omega) = H_u(i\Omega) + H_{u_R} \qquad \text{...(2-11)}$$

A simple and striking demonstration of the use of modal truncation with static convergence is provided by the four degree of freedom oscillator shown in Figure 2-3. The first three masses of the oscillator were taken as component 1 and the end mass was component 2. Component modes computed for component 1 give three pairs of modes with conjugate roots. The highest pair of modes was truncated and the two components were coupled together. Frequency response curves were computed for the end mass using modal truncation with static convergence (Figure 2-4). and using the exact Direct Sub-Systems method for comparison (Figure 2-5). A static unit force applied at the end mass produces a displacement of four since there are four unit springs in series. The exact frequency response curve shows this at low frequency and so does the approximate response curve. The difference between the two curves occurs at the higher frequencies. The approximate curve is accurate for the first two modes as is indicated by the similarity in the first two peaks in the response curves. The fourth mode is missing, however, and the third mode is between the third and fourth modes of the exact solution. At frequencies past the highest mode the response of the approximate solution levels off at an amount equal to the static correction H_{u_R}. This is markedly

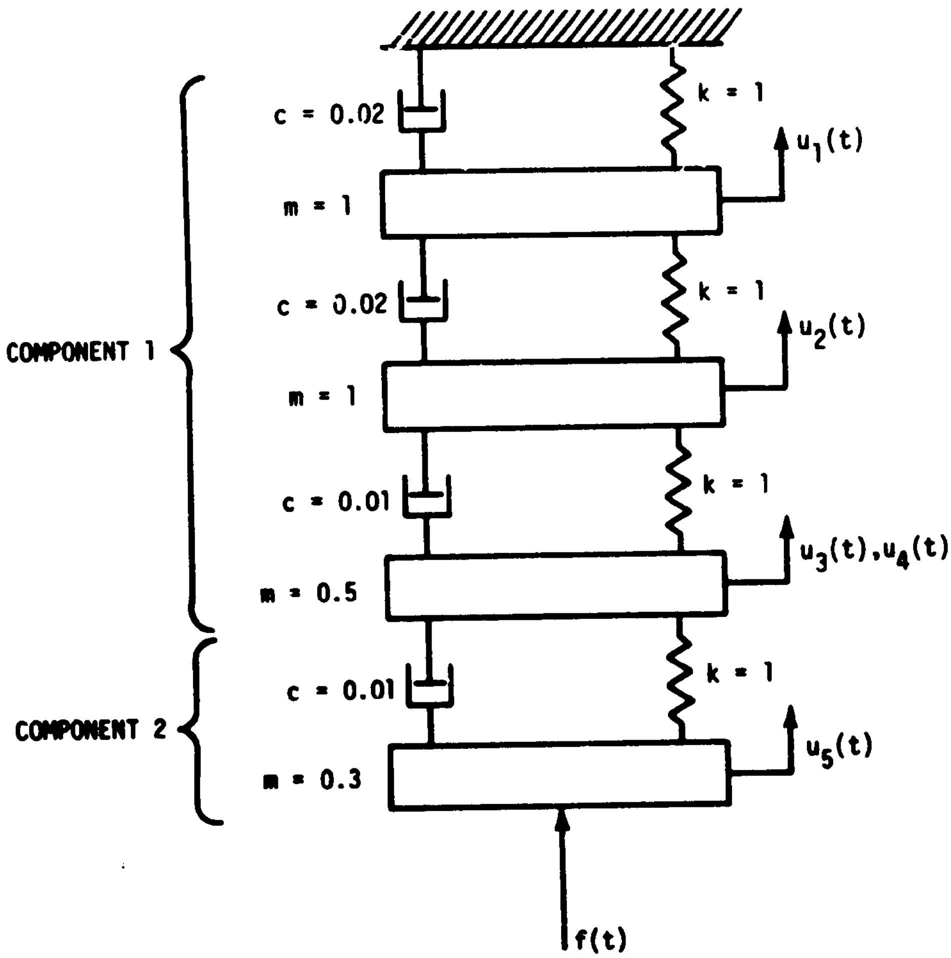


Figure 2-3. Four Degree of Freedom Test Problem

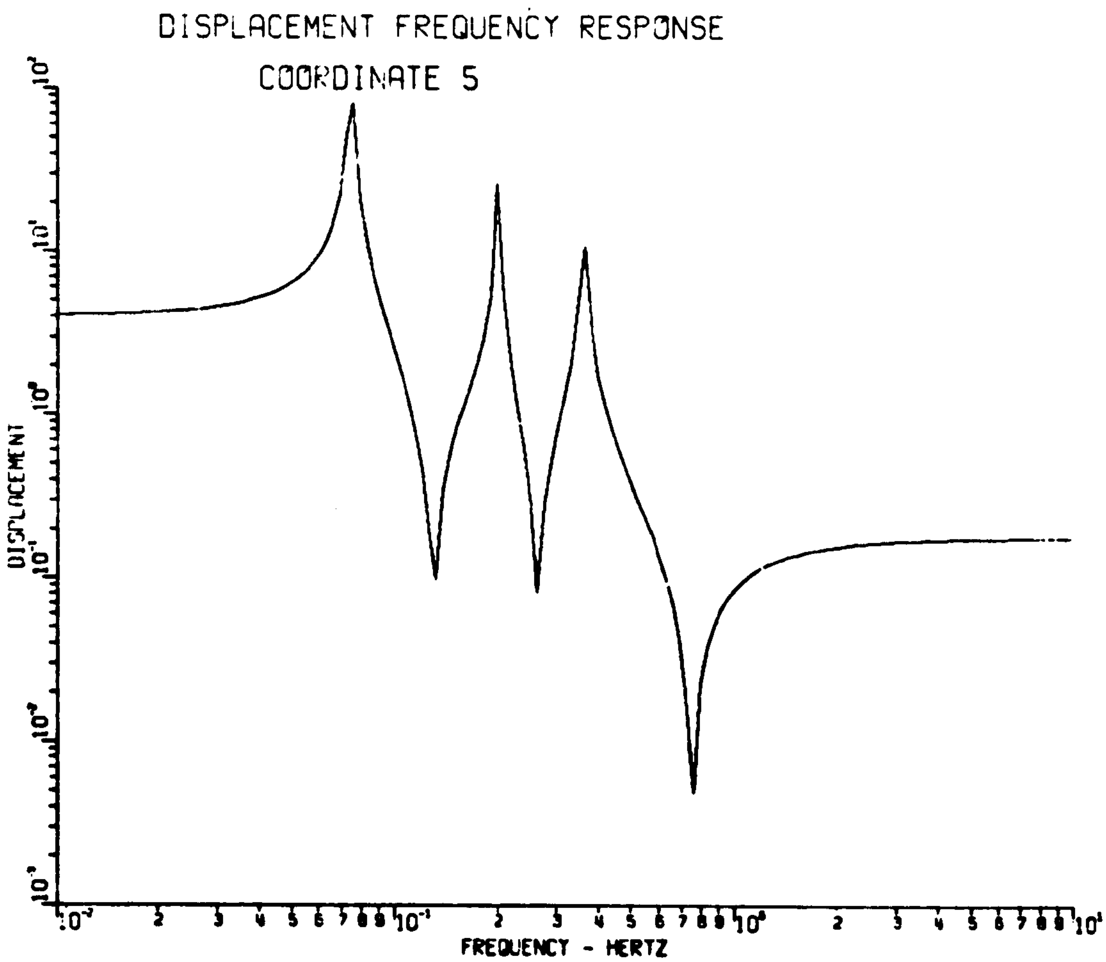


Figure 2-4. Frequency Response For Four Degree of Freedom Test Problem (Modal Truncation with Low Frequency Convergence)

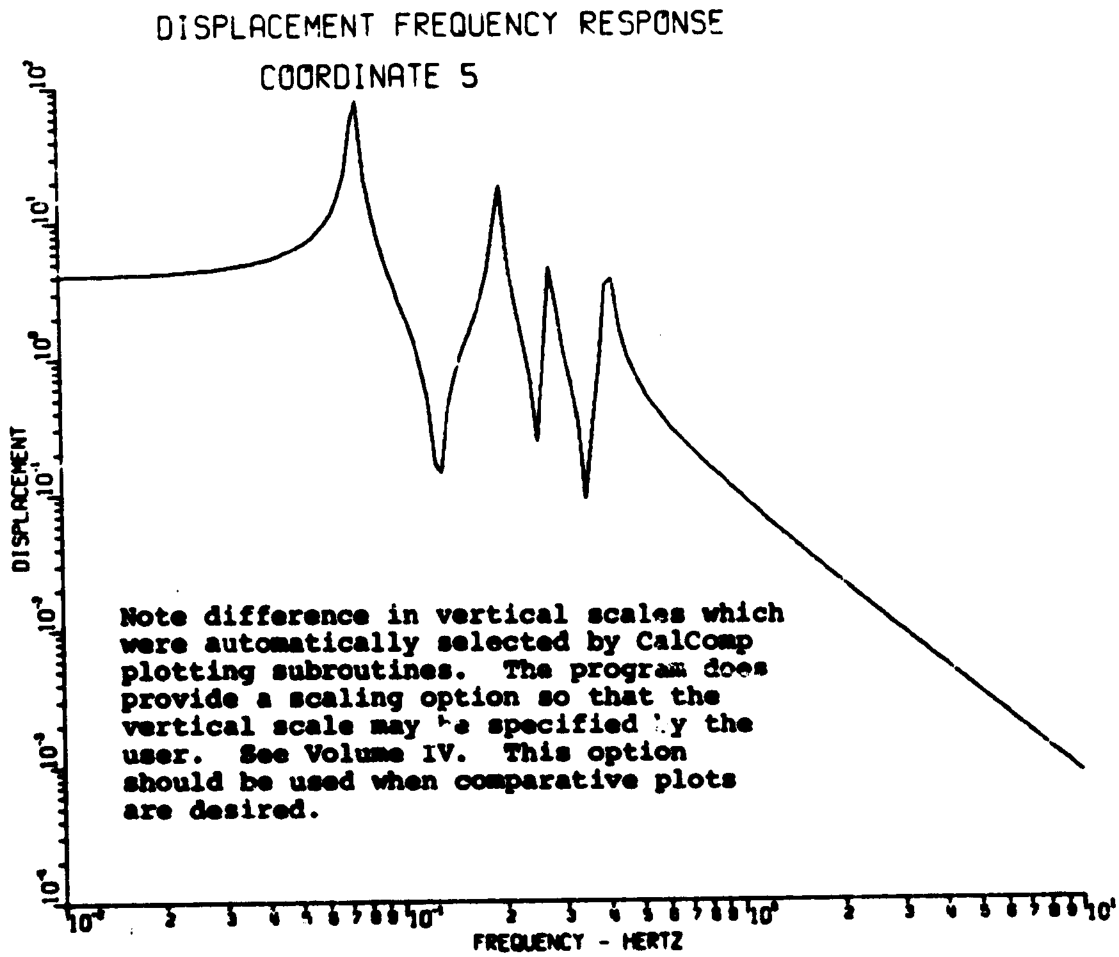


Figure 2-5. Frequency Response For Four Degree of Freedom Test Problem (No Modal Truncation)

different from the response of the exact system, which drops rapidly at high frequencies. Thus we see that the use of modal truncation with static convergence provides good results at frequencies up to the frequency of the highest mode included for lightly damped systems such as this.

An application of static convergence to a lateral rail vehicle is provided in Sample Problems 1 and 3 given in the REVISED USER MANUAL. Both of these problems model a vehicle having two six degree of freedom trucks linked to a four degree of freedom flexible carbody. The response points plotted are the lateral displacement of the center of the carbody, coordinate 10, and the lateral displacement of the rear truck, coordinate 20. Figures 2-6 and 2-7 show the displacement frequency response for lateral motion of the car and truck, respectively, for the exact solution. Figures 2-8 and 2-9 show these responses where only the lowest four conjugate pairs of roots out of six have been retained per truck. A listing of the printed output for the exact solution is given in Appendix C of Volume IV. The response found is due to a lateral track irregularity of decreasing wavelength and unit amplitude. The static lateral response of the vehicle is thus seen to be unity. The response given by the approximate solution is seen to compare quite favorably with the exact response up to a frequency of about 10 Hertz, which is the frequency of the highest truck mode. Above this frequency the static correction plays an increasingly large role in the response of the approximate solution. The peaks in the response of the car are: car yaw and sway modes at .7 and .8 Hz, truck hunting mode at 4 Hz, first car bending mode at 5 Hz and second car bending mode at 14 Hz.

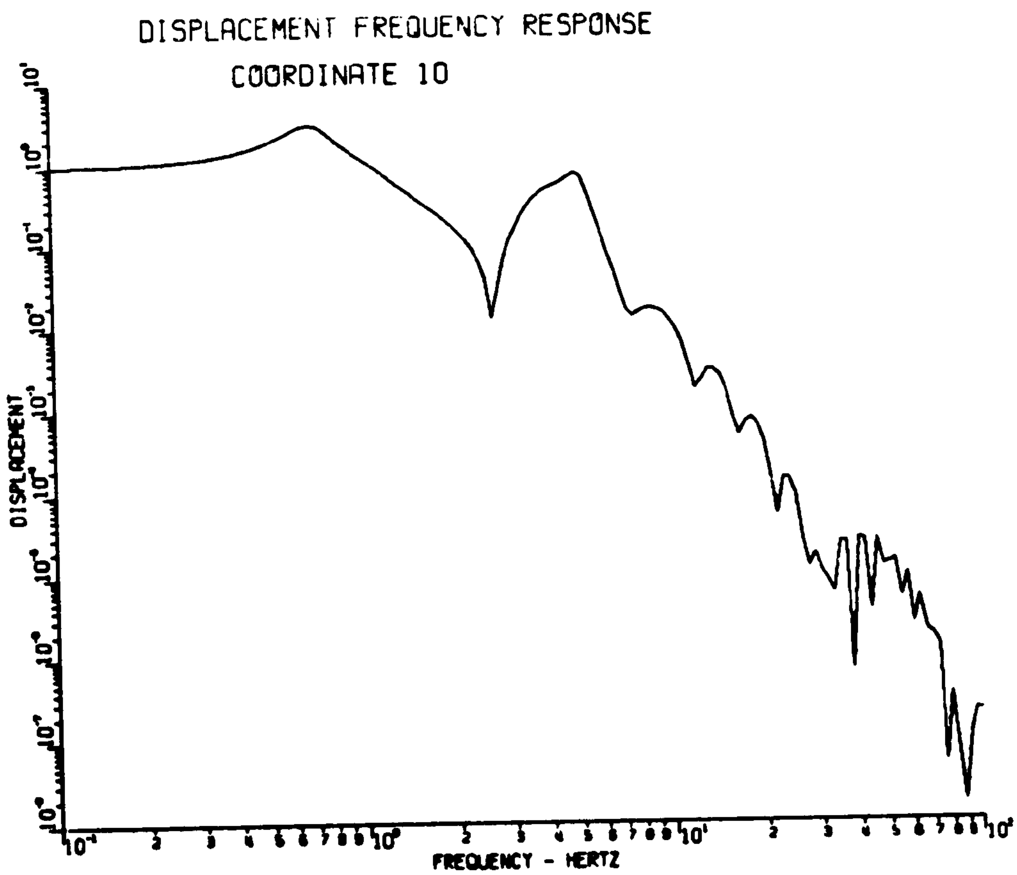


Figure 2-6. Displacement Frequency Response, Flexible Car Lateral Model, Sway at Center of Car, Exact

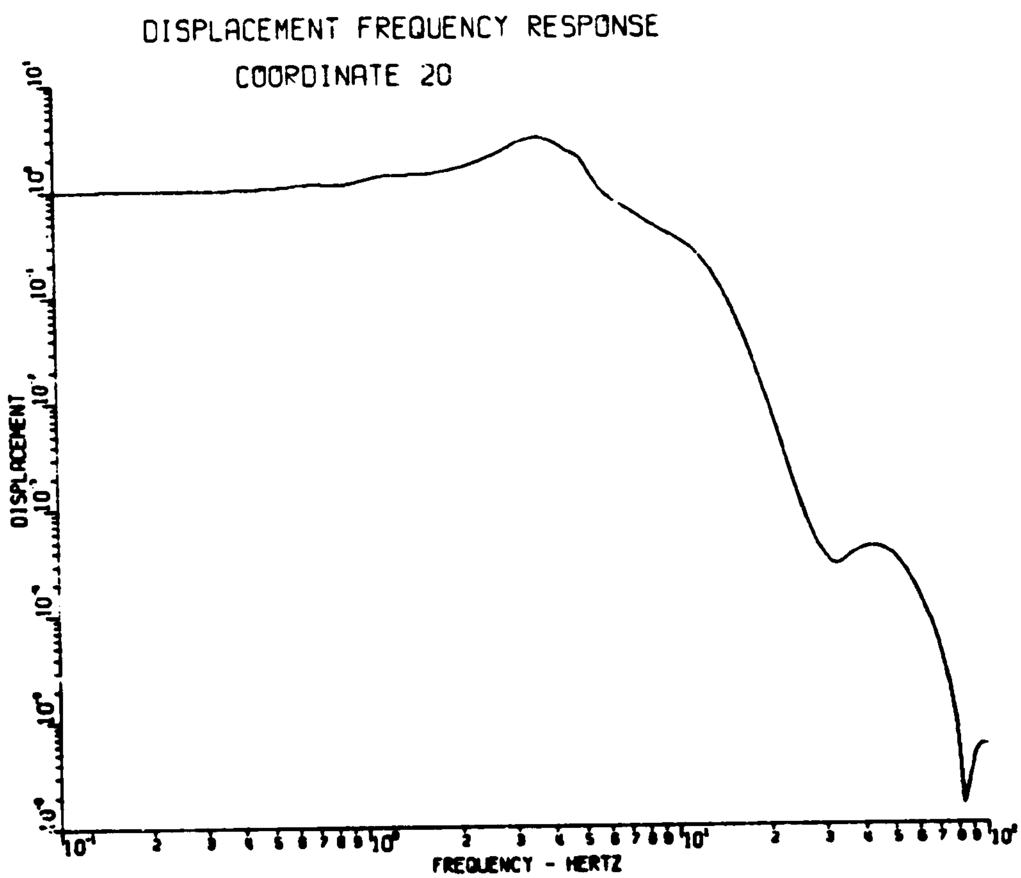


Figure 2-7. Displacement Frequency Response, Flexible Car Lateral Model, Sway at Rear Truck, Exact

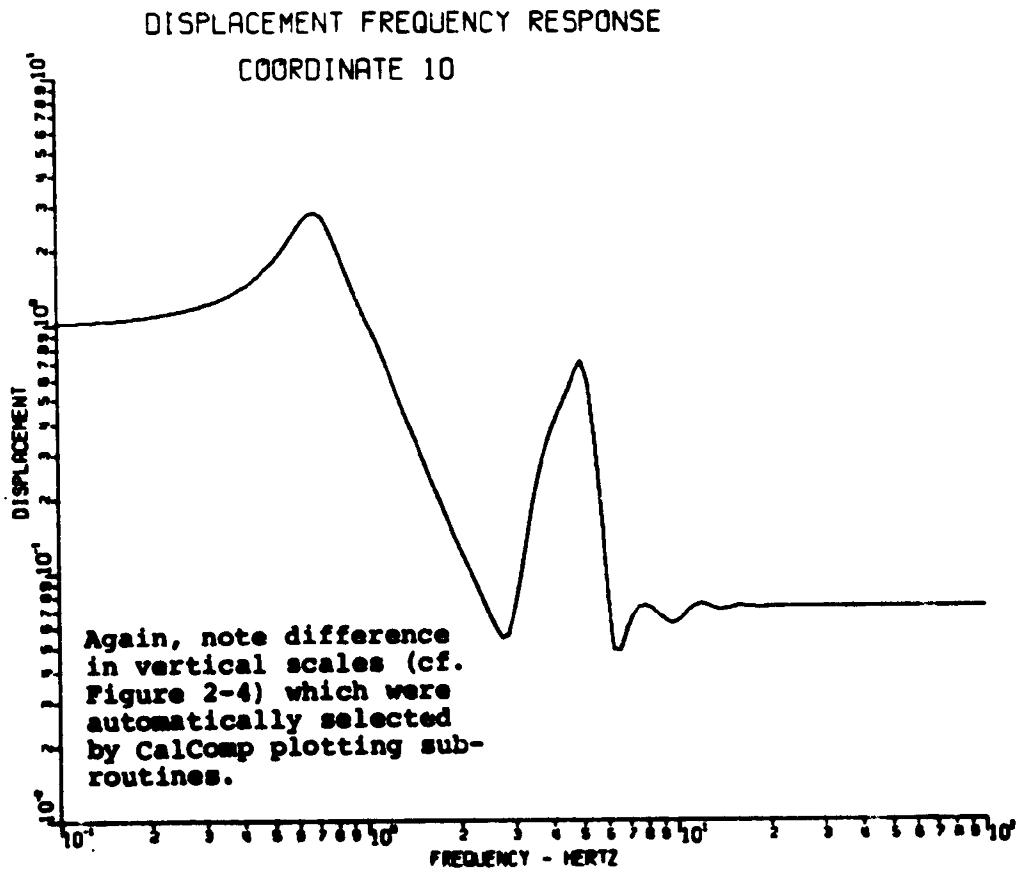


Figure 2-8. Displacement Frequency Response, Flexible Car Lateral Model, Sway at Center of Car, Four Conjugate Pairs of Modes Retained per Truck

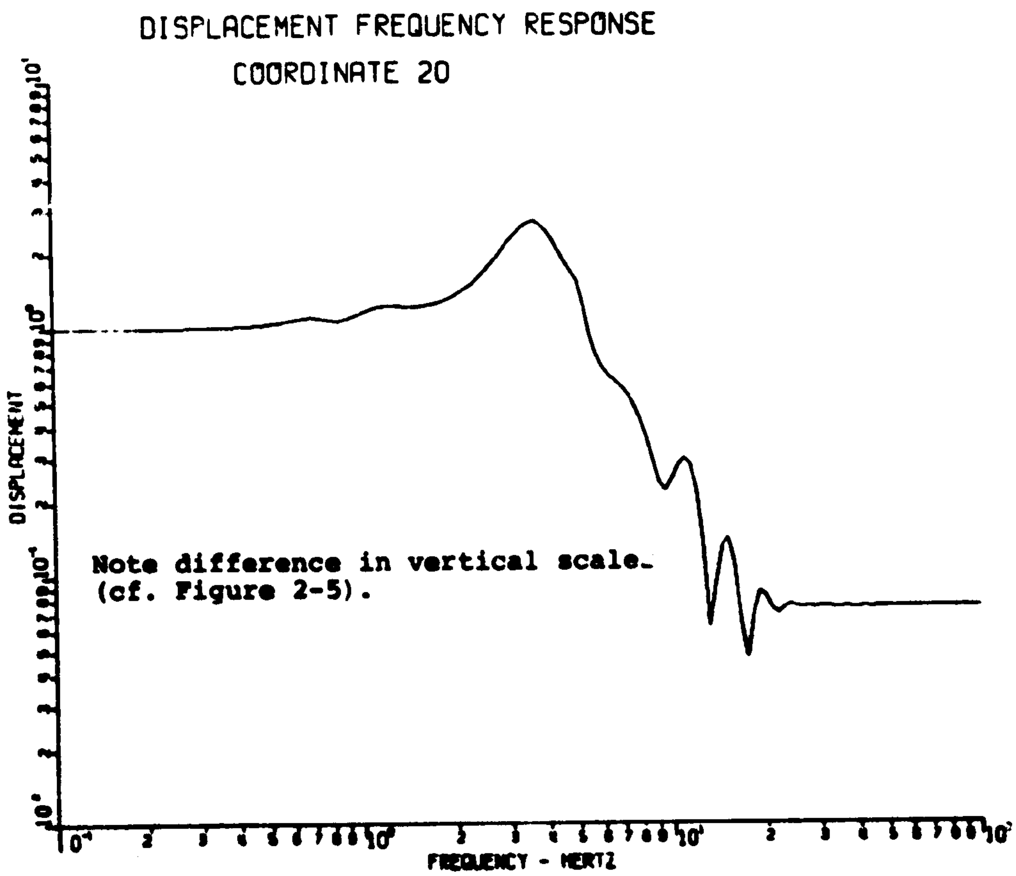


Figure 2-9. Displacement Frequency Response, Flexible Car Lateral Model, Sway at Rear Truck, Four Conjugate Pairs of Modes Retained per Truck

The notches at the high frequencies are due to canceling of forces at the wheelsets due to phase shifts. The peaks in the response of the truck at 4 and 10 Hz are both truck hunting modes.

2.6 Periodic and Transient Response

The ability to analyze the response of a dynamic system to a force which varies with frequency gives us a power which goes beyond mere frequency response. Given a random force whose power varies with frequency we can find the power spectral density of response. We can integrate this power over its frequency spectrum and compute the average response and we may also determine the probabilities that certain levels of response may be reached. We can also use the frequency response tool to analyze response to a deterministic force, if that force repeats itself as a function of time. If the period of repetition of the force is of sufficient duration then the response of the system to a transient input may be found.

There are three steps involved in computing the response of a system to a periodic excitation. First the waveform of the periodic input must be translated into a Fourier series which gives the amplitudes of the harmonics of the waveform. Using the frequency response function the response of the system at each harmonic frequency is then determined. The Fourier coefficients of the response are then converted into a response waveform from which peak response is determined. The Fourier coefficients of response are also used to determine root mean square response.

2.6.1 Fourier Excitation

A periodic excitation waveform $\delta(t)$ defined over a period of time T may be approximated in the form of a Fourier series as

$$\delta(t) = \frac{a_0}{2} + \sum_{n=1}^{n_F} (a_n \cos \omega_n t + b_n \sin \omega_n t) \qquad \ldots(2\text{-}12)$$

$$\omega_n = \frac{2\pi n}{T}$$

where ω_n are the harmonic frequencies and n_F is the number of terms to be used in the approximation. The coefficients of the cosine and sine terms, a_n and b_n, respectively, may be found using the following formulas.

$$a_n = \frac{2}{T}\int_0^T \delta(t) \cos \omega_n t \, dt \qquad n = 0,1,2,\ldots,n_F \quad \ldots(2\text{-}13a)$$

$$b_n = \frac{2}{T}\int_0^T \delta(t) \sin \omega_n t \, dt \qquad n = 1,2,\ldots,n_F \quad \ldots(2\text{-}13b)$$

The excitation waveform is input to the program as a series of amplitudes at specified times. $\delta(t)$ is assumed to be a piecewise linear function of time between the points input. Given n_δ excitation data points we may express $\delta(t)$ as

$$\delta(t) = \{c_j + d_j t \; : \; t_j \le t \le t_{j+1}; \; j=1,\ldots,n_\delta - 1\}$$

$$d_j = \frac{\delta(t_{j+1}) - \delta(t_j)}{t_{j+1} - t_j}; \qquad c_j = \delta(t_j) - d_j t_j$$

The Fourier coefficients may thus be found by integrating the piecewise linear function giving

$$\frac{a_0}{2} = \frac{1}{2T} \sum_{j=1}^{n_\delta - 1} [\delta(t_{j+1}) + \delta(t_j)](t_{j+1} - t_j) \qquad \ldots(2\text{-}14a)$$

$$a_n = \frac{2}{T} \sum_{j=1}^{n_\delta - 1} \frac{1}{\omega_n}\left[c_j \sin\omega_n t + d_j t \sin\omega_n t + \frac{d_j}{\omega_n} \cos\omega_n t\right]_{t_j}^{t_{j+1}} \qquad \ldots(2\text{-}14b)$$

$$n = 1, \ldots, n_F$$

$$b_n = \frac{2}{T} \sum_{j=1}^{n_\delta - 1} \frac{1}{\omega_n}\left[-c_j \cos\omega_n t - d_j t \cos\omega_n t + \frac{d_j}{\omega_n} \sin\omega_n t\right]_{t_j}^{t_{j+1}} \qquad \ldots(2\text{-}14c)$$

$$n = 1, \ldots, n_F$$

Fourier series of up to one hundred terms may be computed. This allows fairly complex waveforms to be analyzed. A complex wave form as might be recorded by a track geometry car will not be modeled accurately and is better represented by random excitation.

2.6.2 Fourier Response

Periodic response is computed at the physical u-coordinates of the system. To find it, the complex frequency response function, $H_u(i\omega_n)$, must be known for each of the harmonic frequencies used in the Fourier series approximation. The Fourier coefficients of response are found by multiplying the coefficients of the excitation by $H_u(i\omega_n)$. Since $H_u(i\omega_n)$ is complex, phase shifts occur in this transformation, which are accounted for in the following manner. A sine wave is a cosine wave shifted back by 90°. Since multiplying a number by i results in a shift forward of 90° we may model the excitation at the frequency ω_n as $a_n - ib_n$. This quantity is multiplied by $H_u(i\omega_n)$ and the Fourier coefficients of response a_{u_n} and b_{u_n} are obtained from the real and imaginary parts of the product.

$$a_{u_n} = \text{Real}\ [H_u(i\omega_n)\cdot(a_n - ib_n)]; \qquad n=0,1,\ldots,n_F \qquad \ldots(2\text{-}15a)$$

$$b_{u_n} = -\text{Imag}\ [H_u(i\omega_n)\cdot(a_n - ib_n)]; \qquad n=1,\ldots,n_F \qquad \ldots(2\text{-}15b)$$

The response at a physical coordinate, u(t), over one period is then obtained as

$$u(t) = \frac{a_{u_0}}{2} + \sum_{n=1}^{n_F} (a_{u_n} \cos\omega_n t + b_{u_n} \sin\omega_n t) \qquad \ldots(2\text{-}16)$$

An efficient way to compute u(t) over one period is to first evaluate the Fourier series over a quarter period for both the even and the odd terms of the series and to then combine them as follows

$$u(t) = \frac{a_{u_0}}{2} + \sum_{n=1}^{n_F} [a_{u_n} \cos\omega_n t + b_{u_n} \sin\omega_n t]$$

$$u(\tfrac{T}{2}-t) = \frac{a_{u_0}}{2} + \sum_{n=1}^{n_F} \left| (-1)^n a_{u_n} \cos\omega_n t + (-1)^{n+1} \sin\omega_n t \right|$$

$$u(\tfrac{T}{2}+t) = \frac{a_{u_0}}{2} + \sum_{n=1}^{n_F} (-1)^n \left| a_{u_n} \cos\omega_n t + b_{u_n} \sin\omega_n t \right|$$

$$u(T-t) = \frac{a_{u_0}}{2} + \sum_{n=1}^{n_F} [a_{u_n} \cos\omega_n t - b_{u_n} \sin\omega_n t]$$

$$\text{for } 0 \le t \le \frac{T}{4}$$

Doing this cuts computation for conversion to the time domain by a factor of four. Further savings are achieved by calculating the sine and cosine terms at each time increment recursively. It is also rather simple to calculate the root mean square value of response, σ_u, from the coefficients of the series. We define the dynamic R.M.S. response over one period as

$$\sigma_u^2 = \frac{1}{T}\int_0^T \left[\sum_{n=1}^{n_F} (a_{u_n} \cos\omega_n t + b_{u_n} \sin\omega_n t)\right]^2 dt \quad \ldots(2\text{-}17)$$

Note that the static term a_{u_0} has been left out since it contributes nothing to motion. By orthogonality relations this is reduced to

$$\sigma_u = \left[\frac{1}{2} \sum_{n=1}^{n_F} (a_{u_n}^2 + b_{u_n}^2)\right]^{1/2} \quad \ldots(2\text{-}18)$$

As an example of the steps in the process of computing periodic response, consider the four degree of freedom test problem shown in Figure 2-3. This oscillator has a force applied at its end mass which gives the frequency response curve of Figure 2-5. A periodic excitation is applied at the end mass in the form of a terminal peak sawtooth wave which starts at 0. and increases to 3.0 at 12 seconds. The Fourier approximation of this wave using just twenty terms in the series is shown in Figure 2-10. Note the divergence of the series at the discontinuities (endpoints) of the wave. This is known as the Gibbs phenomenon. The response of the end mass to this excitation is shown in Figure 2-11. As can be seen from the frequency response curve of Figure 2-3 the fundamental frequency of the oscillator is at 0.075 Hz. The fundamental frequency of the input is close to this at 1/12 sec = 0.0833 Hz. Thus the sawtooth wave excites mainly the first mode of the oscillator and, as may be seen from the periodic response curve, very few terms of the Fourier series are needed to adequately represent response. Most of the high frequency content of the excitation is filtered out.

2.6.3 Transient Response

To demonstrate the use of the periodic response capability in solving a transient problem the two degree of freedom problem shown in Figure 2-12 was run. The oscillator is excited at its base by a triangular impulse of duration 10 seconds followed by a pause of 90 seconds. The triangular impulse, shown in Figure 2-13 is modeled with 50 terms. The complex eigenvalue of the first mode of the oscillator is $\lambda = -.0764 + i.613$. Thus the first mode has a period of approximately 10 seconds. The damping reduces the oscillator's amplitude by approximately one half in one cycle. The pause of 90 seconds following the impulse should be enough to reduce the oscillator's motion to a negligible amount before the next impulse excites the oscillator. This is indeed the case as may be seen in Figure 2-14. It is possible to compute transient response for any system so long as the system has sufficient damping and there exists a dead zone in the excitation, following the initial transient excitation, which will allow the motion of the system to die down.

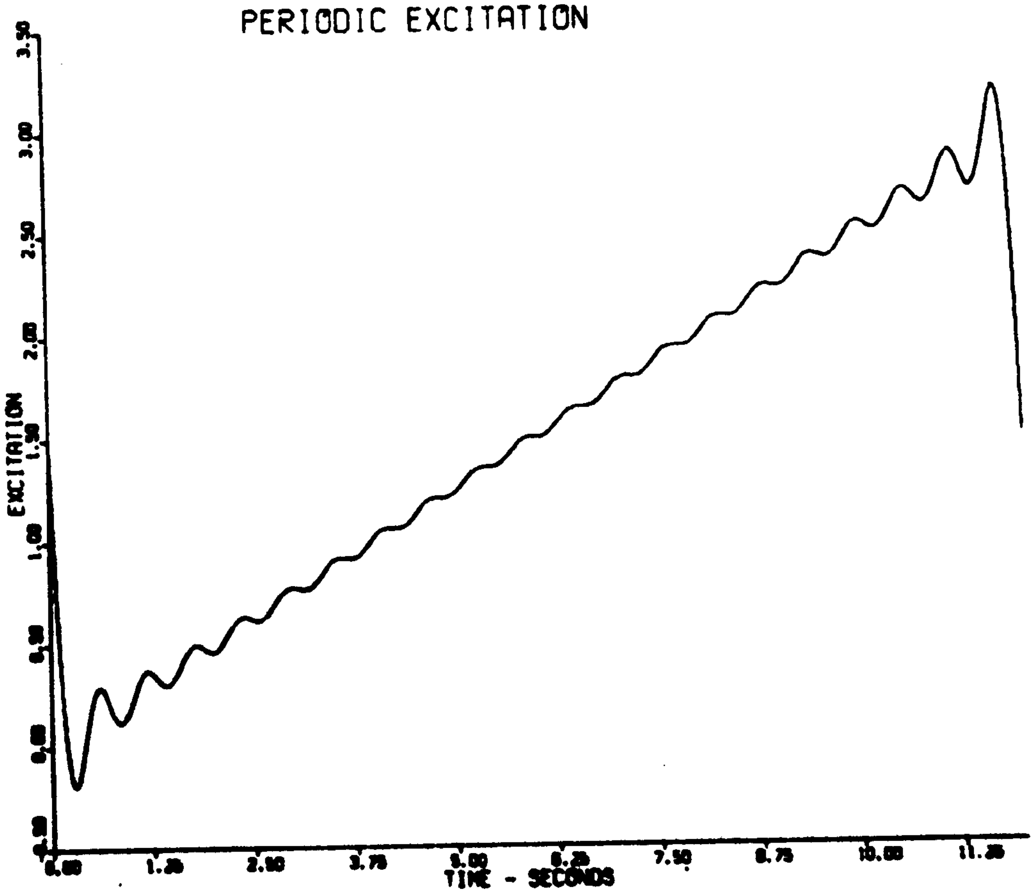


Figure 2-10. Fourier Approximation of Sawtooth Wave (20 Terms)

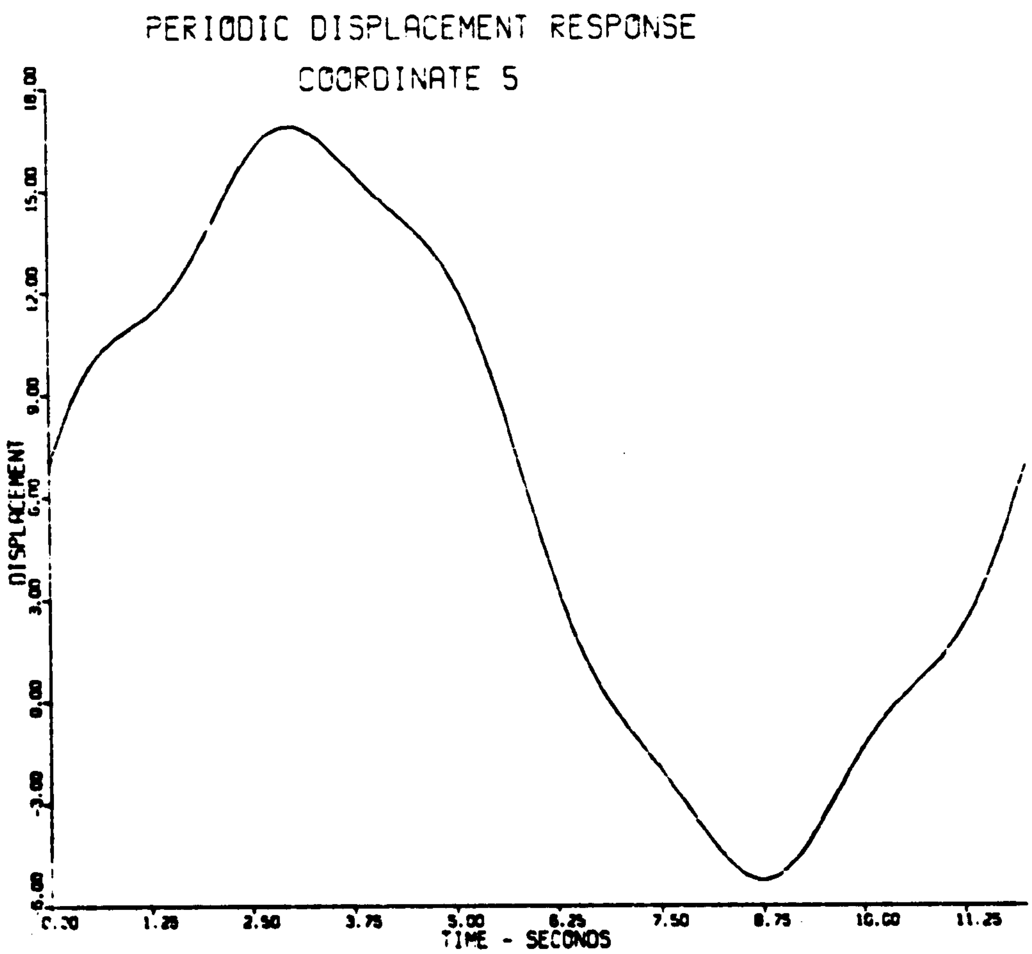


Figure 2-11. Response of End Mass of 4-DOF Oscillator to Sawtooth Wave (No Modal Truncation)

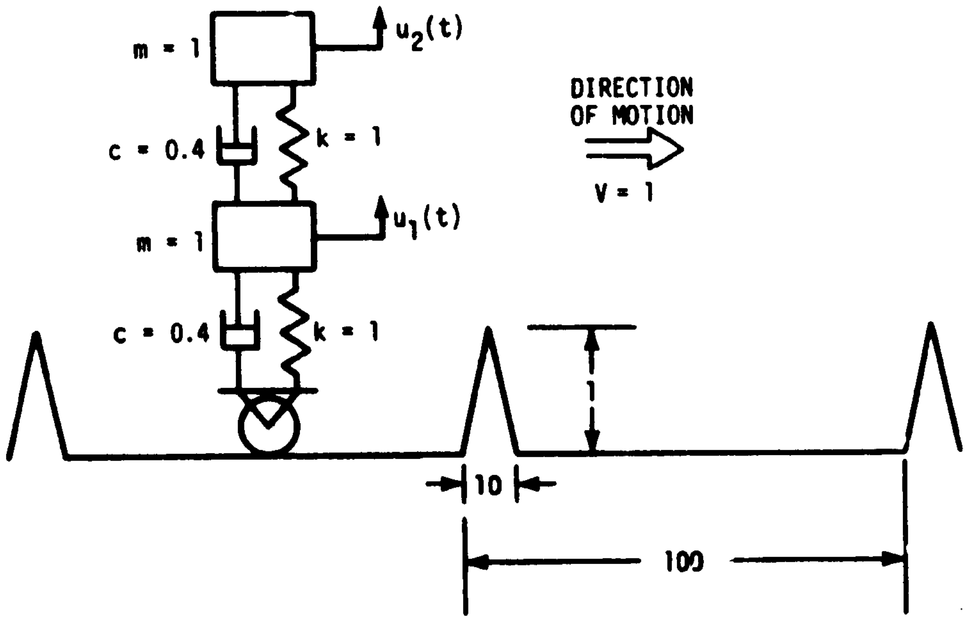


Figure 2-12. Two Degree of Freedom Transient Test Problem

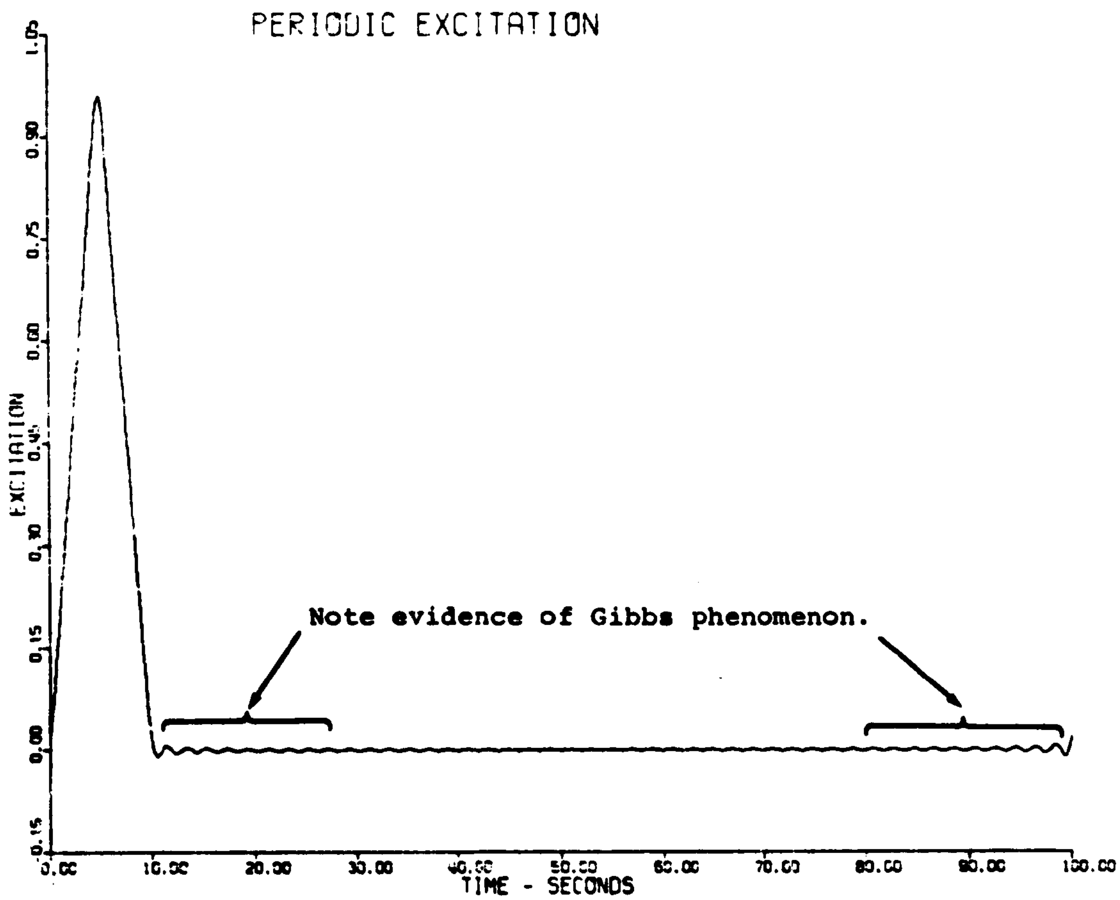


Figure 2-13. Fourier Approximation of Transient Impulse

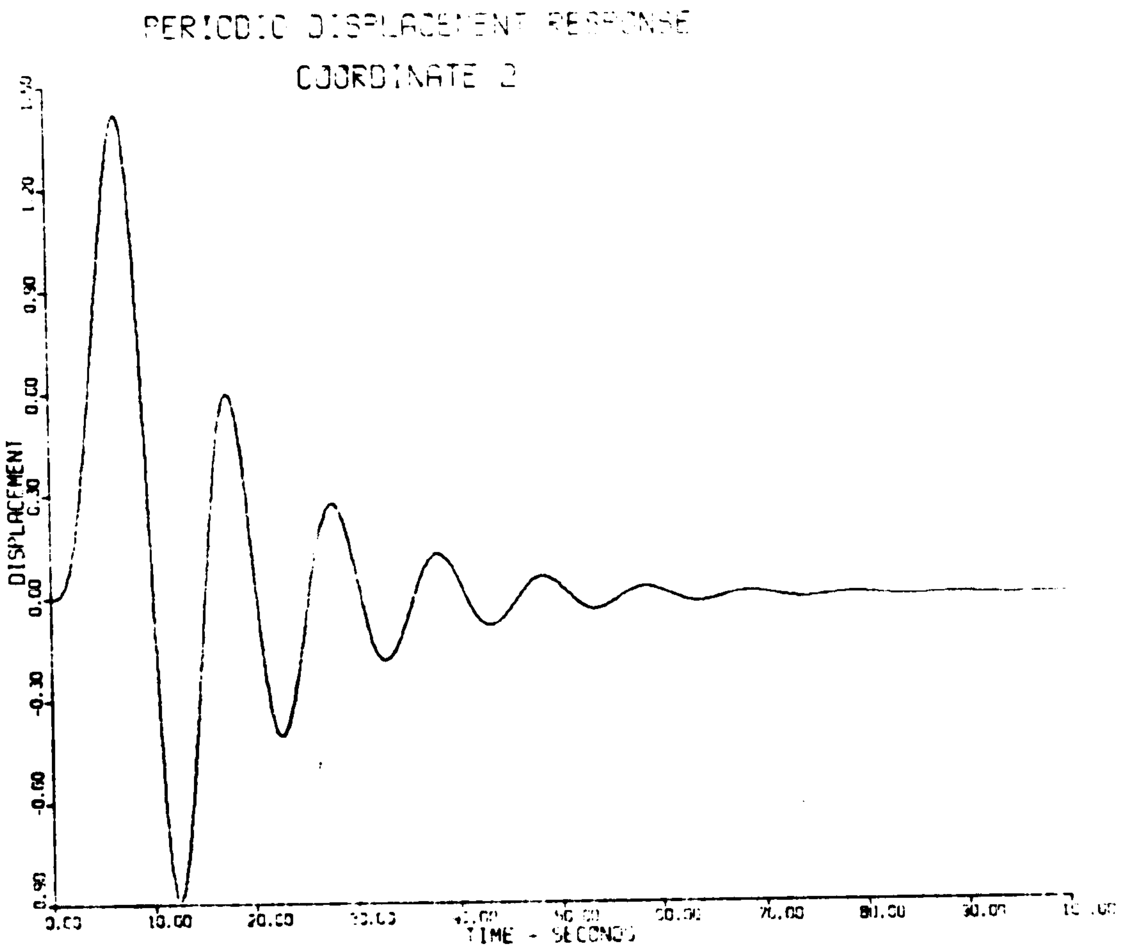


Figure 2-14. Transient Response of End Mass of 2-DOF Test Problem

For simple waveforms such as this, the Fourier series can give an accurate representation of transient response. By using up to 100 terms in the series and recognizing that most systems will attenuate the higher frequency content of the excitation, one can represent transient waveforms of considerably greater complexity and compute transient response with sufficient accuracy.

3. OTHER PROGRAM MODIFICATIONS

The present version of DYNALIST II contains many refinements over the previous version, which makes the program more general, easier to use and which gives more usable results. A brief list of some improvements is given here: printed output for frequency response now gives the frequencies in Hertz; plotted output for frequency response gives an arithmetic scale; user controlled scaling options are provided for all response plots; an upper frequency bound may be given for integration of the power spectral density function; component eigenvectors are printed in both the u- and p-coordinate systems; more of the namelist variables have been given default values. Some of the more important modifications are discussed in the next four sections.

3.1 Modeling Options

A dynamic system can be modeled in three different ways. The system may be modeled as one component, which is called the Direct System (DS) method. Two or more components may be directly linked with constraint equations without having computed any component modes, which is called the Direct Sub-Systems (DSS) method. Two or more components may have their component modes computed prior to being linked together, which is called the Modal Synthesis (MS) method. (Note that, from a response point of view, the MS method is not advantageous unless component modes are to be truncated. From a stability point of view, however, the MS method permits each component to be investigated individually.) These three modeling options may be used to fit the model to the problem being solved, attacking the problem in parts or all at once. After modeling, the system modes are computed and from these response is found. The option of truncating system modes for response computation has been removed from the program. This option was removed because it tends to introduce additional truncation error and does not allow for the solution of a larger problem than was already possible. The removal of this option also simplified the response segment of the program somewhat.

3.2 Definition of Constrained Coordinates

The prior version of the program allowed the p-coordinates which are constrained in the component eigenvalue problem to be classified as either "constraint" or "rigid body" coordinates. As there is no real mathematical significance to this distinction, they have now been lumped together under the category of "constrained" coordinates.*

3.3 Modified Sine Input

The program now allows system response to be calculated due to a sinusoidal excitation whose amplitude is a function of the excitation frequency. The sinusoidal excitation amplitude, $\delta(f)$, is input in the same fashion as the power spectral density function is. The amplitude, $\delta(f)$, is given at

*The category of constrained coordinates should be thought of as a complementary category to the free coordinates; that is, constrained = not free. Recalling that the sole purpose of the free coordinate category is to partition component matrices for the component eigenproblem solution, one should recognize that constrained coordinates are used for <u>all other</u> purposes; e.g., to constrain components whose equations of motion are already in diagonal form, or to constrain components which would otherwise be free-free and would thus have rigid body modes which DYNALIST will not compute. It is emphasized that the constraints discussed here apply only to the <u>isolated component</u> phase of analysis, and that all such constraints are <u>relaxed</u> when evaluating the motion of the <u>composite system</u>. The constraints that actually apply to the system are specified in the <u>constraint matrix</u>, G, with the resulting dependent system coordinates defined in KDEP.

discrete frequencies (Hertz) across the frequency spectrum being investigated. $\delta(f)$ is then assumed to be piecewise linear between these points. The amplitude of response, $U(if)$, is computed quite easily from the frequency response function $H_u(if)$, where $f = \Omega/2\pi$, as

$$|U(if)| = |H_u(if)|\delta(f) \qquad \ldots(3\text{-}1)$$

Examples of excitations which vary with frequency include: the forces on a vehicle traveling on an irregular roadbed where the amplitude of the irregularity changes with wavelength; and vibrational forces due to an unbalanced rotating shaft where the unbalance grows with frequency due to centrifugal effects.

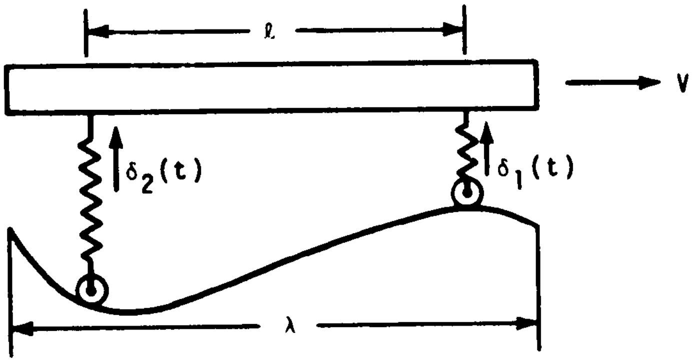
3.4 Phasing of Forces

The prior version of the program allowed the forces applied at the different coordinates to be phased with respect to the excitation, δ. The phase angles of these forces were assumed to be proportional to the frequency, and represented the lag time of the forces. To make this option more general, the present version incorporates a constant phase angle in addition to the frequency dependent phase angle. The form of the force applied at the physical coordinate u is given as

$$F_u(if) = |F_u(if)|e^{-2\pi i(\phi_0 + f\phi_1)} \qquad \ldots(3\text{-}2)$$

Where ϕ_0 represents a constant phase lag, and where $f\phi_1$ represents a frequency dependent phase lag (ϕ_1 then represents the constant time lag of the force in seconds, and f is the frequency in Hertz).

The vertical train model traveling over a track having an irregularity of wavelength λ serves to give an example of both kinds of phased inputs. From Figure 3-1, we see that the phase lag, ϕ, of the force at the trailing axle with respect to the lead axle is ℓ/λ. The frequency of the excitation is given by $f=V/\lambda$. In the case where the frequency of the excitation is varied by holding λ constant while varying train speed, V, this phase lag remains spatially constant ($\phi=\ell/\lambda=\phi_0$). Even though the bumps will be encountered at a higher frequency as speed increases, the phase lag will still be a constant portion of the irregularity wavelength. In the case where the frequency of the excitation is varied by holding train speed constant and varying the the wavelength of the irregularity instead, the phase lag becomes frequency dependent. Since $f=V/\lambda$ and $\phi=\ell/\lambda$, substituting for λ, one obtains $\phi=f\ell/V=f\phi_1$. Thus, $\phi_1=\ell/V$ represents a constant time lag of the force.



$$\delta_2(t) = \delta_1(t)e^{-2\pi i\phi} \text{ [where } \phi=\ell/\lambda]$$

Constant λ, Variable V : $\phi = \ell/\lambda = \phi_0$

Variable λ, Constant V : $\phi = \ell/\lambda = f(\ell/V) = f\phi_1$

Figure 3-1. Example of Phased Inputs for Vertical Train Model

4. CONCLUSIONS AND RECOMMENDATIONS

DYNALIST II has been further modified and extended during the present effort to enhance its effectiveness from a user standpoint. New modeling capabilities and response options have been added and many significant improvements have been made to the program. Conclusions and recommendations resulting from this effort are summarized below.

4.1 Conclusions

Three major new capabilities have been added. The building block approach from which the program was originally designed has been extended from a two tier to a three tier logical structure. Basic elements including rigid bodies, flexural bodies, wheelsets, springs, dampers, and nodal masses can now be used to synthesize linear, three-dimensional component models. Components, in turn, are used to synthesize complete track/train systems.

A periodic response capability has been added to enable the computation of response anywhere in the system to a periodic waveform input in tabular form. RMS response as well as time histories for a typical period are computed.

The periodic response capability is sufficiently general to provide a transient response capability also. Using a Fourier series approximation of the Fourier integral approach, transient waveforms can be specified with up to 100 points, and response is computed on the basis of up to 100 terms of a Fourier series. Points selected to specify the input waveform can be arbitrarily spaced.

The current version of DYNALIST is more user oriented. The matrix coefficients of the equations of motion can be automatically generated by specifying the system topology and the basic elements comprising the system (springs, dampers, wheelsets, masses, etc.), as opposed to the previous version which required the matrix coefficients to be input directly. Wheel/rail force inputs and phase lag terms are now generated automatically instead of having to be user input. Plots can be scaled by the user and are labeled in arithmetic rather than logarithmic units. Parametric studies are facilitated by a new component update capability. In addition, recent testing of the program has led to the development of user guidelines which are helpful in the formulation of practical problems.

Computational improvements have also been made. The Direct System and Direct Subsystem options have been debugged and are now fully operational. These options provide useful alternatives to the Modal Synthesis option.

An eigenvector orthogonality check is now made automatically to confirm diagonalization of the state equations. If the computed eigenvalues are all stable and the associated eigenvectors diagonalize the equations, subsequent response calculations can be relied upon.

In the previous version of the program, modal truncation sometimes resulted in a residual error at the low frequency end of the displacement response spectrum. This residual term is now computed internally and added to the truncated mode solution to guarantee convergence at low frequencies. Since computation of RMS response for stationary random inputs requires integration over a range of frequencies, it is sometimes desirable to specify an upper limit on this range separately from the range of computed frequency response and response PSD, particularly when the use of modal synthesis introduces truncation error in the high frequency range. This capability has been added.

4.2 Recommendations

Based on experience gained with DYNALIST II to date, the following recommendations are made with regard to its future use:

- Observe the guidelines listed in Section 2.3.4 of this report with regard to problem formulation.
- Consider the advantages of the Direct modeling options as opposed to Modal Synthesis when running small systems, systems with many real roots, or systems which do not lend themselves to natural subdivision into components with at least six degrees of freedom.
- Consider the advantages of Modal Synthesis for modeling large systems, particularly trains consisting of several cars where high frequency truck and wheel modes can be truncated for each car, or for a car model having non-rigid truck assemblies and many degrees of freedom. The truncation of high frequency component modes can result in significant computational savings without sacrificing needed accuracy.
- Ensure the stability of system eigenvalues and the orthogonality of system eigenvectors in verifying response computations.

Finally, it should be recognized that DYNALIST II provides the modaling capability to include track structure as part of a track/train system. Either lumped parameter or distributed parameter track models can be specified. The static deformation of track structure under nominal wheel loading can be used to define distributed (or modal) coordinates for use in dynamic analysis based on Rayleigh-Ritz principles.

As experience with this program grows, it is anticipated that many new applications will emerge. User feedback will continue to enhance its practical worth.

APPENDIX A

CHARACTERIZATION OF FORCING FUNCTIONS

Some of the user feedback on DYNALIST II has revealed that the form of the forcing function built into the program is not immediately apparent, and at least to some extent, confusion has been generated as a consequence. This appendix has been added in an attempt to clarify this area. Whereas an introduction to the forcing function in Reference [1] was by way of specific examples, the approach taken here is to begin with the general and then proceed to the specific. Hopefully this treatment will help to resolve some of the difficulty.

In pursuing this objective, some different notation will be introduced. While it is usually desirable to maintain the same notation within a given subject area, the changes here are to some degree necessary for the generalization, and are furthermore intended to interrupt any thought patterns which may have lead to the initial confusion. In this regard, an attempt is made to recast the subject in a different light. Thus, the forces discussed here can be visualized as wheel/rail forces acting on a train, seismic forces on a building, hydrodynamic forces on a ship, or simple point forces acting on a beam. In fact, different examples will be presented to illustrate the generality of the forcing function capability within DYNALIST II.

A.1 General Form

Before taking up the characterization of a particular force environment such as seismic or wave, it will be useful to consider the general form of the forcing function built into DYNALIST II. In simplest terms, a distributed force, $f(u,t)$, where u denotes position or spatial dependency, and t denotes time dependency, is assumed to be variable-separable so that

$$f(u,t) = P(u)g(t) \tag{A-1}$$

where $P(u)$ is a function depending only on position (not to be confused with the lower case "p" used to denote generalized coordinates) and $g(t)$ is a scalar function depending only on time.

Actually, the functional form used in DYNALIST II is somewhat more general. In particular,

$$f(u,t) = \sum_{k=0}^{2} P^{(k)}(u)\frac{d^k}{dt^k}[g(t)] \tag{A-2}$$

Since DYNALIST II is formulated on the basis of discrete variables rather than continuous variables, a vector form is used instead of (A-2):

$$\{f_u(t)\} = \sum_{k=0}^{2} \{P_u{}^{(k)}\} \frac{d^k}{dt^k} [g(t)]$$

$$= \{P_u{}^{(0)}\} g(t) + \{P_u{}^{(1)}\} \dot{g}(t) + \{P_u{}^{(2)}\} \ddot{g}(t) \tag{A-3}$$

Typical force distributions are shown in Figure A-1, for example. In part (a) of that figure, a cantilever beam is subjected to a distributed load which varies sinusoidally with time, similar

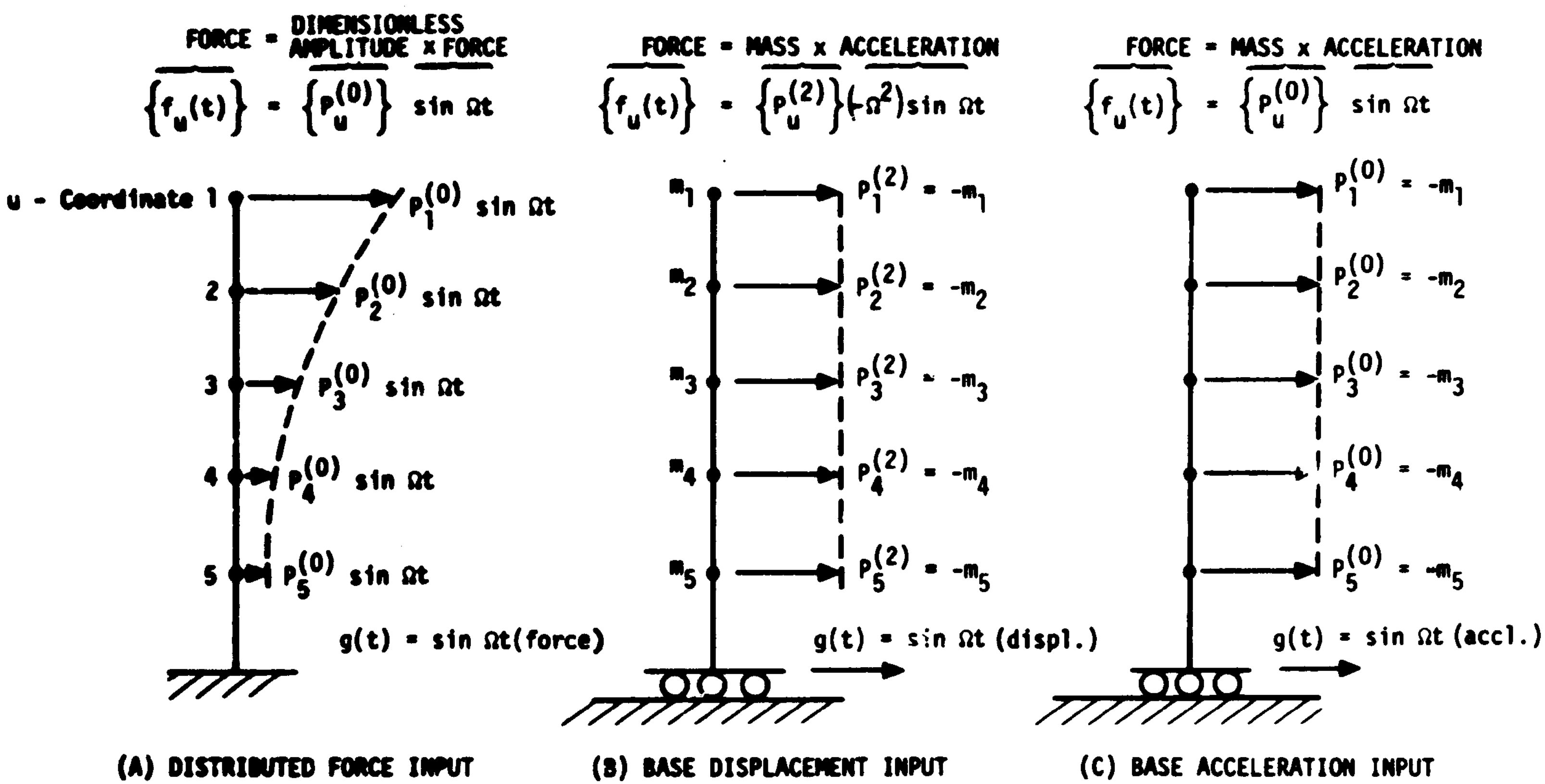


Figure A-1. Examples of Typical Force Vectors Used in DYNALIST II to wave action on a pier. In part (b) the cantilever beam is subjected to base motion whose waveform is defined in terms of displacement. In part (c) the same beam is subjected to base motion whose waveform is defined in terms of acceleration. In all cases, equations of motion are of the form

$$[m]\{\ddot{u}\} + [c]\{\dot{u}\} + [k]\{u\} = \{f_u(t)\} \quad . \qquad \text{(A-4)}$$

where the u-coordinates numbered 1 through 5 denote lateral displacements of the beam relative to its base, and where in parts (b) and (c) the inertial forces due to base acceleration have been transposed to the right hand side of the equations.

A.2 Frequency Domain Analysis

DYNALIST II solves equations of motion in the frequency domain, rather than in the time domain. This makes it convenient to use Equation (A-3) in a form where the vectors $\{P_u^{(0)}\}$, $\{P_u^{(1)}\}$ and $\{P_u^{(2)}\}$ are complex, i.e. have both real and imaginary parts. In other words, the force distribution functions allow phase distribution in addition to amplitude distribution. In order to pursue this discussion further, it is helpful to consider the equations of motion given by (A-4) and transform them to the frequency domain. This is done by first taking the Laplace transform of the equations, which maps them from the time variable t into the complex variable $s = \sigma + i\Omega$, and then let $\sigma \rightarrow 0$. The resulting equations in the frequency domain are

$$\left([k] + i\Omega[c] - \Omega^2[m]\right)\{U(i\Omega)\} = \{F_u(i\Omega)\} \qquad \text{(A-5)}$$

where $\{U(i\Omega)\}$ and $\{F_u(i\Omega)\}$ denote the transformed vectors $\{u(t)\}$ and $\{f(t)\}$ respectively. The vectors $\{U(i\Omega)\}$ and $\{F(i\Omega)\}$ are the Fourier transforms of their time dependent counterparts whenever the Fourier integrals exist.* Transformation of $\{f_u(t)\}$, as expressed in (A-3), to the frequency domain leads to

$$\{F_u(i\Omega)\} = \left(\{P_u^{(0)}\} + (i\Omega)\{P_u^{(1)}\} + (i\Omega)^2\{P_u^{(2)}\}\right) c(i\Omega) \qquad \text{(A-6)}$$

where the P_u-vectors are still complex. Finally the P_u-vectors are resolved into amplitude and phase vectors of the form

$$\{P_u^{(0)}\} + (i\Omega)\{P_u^{(1)}\} + (i\Omega)^2\{P_u^{(2)}\}$$

$$= \left(\{\bar{P}_u^{(0)}\} + (i\Omega)\{\bar{P}_u^{(1)}\} + (i\Omega)^2\{\bar{P}_u^{(2)}\}\right)e^{-i[\{\theta_u^{(0)}\} + \Omega\{\theta_u^{(1)}\}]} \qquad \text{(A-7)}$$

where the "P-bars" are now real and the phase shifts implied by $\{\theta_u^{(0)}\}$ and/or $\{\theta_u^{(1)}\}$ operate on the three force vectors - $\{\bar{P}_u^{(0)}\}$, $\{\bar{P}_u^{(1)}\}$ and $\{\bar{P}_u^{(2)}\}$ - simultaneously.** The superscript notation (k), where k=0,1,2, may be associated with multiplication by Ω to kth power. See examples which follow in Tables A-1 and A-2.

A.3 Position Dependency (Force Distribution)

The frequency domain formulation given in the preceding section is particularly well suited to the dynamic response analysis of systems subjected to wave environments, either traveling waves or standing waves, where the forces acting on different parts

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*Although the Fourier integral transform of a periodic function is not defined, a Fourier series representation does exist so that the formal treatment given here is physically meaningful and practically useful.

**If at a particular u-location, multiple forces exist which have different phases, then the simplest approach would be to multiply define identical u-coordinates at that location, and to apply one force to each such coordinate.

of the system may all be assumed to have the same time dependency, but have different amplitudes and phase angles. Recalling the rail-vehicle application for which the program was originally designed, one may visualize forces acting on the vehicle through each wheel. In the two-dimensional case where differences between rails are ignored, one can show that the driving forces acting on all of the wheels have the same time dependency as determined by the velocity of the vehicle and the profile of the rail surface. Aside from differences in wheel geometry, mass, and suspension parameters which are system dependent and show up in the force distribution, each trailing wheel sees the same input as the lead wheel with a time lag equal to its distance behind the lead wheel divided by the forward velocity of the vehicle, i.e., ℓ/V. In the frequency domain, this lag term is represented by the phase angle vector $\Omega\{\theta_u^{(1)}\}$ where Ω is the circular frequency associated with the wavelength of the rail surface profile irregularity and the vehicle's forward velocity.

A similar situation prevails in the case of an offshore platform excited by traveling sea waves. If the wave displacement profile is represented by the scalar time function g(t), the wave forces acting on the platform can be separated into

- Hydrostatic forces $\{P_u^{(0)}\}$
- Drag forces $\{P_u^{(1)}\}$
- Inertia forces $\{P_u^{(2)}\}$

and their phasing relative to wherever g(t) is measured will be specified by the phase angle vector $\Omega\{\theta_u^{(1)}\}$, where in this case Ω is associated with wave length and propagation velocity (celerity), and $\{\theta_u^{(1)}\}$ will again be of the form ℓ/V. This application is illustrated by Figure A-2 and Table A-1, except in this particular case, the hydrostatic forces are zero.

If the platform were situated in a standing wave environment, the phase angle distribution would be specified in terms of $\{\theta_u^{(0)}\}$ instead of $\{\theta_u^{(1)}\}$. Another example of the use of $\{\theta_u^{(0)}\}$ is in the vibration of machinery due to rotor unbalance, where the relative phase angles among different eccentric masses are independent of the frequency of rotation, Ω.

Force distribution is input to DYNALIST II via the NAMELIST variable arrays

FORCØ = $\{P_u^{(0)}\}$

FORC1 = $\{P_u^{(1)}\}$

FORC2 = $\{P_u^{(2)}\}$

PHASØ = $\{\theta_u^{(0)}\}/2\pi$ (units in cycles)

PHAS1 = $\{\theta_u^{(1)}\}$ (units in seconds)

as explained in Reference [2].

A.4 Time Dependency (Waveform)

It has already been stated that DYNALIST II offers the capability to compute dynamic response to

- Sinusoidal,
- Periodic,
- Transient, and
- Random

forcing functions. These categories relate to different ways of characterizing the time dependency of $\{f_u(t)\}$ as embodied in the scalar waveform function $g(t)$. Figure A-3 illustrates

TIME DOMAIN FORCES

$$\{f_u(t)\} = \{P_u^{(1)}\} \dot{g}(t) + \{P_u^{(2)}\} \ddot{g}(t)$$

$$g(t) = \begin{cases} \text{SINUSOIDAL} \\ \text{PERIODIC} \\ \text{TRANSIENT} \\ \text{RANDOM} \end{cases}$$

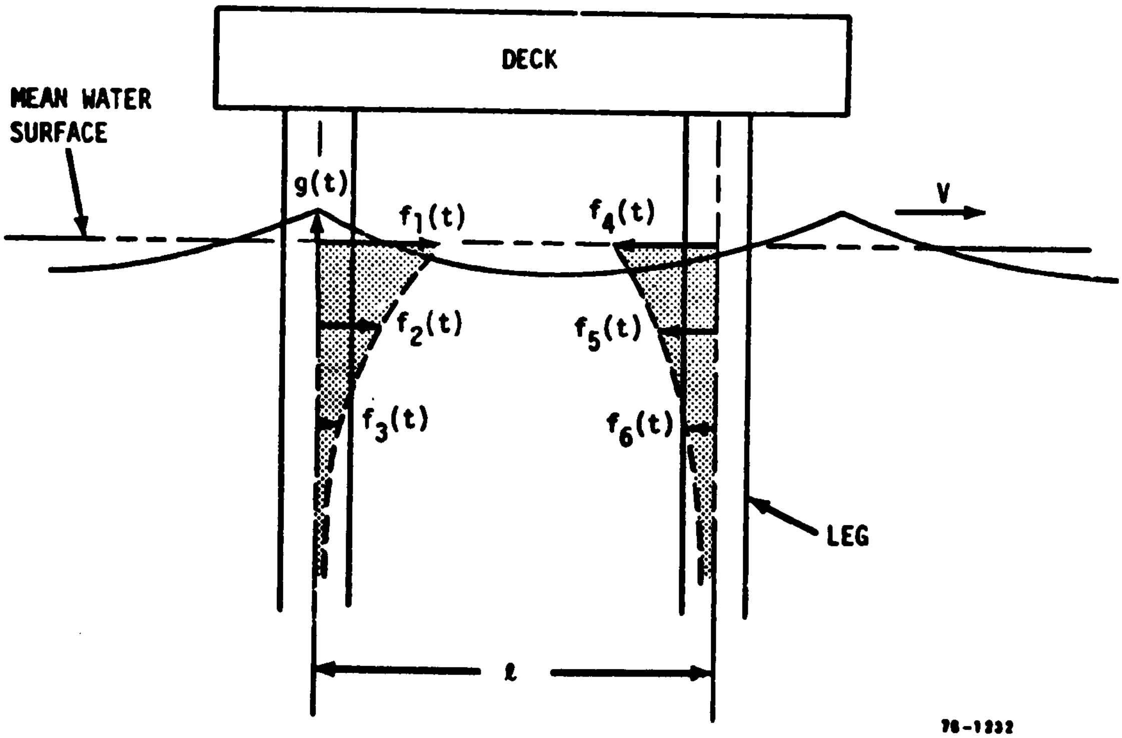


Figure A-2. Force Distribution for Wave Excitation

TABLE A-1. CHARACTERIZATION OF WAVE FORCES IN FIGURE A-2

FREQUENCY DOMAIN FORCES

$$F_1(i\Omega) = \left[(i\Omega)\bar{P}_1^{(1)} + (i\Omega)^2\bar{P}_1^{(2)}\right] G(i\Omega)$$

$$F_2(i\Omega) = \left[(i\Omega)\bar{P}_2^{(1)} + (i\Omega)^2\bar{P}_2^{(2)}\right] G(i\Omega)$$

$$F_3(i\Omega) = \left[(i\Omega)\bar{P}_3^{(1)} + (i\Omega)^2\bar{P}_3^{(2)}\right] G(i\Omega)$$

$$F_4(i\Omega) = \left[(i\Omega)\bar{P}_4^{(1)} + (i\Omega)^2\bar{P}_4^{(2)}\right] e^{-i\Omega\ell/V} G(i\Omega)$$

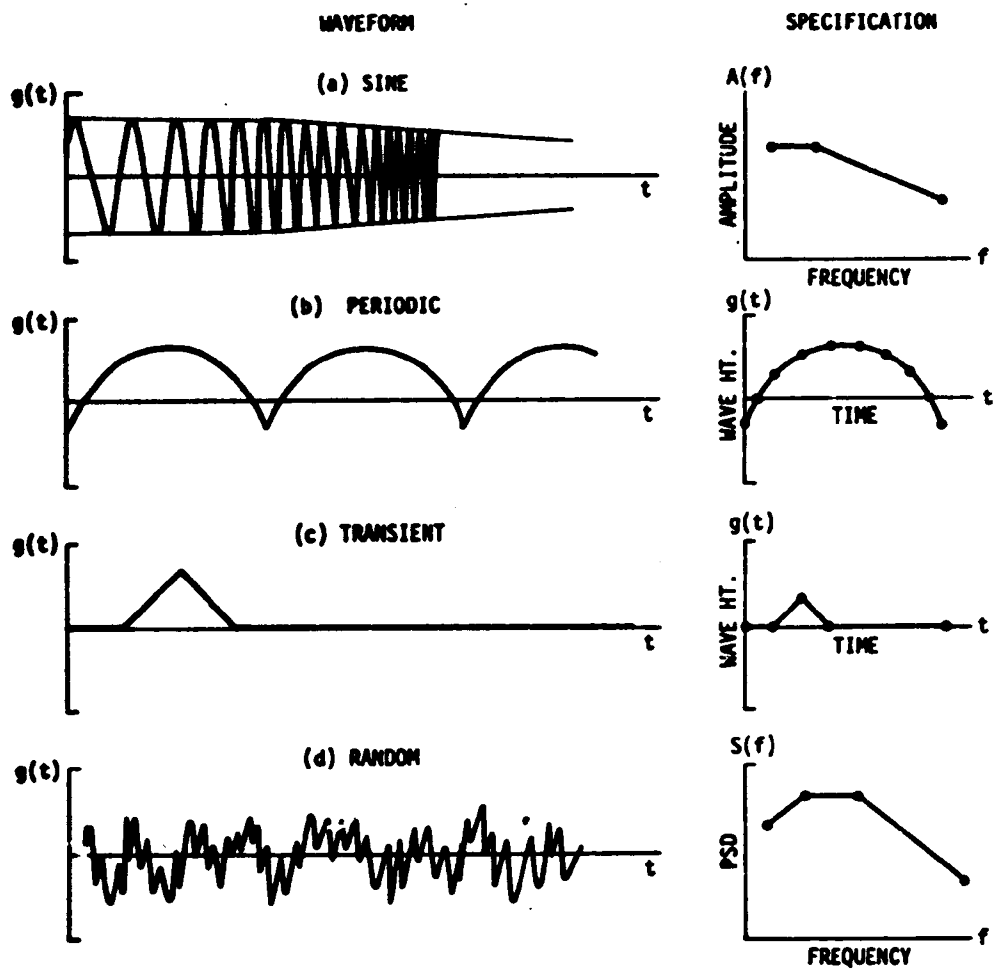
$$F_5(i\Omega) = \left[(i\Omega)\bar{P}_5^{(1)} + (i\Omega)^2\bar{P}_5^{(2)}\right] e^{-i\Omega\ell/V} G(i\Omega)$$

$$F_6(i\Omega) = \left[(i\Omega)\bar{P}_6^{(1)} + (i\Omega)^2\bar{P}_6^{(2)}\right] e^{-i\Omega\ell/V} G(i\Omega)$$

FORCE DISTRIBUTION INPUT TO DYNALIST II

u-COORD	FORC0	FORC1	FORC2	PHAS0	PHAS1
1	0	$\bar{P}_1^{(1)}$	$\bar{P}_1^{(2)}$	0	0
2	0	$\bar{P}_2^{(1)}$	$\bar{P}_2^{(2)}$	0	0
3	0	$\bar{P}_3^{(1)}$	$\bar{P}_3^{(2)}$	0	0
4	0	$\bar{P}_4^{(1)}$	$\bar{P}_4^{(2)}$	0	ℓ/V
5	0	$\bar{P}_5^{(1)}$	$\bar{P}_5^{(2)}$	0	ℓ/V
6	0	$\bar{P}_6^{(1)}$	$\bar{P}_6^{(2)}$	0	ℓ/V

Note: If the two legs shown in Figure A-2 are identical, then $F_4 = F_1$, $F_5 = F_2$, and $F_6 = F_3$.



NAMELIST VARIABLES

A(f) = AMP
f = CYCLS

[If A(f) = Constant, then Frequency Response Option may be used with constant scale factor in mind.]

g(t) = WAVE
t = TIME

g(t) = WAVE
t = TIME

S(f) = AMP
f = CYCLS

Figure A-3. Waveform Specifications in DYNALIST II

the four basic waveform input options, and the means by which they are entered in DYNALIST II.

In the case of the amplitude varying sinusoidal waveform, steady state response is assumed. Thus, even though the waveform in Part (a) of the figure is actually changing with respect to both frequency and time, the implication is that the variations with time are slow so that the system is able to "track" the variations with virtually steady state response. Input to the program can therefore be specified in terms of excitation amplitude vs. frequency as shown in the figure.

Periodic excitation is specified in terms of the typical waveform over one fundamental period of the excitation. Thus, where the actual waveform repeats itself over every successive period, all of the information is contained in one period of the motion. The same argument applies to response. Only one peroid of the response time-history is computed, and only one period is plotted.

Transient excitation and response is treated by Fourier series approximation and is therefore considered to be a periodic function with a very long fundamental period as illustrated in Part (c) of the figure. The length of this period is artificial and is chosen on the basis of the time constants of the system, i.e., natural frequencies and damping rates. The artificial period must be long enough to allow all motion from one transient to damp out before another transient (in a successive period) is encountered. Since transient excitation and response are treated as periodic functions, program input and output are identical in form to the periodic case. The program in fact recognizes no distinction between the two.

Random excitation is assumed to be stationary. It is specified in terms of the power spectral density (PSD) function as illustrated in Part (d) of Figure A-3. Since the waveform is random, stationary and assumed to be ergodic (establishing statistical equivalence between one sample function and an ensemble), no phase information and no particular wave shape characteristics are implied. Only the power (energy type information) distribution with respect to frequency is specified.

A.5 Notation

Now that the general form of the forcing function has been developed in terms of some different notation, we can go back to relate the original notation of Reference [1] to the notation of this appendix.

The system force vector $\{f_u(t)\}$ shown in Equation (A-4) of this appendix is related to the ℓth component force vector $\{f_u^\ell(t)\}$ of Reference [1], Equation (2-5) as follows

$$\{f_u(t)\} = \begin{Bmatrix} f_u^1(t) \\ f_u^2(t) \\ \vdots \\ f_u^\ell(t) \\ \vdots \\ f_u^N(t) \end{Bmatrix} ; \quad \text{where } f_u^\ell(t) \equiv \{f_u^\ell(t)\} \quad \text{for } 1 \leq \ell \leq N$$

The Fourier transform of $\{f_u(t)\}$ as denoted by $\{F_u(i\Omega)\}$ in (A-5) is similarly related to the $\{F_u^{\ell}(i\Omega)\}$ in Reference [1], Equation (2-37) by

$$\{F_u(i\Omega)\} = \begin{Bmatrix} F_u^1(i\Omega) \\ F_u^2(i\Omega) \\ \vdots \\ F_u^{\ell}(i\Omega) \\ \vdots \\ F_u^N(i\Omega) \end{Bmatrix} ; \text{ where } F_u^{\ell}(i\Omega) \equiv \{F_u^{\ell}(i\Omega)\} \text{ for } 1 \leq \ell \leq N$$

In the case where the excitation waveform corresponds to the rail irregularity (track geometry profile) $\delta(t) = \delta_1^{\ell}(t)$* as implied by Reference [1], Equation (2-34), it follows that the function g(t) in (A-3) becomes

$$g(t) = \delta(t) \equiv \delta_1^{\ell}(t)$$

Similarly, in the frequency domain,

$$G(i\Omega) = \Delta(i\Omega) \equiv \Delta_1^{\ell}(i\Omega)$$

* This equality implies that the general waveform $\delta(t)$ is measured with respect to a particular axle "1" of a particular component "ℓ", i.e., a fixed point on the vehicle.

This "G" of course bears no relationship to the constraint matrix "G" used in Reference [1], Equation (2-1).

In the case of the train application, the complex vectors $\{P_u^{(0)}\}$, and $\{P_u^{(1)}\}$ and $\{P_u^{(2)}\}$ of (A-3) are

$$\{P_u^{(0)}\} \equiv \{F_0(i\Omega)\}$$

$$\{P_u^{(1)}\} \equiv \{F_1(i\Omega)\}$$

$$\{P_u^{(2)}\} \equiv \{F_2(i\Omega)\}$$

where the terms on the right hand side appear in Reference [1], Equation (2-38). Again, these capital "P's" bear no relation to the lower case "p's" of Reference [1], Equation (2-14).

Referring back to the truck example of Reference [1], Figure 2-2, and the force vector given by Equation (2-37) which follows that figure, we find that we can generate another table analogous to Table A-1, for the truck model. See Table A-2. Either the sine, periodic, transient or random type waveform input can be specified for this force distribution.

TABLE A-2. CHARACTERIZATION OF WHEEL/RAIL FORCES IN REFERENCE [1], FIGURE 2-2

FREQUENCY DOMAIN FORCES

$$F_1(i\Omega) = 0$$

$$F_2(i\Omega) = P_1^{(0)}\ G(i\Omega) = \left(\frac{2f\ \lambda_o\ L_o}{r_o}\right) \Delta(i\Omega)$$

$$F_3(i\Omega) = 0$$

$$F_4(i\Omega) = 0$$

$$F_5(i\Omega) = 0$$

$$F_6(i\Omega) = P_6^{(0)}\ e^{-i\Omega L/V}\ G(i\Omega) = \left(\frac{2f\ \lambda_o\ L_o}{r_o}\right)\ e^{-i\Omega L/V}\ \Delta(i\Omega)$$

FORCE DISTRIBUTION INPUT TO DYNALIST II

u-COORD	FORC0	FORC1	FORC2	PHAS0	PHAS1
1	0	0	0	0	0
2	$\left(\frac{2f\ \lambda_o\ L_o}{r_o}\right)$	0	0	0	0
3	0	0	0	0	0
4	0	0	0	0	0
5	0	0	[illegible]	0	0
6	$\left(\frac{2f\ \lambda_o\ L_o}{r_o}\right)$	0	0	0	L/V

APPENDIX B

REPORT OF INVENTIONS

In accordance with the patent rights clause of the terms and conditions of this contract, and after comprehensive review of the work performed, it was found that no new patentable items were produced under this contract. However, significant innovations and improvements were made relative to the DYNALIST computer program and its documentation, as summarized in Section 1 of this volume. In particular these include: (1) a component matrix generator which operates as a 3-D finite element modeling program where elements consist of rigid bodies, flexural bodies, wheelsets, suspension elements, and point masses assembled on a nodal skeleton; (2) a periodic and transient time-history response capability; (3) a component update capability for parametric studies; (4) an orthogonality check on component and system complex eigenvectors; (5) an option for improving low-frequencey convergence under modal truncation; (6) a more general sine-amplitude forcing function capability; (7) automatic phase lag generation; (8) user controlled scaling options on all response plots; and a number of additional minor improvements. The overall utility of the program has been enhanced accordingly.

REFERENCES

1. Hasselman, T. K., Bronowicki, Allen, and Hart, Gary C., "DYNALIST II - A Computer Program for Stability and Dynamic Response Analysis of Rail Vehicle Systems, Volume I: Technical Report," Report No. FRA-OR&D-75-22.I, prepared for the U. S. Department of Transportation, Federal Railroad Administration, Office of Research and Development, February 1975.

2. Bronowicki, Allen, and Hasselman, T. K., "DYNALIST II - A Computer Program for Stability and Dynamic Response Analysis of Rail Vehicle Systems, Volume IV: Revised User's Manual," Report No. FRA-OR&D-75-22.IV, prepared for the U. S. Department of Transportation, Federal Railroad Administration, Office of Research and Development, July 1976.