EFFECT OF TORSIONAL FASTENER RESISTANCE ON THE LATERAL RESPONSE OF A RAIL-TIE STRUCTURE

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INTERIM REPORT

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16. Abstract

The use of the classical beam bending equations for the analysis of the track response in the lateral plane is of questionable validity, when the used fasteners exhibit a noticeable torsional resistance. To remedy this situation, recently a variety of other track equations were proposed and used. The purpose of the present study is to establish the effect of fastener resistance on the lateral response of the rail-tie structure and also to determine whether a fourth order differential equation, which includes a rotational resistance term, is sufficiently accurate for describing its lateral response. To achieve this aim deflection tests were conducted on a rail-tie structure with adjustable fastener rigidities, then this test-structure was analyzed using a fourth order equation with and without a rotational resistance term, and subsequently the analytical and tests results were compared. The test results revealed that with an increasing rotational resistance of the fasteners, the deviation of the test curves, from the case of zero fastener resistance, also increases; thus, the beam bending equation is not suitable, in general, for the analysis of the lateral track response. The comparison of the analytical and test result showed that the measured deflection shapes of the test structure, for a variety of fastener rigidities, agree closely with the deflection shapes obtained using a fourth order differential equation which includes a rotational resistance term, provided the coefficient of this additional term contains the effect of the fastener rigidity and the bending rigidity of the cross-ties.

17. Key Words Railroad Track Rail-Tie Structure Fasteners Lateral Deformations of Track Railroad Track Equations	18. Distribution Statement Document Available from: National Technical Information Service Springfield, Virginia 22151

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PREFACE

This report was prepared as part of the contract
DOT-TSC-1149. It is a partial result of a research effort
whose aim is to create a basis for the rational design, construction,
and maintenance of railroad tracks. This research program was
sponsored by the Federal Railroad Administration, Office of Research
and Developemnt, with the Transportation Systems Center as program
manager. Dr. Andrew Kish, was the technical monitor.

The present report contains an experimental and analytical
study of the lateral response of the rail-tie structure. The
purpose of this study is to determine the proper equations needed
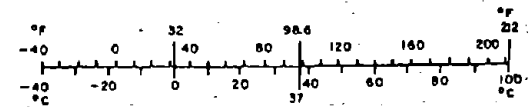for the analysis of the lateral track response.

METRIC CONVERSION FACTORS
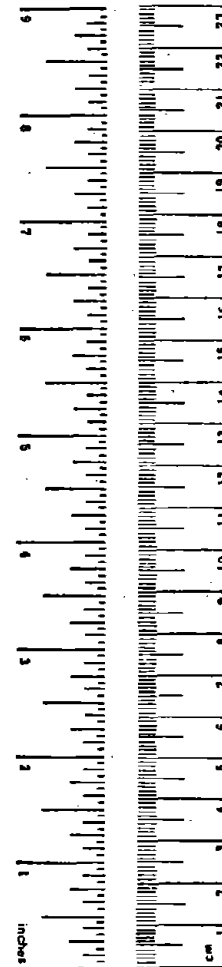
Approximate Conversions to Metric Measures

Symbol	When You Know	multiply by	to Find	Symbol
		LENGTH		
in	inches	2.5	centimeters	cm
ft	feet	30	centimeters	cm
yd	yards	0.9	meters	m
mi	miles	1.6	kilometers	km
		AREA		
in^2	square inches	6.5	square centimeters	cm^2
ft^2	square feet	0.09	square meters	m^2
yd^2	square yards	0.8	square meters	m^2
mi^2	square miles	2.6	square kilometers	km^2
	acres	0.4	hectares	ha
		MASS (weight)		
oz	ounces	28	grams	g
lb	pounds	0.45	kilograms	kg
	short tons (2000 lb)	0.9	tonnes	t
		VOLUME		
tsp	teaspoons	5	milliliters	ml
Tbsp	tablespoons	15	milliliters	ml
fl oz	fluid ounces	30	milliliters	ml
c	cups	0.24	liters	l
pt	pints	0.47	liters	l
qt	quarts	0.95	liters	l
gal	gallons	3.8	liters	l
ft^3	cubic feet	0.03	cubic meters	m^3
yd^3	cubic yards	0.76	cubic meters	m^3
		TEMPERATURE (exact)		
°F	Fahrenheit temperature	5/9 (after subtracting 32)	Celsius temperature	°C

Approximate Conversions from Metric Measures

Symbol	When You Know	Multiply by	To Find	Symbol
		LENGTH		
mm	millimeters	0.04	inches	in
cm	centimeters	0.4	inches	in
m	meters	3.3	feet	ft
m	meters	1.1	yards	yd
km	kilometers	0.6	miles	mi
		AREA		
cm^2	square centimeters	0.16	square inches	in^2
m^2	square meters	1.2	square yards	yd^2
km^2	square kilometers	0.4	square miles	mi^2
ha	hectares (10,000 m^2)	2.5	acres	
		MASS (weight)		
g	grams	0.035	ounces	oz
kg	kilograms	2.2	pounds	lb
t	tonnes (1000 kg)	1.1	short tons	
		VOLUME		
ml	milliliters	0.03	fluid ounces	fl oz
l	liters	2.1	pints	pt
l	liters	1.06	quarts	qt
l	liters	0.26	gallons	gal
m^3	cubic meters	35	cubic feet	ft^3
m^3	cubic meters	1.3	cubic yards	yd^3
		TEMPERATURE (exact)		
°C	Celsius temperature	9/5 (then add 32)	Fahrenheit temperature	°F

```
°F   -40    0    32  40   80  98.6 120  160  200  212 °F
      |-----|----|----|----|----|----|----|----|----|
     -40  -20    0   20   37 40   60   80  100 °C
```

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1. INTRODUCTION

The early analyses of the response of railroad tracks in the lateral plane were based on the assumption that the rail-tie structure responds like a beam in bending. The corresponding equation is

$$EI \frac{d^4 v}{dx^4} + N \frac{d^2 v}{dx^2} = q(x) \tag{1.1}$$

where $v(x)$ is the lateral displacement of the track axis at point x, N is the axial compression force, and $q(x)$ is the lateral resistance and/or the lateral load, as shown in Fig. 1. A review of several track analyses which used eq. (1.1), is contained in Ref. [1], Section 3.

If the bending rigidity is chosen as the sum of the rigidities of the two rails with respect to their vertical centroidal axes, namely $EI = E(2I_r)$, then this implies that the torsional resistance the fasteners exert on the rails is neglected. Although this may be justified for tracks with cut-spike fasteners which were subjected to extensive traffic, this is definitely not the case for tracks with the more rigid K-type fasteners or the spring-type fasteners. To remedy this situation a number of investigations [2 - 4] used a "substitute" bending rigidity for EI, which was chosen to be higher than the $2EI_r$ -value.

A more recent approach is to split the first term in eq. (1.1) into two or more terms. The first term representing the bending rigidity of the two rails with respect to their vertical centroidal axes and the remaining terms representing the "continuous" effect the fasteners exert on the rails in the lateral plane. So, for example, some authors [5 - 8] assumed that these continuous effects are resistance moments, which are proportional, at each point x, to the angle of rotation of the track axis, as shown in Fig. 2 and
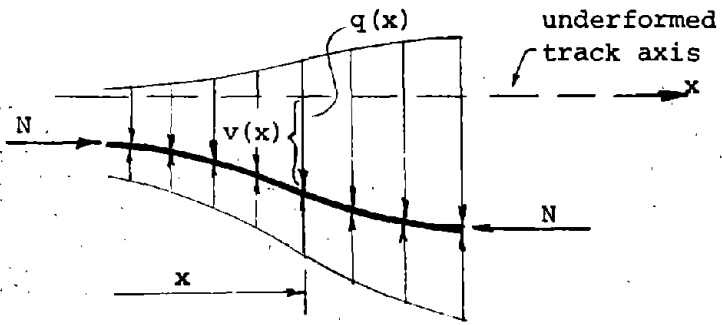
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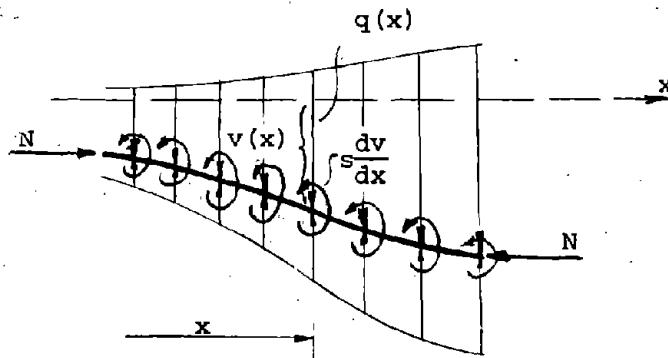FIG. 1   SIGN CONVENTION FOR EQ. (1.1)

FIG. 2   SIGN CONVENTION FOR EQ. (1.2)

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proposed, instead of eq. (1.1), the equation

$$2EI_r \frac{d^4v}{dx^4} + (N - 2s)\frac{d^2v}{dx^2} = q(x) \qquad (1.2)$$

In eq. (1.2), s is the proportionality constant between the resistance moment (per unit length of rail) and the rotation of the track axis at x. The parameter s was determined experimentally by a number of investigators [5, 6, 9] by rotating a section of rail mounted by a fastener to a rigidized tie and then by recording the dependence of the applied torsional moment to the angle of rotation of the rail at the fastener, as shown in Fig. 3.

Recently it was noted by Kerr [1] that equ. (1.2) in conjunction with this method of determining the s coefficient, does not include the effect of the gauge on the lateral track response, which is mechanically incorrect. For example, according to this formulation the lateral rigidity of a rail-tie structure, consisting of two rails fastened to closely spaced cross-ties, will be the same regardless of the gauge. This situation takes place, indeed, when the torsional regidity of the fasteners is non-existent, since then the cross-ties act merely as spacers and a bending moment of the rail-tie structure is carried by the bending stresses of the two rails. However, when the fasteners exert a resistance against rotation, the bending moment is carried not only by the bending stresses of each rail, but also by axial forces in the rails, as shown in Fig. 4. Note that for lateral deformations also the effect of the tie rigidity has to enter the analysis of a track structure.

This consideration led to a study aimed at the derivation of the proper differential equations for the response of the rail-tie structure in the lateral plane [10]. In this study, at first, the
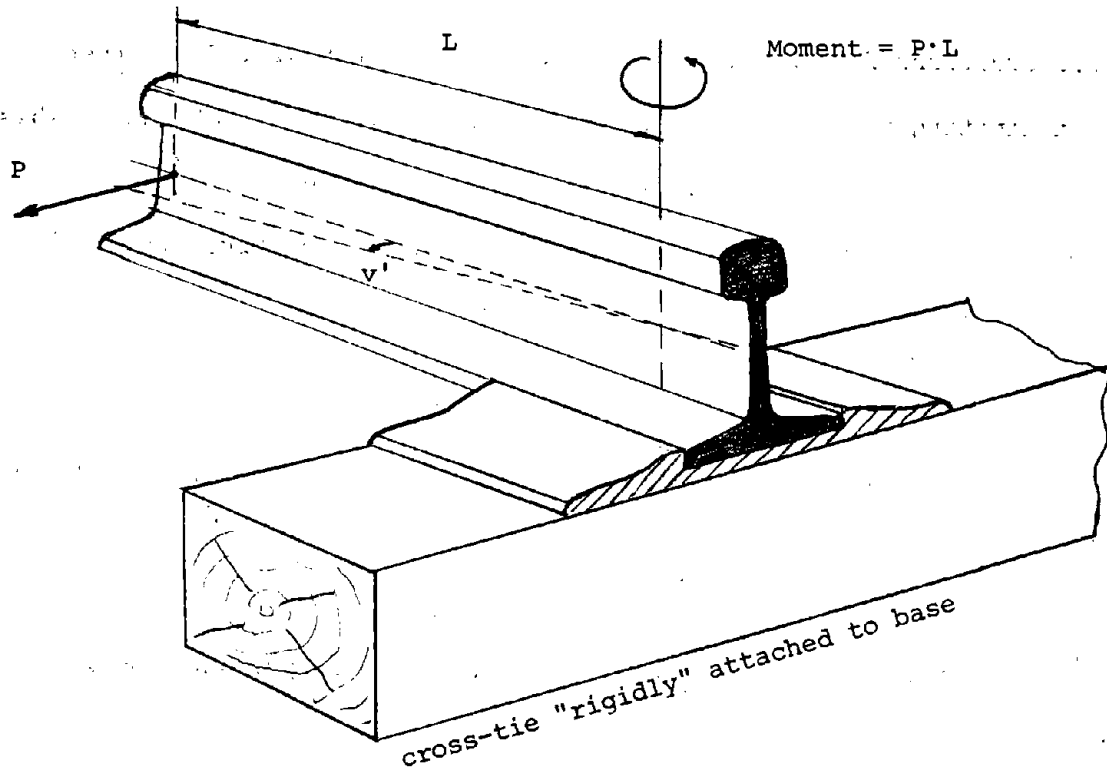
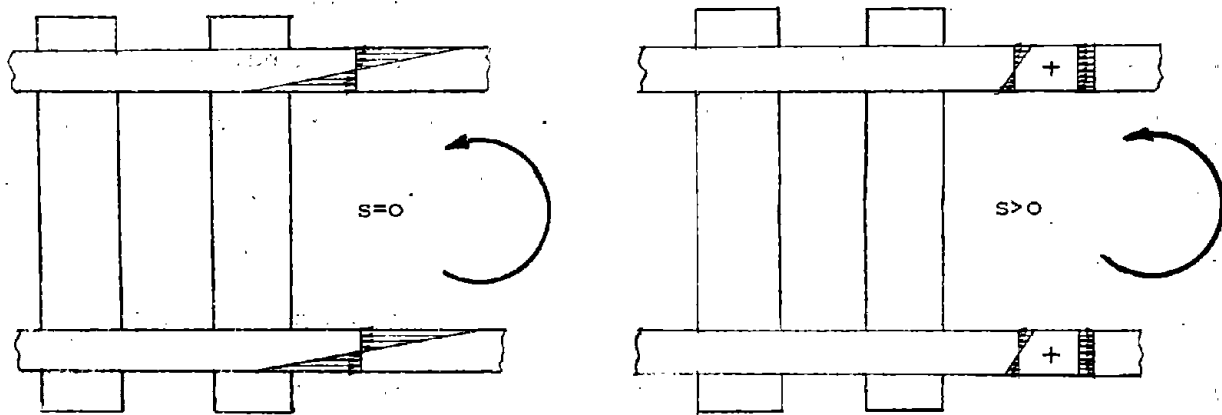FIG. 3   TEST SET-UP FOR DETERMINATION OF S.

FIG. 4   EFFECT OF s ON THE STRESS DISTRIBUTION IN RAILS.

difference equations for the rail-tie structure were formulated and then, through a limiting process, the differential equations were obtained. This approach yielded equations with well defined coefficients.

Using this procedure, it was found that the rail-tie structure shown in Fig. 5 is governed (for N = 0) by two simultaneous differential equations

$$2EI_r v^{IV} - \frac{12\alpha s}{6\alpha+s} v'' - \frac{24\alpha s}{h(6\alpha+s)} u' = q \qquad (1.3)$$

$$EAu'' - \frac{24\alpha s}{h^2(6\alpha+s)} u - \frac{12\alpha s}{h(6\alpha+s)} v' = 0 \qquad (1.4)$$

where u and v are the axial and lateral displacements, respectively, and

$$\alpha = \frac{E'I'}{he}; \quad s = \frac{S}{e}; \quad q = \frac{Q}{e} \qquad (1.5)$$

Note that without the third term in eq. (1.3), this equation is identical, except for the coefficient in the second term, with eq. (1.2) for N = 0. However, it should be noted that,whereas the coefficient in eq. (1.3) contains the effect of the rotational fastener resistance, the gauge, and the effect of the cross-ties, the corresponding coefficient in eq. (1.2) represents only the rotational fastener resistance. Note also that for the case of infinite fastener resistance, that is for s = ∞, eq. (1.2) reduces to an equation of second order, whereas eq. (1.3) and eq. (1.4), reduce to

$$2EI_r v^{IV} - 12\alpha v'' - (24\alpha/h) u' = q \qquad (1.6)$$

$$EAu'' - (24\alpha/h^2) u - (12\alpha/h) v' = 0 \qquad (1.7)$$

thus, they retain their order. This point will be of interest in Section 3, when interpreting the test data for a test structure with "rigid" fasteners.

A track analysis which is based on the two simultaneous equations
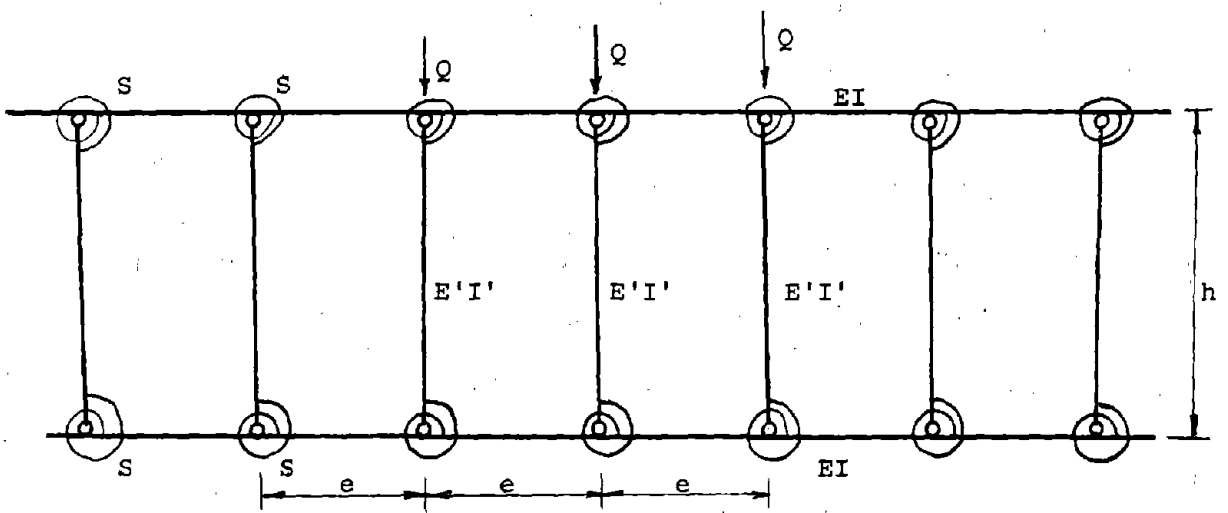
FIG. 5 ANALYTICAL MODEL FOR THE RAIL-TIE STRUCTURE.

(1.3) and (1.4) is more involved than an analysis based on the one equation (1.2). Therefore, it is desirable to establish the effect the third term in eq. (1.3) has on the track response and, if small, whether it could be neglected in track analyses.

As a step in this direction, a model test track, with adjustable fasteners, was built and then subjected to lateral loads. The lateral deflections were recorded for a variety of fastener rigidities. The obtained test results were then compared with corresponding analytical results. This study is described in the following sections.

2. TEST PROGRAM

The test structure used is shown in Fig. 6. It consists of two aluminum rails (about 1 inch high) which were attached to wooden cross-ties (oak, 3.5 x 3.5 x 50.8 cm) by means of custom made fasteners.[*] Each of the fasteners, shown in Fig. 7, consists of two ⌐ elements. The larger element is attached rigidly to the cross-tie and the smaller inner element is free to rotate about a pin. The desired torsional rigidity of the fastener is established by inserting rectangular pads of prescribed material properties between the ⌐ elements, as shown in Fig. 7.

The test structure is 3.1 meters long. The used center to center tie spacing was 7.6 centimeters.

The rail-tie structure rested on a rigid horizontal base. To eliminate the friction between the rail-tie structure and the base, two "frictionless" ball bearings were attached to the bottom surface of every second tie. The "simply supported" end conditions for the test structure were created by drilling a hole through the center of the first and last tie and by inserting a pin which was attached to the base through each of these two holes. Thus, the length of the test section is equal to the distance between the pins. For the used test it was 3.03 meters.

The simply supported test structure was subjected at the center of the span to a concentrated force, by means of a dynamometer which automatically recorded the force, as shown in Fig. 6. The lateral

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[*] This rail-tie structure was built originally for demonstration purposes. Its construction was sponsored by a grant from the Association of American Railroads (AAR), Washington, D.C.
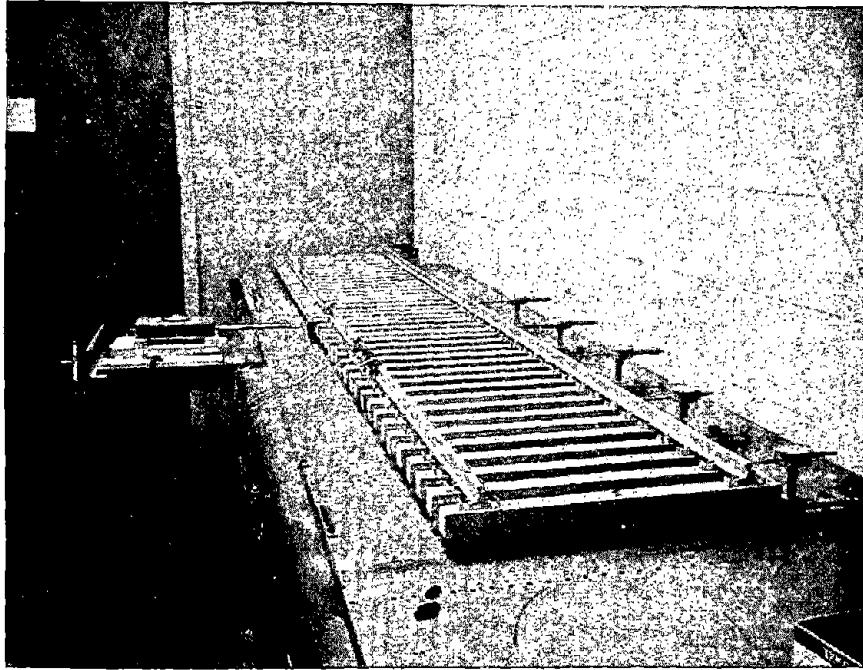
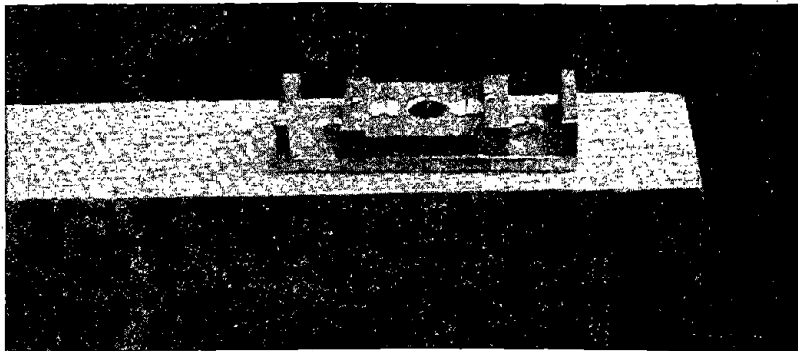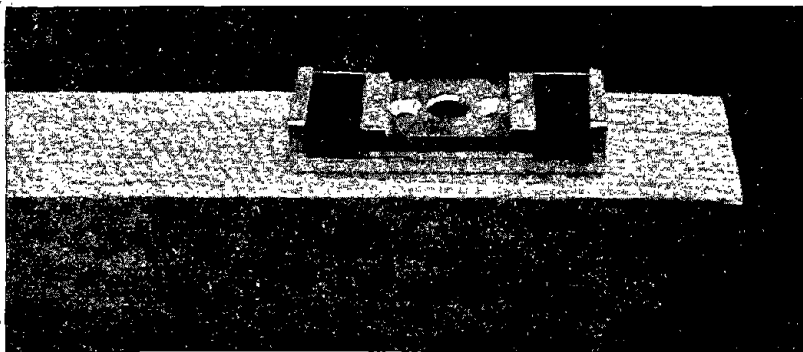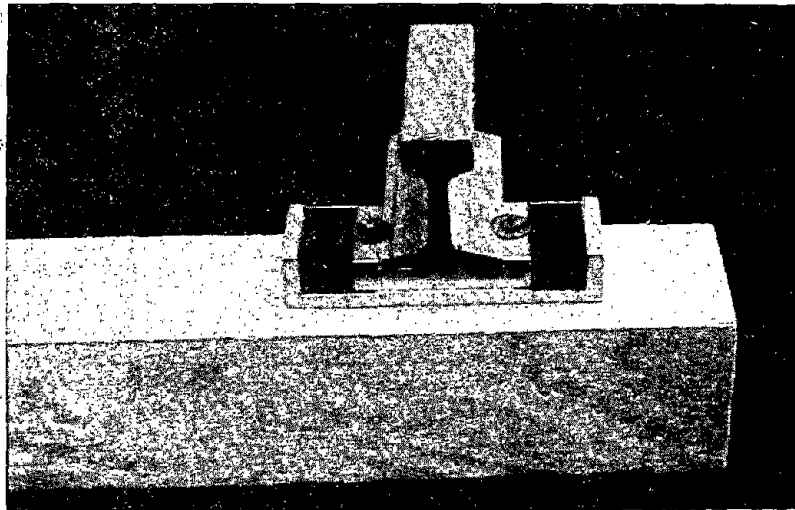FIG. 6   RAIL-TIE STRUCTURE USED IN TESTS

FIG. 7   FASTENER OF TEST STRUCTURE

displacements were recorded by mechanical dial gages. Dial gages were also placed at both ends of the test structure to record possible support displacements.

Preliminary tests showed no noticeable time dependent deformations of the test structure, when the fasteners had hard rubber or metal inserts. Another group of tests established that for the range of caused deflections, the load-displacement relations were nearly linear as shown in Fig. 8. This finding was an indication that a linear formulation may be used to describe the response of the test structure.

The recorded deflections normalized with respect to their deflection at the center, for four different fastener rigidities and for different load intensities are shown in Fig. 9. The four fastener rigidities correspond to:(1) no inserts, (2) hard rubber inserts, (3) two screws through both ⊔ shapes, and (4) metal inserts. Each test point is a result of several measurements which showed very little scatter. Note that only the normalized deflections are shown , since for the present study only the comparison of the deformation characteristics of the curves is of interest and not the magnitude of the deflections.
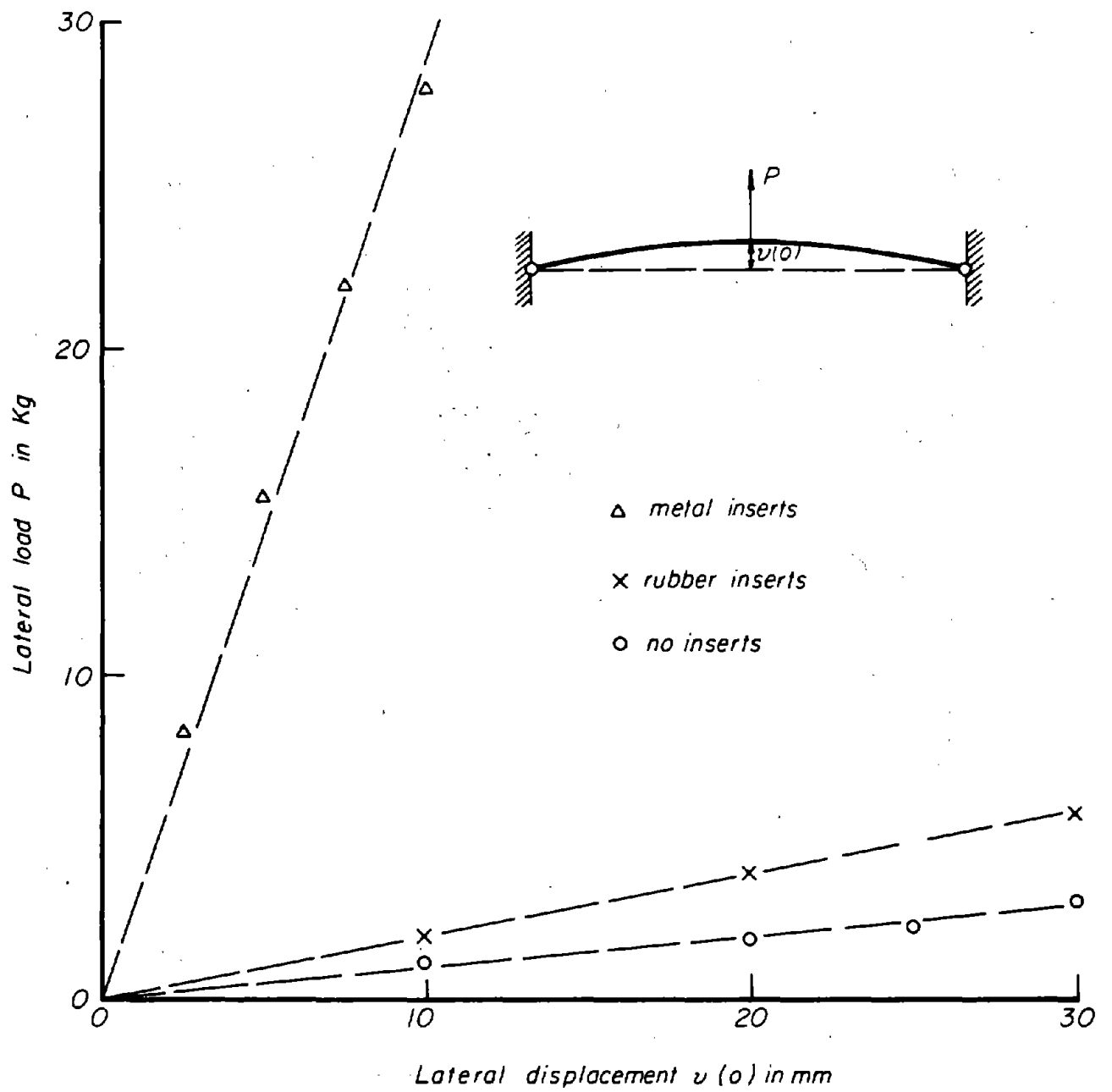
FIG. 8 RECORDED LOAD-DISPLACEMENT RELATION OF RAIL-TIE STRUCTURE
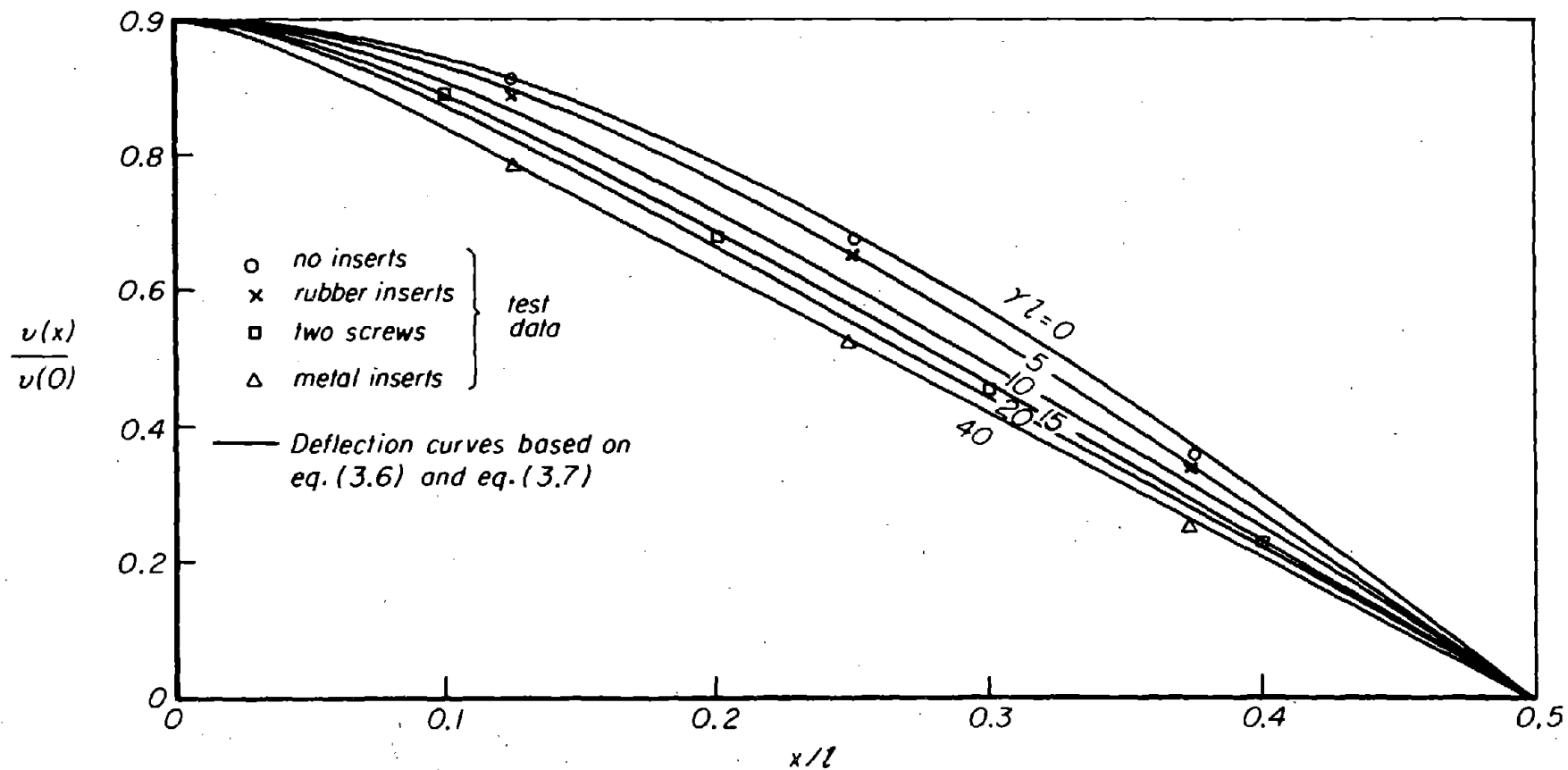FOR VARIOUS FASTENER RIGIDITIES

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FIG. 9 COMPARISON OF MEASURED AND CALCULATED LATERAL DISPLACEMENTS.

3. ANALYSIS OF TEST TRACK AND COMPARISON OF RESULTS

Because of symmetry of the anticipated deflection curves, the origin of the reference axis x, was placed at midspan, as shown in Fig. 10. For this problem eq. (1.3) without the third term reduces to

$$2EI_r v^{iv} - \frac{12\alpha\ s}{6\alpha + s}\ v'' = 0 \qquad\qquad 0 \le x \le l/2 \qquad\qquad (3.1)$$

where $(\ )' = d/dx$, etc. The corresponding boundary conditions are

$$
\begin{aligned}
v'(0) &= 0 & v(l/2) &= 0 \\
v''(0) &= P/4EI_r & v''(l/2) &= 0
\end{aligned}
\right\} \qquad\qquad (3.2)
$$

The general solution of (3.1) is

$$v(x) = A_1 + A_2 x + A_3 \cosh(\gamma x) + A_4 \sinh(\gamma x) \qquad\qquad (3.3)$$

where

$$\gamma = \sqrt{\frac{3\alpha s}{EI_r (6\alpha + s)}} \qquad\qquad (3.4)$$

Determining the constant A_1 to A_4 from the boundary conditions in (3.2), the solution to (3.1) and (3.2) may be written as

$$v(x) = \frac{P}{4EI_r \gamma^3} \left[\frac{\gamma l}{2} - \gamma x - \text{tgh}\left(\frac{\gamma l}{2}\right) \cosh(\gamma x) + \sinh(\gamma x) \right] \qquad\qquad (3.5)$$

Normalizing the deflection expression with respect to $v(0)$ we obtain

$$\frac{v(x)}{v(0)} = \frac{\frac{\gamma l}{2} - \gamma x/2 - \text{tgh}(\gamma l/2) \cosh(\gamma x/2) + \sinh(\gamma x/2)}{\gamma l/2 - \text{tgh}(\gamma l/2)} \qquad\qquad (3.6)$$

Note that γl is the only parameter which enters eq. (3.6); $\gamma l = 0$ being
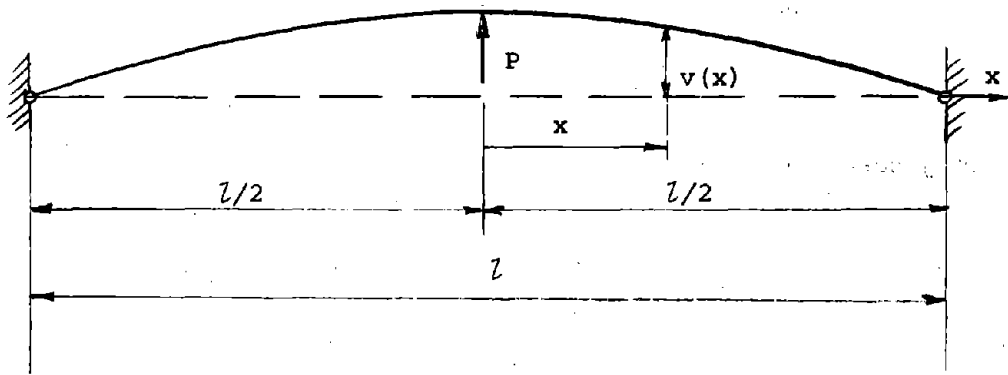
FIG. 10 SIGN CONVENTION FOR ANALYSIS OF TEST STRUCTURE.

the case of zero rotational resistance of the fasteners.

For zero fastener resistance, eq. (3.6) reduces to

$$\frac{v(x)}{v(o)} = 1 - 6(x/l)^2 + 4(x/l)^3 \qquad (3.7)$$

Equations (3.6) and (3.7) were evaluated numerically for $\gamma l = 0, 5, 10, 15, 20, 40$ and the results are presented as graphs in Fig. 9.

From Fig. 9 it follows that the analytically obtained deflection curve for $\gamma = 0$, i.e. from eq. (3.7), and the recorded deflections of the test structure with no inserts in the fasteners, agree closely. This was to be anticipated, since for this fastener case the two rails act independently as single beams. Note, however, that with increasing rotational resistance of the fasteners the deviations from the $\gamma = 0$ case increase. Thus, for these cases the classical beam equation (1.1) is not suitable for describing the lateral response of the test structure.

Of great interest is the finding that, when the fasteners exert a rotational resistance, the recorded deflections agree with those of the analytical results for $\gamma \neq 0$. Namely, when the fasteners had rubber inserts the measured deflections agree with the analytical curve for $\gamma l = 5$. When the fasteners were rigidized by means of two screws sunk through both ⊔ shapes into the tie, the test results agree with the analytical curve for $\gamma l = 16$. Furthermore, when the fasteners were made "rigid" by locking the fasteners with metal inserts, as shown in Fig. 7, the measured deflection shape agrees with the analytical curve for $\gamma l = 40$.

For a better interpretation of the above test results, let us also consider two limiting cases. When the fastener is rigid, then $s = \infty$, and eq. (3.1) reduces to

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$$2EI_r \, v^{IV} - 12\left(\frac{E'I'}{he}\right) \, v'' = 0 \qquad\qquad (3.8)$$

whereas eq. (1.2), reduces to a differential equation of second order.
Thus, when the rails are rigidly attached to the ties, eq. (1.2) is
not suitable for track analyses. However, when the rotational rigidity
of the fasteners is small compared to the stiffness of the cross-ties,
namely when $s \ll \alpha$ (but not zero), then eq. (3.1) reduces to

$$2EI_r \, v^{IV} - 2s \, v'' = 0 \qquad\qquad (3.9)$$

which is identical with eq. (1.2) for N=0 and q=0.

4. CONCLUSION AND RECOMMENDATIONS

The comparison of the analytical and the test results presented in Fig. 9 shows that for the tested rail-tie structure: (1) the classical beam bending equation closely represents the lateral deflection shape when the torsional resistance of the fasteners is nearly zero (2) the nature of the test curves noticeably deviates from the zero case with increasing fastener resistance, thus when the fasteners exhibit a torsional resistance the classical bending equation, even with a modified EI value, are not suitable for the analysis of the rail-tie structure, and (3) the measured deflection shapes for a variety of fastener rigidities agree closely with the deflection shapes based on eq. (1.3) without the third term. The close agreement, for all four types of fastener rigidities, suggests that an equation of the type (1.3) may be suitable for the analysis of railroad tracks in the lateral plane. This is an indication of a possibility. To establish whether this is true in general, will require additional tests (also on full scale track structures), corresponding analyses, and comparisons.

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[9] McConnell, D. P., "On the torsional rigidity of rail fasteners in the lateral plane", DOT/FRA Report, NYU-AA-72-35, 1972.

[10] Zarembski, A. M., "On the formulation of the equations for the railroad track response in the horizontal plane", Ph.D. Thesis, Princeton University, 1975.

APPENDIX:

REPORT OF INVENTIONS

The purpose of this project was to study the lateral response
of the rail-tie structure, experimentally and analytically, in order
to establish the proper equations for the lateral track response.
After a review of the work performed under this phase of the contract,
it was determined that no technical innovation or invention has been
made.

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