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A SURVEY OF VARIABLE VOLTAGE POWER CONDITIONERS FOR APPLICATION TO THE TRACKED AIR CUSHION VEHICLE

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INTRODUCTION

The linear induction motor (LIM) will be used for the propulsion of tracked air cushioned vehicles (TACV). Speed control of the LIM is essential to vehicle operation. Speed control would generally be accomplished in the power conditioner unit used to convert wayside power into a form desirable for control and drive of the LIM.

The electrical drive requirements of the LIM are similar to those of the rotary induction motor. The maximum, or synchronous speed of the motor is related to the drive frequency by:

$$\text{Speed} = \frac{\text{Constant x Drive Frequency}}{\text{Poles Per Phase}}$$

and is nearly achieved under unloaded conditions. The motor delivers no thrust (torque) at synchronous speed, and is normally operated at a lower speed where the thrust output is high. Curves are shown in Figures 1 and 2 showing thrust vs. speed for a constant frequency drive and a variable frequency drive. From Figure 1, it is apparent that the starting thrust can be made variable by varying the drive voltage only. The thrust is proportional to the square of the voltage, so changing the voltage over a small range will have a pronounced effect on starting thrust. Once the vehicle has achieved the desired speed, the thrust requirement is reduced, since aerodynamic drag is the main counterforce and is much less than the thrust needed for vehicle acceleration. In general, the motor will not be operated near its maximum torque point using a voltage control only.

To maximize thrust at all speeds, and to fully utilize the motor capability, the difference between the actual motor speed and the synchronous speed must be small. As a result, the drive frequency should vary from zero at start to nearly 200 Hertz at full speed. If the voltage is made proportional to drive frequency, a series of thrust vs. speed curves result as shown in Figure 2. It is apparent that constant thrust occurs by increasing the drive frequency as the vehicle accelerates, until the desired speed is achieved. Then the voltage may be lowered such that only the thrust needed to overcome drag is generated.

A means of braking the vehicle is provided by allowing the LIM to convert the kinetic energy of the vehicle into electrical energy. The rate at which this energy is either expended, or returned to the wayside line determines the braking profile of the vehicle. The drive source requirement under this condition is to supply a small amount of three phase power to the LIM for field excitation, at a frequency slightly less than the synchronous frequency determined by vehicle speed. Power will now flow out of the LIM, and can be directed to a resistor bank for

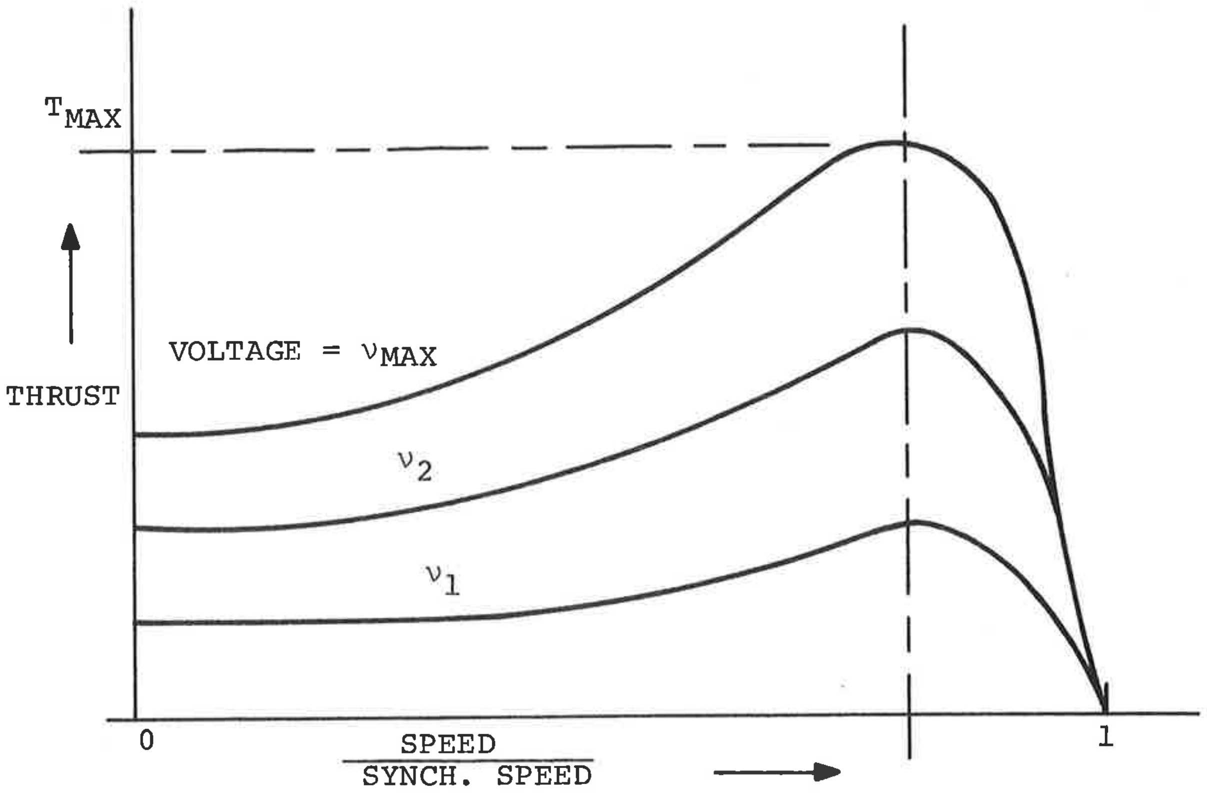

Figure 1. LIM Thrust vs. Speed at Various Drive Voltages, Constant Frequency

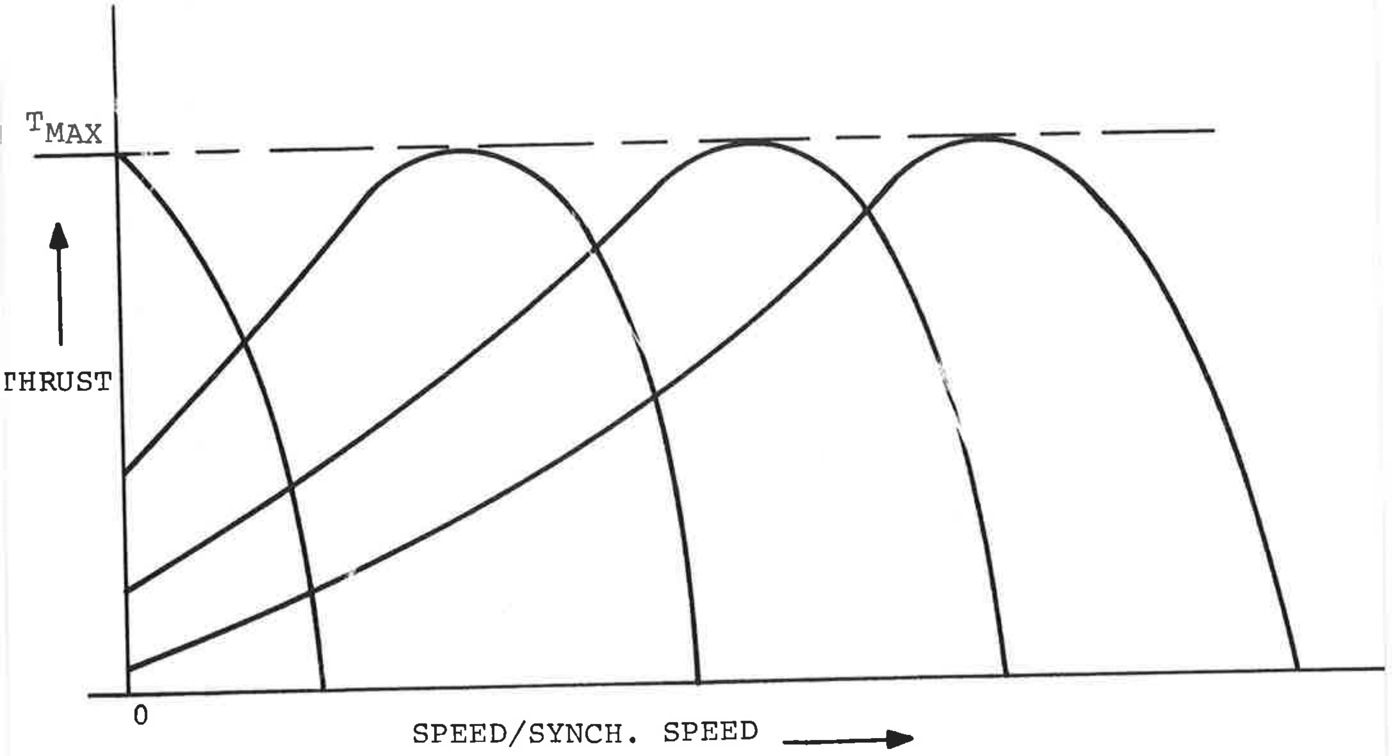

Figure 2. LIM Thrust vs. Speed for Various Drive Frequencies, Voltage Proportional to Frequency

dynamic braking. If the drive source can operate bilaterally, the power is returned to the wayside line for regenerative braking.

Further requirements are placed upon the drive source due to the inductive nature of the LIM load. Since the LIM power factor can vary from near 0.2 to 0.7 over its speed range, considerable reactive energy flows to and from the LIM on a cyclic basis. The drive source should be able to supply and store the energy.

In general, highest performance of the LIM is obtained using a three phase variable voltage, variable frequency bilateral drive source. This will result in the smallest motor, maximum thrust at any speed, and complete control of speed and thrust at all speeds. Drive source sophistication can be reduced at the expense of control, efficiency and motor size. Here, variable voltage only, fixed voltage/frequency, and pole changing are possibilities.

The reaction rail of the LIM may be tailored in resistivity to meet thrust requirements at specific speeds. Referring to Figure 3, the thrust of the motor is shown to be a function of reaction rail resistance. The rail would have higher resistance farther down the track where the vehicle had a greater speed. Trade-offs here are low efficiency and rail hearing at the station, and possible starting problems farther down the track if the vehicle made an emergency stop where the rail resistance is low.

The next section describes techniques for obtaining variable voltage at constant frequency for the LIM.

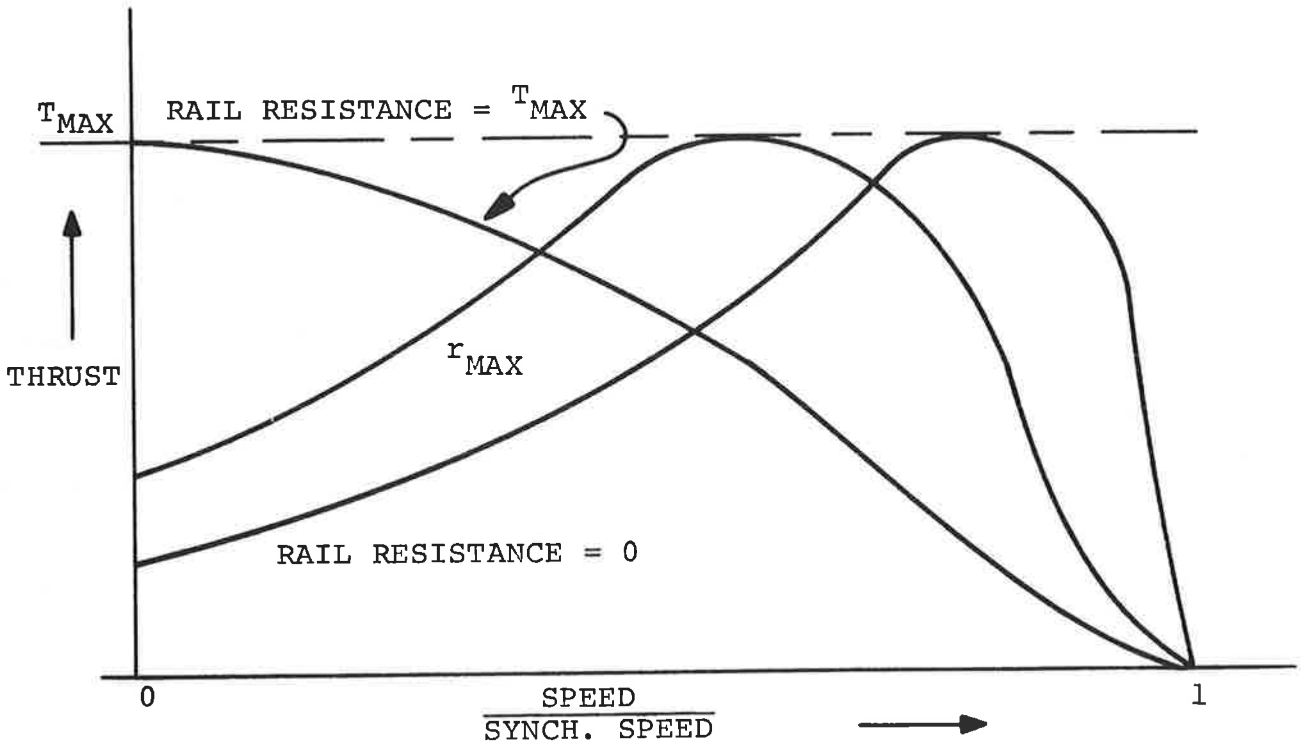

Figure 3. LIM Thrust vs. Speed for Various Rail Resistances, Constant Voltage and Frequency

TECHNICAL DISCUSSION

The following techniques of power conditioning for the TACV are discussed in this section.

1. Forced Commutated Inverter
2. Motor-Alternators
3. Line Choppers
4. DC-AC Links
5. Resistor Switching
6. Transformer Switching
7. Induction Regulator
8. Series Regulator

VARIABLE VOLTAGE SOLID STATE POWER CONDITIONERS

FORCED COMMUTATED INVERTER

A block diagram of a power conditioner using a forced commutated inverter (FCI) is shown in Figure 4. A rectifier provides dc power from the three phase 60 Hertz wayside line. The dc can be fixed or variable voltage, depending on the inverter requirements. The inverter provides three phase variable voltage power to the LIM. The power generated by the LIM during braking can be directed to a resistor bank. The resistor bank can dissipate this power in a controlled manner to provide dynamic braking.

The rectifier is shown in Figure 5, with silicon controlled rectifiers (SCR's) as the switching elements. If ordinary diodes are used, the dc output voltage is uncontrolled and is maximum. SCR's allow a variable dc voltage to be produced by changing the firing angle of the devices. SCR's also allow the rectifier to operate reversibly, transferring dc power applied to the output terminals back to the three phase input line. This dc must have the opposite polarity from the normal operating mode. If polarity reversal is not possible (determined by inverter operation during braking), the rectifier of Figure 6 can be used. Here additional SCR's connected across the first SCR's in reverse fashion allow power transfer from the dc to the ac side without polarity reversal. This doubles the number of SCR's needed, however. Inverter operation is not required for the present task, since regenerative braking will probably not be used.

The basic FCI is shown in Figure 7. The SCR's are turned on and off as shown, and generate the illustrated output voltage waveforms. By suitable control of the SCR's, the family of output waveforms, shown in Figure 8 are formed. The output voltage is reduced at the expense of increased harmonic content in the waveforms. No control of the dc voltage is required, and, therefore, the commutating circuits can be operated from the main dc

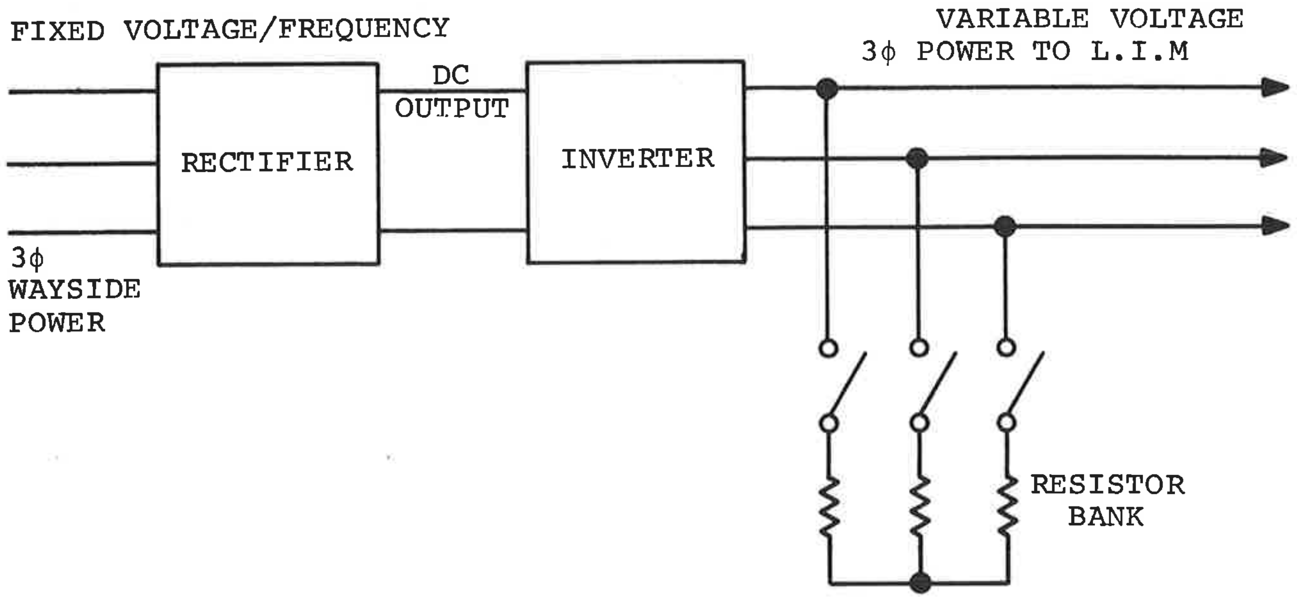

Figure 4. Forced Commutated Inverter Power Conditioner

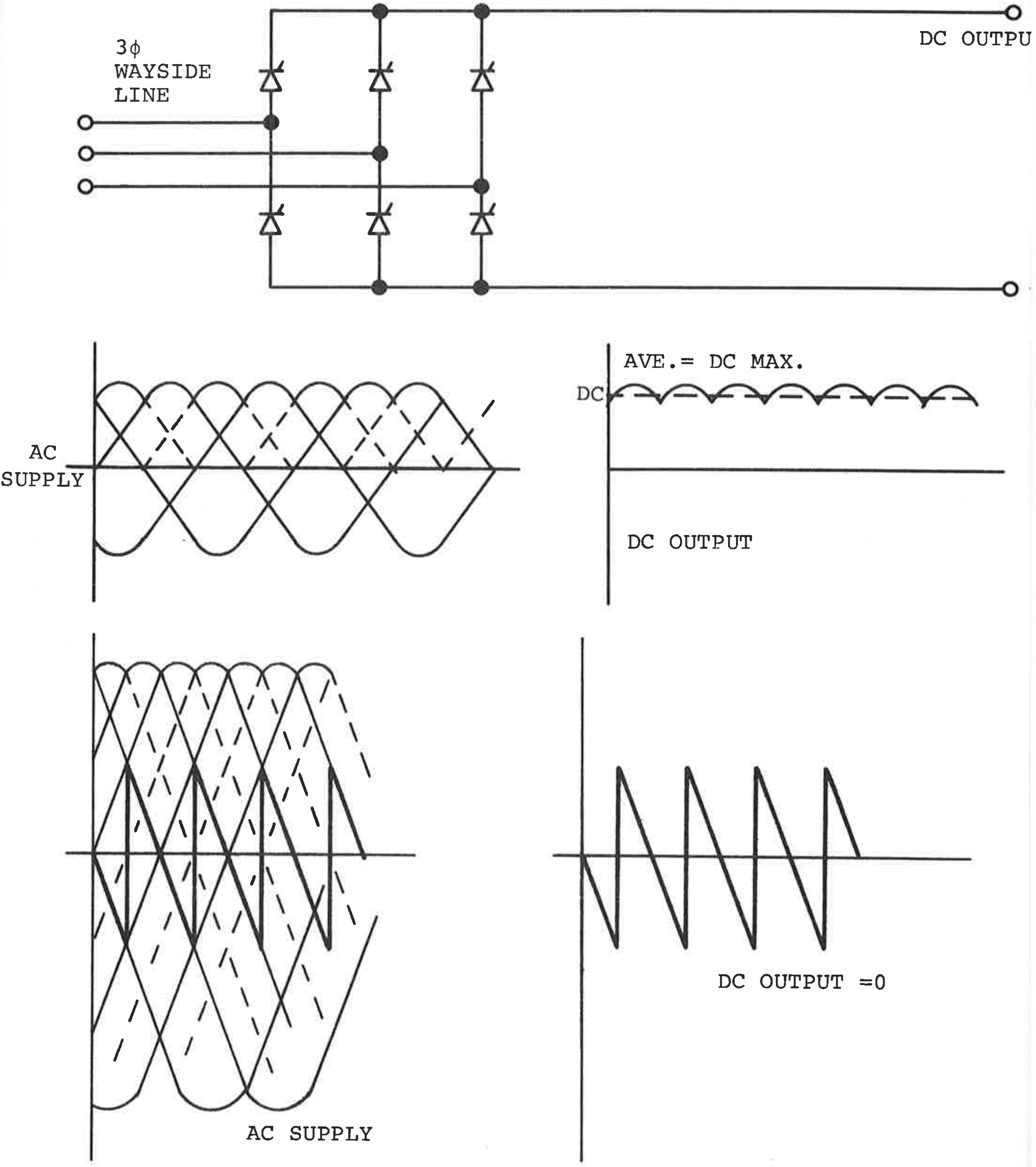

Figure 5. 3 Phase Bridge Rectifier and Output Waveforms

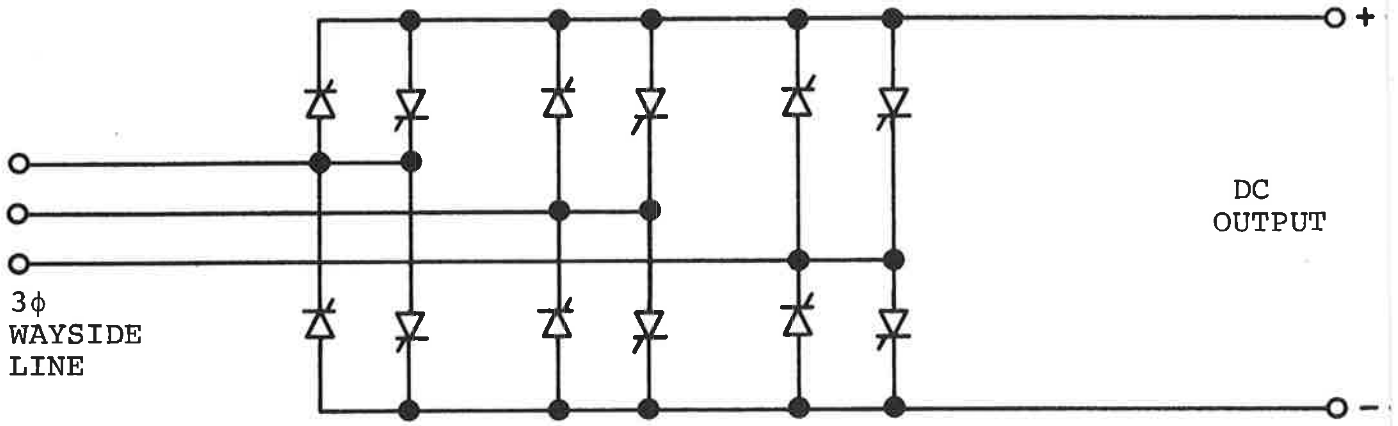

Figure 6. 3 Phase Bridge Rectifier for Reversible Uni-polarity Power Flow

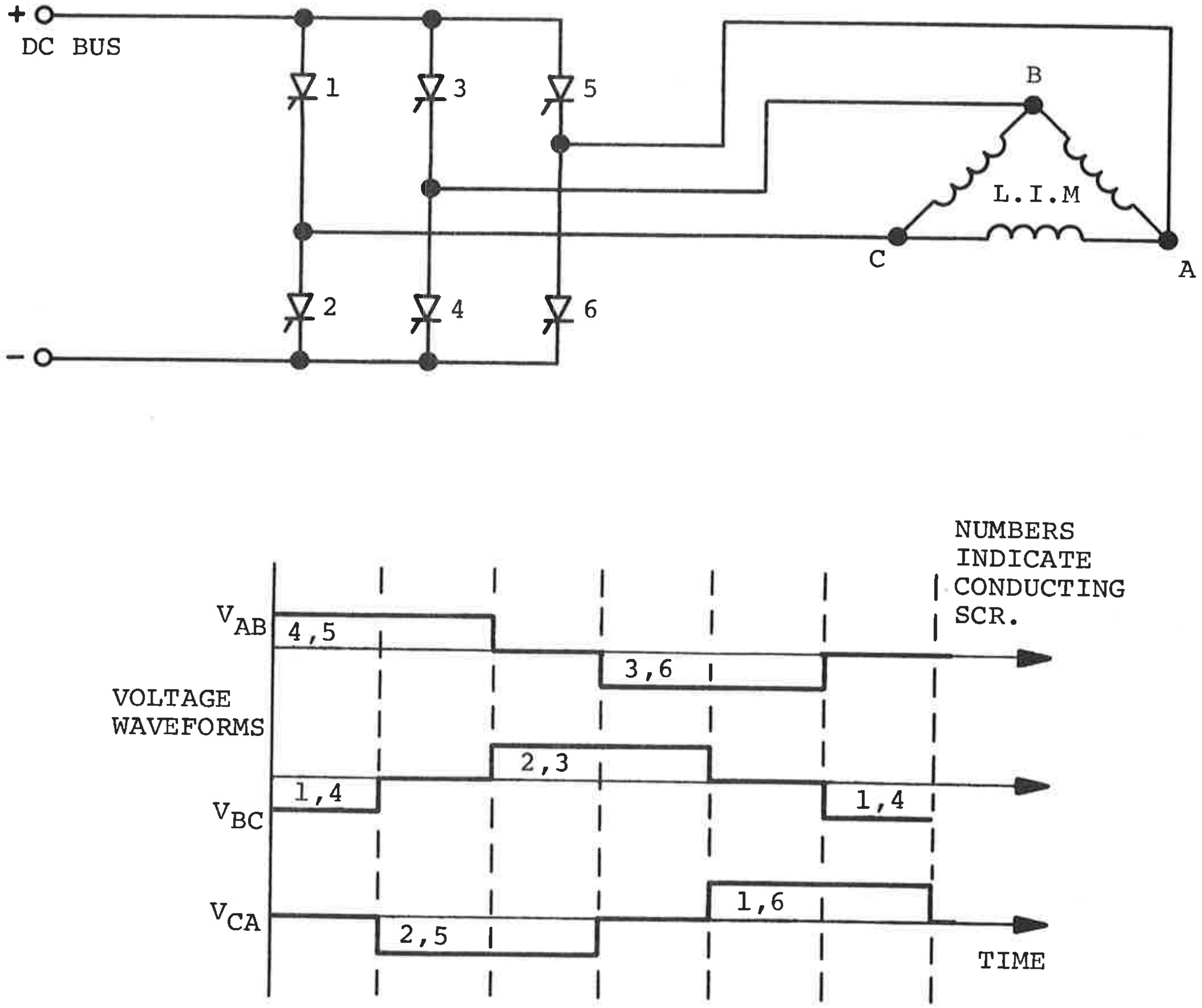

Figure 7. Basic Forced Commutated Inverter and Output Waveforms

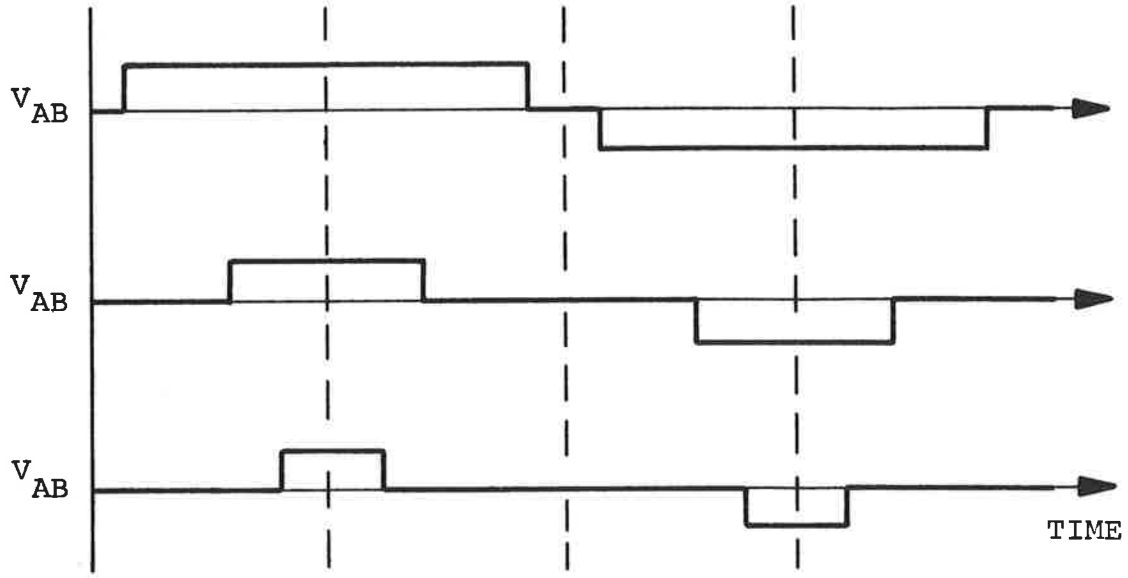

Figure 8. Output Voltage Control of FCI Using Fixed dc Bus

bus. If a variable dc bus is used, the commutating circuits need a separate supply for operation when the dc bus is low. A variable dc bus allows the waveforms of Figure 7 to be generated regardless of output voltage or frequency, keeping the harmonic content to a minimum. The circuit of the FCI takes the form of Figure 9, showing all commutating elements and the diodes "D" needed for reactive power flow. The action of the commutating circuit is to reverse bias a conducting SCR, terminating its conduction. The strong possibility of commutation failure exists in the FCI, with large fault currents resulting if both SCR's in the same branch are conducting. If the rectifier unit uses SCR's also, it can be shut down to clear the fault in the inverter.

Another method of operation of the FCI is multiple pulse width modulation (MPWM). The SCR's of the FCI shown in Figure 9 are operated to obtain the waveforms shown in Figure 10. The dc bus is not variable, but the inverter output voltage is variable with little harmonic current penalty. Reduction and elimination of specific harmonics is the advantage of MPWM. A disadvantage is the increased number of commutations per cycle for each SCR, since losses in the SCR's are greatest during switching.

MOTOR ALTERNATORS

Motor-alternators for a variable voltage, fixed frequency output could be used if there was an advantage to using some fixed frequency other than 60 Hertz. The size and weight penalty of this scheme is so great that it would most likely be rejected in favor of using the wayside 60 Hertz power.

LINE CHOPPERS

Variable voltage, constant frequency power for the LIM can be obtained by using a line chopper, shown in Figure 11. By controlling the firing angle of the SCR's, an RMS voltage from zero to the value of the input line can be applied to the LIM. This is done at the input line frequency. The actual voltage pulses applied to the LIM are portions of sinewaves with spaces of zero volts between the pulses. This makes the power factor poor at low output voltages. High harmonic content under these conditions may overheat the LIM. An advantage is that no intermediate dc link is needed. No commutation circuits are needed since the SCR's are commutated by polarity reversal of the input line.

DC-AC LINKS

A variable voltage constant frequency drive for the LIM can

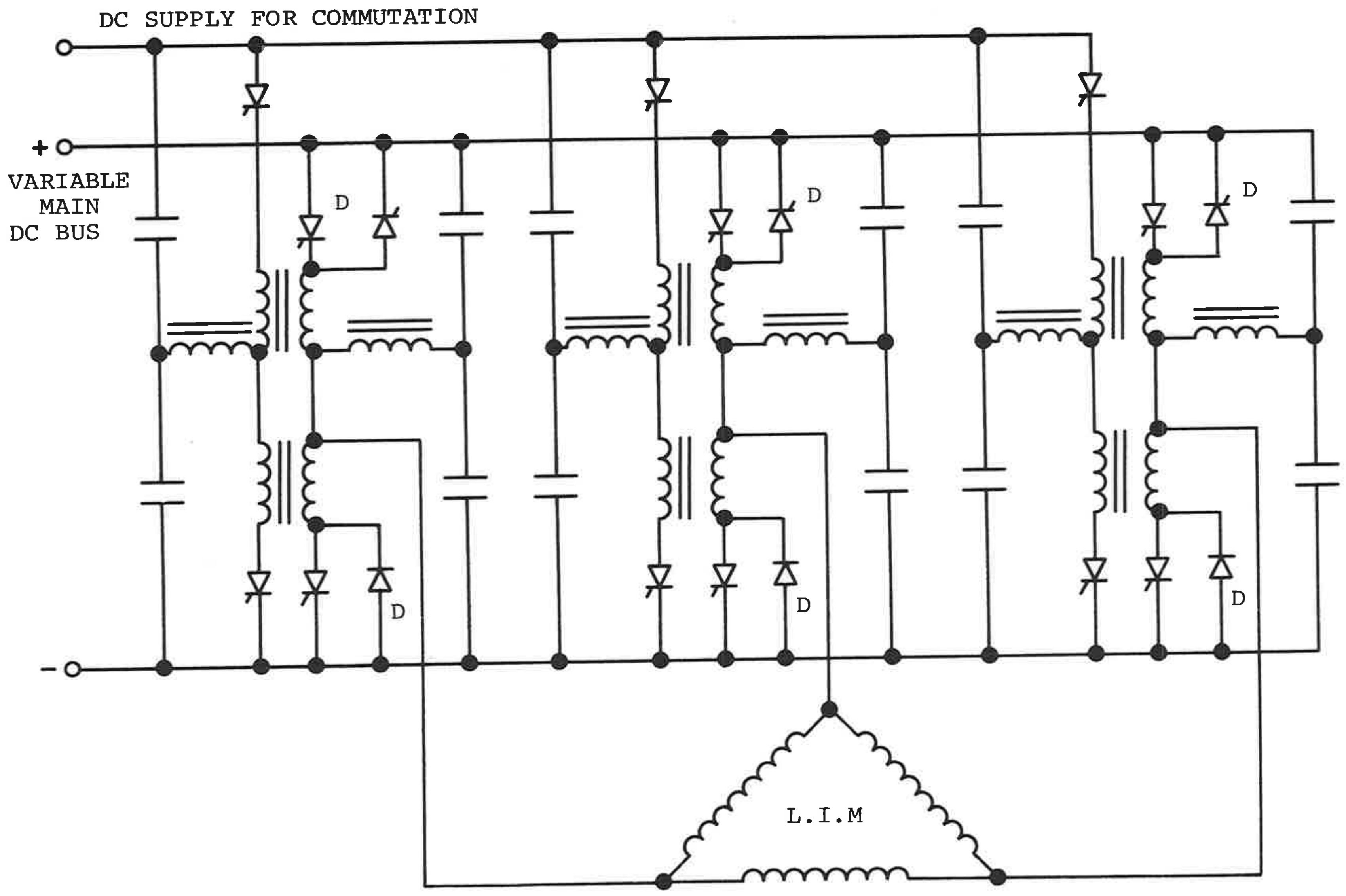

Figure 9. Forced Commutated Inverter Showing Commutation Circuits

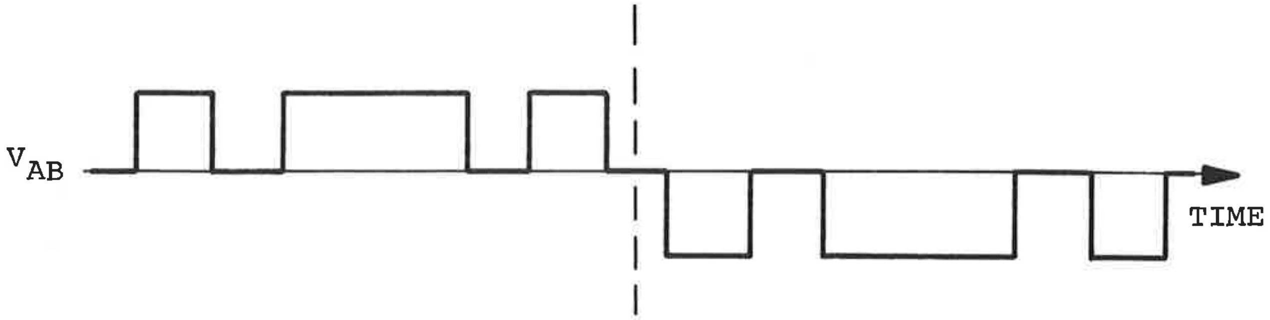

Figure 10. Output Voltage Waveform of MPWM for Voltage Control and Harmonic Reduction

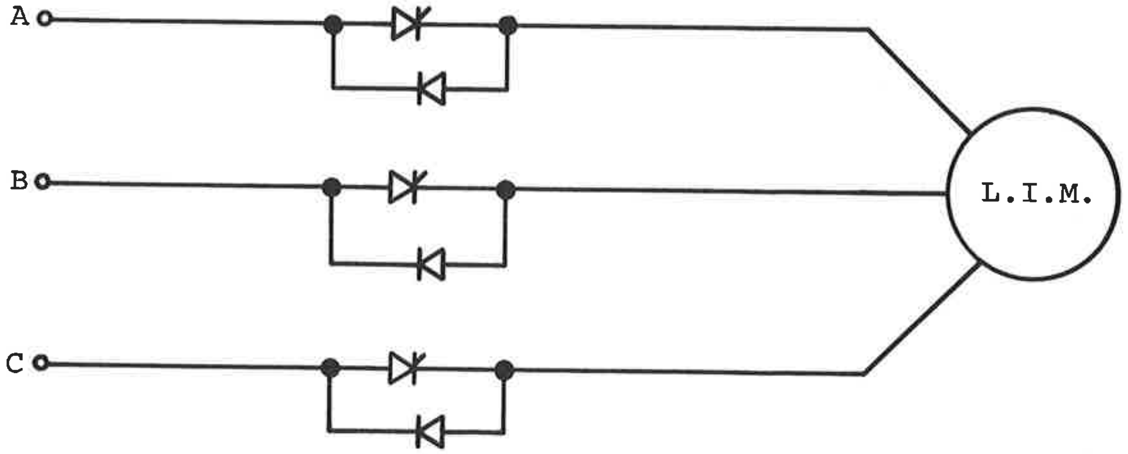

Figure 11. Line Chopper

be generated by using a dc-ac link processor. The processor configuration is shown in Figure 12. The three phase wayside line power is first converted to variable dc using a phase delay rectifier. The dc powers one of two types of inverters. The first type is of the forced commutated inverter class described earlier. Although these inverters are generally capable of variable frequency operation, fixed frequency operation of the inverters may be advantageous. For example, a separate commutating circuit power supply is not required if the dc input to the inverter is not reduced for lower frequency operation. Power dissipation in the semiconductor switching elements is less if the frequency is low, since commutation energy is used fewer number of times per cycle.

The second type of inverter is the naturally commutated or series capacitor commutated inverter. This is shown in Figure 13. This is a fixed frequency inverter. The output frequency is determined by the natural resonant frequency of inductor L and capacitor C. These elements are not easily variable at the frequencies and power levels required, so operation is essentially confined to a single frequency. Operation is as follows. Consider one phase of the load, represented as "R" in the figure. Both SCR's 1 and 4 are triggered on. Current flows from dc + through SCR-1, inductor L, capacitor C, and the load through SCR-4. This current is basically a half-sinusoidal pulse due to the resonance of L and C. When the current attempts to reverse direction (the normal condition for an underdamped resonant circuit), SCR's 1 and 4 become reversed biased and cease to conduct. Now SCR's 2 and 3 are turned on. These SCR's allow the current to flow in the reverse direction through L, C and R, completing a full cycle of sinusoidal current through the load. Firing of the other SCR's in the circuit are synchronized such that a three phase supply to the load exists.

Advantages of this scheme are that no extra commutation circuits are needed, since the SCR's commutate naturally as a result of load current reversal. In the event that load current reversal does not occur, the series capacitor cannot pass dc, so the SCR's in question will eventually turn off. The motor drive waveforms are nearly sinusoidal, eliminating the dissipation of harmonic energy in the motor.

PASSIVE CONTROL

RESISTOR SWITCHING

Resistor switching can be used to obtain variable voltage, constant frequency power for the LIM. Resistors are placed in series with the lines to the LIM. As the vehicle accelerates from start, the resistance value is decreased by means of tap switches until no resistance remains in the circuit at full speed.

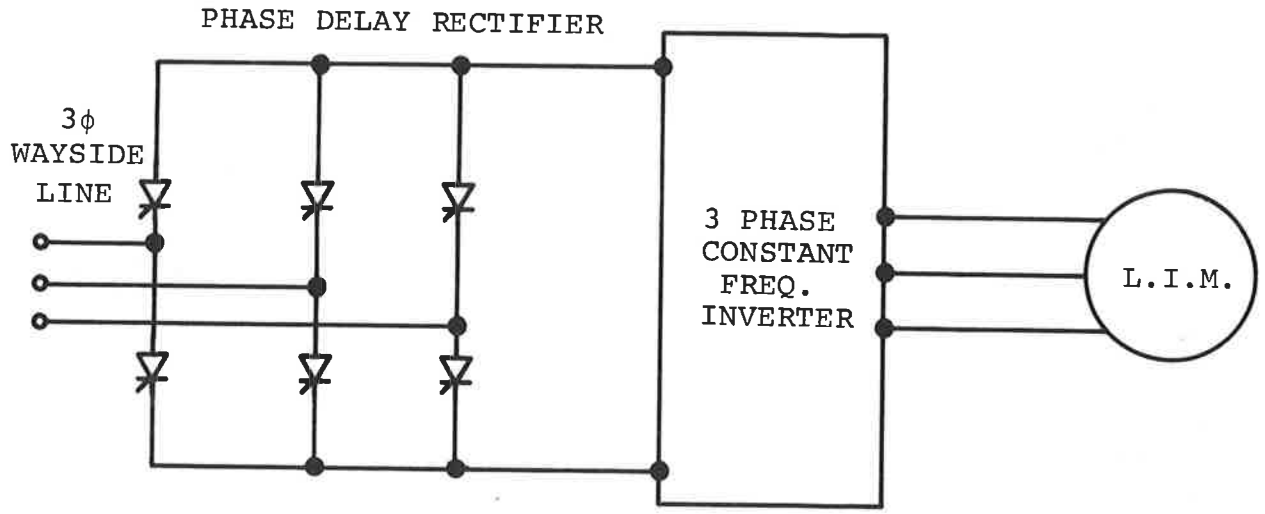

Figure 12. DC-AC Link Power Conditioner

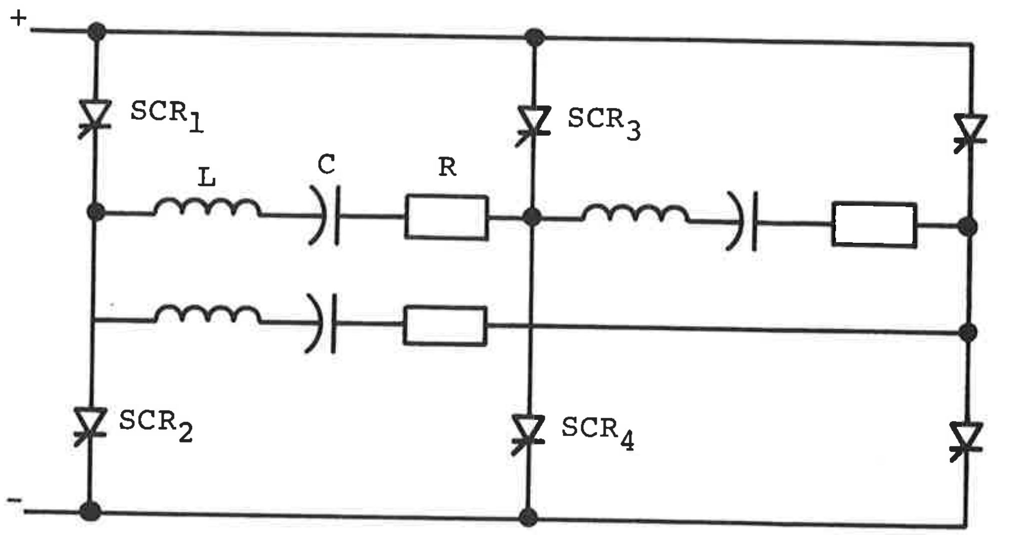

Figure 13. 3 Phase Naturally Commutated Series Inverter

These resistors can also be used as the dynamic braking resistors. It has been found that the thermal problems associated with the dynamic braking resistors on the Metroliner have been particularly significant. Since the speed of the TACV is greater than that of the Metroliner, it is expected that the use of resistor switching for speed control and dynamic braking will need considerable developmental effort to insure reliable operation.

TRANSFORMER SWITCHING

Transformer switching, or tap changing is a practical means of obtaining variable voltage, constant frequency power. A circuit diagram is shown in Figure 14. The transformer consists of primary windings A, B, C, one for each input phase. Secondary windings, a, b, c are arranged such that their induced voltages can either add to or subtract from the line voltage. The various taps on each secondary winding allow voltage output changes in small increments. To prevent power interruption when changing taps, a small inductor L is used. One contactor remains connected to the previous tap while the other is changed to the new, desired tap. A short circuit would exist between these taps if the inductor were not present. The inductor is designed to saturate at the normal load current such that it offers the minimum impedence in series with the load. Tap switching is normally used over a ±10% voltage range, although wider ranges are practical.

INDUCTION REGULATOR

The induction regulator can be used to vary the three phase wayside voltage over a range of ±10%. Since the LIM torque is proportional to the square of applied voltage, this gives a ±20% torque variation. The circuit for the induction regulator is shown in Figure 15. Terminals A, B, and C are the wayside line, while terminals a, b, c, are connected to the LIM. The construction of the induction regulator is similar to a rotary wound rotor induction motor; primary windings A, B, and C are on the stator, while secondary windings a, b, and c are on the rotor. The primary current produces a rotating magnetic field in the air gap. This field has constant amplitude. It induces voltages of constant amplitude in the secondary windings. The phase difference of the secondary and primary voltages changes when the rotor position is changed. Since the output of the regulator is taken from the series connection of both the primary and secondary windings, the output is the vector sum of the primary and secondary voltages. The output voltage is adjusted by turning the rotor.

SERIES REACTOR

Another method that provides variable voltage control for

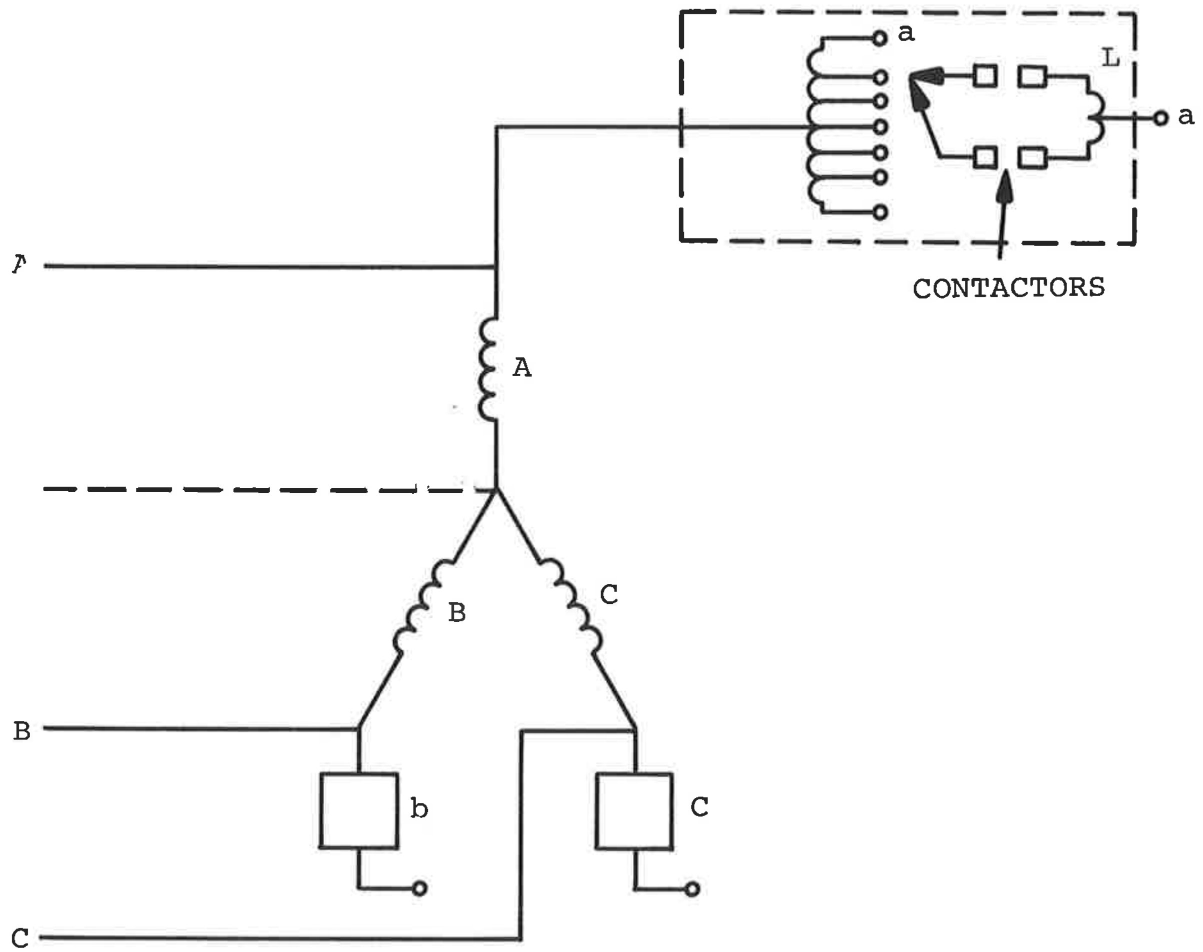

Figure 14. Transformer Tap Switching

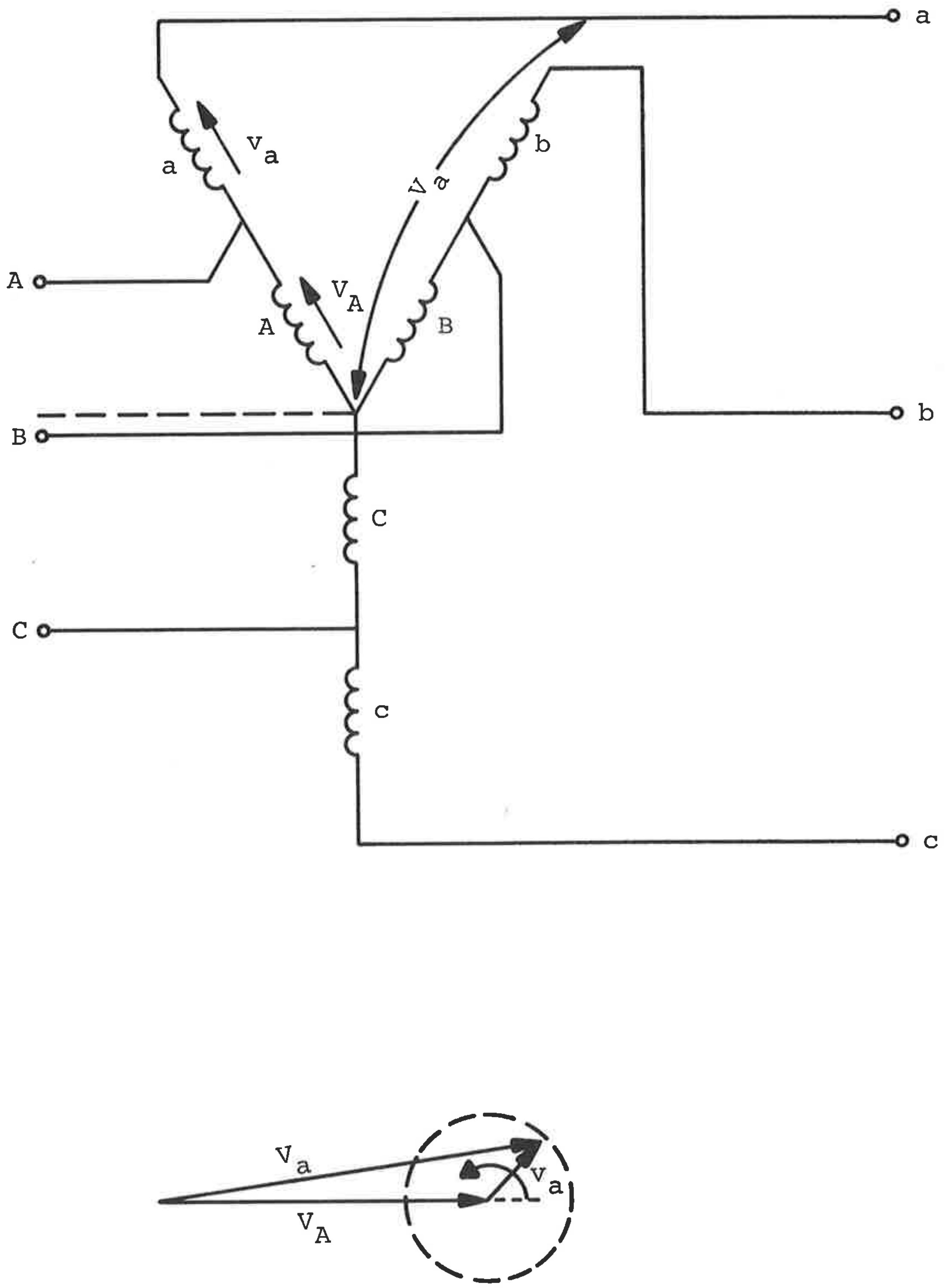

Figure 15. Induction Regulator

the LIM is the series reactor. The reactor can provide an ac voltage drop without significantly lowering the efficiency of the drive system. It does this at the expense of lowering the power factor however.

The approach is shown in Figure 16. Two reactors are needed. The voltage drop across either reactor is

$$V_{Drop} = 2\pi fLI$$

where f is the frequency in Hertz, L the inductance in henries, and I is the current in amperes. At start, the motor current is normally maximum, so V_{Drop} would be high and little thrust is produced. Consequently, the taps on the reactors are set for low inductance at start. As the LIM increases speed, the inductance is raised to prevent too great an acceleration. Maximum inductance is used once the vehicle attains the desired speed. Some form of dynamic braking can be used for deceleration, or plugging can be used. Plugging is a braking action that is derived by using two LIM's, one attempting to power the other in the reverse direction, slowing the vehicle. The advantage of the series reactor is that large voltage variation can be obtained efficiently. Unfortunately, the power factor is reduced also.

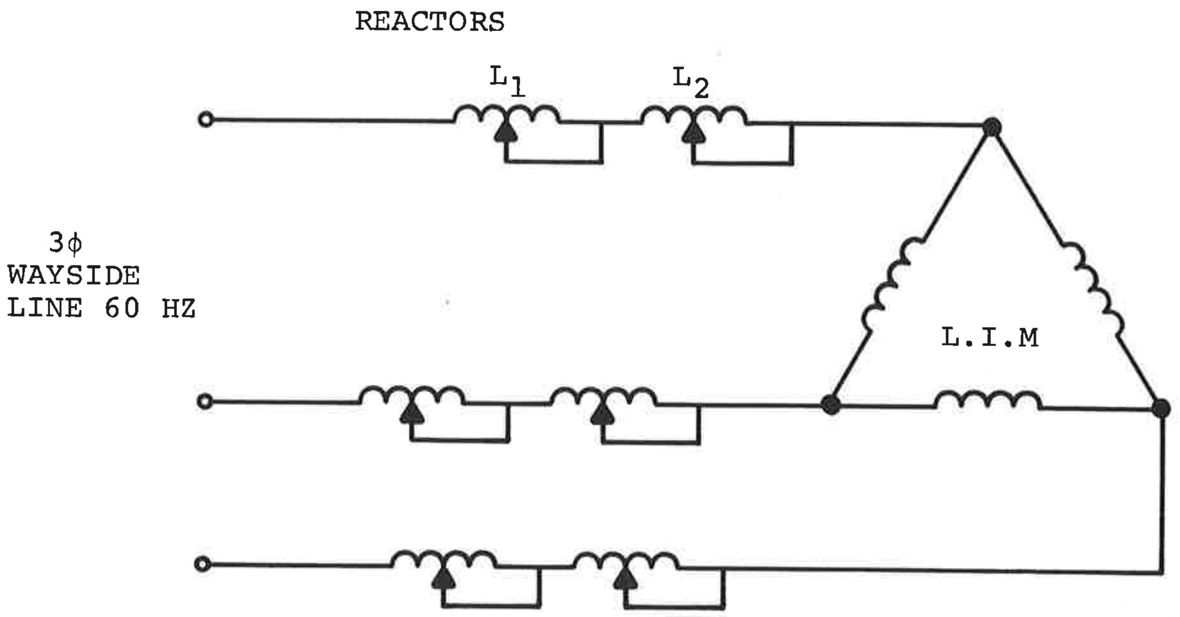

Figure 16. Series Reactor Control

CONCLUSIONS

This report has surveyed the potential power conditioners for the TACV. No attempt has been made to evaluate these power conditioners against vehicle performance. Some general conclusions can be made, however.

1. Variable voltage, fixed frequency systems that require no semiconductors need little development effort. The passive techniques are easily within the state-of-the art.
2. Graded reaction rail resistance techniques require little or no power conditioning. The graded rail technique depends on a nominal vehicle speed profile being maintained, the performance suffering if emergency stops are needed.
3. Pole-changing is a possible method for speed control. Since the wayside line will probably be at 60 Hertz and the optimum maximum frequency for most LIM's is near 200 Hertz, it appears that pole-changing alone will not provide optimum speed control. Little information on pole-changing for LIM's is presently available.

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TABLE 1

MDELTA-W/R EQS. FOR ELECTROHYDRAULIC SERVOSYSTEMS IN POLYNOMIAL FORM

1.0 $$\left(M_D s^2+R_D s+K_D\right) X_D - \left(R_D s+K_D\right) X_e = 0$$

2.0 $$F_{ex} - \left(M_L s^2+\left(R_L+R_D\right) s + \left(K_L+K_D\right)\ \right) X_e + \left(R_D s+K_D\right) X_D = 0$$

3.0 $$K_s \dot{X}_s - A_e s^2 X_e - \left(\frac{C_e}{2A_e} s^2+\frac{1}{R_{Le} A_e} x\ s \right) F_{ex} = 0$$

$$K_s X_s - A_e s X_e - \left(\frac{C_e}{2A_e} s + \frac{1}{R_{Le} A_e} \right) F_{ex} = 0$$

4.0 $$\left[\left(\frac{R_s C_s}{2A_s{}^2} + \frac{M_s}{R_{Ls} A_s{}^2}\right) s^2 + \frac{M_s C_s}{2A_s{}^2} s + \left(1+\frac{R_s}{A_s{}^2 R_{Ls}}\right) s\right] \dot{X}_s - \frac{K_p}{A_s} \dot{X}_p = 0$$

5.0 $$(BL) \frac{C_p}{R_e} \cdot s \cdot E_p - \left[M_p C_p s^2+\left(R_p C_p+\frac{(BL)^2 C_p}{8.85 R_e}\right) s + 1\right] \dot{X}_p = 0$$

6.0 $$-\left[T_{13} T_{15} s^3+\left(T_{13}+T_{15}\right) s^2+s\right] E_1 - \frac{E_p}{2} \frac{R_G}{R_G+R_{15}} \times \frac{R_{17}}{R_9} \left[T_5 T_{13} T_{15} s^4\right.$$

$$\left. + \left(T_{13} T_{15}+T_5 T_{15}+T_5 T_{13}\right) s^3 + \left(T_5+T_{13}+T_{15}\right) s^2+s\right]$$

$$- F_{ex} \times F_f G_f \times \frac{R_{17}}{R_5} \left[T_{13} T_{15} s^3+\left(T_{13}+T_{15}\right) s^2+s\right]$$

$$- 1x_e F_e G_e \times \frac{R_{17}}{R_1} \times \left[T_1 T_{15} s^3+\left(T_1+T_{15}\right) s^2+s\right]$$

$$- \dot{x}_s \times F_s G_s \times \frac{R_{17}}{R_3} \left[T_{13} T_{15} s^2+\left(T_{13}+T_{15}\right) s+1\right]$$

$$- \dot{x}_p\ F_p G_p \frac{R_{17}}{R_4} T_{15} \left(T_{13} s^3+s^2\right) = 0$$

7.0 $$+ E_1 + (KDUM)x_C + (KDUM2)x_D + (KDUM3)\dot{x}_s + (KDUM4)\dot{x}_p = F_1$$

6.0 $$0 = E_1 + \frac{E_p}{2} \text{ x } \frac{R_G}{R_G+R_{15}} \text{ x } \frac{R_{17}}{R_9}\left(T_5 s+1\right) + F_{ex}F_fG_f \frac{R_{17}}{R_5}$$

$$+ x_e \text{ x } F_eG_e \frac{R_{17}}{R_1}\frac{\left(T_1 s+1\right)}{\left(T_{13}s+1\right)} + x_sF_sG_s \frac{R_{17}}{R_3}$$

$$+ \dot{x}_p F_pG_p \frac{R_{17}}{R_4}\frac{T_{15}s}{\left(T_{15}s+1\right)}$$

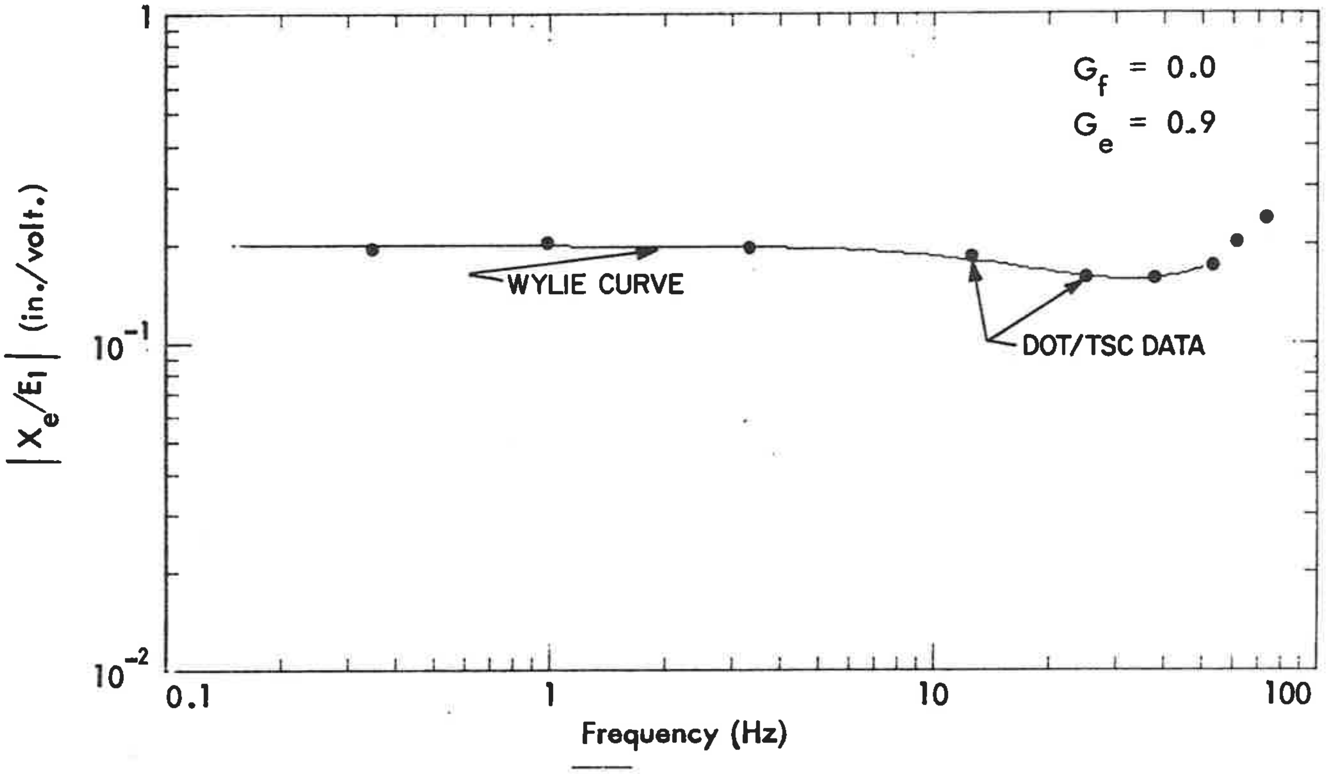

Figure 7. Amplitude of the Overall Transfer Function X_e/E_1 with Exciter Displacement Feedback (Wylie Figure 14, Appendix D, Ref. 1)

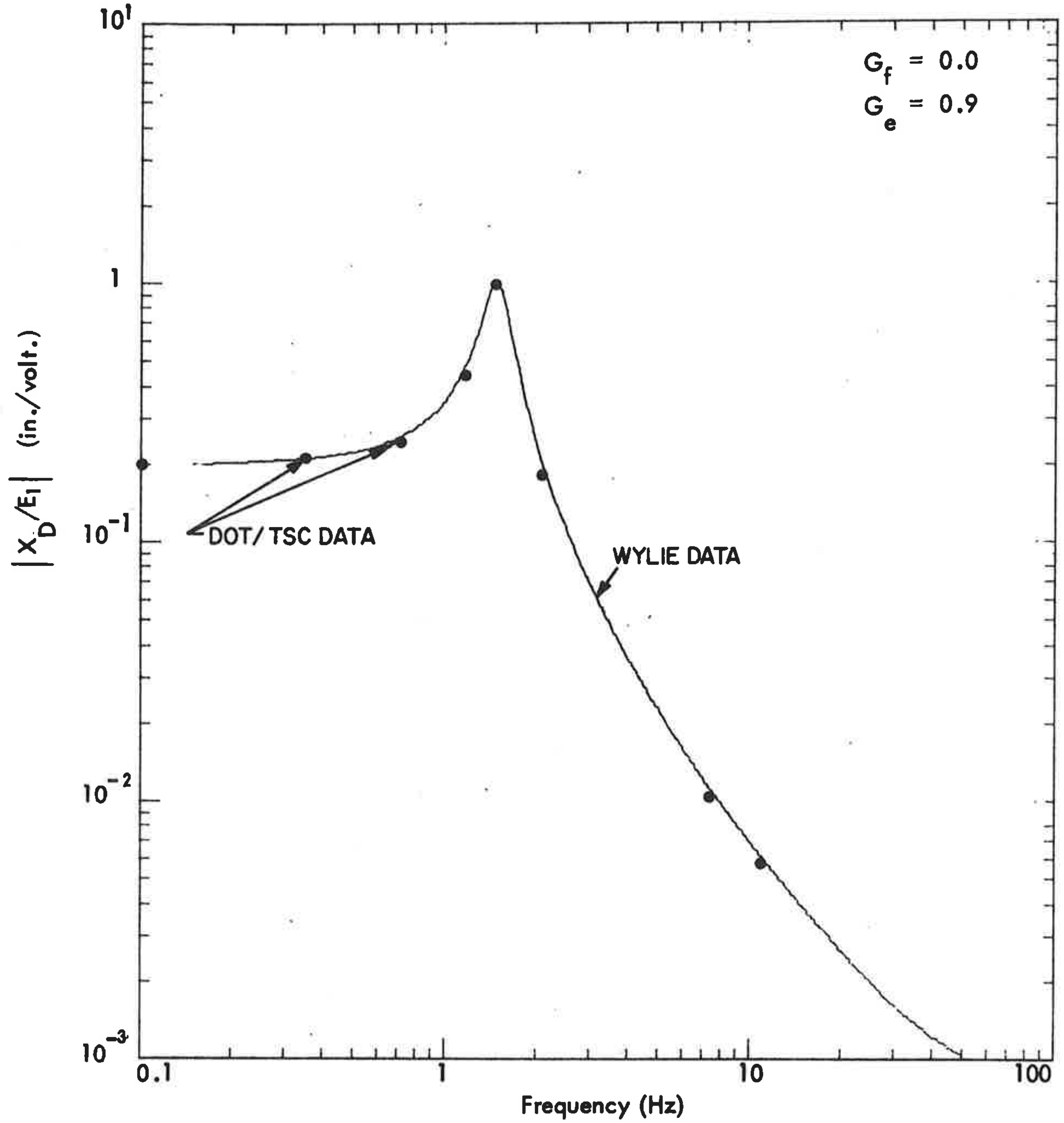

Figure 8. Amplitude of the Overall Transfer Function X_D/E_1 with the Exciter Displacement Feedback (Wylie Figure 15, Appendix D, Ref. 1)

TABLE 2. "A" MATRIX FOR POLYNOMIAL INPUT OF W/R EQS.

2#/PARAMETER	X_D	X_e	F_{es}	$\dot{X}_s$	$\dot{X}_p$	E_p	E_1
1.0	$M_D s^2+R_D s+K_D$	$-\left(R_D s+K_D\right)$					
2.0	$R_D s+K_D$	$-\left[M_L s^2+\left(R_L+R_D\right)s + K_L+K_D\right]$	1				
3.0		$-A_e s^2$	$-\left(\frac{C_e}{2A_e}s^2+\frac{s}{R_{Le}A_e}\right)$	K_s			
4.0				$+\left[\frac{M_s C_s}{2A_s^2}s^3+\left(\frac{R_s C_s}{2A_s^2}+\frac{M_s}{R_{Ls}A_s^2}\right)s^2 + 1+\left(\frac{R_s}{A_s^2 R_{Ls}}\right)\right]s$	$\frac{-K_p}{A_s}$		
5.0					$-\left[M_p C_p s^2+\left(R_p C_p+\frac{(BL)^2 C_p}{8.85R_e}\right)s+1\right]$	$+\frac{(BL)C_p}{R_e}$	
6.0		$-F_e G_e \frac{R_{17}}{R_1}\left[T_1 T_{15}s^3+\left(T_1+T_{15}\right)s^2 +s\right]$	$-F_f G_f \frac{R_{17}}{R_5}\left[T_{13}T_{15}s^3 + \left(T_{13}T_{15}\right)s^2+s\right]$	$-F_s G_s \frac{R_{17}}{R_3}\left[T_{13}T_{15}s^2 +\left(T_{13}+T_{15}\right)s+1\right]$	$-F_p G_p \frac{R_{17}}{R_4}\left(T_{15}\ T_{13}s^3+s^2\right)$	$-\frac{1}{2}\frac{R_G}{R_G+R_{15}}\times\frac{R_{17}}{R_9}\left[T_5 T_{13}T_{15}s^4 + \left(T_{13}T_{15}+T_5 T_{15}+T_{13}T_{15}\right)s^3 + \left(T_5+T_{13}+T_{15}\right)s^2+s\right]$	$-\left[T_{13}T_{15}s^3 + \left(T_{13}+T_{15}\right)s^2 +s\right]$
7.0	KDUM2	KDUM		KDUM3	KDUM4		

TABLE 3
FREQUENCY RESPONSE
WYLIE FIG. 14, Ge = 0.9
SYSTEMS IS UNSTABLE

```
               DENOMINATOR ROOTS

  EIGENVALUE EVALUATION BY TARNOVES METHOD

     DEGREE OF POLYNOMIAL ELEMENTS=  4
                 REAL MATRIX ORDER=  7



 ROOT                    LAMBDA
 NUMBER          REAL             IMAGINARY

    1       -2.167653E-07   -2.328306E-07
    2       -9.466508E-01   -9.394233E 00
    3       -9.466508E-01    9.394233E 00
    4       -1.160816E 02   -8.680502E-09
    5       -3.399001E 02    3.141917E 02
    6       -3.399001E 02   -3.141917E 02
    7        5.773792E 01    5.368186E 02
    8        5.773792E 01   -5.368186E 02
    9       -9.285073E 02   -1.724691E-03
   10       -7.932402E 02    4.564016E-05
   11       -1.116989E 04    2.893741E 03
   12       -1.116989E 04   -2.893741E 03
   13       -1.607565E 04    2.106845E 03
   14       -1.607565E 04   -2.106845E 03
```

TABLE 4
STEP FUNCTION INPUT
TIME RESPONSE FOR XE

```
                     ROOT                                              RESIDUE
                REAL           IMAGINARY                   REAL           IMAGINARY     EXP

       1.)   9.999999E-04    0.0                        2.975834E 00   -1.655243E-05    0
       2.)  -9.999999E-04    0.0                        2.976671E 00   -1.656268E-05    0
       3.)  -7.349698E 00    0.0                       -6.100557E 00    3.516357E-05    0
       4.)  -9.456432E-01   -9.393666E 00              -4.872193E-04   -1.787531E-06    0
       5.)  -9.456432E-01    9.393666E 00              -4.872193E-04    1.793029E-06    0
       6.)  -2.720078E 02    0.0                        1.845930E-01   -1.029916E-06    0
       7.)  -2.363294E 02    0.0                       -9.586626E-02    5.376951E-07    0
       8.)  -1.374303E 00   -6.249041E 02               1.872918E-02   -1.633651E-03    0
       9.)  -1.374303E 00    6.249041E 02               1.872920E-02    1.633442E-03    0
      10.)  -9.418142E 02    2.790020E 02               1.142217E-02   -1.135723E-02    0
      11.)  -9.418142E 02   -2.790020E 02               1.142230E-02    1.135710E-02    0
      12.)  -1.116992E 04    2.893815E 03               5.211498E-07   -7.519825E-07    0
      13.)  -1.116992E 04   -2.893815E 03               5.211593E-07    7.519764E-07    0
      14.)  -1.607567E 04    2.106841E 03               1.553045E-07   -2.168716E-08    0
      15.)  -1.607567E 04   -2.106841E 03               1.553047E-07    2.168522E-08    0


IME           XE                       TIME            XE                           TIME


           4.176678E-06             2.999998E-01    5.295280E 00                6.000000E-01
00E-02     1.762175E 00             3.499997E-01    5.497773E 00                6.999999E-01
96E-02     3.058354E 00             3.999997E-01    5.636742E 00                7.999999E-01
99E-01     3.954513E 00             4.499996E-01    5.731890E 00                8.999999E-01
99E-01     4.573626E 00             4.999996E-01    5.796903E 00                9.999998E-01
98E-01     5.000870E 00             5.000000E-01    5.796907E 00
```

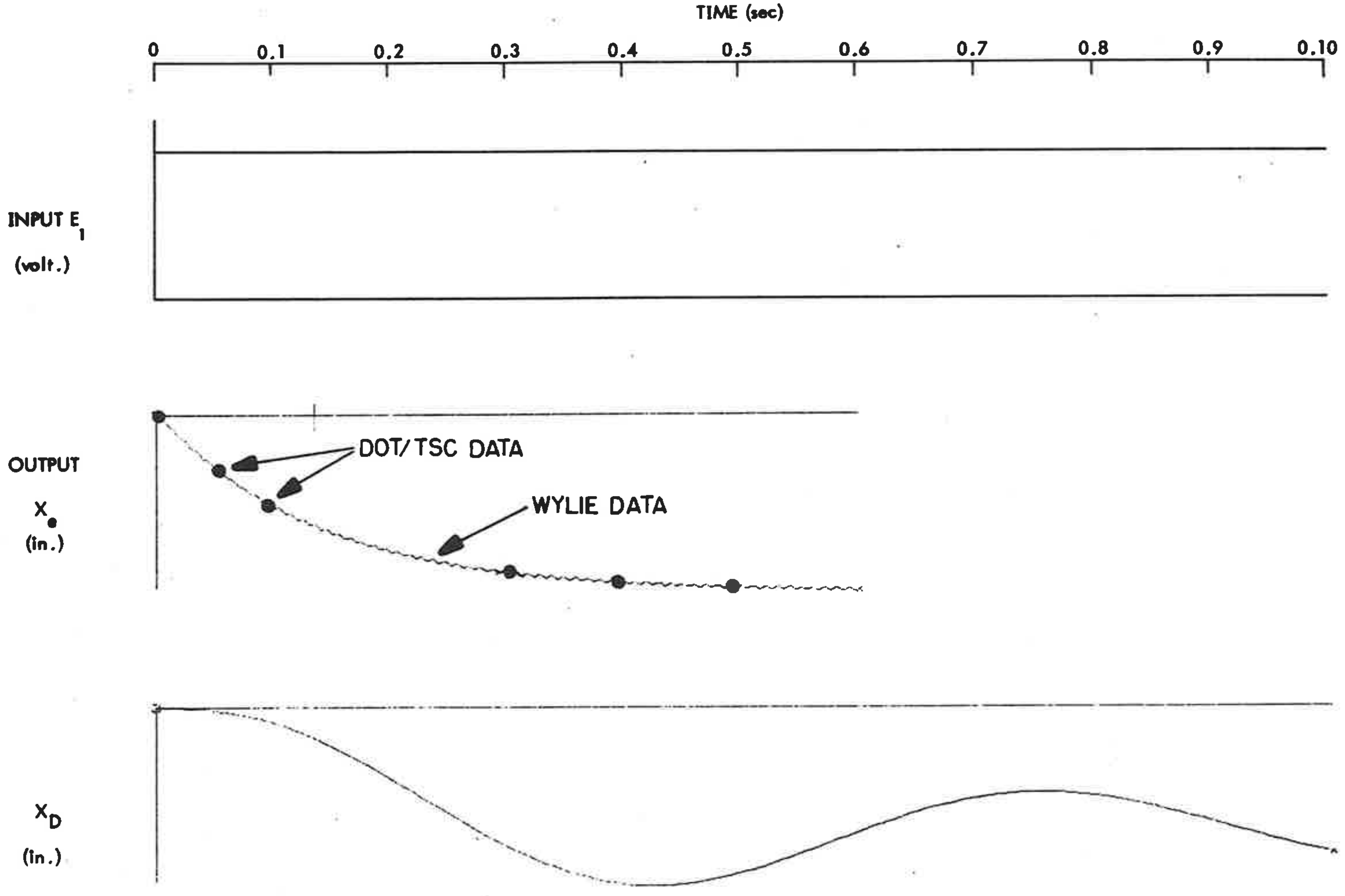

Figure 9. Transient Response to a Unit Step Input for a Typical EHSA System (Example 1) (Wylie Figure 18, Appendix D, Ref. 1)

TABLE 5
ROOTS FOR STEP FUNCTION RESPONSE POLYNOMIAL INPUT METHOD
WYLIE FIG. 18 (Ex #1) Ge = 0.03 SYSTEM IS STABLE

```
              DENOMINATOR EIGENVALUES

    EIGENVALUE EVALUATION BY TARNOVES METHOD

        DEGREE OF POLYNOMIAL ELEMENTS=  4
                    REAL MATRIX ORDER=  6



     ROOT                      LAMBDA
    NUMBER           REAL              IMAGINARY

        1       -5.124602E-07    1.343869E-07
        2       -7.349698E 00   -1.864391E-07
        3       -9.456432E-01   -9.393666E 00
        4       -9.456432E-01    9.393666E 00
        5       -2.720078E 02    1.004357E-03
        6       -2.363294E 02   -1.560917E-05
        7       -1.374303E 00   -6.249041E 02
        8       -1.374303E 00    6.249041E 02
        9       -9.418142E 02    2.790020E 02
       10       -9.418142E 02   -2.790020E 02
       11       -1.116992E 04    2.893815E 03
       12       -1.116992E 04   -2.893815E 03
       13       -1.607567E 04    2.106841E 03
       14       -1.607567E 04   -2.106841E 03
```

TABLE 6

1) $$X_d = -\frac{R_D}{M_D}\dot{X}_D - \frac{K_D}{M_D}X_D + \frac{R_D}{M_D}\dot{X}_e + \frac{K_D}{M_D}X_e$$

2) $$X_e = -\frac{R_L+R_D}{M_L}\dot{X}_e - \frac{K_L+K_D}{M_L}X_e + \frac{R_D}{M_L}\dot{X}_D + \frac{K_D}{M_L}X_D + F_{ex}$$

3) $$K_sX_k - A_esX_e - \left(\frac{C_e}{2A_e}s + \frac{1}{R_{Le}A_e}\right)F_{ex} = 0$$

$$\dot{F}_{ex} = -\frac{2}{R_{Le}C^e}F_{ex} - \frac{2A_e^2}{C_e}\dot{X}_e + \frac{2A_e}{C_e}K_s$$

4) $$\frac{R_sC_s}{2A_s{}^2}\dddot{X}_s + \frac{M_sC_s}{2A_s{}^2}\ddddot{X}_s + \ddot{X}_s - \frac{K_p}{A_s}\dot{X}_p = 0$$

$$\ddddot{X}_s = -\frac{R_s}{M_s}\dddot{X}_s - \frac{2A_s{}^2}{M_sC_s}\ddot{X}_s + \frac{2A_s}{M_sC_s}K_p\dot{X}_p$$

5) $$(BL)\frac{C_p}{R_e}\dot{E}_p - M_pC_p\dddot{X}_p - R_pC_p + \frac{(BL)^2C_p}{8.85R_e}\ddot{X}_p - \dot{X}_p = 0$$

$$\dddot{X}_p = -\frac{1}{M_p}\left(R_p + \frac{(BL)^2}{8.85R_e}\right)\ddot{X}_p - \frac{1}{M_pC_p}\dot{X}_p + \frac{(BL)}{M_pR_e}\dot{E}_p$$

TABLE 6 (Cont)

$$
\begin{array}{lll}
6) \quad -E_1 = \frac{E_p}{2} \times \frac{R_G}{R_G+R_{15}} \quad \frac{R_{17}}{R_9}\left(T_5 s+1\right) \dot{E}_p & = & -2 \times \overbrace{\frac{R_G+R_{15}}{R_G}\,\frac{R_p}{R_{17}}}^{C'} \times \frac{1}{T_5} \times E_1 \\
 & & - G_p/T_5 \\
\qquad + F_{ex}\, F_f G_f \frac{R_{17}}{R_5} & & - C' F_f G_f \frac{R_{17}}{R_5} F_{ex} \\
\qquad + X_e\, F_e G_e \frac{R_{17}}{R_1} \frac{(T_1 s+1)}{(T_{13} s+1)} & & - C' F_e G_e \frac{R_{17}}{R_1} \quad \frac{(T_1 s+1)}{(T_{13} s+1)} X_e \\
\qquad + X_s\, F_s G_s \frac{R_{17}}{R_3} & & - C' F_s G_s \frac{R_{17}}{R_3} X_s \\
\qquad + \dot{X}_p\, F_p G_p \frac{R_{17}}{R_4} \frac{T_{15} s}{(T_{15} s+1)} & & - C' F_p G_p \frac{R_{17}}{R_4} \frac{T_{15} s}{T_{15} s+1} \dot{X}_p
\end{array}
$$

$$M_D\ddot{X}_D + R_D\dot{X}_D + K_DX_D - R_D\dot{X}_e - K_DX_e = 0$$

$$\ddot{X}_D = -\frac{R_D}{M_D}\dot{X}_D - \frac{K_D}{M_D}X_D + \frac{R_D}{M_D}\dot{X}_e + \frac{K_D}{M_D}X_e$$

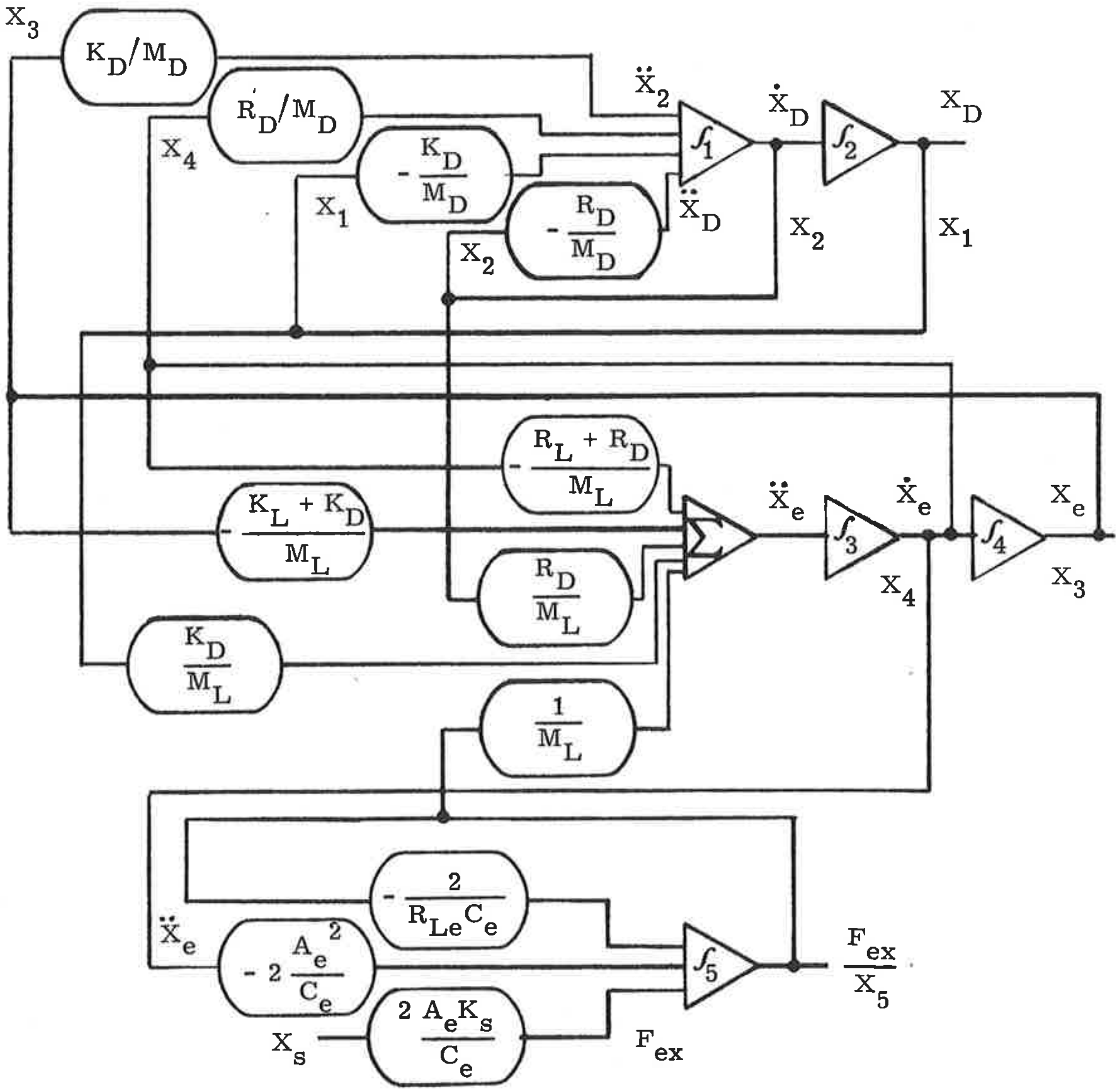

Figure 10. State Variables by Block Diagrams

]

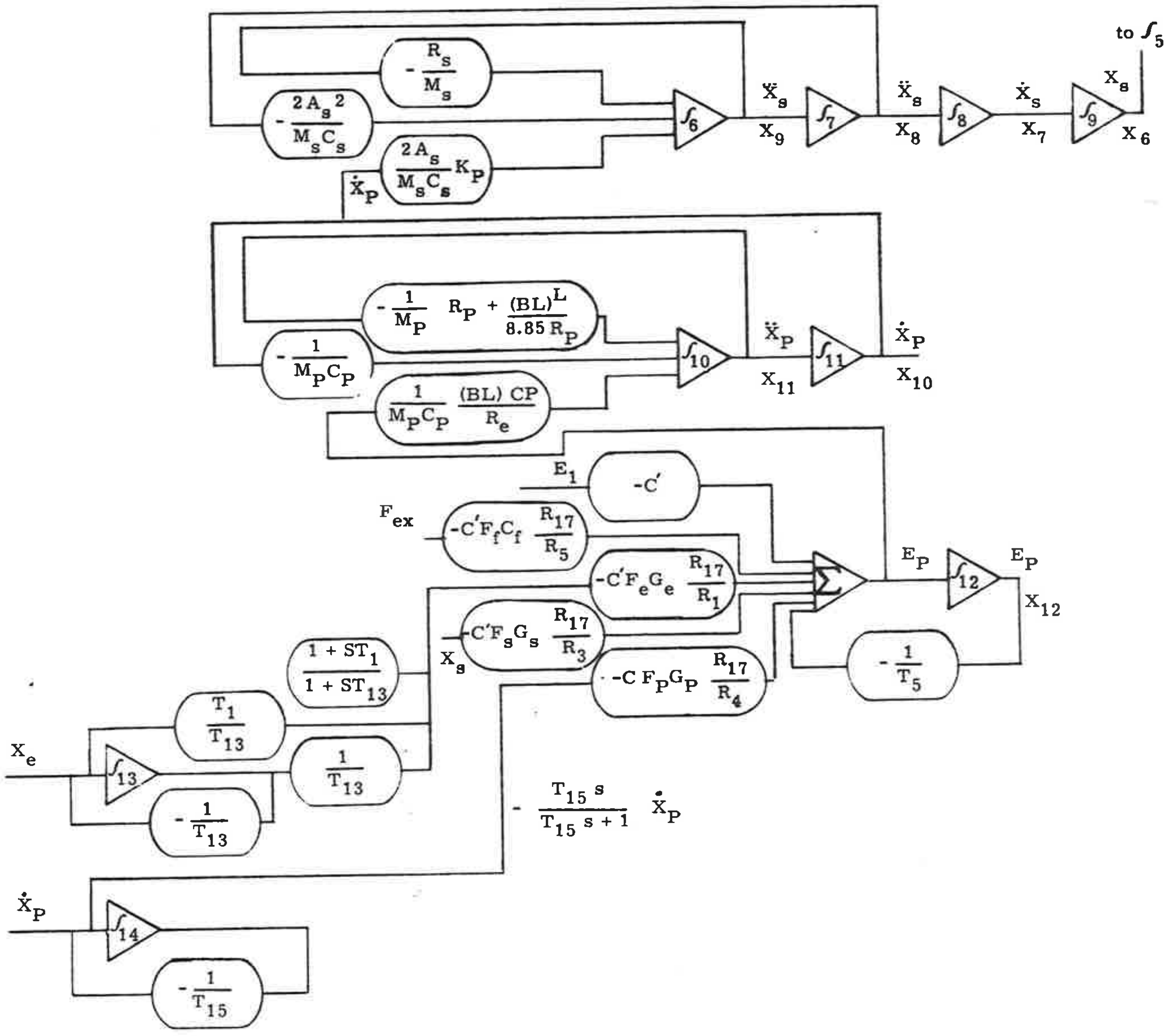

Figure 10. State Variables by Block Diagram.

TABLE 7. ANALOGUE DIAGRAM DEVELOPED LIST OF STATE VARIABLES RELATIONSHIPS

State Variable Rate State Variable		(X_D) X_1	$(\dot X_D)$ X_2	(X_e) X_3	$(\dot X_e)$ X_4	(F_{ex}) X_5	(X_A) X_6	$(\dot X_A)$ X_7	$(\ddot X_A)$ X_8	(X_a) X_9	$(\dot X_P)$ X_{10}	$(\ddot X_P)$ X_{11}	E_P X_{12}	X_{13}	X_{14}	E_1
From Dif. Eq's	$\dot x_1$		1													
	$\dot x_2$	$-K_D/M_D$	$-R_D/M_D$	$+K_D/M_D$	$+R_D/M_D$											
	$\dot x_3$				1											
	$\dot x_4$	K_D/M_L	R_D/M_L	$-(K_L+K_D)/M_L$	$-(R_L+R_D)/M_L$	$1/M_L$										
	$\dot x_5$				$-2A_e^2/C_e$	$-2/C_e \cdot R_{Le}$	$+2A_e/C_e \times K_a$									
	$\dot x_6$							1								
	$\dot x_7$								1							
	$\dot x_8$									1						
	$\dot x_9$								$-2A_a^2/M_aC_a$	$-R_a/M_a$	$+\frac{2A_a}{M_aC_a} \times K_P$					
	$\dot x_{10}$											1				
	$\dot x_{11}$			$-\frac{C'}{R_{17}} \times K_{DUM}$		$C \times \frac{G_fF_f}{R_5}$	$C \times \frac{G_aF_a}{R_3}$				$-\frac{1}{M_PC_P}$	$-\frac{1}{M_PC_P} \times \left(R_PC_P + \frac{(BL)^2C_P}{8.85\,R_e}\right)$	$C \times \frac{R_G}{R_G+R_{15}} \times \frac{1}{2R_9}$	C	C	$\frac{C}{R_{17}}$
	$\dot x_{12}$			$\frac{C}{R} \times K_{DUM}$	$\frac{T_1}{T_{13}}$	$+\frac{C' \times R_{17}}{R_5} G_fF_f$	$+\frac{C'R_{17}}{R_3} G_aF_a$						$-\frac{1}{T_5}$	$-C' \times R_{17}$	$-C' \times R_{17}$	$-C'$
	$\dot x_{13}$			$\frac{1}{R_1R_2C_1} \times G_eF_e$	$\frac{R_1+R_3}{R_1R_3} G_eF_e$									$+\frac{1}{R_{29}}$		
	$\dot x_{14}$											$\frac{G_PF_P}{R_4}$			$-\frac{1}{R_{22}C_8}$	
From Block Diagram	$\dot x_{11}$			$-\frac{C}{R_1} F_eG_e \frac{T_1}{T_{13}}$ $-\frac{C}{R_{17}} \times K_{DUM}$		$\frac{C}{R_5} G_fF_f$	$\frac{C}{R_3} F_aG_a$				$-\frac{1}{M_PC_P} + \frac{C}{R_4} F_PG_P$	$-\frac{1}{M_PC_P} \times \left(R_PC_P - \frac{(BL)^2C_P}{8.85\,R_e}\right)$	$-\frac{1}{M_PC_P} \frac{BL\,C_P}{R_e} \frac{1}{T_5}$	$\frac{C}{R_1} F_eG_e \frac{1}{T_{13}} \left(1-\frac{T_1}{T_{13}}\right)$	$-\frac{C'}{R_4} F_PG_P \left(\frac{1}{T_{15}}\right)$	$\frac{C}{R_{17}}$
	$\dot x_{12}$			$C' \times K_{DUM} -$ $C' \times F_eG_e \frac{R_{17}}{R_1} \frac{T_1}{T_{13}}$		$-C'F_fG_f \frac{R_{17}}{R_5}$	$-C'\frac{R_{17}}{R_3} F_aG_a$				$-C'F_PG_P \frac{R_{17}}{R_4}$		$-\frac{1}{T_5}$	$-C'F_eG_e \frac{R_{17}}{R_1} \times$ $\frac{1}{T_{13}}\left(1-\frac{T_1}{T_{13}}\right)$	$+C'F_PG_P \frac{R_{17}}{R_4} \frac{1}{T_{15}}$	$-C'$
	$\dot x_{13}$													$-\frac{1}{T_{13}} X_{13}'$		
	$\dot x_{14}$														$-\frac{1}{T_{15}} X_{14}'$	

NOTES

C' is called F1 in Parameter Values

Watch out that F1 is FP input notation

$$C' = \frac{R_G' - R_{15}}{R_G} \times \frac{2}{C_4} = \frac{1}{R_{17}} \cdot \frac{20.1}{2.1} \times \frac{2}{470 \times 10^{-6}} \times \frac{1}{.056} = 1.05 \times 10^6$$

$$C' \times R_{17} = 1.05 \times 10^6 \times .056 = .059 \times 10^6$$

$$C = -\frac{1}{M_P} \times \frac{B_L}{R_e} \times \frac{R_G + R_{15}}{R_G} \times \frac{2}{C_4} = -\frac{1}{1.3 \times 10^{-4}} \times \frac{40}{400} \times \frac{20.1}{2.1} \times \frac{2}{470 \times 10^{-6}}$$

$$C = -27.1 \times 10^6$$

DEVELOPMENT OF STATE VARIABLES FROM DIFFERENTIAL EQUATIONS

1) $M_D X_D + R_D \dot{X}_D + K_D X_D - R_D \dot{X}_e - K_D X_e = 0$

let $X_1 = X_D$ $X_2 = \dot{X}_1$, $X_3 = X_e$, $X_4 = \dot{X}_3$

$$M_D \dot{X}_2 + R_D X_2 + K_D X_1 - R_D X_4 - K_D X_3 = 0$$

$$\underline{\dot{X}_1 = X_2}$$

$$\underline{\dot{X}_2 = \frac{1}{M_D}} \left(R_D X_2 + K_D X_1 - R_D X_4 - K_D X_3\right)$$

$$\underline{\dot{X}_3 = X_4}$$

2) $F_{ex} - \left(M_L s^2 + (R_L + R_D)s + K_L + K_D\right) X_e + \left(R_D s + K_D\right) X_D = 0$

let $X_5 = F_{ex}$

$$X_5 - M_L \dot{X}_4 - (R_L + R_D) X_4 - (K_L + K_D) X_3 + R_D X_2 + K_D X_1 = 0$$

$$\dot{X}_4 = + \frac{K_D}{M_L} X_1 + \frac{R_D}{M_L} X_2 - \left(\frac{K_L + K_D}{M_L}\right) X_3 - \left(\frac{R_L + R_D}{M_L}\right) X_4 + \frac{X_5}{M_L}$$

Eq3) $K_s \dot{X}_s - A_e s^2 X_e - \left(\frac{C_e}{2A_e} s^2 + \frac{1}{R_{Le} A_e} s\right) F_{ex} = 0$

also $K_s X_s - A_e s X_e - \left(\frac{C_e}{2A_e} s + \frac{1}{R_{Le}}\right) F_{ex} = 0$

$X_6 = X_s$

$$K_s X_6 - A_e X_4 - \frac{C_e}{2A_e} \dot{X}_5 - \frac{1}{R_{Le} A_e} X_5 = 0$$

$$\dot{X}_5 = \left[K_s X_6 - \frac{1}{R_{Le} A_e} X_5 - A_e X_4\right] \frac{2A_e}{C_e}$$

$$\underbrace{\dot{X}_5 = \frac{2A_e}{C_e}\left[-A_e\left(+\frac{K_D}{M_L}X_1+\frac{R_D}{M_L}X_2 - \frac{(K_L+K_D)}{M_L}X_3 - \frac{(R_L+R_D)}{M_L}\right.\right. \quad \left.\left.\frac{1}{R_{Le}A_e}X_5\right) + K_sX_6\right]}_{\text{for } R_{Ls} = \infty}$$

4) $$\left(\left(\frac{R_sC_s}{2A_s^2}\right)s^2 + \frac{M_sC_s}{2A_s^2}s^3 + s\right)\dot{X}_s - \frac{K_p}{A_s}\dot{X}_p = 0$$

let $X_6 = X_s$, $X_7 = \dot{X}_6$, $X_8 = \dot{X}_7$, $X_9 = \dot{X}_8$, $X_{10} = \dot{X}_9$

$$\frac{M_sC_s}{2A_s^2}\dot{X}_9 + \frac{R_sC_s}{2A_s^2}X_9 + X_8 - \frac{K_p}{A_s}X_{10} = 0$$

$$\underline{\dot{X}_6 = X_7}$$

$$\underline{\dot{X}_7 = X_8}$$

$$\underline{\dot{X}_8 = X_9}$$

$$\underline{\dot{X}_9 = -\frac{2A_s^2}{M_sC_s}X_8 - \frac{R_s}{M_s}X_9 + \frac{2A_2}{M_sC_s}K_p X_{10}}$$

5) $$\frac{(BL)C_p}{R_e}s\,E_p - \left(M_pC_ps^2+\left(R_pC_p+\frac{(BL)^2C_p}{8.85R_e}\right)s+1\right)\dot{X}_p = 0$$

$X_{10} = \dot{X}_p$, $X_{11} = \dot{X}_{10}$, $\dot{X}_{11} = s^2\dot{X}_p$, $X_{13} = E_p$, $\dot{X}_{12} = \dot{E}_p$

$$\dot{X}_{10} = X_{11}$$

$$\dot{X}_{11} = \frac{1}{M_pC_p}\left[\frac{(BL)C_p}{R_e}\dot{X}_{12} - \left(R_pC_p + \frac{(BL)^2C_p}{8.85R_e}\right)X_{11} - X_{10}\right]$$

from eq. 6

$$\dot{X}_{11} = \frac{1}{M_pC_p}\left\{\frac{(BL)C_p}{R_e}\left(-\frac{R_G+R_{15}}{R_G}\frac{2}{C_4}\right)\left[X_{13}+\frac{G_fF_f}{R_5}X_5-\frac{G_sF_f}{R_3}X_6+X_{14}\right.\right.$$

$$\left.\left.+\frac{R_G}{R_G+R_{15}}\frac{1}{2R_9}X_{12}+\frac{E_1}{R_{17}}\right]-\left(R_pC_p+\frac{(BL)^2C_p}{8.85R_e}\right)X_{11}-X_{10}\right\}$$

for eq. 6 derive by equating i s = 0 into mode.

$$V_e = G_eF_e \times X_e \quad , \quad i_e = V_e \Big/ \frac{R_1\left(R_2+\frac{1}{Gs}\right)}{R_1+R_2+\frac{1}{Gs}}$$

$$R_1\left(R_2+\frac{1}{Gs}\right)i_e = V_c\left(R_1+R_2+\frac{1}{Gs}\right)$$

$$(R_1R_2C_1s+R_1)\,i_e = V_e\left((R_1+R_2)Gs+1\right)$$

$$\dot{i}_e = -\frac{1}{R_2C_1}i_e+\frac{1}{R_1R_2C_1}V_e+\frac{R_1+R_2}{R_1R_2}\dot{V}_e$$

let $X_{13} = i_e$, $\dot{X}_{13} = \dot{i}_e$

$$\dot{X}_{13} = -\frac{1}{R_2C_1}X_{13}+\frac{1}{R_1R_2C_1}G_eF_e\,X_3+\frac{R_1+R_2}{R_1R_2}G_eF_e\,X_4$$

$$V_f = G_fF_f \times F_{ex} \quad , \quad i_f = \frac{V_f}{R_5} = \frac{G_fF_f}{R_5}F_{ex} = \frac{G_fF_f}{R_5}X_5$$

$$i_s = \frac{G_sF_s}{R_3}X_s = \frac{G_sF_s}{R_3}X_s$$

$$V_p = \frac{R_{22}}{R_{22} + \frac{1}{C_6 s}} G_p F_p \dot{X}_p \quad , \qquad i_p = \frac{V_p}{R_4}$$

$$i_p = \frac{1}{R_4} \frac{R_{22} C_6 s}{1 + R_{22} C_6 s} G_p F_p \dot{X}_p$$

$$i_p + R_{22} C_6 i_p = \frac{R_{22}}{R_4} C_6 G_p F_p \; s \; \dot{X}_p$$

let $X_{14} = \dot{X}_p \quad X_{14} = i_p$

$$\dot{X}_{14} = \frac{1}{R_4} G_p F_p \; X_{11} - \frac{1}{R_{22} C_6} X_{14}$$

$$i_1 = \frac{E_1}{R_{17}}$$

$$V_g = \frac{R_G}{R_G + R_{15}} \times \frac{E_p}{2} \quad , \qquad i_g = \frac{V_g}{\dfrac{R_9 \times \frac{1}{C_4 s}}{R_9 + \frac{1}{C_4 s}}}$$

$$i_g \times \frac{R_4}{C_4 s} = V_g \left(R_9 + \frac{1}{C_4 s} \right) \quad , \qquad i_g = V_g \left(\frac{1}{R_9} + C_4 s \right)$$

$$i_g = \frac{R_G}{R_G + R_{15}} \times \frac{1}{2} \times \frac{1}{R_9} X_2 + \frac{R_G}{R_G + R_{15}} \times \frac{C_4}{2} \times \dot{X}_{12}$$

$\Sigma \; i \; s = 0 \quad , \; i_e + i_f + i_s + i_p + i_g + i_1 = 0$

$$X_{13} + \frac{G_f F_f}{R_5} X_5 + \frac{G_s F_s}{R_3} X_6 + X_{14} + \frac{R_G}{(R_G + R_{15})} \frac{1}{2R_9} X_{12}$$

$$+ \frac{R_G}{R_G + R_{15}} \frac{C_4}{2} \dot{X}_{12} + \frac{E_1}{R_{17}} = 0$$

$$\dot{X}_{12} = \underbrace{- \frac{R_G + R_{15}}{R_G}}_{-C' \text{ x } R_{17}} \frac{2}{C_4} \left[X_{13} + \frac{G_f F_f}{R_5} X_5 + \frac{G_s F_s}{R_3} X_6 + X_{14} + \frac{R_G}{(R_G + R_{15})} \frac{1}{2R_9} X_{12} + \frac{E_1}{R_{17}} \right]$$

TABLE 9
STATE VARIABLE METHOD
PULSE INPUT
TIME RESPONSE EVALUATION

```
                                                              TIME= 2.000000E-03
    X1      =-5.088396E-07       X3      =-8.423063E-04                .002

                                                              TIME= 3.999997E-03
    X1      =-3.091950E-05       X3      =-2.160228E-02                .004

                                                              TIME= 5.999994E-03
    X1      =-2.483400E-04       X3      =-9.859586E-02                .006

                                                              TIME= 7.999990E-03
    X1      =-8.904601E-04       X3      =-2.196671E-01                .008

                                                              TIME= 9.999987E-03
    X1      =-2.052718E-03       X3      =-3.237454E-01                .010

                                                              TIME= 1.500000E-01
    X1      =-2.586845E-01       X3      =-1.526415E-01                .15

                                                              TIME= 1.999999E-01
    X1      =-3.221382E-01       X3      =-1.049034E-01                .2

                                                              TIME= 2.499999E-01
    X1      =-3.313633E-01       X3      =-7.187831E-02                .25

                                                              TIME= 2.999998E-01
    X1      =-2.834721E-01       X3      =-4.904082E-02                .3

                                                              TIME= 3.499998E-01
    X1      =-1.855629E-01       X3      =-3.326092E-02                .35
```

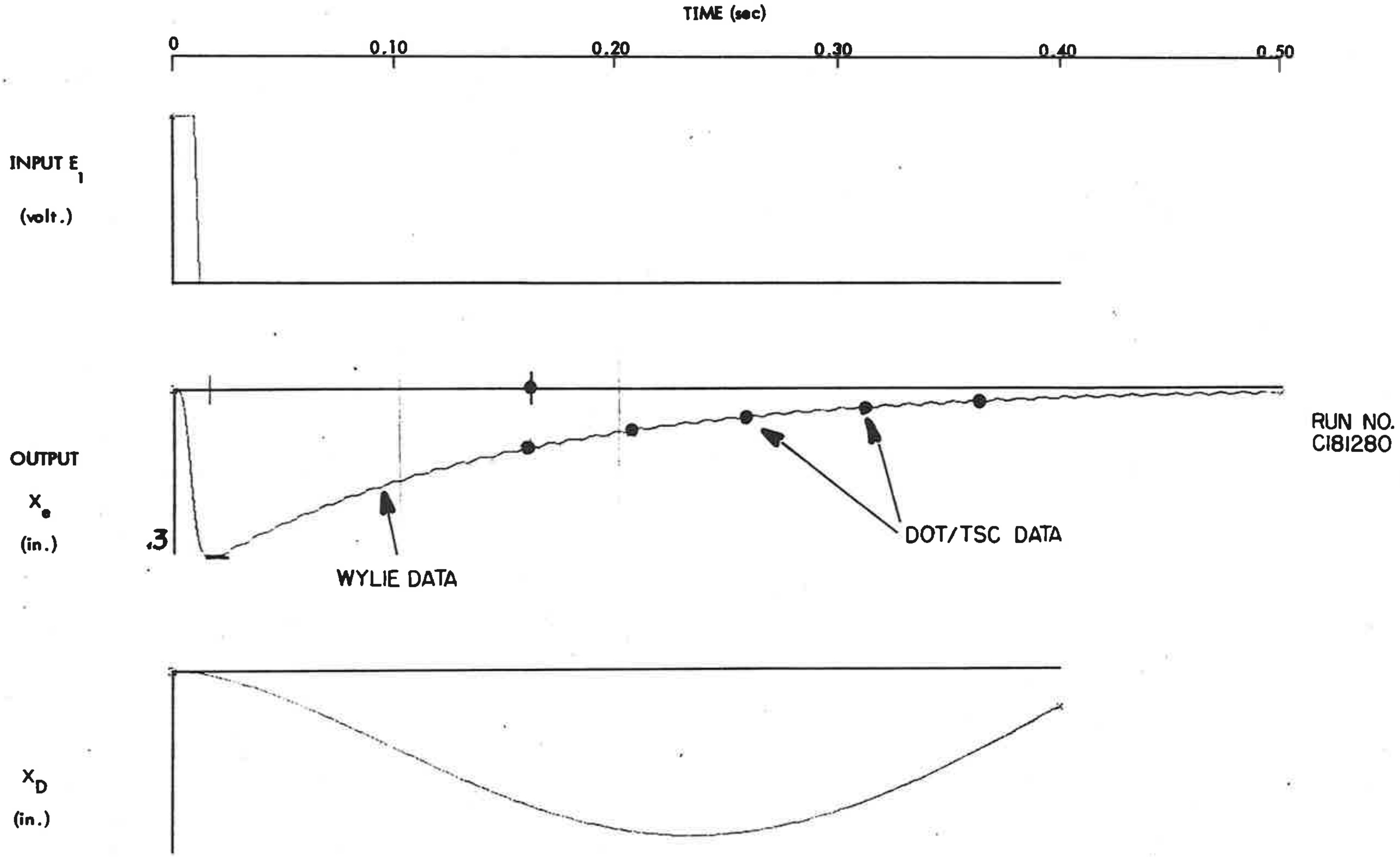

Figure 12. Transient Response to a Unit Impulse Input Signal (Example 1) (Wylie Figure 17, Appendix D. Ref. 1)

TABLE 10
ROOTS FOR PULSE INPUT STATE VARIABLE METHOD
Ge = 0.03 WYLIE FIG. 17

```
                 DENOMINATOR ROOTS

        EIGENVALUE EVALUATION BY TARNOVES METHOD

          DEGREE OF POLYNOMIAL ELEMENTS=  1
                      REAL MATRIX ORDER= 14


       ROOT                         LAMBDA
      NUMBER            REAL                 IMAGINARY

         1         -2.729141E-02     -6.225663E-05
         2         -7.365518E 00      2.512501E-07
         3         -9.456441E-01      9.393661E 00
         4         -9.456441E-01     -9.393661E 00
         5         -2.730332E 02      4.770387E-03
         6         -2.345472E 02     -3.578602E-04
         7         -9.399922E 02      2.836006E 02
         8         -9.399922E 02     -2.836006E 02
         9         -1.372005E 00      6.248970E 02
        10         -1.372005E 00     -6.248970E 02
        11         -1.117550E 04      2.916989E 03
        12         -1.117550E 04     -2.916989E 03
        13         -1.607169E 04      2.108917E 03
        14         -1.607169E 04     -2.108917E 03
```

REFERENCES

1. Technical Proposal, Prototype Control System for WRDRF, to to FRA, DOT: by Wylie Labs. East Operations, Huntsville, Ala., Feb. 12, 1971.

APPENDIX A

APPENDIX A

TABLE 1

COMPONENT DESCRIPTION AND VALUES FOR EHSA MECHANICAL ANALOG CIRCUITS

Pilot Valve (Figure 2)

E_p	Load Voltage Across Armature	Volts
BL'	Electrodynamic Coupling Constant	lb/amp
C_p	Compliance of Armature Coil	in/lb
L_e	Inductance of Armature Coil	henries
M_p	Mass of Armature and Pilot Valve	lb-sec^2/in
r_e	Electrical Resistance of Armature Coil	ohms
R_p	Mechanical Resistance of Pilot Valve	lb-sec/in
X_p	Velocity of Pilot Valve	in/sec
K_p	Flow Control Constant for Pilot Valve	in^3/sec-in

Slave Valve (Figure 3)

A_s	Effective End Area of Slave Spool	in^2
C_s	Hydraulic Compliance of a Single end Cavity	in^5/lb
M_s	Mass of Slave Spool	lb-sec^2/in
R_{Ls}	Hydraulic Leakage Resistance around Slave Spool	lb-sec/in^5
R_s	Mechanical Resistance of Slave Spool	lb-sec/in
X_s	Velocity of Slave Spool	in/sec
K_s	Flow Control Constant for Slave Valve	in^3/sec-in

TABLE 1 (Continued)

Exciter (Figure 6)

A_e	Effective End Area of Exciter Piston	in^2
C_e	Hydraulic Compliance of a Single End Cavity	in^5/lb
F_{ex}	Mechanical Force Output of Exciter	lb
R_{Le}	Hydraulic Leakage Resistance Around Exciter	$lb\text{-}sec/in^5$
M_e	Mass of Exciter Piston	$lb\text{-}sec^2/in$
R_e	Mechanical Resistance of Exciter Piston	lb-sec/in
X_e	Velocity of Exciter Piston	in/sec
F_L	Mechanical Force on Load	lb
M'_L	Mass of External Load	$lb\text{-}sec^2/in$
R'_L	Mechanical Resistance of External Load	lb-sec/in
$1/K_L$	Compliance of External Load	in/lb
M_D	Mass of External Load	$lb\text{-}sec^2/in$
R_D	Mechanical Resistance	lb-sec/in
$1/K_D$	Compliance of External Load	in/lb
X_D	Displacement of External Load	in

TABLE 2

PARAMETERS TO BE USED FOR SERVO-AMPLIFIER ANALYSIS

Circuit Elements in Figure 8

R_1 = 100.0 kilohms

R_2 = 6.8 kilohms

R_3 = 100.0 kilohms

R_4 = 100.0 kilohms

R_5 = 56.0 kilohms

R_9 = 100.0 kilohms

R_{11} = 10.0 kilohms

R_{12} = 1.8 kilohms

R_{13} = 2.5 kilohms

R_{14} = 2.5 kilohms

R_{15} = 27.0 kilohms

R_{17} = 56.0 kilohms

R_{22} = 10.0 kilohms

C_1 = 0.68 microfarads

C_4 = 0.00047 microfarads

C_6 = 0.22 microfarads

Time Constants

$T_1 = (R_1 + R_2)\, C_1 = 0.00726$ sec

$T_5 = R_9 C_4 = 0.000047$ sec

$T_{13} = R_2 C_1 = 0.0046$ sec

$T_{15} = R_{22} C_6 = 0.0022$ sec

Transducer Constants

F_f = 3.67 volt/lb

F_e = 10 volts/in

F_s = 60 volts/in

F_p = 0.7 volts/in/sec (including transducer amplifier gain of 26)

Nominal Gain Settings

$G_f = 1.0$

$G_e = 0.9$

$G_s = 0.5$

$G_p = 0.3$

$G_A = 0.2$

TABLE III

COMPONENT VALUES FOR EHSA SYSTEM

<u>Pilot Valve</u>	<u>Example 1</u>	<u>Example 2</u>
BL' (lb/amp)	40	
C_p (in/lb)	0.0005	
r_e (ohms)	490	
R_p (lb-sec/in)	0.24	
M_p (lb)	0.05	
K_p (in^3/sec-in)	3200	
<u>Slave Valve</u>		
A_s (in^2)	0.375	
C_s (in^5/lb)	0.625×10^{-6}	
M_s (lb)	0.75	
R_{Ls} (lb-sec/in^5)	$\left.\right\} \delta_s \simeq 1.0$	
R_s (lb-sec/in)		
K_s (in^3/sec-in)	1.5×10^{-4}	
<u>Exciter</u>		
A_e (in^2)	20.5	
C_e (in^5/lb)	5.5×10^{-5}	
R_{Le} (lb-sec/in^5)	∞	
M_e (lb)	150	
R_e (lb-sec/in)	3.44×10^{-8}	

TABLE III (Continued)

External Loads	Example 1	Example 2
M_L (lb)	1.5×10^4	0.5×10^4
R_L (lb-sec/in)	0.549×10^2	0.183×10^2
K_L (lb/in)	0.777×10^4	0.259×10^4
M_D	2.0×10^4	0.667×10^4
R_D (lb-sec/in)	0.977×10^2	0.326×10^2
K_D (lb/in)	0.460×10^4	0.153×10^4
Nominal Gain Settings		
G_f	0.0	0.0
G_e	0.03	1.0
G_s	0.5	0.5
G_p	0.3	0.3
G_A	0.2	0.2

NOTE: In Example 2, the same EHSA system was used as in Example 1, except the external loads which were excited by three servo actuators simultaneously.